Electromagnetics

<Chap. 11> Antennas and Radiating systems

Section 11.1 ~ 11.3

(1st of **week 13**)

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Textbook: Field and Wave Electromagnetics, 2E, Addison-Wesley

Chap. 11 | Contents

Chap. 11 | Introduction to Chapter

- **Antennas**
  - Structures designed for radiating EM energy in a prescribed manner (i.e., Directivity. Otherwise, huge loss!)
  - Used for wireless transmission of info over long distances
  - Examples
    - Single straight wire
    - Conducting loop
    - Aperture at the end of waveguide
    - Complex array of these elements

- **What will we learn?**
  - How $\boldsymbol{E}$ and $\boldsymbol{H}$ generated by antenna of particular geometry
  - How $\boldsymbol{E}$ and $\boldsymbol{H}$ “directed” in a source-free region over long distances

- **How do we do?**
  - Obtain $\boldsymbol{E}$ and $\boldsymbol{H}$ by solving for $\boldsymbol{A}$ and V of antenna with sources ($\boldsymbol{J}$ and ρ)

Img src: NETGEAR Nighthawk, HomeDepot, Wikipedia

Chap. 11 | Review of time-varying EM fields

- Time-varying EM waves originate from time-varying sources

  - ***E*** and ***H*** in terms of electric and magnetic potentials

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$ *(∵Divergence-less)*

$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

spatial distribution of charges $-\nabla V$

Time-varying magnetic field $-\frac{\partial \boldsymbol{A}}{\partial t}$ ***(∵Faraday's law)***

  - By plugging above into Maxwell's equations, we get ***non-homogeneous equations*** in ***A*** and *V*:

$$\begin{cases} \nabla^2 \boldsymbol{A} - \mu\varepsilon \dfrac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu \boldsymbol{J} \\ \nabla^2 V - \mu\varepsilon \dfrac{\partial^2 V}{\partial t^2} = -\dfrac{\rho}{\varepsilon} \end{cases}$$

, if ***A*** and V satisfies

$$\boxed{\nabla \cdot \boldsymbol{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0}$$

Lorentz Condition

$$\Rightarrow \boxed{\begin{aligned} \boldsymbol{A}(R,t) &= \frac{\mu}{4\pi}\int_{V'} \frac{\boldsymbol{J}(t - R/u)}{R} dv' \\ V(R,t) &= \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho(t - R/u)}{R} dv' \end{aligned}}$$

Solutions: Retarded potentials

  - In phasor notation, solutions are given as

$$\boxed{\begin{aligned} \boldsymbol{A} &= \frac{\mu}{4\pi}\int_{V'} \frac{\boldsymbol{J}e^{-jkR}}{R} dv' \\ V &= \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho e^{-jkR}}{R} dv' \end{aligned}}$$

Q: Do we have to evaluate these two integrals?

Chap. 11 | Procedures to obtain *E* and *H* for antenna

- **Duality between *E* and *H***

  - Potentials (***A*** and *V*) related by ***Lorentz condition*** as

  $$\nabla \cdot \boldsymbol{A} + j\omega\mu\varepsilon V = 0$$

  - Sources (***J*** and *ρ*) related by ***Equation of Continuity*** as

  $$\nabla \cdot \boldsymbol{J} = -j\omega\rho$$

  ∴we need to evaluate ***only one integral for A to obtain H***, then use Maxwell's equations to obtain ***E***

  $$\boldsymbol{A} = \frac{\mu}{4\pi}\int_{V'} \frac{\boldsymbol{J}e^{-jkR}}{R}dv' \Rightarrow \boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A} \Rightarrow \nabla \times \boldsymbol{H} = \cancel{\boldsymbol{J}} + j\omega\varepsilon\boldsymbol{E}$$

  (∵E and H in "source-free" region!
  → How waves propagate in free space)

- **Three steps for determining EM fields from a time-varying current source**

  (1) Determine ***A*** from ***J***: $\boldsymbol{A} = \frac{\mu}{4\pi}\int_{V'} \frac{\boldsymbol{J}e^{-jkR}}{R}dv'$

  (2) Find ***H*** from ***A***: $\boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A}$

  (3) Find ***E*** from ***H***: $\boldsymbol{E} = \frac{1}{j\omega\varepsilon}\nabla \times \boldsymbol{H}$

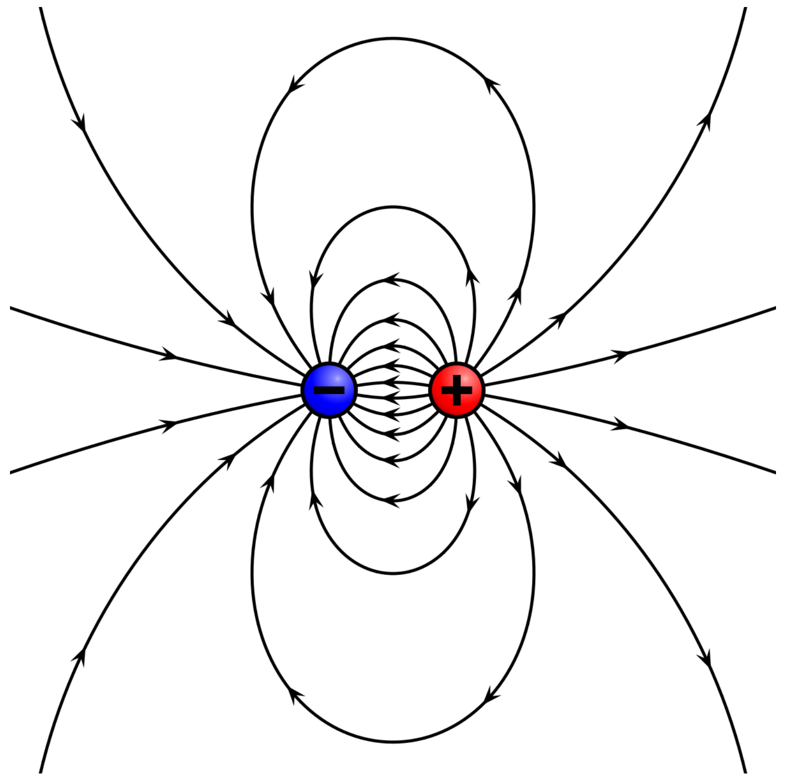
Chap. 11 | Types of Antennas

- What types of antennas will we learn in Chap. 11?
  - Elemental electric dipole
  - Elemental magnetic dipole (i.e. conducting loop)

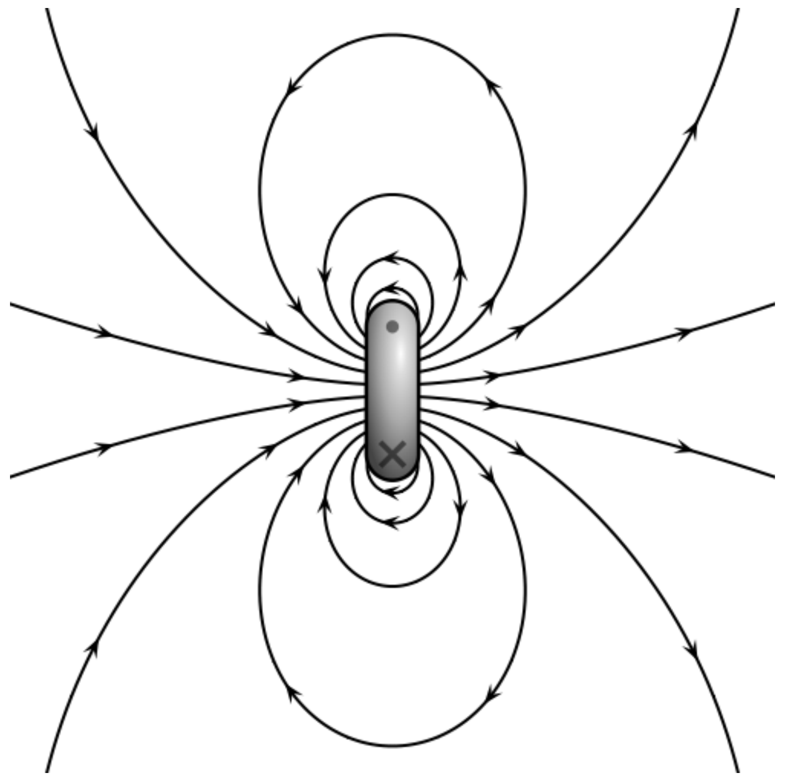
  this class!

  - Finite-length linear antenna
  - Antenna array
    - Higher directivity & desirable radiation properties

- Reciprocity theorem
  - Good ***transmitting*** antenna = Good ***receiving*** antenna

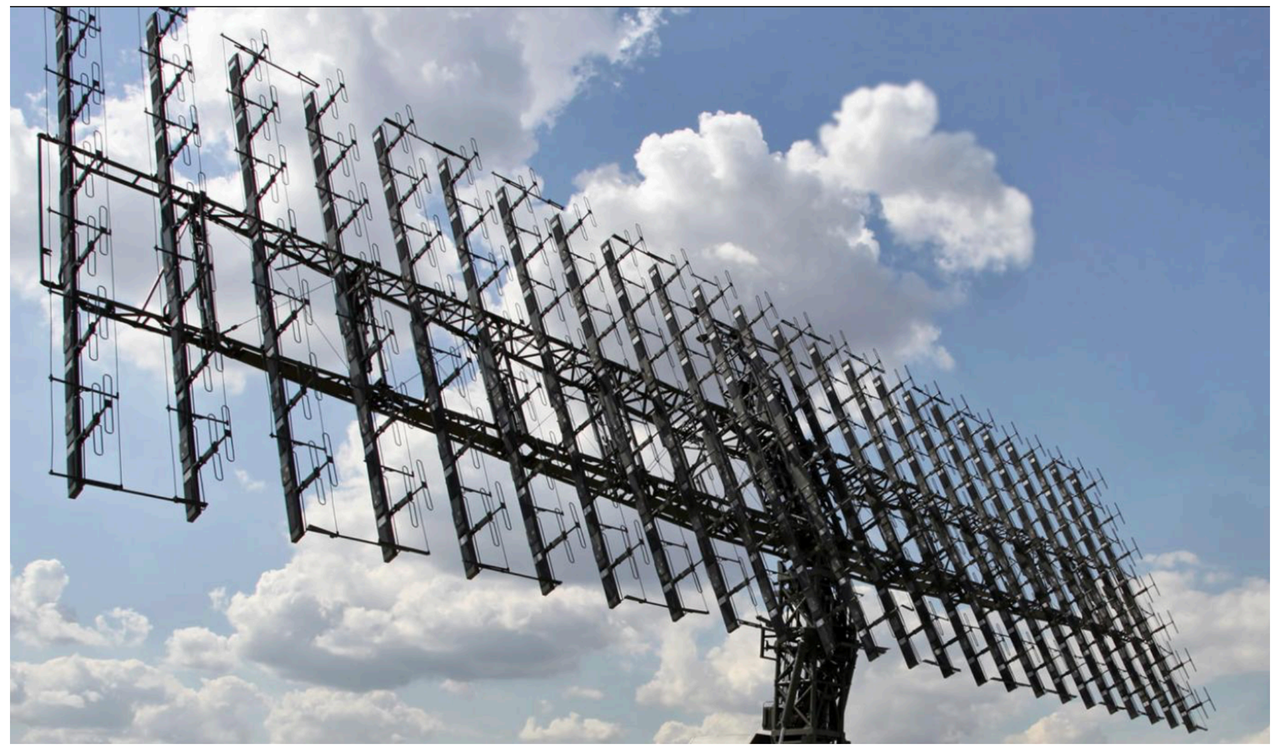


electric dipole



magnetic dipole

Linear Antenna



Antenna array

Img src: Team Blacksheep, mwrf.com

Chap. 11 | Hertzian dipole (1/3)

- **Elemental electric dipole**
  - A short conducting wire of length *dl* terminated with two conductive spheres
  - Uniform, sinusoidal current flowing in the wire

