Foundations and retaining structures – Research and practice

Fondations et structures de soutènement – la recherche et la pratique

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ABSTRACT: This paper presents a broad review of shallow and deep foundations and retaining structures and the most significant methods developed to predict their behaviour. Static and some cyclic loading effects are considered, but dynamic behaviour has been excluded. Emphasis has been placed on methods that have been validated or found to be reliable for use in engineering practice. These include some well-tried and tested methods and others that have been suggested and validated in recent times by geotechnical researchers. Recommendations are made about preferred methods of analysis as well as those whose use should be discontinued. In addition, some observations are made about the future directions that the design of foundations and retaining walls may take, as there are still many areas where uncertainty exists. Some of the latter have been identified.

RÉSUMÉ: Ce papier a pour objet donner une vue générale des types de fondations profondes et peu profondes et des structures de soutènement ainsi que des méthodes les plus significatives développées pour prédire leur comportement. Les effets de charges statiques et cycliques sont considérés mais le comportement dynamique a été exclu. L'accent a été mis sur les méthodes qui ont été validées ou reconnues fiables dans la pratique de l'ingénierie. Ce qui inclus tant quelques méthodes qui ont fait leurs preuves que des méthodes qui ont été suggérées et testées récemment par des chercheurs en géotechnique. Des préconisations sont formulées sur les méthodes d'analyse à privilégier et sur celles qui devraient être abandonnées. De plus, quelques pistes quant aux futures directions que devrait prendre le design des fondations et des structures de soutènement sont proposées, car dans de nombreux domaines des incertitudes subsistent. Parmi ces dernières, certaines sont identifiées.

1 INTRODUCTION

The design of foundations and retaining structures constitutes one of the most enduring and frequent series of problems encountered in geotechnical engineering. Rational design methods based on soil mechanics principles were established over 50 years ago, and the classic book by Terzaghi and Peck (1948) crystallized the broad design techniques of the time, providing practitioners with an invaluable source of knowledge and experience to apply to their problems. Since the publication of that book, an enormous amount of research has been carried out to improve and refine methods of design, and to gain a better understanding of foundation behaviour and the factors which govern this behaviour. Despite this vast volume of research, many practitioners still rely on the traditional methods of design, and are not aware of some of the research developments which have occurred. In some cases, these developments have verified the traditional design methods, but in other cases, some of those methods have been found to be inaccurate or inappropriate. Examples of the latter category of cases are Terzaghi's bearing capacity theory, which tends to over-estimate the capacity of shallow foundations, and the method of settlement analysis of piles suggested by Terzaghi (1943) which focuses, inappropriately, on consolidation rather than shear deformation.

The reluctance to adopt new research into practice is not surprising, as the concerns of the practitioner tend to be rather different to those of the researcher. The concerns of the practitioner include finding answers to the following questions:

- How can I characterize the site most economically?
- How can I carry out the most convenient design?
- How can I estimate the required design parameters?
- How can I optimize the cost versus performance of the foundation or retaining structure?
- How can I ensure that the design can be constructed effectively?

In contrast, questions, which the researcher tries to address, include:

- What are the main features of behaviour of the particular foundation or retaining structure?
- What are the key parameters affecting this behaviour?
- How can I refine the analysis and design method to incorporate these parameters?
- How can I describe the behaviour most accurately?

While there is of course some overlap in some of these questions, there is often an over-riding emphasis on cost and speed of design by the practitioner, which appears to be excessively commercial to the researcher. Conversely, the researcher often tends to focus on detail, which appears to be unimportant to the practitioner. In addition, the practitioner all too rarely can afford the luxury of delving into the voluminous literature that abounds in today's geotechnical world. As a consequence, there appears to be an ever-increasing "gap" between the researcher and the practitioner.

This paper attempts to decrease the gap between research and practice, and has two main objectives:

- To summarize some of the findings of research in foundation and retaining structure engineering over the past 30 years or so;
- To evaluate the applicability of some of the commonly used design approaches in the light of this research.

Because of the broad scope of the subject, some limitation of scope is essential. Thus, attention will be concentrated on design of foundations and retaining structures under static loading conditions. The important issue of design for dynamic loading will not be considered herein, nor will issues related to construction be addressed in detail. The following subjects will be dealt with:

- Design philosophy and design criteria;
- Bearing capacity of shallow foundations;
- Settlement of shallow foundations;
- Pile foundations;

– Raft and piled raft foundations;
– Retaining structures, with an emphasis on the assessment of earth pressures and the design of flexible structures;
– Assessment of geotechnical parameters.

At the dawn of a new millennium, it seems appropriate to attempt to make an assessment of those traditional design methods that should be discarded, those that should be modified, and those that should be retained. The conclusions will therefore summarize some methods in these three categories. In addition, the conclusions will propose a number of topics that deserve further research, and conversely, some which may be considered to be too mature for further extensive investigation. Clearly, such suggestions represent the subjective opinions of the authors and may be subject to challenge by others.

2 DESIGN APPROACHES AND DESIGN CRITERIA

2.1 *Design philosophy for design against failure*

When designing against failure, geotechnical engineers generally adopt one of the following procedures:

1. The overall factor of safety approach
2. The load and resistance factor design approach (LRFD)
3. The partial safety factor approach
4. A probabilistic approach.

Each of these procedures is discussed briefly below.

2.1.1 *Overall factor of safety approach*

It was customary for most of the 20th century for designers to adopt an overall factor of safety approach when designing against failure. The design criterion when using this approach can be described as follows:

$$R_u / F \geq \Sigma P_i \tag{2.1}$$

where P_i = applied loading; R_u = ultimate load capacity or strength; F = overall factor of safety.

Factors of safety were usually based on experience and precedent, although some attempts were made in the latter part of the century to relate safety factors to statistical parameters of the ground and the foundation type. Typical values of F for shallow foundations range between 2.5 and 4, while for pile foundations, values between 2 and 3.5 have been used. Figure 2.1 shows typical values for a variety of geotechnical situations (Meyerhof, 1995a).

2.1.2 *LRFD approach*

In recent years, there has been a move towards a limit state design approach. Such an approach is not new, having been proposed by Brinch Hansen (1961) and Simpson *et al.* (1981), among others. Pressure from structural engineers has hastened the application of limit state design to geotechnical problems.

One approach within the limit state design category is the LRFD approach, which can be represented by the following design criterion:

$$\Phi R_u \geq \Sigma a_i P_i \tag{2.2}$$

where Φ = strength reduction factor; R_u = ultimate load capacity or strength; P_i = applied loading component i (e.g., dead load, live load, wind load, etc.); a_i = load factor applied to the load component P_i.

Values of a_i are usually specified in codes or standards, while values of Φ are also often specified in such documents. The LRFD approach is sometimes referred to as the "American Approach" to limit state design, because of its increasing popularity in North America.

2.1.3 *Partial factor of safety approach*

In this approach, the design criterion for stability is:

$$R' \geq \Sigma a_i P_i \tag{2.3}$$

where R' = design resistance, calculated using the design strength parameters obtained by reducing the characteristic strength values of the soil with partial factors of safety; a_i, P_i are as defined above.

The partial factor of safety approach is sometimes referred to as the "European Approach" because of the considerable extent of its application in parts of that continent.

2.1.4 *Probabilistic approach*

In this approach, the design criterion can be stated simply as:

$$\textit{Probability of failure} \leq \textit{Acceptable probability} \tag{2.4}$$

Typical values of the acceptable probability of failure are shown in Figure 2.2 for various classes of engineering projects (Whitman, 1984).

Much has been written about the application of probability theory to geotechnical engineering, but despite enthusiastic support for this approach from some quarters, it does not appear to have been embraced quantitatively by most design engineers. Exceptions are within geotechnical earthquake engineering, environmental geotechnics and in some facets of offshore geotechnics, but it is rarely applied in the design of foundations or retaining structures. An excellent discussion of the use of probabilistic methodologies is given by Whitman (2000).

2.1.5 *Discussion of approaches*

While a considerable proportion of design practice is still carried out using the overall factor of safety approach, there is an increasing trend towards the application of limit state design methods. Becker (1996) has explored fully the issues involved in the alternative approaches, and provides a useful comparison of the LRFD and partial safety factor approaches which is shown in Figure 2.3.

Considerable debate has taken place recently in relation to the partial factor (European) approach, and a number of reservations have been expressed about it (Gudehus, 1998). Particular problems can be encountered when it is applied to problems involving soil-structure interaction, and the results of analyses in which reduced strengths do not always lead to the worst cases for design. For example, in the design of a piled raft, if the pile capacity is reduced (as is customary), the negative bending moment within the raft may be underestimated when the pile is not located under a column. Thus, in many cases, it is preferable to adopt the LRFD approach, and compute the design values using the best-estimate geotechnical parameters, after which an appropriate factor can be applied to the computed results.

2.2 *Design loadings and combinations*

Conventional foundation design is usually focussed on static vertical loading, and most of the existing design criteria address foundation response to vertical loads. It is however important to recognize that consideration may need to be given to lateral and moment loadings, and that in some cases, cyclic (repeated) loadings and dynamic loadings may be important. In this paper, the primary focus will be placed on static vertical loads, but some cases involving horizontal static loading and cyclic loading will also be addressed.

Load combinations which need to be considered in design are usually specified in structural loading codes. Typical load combinations are shown in Table 2.1 for both ultimate and serviceability load conditions (Standards Australia, 1993). Other combinations are also specified, including liquid and earth pressure loadings.

Table 2.1. Typical load factors for load combinations (Standards Australia, AS 1170-1993).

Case	Combinations for ultimate limit state	Combinations for serviceability limit state	
		Short Term	Long Term
Dead + live	1.25G + 1.5Q 0.8G + 1.5Q	G + 0.7Q	G + 0.4Q
Dead + live + wind	1.25G + Wu + 0.4Q 0.8G + Wu	G + Ws G +0.7Q+Ws	-
Dead + live + earthquake	1.25G + 1.6E + 0.4Q 0.8G + 1.6E	-	-

Note: G = dead load; Q = live load: Wu = ultimate wind load: Ws = serviceability wind load: E = earthquake load.

2.3 *Design criteria*

Criteria for foundation design typically rely on past experience and field data with respect to both ultimate limit state (stability) design and serviceability design. Some of these criteria are summarized in this section.

2.3.1 *Ultimate limit state design*

Typical values of overall factor of safety for various types of failure are summarized in Table 2.2 (Meyerhof, 1995a). Meyerhof has also gathered data on the factor of safety in the context of the probability of stability failure and of the coefficients of variation of the loads and soil resistance, and these data are shown in Figure 2.1.

Values of partial factors of safety for soil strength parameters for a range of circumstances are summarized in Table 2.3. As indicated above, the use of factored soil strength data can sometimes lead to designs, which are different from those using conventional design criteria, and must be used with caution.

Table 2.4 summarizes typical geotechnical reduction factors (Φ_g) for foundations. These values are applied in the LRFD design approach to the computed ultimate load capacity, to obtain the design load capacity (strength) of the foundation, as per Equation (2.2). The assessment of an appropriate value for design requires the application of engineering judgement, including the level of confidence in the ground information, the soil data and the method of calculation or load test interpretation employed.

Table 2.2 Typical overall factors of safety (Meyerhof, 1995a)

Failure type	Item	Factor of safety
Shearing	Earthworks	1.3 – 1.5
	Earth retaining structures, excavations, offshore foundations	1.5 – 2
	Foundations on land	2 – 3
Seepage	Uplift, heave	1.5 – 2
	Exit gradient, piping	2 – 3
Ultimate pile loads	Load tests	1.5 – 2
	Dynamic formulae	3

Table 2.3 Typical values of partial factors of safety for soil strength parameters (after Meyerhof, 1995a).

Item	Brinch Hansen 1953	Brinch Hansen 1956	Denmark DS165 1965	Euro-code 7 1993	Canada CFEM 1992	Canada NBCC 1995	USA ANSI A58 1980
Friction ($\tan \phi$)	1.25	1.2	1.25	1.25	1.25	See Note 1	See Note 2
Cohesion c (slopes, earth pressures)	1.5	1.5	1.5	1.4-1.6	1.5	"	"
Cohesion c (Spread foundations)	-	1.7	1.75	1.4-1.6	2.0	"	"
Cohesion c (Piles)	-	2.0	2.0	1.4-1.6	2.0	"	"

Note 1: Resistance factor of 1.25-2.0 on ultimate resistance using unfactored strengths.
Note 2: Resistance factor of 1.2-1.5 on ultimate resistance using unfactored strengths.

Table 2.4. Typical values of geotechnical reduction factor Φ_g

Item	Brinch Hansen (1965)	Denmark DS415 (1965)	Euro-code 7 (1993)	Canada CFEM (1992)	Canada NBCC (1995)	Australian Piling Code (1995)
Ultimate Pile Resistance – load tests	0.62	0.62	0.42 - 0.59	0.5 – 0.62	0.62	0.5 – 0.9 *
Ultimate Pile Resistance – dynamic formulae	0.5	0.5	-	0.5	0.5	0.45 – 0.65*
Ultimate pile resistance – penetration tests	-	-	-	0.33-0.5	0.4	0.40 – 0.65*

*Value depends on assessment of circumstances, including level of knowledge of ground conditions, level of construction control, method of calculation, and method of test interpretation (for dynamic load tests).

2.3.2 *Serviceability design*

The general criteria for serviceability design are:

$$\textit{Deformation} \le \textit{Allowable deformation} \tag{2.5a}$$

$$\textit{Differential deformation} \le \textit{Allowable differential deformation} \tag{2.5b}$$

These criteria are usually applied to settlements and differential settlements, but are also applicable to lateral movements and rotations. The following discussion will however relate primarily to vertical settlements.

The following aspects of settlement and differential settlement need to be considered, as illustrated in Figure 2.4:

- Overall settlement;
- Tilt, both local and overall;
- Angular distortion (or relative rotation) between two points, which is the ratio of the difference in settlement divided by the distance between the two points;
- Relative deflection (for walls and panels).

Data on allowable values of the above quantities have been collected by a number of sources, including Meyerhof (1947), Skempton and MacDonald (1955), Polshin and Tokar (1957), Bjerrum (1963), Grant *et al.* (1974), Burland and Wroth (1974), Burland *et al.* (1977), Wahls (1994), Boscardin and Cording (1989), Barker *et al.* (1991) and Boone (1996). Some of the recommendations distilled from this information are summarized in Table 2.5. Information on criteria for bridges is also included in this table as the assessment of such aspects as ride quality and function requires estimates to be made of the deformations and settlements.

Boone (1996) has pointed out that the use of a single criterion, such as angular distortion, to assess building damage excludes many important factors. A more rational approach requires consideration of the following factors:

- Flexural and shear stiffness of building sections
- Nature of the ground movement profile
- Location of the structure within the settlement profile
- Degree of slip between the foundation and the ground
- Building configuration.

