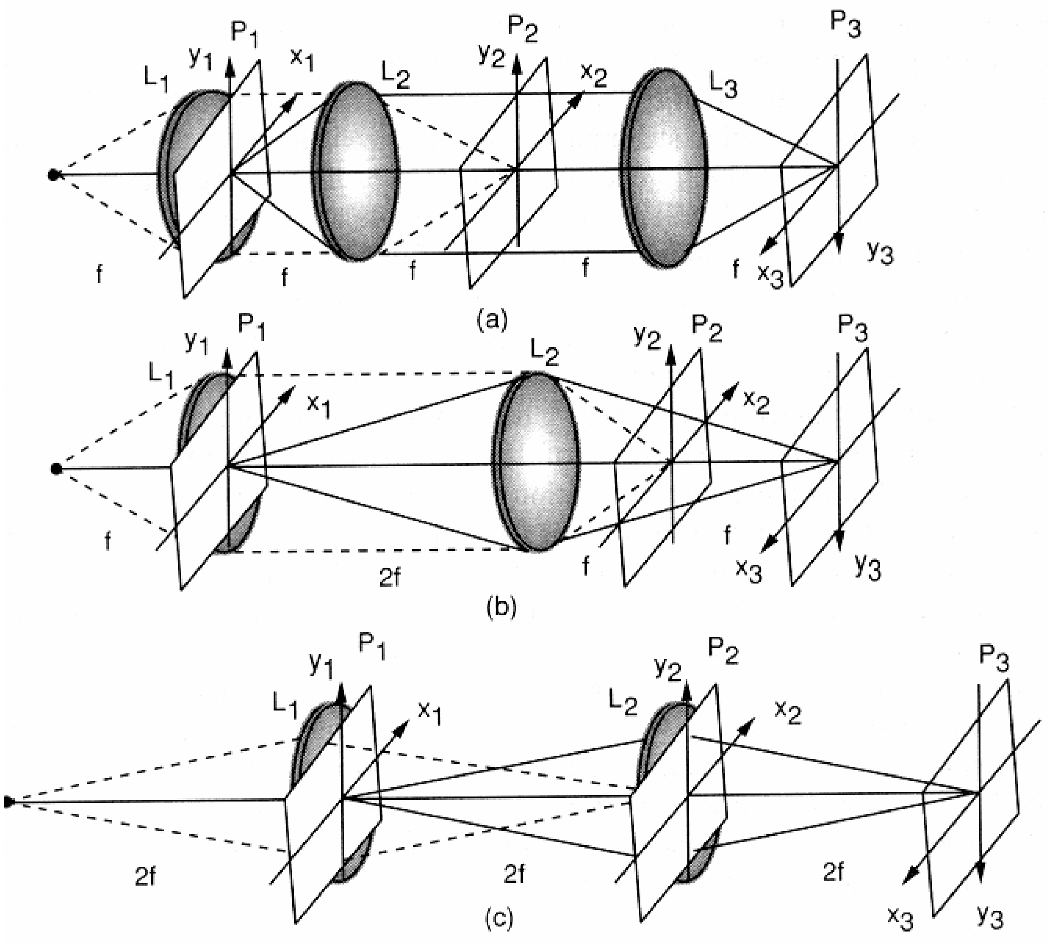
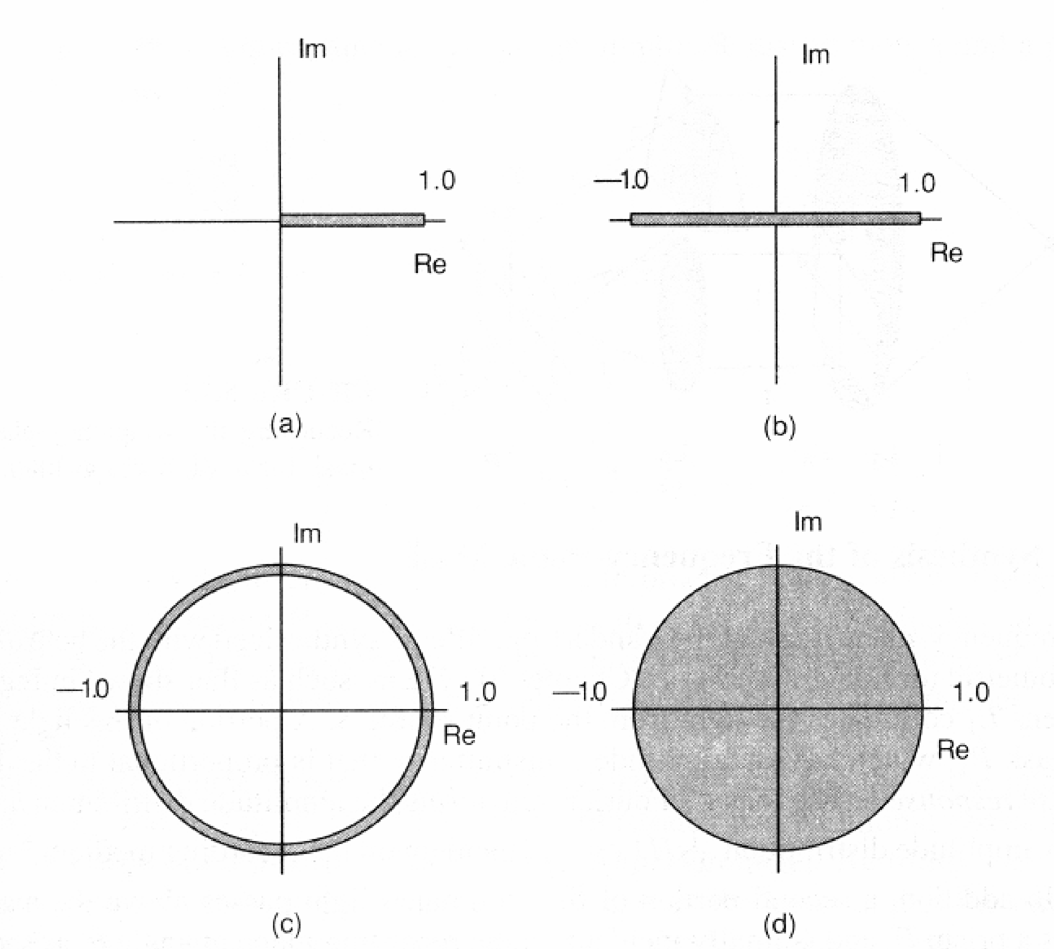
Coherent Optical Processing



