

8. Polarization



Polarization (I)

$$\mathbf{E}(z,t) = \text{Re} \left\{ \mathbf{A} \exp \left[j 2 \pi \nu \left(t - \frac{z}{c} \right) \right] \right\}$$

$$\mathbf{A} = A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}}$$

$$A_x = a_x \exp(j\varphi_x), \quad A_y = a_y \exp(j\varphi_y)$$

x 축, y 축 두개의 직교 편광 성분으로 모든 파동방정식의 해를 기술할 수 있다.

x 축, y 축뿐만 아니라 두가지 원형편광(좌편광, 우편광)등의 어떠한 직교 편광쌍으로도 가능.



Polarization (II)

Different notation

$$\mathbf{E}(z,t) = \text{Re} \left\{ \mathbf{A} \exp \left[-j2\pi\nu \left(t - \frac{z}{c} \right) \right] \right\}$$

$$A_x = a_x \exp(-j\varphi_x), \quad A_y = a_y \exp(-j\varphi_y)$$



Polarization (III)

$$\mathbf{E}(z, t) = E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}}$$

$$E_x = a_x \cos \left[2\pi\nu \left(t - \frac{z}{c} \right) + \varphi_x \right]$$

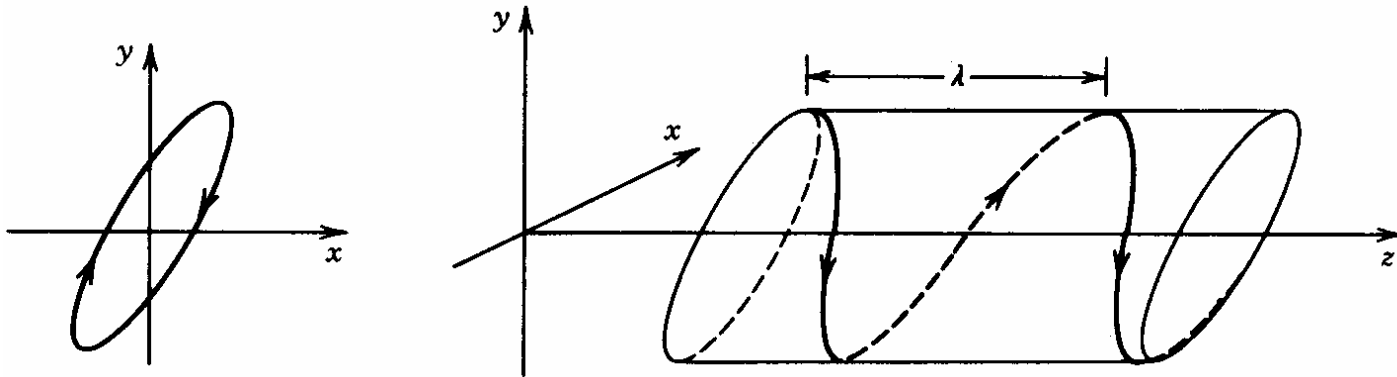
$$E_y = a_y \cos \left[2\pi\nu \left(t - \frac{z}{c} \right) + \varphi_y \right]$$



Polarization Ellipsoid

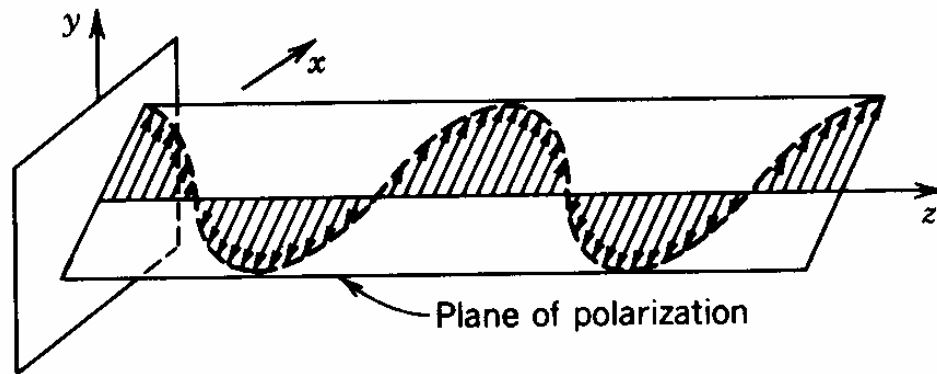
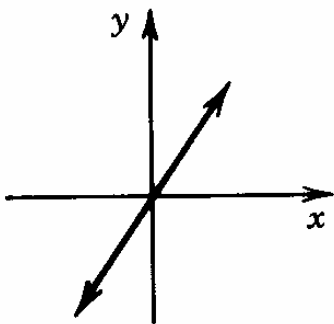
$$\frac{E_x^2}{a_x^2} + \frac{E_y^2}{a_y^2} - 2\cos\varphi \frac{E_x E_y}{a_x a_y} = \sin^2 \varphi$$

