5. Bond, Anchorage, and Development Length

FUNDAMENTALS of FLEXURAL BOND

BOND STRENGTH & DEVELOPMENT LENGTH

KCI CODE PROVISIONS

ANCHORAGE of TENSION by HOOKS

ANCHORAGE REQUIREMENT FOR WEB REBARS

DEVELOPMENT of BARS in COMPRESSION

BAR CUTOFF AND BEND POINT in BEAMS

INTEGRATED BEAM DESIGN EXAMPLE

BAR SPLICES

447.327
Theory of Reinforced Concrete and Lab. I
Spring 2008

5. Bond / Anchorage / Develop. Length

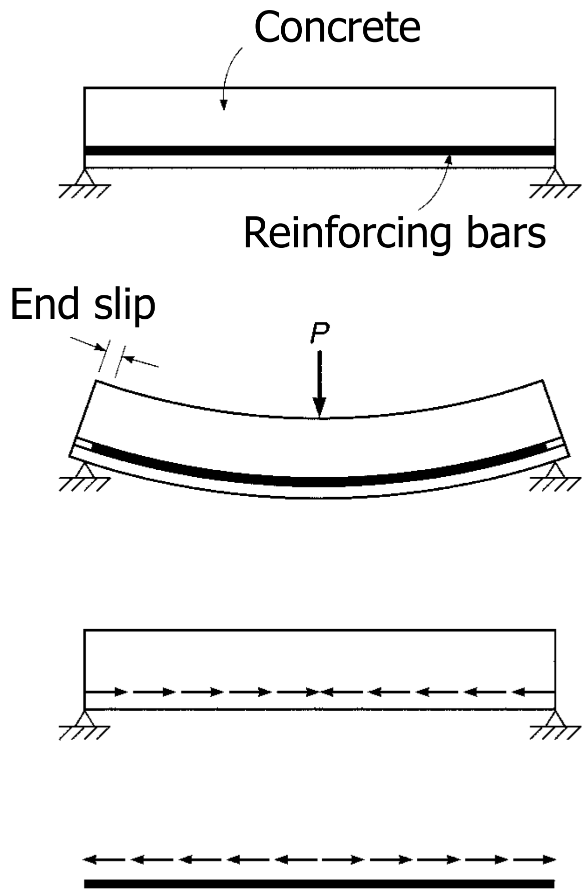
FUNDAMENTALS OF FLEXURAL BOND

(a) beam before loading

(b) unrestrained slip between concrete and steel

(c) bond forces acting on concrete

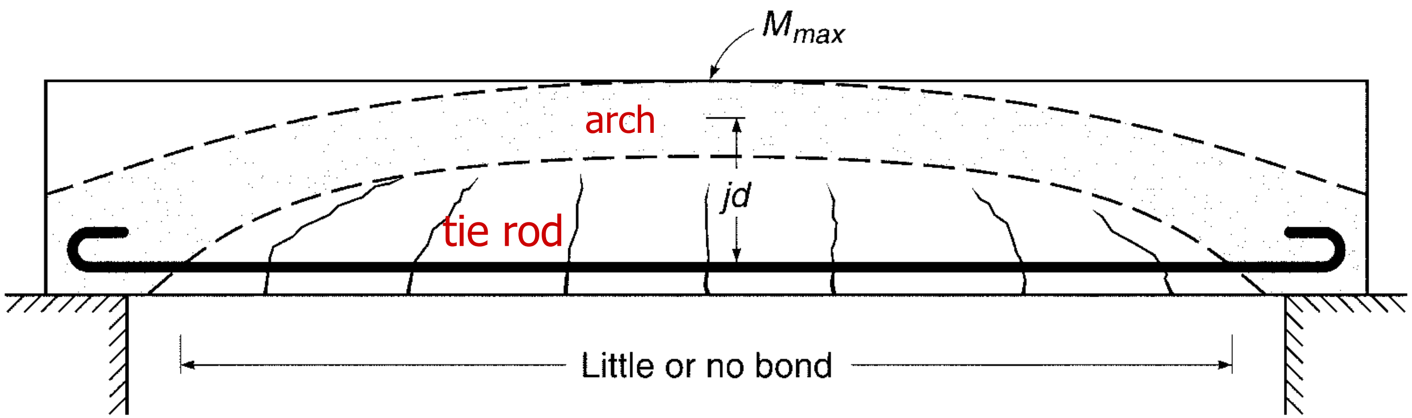
(d) bond forces acting on steel



5. Bond / Anchorage / Develop. Length

- Bond between PLAIN bar and concrete is resisted by chemical adhesion and mechanical friction

⇨ Due to the weakness of bond strength, end ANCHORAGE was provided in the form of HOOKs.



⇨ If the anchorage is adequate, above beam does not collapse even if the bond is broken over the entire length.

5. Bond / Anchorage / Develop. Length

- In this case, the bond is broken over the bar length.

  ; The force in the steel, T, is CONSTANT over the entire unbonded length

$$T = \frac{M_{\max}}{jd} \qquad (1)$$

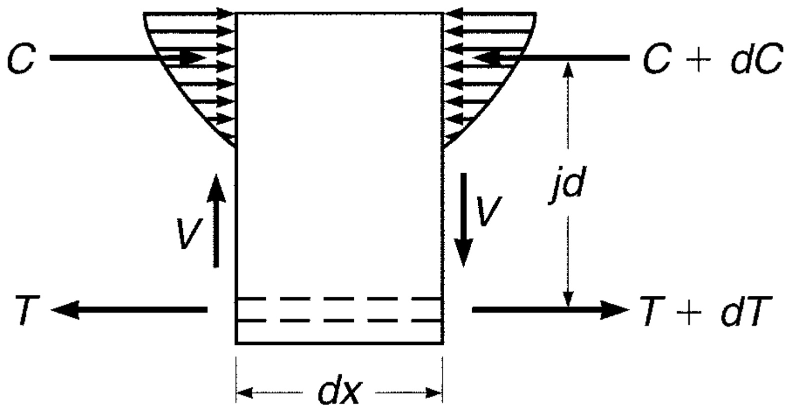
  The total steel elongation is larger than in beam in which bond is preserved.

  ⇨ large deflection and greater crack width

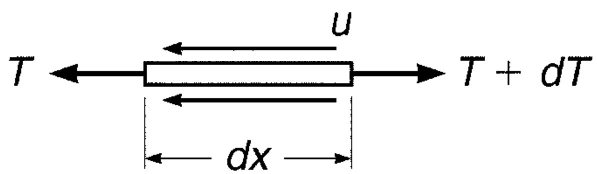
- To improve this situation, deformed bars are provided.

5. Bond / Anchorage / Develop. Length

Bond Force Based on Simple Cracked Section Analysis



(a) Free-body sketch of reinforced concrete element



(b) Free-body sketch of steel element

- Consider a reinforced concrete beam with small length *dx*. The change in bending moment *dM* produces a change in the bar force.

$$dT = \frac{dM}{jd} \qquad (2)$$

This change in bar force is resisted by bond forces at the interface between concrete and steel.

5. Bond / Anchorage / Develop. Length

$$Udx = dT \quad \Rightarrow \quad U = \frac{dT}{dx} \qquad (3)$$

, where U is the magnitude of the local bond force per unit length.

Alternatively,

$$U = \frac{1}{jd}\frac{dM}{dx} \qquad (4)$$

$$\Rightarrow \quad U = \frac{V}{jd} \qquad (5)$$

Eq.(5) is the "elastic cracked section equation" for flexural bond force.

; Bond force per unit length is proportional to the shear at a particular section.

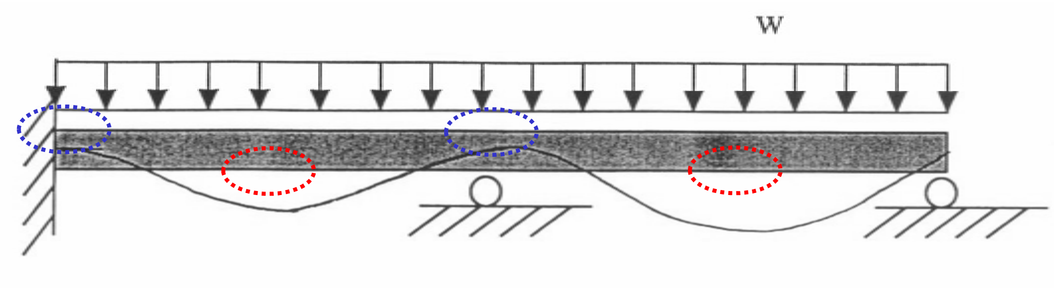
5. Bond / Anchorage / Develop. Length

Bond Force Based on Simple Cracked Section Analysis

Note

– Eq.(5) applies to the tension bars in a concrete zone that is assumed FULLY CRACKED.