Filter Realization



VanderLugt Filter - Recording (I)

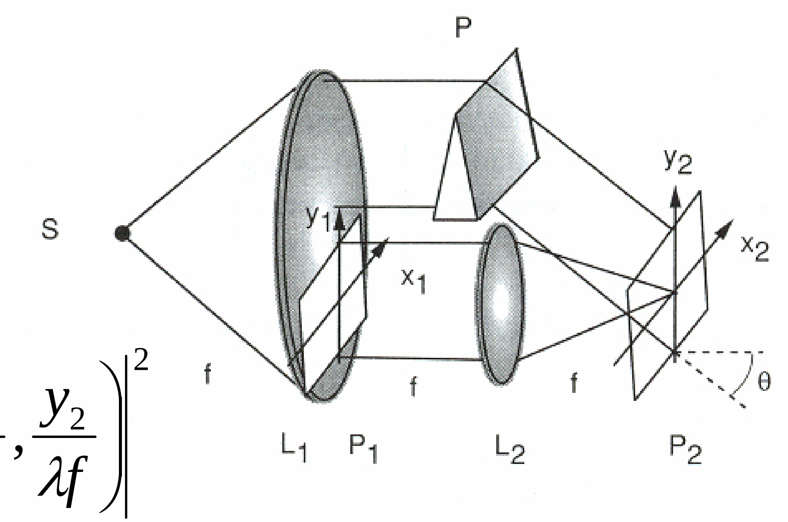
$$U_r(x_2, y_2) = r_o \exp(-j2\pi\alpha y_2)$$

$$\alpha = \frac{\sin\theta}{\lambda}$$

$$I(x_2, y_2) = \left| r_o \exp(-j2\pi\alpha y_2) + \frac{1}{\lambda f} H\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) \right|^2$$

$$= r_o^{\,2} + \frac{1}{\lambda^2 f^2} \left| H\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) \right|^2 + \frac{r_o}{\lambda f} H\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) \exp(j2\pi\alpha y_2)$$

$$+ \frac{r_o}{\lambda f} H^*\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) \exp(-j2\pi\alpha y_2)$$



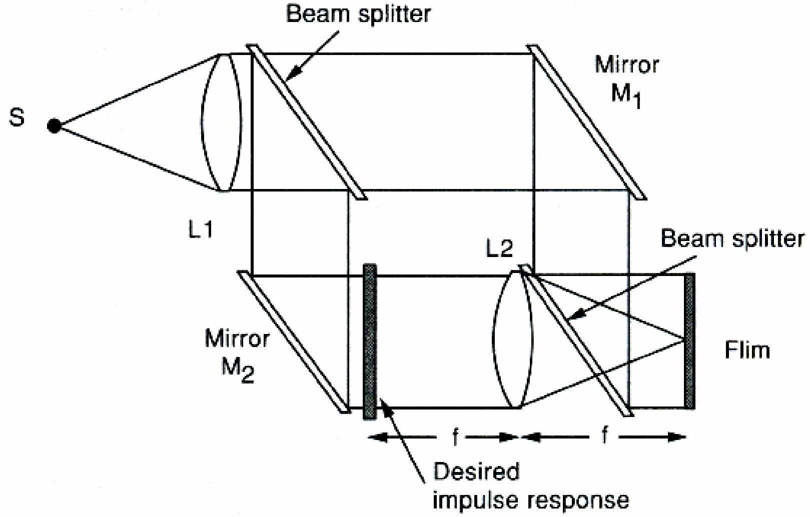
VanderLugt Filter - Recording (II)

$$H\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)=A\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)\exp\left[j\psi\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)\right]$$