$$\varphi = \varphi_y - \varphi_x$$



Linear Polarization

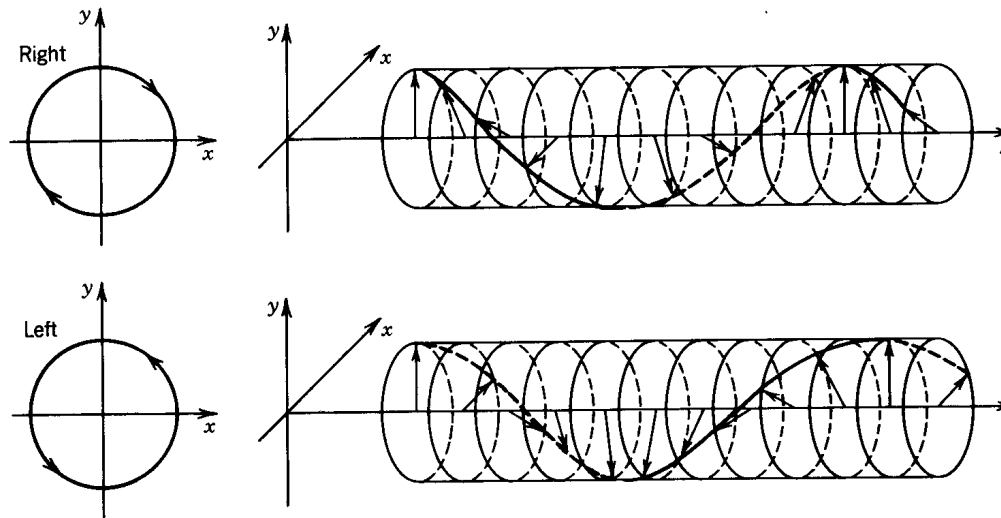
$$\varphi = 0 \text{ or } \pi$$



Circular Polarization

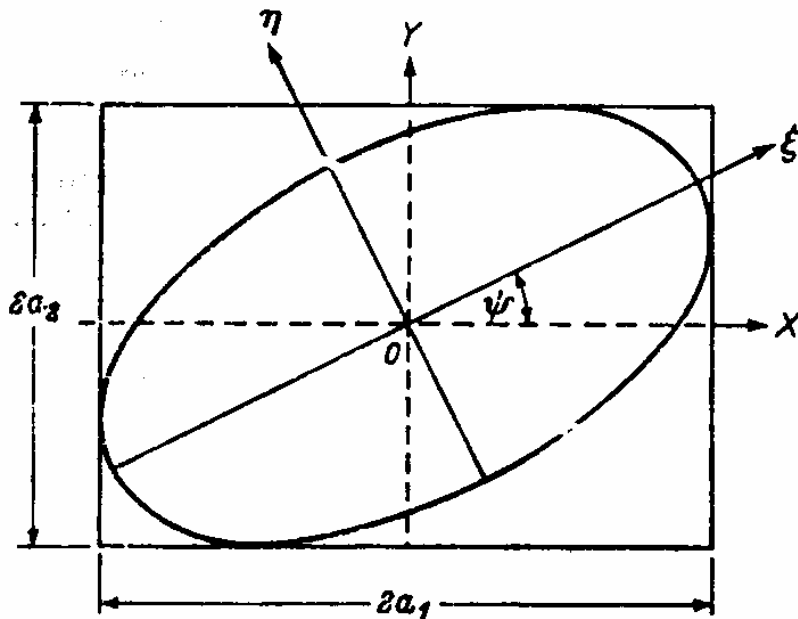
Right Circularly Polarized: $\varphi = \frac{\pi}{2}$

Left Circularly Polarized: $\varphi = -\frac{\pi}{2}$



Elliptical Polarization

다른 표기 $a_x \rightarrow a_1, a_y \rightarrow a_2, \quad \varphi = \varphi_y - \varphi_x \rightarrow \delta = \delta_2 - \delta_1$