; No resist to tension

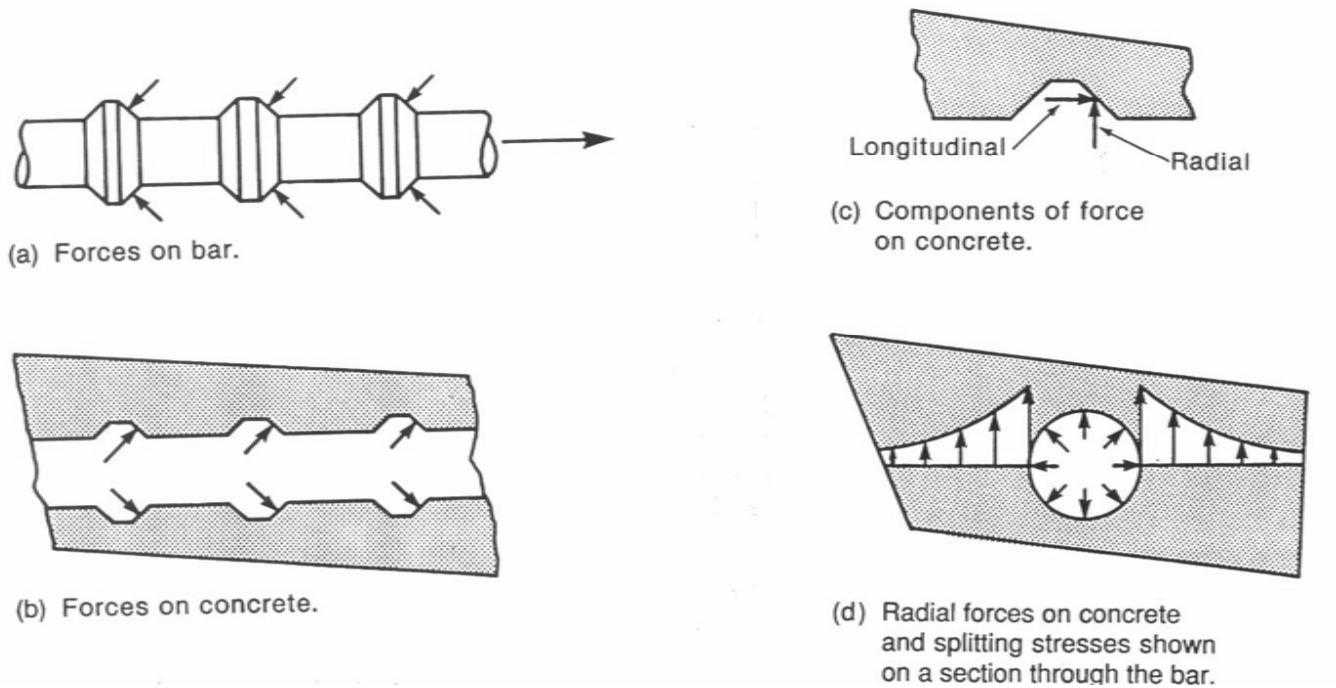


– It does NOT apply to compression reinforcement, for which it can be shown that the flexural bond forces are very low.

5. Bond / Anchorage / Develop. Length

Actual Distribution of Flexural Bond Force

Interlocking mechanism of deformed bar



(a) Forces on bar.

(b) Forces on concrete.

(c) Components of force on concrete.

(d) Radial forces on concrete and splitting stresses shown on a section through the bar.

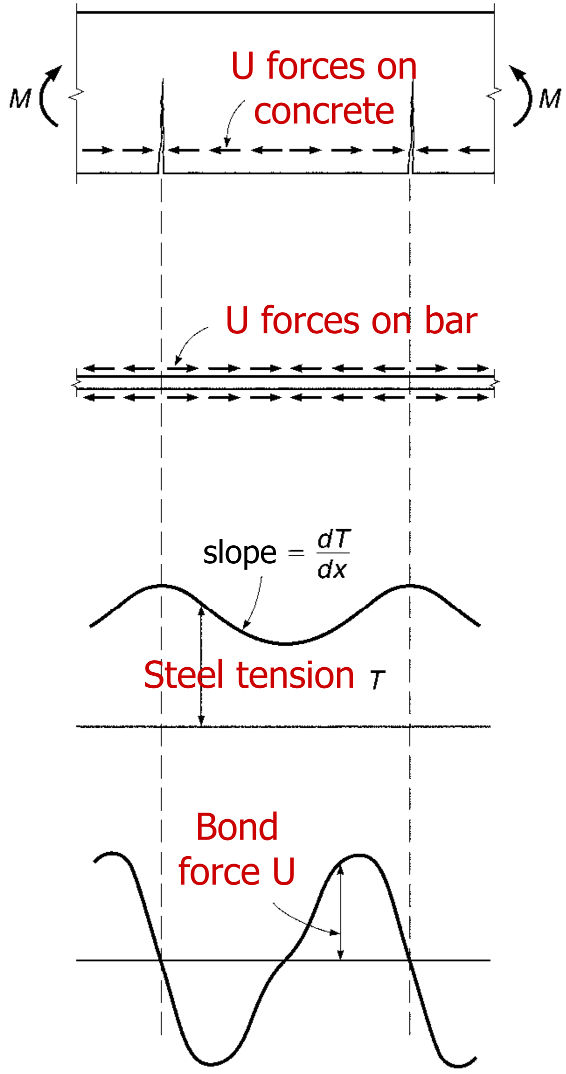
5. Bond / Anchorage / Develop. Length

(a) cracked concrete segment

(b) bond forces acting on reinforcing bar

(c) variation of tensile force in steel

(d) variation of bond force along steel



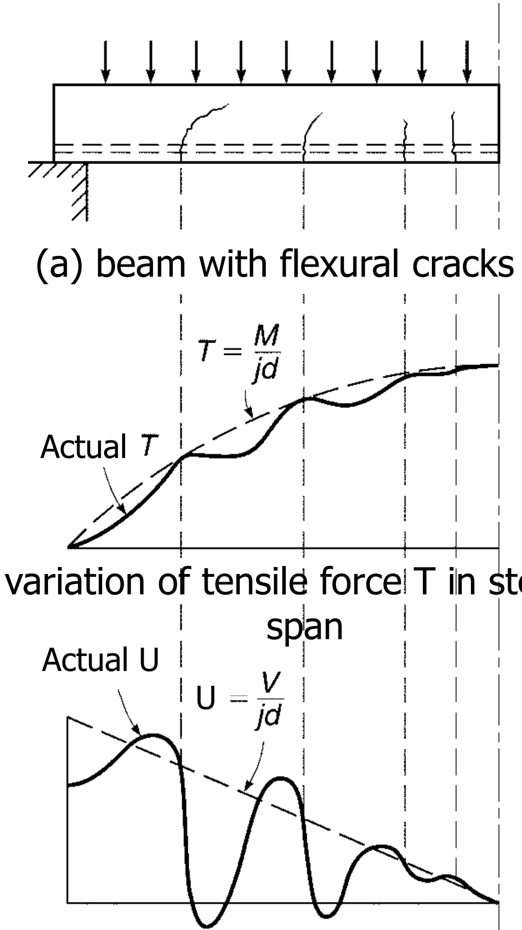
5. Bond / Anchorage / Develop. Length

- Between cracks, the concrete resist moderate amount of tension. (Fig 5.4(a))
  - ⇦ By the bond force acting along the interface. (Fig 5.4(b))
  - ⇨ This reduces the tensile force in the steel. (Fig 5.4(c))
- At crack, the steel tension has the maximum value of $T=M/jd$ (Fig 5.4(c))
- Fig 5.4(d) supports that U is proportional to the ratio of change of bar force (= dT/dx)

5. Bond / Anchorage / Develop. Length

Generalized Consideration (bending combined with shear)

- Actual *T* is less than the predicted except at the actual crack location.
- It is equal to that given from *V/jd* only at the locations where the slope of the steel force diagram equals that of the simple theory.



(a) beam with flexural cracks

(b) variation of tensile force T in steel along span

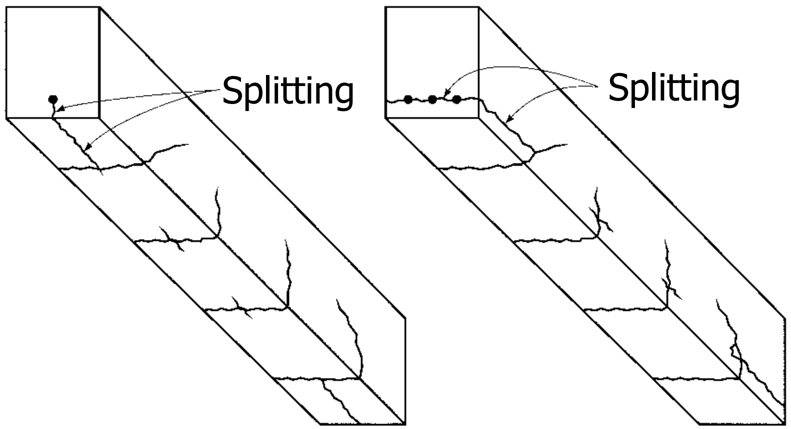
(c) variation of bond force per unit length U along span

BOND STRENGTH & DEVELOPMENT LENGTH

Types of Bond Failure (*Handout 5-1*)

– *Direct pullout* : occurs where sufficient confinement is provided by the surrounding concrete.

– *Splitting of concrete* : occurs along the bar when cover, confinement, or bar spacing is insufficient to resist the lateral concrete tension.



BOND STRENGTH & DEVELOPMENT LENGTH

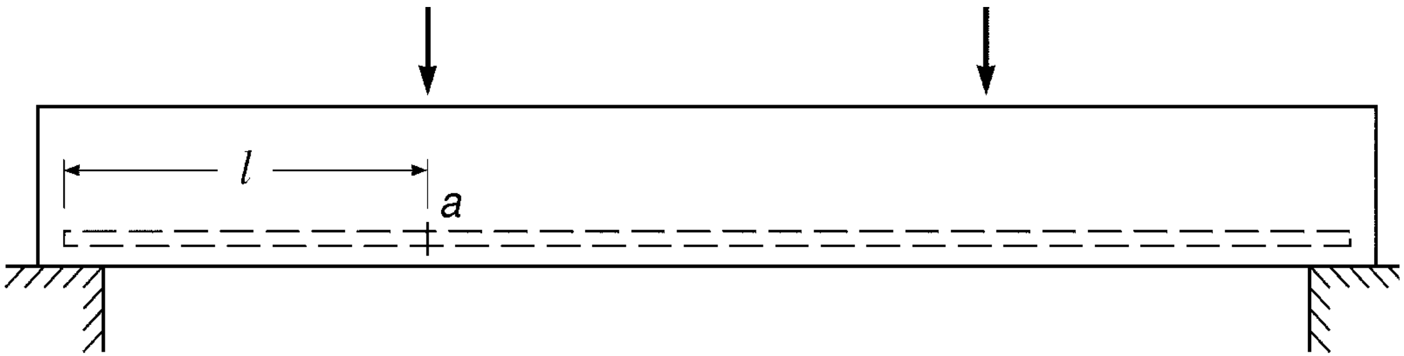
Bond Strength

- When pullout resistance is overcome or when splitting has spread to the end of anchored bar, COMPLETE bond failure occurs.
  - ⇨ Sliding of the steel relative to concrete leads to immediate collapse of the beam.
- Local bond failure adjacent to cracks results in small local slips and widening of cracks and increases of deflections.
  - ⇨ Reliable and anchorage or sufficient extension of rebar can make BOND serve along the entire length of the bar.