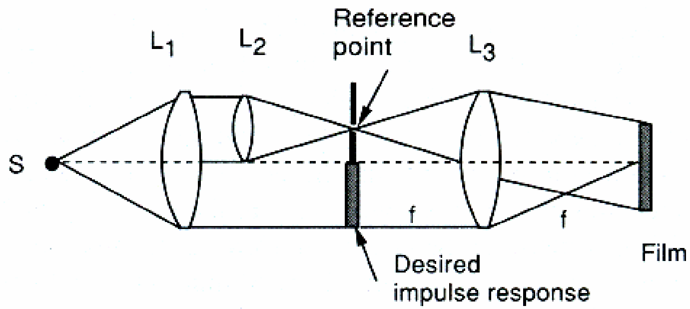
$$\begin{aligned}I(x_2,y_2)&=r_0^2+\frac{1}{\lambda^2 f^2}A^2\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)\\&+\frac{2r_0}{\lambda f}A\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)\cos\left[2\pi\alpha y_2+\psi\left(\frac{x_2}{\lambda f},\frac{y_2}{\lambda f}\right)\right]\end{aligned}$$

$$\begin{aligned}t_A(x_2,y_2)\propto\ & r_0^2+\frac{1}{\lambda^2 f^2}|H|^2+\frac{r_0}{\lambda f}H\ \exp(\ j2\pi\alpha\ y_2)\\&+\frac{r_0}{\lambda f}H^*\exp\left(-\ j2\pi\alpha\ y_2\right)\end{aligned}$$

VanderLugt Filter - Recording (III)



(a)



(b)

VanderLugt Filter - Processing (I)

$$U_2 \propto \frac{r_0^2 G}{\lambda f} + \frac{1}{\lambda^3 f^3} |H|^2 G + \frac{r_0^2}{\lambda^2 f^2} HG \ \exp\left(j 2\pi\alpha y_2\right)$$

$$+ \frac{r_0}{\lambda^2 f^2} H^* G \ \exp(-j 2\pi\alpha y_2)$$

$$U_3(x_3, y_3) \propto r_0^2 g(x_3, y_3) + \frac{1}{\lambda^2 f^2}\left[h(x_3, y_3) \otimes h^*(-x_3, -y_3) \otimes g(x_3, y_3)\right]$$

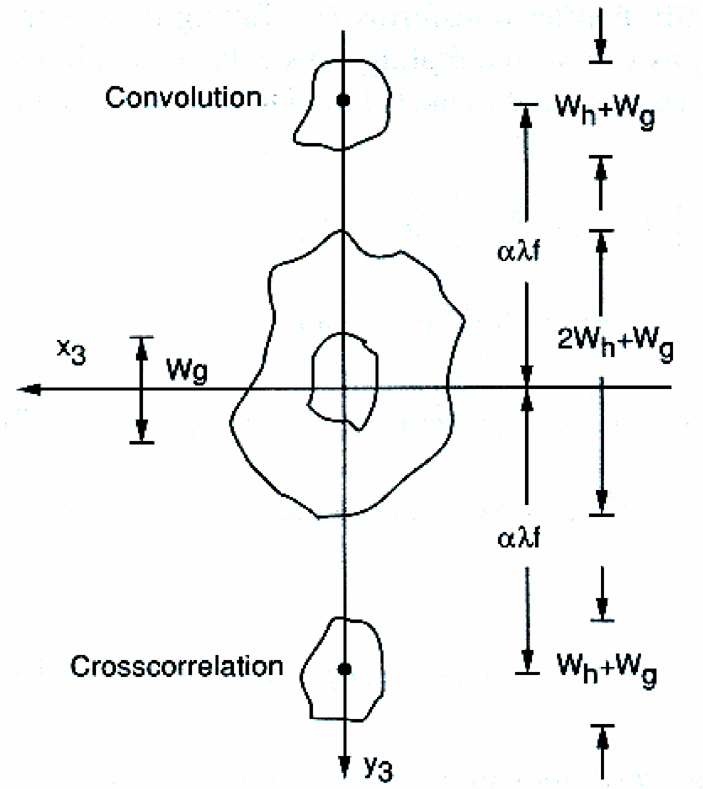
$$+ \frac{r_0}{\lambda f}\left[h(x_3, y_3) \otimes g(x_3, y_3) \otimes \delta(x_3, y_3 + \alpha\lambda f)\right]$$

$$+ \frac{r_0}{\lambda f}\left[h^*(-x_3, -y_3) \otimes g(x_3, y_3) \otimes \delta(x_3, y_3 - \alpha\lambda f)\right]$$

