Finite element modeling and analysis

Finite element modeling and analysis

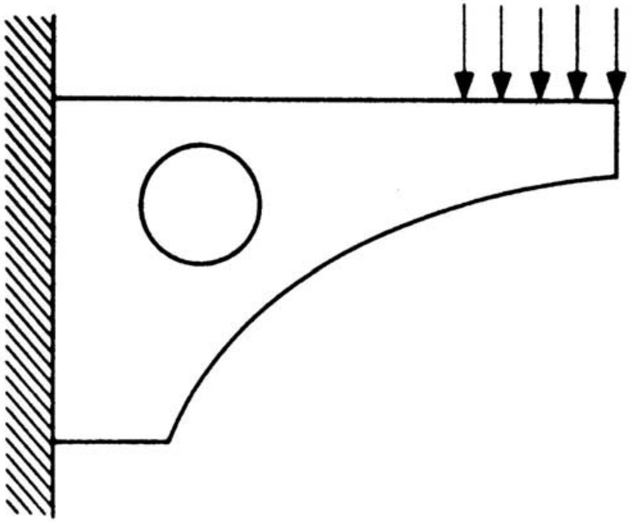
- Structural problems
  - stress, strain distribution
- Heat transfer problems
  - temperature distribution
- Electrostatic potential problems
- Fluid mechanics problems
- Vibration analysis problems

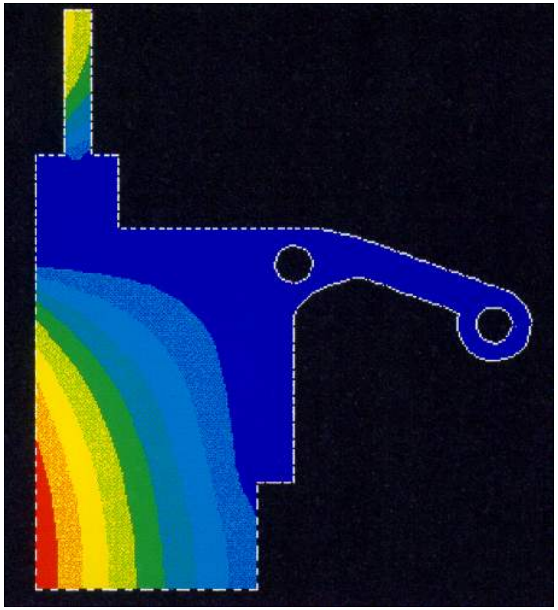
Finite element modeling and analysis – cont’

- When geometry of problem domain is complicated, analytic solution can not be derived
  - Numerical approximate solution

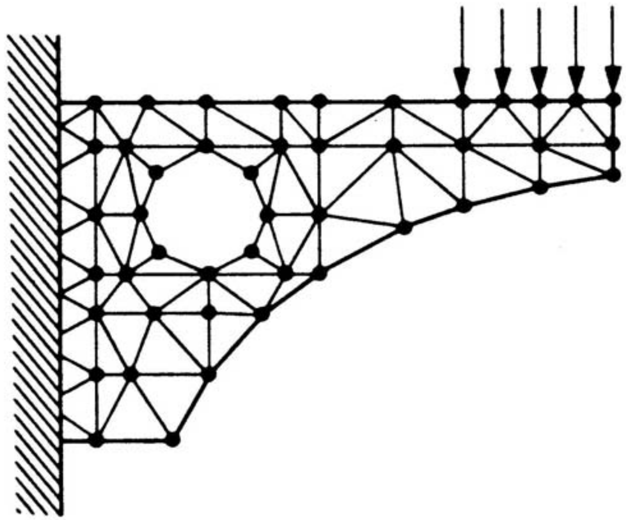
- Even for simple geometry, analytic solution cannot be obtained if the object is composed of composite material