5. Bond / Anchorage / Develop. Length

Development Length

- Definition : length of embedment necessary to develop the full tensile strength of the bar.



- To fully develop the strength of the bar, $A_b f_y$, the distance l should be at least equal to the development length of the bar established by tests.

5. Bond / Anchorage / Develop. Length

Development Length

- Then, the beam will fail in bending or shear rather than by bond failure. (premature failure)
- This is still valid if local slip around cracks may have occurred over small region along the beam.
- However, if the actual available length is inadequate for full development, special ANCHORAGE, such as hooks, must be provided.

5. Bond / Anchorage / Develop. Length

BOND STRENGTH & DEVELOPMENT LENGTH

Factors Influencing Development Length (l_d)

- Tensile strength of the concrete ($\sqrt{f_{ck}}$, f_{sp}, λ)
- Concrete cover distance (c)
- Bar spacing (c)
- Transverse reinforcement (K_{tr})
- Vertical location of longitudinal bar (α)
- Epoxy-coated bars (β)
- Bar size (diameter) (γ)

5. Bond / Anchorage / Develop. Length

Factors Influencing Development Length (l_d)

(1) Tensile strength of the concrete ($\sqrt{f_{ck}}$, f_{sp}, λ)

; most common type of bond failure is splitting as seen previously.

⇨ Development length is a function of $\sqrt{f_{ck}}$

(2) Concrete cover distance (c)

; is defined from the surface of the bar to the nearest concrete face and measured either in the plane of the bars or perpendicular to that plane

⇦ Both influence splitting.

5. Bond / Anchorage / Develop. Length

Factors Influencing Development Length (l_d)

(3) Bar spacing (c)

; if the bar spacing is increased (e.g. if only two instead of three bars are used), more concrete can resist horizontal splitting.

⇨ bar spacing of slabs and footings is greater than that of beams. Thus less development length is required.

(4) Transverse reinforcement (K_{tr})

; confinement effect by transverse reinforcement improves the resistance of tensile bars to both vertical or horizontal splitting.

5. Bond / Anchorage / Develop. Length

Factors Influencing Development Length (l_d)

(5) Vertical location of horizontal bars (α)

; Test have shown a significant loss in bond strength for bars with more than 300mm of fresh concrete cast beneath them.

⇨ excess water and entrapped air accumulate on the underside of the bars.

(6) epoxy-coated reinforcing bars (β)

; less bond strength due to epoxy coating requires longer development length.

5. Bond / Anchorage / Develop. Length

Factors Influencing Development Length (l_d)

(7) Bar size (γ)

; smaller diameter bars require lower development lengths.

5. Bond / Anchorage / Develop. Length

ACI CODE PROVISIONS FOR DEVELOPMENT LENGTH

- The force to be developed in tension reinforcement is calculated based on its yield stress.
- Local high bond forces adjacent to cracks are not considered.
- KCI code provides a basic equation of the required development length for deformed bar in tension, including ALL the influences discussed in previous section.
- KCI code provides simplified equations which are useful for most cases in ordinary design.

5. Bond / Anchorage / Develop. Length

Basic Equation for Development of Tension Bars

$$l_d = \left[\frac{0.9 f_y}{\sqrt{f_{ck}}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b}\right)} \right] d_b \qquad (6)$$

, where

α : reinforcement location factor (placed less than 300mm) ≥ 1.0

β : coating factor (uncoated) ≥ 1.0

γ : reinforcement size factor (D22 and larger) ≤ 1.0

λ : light weight aggregate concrete factor (normal weight) ≥ 1.0

c : spacing or cover dimension use the smaller of EITHER the distance from the center of the bar to the nearest concrete surface OR one-half the center-to-center spacing of the bars

Basic Equation for Development of Tension Bars

K_{tr} : transverse reinforcement index

$$K_{tr} = A_{tr} f_{yt} /(10.7sn) \qquad (7)$$

, where

A_{tr} : total cross-sectional area of all transverse reinforcement that is within the spacing s and that crosses the potential plane of splitting through the reinforcement being developed (mm^2)

f_{yt} : specified yield strength of transverse reinforcement (MPa)

s : maximum spacing of transverse reinforcement within l_d (mm)

n : number of bars being developed along the plane of splitting

5. Bond / Anchorage / Develop. Length

Basic Equation for Development of Tension Bars

- To avoid pullout failure

$$\frac{c + K_{tr}}{d_b} \leq 2.5 \qquad (8)$$

- Values of $\sqrt{f_{ck}}$ are not to be taken greater than 8.37MPa due to the lack of experimental evidence.

Simplified Equations for Development Length

- For the simplicity,

$$\frac{c + K_{tr}}{d_b} = 1.5 \qquad (9)$$

For the following two cases,

(a) Minimum clear cover of $1.0d_b$,
minimum clear spacing of $1.0d_b$, and at least
the Code required minimum stirrups throughout l_d

(b) Minimum clear cover of $1.0d_b$ and
minimum clear spacing of $2.0d_b$

5. Bond / Anchorage / Develop. Length

Simplified Equations for Development Length

- In case of D22 and larger bars

$$l_d = \left(\frac{0.6 f_y \alpha \beta \gamma}{\sqrt{f_{ck}}} \right) d_b \qquad (10)$$

in case of D19 and smaller bars

$$l_d = \left(\frac{0.48 f_y \alpha \beta \gamma}{\sqrt{f_{ck}}} \right) d_b \qquad (11)$$

- Otherwise,

$$\frac{c + K_{tr}}{d_b} = 1.0 \qquad (12)$$

5. Bond / Anchorage / Develop. Length

Simplified Equations for Development Length

	D19 and smaller bars	D22 and larger bars
For case (a) & (b) (previous page)	$l_d = \left(\frac{0.48 f_y \alpha \beta \lambda}{\sqrt{f_{ck}}} \right) d_b$	$l_d = \left(\frac{0.60 f_y \alpha \beta \lambda}{\sqrt{f_{ck}}} \right) d_b$
Other cases	$l_d = \left(\frac{0.72 f_y \alpha \beta \lambda}{\sqrt{f_{ck}}} \right) d_b$	$l_d = \left(\frac{0.90 f_y \alpha \beta \lambda}{\sqrt{f_{ck}}} \right) d_b$

Note Regardless of equations used in calculation, development length may be reduced where reinforcement is in excess of that required by analysis according to the ratio, $A_{s,required}/A_{s,provided}$

5. Bond / Anchorage / Develop. Length

Example 5.1

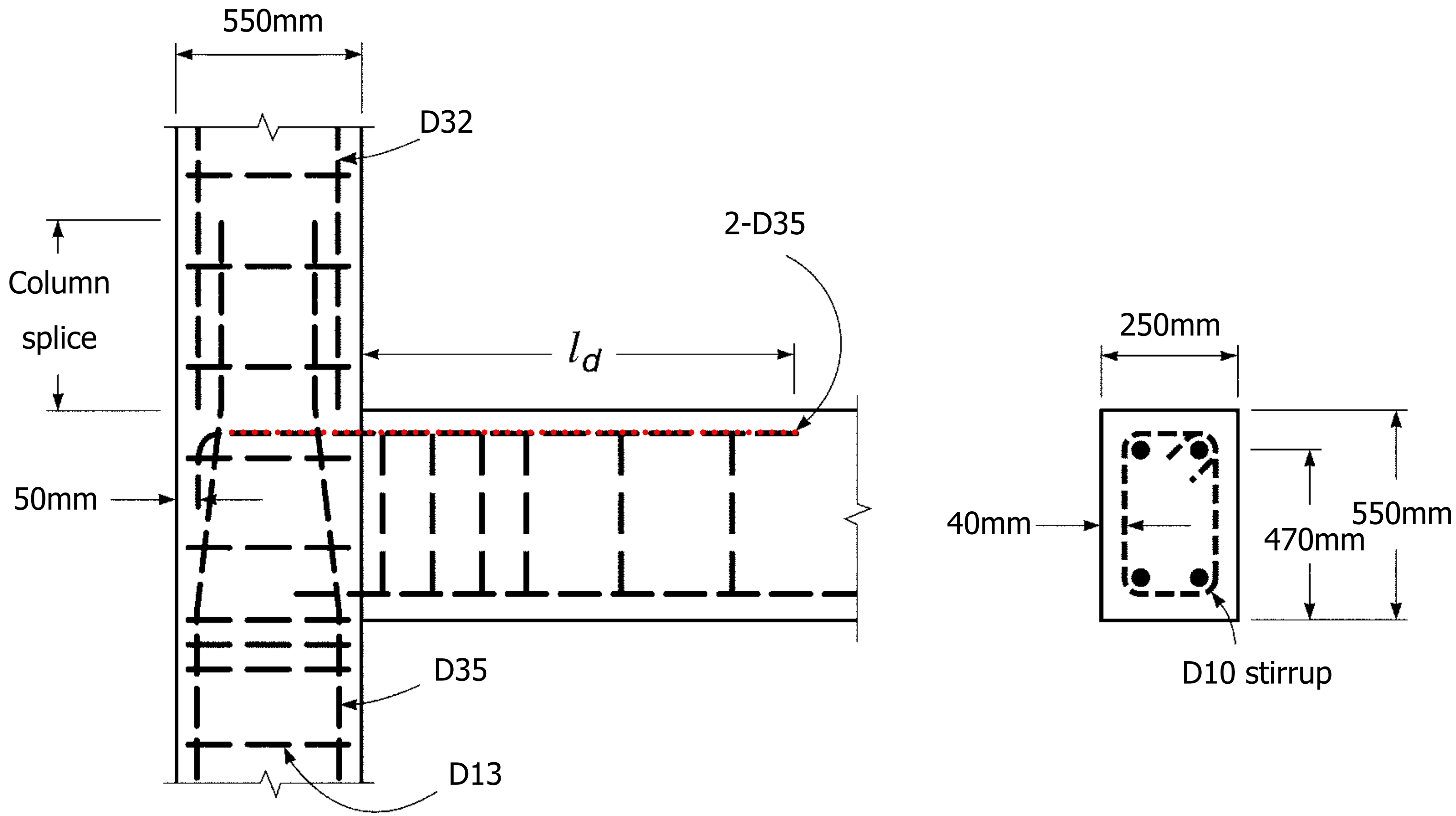
A beam-column joint in a continuous building frame