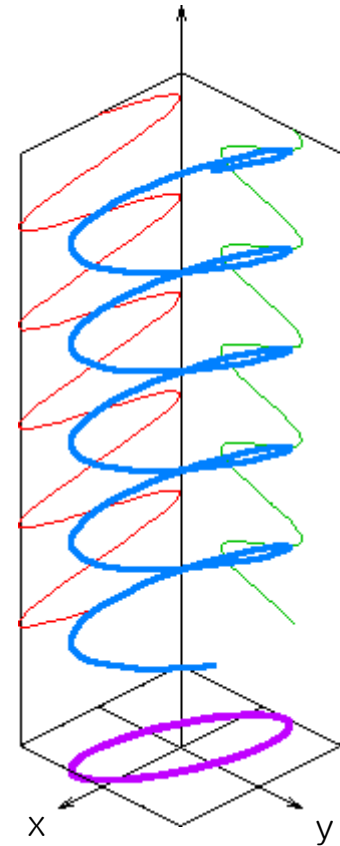
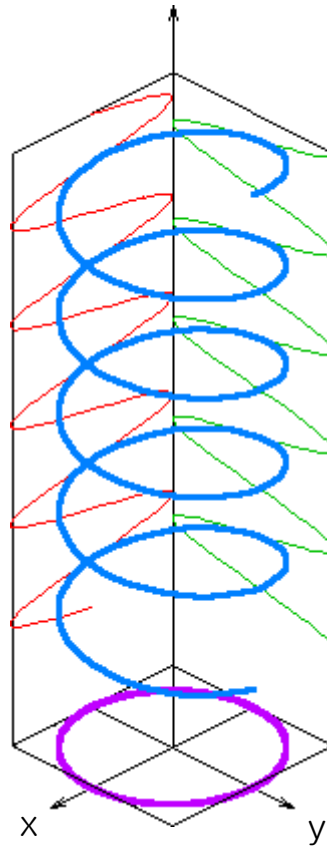
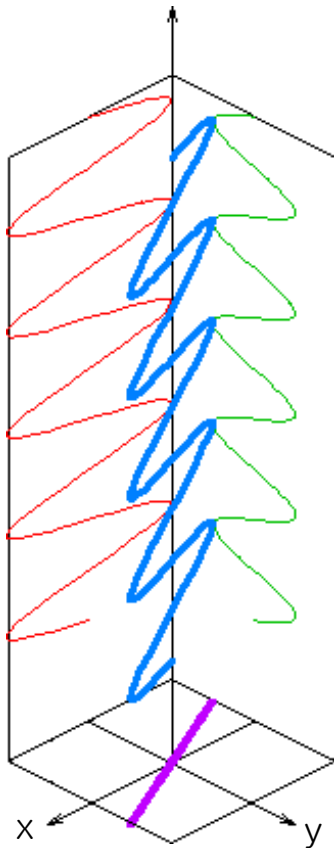


$$\tan \alpha = \frac{a_2}{a_1}, \quad \tan \chi = \mp \frac{b}{a}$$

($2a$: 장축의 길이, $2b$: 단축의 길이)