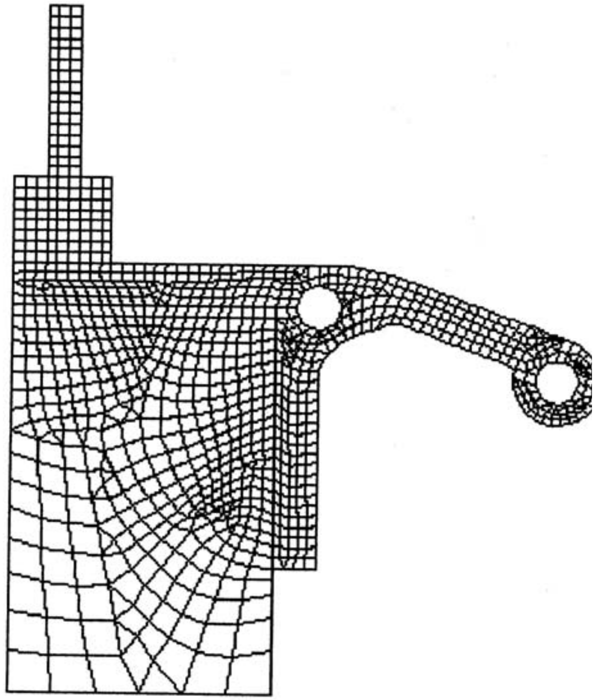
Problems which cannot be solved analytically





Approximation of each object by an assemblage of finite element





Choice of element

- Select element type among those supported by the FEM package in use
- Select element type mostly suitable for the given problem ← know-how
- Determining element size is also a know-how
- Use small elements where rapid change in stress or strain is expected

Analysis procedure

- Input material properties and boundary condition after elements are generated

  -> Specify B.C at element nodes

  B.C known displacement

  known external force

  known temperature

- Pre-processor: generate meshes automatically and allow the interactive input of B.C's

Analysis procedure – cont'

- System equations are composed by FEM package
- System equation
  - Relation between B.C. and unknown (displacement, temperature, ...) at every node

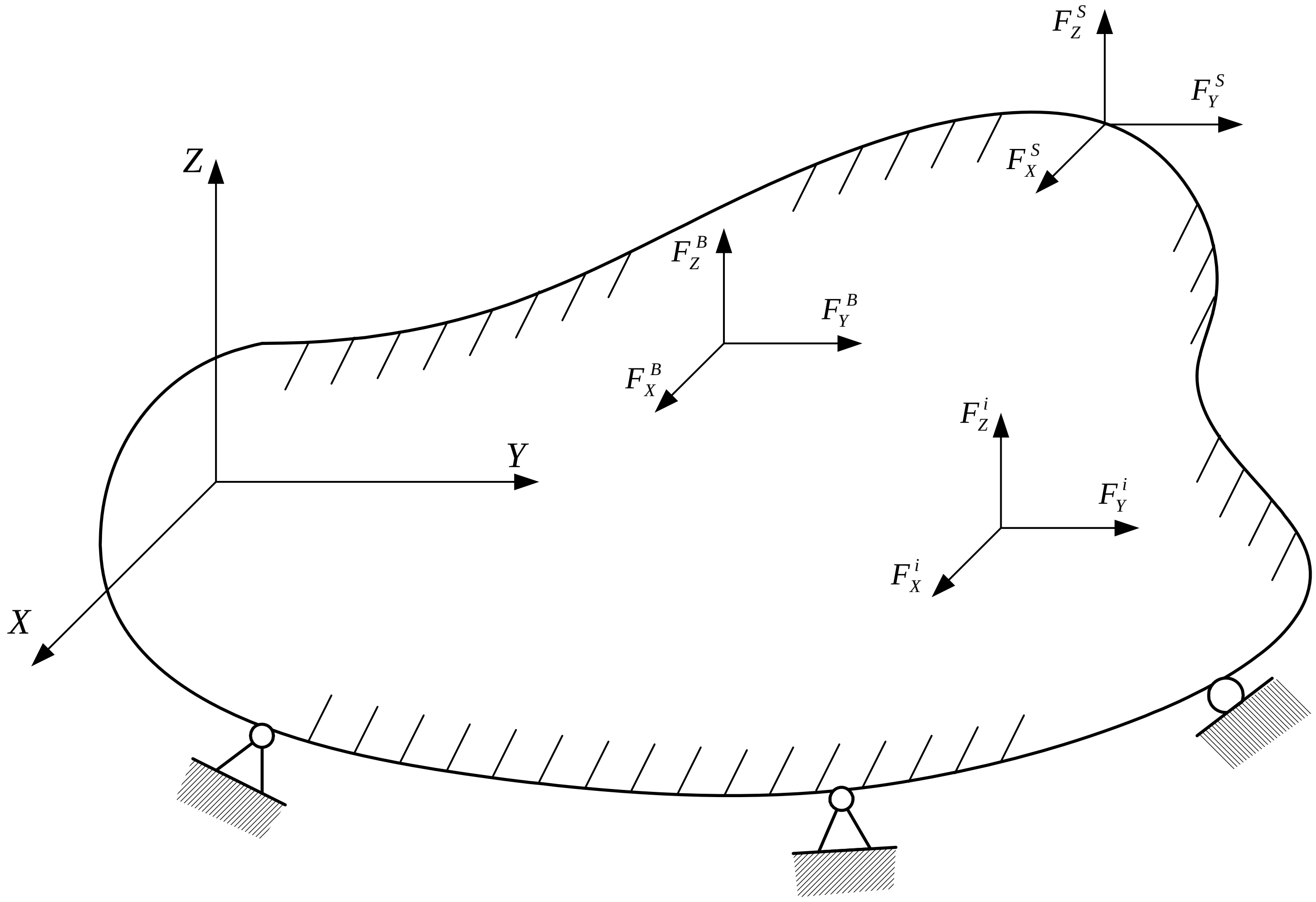
-> By solving system equation, unknowns at each node is obtained.