Based on analysis, the negative steel required at the end of the beam is 1,780mm^2 ; two D35 bars are used (A_s=1,913mm^2)

- b=250mm, d=470mm, h=550mm
- D10 stirrups spaced four 80mm, followed by a constant 120mm spacing in the support region with 40mm clear cover
- Normal density concrete of f_{ck}=27MPa and f_y=400MPa

Find the minimum distance l_d using (a) the simplified equations ,(b) Table A.10 of Appendix, (c) the basic Eq. (6)

5. Bond / Anchorage / Develop. Length



5. Bond / Anchorage / Develop. Length

Solution

1. Method (a) – Approximated equation

- Check which equation can be used in this case

  - clear distance between bars (D35)

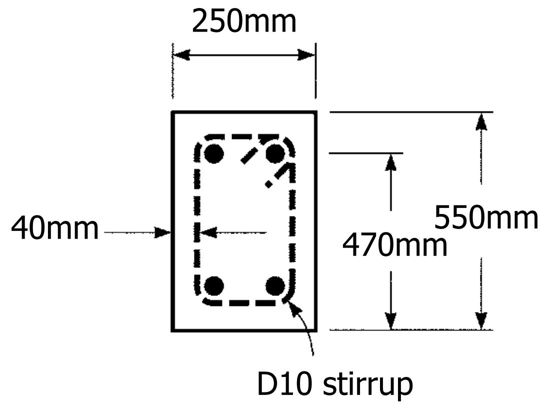
$$250\text{-}2(40+10+35)=80\text{mm}=2.3d_b$$

  - clear cover to the side face of the beam

$$40+10=50\text{mm}=1.4d_b$$

  - clear cover to the top face of the beam

$$(550\text{-}470)\text{-}35/2=63\text{mm}=1.8d_b$$



5. Bond / Anchorage / Develop. Length

- Therefore, we can use a simplified equation

$$l_d = \left(\frac{0.6 f_y \alpha \beta \gamma}{\sqrt{f_{ck}}} \right) d_b$$

, where α=1.3, β=1.0, γ=1.0 for top bars, uncoated bars, and normal-density concrete.

$$\Rightarrow \quad l_d = \frac{(0.6)(400)(1.3)(1.0)(1.0)}{\sqrt{f_{ck}}} = 60 d_b = 2{,}100 mm$$

- This can be reduced by the ratio of steel required to that provided,

$$l_d = (2{,}100)\left(\frac{1{,}780}{1{,}913} \right) = 1{,}954 mm$$

5. Bond / Anchorage / Develop. Length

Solution

2. Method (b) – using design AID

- From the table A.10 (SI unit version) $l_d / d_b = 60$

$$\therefore l_d = (60)(35)\frac{(1,780)}{(1,913)} = 1,954mm$$

5. Bond / Anchorage / Develop. Length

Table A.10 Simplified tension development length l_d/d_b

	f_y MPa	D19 and smaller			D22 and larger		
		f_{ck}, MPa			f_{ck}, MPa		
		21	27	35	21	27	35
(1) Bottom bars							
Case (a) & (b)	300	31	28	24	39	35	30
	400	42	37	32	52	46	41
Other cases	300	47	42	37	59	52	46
	400	63	56	49	79	69	61
(2) Top bars							
Case (a) & (b)	300	41	36	32	51	45	40
	400	55	48	42	68	60	61
Other cases	300	61	54	48	77	68	59
	400	82	72	63	102	90	79

Solution

3. Method (c) – basic equation

- Determination of K_{tr}
  - The center-to-center spacing of the D35 bars is,

$$250-2(40+10+35/2) = 115\text{mm}$$

  - one-half of which is 58mm
  - The side cover to bar center line is

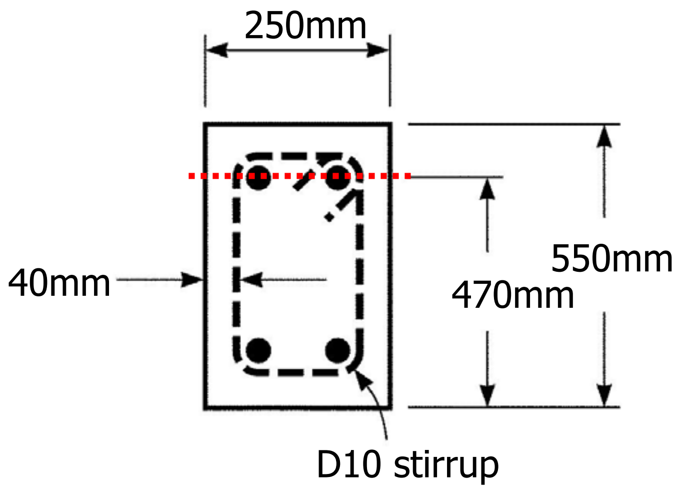
$$40+10+35/2 = 68\text{mm}$$

  - The top cover to bar center line is 80mm

⇨ The smallest of these three distances controls, and c=58mm

5. Bond / Anchorage / Develop. Length

- Potential splitting would be in the horizontal plane of the bars



A_{tr}=2*71=142mm^2 and maximum spacing s=120mm and n=2(two D35)

$$K_{tr} = \frac{(142)(400)}{(10.7)(120)(2)} = 22.1$$

and

$$\frac{c + K_{tr}}{d_b} = \frac{58 + 22.1}{35} = 2.29 < 2.5 \quad \text{O.K !}$$

5. Bond / Anchorage / Develop. Length

- Development length

$$l_d = \left[\frac{0.9 f_y}{\sqrt{f_{ck}}} \frac{\alpha\beta\gamma\lambda}{(\frac{c+K_{tr}}{d_b})} \right] d_b$$

$$= \left[\frac{(0.9)(400}{\sqrt{27}} \frac{(1.3)(1.0)(1.0)(1.0)}{2.29} \right] (35)$$

$$= 1,376mm$$

- Final development length is,

$$l_d = (1,376) \frac{(1,780)}{(1,913)} = 1,280mm$$

5. Bond / Anchorage / Develop. Length

Solution

Summary

- 1,954mm > 1,280mm
  approx. basic.

- More accurate equation permits a considerable reduction in development length

- Even though its use requires more time and effort, it is justified if the design is to be repeated many times in a structure.

5. Bond / Anchorage / Develop. Length

BAR CUTOFF AND BEND POINTS IN BEAMS

Theoretical Points of Cutoff or Bend

- Tensile force to be resisted by the reinforcement at any cross section

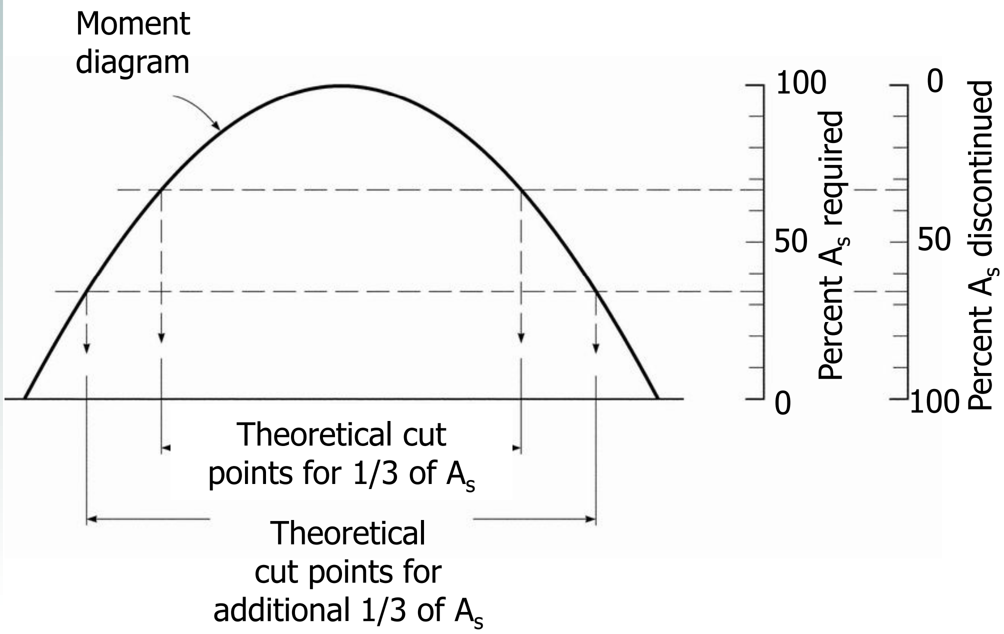
$$T = A_s f_s = \frac{M}{z}$$

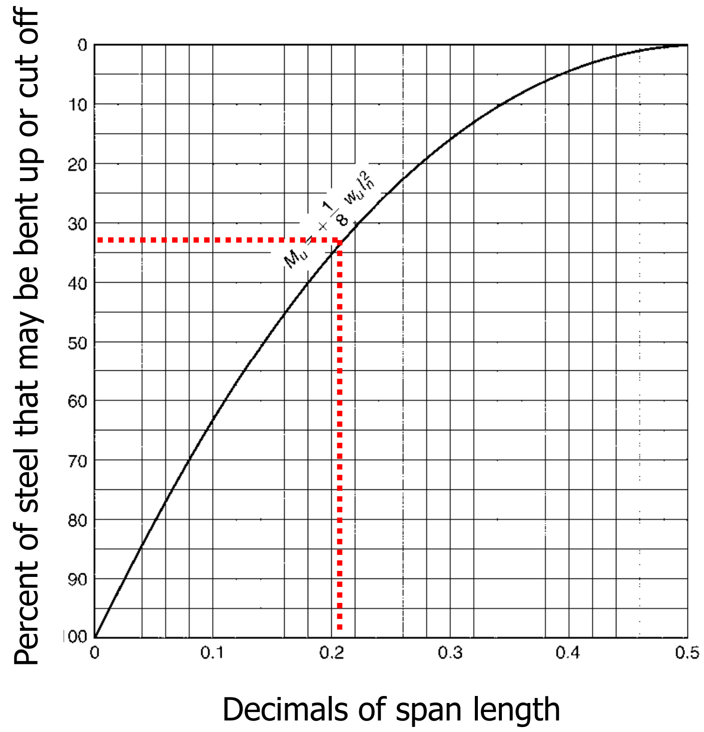
- The internal lever arm varies only within narrow limits
  ⇨ Tensile force can be taken directly proportional to the bending moment.