Third term : Convolution of g and h centered at $(0, -\alpha\lambda f)$

Fourth term : Crosscorrelation of g and h centered at $(0, \alpha\lambda f)$

VanderLugt Filter - Processing (II)



$$\alpha > \frac{1}{\lambda f}\left(\frac{3W_h}{2} + W_g\right)$$

$$\theta > \frac{3}{2}\frac{W_h}{f} + \frac{W_g}{f}$$

with $\sin\theta \approx \theta$

VanderLugt Filter – Processing (III)

Crosscorrelation term

$$h^*(-x_3,-y_3)\otimes g(x_3,y_3)\otimes \delta(x_3,y_3-\alpha\lambda f)$$

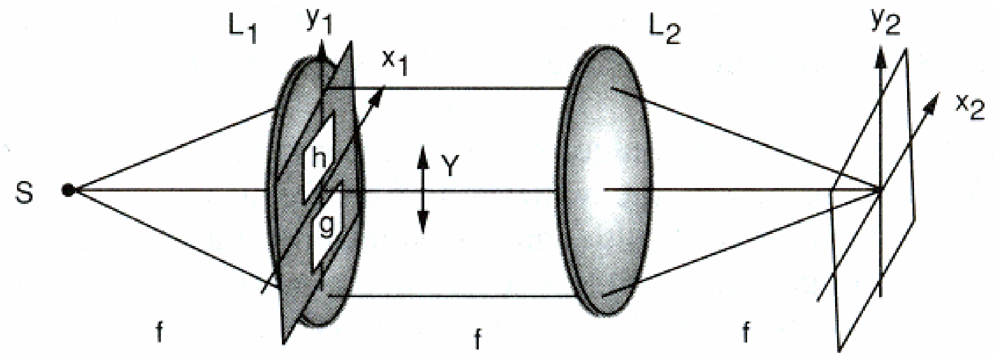
$$=\iint_{-\infty}^{\infty} g(\xi,\eta)h^*(\xi-x_3,\eta-y_3+\alpha\lambda f)d\xi d\eta$$

If the input object function is translated to a new location, the crosscorrelation result is also translated.

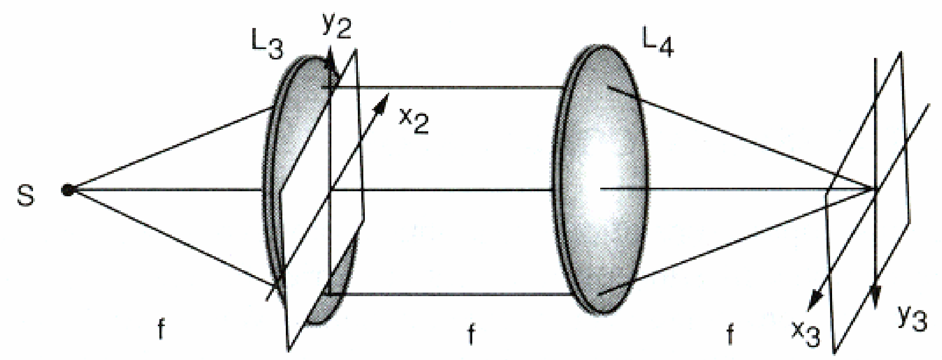
VanderLugt Filter - Advantages

- Impulse response is optically Fourier transformed.
- Generally complicated complex-valued transfer function is synthesized with a single absorbing mask.

Joint Transform Correlator (JTC) (I)



(a)



(b)

JTC (II)

$$U_1(x_1, y_1) = h(x_1, y_1 - Y/2) + g(x_1, y_1 + Y/2)$$

$$U_2(x_2, y_2) = \frac{1}{\lambda f} H\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) e^{-j\pi y_2 Y/\lambda f} + \frac{1}{\lambda f} G\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) e^{+j\pi y_2 Y/\lambda f}$$

$$\begin{aligned} I(x_2, y_2) = \frac{1}{\lambda^2 f^2} \Bigg[& \left| H\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) \right|^2 + \left| G\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) \right|^2 \\ & + H\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) G^*\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) e^{-j\pi y_2 Y/\lambda f} \\ & + H^*\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) G\left(\frac{x_2}{\lambda f}, \frac{y_2}{\lambda f}\right) e^{+j\pi y_2 Y/\lambda f} \Bigg] \end{aligned}$$