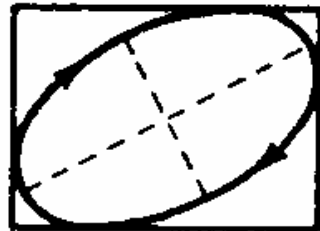
Comparison of Three Polarization Types



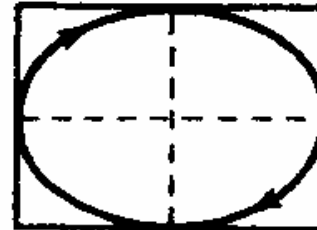
Types of Polarization



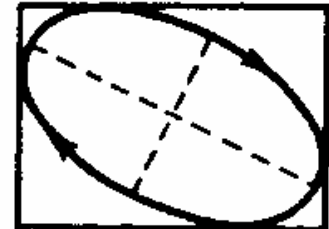
$$\delta = 0$$



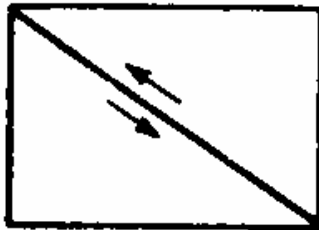
$$0 < \delta < \frac{\pi}{2}$$



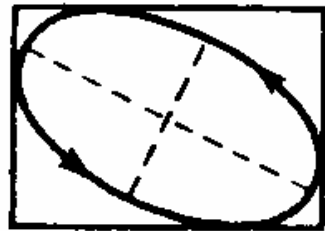
$$\delta = \frac{\pi}{2}$$



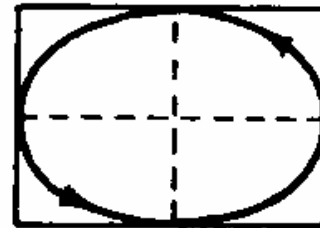
$$\frac{\pi}{2} < \delta < \pi$$



$$\delta = \pi$$



$$\pi < \delta < \frac{3\pi}{2}$$

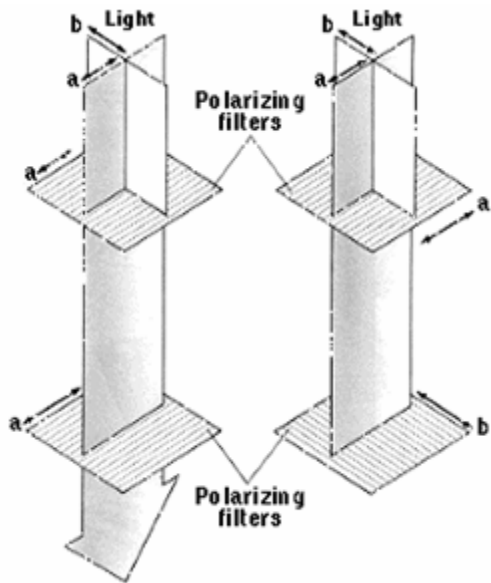


$$\delta = \frac{3\pi}{2}$$

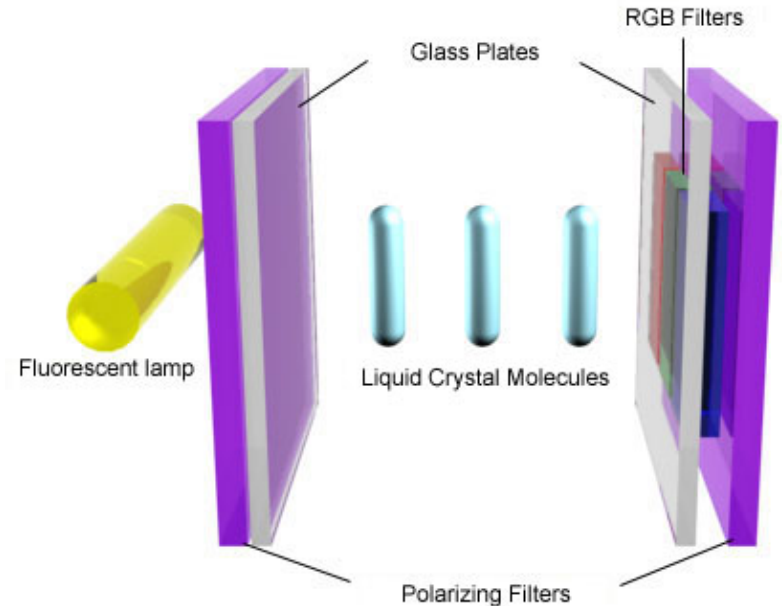


$$\frac{3\pi}{2} < \delta < 2\pi$$

Liquid Crystal Display



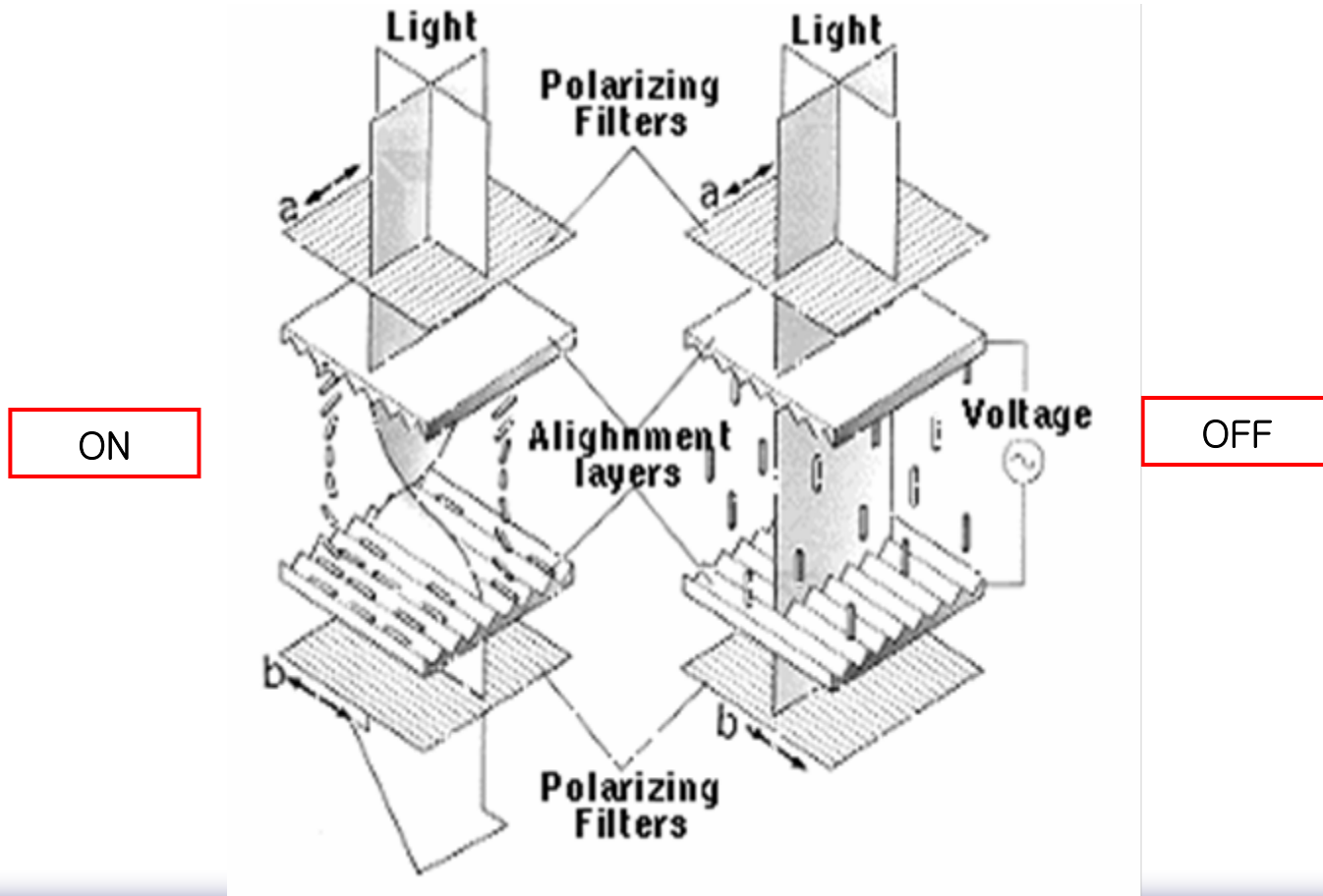
polarizer



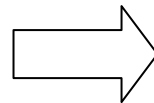
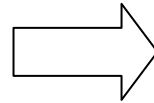
- 빛이 액정을 통과하면서 복굴절 현상으로 인해 빛의 진동축이 회전됨
- LCD는 이러한 빛의 편광 변화를 통해 빛의 세기를 control하는 장치