- Required steel area is nearly proportional to the bending moment.

Theoretical Points of Cutoff or Bend

- The moment diagram for a uniformly loaded "simple" beam

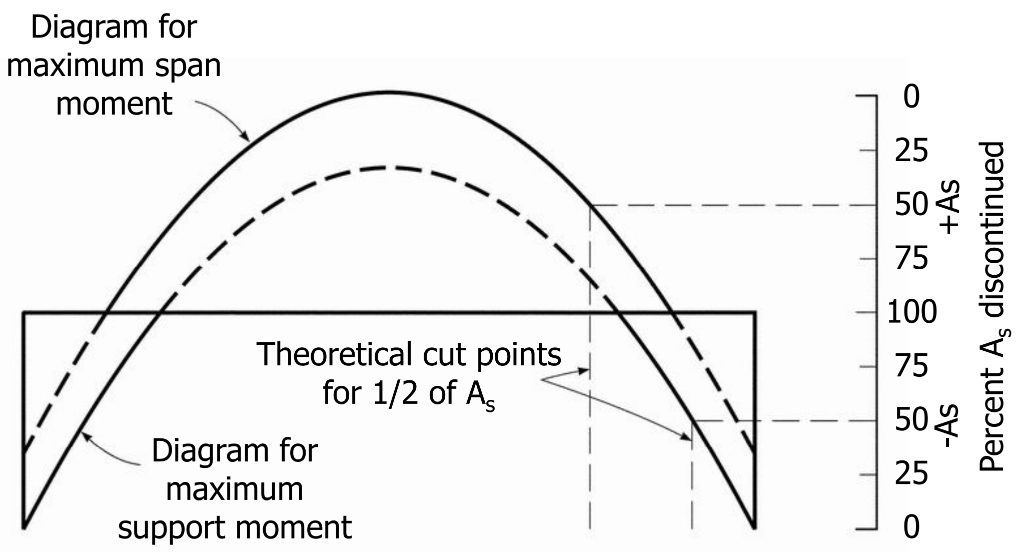




5. Bond / Anchorage / Develop. Length

Theoretical Points of Cutoff or Bend

- The moment diagram for a uniformly loaded "continuous" beam



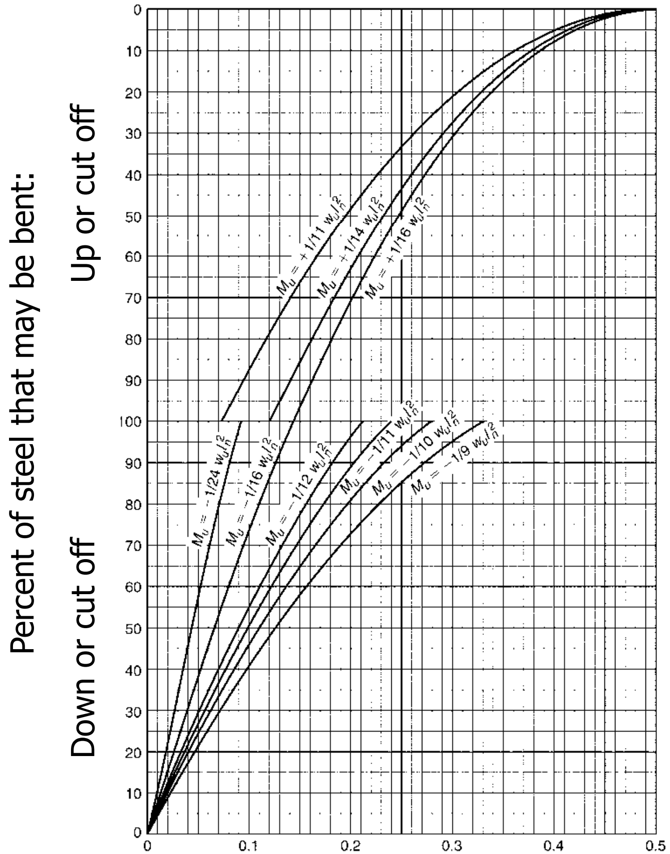


Table 12.1 Moment and shear values using KCI coefficients (Approx.)

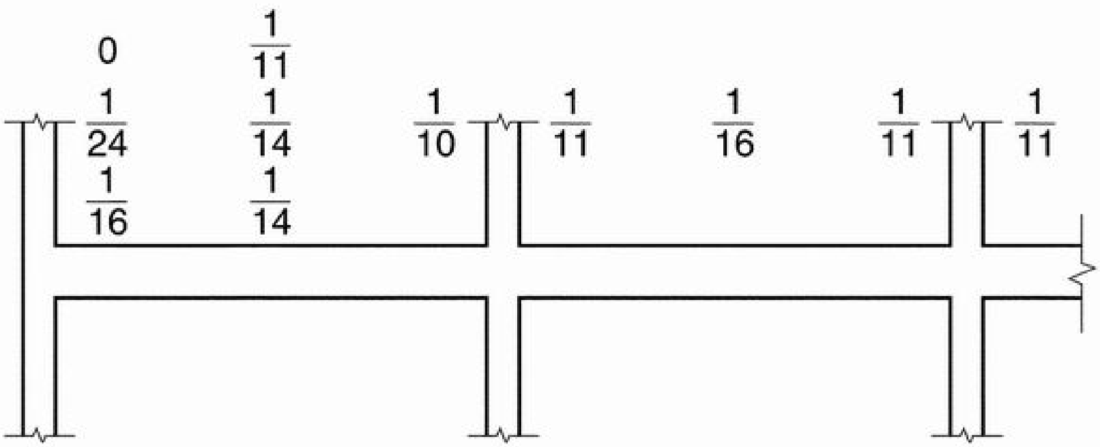
Positive moment	
End spans	
If discontinuous end is unrestrained	$\frac{1}{11} w_u l_n^2$
If discontinuous end is integral with the support	$\frac{1}{14} w_u l_n^2$
Interior spans	$\frac{1}{16} w_u l_n^2$
Negative moment at exterior face of first interior support	
Two spans	$\frac{1}{9} w_u l_n^2$
More than two spans	$\frac{1}{10} w_u l_n^2$
Negative moment at other faces of interior supports	$\frac{1}{11} w_u l_n^2$
Negative moment at face of all supports for (1) slabs with spans not exceeding 10ft and (2) beams and girders where ratio of sum of column stiffness to beam stiffness exceeds 8 at each end of the span	$\frac{1}{12} w_u l_n^2$
Negative moment at interior faces of exterior supports for members built integrally with their supports	
Where the support is a spandrel beam or girder	$\frac{1}{24} w_u l_n^2$
Where the support is a column	$\frac{1}{16} w_u l_n^2$
Shear in end members at first interior support	$1.15 \frac{w_u l_n}{2}$
Shear at all other supports	$\frac{w_u l_n}{2}$

5. Bond / Anchorage / Develop. Length

Discontinuous end unrestrained:

Spandrel:

Column:

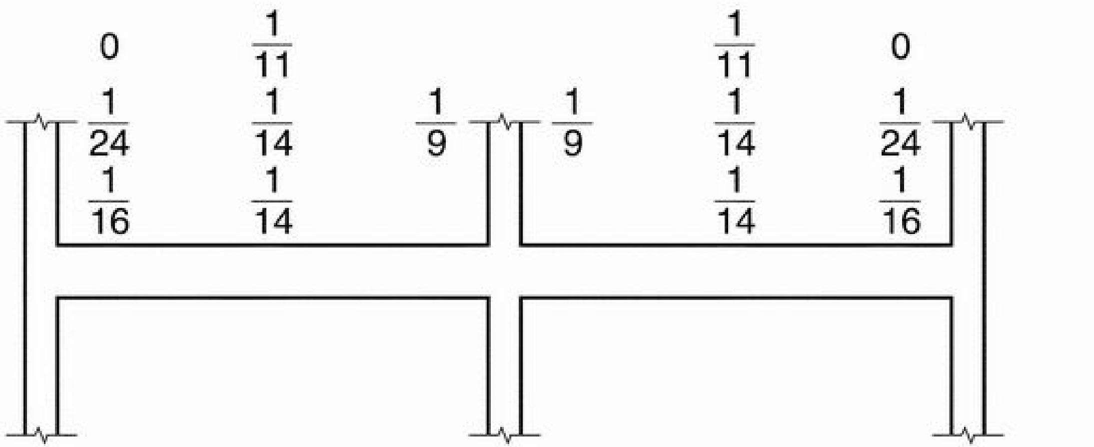


(a) Beams with more than two spans

Discontinuous end unrestrained:

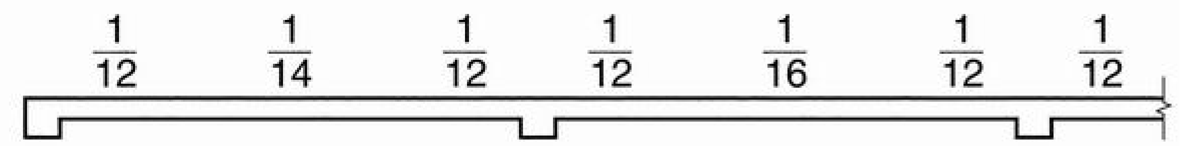
Spandrel:

Column:

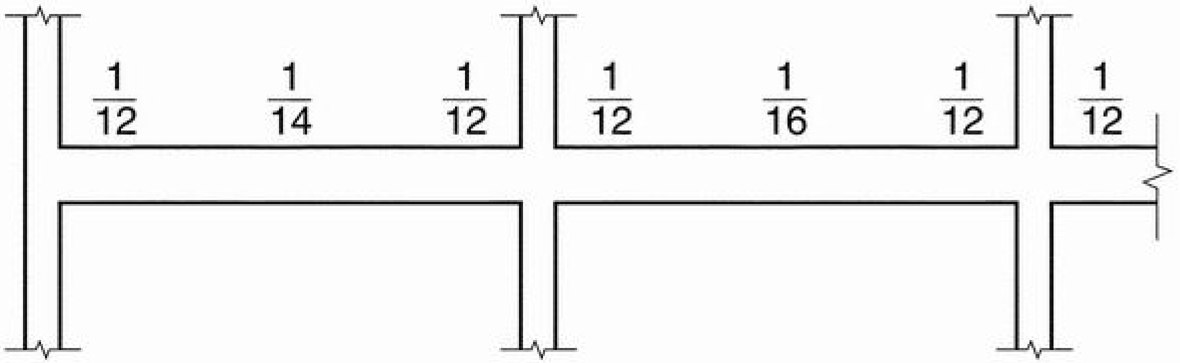


(b) Beams with two spans only

5. Bond / Anchorage / Develop. Length



(c) Slabs with spans not exceeding 3m



(d) Beams in which the sum of column stiffness exceeds 8 times the sum of beam stiffnesses at each end of the span

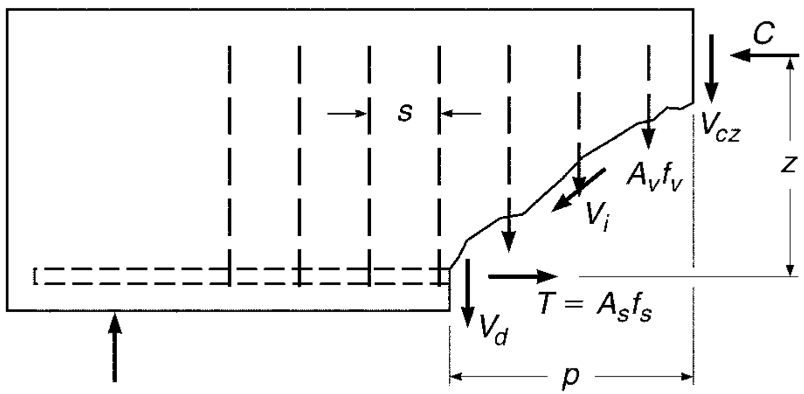
5. Bond / Anchorage / Develop. Length

Practical Consideration and KCI Code Requirement

- Actually, IN NO CASE should the tensile steel be discontinued EXACTLY at the theoretically described points.

  ⇦ Diagonal cracking causes an internal redistribution of forces in a beam.

  ; the tensile force in the steel at the crack is governed by the moment at a section nearer midspan.



5. Bond / Anchorage / Develop. Length

Practical Consideration and KCI Code Requirement

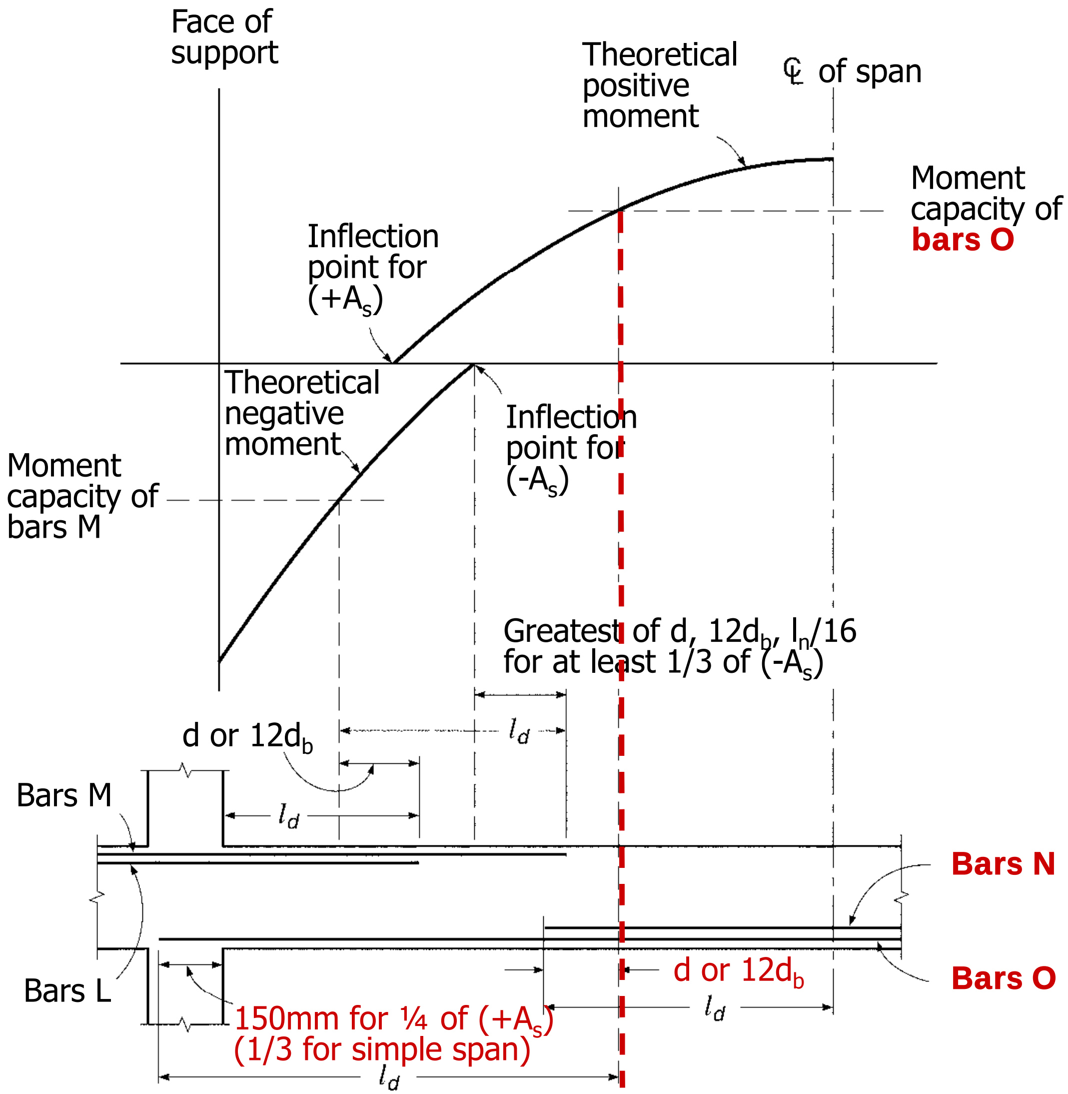
⇦ the actual moment diagram may differ from that used as a design basis due to

- approximation of real loads
- approximations in the analysis
- the superimposed effect of settlement or lateral loads

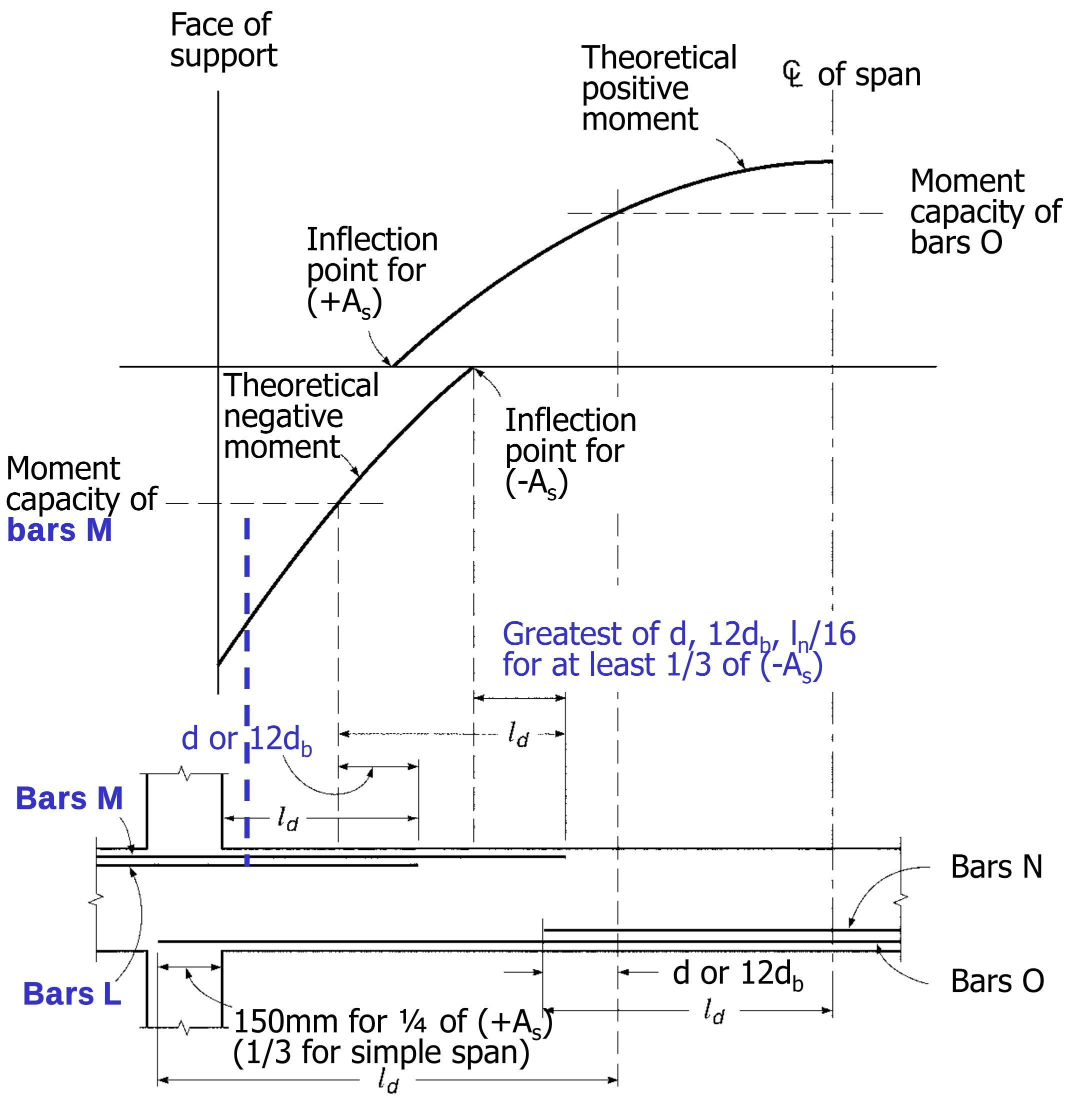
- Therefore, KCI Code 8.5 requires that every bar should extend to the distance of the effective depth d or $12d_b$ (whichever is larger) beyond the point where it is theoretically no longer required to resist stress.