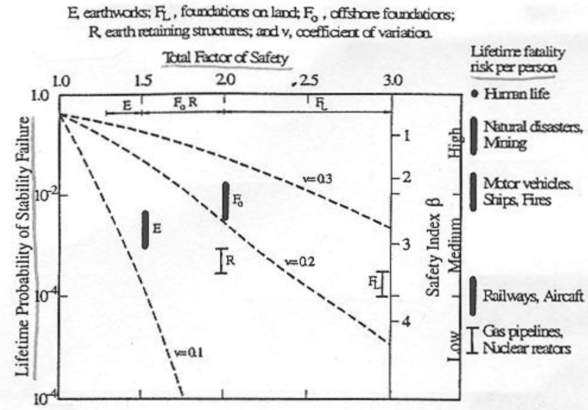


Figure 2.1. Lifetime probability of failures for overall factor of safety approach (Meyerhof, 1995)

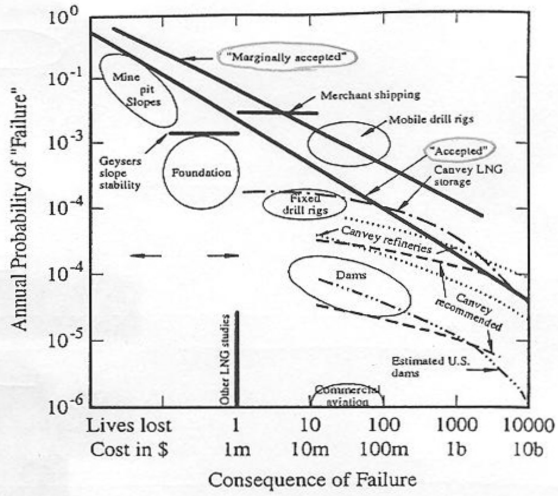
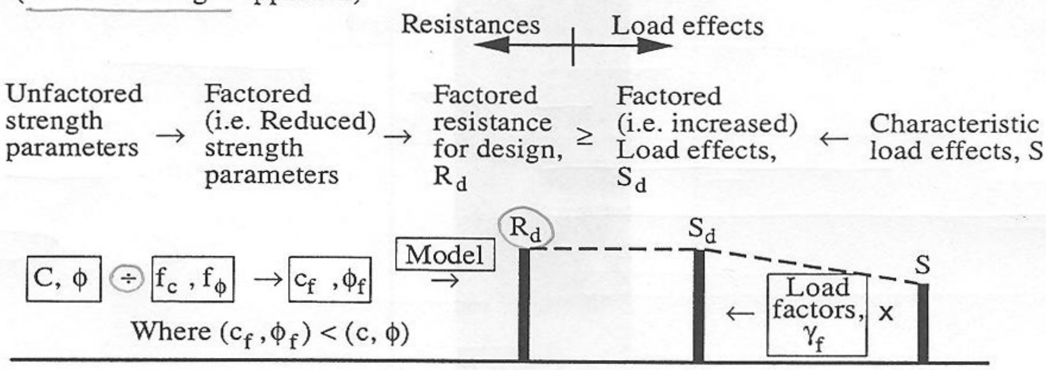


Figure 2.2. Risks for engineering projects (Whitman, 1984).

* European approach:
(factored strength approach)



* North American approach:
(factored resistance approach)

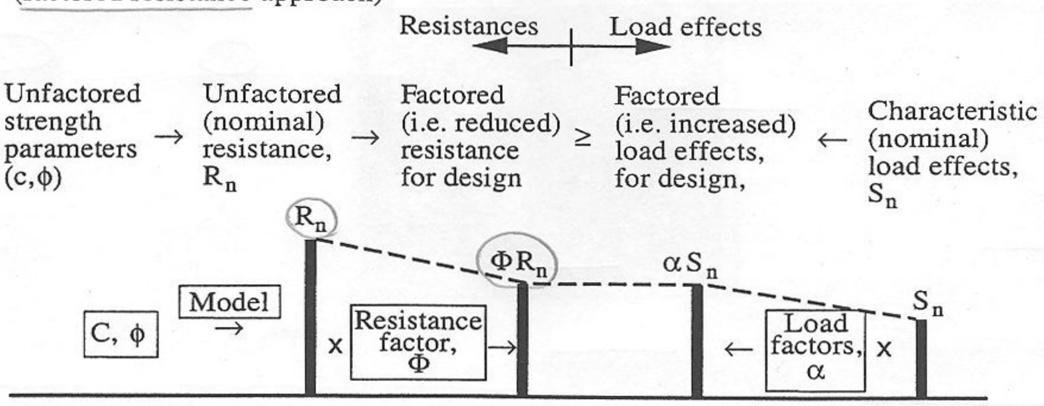


Figure 2.3. LRFD (North American) approach vs partial factor (European) approach (Becker, 1996).

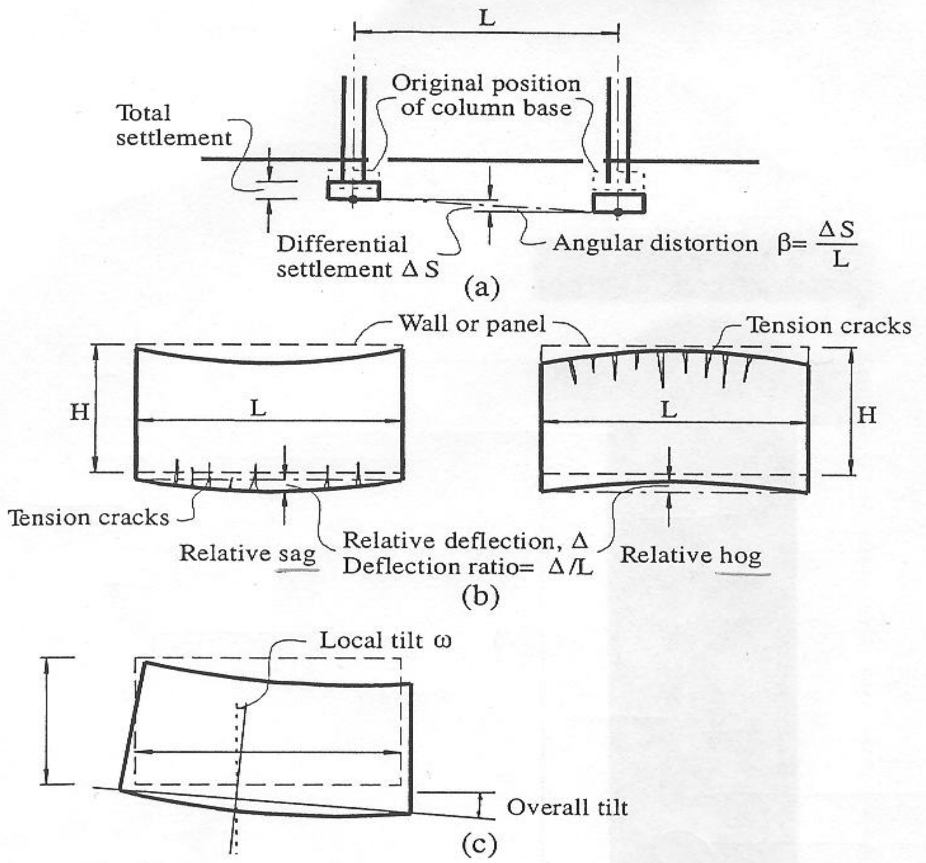


Figure 2.4. Definitions of differential settlement and distortion for framed and load-bearing wall structures (after Burland and Wroth, 1974).

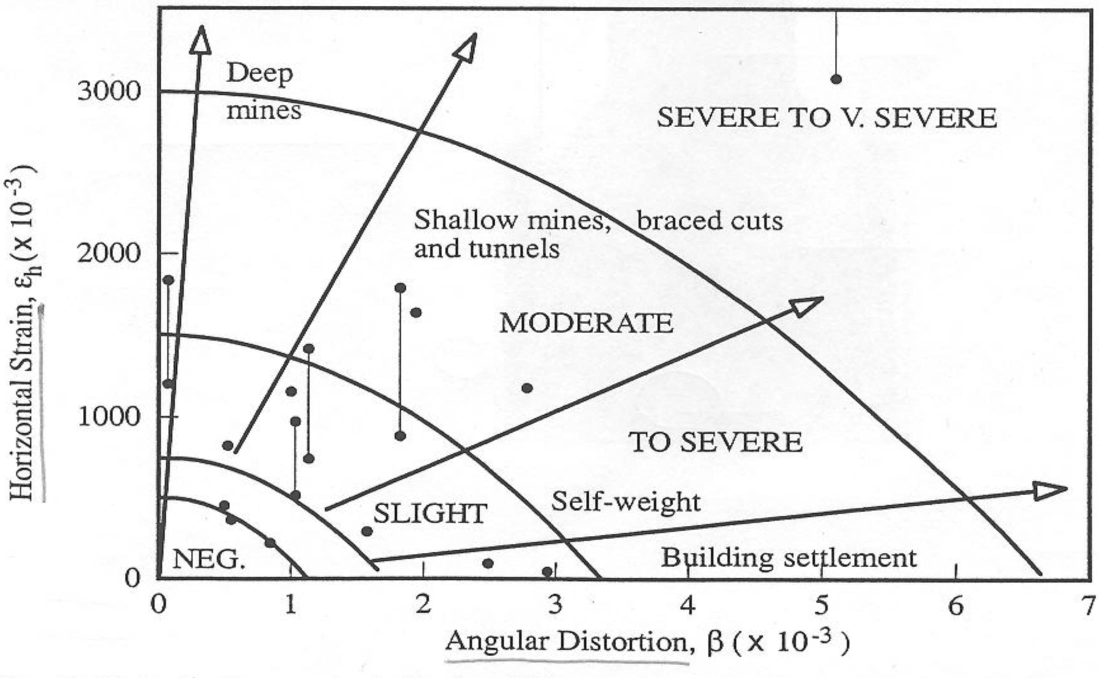


Figure 2.5. Relationship of damage to angular distortion and horizontal extension strain (after Boscardin and Cording, 1989).

Table 2.5. Summary of criteria for settlement and differential settlement of structures.

Type of structure	Type of damage/concern	Criterion	Limiting value(s)
Framed buildings and reinforced load bearing walls	Structural damage	Angular distortion	1/150 – 1/250
	Cracking in walls and partitions	Angular distortion	1/500 (1/1000-1/1400) for end bays
	Visual appearance	Tilt	1/300
	Connection to services	Total settlement	50 – 75 mm (sands) 75 – 135 mm (clays)
Tall buildings	Operation of lifts & elevators	Tilt after lift installation	1/1200 – 1/2000
Structures with unreinforced load bearing walls	Cracking by sagging	Deflection ratio	1/2500 (L/H = 1) 1/1250 (L/H = 5)
	Cracking by hogging	Deflection ratio	1/5000 (L/H = 1) 1/2500 (L/H = 5)
Bridges – general	Ride quality	Total settlement	100 mm
	Structural distress	Total settlement	63 mm
	Function	Horizontal movement	38 mm
Bridges – multiple span	Structural damage	Angular distortion	1/250
Bridges – single span	Structural damage	Angular distortion	1/200

The importance of the horizontal strain in initiating damage was pointed out by Boscardin and Cording (1989), and Figure 2.5 shows the relationship they derived relating the degree of damage to both angular distortion and horizontal strain. Clearly, the larger the horizontal strain, the less is the tolerable angular distortion before some form of damage occurs. Such considerations may be of particular importance when assessing potential damage arising from tunnelling operations. For bridges, Barker *et al.* (1991) also note that settlements were more damaging when accompanied by horizontal movements.

It must also be emphasized that, when applying the criteria in Table 2.5, consideration be given to the settlements which may have already taken place prior to the construction or installation of the affected item. For example, if the concern is related to architectural finishes, then assessment is required only of the settlements and differential settlements which are likely to occur after the finishes are in place.

More detailed information on the severity of cracking damage for buildings is given by Day (2000), who has collected data from a number of sources, including Burland *et al.* (1977), and Boone (1996). Day has also collected data on the relationship between the absolute value of differential settlement Δ_{max} and the angular distortion ($\Delta s/L$) to cause cracking, and has concluded that the following relationship, first suggested by Skempton and MacDonald (1956), is reasonable:

$$\Delta_{max} \approx 8900(\Delta s / L)(mm) \tag{2.5c}$$

2.4 *Categories of analysis and design methods*

In assessing the relative merits of analysis and design methods, it is useful to categorize the methods in some way. It has been proposed previously (Poulos, 1989) that methods of analysis and design can be classified into three broad categories, as shown in Table 2.6.

Category 1 procedures probably account for a large proportion of the foundation design performed throughout the world. Category 2 procedures have a proper theoretical basis, but they generally involve significant simplifications, especially with respect to soil behaviour. The majority of available design charts fall into one or other of the Category 2 methods. Category 3 procedures generally involve the use of a site-specific analysis based on relatively advanced numerical or analytical techniques, and require the use of a computer. Many of the Category 2 design charts have been developed from Category 3 analyses, and are then condensed into a simplified form. The most advanced Category 3 methods (3C) have been used relatively sparsely, but increasing research effort is being made to develop such methods, in conjunction with the development of more sophisticated models of soil behaviour.

From a practical viewpoint, Category 1 and 2 methods are the most commonly used. In the following sections, attention will be focussed on evaluating such methods with respect to more refined and encompassing methods, many of which fall into Category 3, or else have been derived from Category 3 analyses.

2.5 *Analysis tools*

Hand calculations and design charts probably still form the backbone of much standard design practice in geotechnical engineering today. However, the designer has available a formidable array of computational tools. Many of the calculations in Categories 1 and 2, which previously required laborious evaluation, can now be carried out effectively, rapidly and accurately with computer spreadsheets and also with mathematical programs such as MATHCAD, MATLAB and MATHEMATICA. The ability of these tools to provide instant graphical output of results is an invaluable aid to the designer.

The development of powerful numerical analyses such as finite element and finite difference analyses now provide the means for carrying out more detailed Category 3 analyses, and of using more realistic models of soil behaviour. In principle, there is virtually no problem that cannot be handled numerically, given adequate time, budget and information on loadings, *in situ* conditions and soil characteristics. Yet, the same limitations that engineers of previous generations faced, still remain. Time is always an enemy in geotechnical engineering practice, and money all too often is limited. Loadings are almost always uncertain, and the difficulties of adequate site characterization are ever-present. Despite substantial research into soil behaviour, mysteries persist in relation to the stress-strain characteristics of soil response to general loading conditions, and the quantitative description of this behaviour. The two-phase behaviour of saturated soils (not to mention the three-phase behaviour of unsaturated soils), also pose a formidable challenge to those who seek to rely solely on high-level numerical analyses for their designs. It must also be recognized that the potential for obtaining irrelevant answers when using complex numerical methods is very great, especially when the user of such methods is relatively inexperienced.

For these reasons, while recognizing the immense contribution of numerical geomechanics to our understanding of the behaviour of foundations and retaining structures, attention will be focussed in this paper on more conventional methods of analysis and design. Such methods are an indispensable part of engineering practice, and are essential in providing a check on the results of more complex numerical analyses whenever the latter are employed.

Table 2.6. Categories of analysis and design.