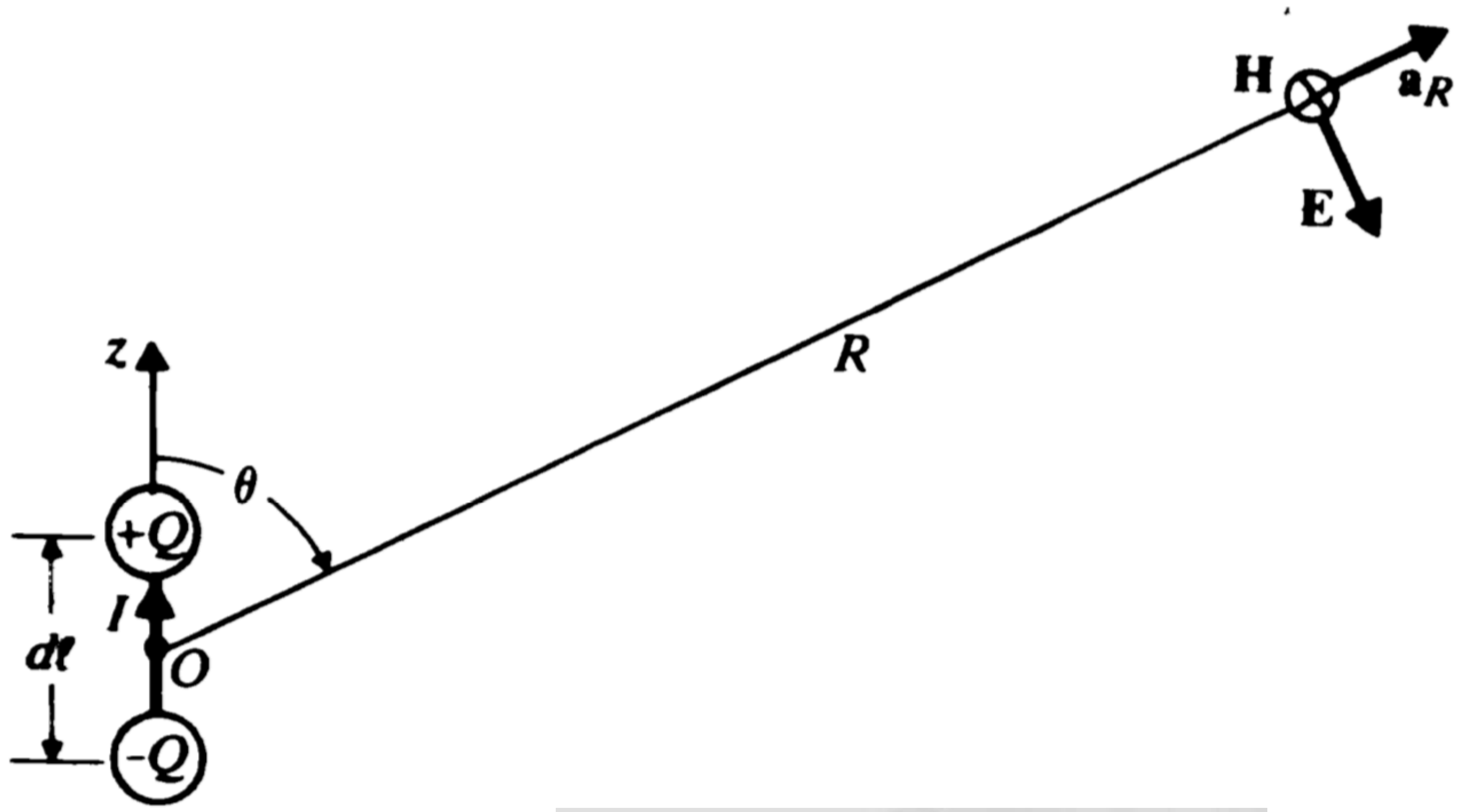
$$i(t) = I\cos\omega t = \mathrm{Re}\left[Ie^{j\omega t}\right]$$

  - Total charge Q oscillating between two spheric ends

$$q(t) = \mathrm{Re}\left[Qe^{j\omega t}\right] \Rightarrow i(t) = \pm\frac{dq(t)}{dt} \Rightarrow I = \pm j\omega Q$$

  - A pair of equal & opposite charges separated by a short distance
  → ***Electric Dipole***

$$\boldsymbol{p} = q\boldsymbol{d} \Rightarrow \boxed{\boldsymbol{p} = \boldsymbol{a}_z Q dl}$$ *"Hertzian" dipole*



Heinrich Hertz
Germany
(1857-1894)
**Proof of EM wave existence theorized by Maxwell*
**First experiment of radio wave using "dipole antenna"*

Img src: Wikipedia

- **Procedures to obtain EM fields by Hertzian dipole**

(1) Determine ***A*** from ***J***

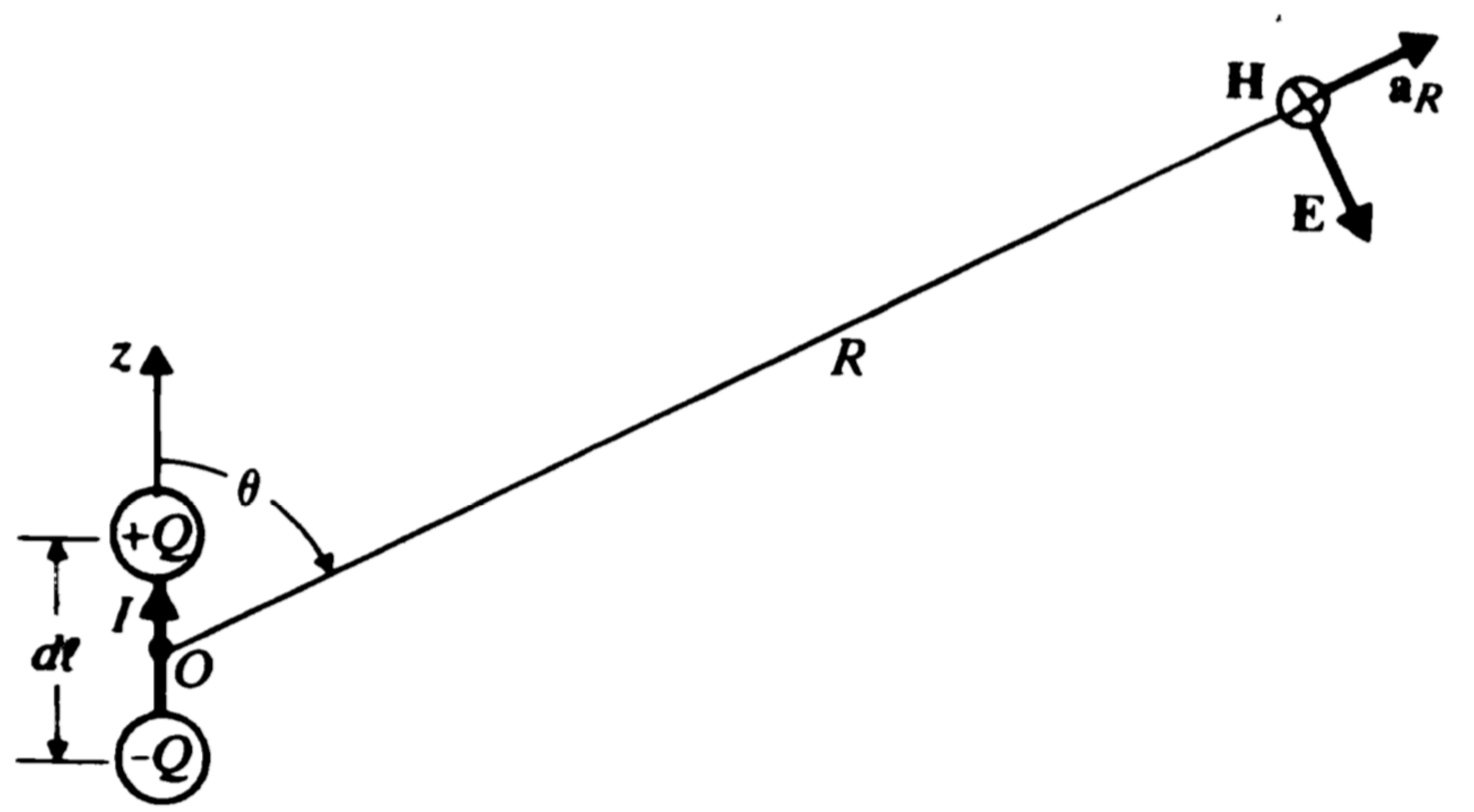
$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_{V'}\frac{\boldsymbol{J}e^{-jkR}}{R}dv' \quad \text{where} \quad \boldsymbol{J}dv' = \boldsymbol{a}_z Idl\cdot\delta(R) \Rightarrow \boldsymbol{A} = \boldsymbol{a}_z\frac{\mu_0 Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)$$

$$\beta = k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

- **Procedures to obtain EM field by Hertzian dipole**

(1) Determine ***A*** from ***J***

$$\boldsymbol{A} = \boldsymbol{a}_z \frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right) \qquad \left(\boldsymbol{a}_z = \boldsymbol{a}_R \cos\theta - \boldsymbol{a}_\theta \sin\theta\right)$$



- Let's express ***A*** in Spherical coordinates

$$\boldsymbol{A} = \boldsymbol{a}_R A_R + \boldsymbol{a}_\theta A_\theta + \boldsymbol{a}_\phi A_\phi \quad \text{where} \quad \begin{cases} A_R = A_z \cos\theta = \dfrac{\mu_0 I dl}{4\pi}\left(\dfrac{e^{-j\beta R}}{R}\right)\cos\theta \\ A_\theta = -A_z \sin\theta = -\dfrac{\mu_0 I dl}{4\pi}\left(\dfrac{e^{-j\beta R}}{R}\right)\sin\theta \\ A_\phi = 0 \end{cases}$$

(2) Determine ***H*** from ***A***

$$\nabla \times \boldsymbol{A} = \frac{1}{R^2 \sin\theta}\begin{vmatrix} \boldsymbol{a}_R & \boldsymbol{a}_\theta R & \boldsymbol{a}_\phi R\sin\theta \\ \dfrac{\partial}{\partial R} & \dfrac{\partial}{\partial \theta} & \cancel{\dfrac{\partial}{\partial \phi}} \\ A_R & R A_\theta & R\sin\theta \cancel{A_\phi} \end{vmatrix}$$

$\Rightarrow$

(∵Azimuthal symmetry)

$$\frac{\partial}{\partial \phi} \to 0, \quad A_\phi = 0$$

$$\boxed{\begin{aligned} \boldsymbol{H} &= \frac{1}{\mu_0}\nabla \times \boldsymbol{A} = \boldsymbol{a}_\phi \frac{1}{\mu_0 R}\left[\frac{\partial}{\partial R}(R A_\theta) - \frac{\partial A_R}{\partial \theta}\right] \\ &= -\boldsymbol{a}_\phi \frac{I dl}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]e^{-j\beta R} \end{aligned}}$$

Hertzian dipole (3/3)

- **Procedures to obtain EM field by Hertzian dipole**

(3) Determine $\boldsymbol{E}$ from $\boldsymbol{H}$

$$\boldsymbol{E} = \frac{1}{j\omega\varepsilon}\nabla\times\boldsymbol{H} = \frac{1}{j\omega\varepsilon_0}\left[a_R\frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(H_\phi\sin\theta\right) - a_\theta\frac{1}{R}\frac{\partial}{\partial R}\left(RH_\phi\right)\right]$$

where

$$\begin{cases} E_R = -\dfrac{Idl}{4\pi}\eta_0\beta^2 2\cos\theta\left[\dfrac{1}{(j\beta R)^2}+\dfrac{1}{(j\beta R)^3}\right]e^{-j\beta R} \\ E_\theta = -\dfrac{Idl}{4\pi}\eta_0\beta^2\sin\theta\left[\dfrac{1}{j\beta R}+\dfrac{1}{(j\beta R)^2}+\dfrac{1}{(j\beta R)^3}\right]e^{-j\beta R} \\ E_\phi = 0 \end{cases}$$

Here, $\eta_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} \cong 120\pi\ (\Omega)$

- **EM fields by Hertzian dipole**

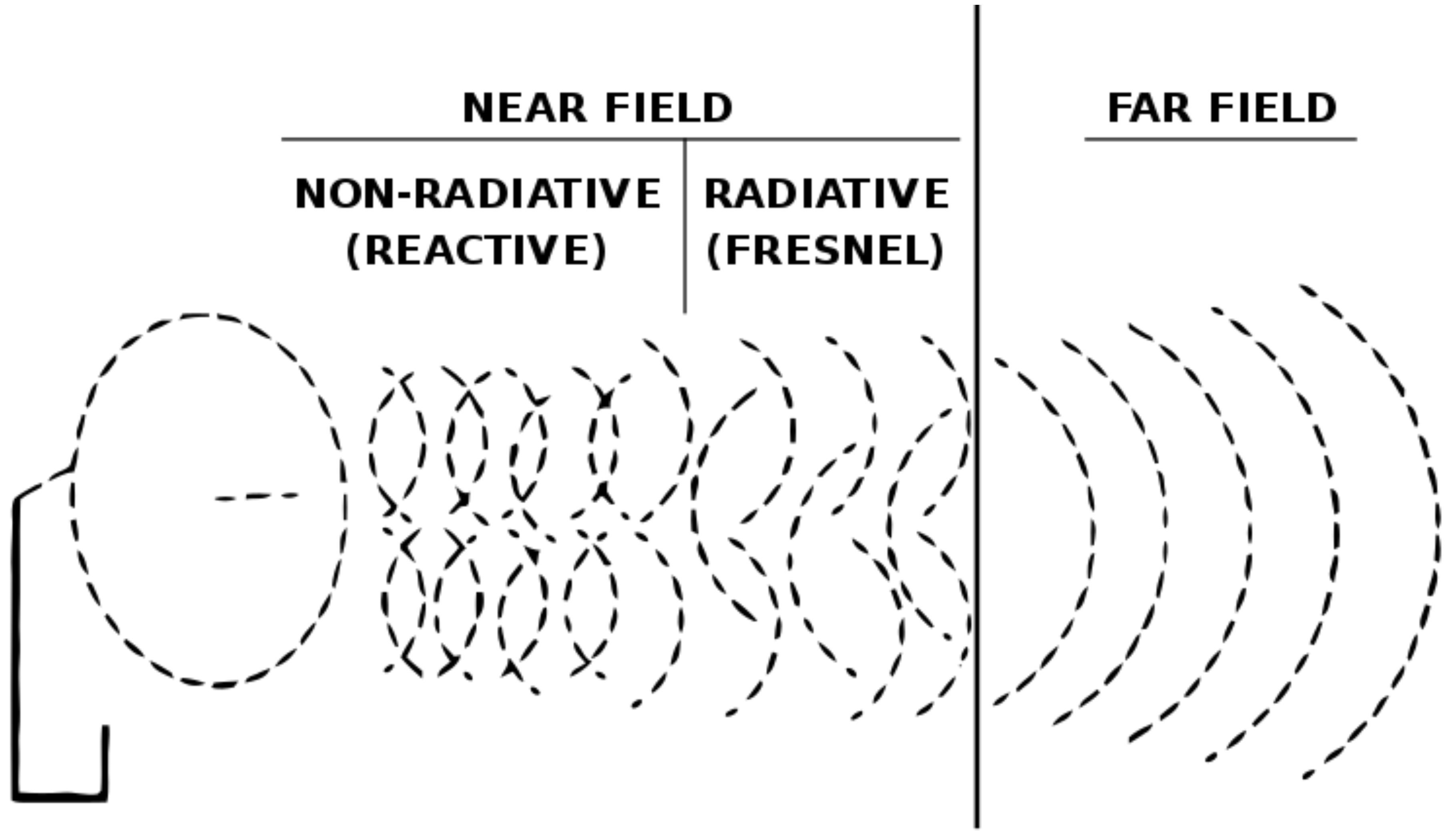
$$H_\phi = -\frac{Idl}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta R}+\frac{1}{(j\beta R)^2}\right]e^{-j\beta R}$$

$$E_R = -\frac{Idl}{4\pi}\eta_0\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2}+\frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

$$E_\theta = -\frac{Idl}{4\pi}\eta_0\beta^2\sin\theta\left[\frac{1}{j\beta R}+\frac{1}{(j\beta R)^2}+\frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

Fairly complicated to analyze!