LCD Cell



Polarization Filter



Jones Vector



R. Clark Jones (1916-2004)

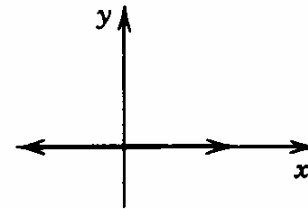
$$A_x = a_x \exp(j\varphi_x), \quad A_y = a_y \exp(j\varphi_y)$$

$$\mathbf{J} = \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

Jones Vectors

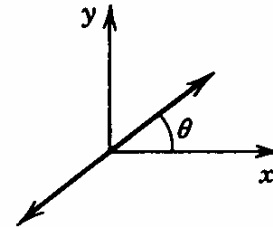
Linearly polarized wave,
in x direction

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



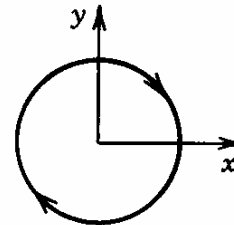
Linearly polarized wave,
plane of polarization making
angle θ with x axis

$$\begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$



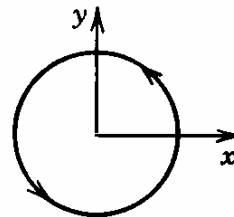
Right circularly polarized

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$$



Left circularly polarized

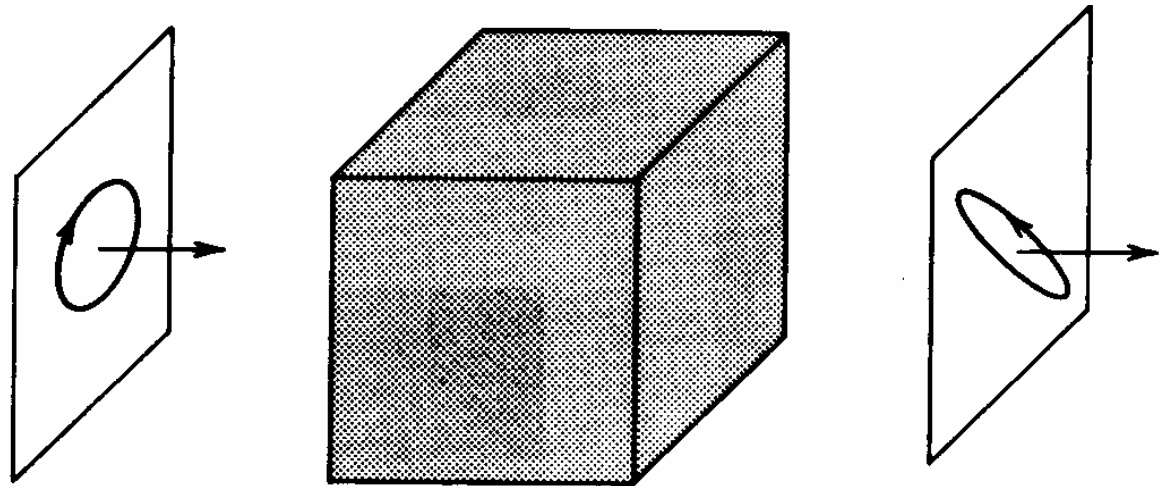
$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$



Jones Calculus

$$\begin{pmatrix} A_{2x} \\ A_{2y} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} A_{1x} \\ A_{1y} \end{pmatrix}$$