Analysis procedure – cont'

- Unknown values (displacement, temperature, stress, strain, ...) at arbitrary location is obtained by interpolating the values at nodes

- Post-processor displays color coding or contour plot

Formulation of system equation



Formulation of system eq. – cont'

$\mathbf{f}^{S}$: external surface tractions

$\mathbf{f}^{B}$: body forces

$\mathbf{F}^{i}$: concentrated external forces

Formulation of system eq. – cont'

- Only displacement at any X,Y,Z

$$U^T = [U(X,Y,Z), V(X,Y,Z), W(X,Y,Z)]$$

needs to be solved

→ Strain and stress are derived from U

$$\varepsilon^T = [\varepsilon_{XX}\ \varepsilon_{YY}\ \varepsilon_{ZZ}\ \gamma_{XY}\ \gamma_{YZ}\ \gamma_{ZX}]$$

$$\tau^T = [\tau_{XX}\ \tau_{YY}\ \tau_{ZZ}\ \tau_{XY}\ \tau_{YZ}\ \tau_{ZX}]$$

Formulation of system eq. – cont'

- Principle of virtual displacement
  - total external virtual work

    = total internal virtual work

$$\int_V \overline{\varepsilon}^T \tau \, dv = \int_V \overline{\mathbf{U}}^T \mathbf{f}^B dV + \int_S \overline{\mathbf{U}}^{S^T} \mathbf{f}^S dS + \sum_i \overline{\mathbf{U}}^{i^T} \mathbf{F}^i$$

' – 'indicates the quantities caused by virtual displacement

Formulation of system eq. – cont'

- Let displacement of all the nodes $\widehat{\mathbf{U}}$

$$\widehat{\mathbf{U}}^{T} = [\mathbf{u}_1\ \mathbf{v}_1\ \mathbf{w}_1\ \mathbf{u}_2\ \mathbf{v}_2\ \mathbf{w}_2 \cdots \mathbf{u}_N\ \mathbf{v}_N\ \mathbf{w}_N]$$
$$= [\mathrm{U}_1\ \mathrm{U}_2\ \mathrm{U}_3\ \cdots \mathrm{U}_n\ ]$$

N : no. of nodes

n=3N : degree of freedom

Formulation of system eq. – cont’

- Displcament at arbitrary location in an element :

–> Is approximated by proper shape function that interpolates the displacements at nodes of the element

–> Thus $\widehat{\mathbf{U}}^{T}$ is the unknown to be solved

Formulation of system eq. – cont'

$\mathbf{u}^{(m)}(x,y,z) = \mathbf{H}^{(m)}(x,y,z)\hat{\mathbf{U}}$: displacement at arbitrary x,y,z in m-element

$\varepsilon^{(m)}(x,y,z) = \mathbf{B}^{(m)}(x,y,z)\,\hat{\mathbf{U}}$:Strain is obtained by differentiating displacement