Near-field & Far-field Approximations!



Chap. 11 | Near-field by Hertzian dipole

- **Near-field approximation**

  - In the region near Hertzian dipole ($\beta R = 2\pi R/\lambda << 1 \;\rightarrow\; 2\pi R << \lambda$)

$$\begin{cases} H_\phi = -\dfrac{Idl}{4\pi}\beta^2 \sin\theta\left[\cancel{\dfrac{1}{j\beta R}} + \dfrac{1}{(j\beta R)^2}\right]e^{-j\beta R} \\ E_R = -\dfrac{Idl}{4\pi}\eta_0\beta^2 2\cos\theta\left[\cancel{\dfrac{1}{(j\beta R)^2}} + \dfrac{1}{(j\beta R)^3}\right]e^{-j\beta R} \\ E_\theta = -\dfrac{Idl}{4\pi}\eta_0\beta^2 \sin\theta\left[\cancel{\dfrac{1}{j\beta R}} + \cancel{\dfrac{1}{(j\beta R)^2}} + \dfrac{1}{(j\beta R)^3}\right]e^{-j\beta R} \end{cases}$$

$$\Rightarrow$$

Only leading term

$$\boxed{\begin{cases} H_\phi \cong \dfrac{Idl}{4\pi R^2}\sin\theta \\ E_R \cong -\dfrac{p}{4\pi\varepsilon_0 R^3} 2\cos\theta \\ E_\theta \cong -\dfrac{p}{4\pi\varepsilon_0 R^3}\sin\theta \end{cases}}$$

where $p = Qdl$

$I = \pm j\omega Q$ was used for derivation

$$\left(e^{-j\beta R} = 1 - j\beta R - \frac{(\beta R)^2}{2} + \cdots \cong 1\right)$$

- **Characteristics of near-field**

  - E_θ and E_R are identical to those by a *static electric dipole* (Chap. 3) $\left(\because \boldsymbol{E} = \dfrac{p}{4\pi\varepsilon_0 R^3}\left(\boldsymbol{a}_R 2\cos\theta + \boldsymbol{a}_\theta \sin\theta\right)\right)$

    ∴Near-fields of oscillating electric dipole = *Quasi-static fields*

Chap. 11 | Far-field by Hertzian dipole

- **Far-field approximation**

  - In the region where $\beta R = 2\pi R/\lambda >> 1 \rightarrow 2\pi R >> \lambda$)

$$
\begin{cases}
H_\phi = -\boldsymbol{a}_\phi \dfrac{Idl}{4\pi}\beta^2 \sin\theta \left[\dfrac{1}{j\beta R} + \cancel{\dfrac{1}{(j\beta R)^2}}\right] e^{-j\beta R} \\
E_R = -\dfrac{Idl}{4\pi}\eta_0 \beta^2 2\cos\theta \left[\cancel{\dfrac{1}{(j\beta R)^2}} + \cancel{\dfrac{1}{(j\beta R)^3}}\right] e^{-j\beta R} \\
E_\theta = -\dfrac{Idl}{4\pi}\eta_0 \beta^2 \sin\theta \left[\dfrac{1}{j\beta R} + \cancel{\dfrac{1}{(j\beta R)^2}} + \cancel{\dfrac{1}{(j\beta R)^3}}\right] e^{-j\beta R}
\end{cases}
$$

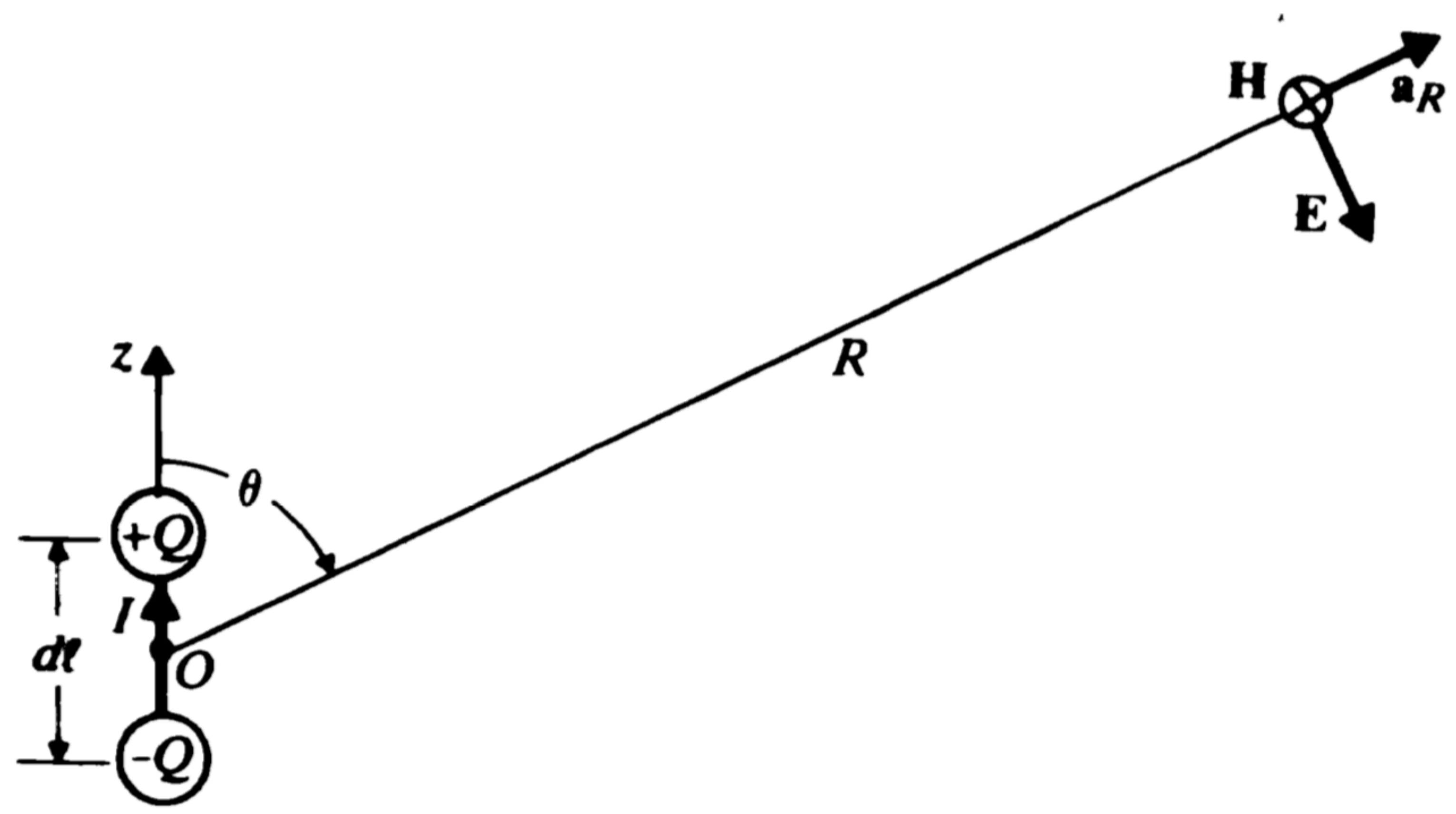
$$
\left(e^{-j\beta R} = 1 - j\beta R - \frac{(\beta R)^2}{2} + \cdots \cong 1 \right)
$$

Only leading term

$$
\boxed{\begin{aligned} H_\phi &= j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta \\ E_\theta &= j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta \end{aligned}}
$$

- **Characteristics of far-field**

  - E_θ and H_ϕ are *in time phase and in space quadrature*
  - Ratio $E_\theta / H_\phi = \eta_0$: Intrinsic impedance of medium → *Far-fields = a plane wave*
  - R (distance from source)↑ → Magnitude of far-fields↓



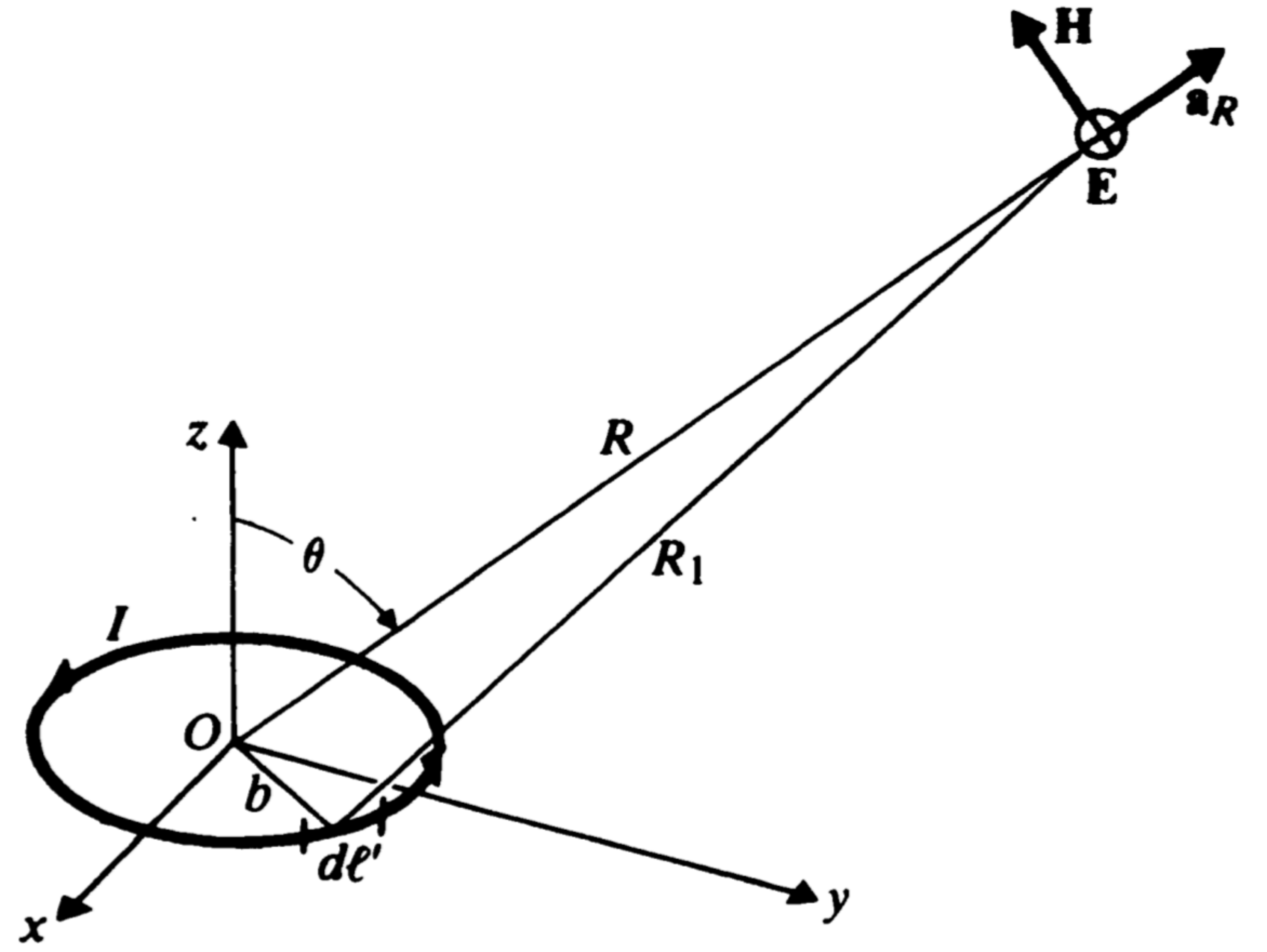
Chap. 11 | Elemental magnetic dipole

- Procedures to obtain EM field by elemental magnetic dipole

  - Small conducting loop of radius *b* carrying time-harmonic current *i*(*t*) = *I*cos*ωt*

  - Vector phasor magnetic moment:

$$\boldsymbol{m} = \boldsymbol{a}_z I\pi b^2 = \boldsymbol{a}_z m \quad \left(A\cdot m^2\right)$$



(1) Determine ***A*** from ***J***

Time-retardation term

$$\boldsymbol{A} = \frac{\mu_0 I}{4\pi}\oint \frac{e^{-j\beta R_1}}{R_1} d\boldsymbol{l}'$$

$$e^{-j\beta R_1} = e^{-j\beta R} e^{-j\beta(R_1 - R)} \cong e^{-j\beta R}\left[1 - j\beta\left(R_1 - R\right)\right]$$

$$\boldsymbol{A} = \frac{\mu_0 I}{4\pi} e^{-j\beta R}\left[(1 + j\beta R)\underbrace{\oint \frac{d\boldsymbol{l}'}{R_1}}_{=\frac{b^2 \sin\theta}{\pi R^2}} - \cancel{j\beta\oint d\boldsymbol{l}'}\right]$$ (∵closed-line integral)

$$\boxed{\boldsymbol{A} = \boldsymbol{a}_\phi \frac{\mu_0 m}{4\pi R^2}(1 + j\beta R)e^{-j\beta R}\sin\theta}$$ where $m = I\pi b^2$