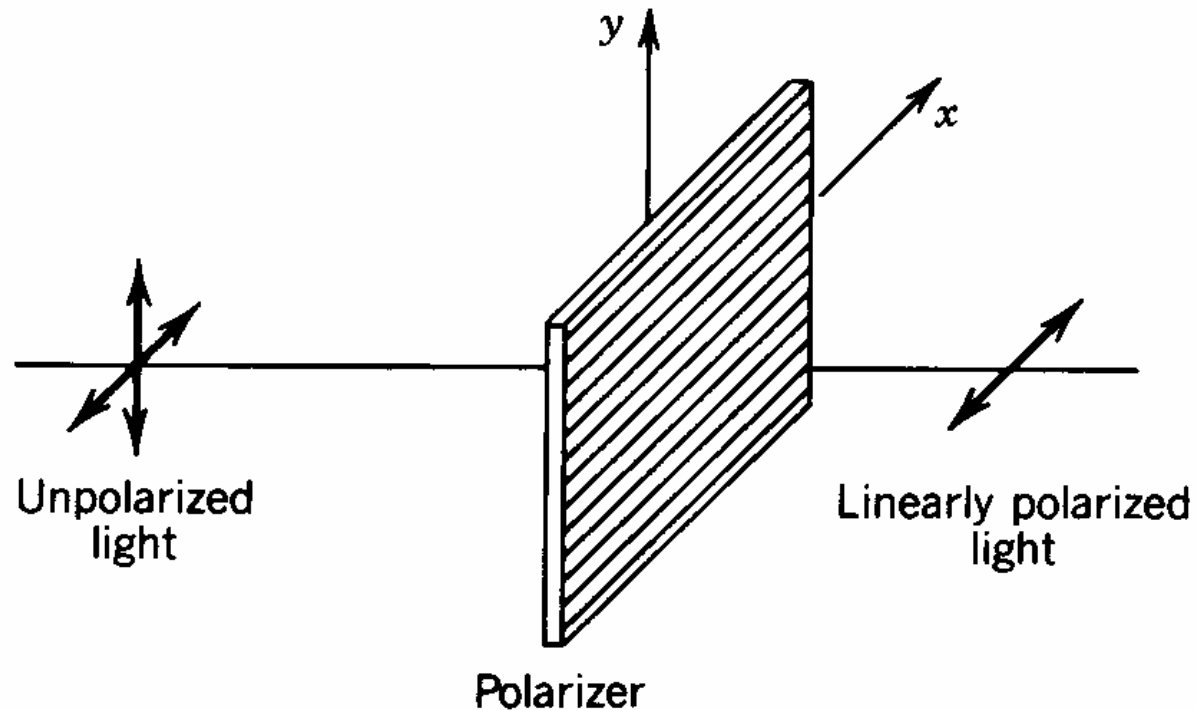
$$\mathbf{J}_2 = \mathbf{TJ}_1$$



Optical system

Jones Matrix for Linear Polarizer

$$T = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

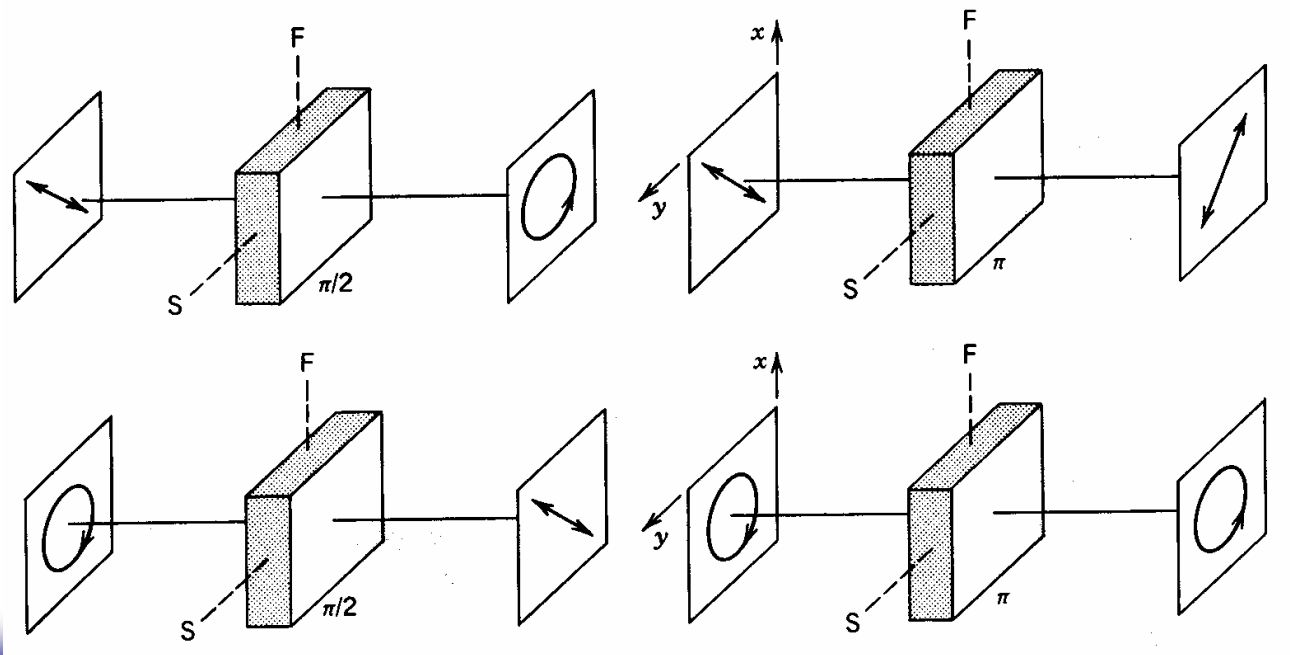


Jones Matrix for Wave Retarders

$$\mathbf{T} = \begin{pmatrix} 1 & 0 \\ 0 & \exp(-j\Gamma) \end{pmatrix}$$

$\Gamma = \frac{\pi}{2}$: Quarter - wave retarder

$\Gamma = \pi$: Half - wave retarder



Jones Matrix for Polarization Rotators

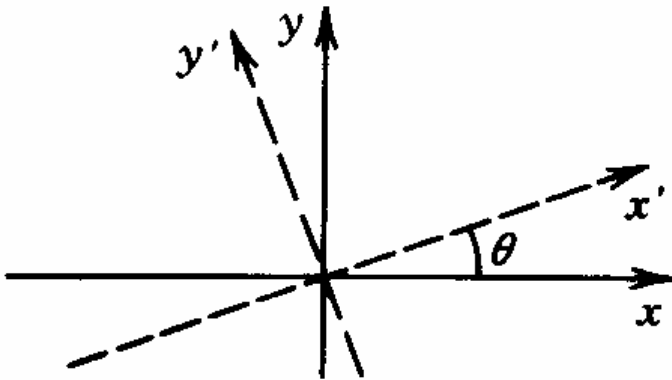
$$\mathbf{T} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$\begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix}$$

$$\theta_2 = \theta_1 + \theta$$



Coordinate Transformation



$$\mathbf{J}' = \mathbf{R}(\theta)\mathbf{J}$$