Category	Subdivision	Characteristics	Method of parameter estimation
1	-	Empirical – not based on soil mechanics principles	Simple in-situ or laboratory tests, with correlations
2	2A	Based on simplified theory or charts – uses soil mechanics principles – amenable to hand calculation; simple linear elastic or rigid plastic soil models	Routine relevant in-situ or laboratory tests – may require some correlations
	2B	As for 2A, but theory is non-linear (deformation) or elasto-plastic (stability)	
3	3A	Based on theory using site-specific analysis, uses soil mechanics principles. Theory is linear elastic (deformation) or rigid plastic (stability)	Careful laboratory and/or in-situ tests which follow the appropriate stress paths
	3B	As for 3A, but non-linearity is allowed for in a relatively simple manner	
	3C	As for 3A, but non-linearity is allowed for via proper constitutive soil models	

3 BEARING CAPACITY OF SHALLOW FOUNDATIONS

3.1 *Design issues*

In relation to shallow foundations, the key design issues include:

* – Estimation of the ultimate bearing capacity of the foundation with, where relevant, appropriate allowance for the combined effects of vertical, horizontal and moment loading;

* – Estimation of the total and differential settlements under vertical and combined loading, including any time-dependence of these foundation movements;

– Estimation of foundation movements due to moisture changes in the underlying soil, where these changes are induced by factors other than the loading applied directly to the foundation;

– Structural design of the foundation elements.

In this section, the first two of these design issues will be addressed, while section 4 deals with settlement issues.

Conventionally, the issues of ultimate capacity and settlement are treated separately in design analyses. For most hand calculation methods this separation is necessary, because to do otherwise would render the analysis intractable. However, in some design applications it may be important to conduct more sophisticated analysis in order to understand fully the characteristic foundation behaviour. Very often these sophisticated analyses will employ numerical techniques requiring computer solution. In this section hand methods of analysis are discussed, and some useful solutions derived from more sophisticated analysis are also identified.

3.2 *Ultimate load capacity*

Prediction of the ultimate bearing capacity of a foundation is one of the most significant problems in foundation engineering, and consequently there is an extensive literature detailing both theoretical and experimental studies of this topic. A list of the principal contributions to the study of this subject may be found, for example, in Vesic (1973), Chen and McCarron (1991) and Tani and Craig (1995).

Bearing capacity failure occurs as the soil supporting the foundation fails in shear. This may involve either a general failure mechanism or punching shear failure. General shear failure usually develops in soils that exhibit brittle stress-strain behaviour and in this case the failure of the foundation may be sudden and catastrophic. Punching shear failure normally develops in soils that exhibit compressible, plastic stress-strain behaviour. In this case, failure is characterised by progressive, downward movement or "punching" of the foundation into the underlying soil. This failure mode is also the mechanism normally associated with deep foundations such as piles and drilled shafts.

Different methods of analysis are used for the different failure modes. For the general shear mode, a rational approach based on the limiting states of equilibrium is employed. The approach is based on the theory of plasticity and its use has been validated, at least in principle, by laboratory and field testing. For the punching shear mode, a variety of approaches have been suggested, none of which is strictly correct from the point of view of rigorous applied mechanics, although most methods predict ultimate capacities which are at least comparable to field test results.

In the discussion that follows, particular emphasis is given to:

– Estimating the ultimate capacity of foundations subjected to combined loading, i.e., combinations of vertical and horizontal forces and moments,

– Estimating the ultimate capacity for cases of eccentrically applied forces, and

– Estimating the ultimate capacity of foundations on non-homogeneous soils including layered deposits.

3.2.1 *Conventional bearing capacity theory*

A rational approach for predicting the bearing capacity of a foundation suggested by Vesic (1975) has now gained quite widespread acceptance in foundation engineering practice. This method takes some account of the stress-deformation characteristics of the soil and is applicable over a wide range of soil behaviour. This approach is loosely based on the solutions obtained from the theory of plasticity, but empiricism has been included in significant measure, to deal with the many complicating factors that make a rigorous solution for the capacity intractable.

For a rectangular foundation the general bearing capacity equation, which is an extension of the expression first proposed by Terzaghi (1943) for the case of a central vertical load applied to a long strip footing, is usually written in the form:

$$q_u = \frac{Q_u}{BL} = cN_c\zeta_{cr}\zeta_{cs}\zeta_{ci}\zeta_{ct}\zeta_{cg}\zeta_{cd} + \frac{1}{2}B\gamma N_\gamma\zeta_{\gamma r}\zeta_{\gamma s}\zeta_{\gamma i}\zeta_{\gamma t}\zeta_{\gamma g}\zeta_{\gamma d} + qN_q\zeta_{qr}\zeta_{qs}\zeta_{qi}\zeta_{qt}\zeta_{qg}\zeta_{qd} \quad (3.1)$$

where q_u is the ultimate bearing pressure that the soil can sustain, Q_u is the corresponding ultimate load that the foundation can support, B is the least plan dimension of the footing, L is the length of the footing, c is the cohesion of the soil, q is the overburden pressure, and γ is the unit weight of the soil. It is assumed that the strength of the soil can be characterised by a cohesion c and an angle of friction ϕ. The parameters N_c, N_γ and N_q are known as the general bearing capacity factors which determine the capacity of a long strip footing acting on the surface of soil represented as a homogeneous half-space. The factors ζ allow for the influence of other complicating features. Each of these factors has double subscripts to indicate the term to which it applies (c, γ or q) and which phenomenon it describes (r for rigidity of the soil, s for the shape of the foundation, i for inclination of the load, t for tilt of the foundation base, g for the ground surface inclination and d for the depth of the foundation). Most of these factors depend on the friction angle of the soil, ϕ, as indicated in Table 3.1. Details of the sources and derivations for them may be found in Vesic (1975), Caquot and Kerisel (1948, 1953), Davis and Booker (1971) and Kulhawy *et al.* (1984). The unusual case of foundations subjected to a combination of a concentric vertical load and a torsional moment has also been studied by Perau (1997).

* Table 3.1. Bearing capacity factors.

Parameter	Cohesion	Self-weight	Surcharge
Bearing Capacity	$N_c=(N_q-1)\cot\phi$ $N_c=2+\pi \quad if\ \phi=0$	$N_\gamma\approx 0.0663e^{9.3\phi}$ Smooth $N_\gamma\approx 0.1054e^{9.6\phi}$ Rough $\phi>0$ in radians $N_\gamma=0 \quad if\ \phi=0$	$N_q=e^{\pi\tan\phi}\tan^2\left(45^o+\frac{\phi}{2}\right)$
Rigidity[1,2]	$\zeta_{cr}=\zeta_{qr}-\left(\frac{1-\zeta_{qr}}{N_c\tan\phi}\right)$ or for $\phi=0$ $\zeta_{cr}=0.32+0.12\left(\frac{B}{L}\right)+0.60\log_{10}I_r$	$\zeta_{\gamma r}=\zeta_{qr}$	$\zeta_{qr}=\exp\left(\left(-4.4+0.6\frac{B}{L}\right)\tan\phi+\left[\frac{3.07\sin\phi\log_{10}2I_r}{(1+\sin\phi)}\right]\right)$
Shape	$\zeta_{cs}=1+\left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$	$\zeta_{\gamma s}=1-0.4\left(\frac{B}{L}\right)$	$\zeta_{qs}=1+\left(\frac{B}{L}\right)\tan\phi$
Inclination[3]	$\zeta_{ci}=\zeta_{qi}-\left(\frac{1-\zeta_{qi}}{N_c\tan\phi}\right)$ or for $\phi=0$ $\zeta_{ci}=1-\left(\frac{nT}{cN_cB'L'}\right)$	$\zeta_{\gamma i}=\left[1-\frac{T}{N+B'L'c\cot\phi}\right]^{n+1}$	$\zeta_{qi}=\left[1-\frac{T}{N+B'L'c\cot\phi}\right]^{n}$
Foundation tilt[4]	$\zeta_{ct}=\zeta_{qt}-\left(\frac{1-\zeta_{qt}}{N_c\tan\phi}\right)$ or for $\phi=0$ $\zeta_{ct}=1-\left(\frac{2\alpha}{\pi+2}\right)$	$\zeta_{\gamma t}=(1-\alpha\tan\phi)^2$	$\zeta_{qt}\approx\zeta_{\gamma t}$
Surface inclination[5]	$\zeta_{cg}=\zeta_{qi}-\left(\frac{1-\zeta_{qi}}{N_c\tan\phi}\right)$ or for $\phi=0$ $\zeta_{cg}=1-\left(\frac{2\omega}{\pi+2}\right)$	$\zeta_{\gamma g}\approx\zeta_{qg}$ or for $\phi=0$ $\zeta_{\gamma g}=1$	$\zeta_{qg}=(1-\tan\omega)^2$ or for $\phi=0$ $\zeta_{qg}=1$
Depth[6]	$\zeta_{cd}=\zeta_{qd}-\left(\frac{1-\zeta_{qd}}{N_c\tan\phi}\right)$ or for $\phi=0$ $\zeta_{cd}=1+0.33\tan^{-1}\left(\frac{D}{B}\right)$	$\zeta_{\gamma d}=1$	$\zeta_{qd}=1+2\tan\phi(1-\sin\phi)^2\tan^{-1}\left(\frac{D}{B}\right)$

1. The rigidity index is defined as $I_r=G/(c+q\tan\phi)$ in which G is the elastic shear modulus of the soil and the vertical overburden pressure, q, is evaluated at a depth of $B/2$ below the foundation level. The critical rigidity index is defined as:

$$I_{rc}=\frac{1}{2}\exp\left[(3.30-0.45B/L)\cot\left(45^o-\frac{\phi}{2}\right)\right]$$

2. When $I_r>I_{rc}$, the soil behaves, for all practical purposes, as a rigid plastic material and the modifying factors ζ_r all take the value 1. When $I_r<I_{rc}$, punching shear is likely to occur and the factors ζ_r may be computed from the expressions in the table.
3. For inclined loading in the B direction ($\theta=90°$), n is given by: $n=n_B=(2+B/L)/(1+B/L)$. For inclined loading in the L direction ($\theta=0°$), n is given by:

$$n=n_L=(2+L/B)/(1+L/B)$$

For other loading directions, n is given by: $n=n\theta=nL\cos^2\theta+nB\sin^2\theta$. θ is the plan angle between the longer axis of the footing and the ray from its centre to the point of application of the loading. B' and L' are the effective dimensions of the rectangular foundation, allowing for eccentricity of the loading, and T and N are the horizontal and vertical components of the foundation load.
4. α is the inclination from the horizontal of the underside of the footing.
5. For the sloping ground case where $\phi=0$, a non-zero value of the term N_γ must be used. For this case is N_γ negative and is given by:

$$N_\gamma=-2\sin\omega$$

ω is the inclination below horizontal of the ground surface away from the edge of the footing.
6. D is the depth from the soil surface to the underside of the footing.

In Table 3.1 closed-form expressions have been presented for the bearing capacity factors. As noted above, some are only approximations. In particular, there have been several different solutions proposed in the literature for the bearing capacity factors N_γ and N_q. Solutions by Prandtl (1921) and Reissner (1924) are generally adopted for N_c, and N_q, although Davis and Booker (1971) produced rigorous plasticity solutions which indicate that the commonly adopted expression for N_q (Table 3.1) is slightly non-conservative, although it is generally accurate enough for most practical applications. However, significant discrepancies have been noted in the values proposed for N_γ. It has not been possible to obtain a rigorous closed form expression for N_γ, but several authors have proposed approximations. For example, Terzaghi (1943) proposed a set of approximate values and Vesic (1975) suggested the approximation, $N_\gamma \approx 2\ (N_q + 1)\tan\phi$, which has been widely used in geotechnical practice, but is now known to be non-conservative with respect to more rigorous solutions obtained using the theory of plasticity for a rigid plastic body (Davis and Booker, 1971). An indication of the degree of non-conservatism that applies to these approximate solutions is given in Figure 3.1, where the rigorous solutions of Davis and Booker are compared with the traditional values suggested by Terzaghi. It can be seen that for values of friction angle in the typical range from 30° to 40°, Terzaghi's solutions can oversitimate this component of the bearing capacity by factors as large as 3.

Analytical approximations to the Davis and Booker solutions for N_γ for both smooth and rough footings are presented in Table 3.1. These expressions are accurate for values of ϕ greater than about 10°, a range of considerable practical interest. It is recommended that the expressions derived by Davis and Booker, or their analytical approximations presented in Table 3.1, should be used in practice and the continued use of other inaccurate and non-conservative solutions should be discontinued.

Although for engineering purposes satisfactory estimates of load capacity can usually be achieved using Equation (3.1) and the factors provided in Table 3.1, this quasi-empirical expression can be considered at best only an approximation. For example, it assumes that the effects of soil cohesion, surcharge pressure and self-weight are directly superposable, whereas soil behaviour is highly non-linear and thus superposition does not necessarily hold, certainly as the limiting condition of foundation failure is approached.

Recent research into bearing capacity problems has advanced our understanding of the limitations of Equation (3.1). In particular, problems involving non-homogeneous and layered soils, and cases where the foundation is subjected to combined forms of loading have been investigated in recent years, and more rigorous solutions for these cases are now available. Some of these developments are discussed in the following sections.

3.2.2 *Bearing capacity under combined loading*

The bearing capacity Equation (3.1) was derived using approximate empirical methods, with the effect of load inclination incorporated by the addition of (approximate) inclination factors. The problem of the bearing capacity of a foundation under combined loading is essentially three-dimensional in nature, and recent research (e.g., Murff, 1994; Martin, 1994; Bransby and Randolph, 1998; Taiebat and Carter, 2000a, 2000b) has suggested that for any foundation, there is a surface in load space, independent of load path, containing all combinations of loads, i.e., vertical force (V), horizontal force (H) and moment (M), that cause failure of the foundation. This surface defines a failure envelope for the foundation. A summary of recent developments in defining this failure envelope is presented in this section.

Most research conducted to date into determining the shape of the failure envelope has concentrated on undrained failure within the soil (i.e. $\phi = 0$ and $c = s_u =$ the undrained shear strength) and the results are therefore relevant to cases of relatively rapid loading of fine-grained soils, including clays. For these cases several different failure envelopes have been suggested and in all cases they can be written in the following form:

$$f\left(\frac{V}{As_u}, \frac{H}{As_u}, \frac{M}{ABs_u}\right) = 0 \tag{3.2}$$

where A is the plan area of the foundation, B is its width or diameter, and s_u is the undrained shear strength of the soil below the base of the foundation.

Bolton (1979) presented a theoretical expression for the vertical capacity of a strip footing subjected to inclined load. Bolton's expression, modified by the inclusion of a shape factor of ζ_s, provides the following expression for the ultimate capacity:

$$f = \left(\frac{V}{A}\right) - \zeta_s s_u \left(\frac{1 + \pi - \mathrm{Arcsin}\left(\frac{H}{As_u}\right) + }{\sqrt{1 - \left(\frac{H}{As_u}\right)^2}} \right) = 0 \tag{3.3}$$

For square and circular foundations it is reasonable to adopt a value of $\zeta_s = 1.2$.