 Duality: electric and magnetic dipoles

- Procedures to obtain EM field by Magnetic dipole

(2) Obtain ***E*** and ***H*** from ***A***

$$\boldsymbol{H} = \frac{1}{\mu}\nabla\times\boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = \frac{1}{j\omega\varepsilon}\nabla\times\boldsymbol{H}$$

EM fields by oscillating "Magnetic" dipole

$$\begin{cases} E_\phi = -\dfrac{j\omega\mu_0 m}{4\pi}\beta^2\sin\theta\left[\dfrac{1}{j\beta R}+\dfrac{1}{(j\beta R)^2}\right]e^{-j\beta R} \\ H_R = -\dfrac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2\cos\theta\left[\dfrac{1}{(j\beta R)^2}+\dfrac{1}{(j\beta R)^3}\right]e^{-j\beta R} \\ H_\theta = -\dfrac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2\sin\theta\left[\dfrac{1}{j\beta R}+\dfrac{1}{(j\beta R)^2}+\dfrac{1}{(j\beta R)^3}\right]e^{-j\beta R} \end{cases}$$

EM fields by oscillating "Electric" dipole

$$\begin{cases} H_\phi = -\dfrac{Idl}{4\pi}\beta^2\sin\theta\left[\dfrac{1}{j\beta R}+\dfrac{1}{(j\beta R)^2}\right]e^{-j\beta R} \\ E_R = -\dfrac{Idl}{4\pi}\eta_0\beta^2 2\cos\theta\left[\dfrac{1}{(j\beta R)^2}+\dfrac{1}{(j\beta R)^3}\right]e^{-j\beta R} \\ E_\theta = -\dfrac{Idl}{4\pi}\eta_0\beta^2\sin\theta\left[\dfrac{1}{j\beta R}+\dfrac{1}{(j\beta R)^2}+\dfrac{1}{(j\beta R)^3}\right]e^{-j\beta R} \end{cases}$$

If $Idl = j\beta m$,

$$\boxed{\begin{aligned} \boldsymbol{E}_e &= \eta_0\boldsymbol{H}_m \\ \boldsymbol{H}_e &= -\frac{\boldsymbol{E}_m}{\eta_0} \end{aligned}}$$

Principle of Duality

Both are solutions of Maxwell's Equations

Observations

- E_θ for electric dipole and E_ϕ for magnetic dipole have the same pattern function $|\sin\theta|$
- Both space and time quadrature
- Combination of two → ***Antenna with circular polarization (Good for signal reception in satellite communication!)***

Electromagnetics

<Chap. 11> Antennas and Radiating systems

Section 11.1 ~ 11.3

(2nd of **week 13**)

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Textbook: Field and Wave Electromagnetics, 2E, Addison-Wesley

Chap. 11 | Contents

Sec 3. Antenna Patterns and Antenna Parameters

- Radiation patterns
- Characteristic parameters
    - Main beam width
    - Side lobes level
    - Directivity
    - Power gain
    - Radiation efficiency

Radiation pattern of Antenna (1/2)

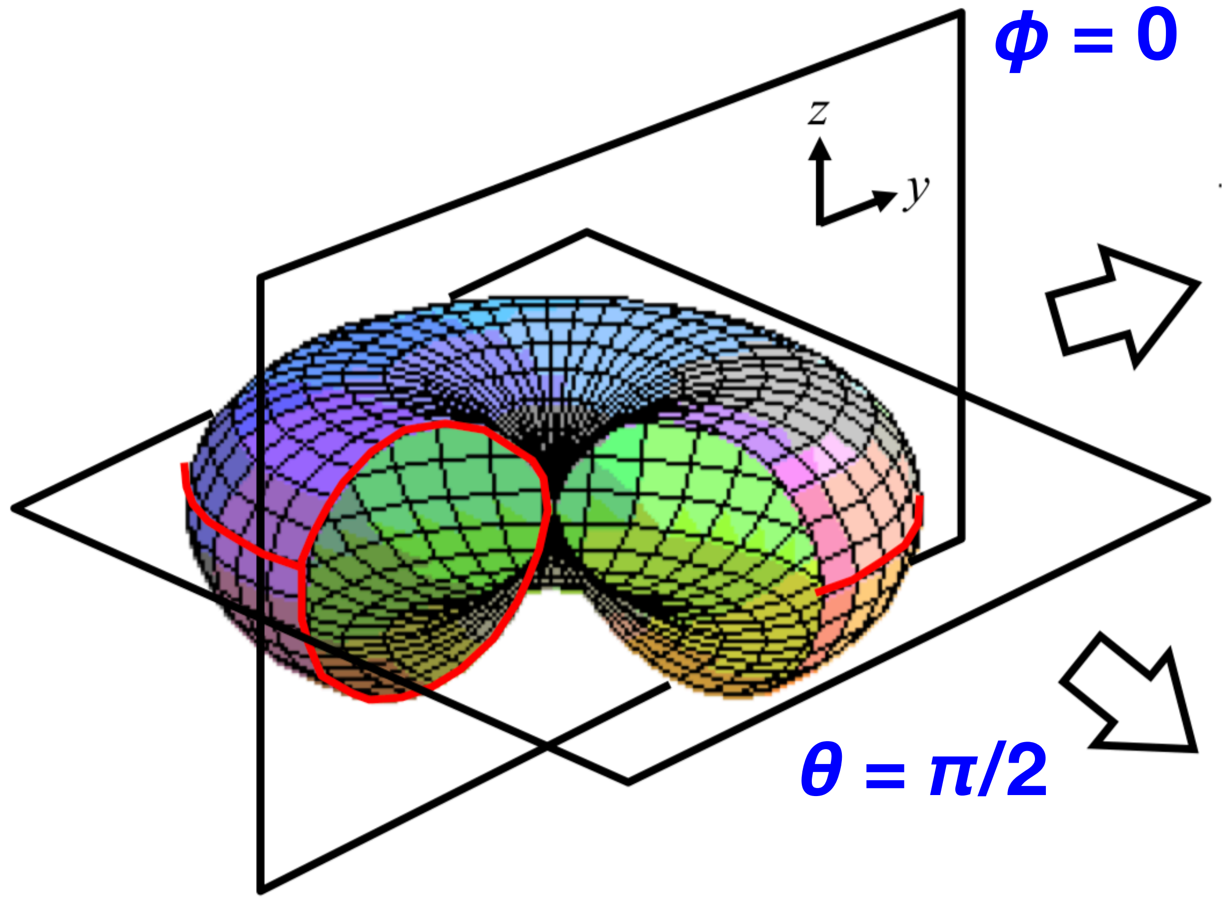
- **Radiation pattern of Antennas**
  - Our major interest: Far fields = Radiation fields
  - Radiation pattern
    - ***Relative far-field strength vs. direction*** at a fixed distance (R) from antenna
    - Three dimensional (varying with θ and ϕ in spherical coordinate)

- **Visualization of radiation pattern in practice**
  - *E*-plane: Plane containing ***E-field*** vector
    - |Normalized field strength| vs. ***θ for constant ϕ***
  - *H*-plane: Plane containing ***H-field*** vector
    - |Normalized field strength| vs. ***ϕ for constant θ***

- **Example: Hertzian dipole**

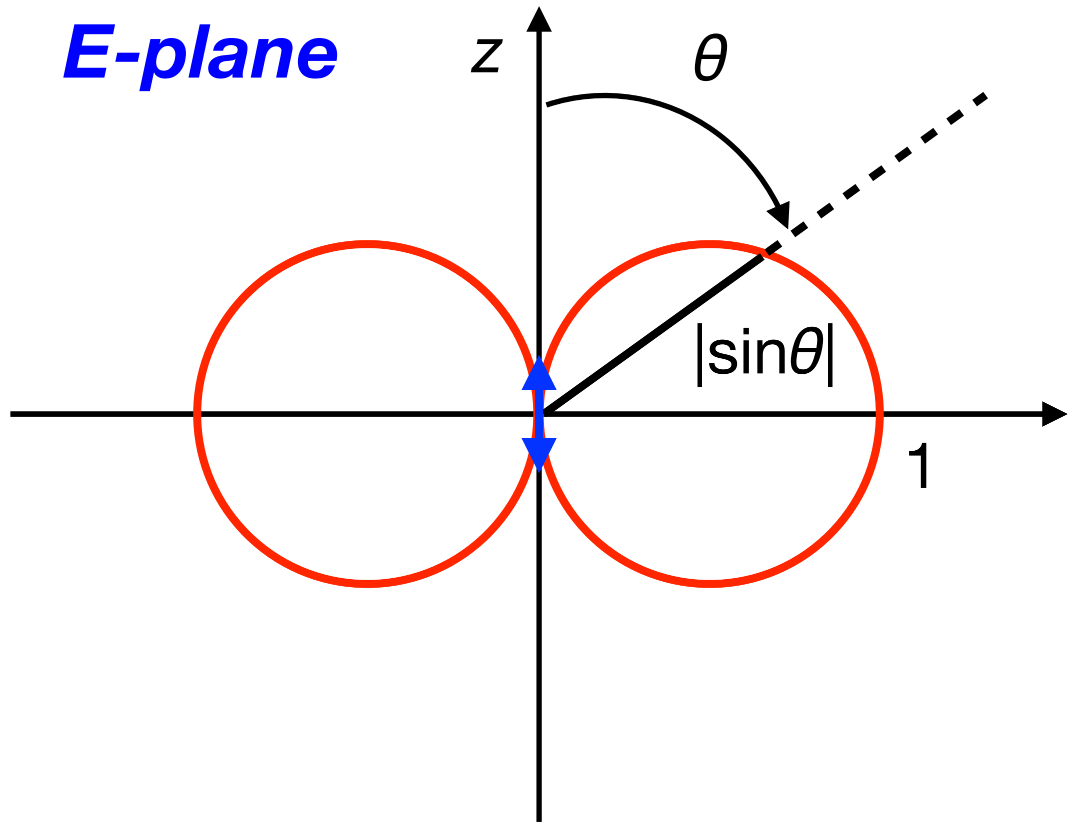
$$E_\theta = j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta \quad\rightarrow\quad |E_\theta| \propto |\sin\theta| \quad \text{(for } \phi = 0\text{)}$$

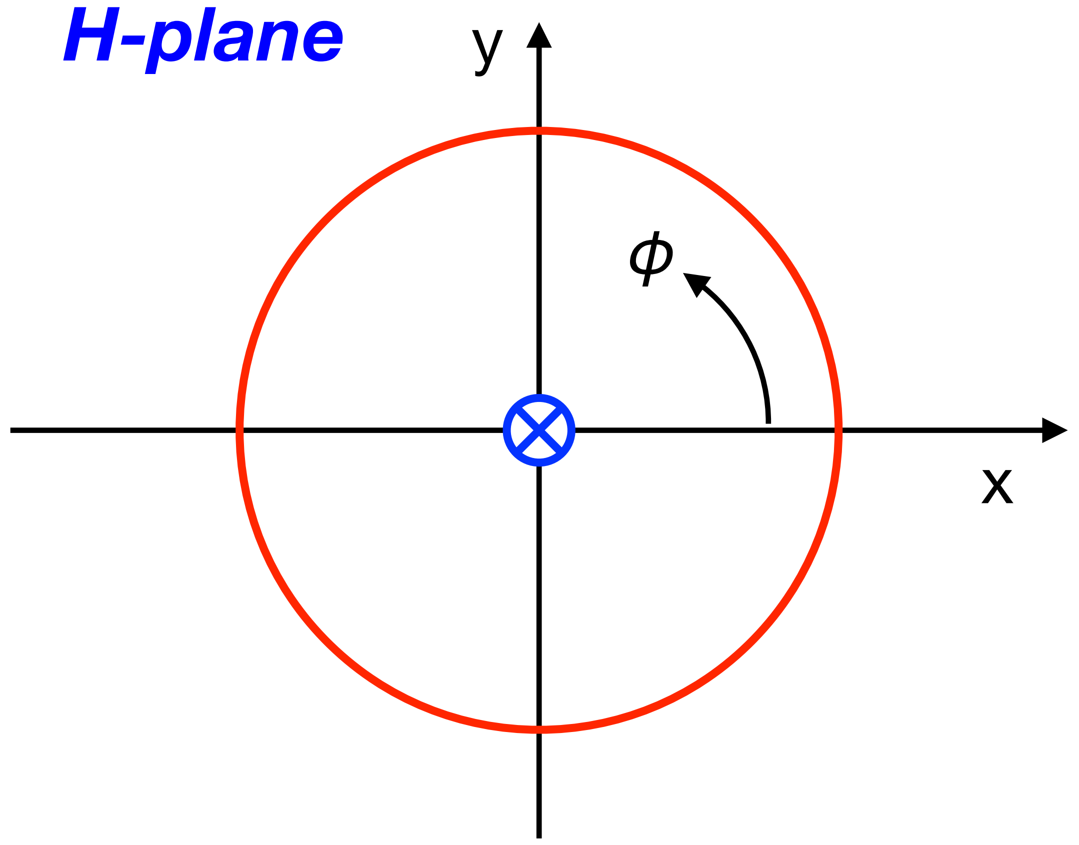
$$H_\phi = j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta \quad\rightarrow\quad |H_\phi| \propto 1 \quad \text{(for } \theta = \pi/2\text{)}$$



3D cutaway view

*** No power radiation along antenna's axis*





Chap. 11 | Radiation pattern of Antenna (2/2)