$\tau^{(m)} = \mathbf{C}^{(m)}\varepsilon^{(m)} + \tau^{I(m)}$: stress – strain relation

element initial stress

Formulation of system eq. – cont'

$$\sum_m \int_{V^{(m)}} \bar{\boldsymbol{\varepsilon}}^{(m)^T} \boldsymbol{\tau}^{(m)} dV^{(m)} = \sum_m \int_{V^{(m)}} \bar{\mathbf{u}}^{(m)^T} \mathbf{f}^{B(m)} dV^{(m)} + \sum_m \int_{S^{(m)}} \bar{\mathbf{u}}^{S(m)^T} \mathbf{f}^{S(m)} dS^{(m)} + \sum_i \bar{\mathbf{U}}^{i^T} \mathbf{F}^i$$

$$\bar{\hat{\mathbf{U}}}^T \left[\sum_m \int_{V^{(m)}} \mathbf{B}^{(m)^T} \mathbf{C}^{(m)} \mathbf{B}^{(m)} dV^{(m)} \right] \hat{\mathbf{U}} = \bar{\hat{\mathbf{U}}}^T \left[\left\{ \sum_m \int_{V^{(m)}} \mathbf{H}^{(m)^T} \mathbf{f}^{B(m)} dV^{(m)} \right\} + \left\{ \sum_m \int_{S^{(m)}} \mathbf{H}^{S(m)^T} \mathbf{f}^{S(m)} dS^{(m)} \right\} - \left\{ \sum_m \int_{V^{(m)}} \mathbf{B}^{(m)^T} \boldsymbol{\tau}^{I(m)} dV^{(m)} \right\} + \mathbf{F} \right]$$

-> Virtual nodal displacement

Formulation of system eq. – cont'

$$\left[\sum_m \int_{V^{(m)}} \mathbf{B}^{(m)^T} \mathbf{C}^{(m)} \mathbf{B}^{(m)} dV^{(m)}\right] \hat{\mathbf{U}} = \sum_m \int_{V^{(m)}} \mathbf{H}^{(m)^T} \mathbf{f}^{B(m)} dV^{(m)}$$

$$+ \sum_m \int_{S^{(m)}} \mathbf{H}^{S(m)^T} \mathbf{f}^{S(m)} dS^{(m)}$$

$$- \sum_m \int_{V^{(m)}} \mathbf{B}^{(m)^T} \boldsymbol{\tau}^{I(m)} dV^{(m)} + \mathbf{F}$$

$$\mathbf{K}\,\hat{\mathbf{U}} = \mathbf{R}$$

Formulation of system eq. – cont'

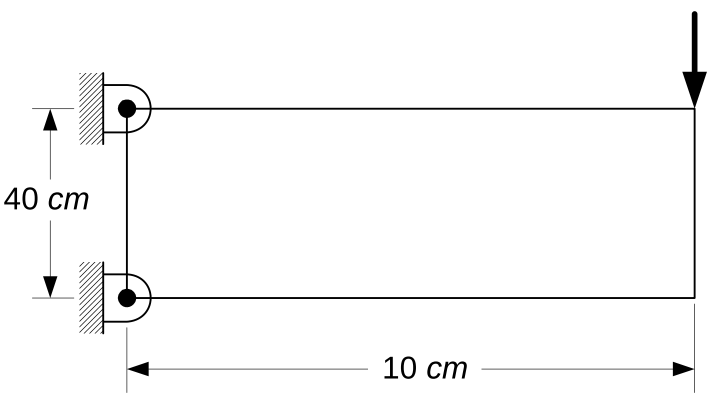
- Role of FEM package is to evaluate the integrals for **K**, **R**

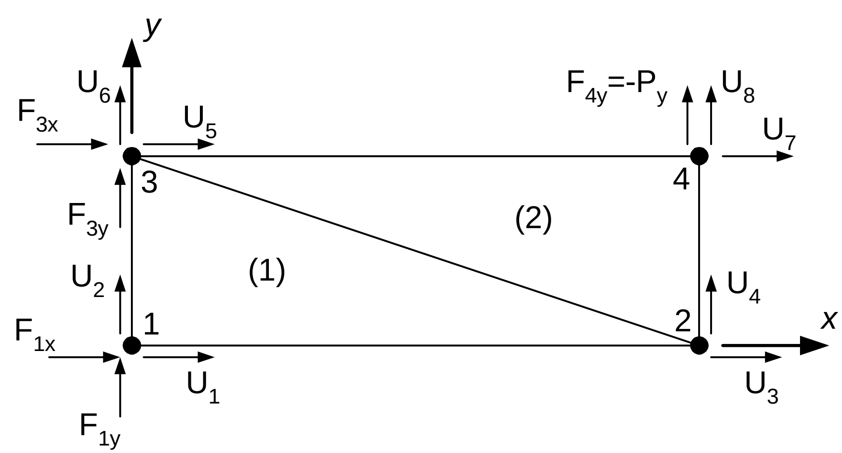
-> Solve for displacements at all the nodes