JTC (III)

$$U_3(x_3, y_3) = \frac{1}{\lambda f}\Big[h(x_3, y_3) \otimes h^*(-x_3, -y_3) + g(x_3, y_3) \otimes g^*(-x_3, -y_3)$$

$$+ h(x_3, y_3) \otimes g^*(-x_3, -y_3) \otimes \delta(x_3, y_3 - Y)$$

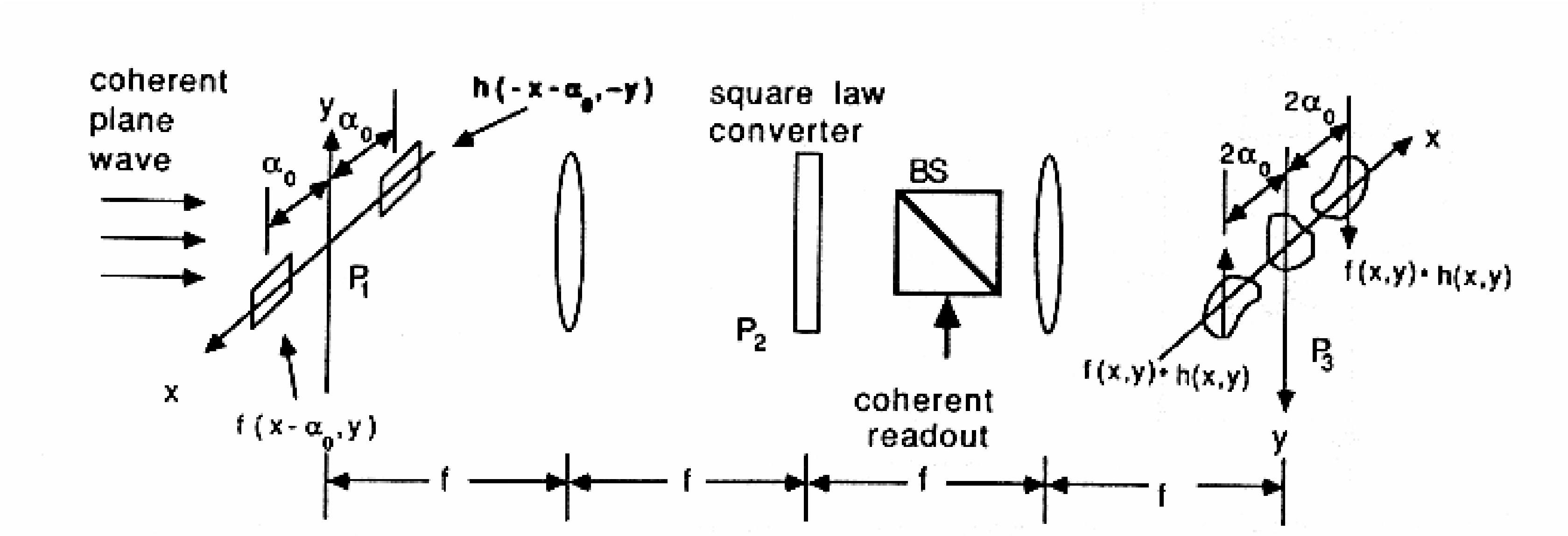
$$+ h^*(-x_3, -y_3) \otimes g(x_3, y_3) \otimes \delta(x_3, y_3 + Y)\Big]$$

Third term : Crosscorrelation of *g* and *h* centered at (0, *Y*)

Fourth term : Crosscorrelation of *g* and *h* centered at (0, -*Y*)

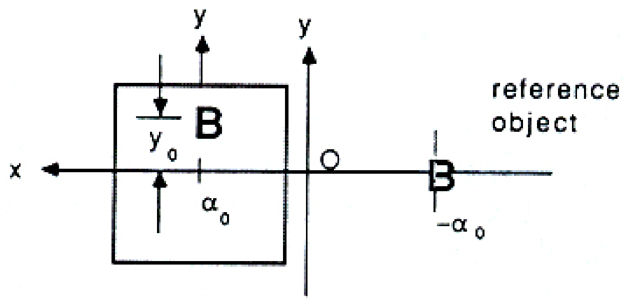
To obtain convolution : One of *g* and *h* should be introduced in the processor with a mirror reflection about its own origin.

JTC (IV)

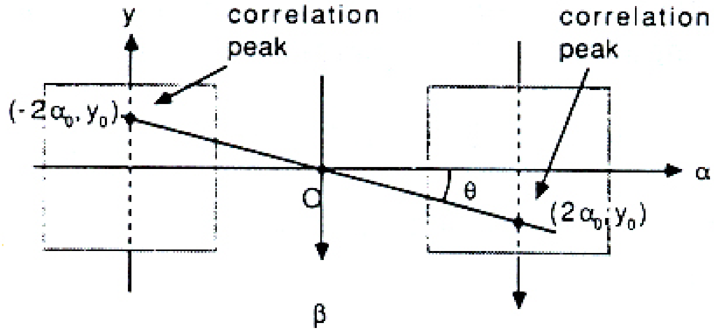


coherent
plane
wave
y
α_0
α_0
h(-x-α_0,-y)
P_1
x
f(x-α_0,y)
square law
converter
P_2
BS
coherent
readout
$2\alpha_0$
$2\alpha_0$
x
f(x,y)•h(x,y)
f(x,y)•h(x,y)
P_3
y
f
f
f
f