Based on the results of experimental studies of circular foundations performed by Osborne *et al.* (1991), Murff (1994) suggested a general form of three-dimensional failure locus as:

$$f = \sqrt{\left(\frac{M}{D}\right)^2 + \alpha_1 H^2} + \alpha_2 \left(\frac{V^2}{V_c} - V\left(1 + \frac{V_t}{V_c}\right) + V_t\right) = 0 \tag{3.4}$$

where α_1 and α_2 are constants, M is the moment applied to the foundation, V_c and V_t are respectively the compression and tension capacities under pure vertical load. A finite value of V_t could be mobilised in the short term due to the tendency to develop suction pore pressures in the soil under the footing. A simple form of Equation (3.4), suitable for undrained conditions, assuming $V_t = -V_c = -V_u$, is as follows:

$$f = \sqrt{\left(\frac{M}{\alpha_3 V_u D}\right)^2 + \left(\frac{H}{\alpha_4 V_u}\right)^2} + \left(\frac{V}{V_u}\right)^2 - 1 = 0 \tag{3.5}$$

It may be seen that $\alpha_3 V_u D$ and $\alpha_4 V_u$ represents the capacity of the foundation under pure moment, M_u, and pure horizontal load, H_u, respectively. Therefore Equation (3.5) can also be expressed as:

$$f = \sqrt{\left(\frac{M}{M_u}\right)^2 + \left(\frac{H}{H_u}\right)^2} + \left(\frac{V}{V_u}\right)^2 - 1 = 0 \tag{3.6}$$

A finite element study to determine the failure locus for long strip foundations on non-homogeneous clays under combined loading was presented by Bransby and Randolph (1998). The results of the finite element analyses were supported by upper bound plasticity analyses, and the following failure locus was suggested for rapid (undrained) loading conditions:

$$f = \left(\frac{V}{V_u}\right)^2 + \sqrt[\alpha_3]{\left(\frac{M^*}{M_u}\right)^{\alpha_1} + \left(\frac{H}{H_u}\right)^{\alpha_2}} - 1 = 0 \tag{3.7}$$

in which

$$\frac{M^*}{ABs_{uo}} = \frac{M}{ABs_{uo}} - \left(\frac{Z}{B}\right)\left(\frac{H}{As_{uo}}\right) \tag{3.8}$$

where M^* is the moment calculated about a reference point above the base of the footing at a height Z, B is the breadth of the strip footing, α_1, α_2 and α_3 are factors depending on the degree

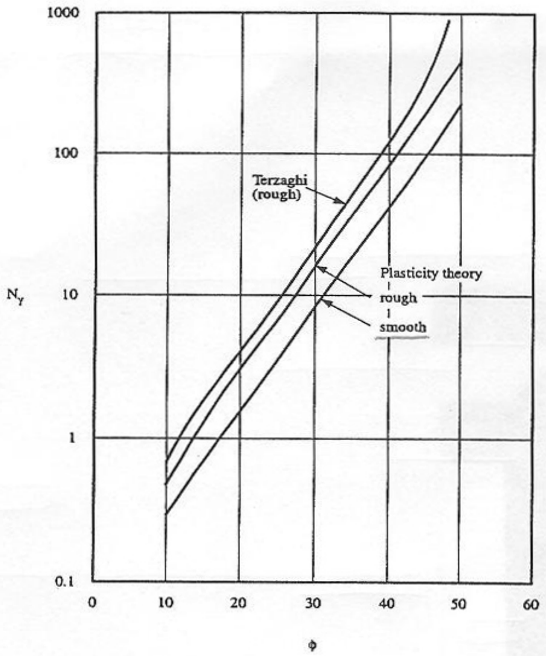


Figure 3.1. Bearing capacity factor $N\gamma$ (after Davis and Booker, 1971).

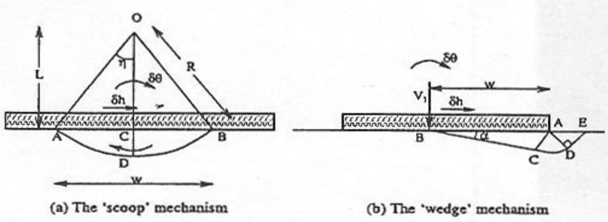


Figure 3.2. 'Scoop' and 'wedge' mechanisms proposed by Bransby and Randolph (1997).

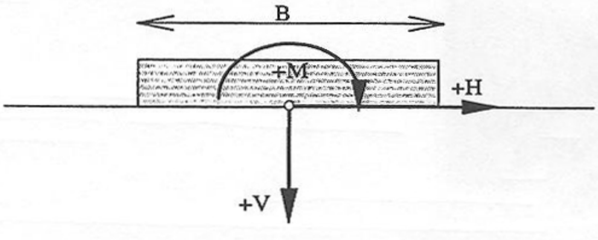


Figure 3.3. Conventions for loads and moment applied to foundations.

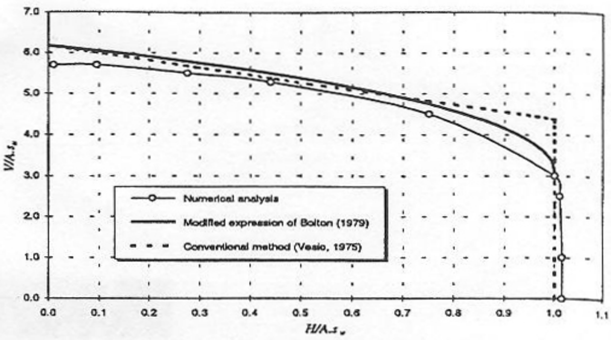


Figure 3.4. Failure loci for foundations under inclined loading (M = 0).

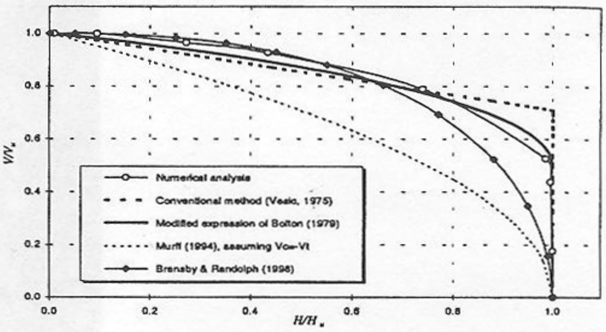


Figure 3.5. Non-dimensional failure loci in the V-H plane (M=0).

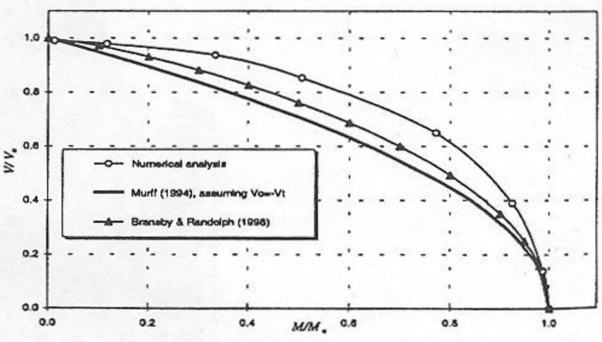


Figure 3.6. Non-dimensional failure loci in the V-M plane (H=0).

of non-homogeneity, and s_{uo} is the undrained shear strength of the soil at the level of the foundation base. Bransby and Randolph (1998) proposed that the collapse mechanism for a footing under combined loading is based on two different component mechanisms: the 'scoop' mechanism and the 'wedge' mechanism, as illustrated in Figure 3.2.

Three-dimensional analyses of circular foundations subjected to combined loading under undrained conditions has been described by Taiebat and Carter (2000a). They compared predictions of a number of the failure criteria described previously with their three-dimensional finite element predictions of the failure surface. The sign conventions for loads and moment used in this study are based on the right-handed axes and clockwise positive conventions, (*V, M, H*), as described by Butterfield *et al.* (1997) and shown in Figure 3.3.

Vertical-Horizontal (V-H) Loading

Taiebat and Carter's finite element prediction of the failure envelope in the *V-H* plane is presented in Figure 3.4, together with the conventional solution of Vesic (1975), Equation (3.1), and the modified expression of Bolton (1979), Equation (3.4) with a shape factor $\zeta_s = 1.2$. Comparison of the curves in Figure 3.4 shows that the numerical analyses generally give a more conservative bearing capacity for foundations subjected to inclined load. The results of the numerical analyses are very close to the results of the modified theoretical expression of Bolton (1979).

All three methods indicate that there is a critical angle of inclination, measured from the vertical direction, beyond which the ultimate horizontal resistance of the foundation dictates the failure of the foundation. Where the inclination angle is more than the critical value, the vertical force does not have any influence on the horizontal capacity of the foundation. For circular foundations the critical angle is predicted to be approximately 19° by the numerical studies and from the modified expression of Bolton (1979), compared to 13° predicted by the conventional method of Vesic (1975). In Figure 3.5, the non-dimensional form of the failure envelope predicted by the finite element analyses is compared with those of Vesic (1975), Equation (3.1), Bolton (1979), Equation (3.4), Murff (1994), Equation (3.6), and Bransby and Randolph (1998), Equation (3.7). The shape of the failure locus predicted by the numerical analyses is closest to the modified expression of Bolton (1979). It can be seen that the conventional method gives a good approximation of the failure locus except for high values of horizontal loads. The failure locus presented by Murff (1994) gives a very conservative approximation of the other failure loci.

Vertical-Moment (V-M) Loading

For a circular foundation on an undrained clay subjected to pure moment, an ultimate capacity of $M_u = 0.8A.D.s_u$ was predicted by the finite element analysis of Taiebat and Carter (2000). In Figure 3.6 the predicted failure envelope is compared with those of Murff, Equation (3.6), and Bransby and Randolph, Equation (3.7). The failure envelopes approximated by Murff (1994) and Bransby and Randolph (1998) are both conservative with respect to the failure envelope predicted by the numerical analyses. It is noted that the equations presented by Bransby and Randolph were suggested for strip footings, rather than the circular footing considered here.

Horizontal-Moment (H-M) Loading

The failure locus predicted for horizontal load and moment is plotted in Figure 3.7. A maximum moment capacity of $M = 0.89A.D.s_u$ is coincident with a horizontal load of $H = 0.71A.s_u$. This is 11% greater than the capacity predicted for the foundation under pure moment.

A non-dimensional form of the predicted failure locus and the suggestions of Murff (1994) and Bransby and Randolph (1998) are plotted in Figure 3.8. It can be seen that the locus suggested by Murff (1994) is symmetric and the maximum moment coincides with zero horizontal loading, whereas the numerical analyses show that the maximum moment is sustained with a positive horizontal load, as already described. The failure locus obtained from Murff's equation becomes non-conservative when $M \times H \leq 0$. Bransby and Randolph (1998) identified two different upper bound plasticity mechanisms for strip footings under moment and horizontal load, a scoop mechanism and a scoop-wedge mechanism (Figure 3.2). The latter mechanism results in greater ultimate moment capacity for strip footings, supporting the finite element predictions.

General Failure Equation

An accurate three-dimensional equation for the failure envelope in its complete form, which accounts for both the load inclination and eccentricity, is likely to be a complex expression. Some degree of simplification is essential in order to obtain a convenient form of this failure envelope. Depending on the level of the simplification, different classes of failure equations may be obtained.

In the previous section, the failure envelopes suggested by different methods were compared in two-dimensional loading planes. It was demonstrated that the failure equation presented by Murff (1994) has simplicity in its mathematical form, but does not fit the failure envelopes produced by the conventional and numerical analyses. The failure equation presented by Bransby and Randolph (1998) for strip footings matches the data for circular footings in two planes, but does not give a suitable answer in three-dimensional load space.

A new equation describing the failure locus in terms of all three components of the load has been proposed recently by Taiebat and Carter (2000a). In formulating this equation, advantage was taken of the fact that the moment capacity of the foundation is related to the horizontal load acting simultaneously on the foundation. The proposed approximate failure equation is expressed as:

$$f = \left(\frac{V}{V_u}\right)^2 + \left(\frac{M}{M_u}\left(1 - \alpha_1 \frac{H.M}{H_u|M|}\right)\right)^2 + \left|\left(\frac{H}{H_u}\right)^3\right| - 1 = 0 \quad (3.9)$$

where α_1 is a factor that depends on the soil profile. For a homogeneous soil a value of $\alpha_1 = 0.3$ provides a good fit to the bearing capacity predictions from the numerical analysis. Perhaps inevitably, the three-dimensional failure locus described by Equation (3.9) will not tightly match the numerical predictions over the entire range of loads, especially around the abrupt changes in the failure locus that occur when the horizontal load is high. However, overall the approximation is satisfactory, conservative and sufficient for many practical applications. Equation (3.9) is shown as a contour plot in Figure 3.9.

3.2.3 *Bearing capacity under eccentrically applied loading*

There is no exact expression to evaluate the effects of eccentricity of the load applied to a foundation. However, the effective width method is commonly used in the analysis of foundations subjected to eccentric loading (e.g., Vesic, 1973; Meyerhof, 1951, 1953). In this method, the bearing capacity of a foundation subjected to an eccentrically applied vertical loading is assumed to be equivalent to the bearing capacity of another foundation with a fictitious effective area on which the vertical load is centrally applied.

Studies aimed at determining the shape of the failure locus in (*V-M*) space (e.g., Taiebat and Carter, 2000a, 2000b; Houlsby and Puzrin, 1999) are relevant, because this loading case is also directly applicable to the analysis of a footing to which vertical load is eccentrically applied.

Finite element modelling of the problem of the bearing capacity of strip and circular footings on the surface of a uniform homogeneous undrained clay layer, subjected to vertical load and moment was described by Taiebat and Carter (2000b). It was also assumed in this particular study that the contact between the footing and the soil was unable to sustain tension. The failure

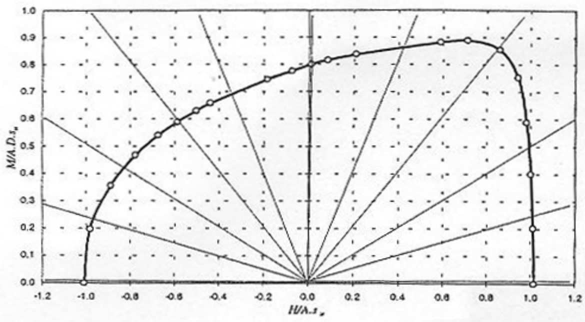


Figure 3.7. Failure loci in the *M-H* plane (*V*=0).

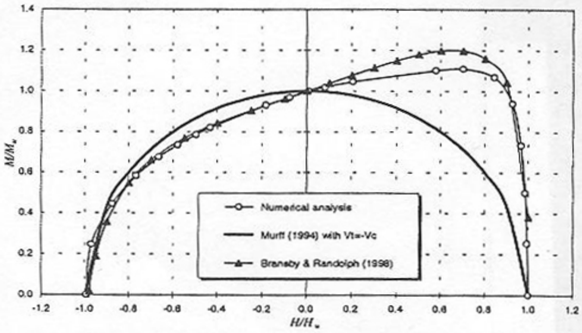


Figure 3.8. Non-dimensional failure loci in the *M-H* plane (*V*=0).