-> Evaluate displacement, stress, strain at any location in each element using shape function

Example of Formulation

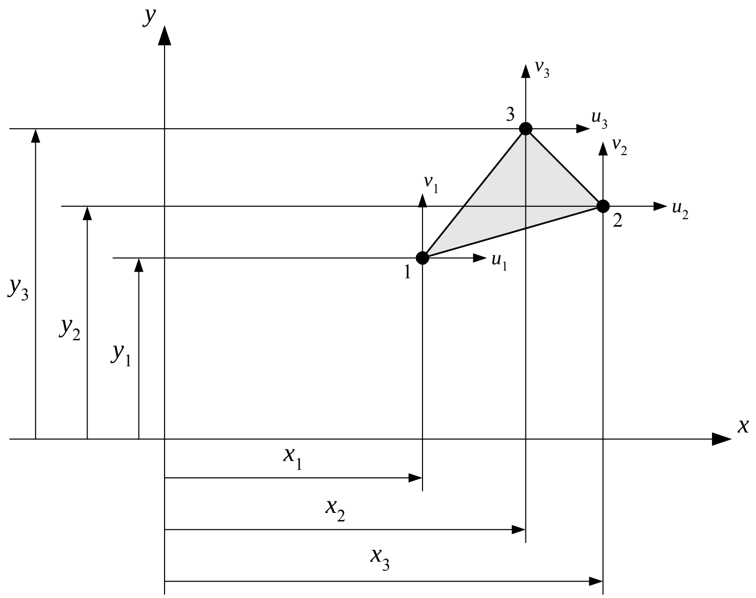
- Derive the system equations of the plate using a two-element model
- Thickness 1 *cm*. Load P_y is applied very slowly so that inertial effect is ignored





Example of Formulation – cont’

- Deriving the displacement transformation matrix $\mathbf{H}^{(i)}$ of any element



$$u = \alpha_1 + \alpha_2 x + \alpha_3 y$$

$$v = \beta_1 + \beta_2 x + \beta_3 y$$

the displacements of each node

$$u_1 = \alpha_1 + \alpha_2 x_1 + \alpha_3 y_1 \qquad v_1 = \beta_1 + \beta_2 x_1 + \beta_3 y_1$$

$$u_2 = \alpha_1 + \alpha_2 x_2 + \alpha_3 y_2 \qquad v_2 = \beta_1 + \beta_2 x_2 + \beta_3 y_2$$

$$u_3 = \alpha_1 + \alpha_2 x_3 + \alpha_3 y_3 \qquad v_3 = \beta_1 + \beta_2 x_3 + \beta_3 y_3$$

Example of Formulation – cont'

- the constants α_i, β_i can be determined in terms of u_i, v_i and x_i, y_i

$$\alpha_1 = \frac{a_1u_1 + a_2u_2 + a_3u_3}{2a} \qquad \beta_1 = \frac{a_1v_1 + a_2v_2 + a_3v_3}{2a}$$

$$\alpha_2 = \frac{b_1u_1 + b_2u_2 + b_3u_3}{2a} \qquad \beta_2 = \frac{b_1v_1 + b_2v_2 + b_3v_3}{2a}$$

$$\alpha_3 = \frac{c_1u_1 + c_2u_2 + c_3u_3}{2a} \qquad \beta_3 = \frac{c_1v_1 + c_2v_2 + c_3v_3}{2a}$$

$$a_1 = x_2y_3 - x_3y_2 \quad b_1 = y_2 - y_3 \quad c_1 = x_3 - x_2 \qquad 2a = a_1 + a_2 + a_3$$

$$a_2 = x_3y_1 - x_1y_3 \quad b_2 = y_3 - y_1 \quad c_2 = x_1 - x_3$$

$$a_3 = x_1y_2 - x_2y_1 \quad b_3 = y_1 - y_2 \quad c_3 = x_2 - x_1$$

Example of Formulation – cont'