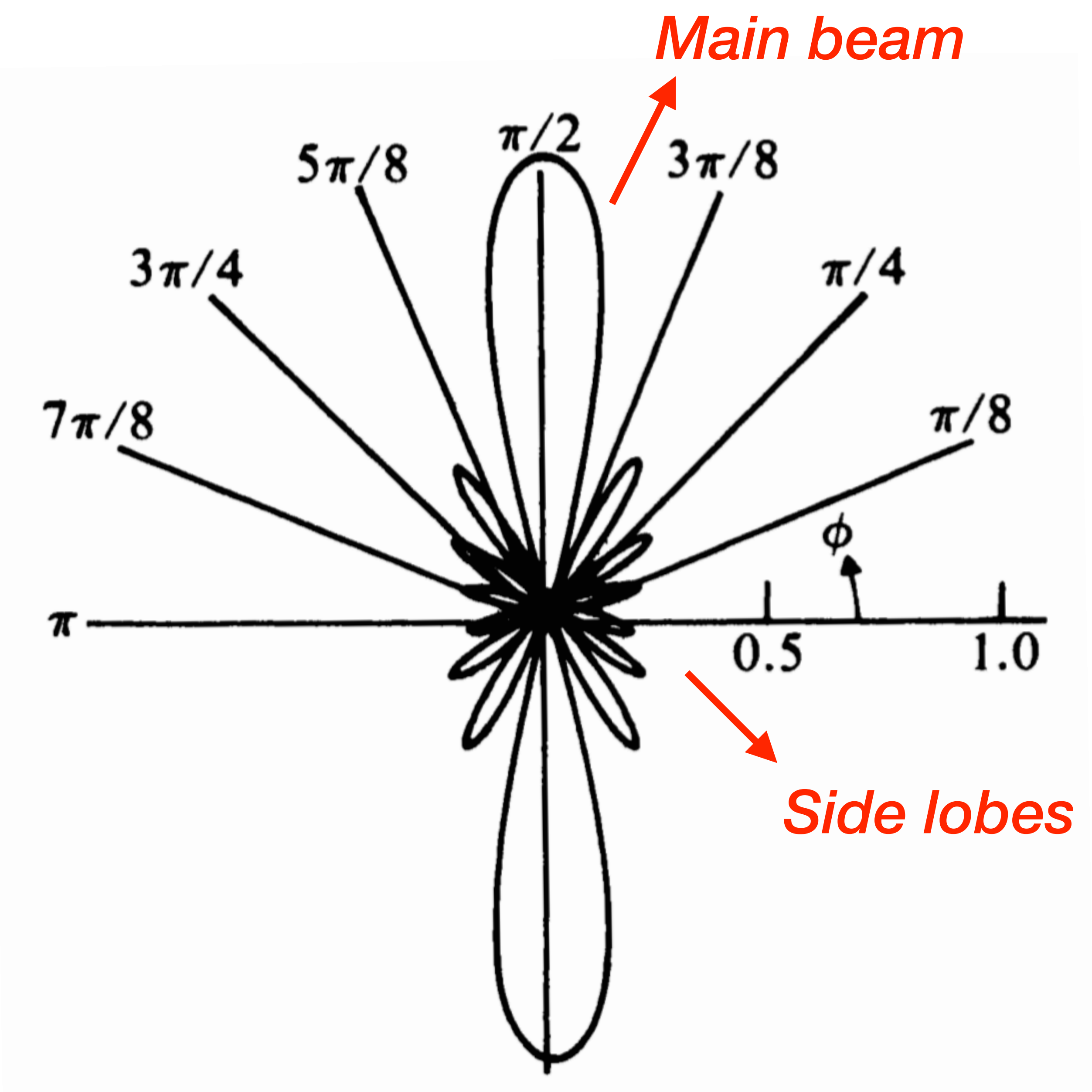
- **Radiation pattern of practical antennas**
  - Quite complicated!
  - Comprised of major maximum (Main beam) & several minor maxima (side lobes)
  - Useful to plot in rectangular coordinates in dB scale

- **Important characteristic parameters**

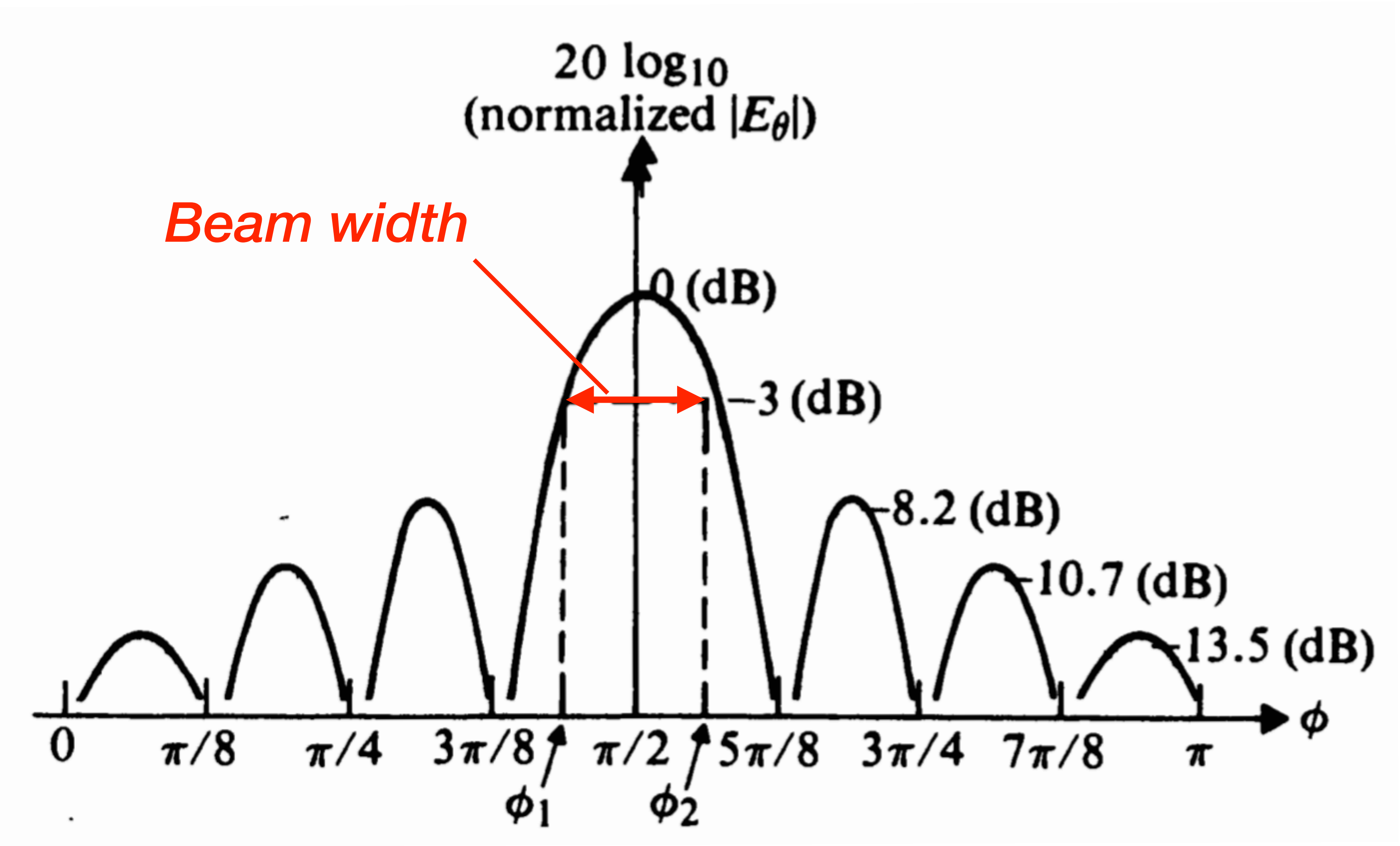
  (1) Width of main beam
  - Sharpness of main radiation region (the narrower, the better!)
  - Angular width between half-power (i.e. -3 dB) points

  (2) Levels of side lobes
  - Regions of unwanted radiation (the smaller, the better!)
  - In modern antennas, maintained at -40dB or even smaller

  (3) Directivity gain (**)
  - A measure of ability to concentrate radiated power in a given direction



<Typical H-plane pattern>



*<H-plane pattern **in dB scale**>*
(in rectangular coordinates)

Chap. 11 | Directivity of antenna (1/2)

- Directive gain
  - A measure of ability to concentrate radiated power in a given direction

$$G_D(\theta,\phi) \triangleq \frac{\text{Radiation intensity in a given direction with } (\theta,\phi)}{\text{Average radiation intensity}}$$

- ***Radiation "intensity"***: Time-average radiated power per unit ***solid angle*** (W/sr)
- ***Solid angle***: 3D angle of…
  - A measure of field-of-view from a particular observing point
  - A measure of how large the object appears to an observer looking from that point
  - Unit: Steradian [sr]

$$\Omega \triangleq \frac{a}{R^2}$$

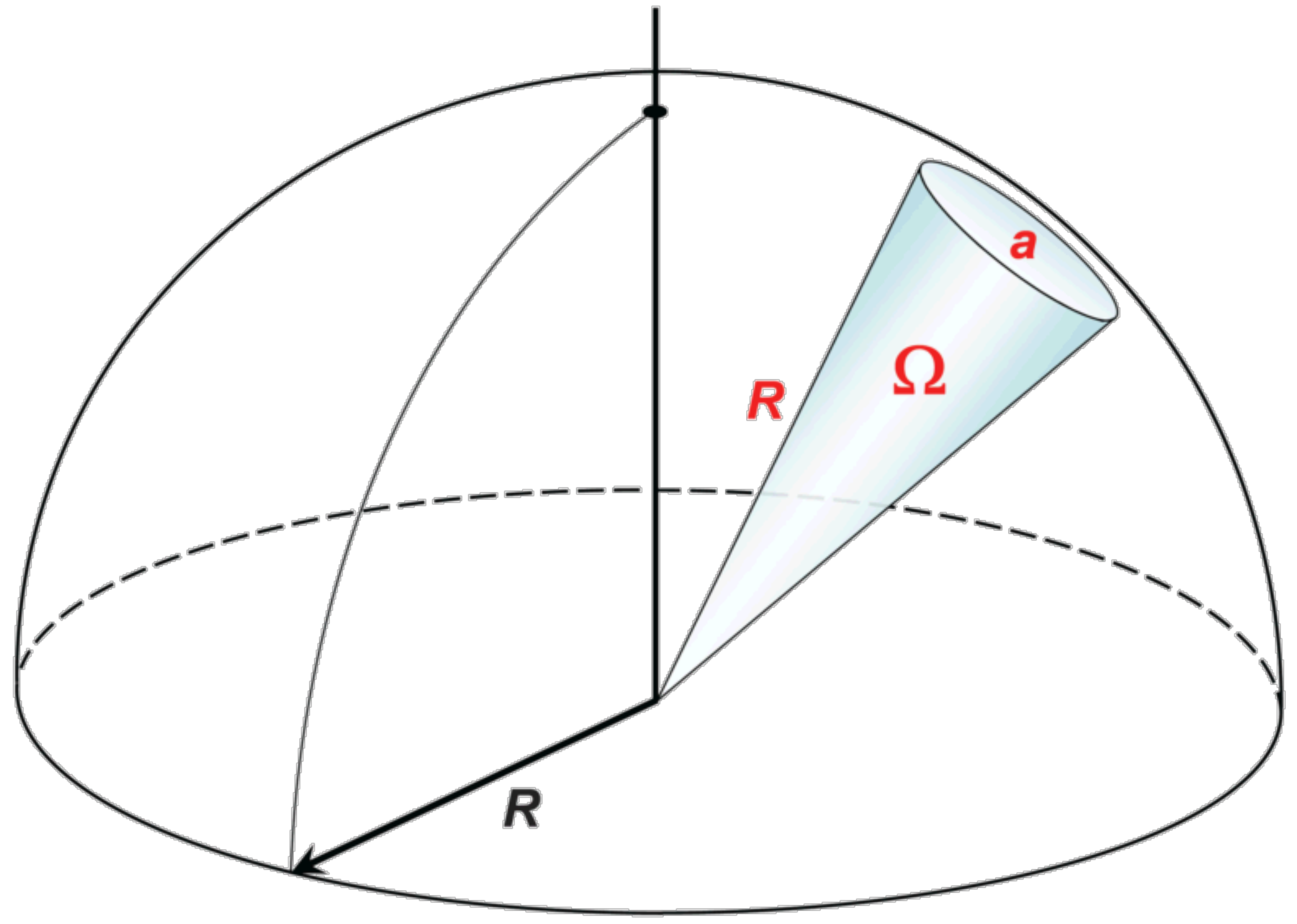
where *a* is spherical surface area and *R* is the radius of the sphere