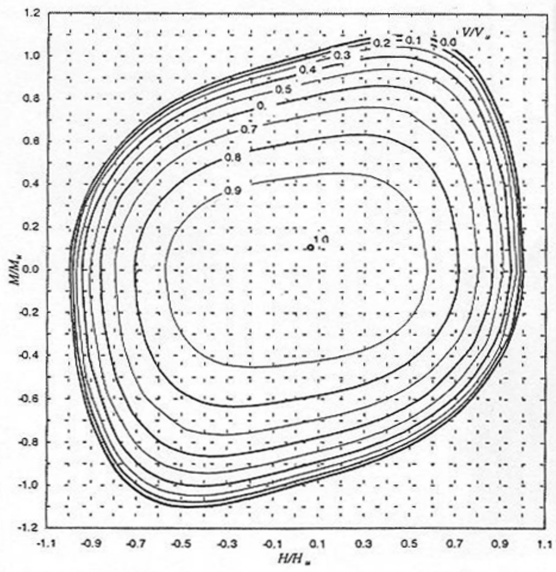


Figure 3.9. Representation of the proposed failure equation in non-dimensional *V-M-H* space.

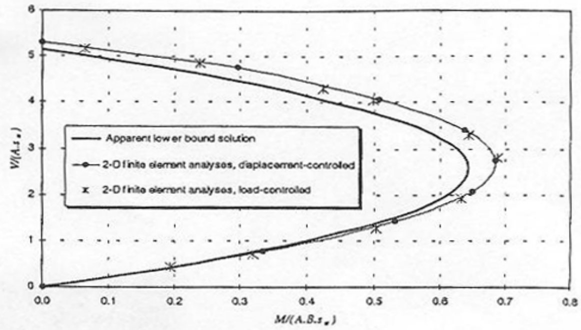


Figure 3.10. Failure loci for a strip footing under eccentric loading.

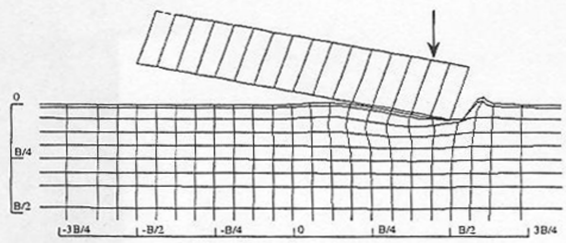


Figure 3.11. Deformed shape of the soil and the strip footing under an eccentric load.

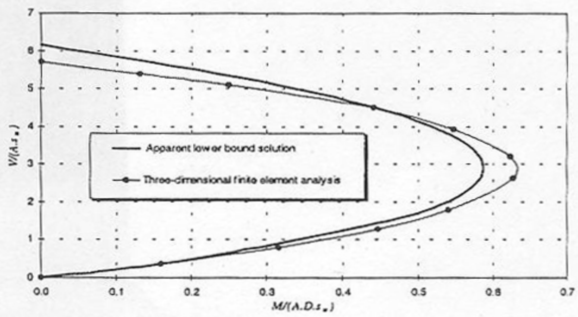


Figure 3.12. Failure loci for a circular footing under eccentric loading.

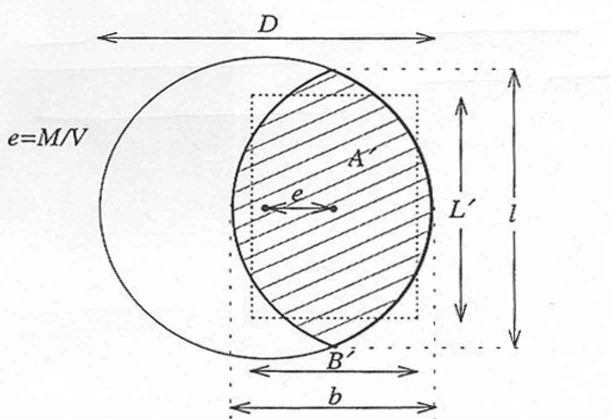


Figure 3.13. Effective area of a circular footing subjected to eccentric load.

envelopes predicted by Taiebat and Carter (2000b) have also been compared with the solutions obtained using the lower bound theorem of plasticity. The "lower bound" solutions satisfy equilibrium and do not violate the yield criterion. However, some of the solutions may not adhere strictly to all requirements of the lower bound theorem. For example, loss of contact at the footing-soil interface implies that the normality principle is not always obeyed. Therefore, the term "apparent lower bound" is used to describe these solutions, as suggested by Houlsby and Purzin (1999).

Strip footings

The failure envelope predicted by two-dimensional finite element analysis of a strip footing under both vertical load and moment is presented in Figure 3.10. Also shown in this figure is the failure envelope resulting from the apparent lower bound solutions of Houlsby and Purzin (1999), which is described by the following equation:

$$\frac{V}{A} = (\pi + 2)\left(1 - \frac{2M}{VB}\right)s_u \tag{3.10}$$

where V is the vertical load, A is the area of the foundation, and M is the moment applied to the foundation. The failure envelope predicted by the finite element method is in good agreement with the envelope obtained from the apparent lower bound solutions. Figure 3.11 shows the deformed shape of the soil and the strip foundation under a vertical load applied with relatively large eccentricity.

Circular footings

The failure envelope predicted by three-dimensional finite element analyses and lower bound analyses of circular footings subjected to both vertical load and moment are presented in Figure 3.12. Good agreement between the two solutions is evident in this figure. It is noted that the apparent lower bound solution presented by Houlsby and Purzin (1999) is for conditions of plane strain only. For the three-dimensional case, the apparent lower bound solutions shown in Figure 3.12 have been obtained based on the following considerations.

A circular foundation, subjected to a vertical load applied with an eccentricity $e = M/V$, can be regarded as an equivalent fictitious foundation with a centrally applied load (Figure 3.13) as suggested by Meyerhof (1953) and Vesic (1973). For a circular footing the area of the fictitious foundation, A', can be calculated as:

$$A' = \frac{D^2}{2}\left(\text{Arccos}\frac{2e}{D} - \frac{2e}{D}\sqrt{1-\left(\frac{2e}{D}\right)^2}\right) \tag{3.11}$$

The aspect ratio of the equivalent rectangular area can also be approximated as the ratio of the lengths b and l, as shown in Figure 3.13, i.e.,

$$\frac{B'}{L'} = \frac{b}{l} = \sqrt{\frac{D-2e}{D+2e}} \tag{3.12}$$

Therefore, in this case the shape factor for the fictitious rectangular foundation is given by (Vesic, 1973):

$$\zeta_s = 1 + 0.2\sqrt{\frac{DV - 2M}{DV + 2M}} \tag{3.13}$$

Hence, the bearing capacity of circular foundations subjected to eccentric loading can be obtained from the effective width method as:

$$V = \zeta_s A'(2+\pi)s_u \tag{3.14}$$

Note that based on Vesic's recommendation the shape factor for circular footings under the pure vertical load is usually adopted as $\zeta_s = 1.2$. However, exact solutions for the vertical bearing capacity of circular footings on uniform Tresca soil (Shield, 1955; Cox, 1961) suggest the ultimate bearing capacity of 5.69 $A.s_u$ and 6.05 $A.s_u$ for smooth and rough footings, respectively. Therefore, the appropriate shape factors are actually 1.11 and 1.18 for smooth and rough footings.

In summary, it is clear from the comparisons presented in this section that the effective width method, commonly used in the analysis of foundations subjected to eccentric loading, provides good approximations to the collapse loads. Its continued use in practice therefore appears justified.

3.2.4 *Bearing capacity of non-homogeneous soils*

Progress has also been made in recent decades in predicting reliably the ultimate bearing capacity of foundations on non-homogeneous soils. A particular example is the important case that arises often in practice where the undrained shear strength of the soil varies approximately linearly with depth below the soil surface, i.e.,

$$s_u = c_0 + \rho z \tag{3.15}$$

or below a uniform crust, i.e.,

$$\begin{aligned} s_u &= c_0 \quad \text{for } z < \frac{c_0}{\rho} \\ s_u &= \rho z \quad \text{for } z > \frac{c_0}{\rho} \end{aligned} \tag{3.16}$$

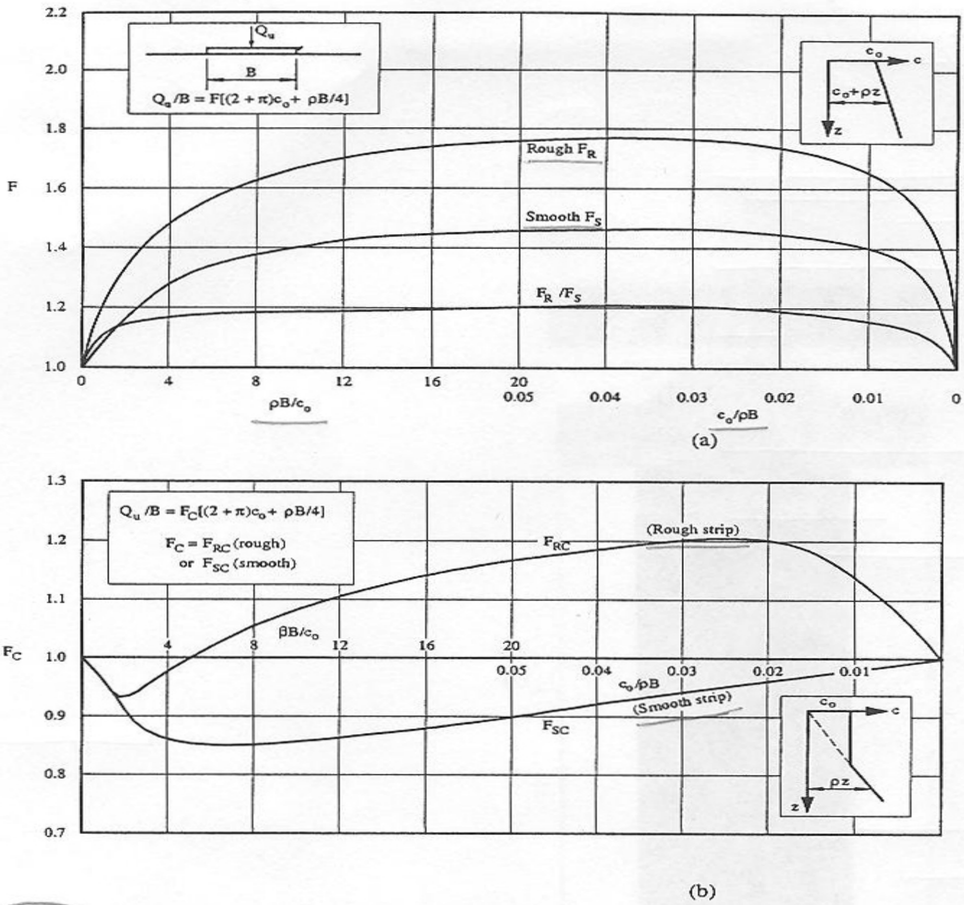
in which c_0 is the undrained shear strength of the soil at the surface, ρ is the strength gradient and z is the depth below the soil surface. Several theoretical approaches attempted to take account of this effect, most notably the work by Davis and Booker (1973) and Houlsby and Wroth (1983). Both used the method of stress characteristics from the theory of plasticity. Assuming the soil to obey the Tresca yield criterion, Davis and Booker's plane strain solution has been expressed as:

$$q_u = \frac{Q_u}{B} = F\left[(2+\pi)c_o + \frac{\rho B}{4}\right] \tag{3.17a}$$

where F is a function of the soil strength non-homogeneity ($\rho B/c_0$) and the roughness of the foundation-soil interface. Values of the bearing capacity factor F are reproduced in Figure 3.14 for two different undrained strength profiles. It is worth noting that the solutions of Davis and Booker (1973) can also be adapted to circular footings, at least approximately, by the replacement of Equation (3.17a) by:

$$q_u = \frac{Q_u}{B} = F\left[6c_o + \frac{\rho B}{4}\right] \tag{3.17b}$$

Tani and Craig (1995) also investigated the case where the undrained shear strength is proportional to depth using plasticity theory. They made predictions of the bearing capacity of skirted offshore gravity structures with skirt length to diameter ratios of 0 to 0.3. They found that their results for surface and circular footings (for both smooth and rough interfaces) obtained using the method of stress characteristics agree very well with the solutions obtained by Davis and Booker (1973) and Houlsby and Wroth (1983). Tani and Craig's results for the computed bearing capacity of an embedded rough footing are reproduced here in Figure 3.15. In this figure c_{Df} is the undrained strength of the clay at the level of the footing base, k is the rate of increase of strength with depth, Df is the depth to the base of the footing, and B and D are the width and diameter of the strip and circle, respectively. Tani and Craig found that the ultimate bearing capacity of the foundation was governed largely by the strength of the soil at the skirt tip rather than at surface level. Soil above this level has little influence on the bearing capacity of the footing. It may be concluded from these results that the increase in



Eq. 3.15

Eq. 3.16

Figure 3.14. Bearing capacityof a strip footing on non-homogeneous clay (after Davis and Booker, 1973).

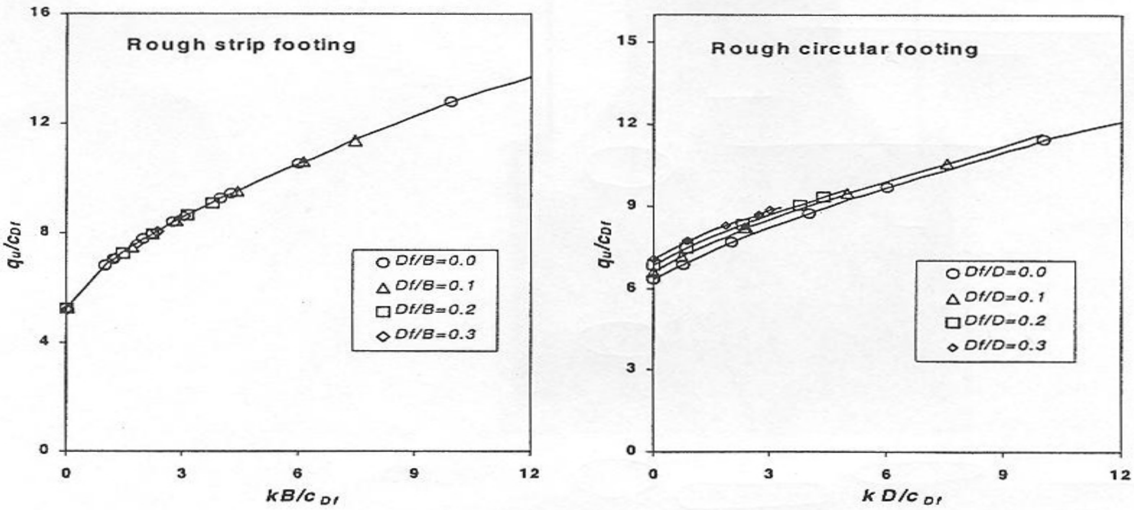


Figure 3.15. Bearing capacity of rough strip footing and circular footing on non-homogeneous clay (after Tani and Craig, 1995.)