- Substituting the values of $x_1, x_2, x_3, y_1, y_2, y_3$ of element 1 into the previous equations

$$u = u_1 + (-\frac{1}{10}u_1 + \frac{1}{10}u_2)x + (-\frac{1}{4}u_1 + \frac{1}{4}u_3)y$$

$$v = v_1 + (-\frac{1}{10}v_1 + \frac{1}{10}v_2)x + (-\frac{1}{4}v_1 + \frac{1}{4}v_3)y$$

Example of Formulation – cont'

$$\begin{bmatrix} u \\ v \end{bmatrix}^{(1)} = \begin{bmatrix} (1-\frac{1}{10}x-\frac{1}{4}y) & 0 & \frac{1}{10}x & 0 & \frac{1}{4}y & 0 & 0 & 0 \\ 0 & (1-\frac{1}{10}x-\frac{1}{4}y) & 0 & \frac{1}{10}x & 0 & \frac{1}{4}y & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix}^{(2)} = \begin{bmatrix} 0 & 0 & (1-\frac{y}{4}) & 0 & (1-\frac{x}{10}) & 0 & (-1+\frac{x}{10}+\frac{y}{4}) & 0 \\ 0 & 0 & 0 & (1-\frac{y}{4}) & 0 & (1-\frac{x}{10}) & 0 & (-1+\frac{x}{10}+\frac{y}{4}) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix}$$

Example of Formulation – cont'

- Determining the strains or the matrix $\mathbf{B}^{(m)}$ using the following two-dimensional strain-displacement relations

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Example of Formulation – cont'

$$\boldsymbol{\varepsilon}^{(1)} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}^{(1)} = \begin{bmatrix} -\frac{1}{10} & 0 & \frac{1}{10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ -\frac{1}{4} & -\frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{4} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix}$$

$$= \mathbf{B}^{(1)}\mathbf{U}$$

$$\boldsymbol{\varepsilon}^{(2)} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & -\frac{1}{10} & 0 & \frac{1}{10} & 0 \\ 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -\frac{1}{4} & 0 & 0 & -\frac{1}{10} & \frac{1}{4} & \frac{1}{10} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix}$$

$$= \mathbf{B}^{(2)}\mathbf{U}$$

Example of Formulation – cont'

- The stress–strain relations for a homogeneous, isotropic plane–stress element are given by

$$\boldsymbol{\tau}^{(m)} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Example of Formulation – cont’

- We assumed that there is no initial stress on the structure
- The stiffness matrix of each element is derived as follows by substituting the previous results
- Since two elements are made of the same material, the expression for $\mathbf{C}^{(m)}$ in Equation $\boldsymbol{\tau}^{(m)}$ can be used for both the elements

Example of Formulation – cont’

$$\mathbf{K}^{(1)} = \int_{V^{(1)}} \mathbf{B}^{(1)^T} \mathbf{C}^{(1)} \mathbf{B}^{(1)} dV^{(1)}$$

$$= \int_{A^{(1)}} \mathbf{B}^{(1)^T} \mathbf{C}^{(1)} \mathbf{B}^{(1)} dA^{(1)}$$

$$= \int_{0}^{10} \mathbf{B}^{(1)^T} \mathbf{C}^{(1)} \mathbf{B}^{(1)} (4 - \frac{4}{10} x) dx$$

Example of Formulation – cont'

$$
= \frac{E}{1-\nu^2}\int_0^{10}
\begin{bmatrix}
-\frac{1}{10} & 0 & \frac{1}{10} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\
-\frac{1}{4} & -\frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{4} & 0 & 0 & 0
\end{bmatrix}^T
\begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix}
\begin{bmatrix}
-\frac{1}{10} & 0 & \frac{1}{10} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\
-\frac{1}{4} & -\frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{4} & 0 & 0 & 0
\end{bmatrix}
(4-\frac{4}{10}x)\,dx
$$