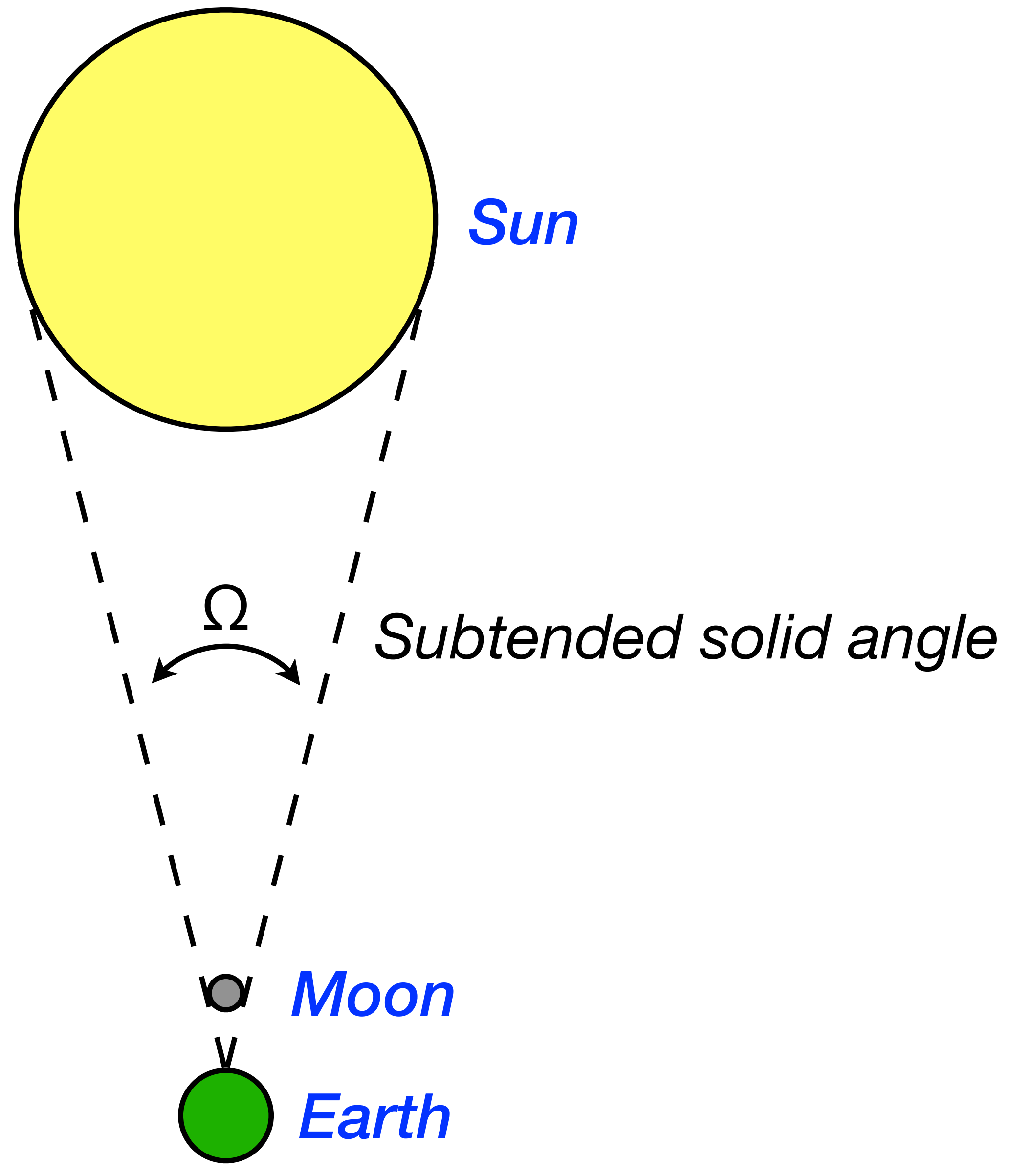
◉ $\Omega = 1$ (sr) when $a = R^2$ (i.e. Unit solid angle)

◉ $\Omega = 4\pi$ (sr) when $a = 4\pi R^2$ (i.e. Max solid angle)

- ***Radiation intensity***:

$$U(\mathrm{W/sr}) = \underbrace{P_{av}}_{\text{Time-average power per unit area (W/m}^2\text{)}} \cdot \underbrace{a}_{\text{Spherical area per unit solid angle, i.e. } a = R^2 \text{ (m}^2\text{/sr)}} = P_{av} \cdot R^2$$





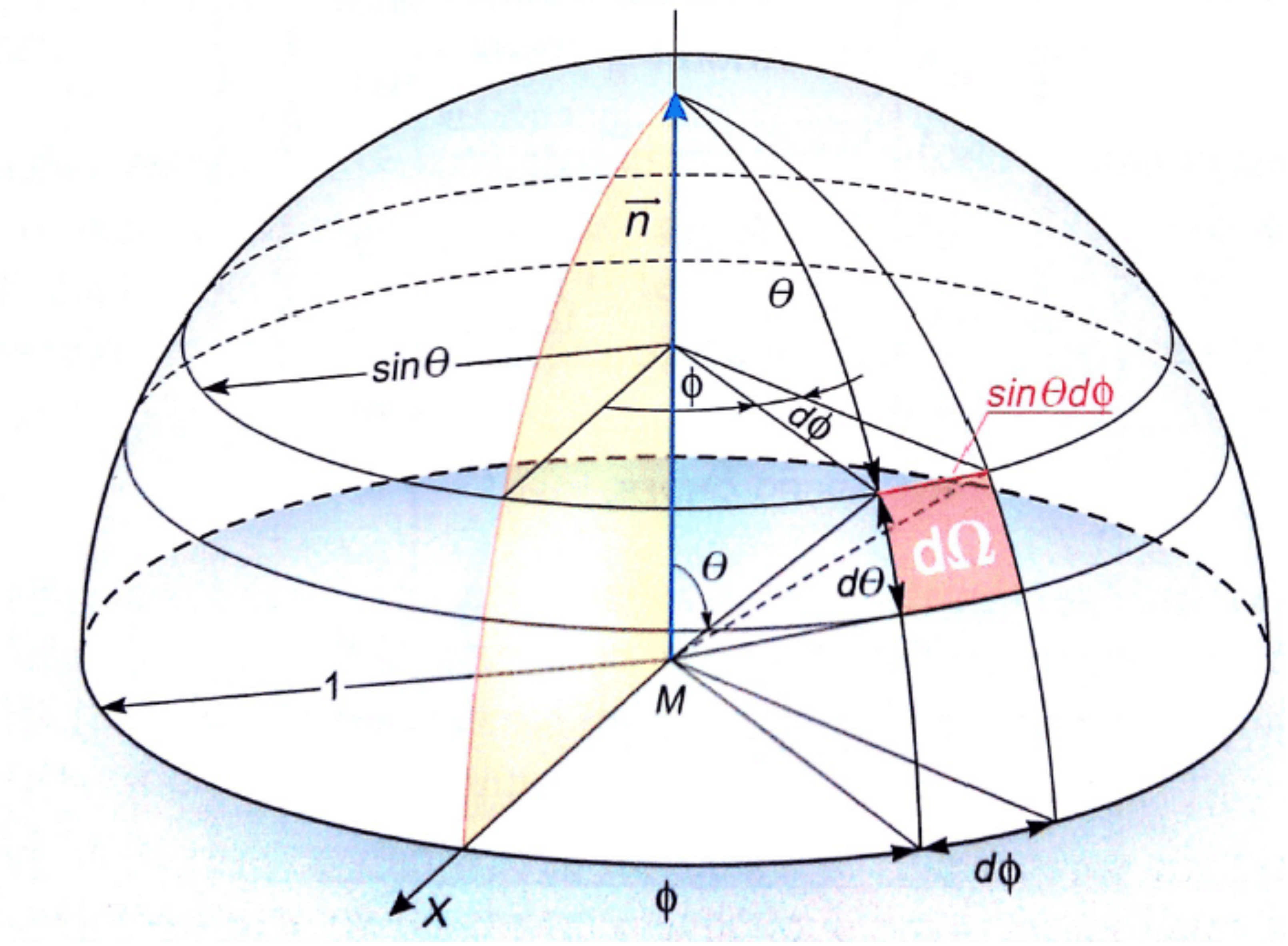
Why Moon and Sun appear to be same size from Earth? Why do you see a solar eclipse?

Directivity of antenna (2/2)

- **Directive gain**
  - Radiation intensity: $U(\mathrm{W/sr}) = P_{av} \cdot a = P_{av} \cdot R^2$
  - "total" time-average radiated power is given by

$$P_r = \oint U\, d\Omega = \oint \boldsymbol{P}_{av} \cdot d\boldsymbol{s} \quad (\mathrm{W})$$ where $d\Omega = \sin\theta d\theta d\phi$

Differential solid angle



  - Thus, directive gain is given by

$$G_D(\theta,\phi) \triangleq \frac{\text{Radiation intensity in a given direction with } (\theta,\phi)}{\text{Average radiation intensity}}$$

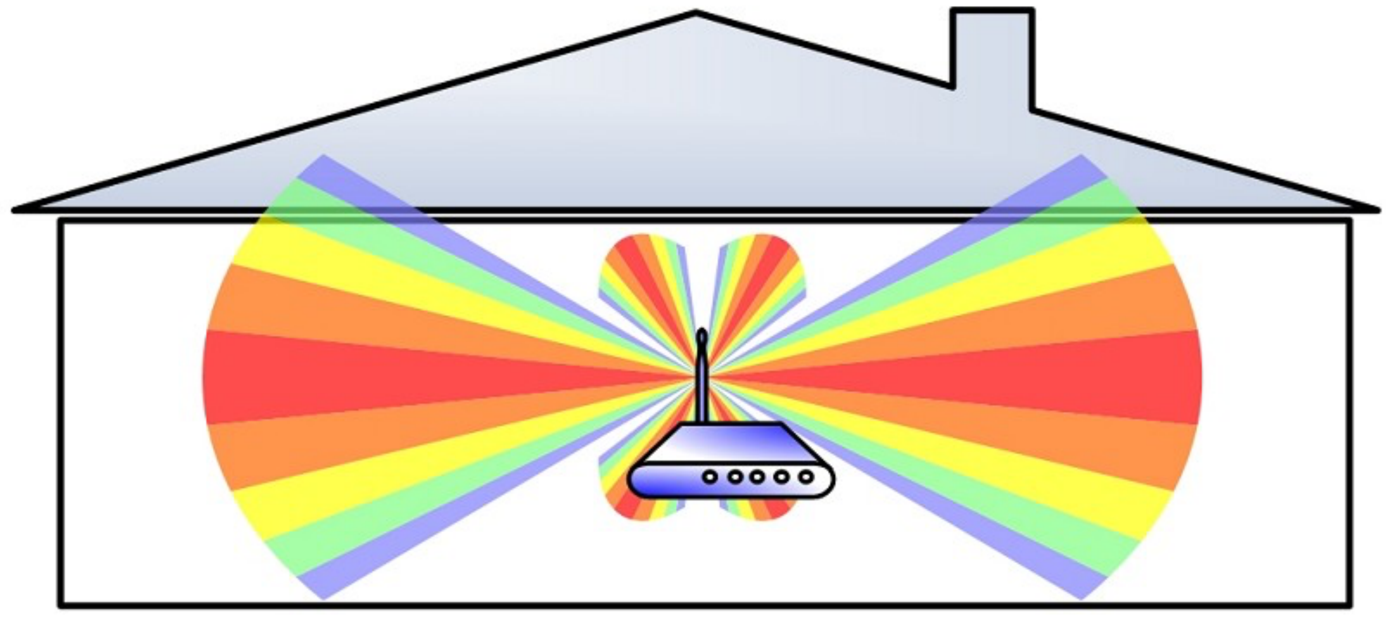
$$\boxed{G_D(\theta,\phi) = \frac{U(\theta,\phi)}{\frac{1}{4\pi}\oint U(\theta,\phi)\, d\Omega} = \frac{4\pi U(\theta,\phi)}{P_r}}$$

* What if antenna is isotropic (= Omni-directional)?

→ $G_D(\theta, \phi) = 1$ [But, practically not useful!)

- **Directivity = Maximum directive gain**

$$\boxed{D \triangleq \max\left[G_D(\theta,\phi)\right] = \frac{4\pi U_{max}}{P_r}}$$



Good example of directive antenna → wifi router!

Img src: Researchgate, countrymilewifi.com

Chap. 11 | Example for directivity

- **Obtain directive gain & directivity of Hertzian dipole.**

  - Directive gain (G_D)

$$G_D(\theta,\phi) = \frac{U(\theta,\phi)}{\frac{1}{4\pi}\oint U(\theta,\phi)\,d\Omega} = \frac{4\pi U(\theta,\phi)}{\oint U(\theta,\phi)\,d\Omega} = \frac{4\pi U(\theta,\phi)}{\mathrm{P}_r}$$ where U is radiation intensity (W/sr) and P_r is total radiated power (W)

‣ (Numerator) Radiation intensity U is given by

$$U(\theta,\phi) = P_{av}(\theta,\phi)\cdot R^2 \quad \text{where} \quad P_{av}(\theta,\phi) = \frac{1}{2}\mathrm{Re}\left|\boldsymbol{E}\times\boldsymbol{H}^*\right| = \frac{1}{2}\left|E_\theta\right|\left|H_\phi\right|$$ [time-average power per unit area in direction of (θ, ϕ)]

Here, $$\begin{cases} E_\theta = j\dfrac{Idl}{4\pi}\left(\dfrac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta \\ H_\phi = j\dfrac{Idl}{4\pi}\left(\dfrac{e^{-j\beta R}}{R}\right)\beta\sin\theta \end{cases}$$ for a Hertzian dipole

$$P_{av}(\theta,\phi) = \frac{(Idl)^2}{32\pi^2}\frac{\eta_0\beta^2}{R^2}\sin^2\theta \quad \rightarrow \quad \boxed{U(\theta,\phi) = P_{av}(\theta,\phi)\cdot R^2 = \frac{(Idl)^2}{32\pi^2}\frac{\eta_0\beta^2}{R^2}\sin^2\theta \quad (\mathrm{W})}$$

‣ (Denominator) Total radiated power can be evaluated as

$$\oint U(\theta,\phi)\,d\Omega = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\,d\theta\,d\phi = \frac{(Idl)^2}{32\pi^2}\frac{\eta_0\beta^2}{R^2}\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\,d\theta\,d\phi = \frac{(Idl)^2}{32\pi^2}\frac{\eta_0\beta^2}{R^2}\cdot\frac{4}{3}$$