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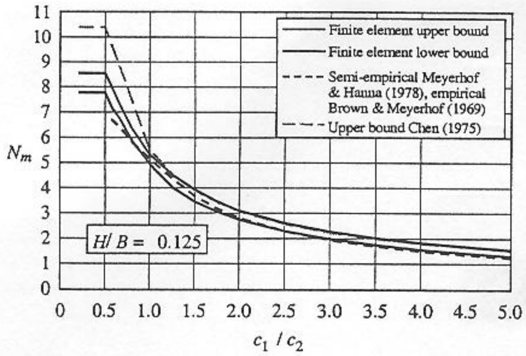
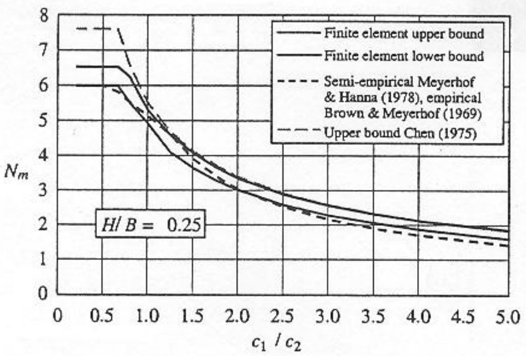
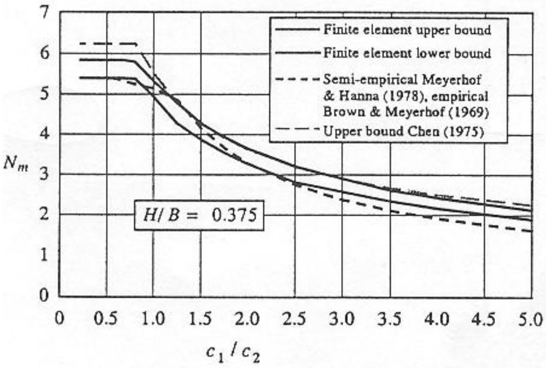
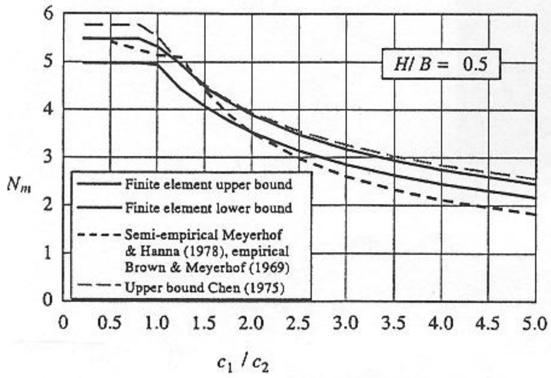








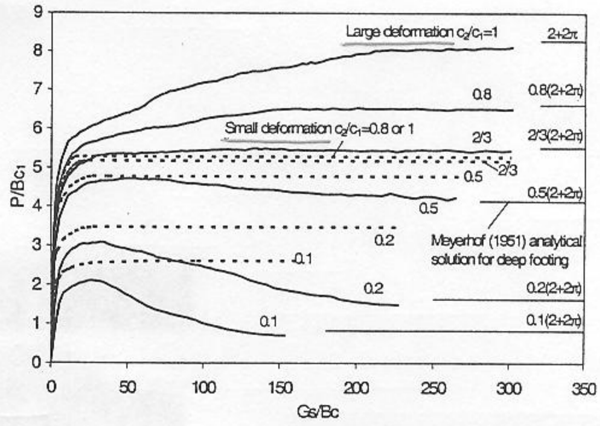
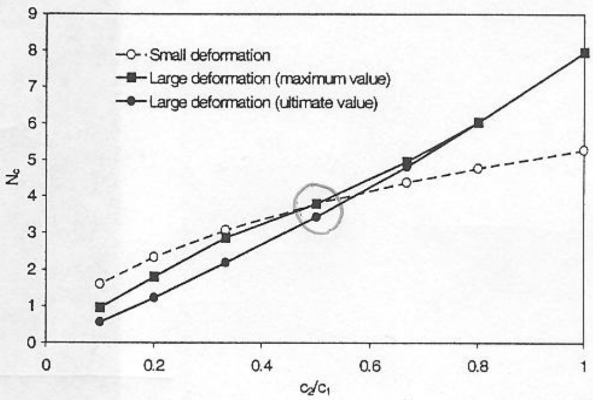
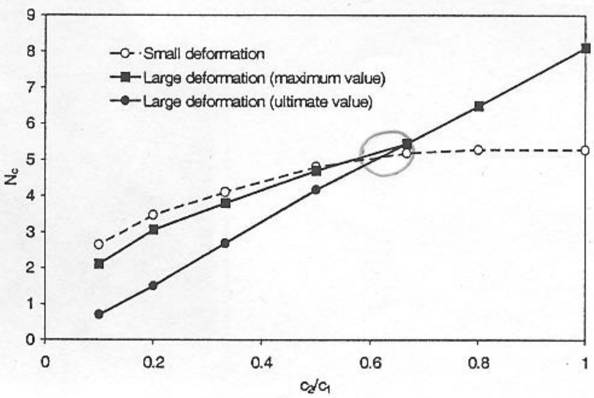


Figure 3.17. Normalised load-settlement curves for layered clay (G/c = 67, H/B = 1) (after Wang, 2000).



(a) H/B = 0.5



(b) H/B = 1

Figure 3.18. Bearing capacity factors for a rigid strip footing on a two-layered clay (after Wang, 2000).

bearing capacity with increasing embedment can be attributed to the higher shear strength at the tip level for the most part, and very little to the shearing resistance of the soil above tip level.

3.2.5 *Two-layered soils*

Natural soil deposits are often formed in discrete layers. For the purpose of estimating the ultimate bearing capacity of a foundation on a layered soil it is often appropriate to assume that the soil within each layer is homogeneous. If a footing is placed on the surface of a layered soil and the thickness of the top layer is large compared with the width of the footing, the ultimate bearing capacity of the soil and the displacement behaviour of the footing can be estimated to sufficient accuracy using the properties of the upper layer only. However, if the thickness of the top layer is comparable to the footing width, this approach introduces significant inaccuracies and is no longer appropriate. This is because the zone of influence of the footing, including the potential failure zone, may extend to a significant depth, and thus two or more layers within that depth range will affect the bearing behaviour of the footing. Examples include offshore foundations of large physical dimensions, and vehicle loads applied to unpaved roads over soft clay deposits.

The case of a footing on a stronger soil layer overlying a weaker layer is of particular interest because of the risk of punch-through failure occurring. Such failures are normally sudden and brittle, with little warning, and methods of analysis that can predict the likelihood of this type of behaviour are of great value in practice.

Methods for calculating the bearing capacity of multi-layer soils range from averaging the strength parameters (Bowles, 1988), using limit equilibrium considerations (Button, 1953; Reddy and Srinivasan, 1967; Meyerhof, 1974), to a more rigorous limit analysis approach based on the theory of plasticity (Chen and Davidson, 1973; Florkiewicz, 1989; Michalowski and Shi, 1995; Merifield, *et al.*, 1999). Semi-empirical approaches have also been proposed based on experimental studies (e.g., Brown and Meyerhof, 1969; Meyerhof and Hanna, 1978). The finite element method, which can handle very complex layered patterns, has also been applied to this problem. (e.g., Griffiths, 1982; Love *et al.*, 1987; Burd and Frydman, 1997; Merifield, *et al.*, 1999).

However, almost all these studies are limited to footings resting on the surface of the soil and are based on the assumption that the displacement of the footing prior to attaining the ultimate load is relatively small. In some cases, such as those where the underlying soil is very soft, the footings will experience significant settlement, and sometimes even penetrate through the top layer into the deeper layer. Penetration into the seabed of spud-can footings supporting a jack-up platform provides a particular example of this behaviour. For these cases, the small displacement assumption is no longer appropriate, and a large displacement theory should be adopted. In all cases, the consequences for the load-deformation response of a non-homogeneous soil profile should be understood, because such profiles can be associated with a brittle foundation response. The prediction of brittleness in these cases can only be made using a large deformation analysis.

A brief review of recent small and large deformation analyses applied to foundations on layered soils is therefore presented in this section. In particular, the behaviour of rigid strip and circular footings penetrating two-layered clays is discussed first, followed by the problem of a sand layer overlying clay. In most cases, the upper layer is at least as strong as the lower layer, so that the issue of punch-through failure can be explored.

Small deformation analysis

In the absence of surcharge pressure, the ultimate bearing capacity, q_u, of a strip or circular footing on a two-layered purely cohesive soil can be expressed as:

$$q_m = c_1 N_m \tag{3.18}$$

where N_m is the modified bearing capacity factor that will depend on the strength ratio of the two layers c_2/c_1 and the relative thickness of the top layer, H/B. c_1 and c_2 denote the undrained shear strengths of the top and bottom layers, respectively, H is the thickness of the top layer and B is the foundation width (or diameter). Equation (3.18) is both a simplification and an extension to account for layering of the general bearing capacity Equation, (3.1). Several researchers have published approximate solutions for the bearing capacity factor N_m appearing in Equation (3.18). For strip footings, Button (1953) and Reddy and Srinivasan (1967) have suggested very similar values for N_m. These include both upper bound plasticity solutions to this problem, and at one extreme they return a bearing capacity factor for a homogeneous soil (considered as a special case of a two-layered soil) of 5.51, i.e., approximately 7% above Prantl's exact solution of $(2+\pi)$. Brown and Meyerhof (1969) published bearing capacity factors based on experimental studies, and their recommendations are in better agreement with the value of $(2+\pi)$ for the case of a homogeneous soil. The Brown and Meyerhof factors can be expressed by the following equation:

$$N_m = 1.5\left(\frac{H}{B}\right) + 5.14\left(\frac{c_2}{c_1}\right) \tag{3.19}$$

$H=0,\ c_1=c_2$

with an upper limit for N_m of 5.14 in this case.

Recently, Merifield *et al.* (1999) calculated rigorous upper and lower bound bearing capacity factors for layered clays under strip footings by employing the finite element method in conjunction with the limit theorems of classical plasticity. The results of their extensive parametric study have been presented in both graphical and tabular form. Some typical results are reproduced in Figure 3.16.

The number of published studies of circular footings on layered cohesive soil is significantly less than for strip footings. Bearing capacity factors for circular footings were given by Vesic (1970) for the case of a relatively weak clay layer overlying a stronger one. These factors were obtained by interpolation between known rigorous solutions for related problems, and they have been published in Chapter 3 of the text by Winterkorn and Fang (1975). Based on the results of model tests, Brown and Meyerhof (1969) suggested that for cases where a stronger clay layer overlies a weaker one, an analysis assuming simple shear punching around the footing perimeter is appropriate. The bearing capacity factors for this case are given by the following equation:

$$N_m = 1.5\left(\frac{H}{B}\right) + 6.05\left(\frac{c_2}{c_1}\right) \tag{3.20}$$

with an upper limit for N_m of 6.05 for a circular footing of diameter B.

Large deformation analysis

The bearing response of strip footings on a relatively strong undrained clay layer overlying a weaker clay layer has been examined by Wang (2000) and Wang and Carter (2000), who compared the results given by both small and large deformation analyses. Cases corresponding to $H/B = 0.5$ and 1, and $c_2/c_1 = 0.1$, 0.2, 1/3, 0.5, 2/3 and 1 (homogeneous soil) were investigated. Normalised load-displacement curves for weightless soil are shown in Figure 3.17, for cases where the width of the footing is the same as the thickness of the stronger upper clay layer, i.e., $H/B = 1$. It is noted that predictions of the large deformation behaviour are also dependent on the rigidity index of the clay, G/c, where G is the elastic shear modulus of the clay. The curves shown in Figure 3.17 correspond to a value of $G/c = 67$.

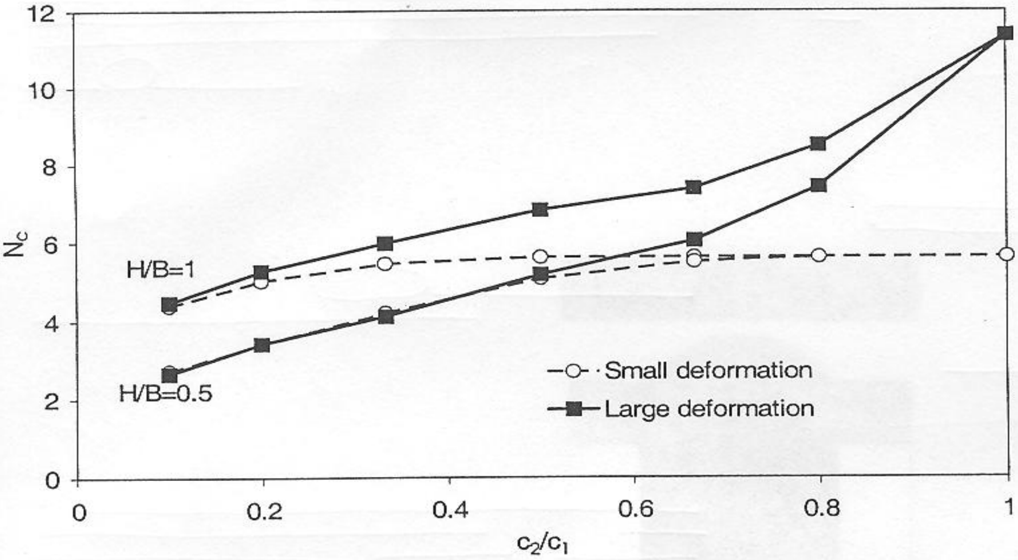


Figure 3.19. Maximum bearing capacity factors for a circular footing on a two-layered clay (after Wang, 2000).

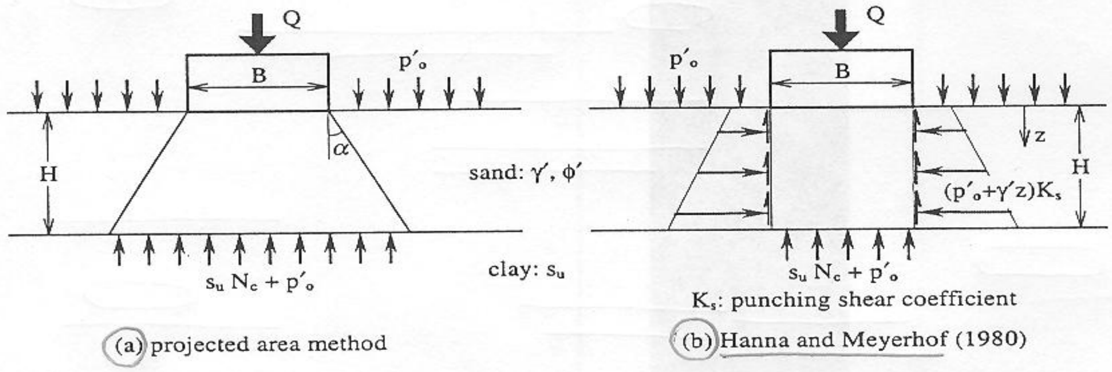


Figure 3.20. Failure mechanisms assumed in two methods of analysis of bearing capacity of a layer of sand over clay.