Example of Formulation – cont'

$$\mathbf{K}^{(1)} = \frac{20E}{1-\nu^2}\begin{bmatrix} \frac{1}{100}+\frac{1-\nu}{32} & \frac{1+\nu}{80} & -\frac{1}{100} & -\frac{1-\nu}{80} & -\frac{1-\nu}{32} & -\frac{\nu}{40} & 0 & 0 \\ \frac{1+\nu}{80} & \frac{1}{16}+\frac{1-\nu}{200} & -\frac{\nu}{40} & -\frac{1-\nu}{200} & -\frac{1-\nu}{80} & -\frac{1}{16} & 0 & 0 \\ -\frac{1}{100} & -\frac{\nu}{40} & \frac{1}{100} & 0 & 0 & \frac{\nu}{40} & 0 & 0 \\ -\frac{1-\nu}{80} & -\frac{1-\nu}{200} & 0 & \frac{1-\nu}{200} & \frac{1-\nu}{80} & 0 & 0 & 0 \\ -\frac{1-\nu}{32} & -\frac{1-\nu}{80} & 0 & \frac{1-\nu}{80} & \frac{1-\nu}{32} & 0 & 0 & 0 \\ -\frac{\nu}{40} & -\frac{1}{16} & \frac{\nu}{40} & 0 & 0 & \frac{1}{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Example of Formulation – cont'

$$\mathbf{K}^{(2)} = \int_{V^{(2)}} \mathbf{B}^{(2)^T} \mathbf{C}^{(2)} \mathbf{B}^{(2)} dV^{(2)}$$

$$= \int_{A^{(2)}} \mathbf{B}^{(2)^T} \mathbf{C}^{(2)} \mathbf{B}^{(2)} dA^{(2)}$$

$$= \int_0^{10} \mathbf{B}^{(2)^T} \mathbf{C}^{(2)} \mathbf{B}^{(2)} \frac{4}{10} x \, dx$$

Example of Formulation – cont'

$$
= \frac{20E}{1-\nu^2}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1-\nu}{32} & 0 & 0 & \frac{1-\nu}{80} & -\frac{1-\nu}{32} & -\frac{1-\nu}{80} \\
0 & 0 & 0 & \frac{1}{16} & \frac{\nu}{40} & 0 & -\frac{\nu}{40} & -\frac{1}{16} \\
0 & 0 & 0 & \frac{\nu}{40} & \frac{1}{100} & 0 & -\frac{1}{100} & -\frac{\nu}{40} \\
0 & 0 & \frac{1-\nu}{80} & 0 & 0 & \frac{1-\nu}{200} & -\frac{1-\nu}{80} & -\frac{1-\nu}{200} \\
0 & 0 & -\frac{1-\nu}{32} & -\frac{\nu}{40} & -\frac{1}{100} & -\frac{1-\nu}{80} & \frac{1}{100}+\frac{1-\nu}{32} & \frac{1+\nu}{80} \\
0 & 0 & -\frac{1-\nu}{80} & -\frac{1}{16} & -\frac{\nu}{40} & -\frac{1-\nu}{200} & \frac{1+\nu}{80} & \frac{1}{16}+\frac{1-\nu}{200}
\end{bmatrix}
$$

Example of Formulation – cont’

$$
\mathbf{K}^{(2)} = \frac{20E}{1-\nu^2}
\begin{bmatrix}
\frac{1}{100}+\frac{1-\nu}{32} & \frac{1+\nu}{80} & -\frac{1}{100} & -\frac{1-\nu}{80} & -\frac{1-\nu}{32} & -\frac{\nu}{40} & 0 & 0 \\
\frac{1+\nu}{80} & \frac{1}{16}+\frac{1-\nu}{200} & -\frac{\nu}{40} & -\frac{1-\nu}{200} & -\frac{1-\nu}{80} & -\frac{1}{16} & 0 & 0 \\
-\frac{1}{100} & -\frac{\nu}{40} & \frac{1}{100}+\frac{1-\nu}{32} & 0 & 0 & \frac{1+\nu}{80} & -\frac{1-\nu}{32} & -\frac{1-\nu}{80} \\
-\frac{1-\nu}{80} & -\frac{1-\nu}{200} & 0 & \frac{1}{16}+\frac{1-\nu}{200} & \frac{1+\nu}{80} & 0 & -\frac{\nu}{40} & -\frac{1}{16} \\
-\frac{1-\nu}{32} & -\frac{1-\nu}{80} & 0 & \frac{1+\nu}{80} & \frac{1}{100}+\frac{1-\nu}{32} & 0 & -\frac{1}{100} & -\frac{\nu}{40} \\
-\frac{\nu}{40} & -\frac{1}{16} & \frac{1+\nu}{80} & 0 & 0 & \frac{1}{16}+\frac{1-\nu}{200} & -\frac{1-\nu}{80} & -\frac{1-\nu}{200} \\
0 & 0 & -\frac{1-\nu}{32} & -\frac{\nu}{40} & -\frac{1}{100} & -\frac{1-\nu}{80} & \frac{1}{100}+\frac{1-\nu}{32} & \frac{1+\nu}{80} \\
0 & 0 & -\frac{1-\nu}{80} & -\frac{1}{16} & -\frac{\nu}{40} & -\frac{1-\nu}{200} & \frac{1+\nu}{80} & \frac{1}{16}+\frac{1-\nu}{200}
\end{bmatrix}
$$