5. Bond / Anchorage / Develop. Length

- Reflecting the possible change in peak-stress location
- When a flexural member is a part of primary lateral load resisting system, positive-moment reinforcement should be extended into support must be anchored to be yielded



5. Bond / Anchorage / Develop. Length



5. Bond / Anchorage / Develop. Length

Practical Consideration and KCI Code Requirement

- When bars are cut off in a tension zone, premature flexural and diagonal tension crack can occur in the vicinity of the cut end. ⇨ reduction of shear capacity.

- Therefore, KCI Code 8.5 requires special precaution ; no flexural bar shall be terminated in a tension zone unless ONE of the following conditions is satisfied.

  1) The shear is not over $(2/3)\ \varphi V_n$
  2) The continuing bars, if D35 or smaller, provide twice the area required for flexure at that point, and shear does not exceed (3/4) φV_n

5. Bond / Anchorage / Develop. Length

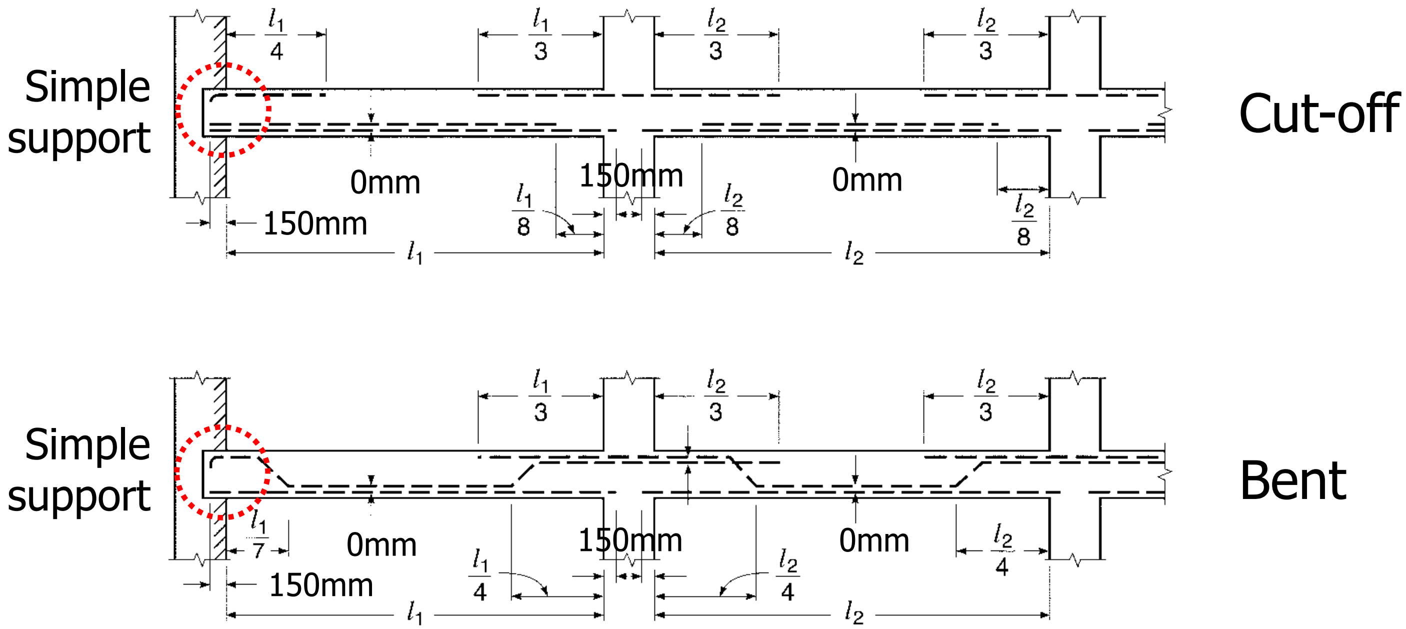
Practical Consideration and KCI Code Requirement

3) Stirrups in excess of those normally required are provided over a distance along each terminated bar from the point of cutoff equal to *¾d*.
And these stirrups amount $A_v \geq 60 b_w s / f_y$.
And stirrup spacing $s \leq d / 8\beta_b$.

5. Bond / Anchorage / Develop. Length

Practical Consideration and KCI Code Requirement

- As an alternative to cutting off, BENDING is also preferable because added insurance is provided against the spread of diagonal tension crack.



Critical Sections in Flexural Member

(Some contents might be repeated)

The critical sections for development of reinforcement in flexural members are:

1. At points of maximum stress
2. At points where tension bars within span are terminated or bent
3. At the face of the support
4. At points of inflection at which moment changes sign.

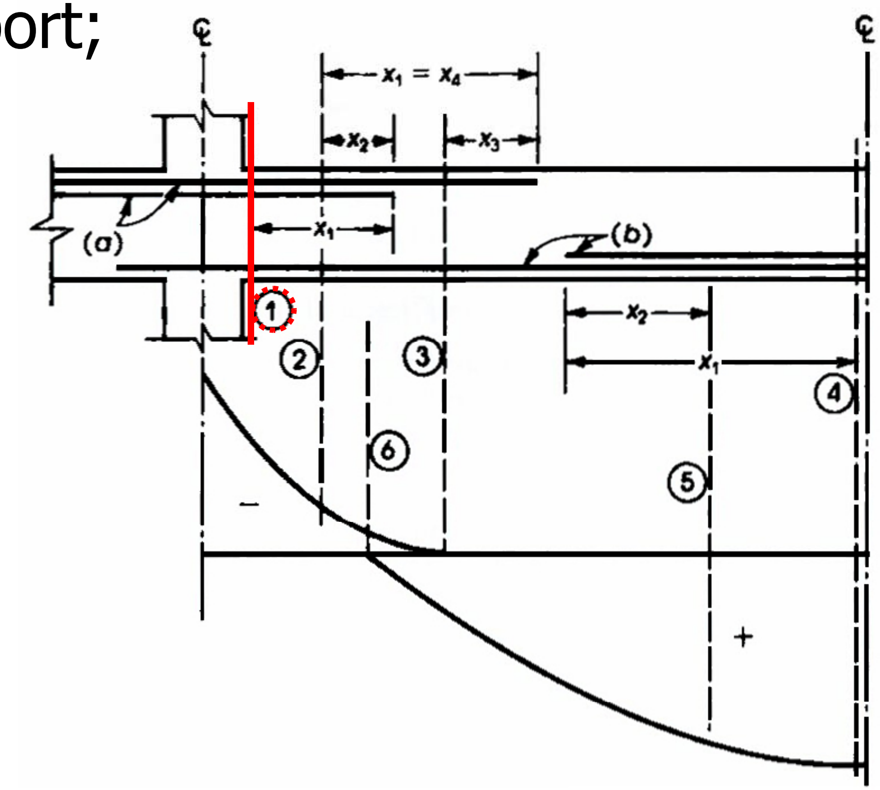
Critical Sections in Flexural Member

Critical sections in NEGATIVE moment reinforcement

Section 1 the face of the support;

the negative moment as well as stress are at maximum value.

Two development lengths, x_1 and x_2 must be checked.



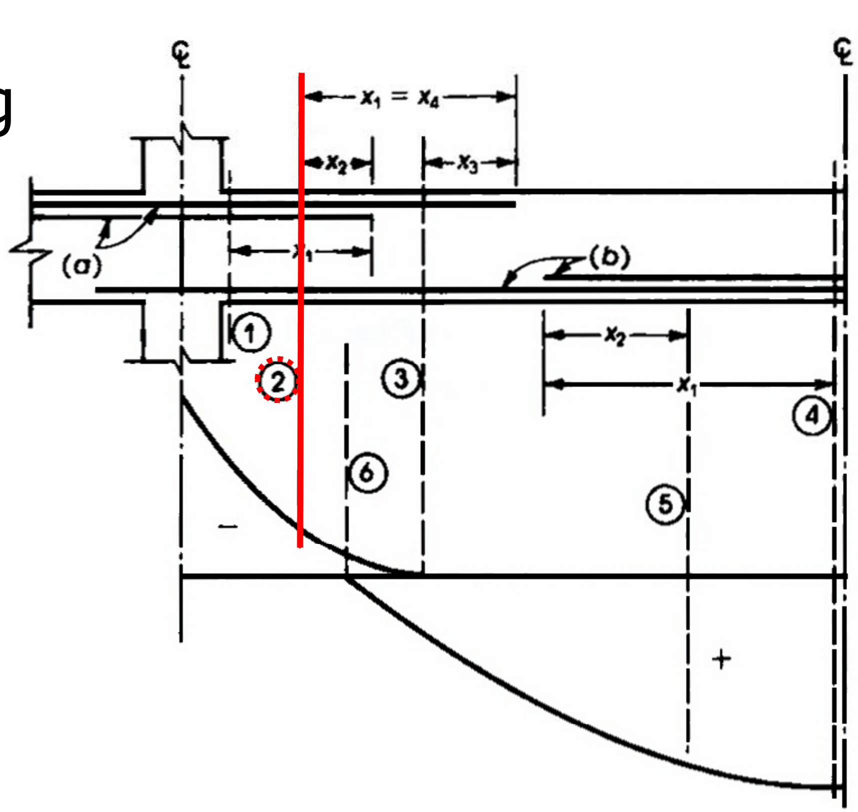
Critical Sections in Flexural Member

Critical sections in NEGATIVE moment reinforcement

Section 2 is the section where part of the *negative* reinforcing bar can be terminated;

To develop full tensile force, the bars should extend a distance x_2 before they can be terminated.

Once part of the bars are terminated the remaining bars develop maximum stress.