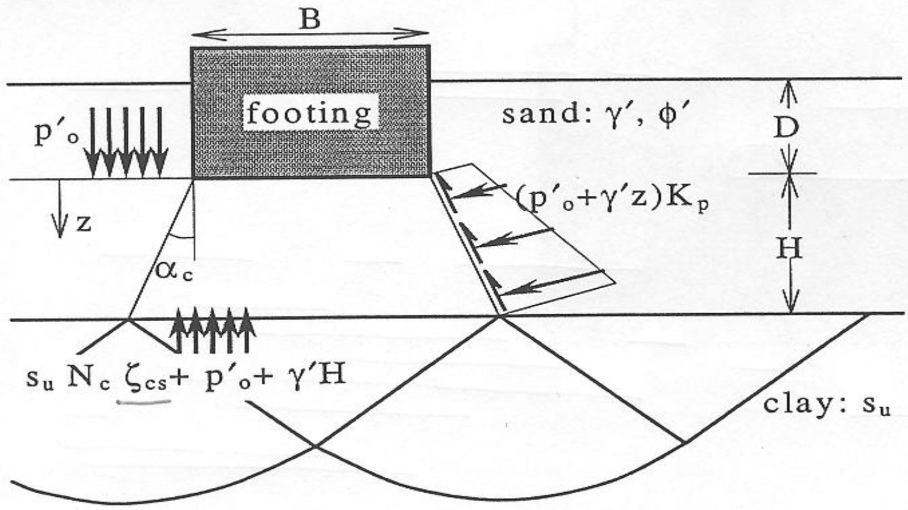


Figure 3.21. Failure mechanisms suggested by Okimura *et al.* (1998) for estimating the bearing capacity of a layer of sand over clay.

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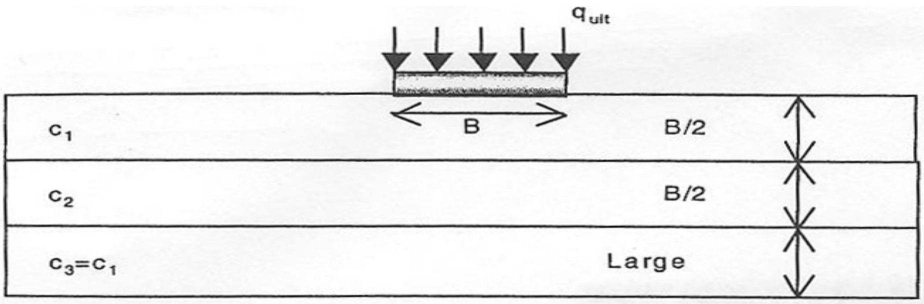

Figure 3.22. Bearing capacity of a strip footing on a layered soil deposit.

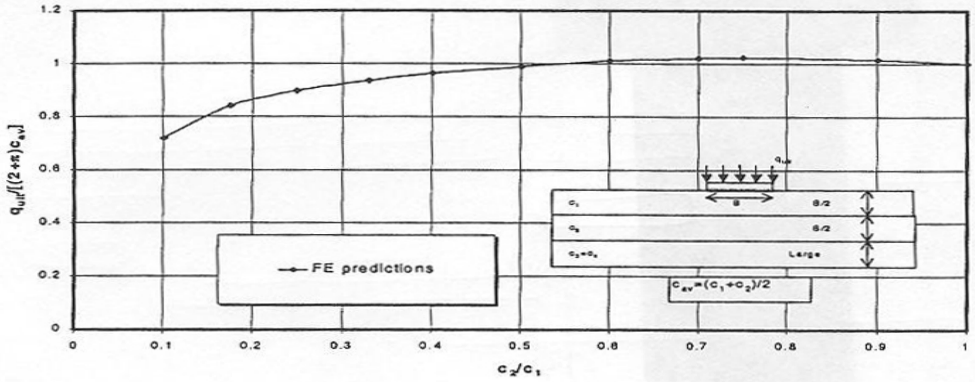

Figure 3.23. Bearing capacity charts for a strip footing on layered clay – soft centre "sandwich".

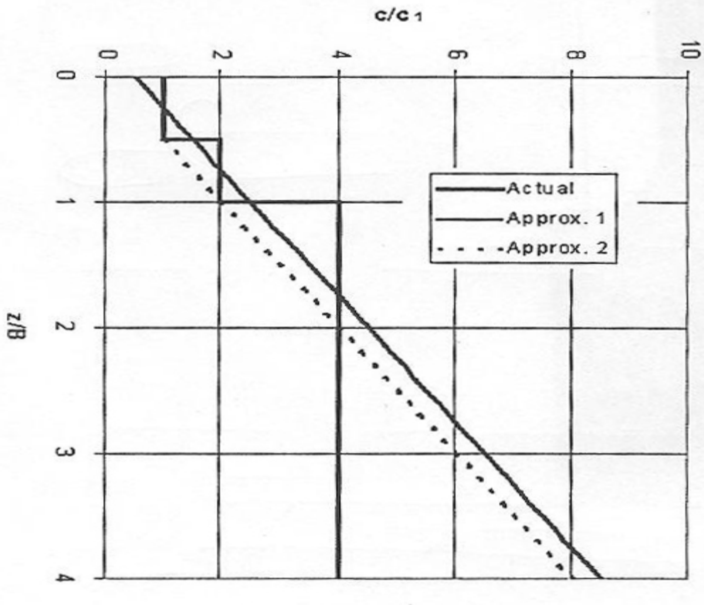

Figure 3.24. Strength profiles for layered clay.

Typically, the curve predicted by the small deformation analysis reaches an ultimate value after a relatively small footing penetration (settlement). Generally, the load-displacement curves predicted by the large deformation analyses are quite different from those given by the small displacement analysis. For cases where a stronger top layer overlies a much weaker bottom layer (e.g., c_2/c_1 = 0.1, 0.2, and 0.5), the overall response is characterised by some brittleness, even though the behaviour of both component materials is perfectly plastic and thus characterised by an absence of brittleness. For these cases, the load-penetration curves given by the large deformation analysis rise to a peak, at which point the average bearing pressure is generally lower than the ultimate bearing capacity predicted by the small deformation analysis. With further penetration of the footing into the clay, it appears that the load-displacement curve approaches an asymptotic value. The peak values of average bearing pressure obtained from these large deformation curves define the maximum bearing capacity factor, and the values reached after large penetration are referred to as the ultimate bearing capacity factor. It is noted that in the small deformation analysis, the maximum and ultimate values of the bearing capacity factor are identical.

It is reasonable to expect that footings exhibiting a brittle response should ultimately behave much like a strip footing deeply buried in the lower clay layer, so that the ultimate value of the average bearing pressure should then be approximately $(2+2\pi)c_2$, where c_2 is the undrained shear strength of the lower layer. These theoretical limits are also indicated on Figure 3.17, and it seems clear that the curves obtained from the large deformation finite element analysis approach closely these limiting values at deep penetrations. In Figure 3.18, values of the bearing capacity factors for cases where $H/B = 0.5$ and $H/B = 1$ have been plotted against the strength ratio, c_2/c_1. Also plotted in this figure are the bearing capacity factors predicted by the small deformation analysis. Wang (2000) has also demonstrated that large deformation effects appear to be even more significant for the case of a circular footing. In all the cases examined, the maximum bearing capacity factors obtained from the large deformation analysis were greater than those obtained from the small deformation analysis. Both sets of values are plotted in Figure 3.19.

Effect of soil self-weight during footing penetration

It is well known that for a surface footing on a purely cohesive soil, the ultimate bearing capacity given by a small strain undrained analysis is independent of the soil density. However, in large deformation analysis, the footing can no longer be regarded as a surface footing once it begins to penetrate into the underlying material, and in this case the self-weight of the soil will also affect the penetration resistance.

The results of the large deformation footing analyses presented in the previous section were obtained by Wang without considering soil self-weight. This was done deliberately in order to investigate the effects of the large deformation analysis exclusively on the bearing capacity factor N_m. However, if soil self-weight is included, the surcharge pressure becomes significant as the footing becomes buried. It was demonstrated by Wang (2000) that it is reasonable to approximate the mobilised penetration resistance for a ponderable soil as that for the corresponding weightless soil supplemented by the overburden pressure corresponding to the depth of footing penetration.

Sand overlying clay

Various theoretical and experimental studies have been conducted into the ultimate bearing capacity of a footing on a layer of sand overlying clay, e.g., Yamaguchi (1963), Brown and Paterson (1964), Meyerhof (1974), Vesic (1975), Hanna and Meyerhof (1980), Craig and Chua (1990), Michalowski and Shi (1995), Vinod (1995), Kenny and Andrawes (1997), Burd and Frydman (1997), Okamura *et al.* (1998). This is an important problem in foundation engineering, as this case often arises in practice and it is one in which punch-through failure may be a genuine concern, particularly for relatively thin sand layers overlying soft clays.

Exact plasticity solutions for this problem have not yet been published, but a number of the existing analyses of the bearing capacity of sand over clay use limit equilibrium techniques. They can be broadly classified according to the shape of the sand block punching into the clay layer and the shearing resistance assumed along the side of the block. Two proposed failure mechanisms are illustrated in Figure 3.20. In each case the strength of the sand is analysed in terms of effective stress, using the effective unit weight (γ') and the friction angle (ϕ'), while the analysis of the clay is in terms of total stress, characterised by its undrained shear strength (s_u). In the case of the "projected area" method an additional assumption about the angle α (Figure 3.20a) is required.

Okamura *et al.* (1998) assessed the validity of these two approaches by comparing their predictions with the results of some 60 centrifuge model tests. This comprehensive series of tests included strip and circular footings on the surface of the sand and embedded in it. Significant differences between observed and calculated bearing capacities were noted and these were attributed to discrepancies in the shapes of the sand block or the forces acting on the sides of the block. To overcome these problems Okamura *et al.* proposed an alternative failure mechanism, which can be considered as a combination of the two mechanisms shown in Figure 3.20. Their new limit equilibrium mechanism is illustrated in Figure 3.21, in which it is noted that K_p is Rankine's passive earth pressure coefficient for the sand, i.e., $K_p = (1 + \sin\phi)/(1 - \sin\phi)$. In this method it is assumed that a normal stress of K_p times the vertical overburden pressure acts on the inclined sides of the sand block. Consideration of equilibrium of the forces acting on the sand block, including its self-weight, provides the following bearing capacity formulae:

(i) for a strip footing;

$$
\begin{aligned}
q_u = & \left(1 + 2\frac{H}{B}\tan\alpha_c\right)(s_u N_c + p'_o + \gamma' H) \\
& + \left(\frac{K_p \sin(\phi' - \alpha_c)}{\cos\phi' \cos\alpha_c}\right) \\
& \times \left(\frac{H}{B}\right)(p'_o + \gamma' H) \\
& - \gamma' H\left(1 + \frac{H}{B}\tan\alpha_c\right)
\end{aligned} \tag{3.21}
$$

(ii) for a circular footing;

$$
\begin{aligned}
q_u = & \left(1 + 2\frac{H}{B}\tan\alpha_c\right)^2 (s_u N_c \zeta_s + p'_o + \gamma' H) \\
& + \left(\frac{4K_p \sin(\phi' - \alpha_c)}{\cos\phi' \cos\alpha_c}\right) \\
& \times \left\{ \begin{aligned} & \left(\frac{H}{B}\right)\left(p'_o + \frac{\gamma' H}{2}\right) \\ & + p'_o \tan\alpha_c \left(\frac{H}{B}\right)^2 + \frac{2}{3}\gamma' H \tan\alpha_c \left(\frac{H}{B}\right)^2 \end{aligned} \right\} \\
& - \frac{\gamma' H}{3}\left\{4\left(\frac{H}{B}\right)^2 \tan^2\alpha_c + 6\frac{H}{B}\tan\alpha_c + 3\right\}
\end{aligned} \tag{3.22}
$$

where ζ_s is the shape factor for a circular footing, which is usually assigned a value of 1.2. In these equations the angle α_c (Figure 3.21) is given as a function of the friction angle of the sand, ϕ', the geometry of the layer and footing and the undrained strength of the clay, s_u, i.e.,

$$\alpha_c = \tan^{-1}\left(\frac{(\sigma_{mc}/s_u)-(\sigma_{ms}/s_u)(1+\sin^2\phi')}{\cos\phi'\sin\phi'(\sigma_{ms}/s_u)+1}\right) \tag{3.23}$$

where

$$\sigma_{mc}/s_u = N_c s_u\left(1+\frac{1}{\lambda_c}\frac{H}{B}+\frac{\lambda_p}{\lambda_c}\right) \tag{3.24}$$

and

$$\sigma_{ms}/s_u = \frac{\sigma_{mc}/s_u-\sqrt{(\sigma_{mc}/s_u)^2-\cos^2\phi'((\sigma_{mc}/s_u)^2+1)}}{\cos^2\phi'} \tag{3.25}$$

The parameters λ_p and λ_c are respectively the normalised overburden pressure and the normalised bearing capacity of the underlying clay, given by:

$$\lambda_p = \frac{p_o'}{\gamma' B} \tag{3.26}$$

and

$$\lambda_c = \frac{s_u N_c}{\gamma' B} \tag{3.27}$$

N_c is the conventional bearing capacity factor for a strip footing on undrained clay ($N_c = 2+\pi$). Okamura *et al.* contended that Equations (3.21) and (3.22) are generally reliable for predicting the bearing capacity of a sand layer overlying clay for cases where $\lambda_c < 26$ and $\lambda_p < 4.8$. However, it is most important to note that the method may not be applicable over the full range of these values, and indeed it may overestimate the capacity if used indiscriminately, without regard to its limitations. Indeed, one very important limitation was highlighted by Okamura *et al.* They recognized that the bearing capacity for a sand layer overlying a weaker clay cannot exceed that of a deep sand layer ($H/B = \infty$). Hence, the bearing capacity values obtained from this method (Equations (3.21) and (3.22)) and from the formula for a deep layer of uniform sand (e.g., Equation (3.1)) must be compared, and the smaller value should be chosen as the bearing capacity of the layered subsoil.

Unlike the case of two clay layers, it seems that no finite element studies of the post-failure behaviour of footings on sand over clay have been published. However, it is expected that a brittle response may also be a possibility for this type of problem, particularly in cases where the self-weight and overburden effects do not dominate the influence of the clay strength.

3.3 *Multiple layers*

In nature, soils are often heterogeneous and in many cases they may be deposited in several layers. For such cases reliable estimation of the bearing capacity is more complicated. Of course, with modern computational techniques such as the finite element method, reliable estimates can ultimately be achieved. However, these methods usually require considerable effort, and the question arises whether simple hand techniques can be devised to provide realistic first estimates of the ultimate bearing capacity of layered soils. In particular, it would be useful to find answers to the following questions, in order to develop reasonably general guidelines for estimating the ultimate bearing capacity of layered soils.