JTC (V) – Space Invariant Property



(a)



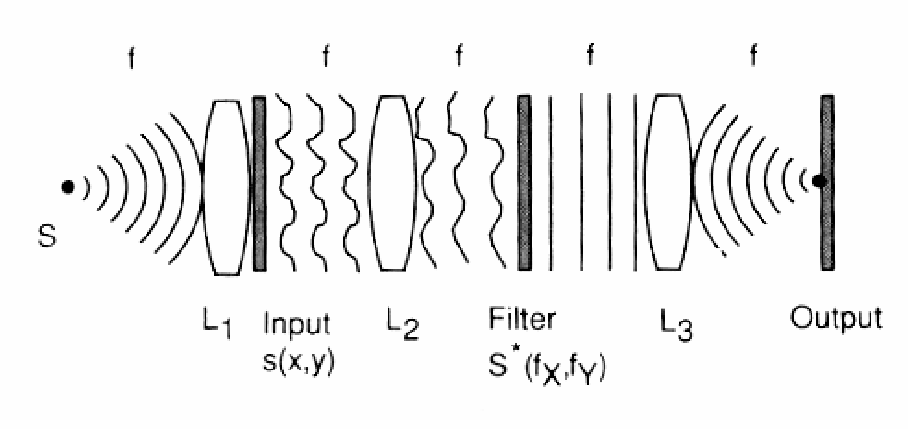
(b)

Matched Filter (I)

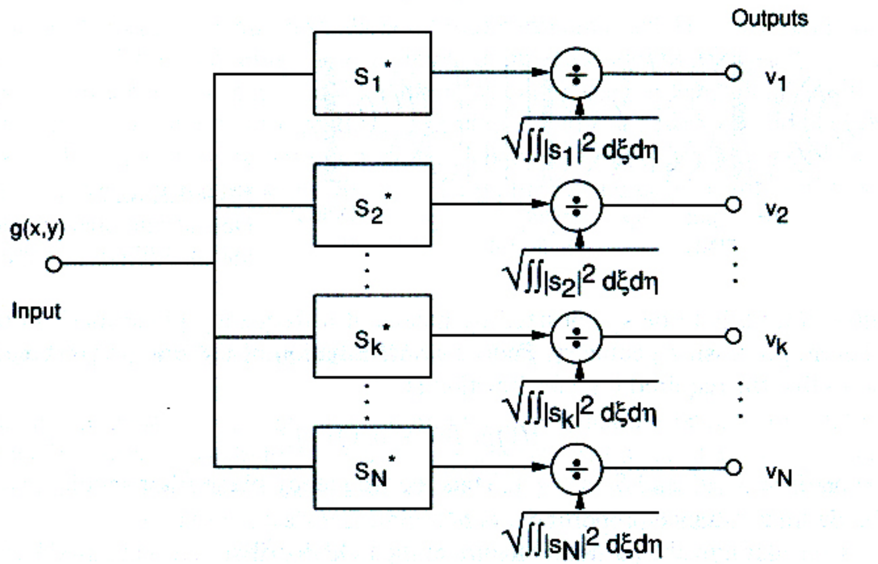
$$h(x,y) = s^*(-x,-y)$$

$$v(x,y) = \int_{-\infty}^{\infty}\int h(x-\xi, y-\eta)\, g(\xi,\eta)\, d\xi d\eta$$

$$= \int_{-\infty}^{\infty}\int g(\xi,\eta)\, s^*(\xi - x, \eta - y)\, d\xi d\eta$$



Matched Filter (II)