Example of Formulation – cont'

- The load vector R is equal to $\mathbf{R}_C$ because there are only concentrated forces on the nodes

$$\mathbf{R} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ 0 \\ 0 \\ F_{3x} \\ F_{3y} \\ 0 \\ -P_y \end{bmatrix}$$

Example of Formulation – cont'

- Solving for the unknown nodal point displacements from the following equation

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & \cdot & \cdot & \cdot & k_{17} & k_{18} \\ k_{21} & k_{22} & k_{23} & \cdot & \cdot & \cdot & k_{27} & k_{28} \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ k_{71} & k_{72} & k_{73} & \cdot & \cdot & \cdot & k_{77} & k_{78} \\ k_{81} & k_{82} & k_{83} & \cdot & \cdot & \cdot & k_{87} & k_{88} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ 0 \\ 0 \\ F_{3x} \\ F_{3y} \\ 0 \\ -P_y \end{bmatrix}$$

Example of Formulation – cont'

- Applying the boundary conditions as below

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & \cdot & \cdot & \cdot & k_{17} & k_{18} \\ k_{21} & k_{22} & k_{23} & \cdot & \cdot & \cdot & k_{27} & k_{28} \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ k_{71} & k_{72} & k_{73} & \cdot & \cdot & \cdot & k_{77} & k_{78} \\ k_{81} & k_{82} & k_{83} & \cdot & \cdot & \cdot & k_{87} & k_{88} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ U_3 \\ U_4 \\ 0 \\ 0 \\ U_7 \\ U_8 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ 0 \\ 0 \\ F_{3x} \\ F_{3y} \\ 0 \\ -P_y \end{bmatrix}$$

Example of Formulation – cont’

- Partitioned into two parts as follows

$$\begin{bmatrix} k_{33} & k_{34} & k_{37} & k_{38} \\ k_{43} & k_{44} & k_{47} & k_{48} \\ k_{73} & k_{74} & k_{77} & k_{78} \\ k_{83} & k_{84} & k_{87} & k_{88} \end{bmatrix} \begin{bmatrix} U_3 \\ U_4 \\ U_7 \\ U_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -P_y \end{bmatrix}$$

$$\begin{bmatrix} k_{13} & k_{14} & k_{17} & k_{18} \\ k_{23} & k_{24} & k_{27} & k_{28} \\ k_{53} & k_{54} & k_{57} & k_{58} \\ k_{63} & k_{64} & k_{67} & k_{68} \end{bmatrix} \begin{bmatrix} U_3 \\ U_4 \\ U_7 \\ U_8 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{3x} \\ F_{3y} \end{bmatrix}$$

Finite Element Modeling

- Preparing data to do finite element analysis
- Building geometry, generating meshes, applying boundary conditions and loads, providing material properties, specifying analysis type
- Done by using preprocessor

Finite Element Modeling – cont'

1. Export CAD data to pre-processor through IGES file or CAD file directly

"Defeaturing" geometry may be necessary to hide features that do not affect the accuracy

Finite Element Modeling – cont'

2. Generate meshes and nodes

Automatic tetrahedral meshing for solid geometry

Quadrilateral or triangular elements for 3D surface, shell, 2D geometry.

Mesh density can be controlled

Finite Element Modeling – cont'

3. Select type of analysis (static/dynamic, linear/non-linear, plane stress, plane strain)
4. Assign boundary conditions: B.C.'s at arbitrary locations can be converted to equivalent B.C's at nodes
5. Assign material properties : Different properties for each element can be assigned
6. Activate analysis code
7. Output results using post-processor