- Can the bearing capacity be estimated by computing the average bearing capacity over a particular depth, e.g., 1 to 2B where B is the footing width?
- If the answer to the previous question is negative, is it possible to assess an average strength of the layers and then use that strength in the bearing capacity calculations for a homogeneous deposit to obtain a reliable estimate of the bearing capacity of the layered soil?
- If neither of the two previous approaches proves reliable, how can the practitioner solve the problem to obtain a reliable estimate of the ultimate bearing capacity?

In order to address some of these issues, consider now the idealised problem of a long strip footing on the surface of a soil deposit consisting of three different horizontal layers. The geometry of the problem is defined in Figure 3.22. The strength of each layer is characterised by the Mohr-Coulomb failure criterion and the conventional strength parameters c and ϕ. Self-weight of the soil is defined by the unit weight, γ. Various cases of practical interest have been identified, as indicated in Table 3.2. In cases where self-weight of the soil has been considered, it has been assumed for simplicity that the initial stress state is isotropic. This assumption should not affect the calculation of the ultimate capacity of a strip footing on the surface of the soil, but of course it will have a significant influence on the computed load-displacement curve. Finite element solutions for the various cases are described in the following subsections.

3.3.1 *Clay "sandwich" - soft centre*

Consider the case where the middle layer is weaker than the overlying and underlying clay layers. Undrained analyses have been conducted using the displacement finite element method and the results of a series of these analyses are presented in Figure 3.23. It is clear from this figure that the use of the simple average of the undrained shear strengths of the top and middle layer, $c = (c_1+c_2)/2$, in the bearing capacity equation for a uniform undrained clay (Equation (3.1)) provides a reasonable estimate of the ultimate load, at least for most practical purposes and provided the strength ratio c_2/c_1 is greater than about 0.5. If this average strength is used for cases where c_2/c_1 is less than 0.5 the bearing capacity will be overestimated. The error for a very weak middle layer ($c_2/c_1 = 0.1$) is approximately 33%. It is also worth noting that the two-layer approach suggested by Brown and Meyerhof (1969) and summarized in Equation (3.19) is generally reasonable for this case only when the strength ratio c_2/c_1 is greater than about 0.7. For strength ratios less than 0.7, Equation (3.19) significantly underestimates the ultimate capacity, presumably because it ignores the higher strength of the bottom layer. It is worth noting that the upper bound solutions for a two-layer system published by Merifield *et al.* (1998) and illustrated in Figure 3.17, provide quite reasonable estimates (typically within 10 to 20%) of the ultimate capacity for this three-layer problem. It should be noted however, that these findings are relevant only for the particular geometry (layer thicknesses) indicated in Figure 3.22. Whether they could be extended to other situations requires further investigation.

3.3.2 *Clay "sandwich" - stiff centre*

For this case the predicted ultimate bearing capacity is only slightly larger (1 to 2%) than the capacity predicted for a uniform clay layer with undrained strength equal to that of the top layer. Clearly, in this case the capacity is derived predominantly from the strength of the material of the top layer and is almost unaffected by the presence of a stronger underlying middle layer. It is expected that this result would not hold for cases where the layer thicknesses and strength ratios are different from those adopted in this example. A more complete description of the effects of layer thickness and strength ratio requires further research. The results presented by Merifield *et al.* (1998) also have significant application to this particular problem.

3.3.3 *Clay – strengthening with depth*

The only significant difference between this case and the one discussed immediately above is that the strengths of the three layers increase monotonically with depth so that the bottom layer is even stronger than the middle layer. Within the accuracy of the finite element predictions the ultimate capacities are essentially the same, and in both cases the capacity is only 1 or 2% more than that of a uniform clay deposit having the same strength as the top layer.

Table 3.2. Bearing capacity cases for soils with three layers.

Case	Layer 1 (Top) c_1 (kPa)	ϕ_1 (°)	γ_1 (kN/m³)	Layer 2 (Middle) c_2 (kPa)	ϕ_2 (°)	γ_2 (kN/m³)	Layer 3 (Bottom) c_3 (kPa)	ϕ_3 (°)	γ_3 (kN/m³)
Clay "Sandwich" - Soft Centre	c_1	0	0	$< c_1$	0	0	$= c_1$	0	0
Clay "Sandwich" - Stiff Centre	c_1	0	0	$> c_1$	0	0	$= c_1$	0	0
Clay – Strengthening With Depth	c_1	0	0	$= 2c_1$	0	0	$= 4c_1$	0	0
Clay – Weakening With Depth	c_1	0	0	$= 0.5c_1$	0	0	$= 0.25c_1$	0	0
Clay "Sandwich" – Sand Centre	100	0	20	0	30	20	100	0	20
Sand "Sandwich" – Clay Centre	0	30	20	100	0	20	0	30	20

It is instructive to investigate whether the solutions presented by Davis and Booker (1973) for strengthening soil profiles can provide reasonably accurate predictions in this case. Two of the most obvious idealizations of the strength profile are:

$$s_u = 0.5c_1 + \left(\frac{2c_1}{B}\right)z \qquad (3.28)$$

and

$$s_u = c_1 \qquad for\ z < B/2$$
$$s_u = \left(\frac{2c_1}{B}\right)z \qquad for\ z > B/2 \qquad (3.29)$$

Equation (3.28) is an approximation assuming strength increasing linearly with depth from a finite value at the surface, while Equation (3.29) is the most obvious approximation for the case of a crust of constant strength overlying material whose strength increases linearly with depth. These profiles are illustrated in Figure 3.24.

For the strength profile indicated in Equation (3.28), Figure 3.14a together with Equation (3.17a) indicates an average value of the average bearing pressure at failure for a smooth strip footing, 1m wide, of approximately $4.1c_1$. This estimate is well below (approximately 80% of) the value predicted by the finite element analysis. A much closer match to the finite element prediction is given by the case depicted in Figure 3.14b, viz. approximately $5.15c_1$, indicating the dominant influence of the strength of the uppermost crustal layer and the relative insignificance in this case of the increase in strength beneath the crust.

3.3.4 *Clay – weakening with depth*

In this slightly unusual case, which could correspond in practice to a clay deposit with a thick desiccated or weathered crust overlying weaker but probably overconsolidated clay, the ultimate capacity predicted by the finite element analysis is approximately $3.1c_1$. This is more than 20% less than the finite element prediction for the case of a soft centre layer discussed in section 3.3.1. It is also approximately 10 to 15% less than the predictions by Brown and Meyerhof and Merifield *et al.* for a two-layered clay deposit. Clearly, in this case the weaker bottom layer has a significant influence on the overall capacity, but it is difficult to devise a simple approach for estimating the capacity of the footing for this type of strength variation.

3.3.5 *Clay "sandwich" – sand centre*

In this case the ultimate capacity computed by the finite element model is not much less than for a uniform clay layer with undrained shear strength of 100 kPa. The sand layer has only a small influence of the overall capacity of the footing.

3.3.6 *Sand "sandwich" – clay centre*

For this case the finite element prediction of the ultimate capacity, q_u, was approximately 98 kPa. This can be compared with the prediction of the bearing capacity for a uniform sand of 86 kPa, obtained using Equation (3.1) and the bearing capacity factors given in Table 3.1. It can be deduced from these results that the presence of the stiff clay beneath the sand has made a small contribution to the bearing capacity, taking it slightly above the value for sand alone.

For this case, it is also instructive to calculate what the capacity would be if the two-layer method proposed by Okamura *et al.*, Equation (3.21), were to be used. Application of Equation (3.21), which in this case is inappropriate, produces an ultimate capacity of 812 kPa., clearly far in excess of the values quoted previously for this problem. The reason for this discrepancy was explained by Okamura *et al.* themselves, when they pointed out that values estimated by their method must always be compared with predictions for a sand layer alone, and the smaller value should be chosen as the bearing capacity of the layered subsoil. This case therefore highlights the necessity of being aware of the limitations of all design methods, if very substantial errors in estimating the bearing capacity are to be avoided.

3.4 *Summary*

From the preceding discussion of the bearing capacity of shallow foundations, the following conclusions can be drawn.

1. The use of conventional theory, based on the original approach suggested by Terzaghi and extended by others such as Vesic, to calculate the bearing capacity of a foundation on homogeneous soil has stood the test of time, and is generally regarded as being reliable for use in engineering practice. Although this approach is approximate and makes a number of simplifying assumptions, as identified above, it is considered acceptable for most practical problems of shallow footings on relatively homogeneous soils. However, the use of the outdated and inaccurate information regarding some of the bearing capacity factors, particularly the factor N_γ, should be discontinued. The factors set out in Table 3.1 are the most reliable values available, at least from a theoretical standpoint, and their use is therefore recommended.
2. Significant developments have been made in recent times concerning methods for estimating the ultimate load capacity of footings subjected to combinations of vertical, horizontal and moment loading. Failure loci such as those expressed by Equation (3.9) have been proposed, and should see increasing use in geotechnical practice. However, although some experimental justification has been provided for them, there is a need for more work of this type.
3. The effective width method, commonly used in the analysis of foundations subjected to eccentric loading, provides good

approximations to the collapse loads. Its continued use in practice therefore appears justified.

4. Improvements have also been made in the area of non-homogeneous and layered soils. Theoretically sound and relatively simple to use design methods are now available for cases involving the following:
   (a) Clays where the undrained shear strength increases linearly with depth,
   (b) Two layers of clay, and
   (c) A layer of sand overlying relatively soft clay.

   The work of Booker and Davis (Figure 3.14), Merifield *et al.* (Figure 3.16) and Okamura *et al.* (Equations (3.21) and (3.22) and Figure 3.21) have been influential with regard to cases (a), (b) and (c) listed above, and the judicious use of their results in practical design is recommended.
5. Sophisticated experimental and theoretical studies have highlighted the brittle nature of footing system behaviour that can occur when relatively thin stronger soils, loaded by surface footings, overlie much weaker materials.
6. It would appear that to date the problem of predicting the bearing capacity of multiple layers of soil lying beneath a footing and within its zone of interest remains beyond the means of relatively simple hand calculation methods. The major reason that relatively little progress has been made to date seems to be the large number of different cases that may be encountered in practice and still require analysis. Some problems of this type were addressed in section 3.3, and only in the simplest case of three layers of clay could simple design rules be deduced. This problem area requires further investigation.

4 SETTLEMENT OF SHALLOW FOOTINGS

4.1 *Introduction*

The main objective of this section is to examine and evaluate some procedures for predicting the settlement of shallow foundations in the light of relatively recent research. Ideally, such an evaluation should consider both the theoretical "correctness" of the methods and also their applicability to practical cases. However, primary attention will be paid here to identifying the shortcomings and limitations of the methods when compared to modern theoretical approaches. Two common problems will be considered:

- Settlement of shallow foundations on clay
- Settlement of shallow foundations on sand.

In each case, an attempt will be made to suggest whether the prediction methods considered can be adopted, or alternatively, adapted to provide an improved prediction capability.

4.2 *Shallow foundations on clay*

Estimation of settlement and differential settlement is a fundamental aspect of the design of shallow foundations. For foundations on clay, Table 4.1 summarizes some of the available techniques and their capabilities. The traditional approach, first developed by Terzaghi, employs the one-dimensional method in which the settlement is assumed to arise from consolidation due to increases in effective stress caused by the dissipation of excess pore pressures. Because of its still widespread use, it is of interest to examine the capabilities and shortcomings of the method, when compared with more complete two- and three-dimensional methods.

The one-dimensional method has the following limitations:

1. It assumes that the foundation loading causes only vertical strains in the subsoil
2. It assumes that all the settlement arises from consolidation, and that settlements arising from immediate shear strains are negligible
3. It assumes that the dissipation of excess pore pressures occurs only in the vertical direction; any lateral dissipation of excess pore pressures is ignored.

Three-dimensional analyses involve, in one form or another, the integration of vertical strains to obtain the settlement of the foundation. In a three-dimensional situation employing the theory of elasticity, the vertical strain ε_1 may be obtained in terms of the increments of applied stress. Alternative forms of the strain versus stress increment relationship are as follows:

$$\varepsilon_1 = (\Delta\sigma_v - \nu(\Delta\sigma_x + \Delta\sigma_y)/E \quad (4.1a)$$

$$\varepsilon_1 = 1/3[(\Delta q/G) + \Delta p'/K] \quad (4.1b)$$

where $\Delta\sigma_v$ = increment in vertical stress; $\Delta\sigma_x$, $\Delta\sigma_y$ = increments in horizontal stresses in x and y direction; Δq = increment in deviator stress; $\Delta p'$ = increment in mean principal effective stress; E = Young's modulus; G = shear modulus; K = bulk modulus.

In applying the above equations, the stress increments are usually computed from elasticity theory. Appropriate values of E and ν must also be used in Equation (4.1): undrained values for immediate (undrained) strains and drained values for total (undrained plus consolidation) strains. In Equation (4.1b), for the undrained case in a saturated soil, $K = \infty$, while in a saturated soil, K can be related to the constrained modulus D by the following elasticity relationship:

$$K = (1+\nu')D/[3(1-\nu')] \quad (4.2)$$

where ν' = drained Poisson's ratio of the soil.

The values of E, G and K in the above equations can be related to stress or strain levels, as discussed in Section 8, so that non-linear soil behaviour can be accounted for in a simple manner. A fuller discussion of these procedures is given by Lehane and Fahey (2000).

4.2.1 *One-dimensional versus three-dimensional settlement analysis*

To examine the possible significance of the limitations of one-dimensional analysis, two very simple hypothetical examples are considered. The first involves a uniformly loaded circular footing resting on the surface of a homogeneous layer of overconsolidated clay, in which the soil stiffness is uniform with depth. The second involves the same footing on a layer of soft normally consolidated clay in which the soil stiffness increases linearly with depth, from a small initial value at the soil surface. The relationship between the one-dimensional compressibility, m_v, and drained Young's modulus, E', (for three-dimensional analysis) is assumed to be that given by elasticity theory for an ideal two-phase elastic soil skeleton, as is the relationship between the undrained Young's modulus E_u and E', i.e.

$$m_v = \frac{(1+\nu')(1-2\nu')}{(1-\nu')E'} \quad (4.3)$$

$$E_u = \frac{3E'}{2(1+\nu')} \quad (4.4)$$

where ν' = drained Poisson's ratio of soil skeleton.

Figure 4.1 shows the ratio of the one – dimensional settlement (excluding creep) to the correct three-dimensional total final settlement (Davis and Poulos, 1968). For the overconsolidated clay layer, the one-dimensional analysis gives a good approximation to the correct total settlement when the drained Poisson's ratio of the soil layer is less than about 0.35, even for relatively deep soil layers. The one-dimensional analysis tends to under-predict the settlement as the drained Poisson's ratio of the soil increases or the relative layer depth increases. For the soft clay layer, the one-dimensional analysis again gives a remarkably good approximation to the total final settlement if the drained Poisson's ratio of the soil layer is 0.35 or less.