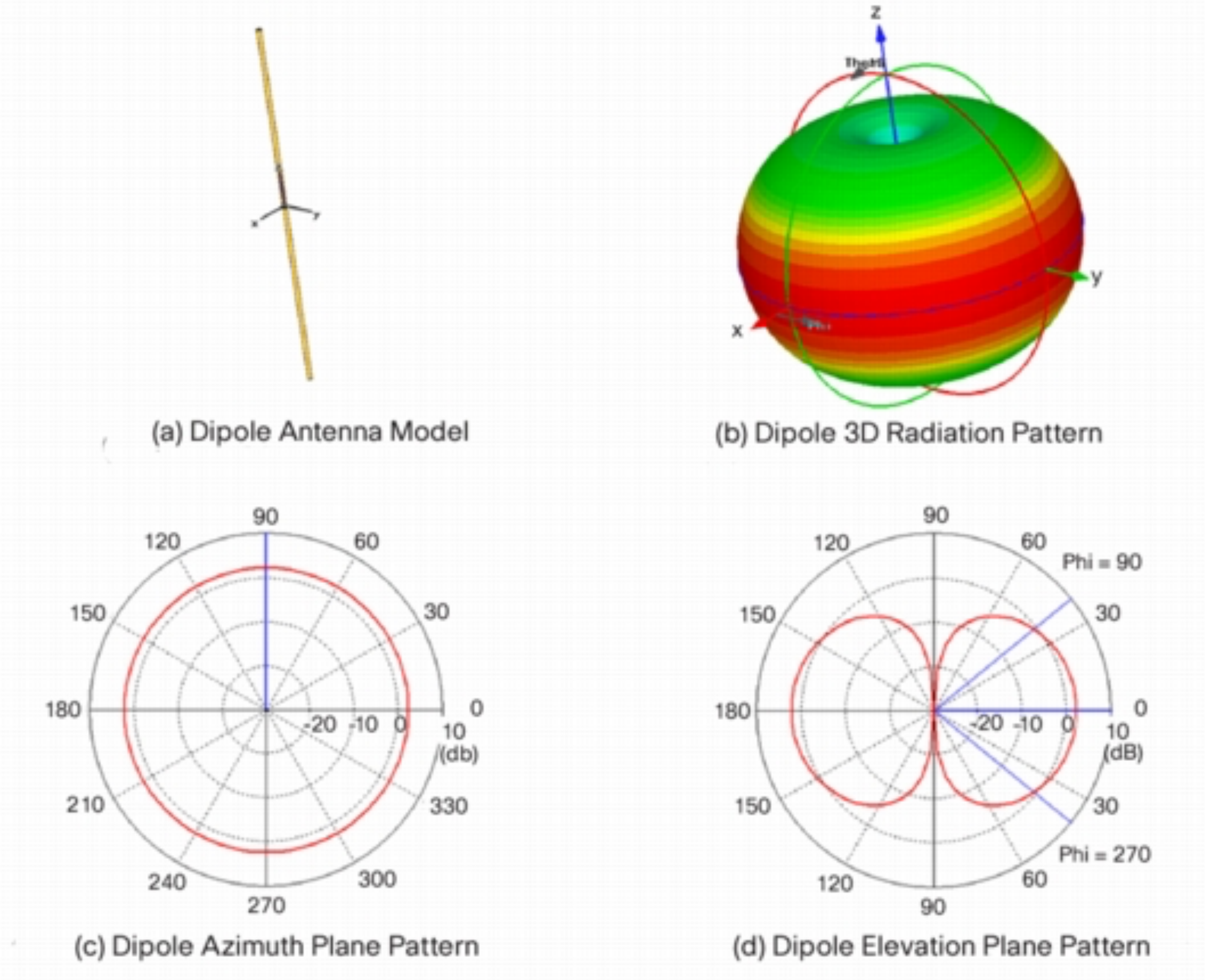
Directive gain

$$\therefore G_D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{\oint U(\theta,\phi)\,d\Omega} = \frac{3}{2}\sin^2\theta$$

Directivity

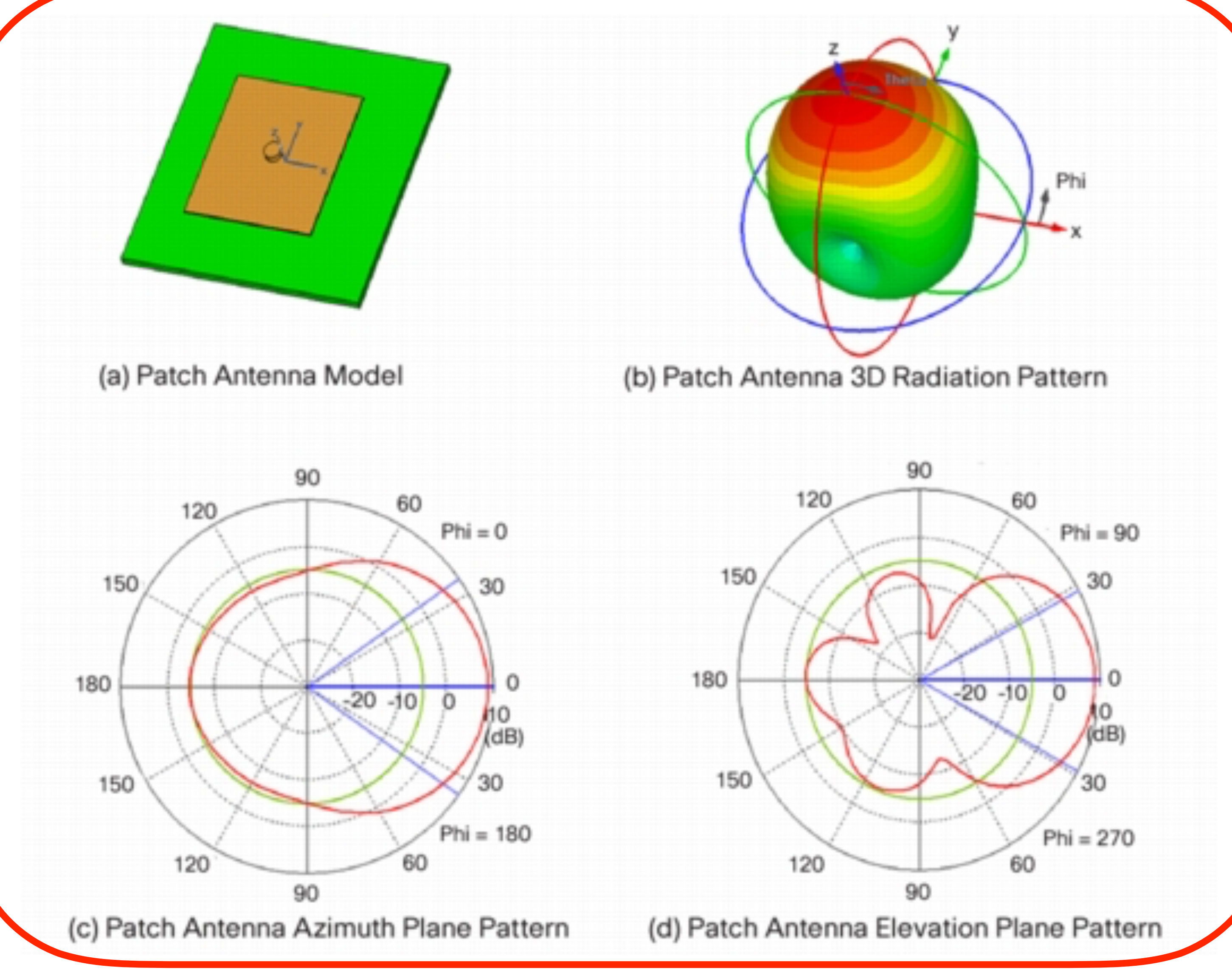
$$\therefore D = \max\left[G_D(\theta,\phi)\right] = \frac{3}{2} \rightarrow 1.76(\mathrm{dB})$$

Directivity of a few antennas

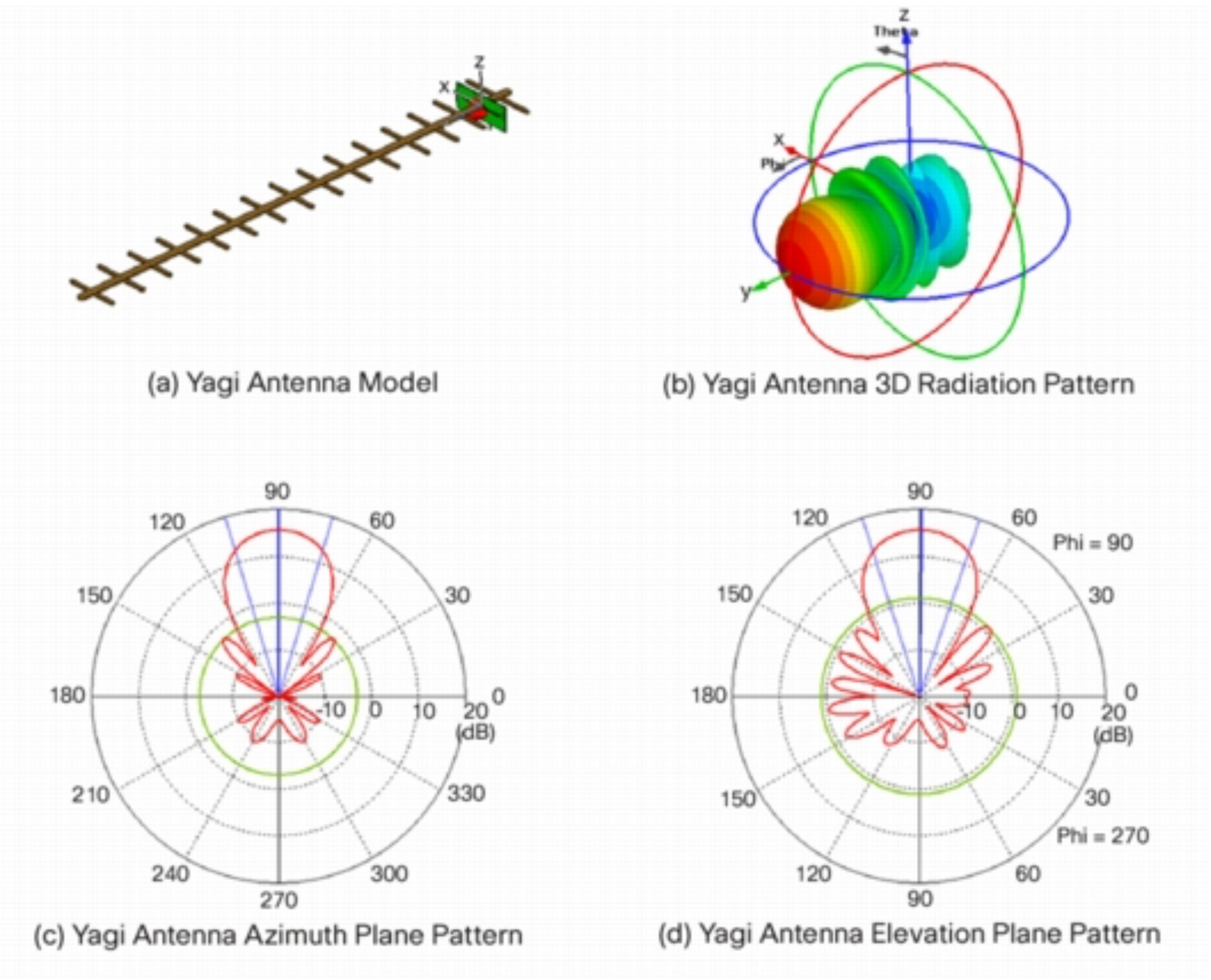


Hertzian antenna = 1.8 (dBi)

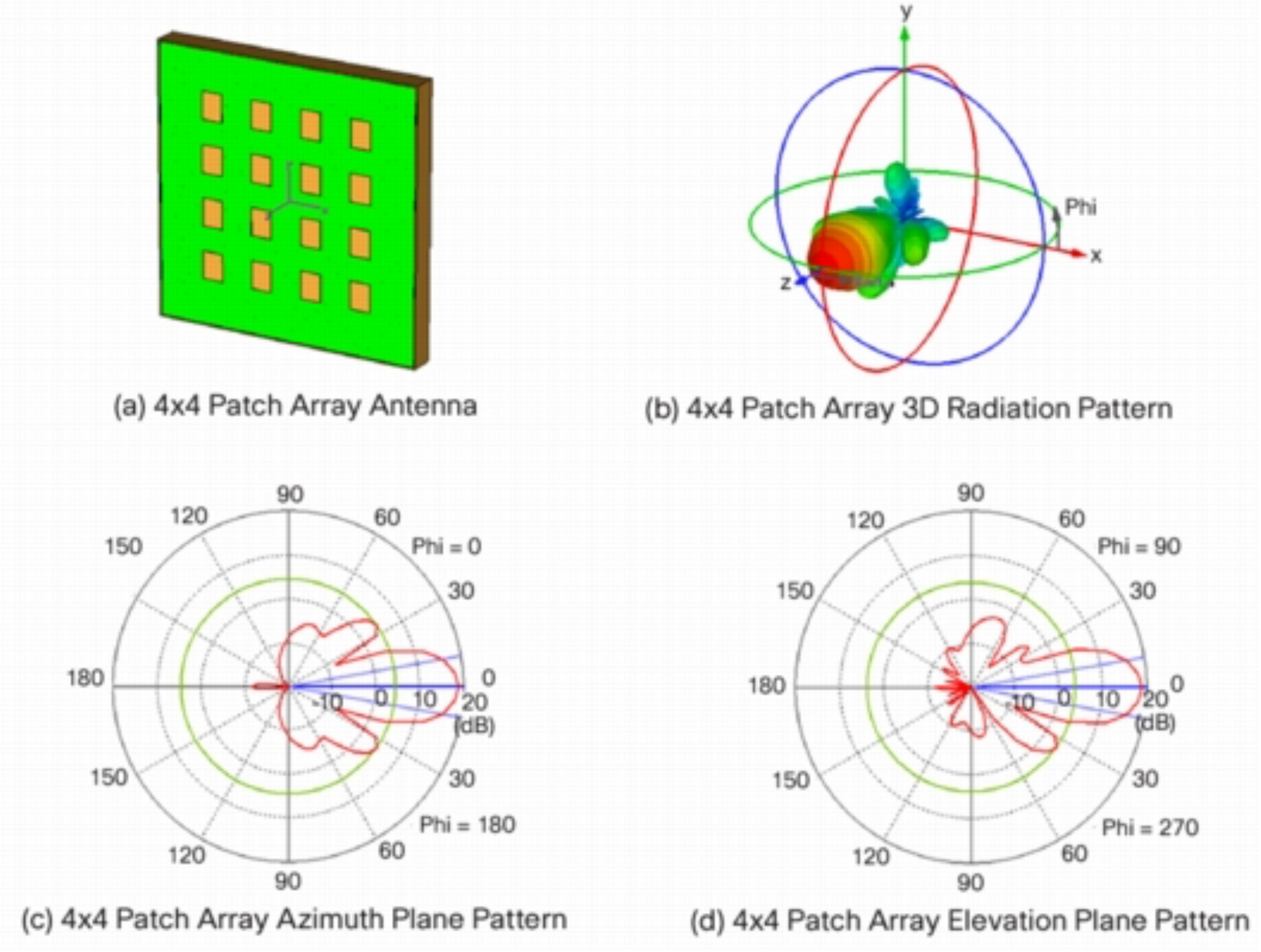
Typically used for mobile phones!