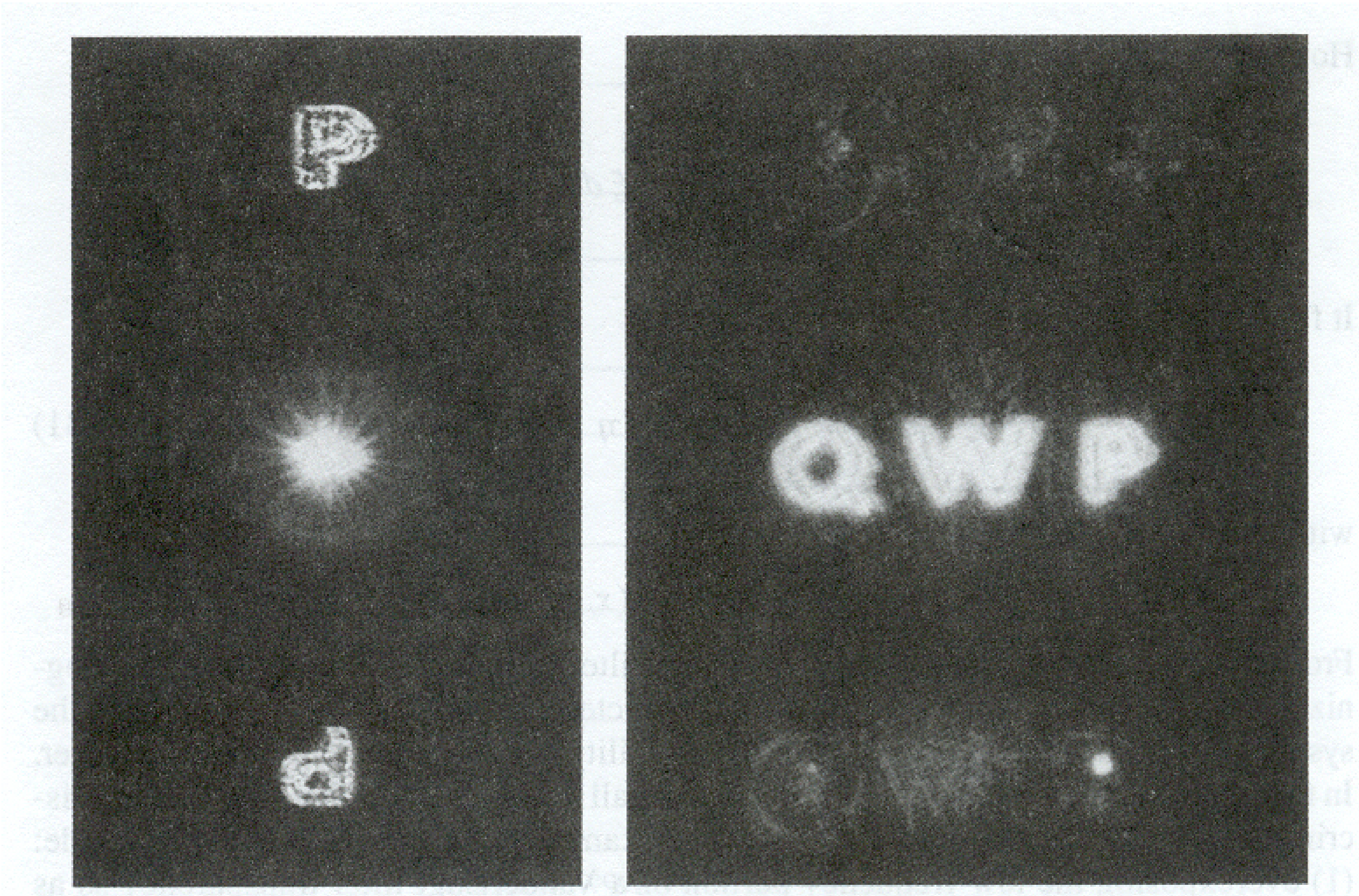
For $g(x, y) = s_k(x, y)$,

$$|v_n|^2 \le \iint_{-\infty}^{\infty} |s_k|^2 \, d\xi d\eta = |v_k|^2$$

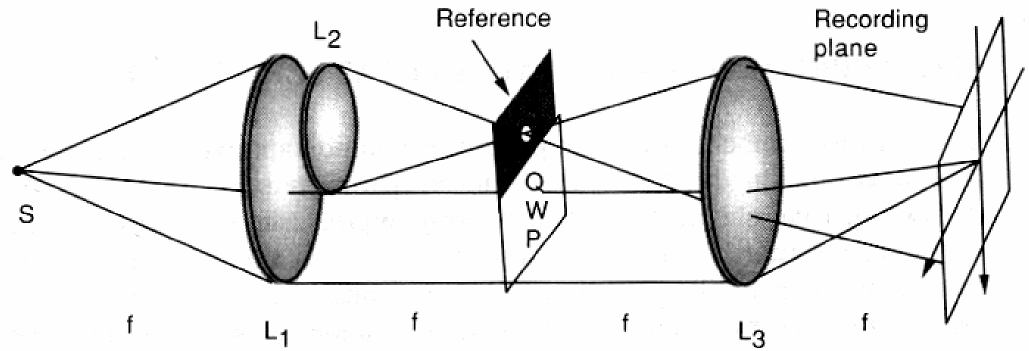
with equality if and only if

$$s_n(x, y) = \kappa s_k(x, y)$$

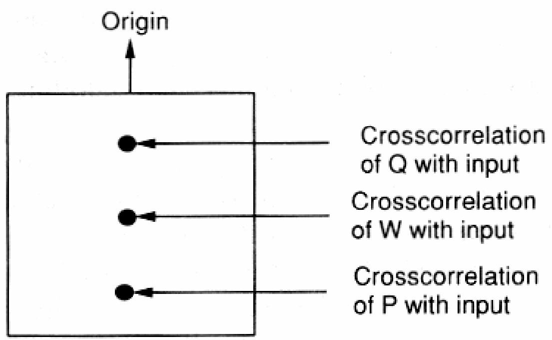
Character Recognition (I)



Character Recognition (II)