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$\mathbf{T}' = \mathbf{R}(\theta)\mathbf{T}\mathbf{R}(-\theta)$$

$$\mathbf{T} = \mathbf{R}(-\theta)\mathbf{T}'\mathbf{R}(\theta)$$



Stokes Parameters

$$s_0 = a_1^2 + a_2^2$$

$$s_1 = a_1^2 - a_2^2$$

$$s_2 = 2a_1a_2\cos\delta$$

$$s_3 = 2a_1a_2\sin\delta$$

$$s_0^2 = s_1^2 + s_2^2 + s_3^2$$



Stokes Parameters (Polarized + Unpolarized) (I)

$$\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

S_0 = Total power(polarized + unpolarized)

S_1 = Power through LH polarizer – power through LV polarizer

S_2 = Power through L+45 polarizer – power through L-45 polarizer

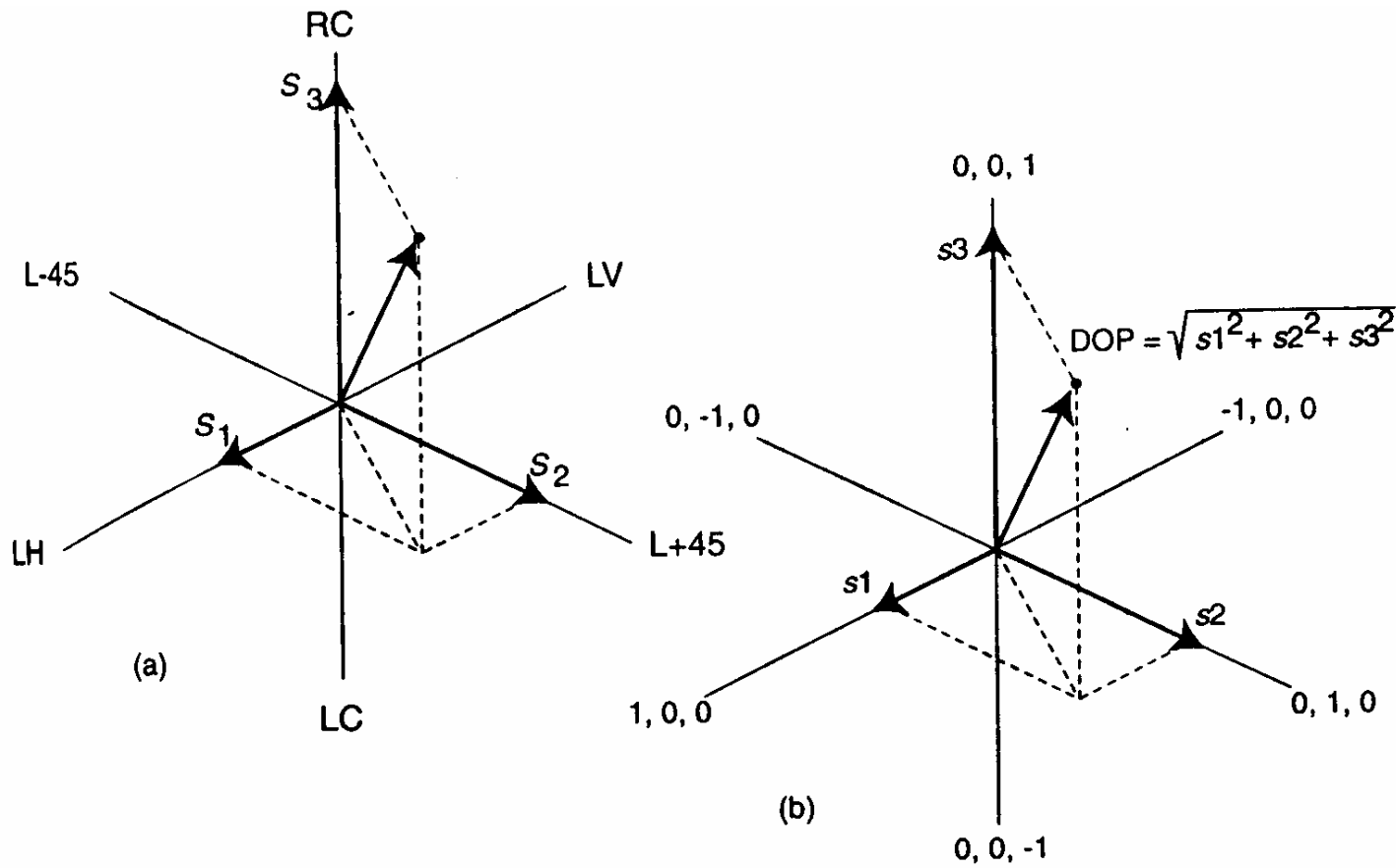
S_3 = Power through RC polarizer – power through LC polarizer