Single patch antenna = 8.8 (dBi)



Yagi antenna = 15 (dBi)



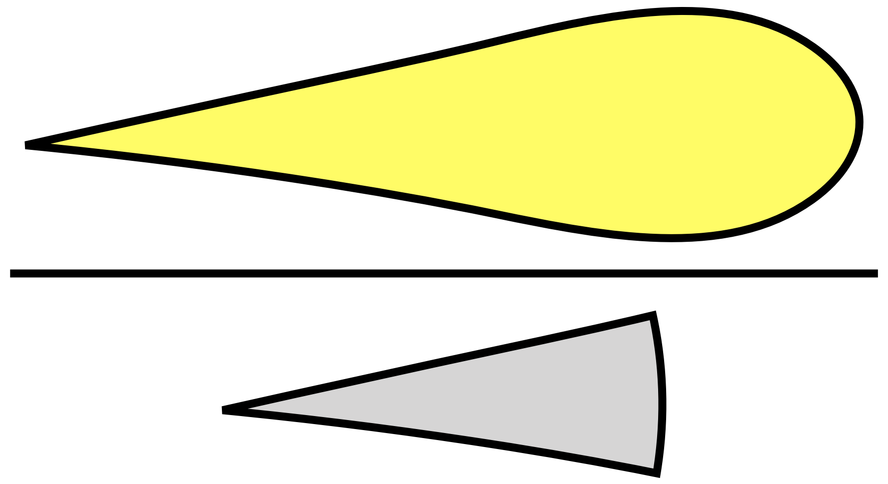
Patch array antenna = 18 (dBi)

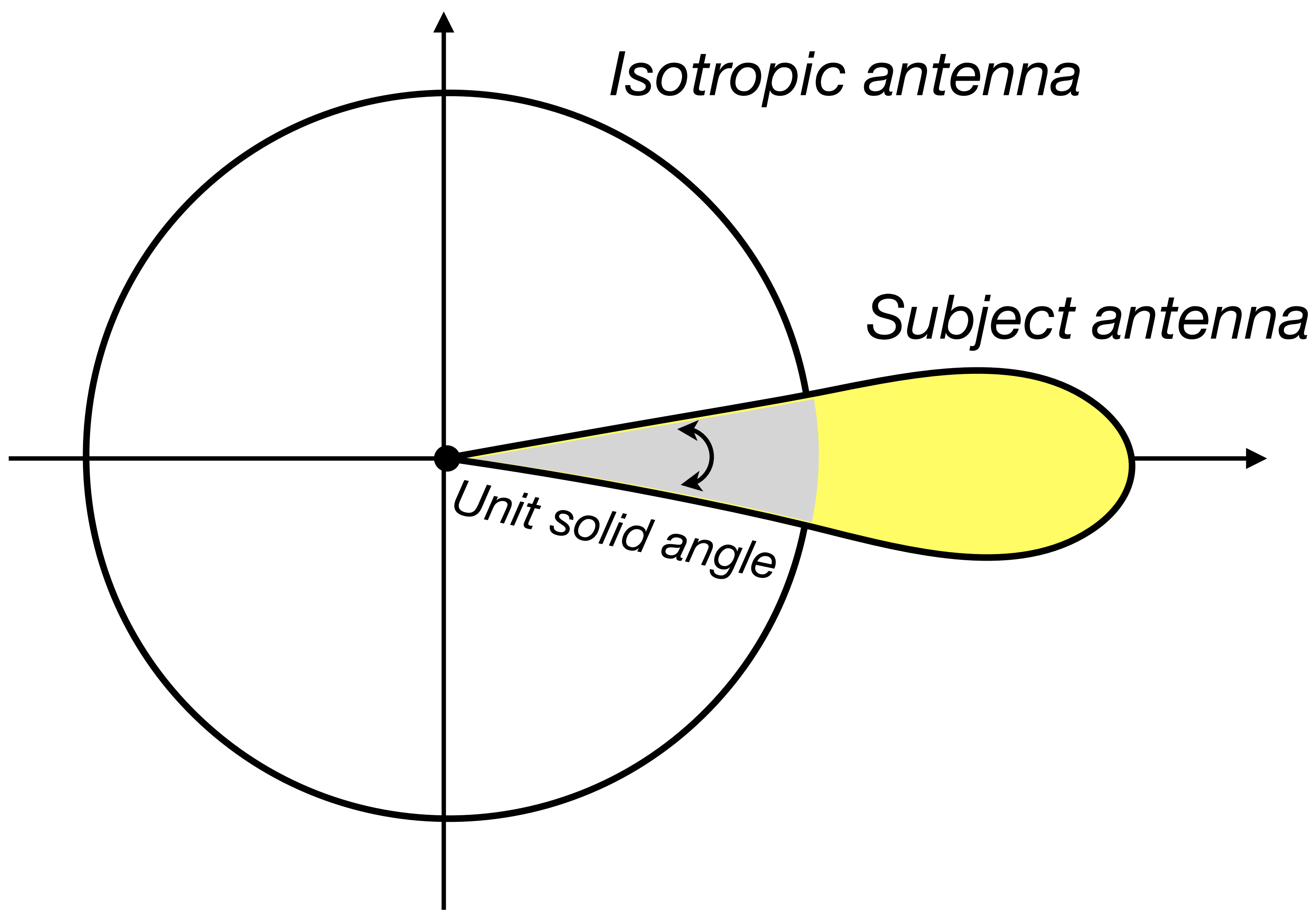
Img src: Cisco

Chap. 11 | Additional characteristic parameters of Antenna

- Power gain (G_p)

$$G_P \triangleq \frac{\text{Max radiation intensity in a particular direction by the subject antenna}}{\text{Radiation intensity in that direction by the isotropic antenna}} = \frac{U_{\max}}{U_{iso}}$$

$=$  (Both antennas excited by ***same power source***)



- Total power generated by the source

$P_i = P_r + P_l$ where P_r is total radiation power and P_l is total ohmic loss

- For isotropic source, P_i radiates isotropically in all directions, $U_{iso} = \dfrac{P_i}{4\pi}$ $(\mathrm{W/sr})$ $\rightarrow$ $\boxed{G_P = \dfrac{4\pi U_{\max}}{P_i}}$

- Radiation efficiency of Antenna (η_r)

$$\eta_r = \frac{P_r}{P_i} = \frac{P_r}{P_r + P_l} \quad (\%)$$

Since $G_P = \dfrac{4\pi U_{\max}}{P_i}$ and $D = \dfrac{4\pi U_{\max}}{P_r}$ $\boxed{\eta_r \triangleq \dfrac{P_r}{P_i} = \dfrac{G_p}{D}}$

(Power gain) *(Directivity)*

- Radiation resistance (R_r)

- Hypothetical resistance that would dissipate radiation power of antenna if current in resistance = max current along antenna
- A measure of amount of power radiated by antenna
- ***The higher, the better!***

$$\boxed{R_r \triangleq \frac{P_r}{I^2}}$$

Example for radiation resistance

- **Example – Obtain radiation resistance of Hertzian dipole if we assume $P_l = 0$ (no ohmic loss).**

$$R_r \triangleq \frac{P_r}{I^2}$$ where P_r is total radiated power and I is maximum current flowing in dipole

i.e., $i(t) = I\mathrm{Re}[e^{j\omega t}] \rightarrow \max[i(t)] = I$

- Total power generated by the source

$$P_r = \oint U\,d\Omega = \oint R^2 P_{av}\,d\Omega \quad \text{where } P_{av}(\theta,\phi) = \frac{1}{2}\mathrm{Re}\left|\boldsymbol{E}\times\boldsymbol{H}^*\right| = \frac{1}{2}\left|E_\theta\right|\left|H_\phi\right| \text{ and } d\Omega = \sin\theta\, d\theta\, d\phi$$

$$\rightarrow P_r = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi} E_\theta H_\phi^* R^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{I^2 (dl)^2}{32\pi^2}\eta_0\beta^2 \int_0^{2\pi}\int_0^{\pi} \sin^3\theta\, d\theta\, d\phi = \frac{I^2 (dl)^2}{12\pi}\eta_0\beta^2 = \frac{I^2}{2}\left[80\pi^2\left(\frac{dl}{\lambda}\right)^2\right]$$

EM fields by Hertzian dipole

$$\begin{cases} E_\theta = j\dfrac{Idl}{4\pi}\left(\dfrac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta \\ H_\phi = j\dfrac{Idl}{4\pi}\left(\dfrac{e^{-j\beta R}}{R}\right)\beta\sin\theta \end{cases}$$

$$\boxed{\therefore R_r = \frac{P_r}{I^2} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 \quad (\Omega)}$$

★ Hertzian dipole vs. half-wave dipole antennas

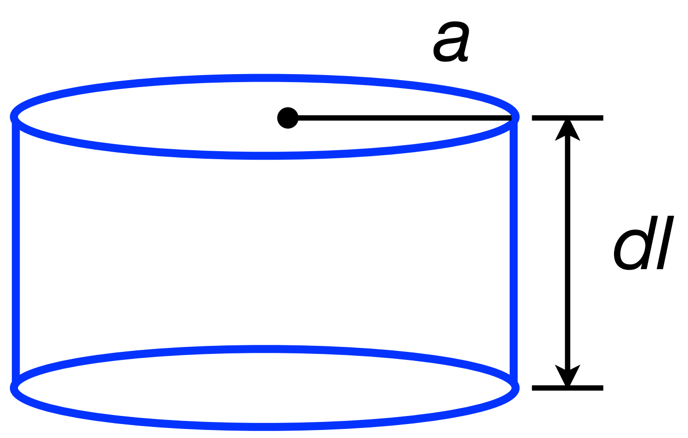
- $dl \ll \lambda \sim 0.01\,\lambda \rightarrow R_r \sim 0.08$ (Ω) for Hertzian dipole
- Input impedance of Hertzian dipole largely capacitive → Hard to match!
- Half-wave dipole ($l = \lambda/2$) → $R_r \sim 73$ (Ω)
- Input impedance of Half-wave dipole purely resistive → Easy to match!

Chap. 11 | Example for radiation efficiency

- Example – Obtain radiation efficiency of Hertzian dipole made of a metal wire of radius *a* and length *d*.

$$P_l = \frac{1}{2} I^2 R_l$$: Total Ohmic loss

$$P_r = \frac{1}{2} I^2 R_r$$: Total radiated power



$$\eta_r = \frac{P_r}{P_r + P_l} = \frac{R_r}{R_r + R_l} = \frac{1}{1 + R_l / R_r}$$ where $R_l = R_s \left(\frac{dl}{2\pi a} \right)$

Here, $R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}}$: Surface resistance

(Effective at high frequency due to skin effect)

$$\eta_r = \frac{P_r}{P_r + P_l} = \frac{R_r}{R_r + R_l} = \frac{1}{1 + R_l / R_r}$$

$$= \frac{1}{1 + \frac{R_s}{160\pi^3} \left(\frac{\lambda}{a} \right) \left(\frac{\lambda}{dl} \right)}$$

For Hertzian dipole under assumption that $\lambda/a << 1$ and $\lambda/dl << 1$

- If f = 1.5 (Mhz), a = 1.8 (mm), dl = 2 (m)

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.5 \times 10^6} = 200 \ (m)$$

$$R_s = \sqrt{\frac{\pi \times (1.5 \times 10^6) \times (4\pi \cdot 10^{-7})}{5.80 \times 10^7}} = 3.20 \times 10^{-4} (\Omega)$$

$$\begin{cases} R_l = \sqrt{\frac{\pi f \mu_0}{\sigma}} = 3.20 \times 10^{-4} \times \left(\frac{2}{2\pi \cdot 1.8 \times 10^{-3}} \right) = 0.057 (\Omega) \\ R_r = 80\pi^2 \left(\frac{dl}{\lambda} \right)^2 = 80\pi^2 \left(\frac{2}{200} \right)^2 = 0.079 (\Omega) \end{cases}$$

$$\boxed{\therefore \eta_r = \frac{R_r}{R_l + R_r} = \frac{0.079}{0.079 + 0.057} = 58\%}$$

c.f.) 95% for half-wave dipole antenna