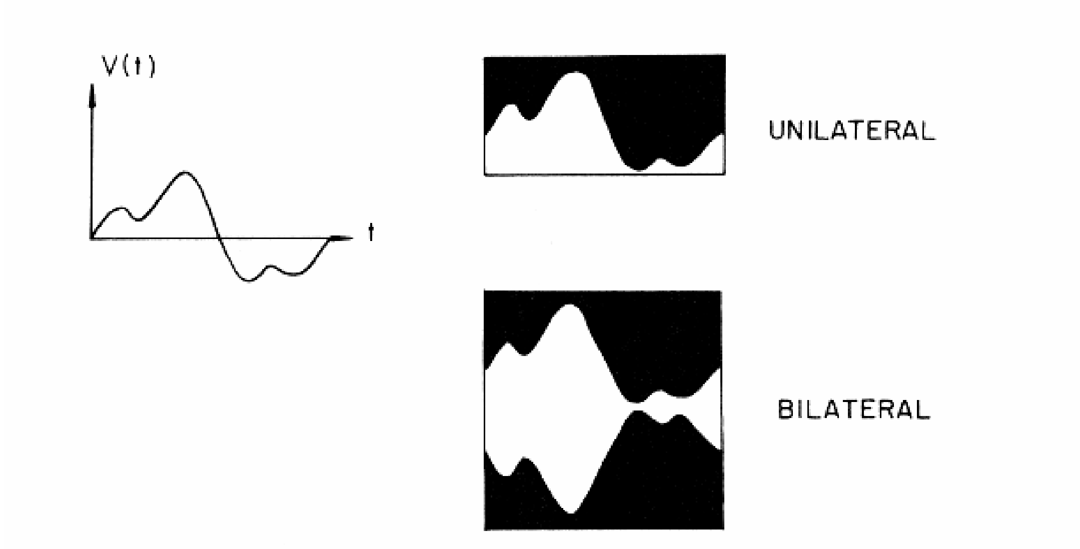


(a)



(b)

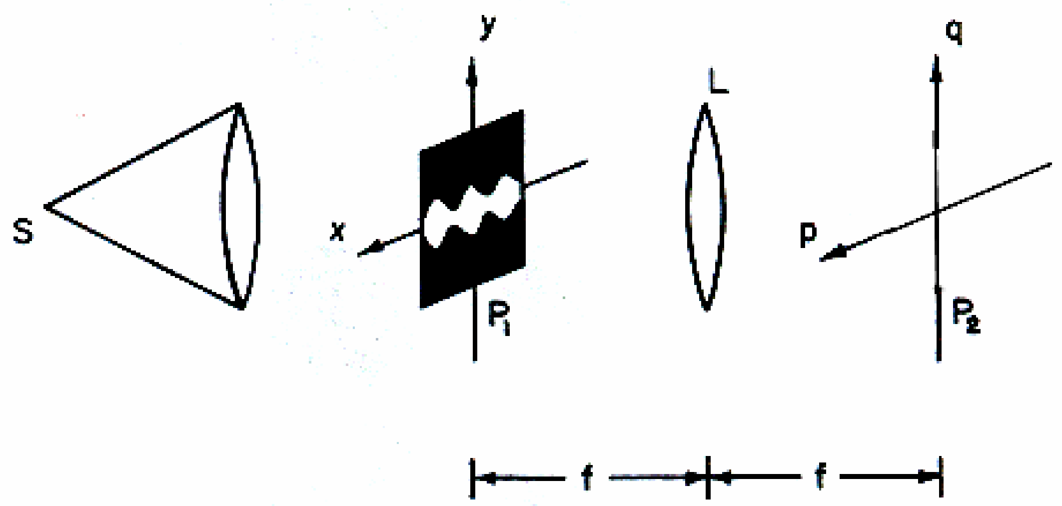
Area Modulation



Bilateral : $$T_1(x,y) = \text{rect}\left(\frac{x}{L}\right)\text{rect}\left\{\frac{y}{2\left[B + f(x)\right]}\right\}$$

Unilateral : $$T_2(x,y) = \text{rect}\left(\frac{x}{L}\right)\text{rect}\left\{\frac{y - \left[f(x) + B\right]/2}{B + f(x)}\right\}$$

Spectrum Analysis with Area Modulation (I)



$$G(p,q) = C\iint T(x,y)\exp[-i(px+qy)]dxdy$$

Spectrum Analysis with Area Modulation (II)

For a bilateral area-modulated signal

$$G_1(p,0) = 2C\int \mathrm{rect}\left(\frac{x}{L}\right)\left[B + f(x)\right]\exp(-ipx)dx$$
$$= 2C\left[\int_{-L/2}^{L/2} B\exp(-ipx)dx + \int_{-L/2}^{L/2} f(x)\exp(-ipx)dx\right]$$

For a unilateral area-modulated signal

$$G_2(p,0) = \frac{1}{2}G_1(p,0)$$