$$P_{\text{polarized}} = \sqrt{S_1^2 + S_2^2 + S_3^2}$$

$$s_1 = \frac{S_1}{S_0} \quad s_2 = \frac{S_2}{S_0} \quad s_3 = \frac{S_3}{S_0}$$



Stokes Parameters (Polarized + Unpolarized) (II)



Degree of Polarization

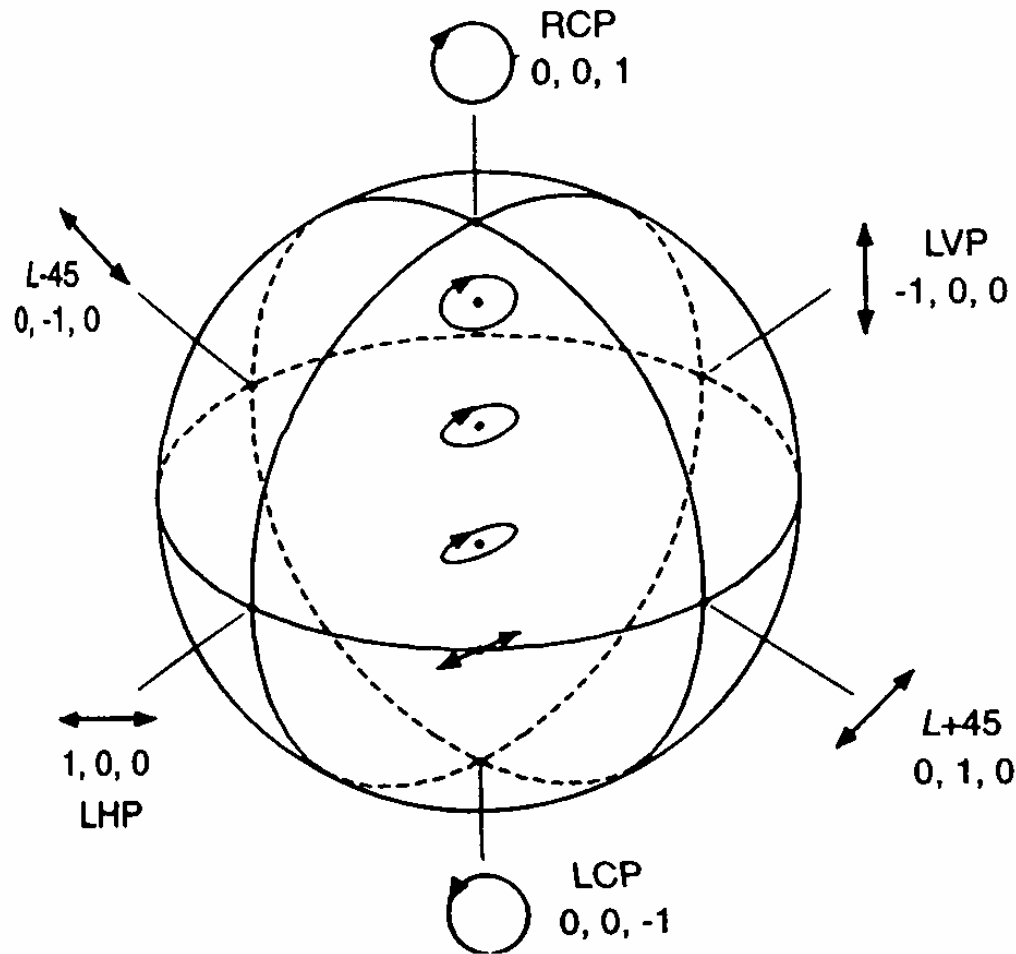
$$\text{DOP} = \frac{P_{\text{polarized}}}{P_{\text{polarized}} + P_{\text{unpolarized}}}$$

$$\text{DOP} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$

$$\text{DOP} = \sqrt{s_1^2 + s_2^2 + s_3^2}$$



Poincaré Sphere (I)



Poincaré Sphere (II)

$$s_1 = s_0 \cos 2\chi \cos 2\psi$$

$$s_2 = s_0 \cos 2\chi \sin 2\psi$$

$$s_3 = s_0 \sin 2\chi$$