5. Bond / Anchorage / Develop. Length

Critical Sections in Flexural Member

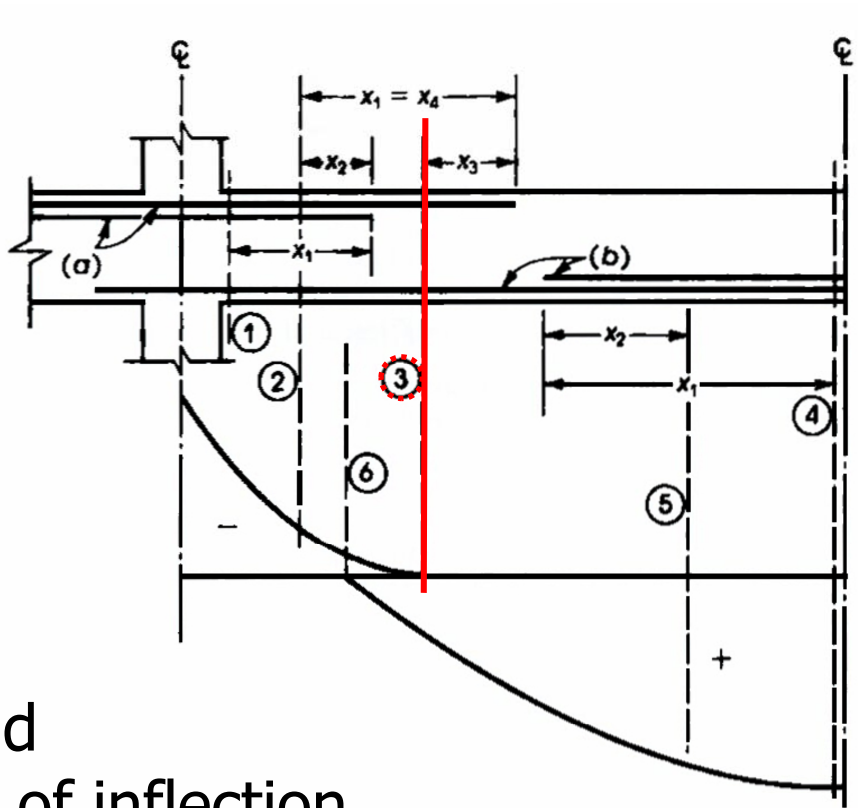
Critical sections in NEGATIVE moment reinforcement

Section 3 a inflection point;

The bars shall extend a distance x_3 beyond section 3

x_3 must be equal to or greater than the effective depth d, $12d_b$ or 1/16 the span, whichever is greater.

At least 1/3 of A_s for negative moment at support shall extend a distance x_3 beyond the point of inflection.



5. Bond / Anchorage / Develop. Length

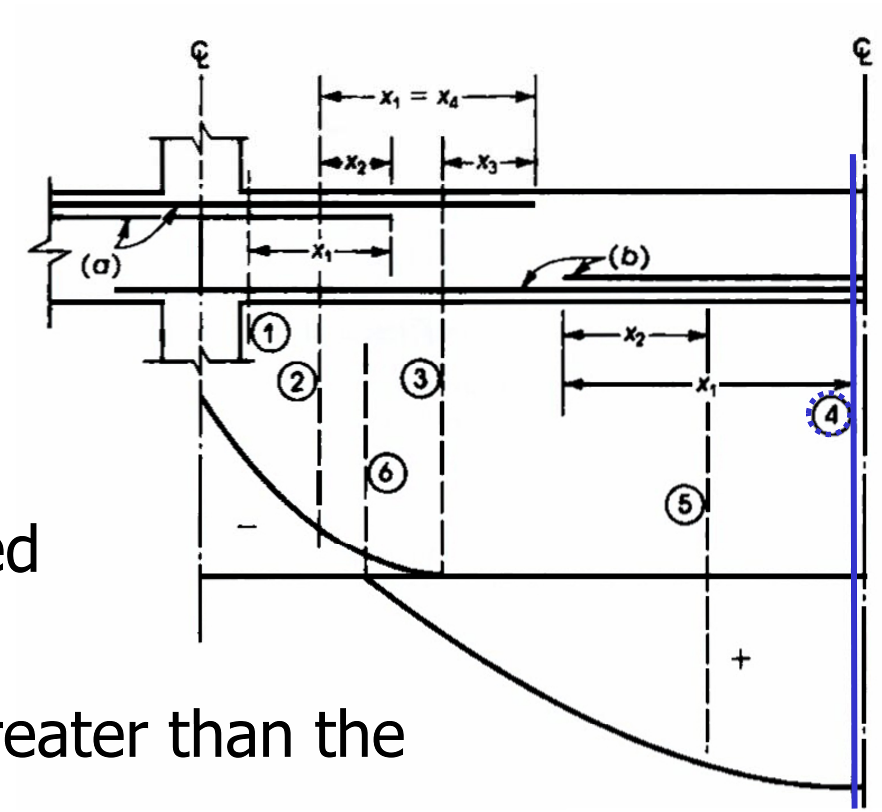
Critical Sections in Flexural Member

Critical sections in POSITIVE moment reinforcement

Section 4 is the section of maximum positive moment and maximum stresses;

Two development lengths x_1 and x_2 have to be checked.

The length x_1 is the development length l_d specified by the KCI Code 8.2.2.



The length x_2 is equal to or greater than the effective depth d, $12d_b$.

5. Bond / Anchorage / Develop. Length

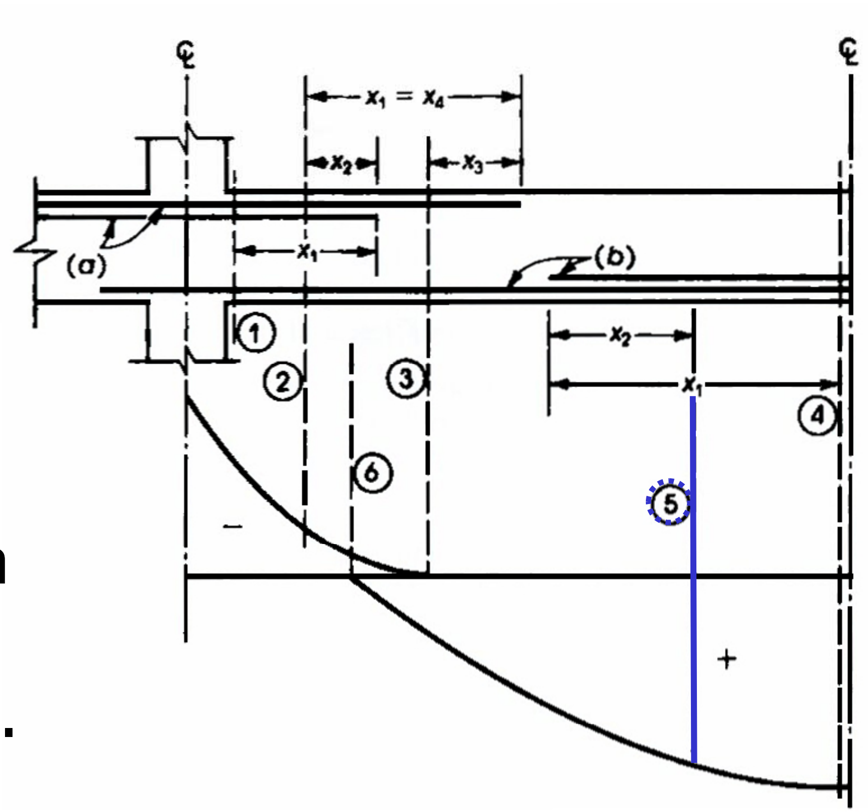
Critical Sections in Flexural Member

Critical sections in POSITIVE moment reinforcement

Section 5 is the section where part of the *positive* reinforcing bar can be terminated;

To develop full tensile force, the bars should extend a distance x_2 .

The remaining bars will have a maximum stress due to the termination of part of the bars.



5. Bond / Anchorage / Develop. Length

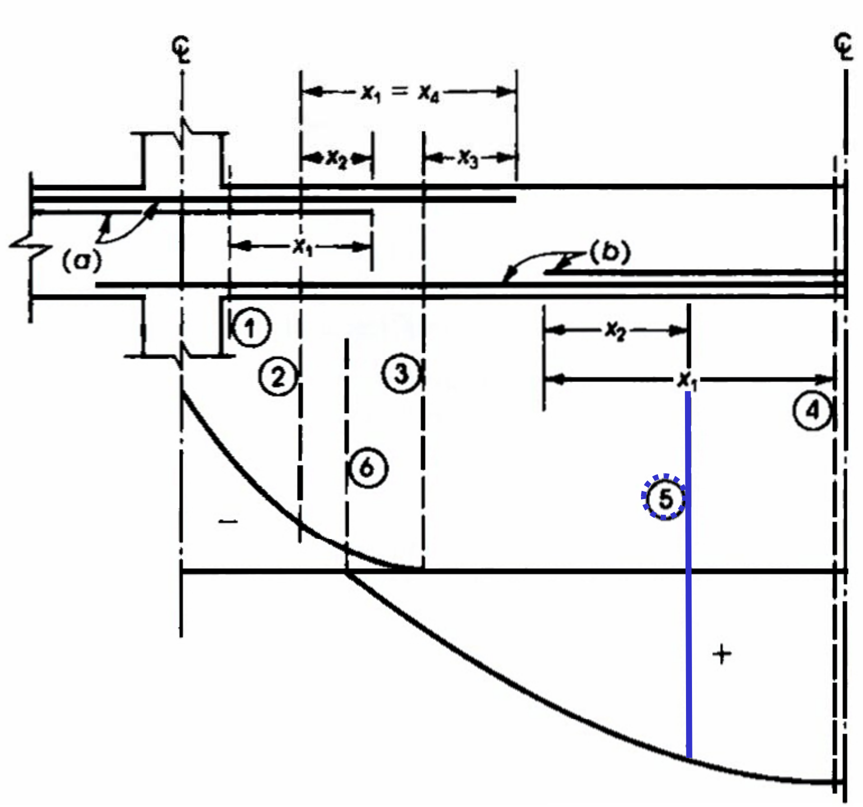
Critical Sections in Flexural Member

Critical sections in POSITIVE moment reinforcement

Section 5 At the face of the support section 1;

At least 1/4 of A_s in continuous members shall expend along the same face of the member a distance at least 150mm into the support.

For simple members at least 1/3 of the reinforcement shall extend into the support.

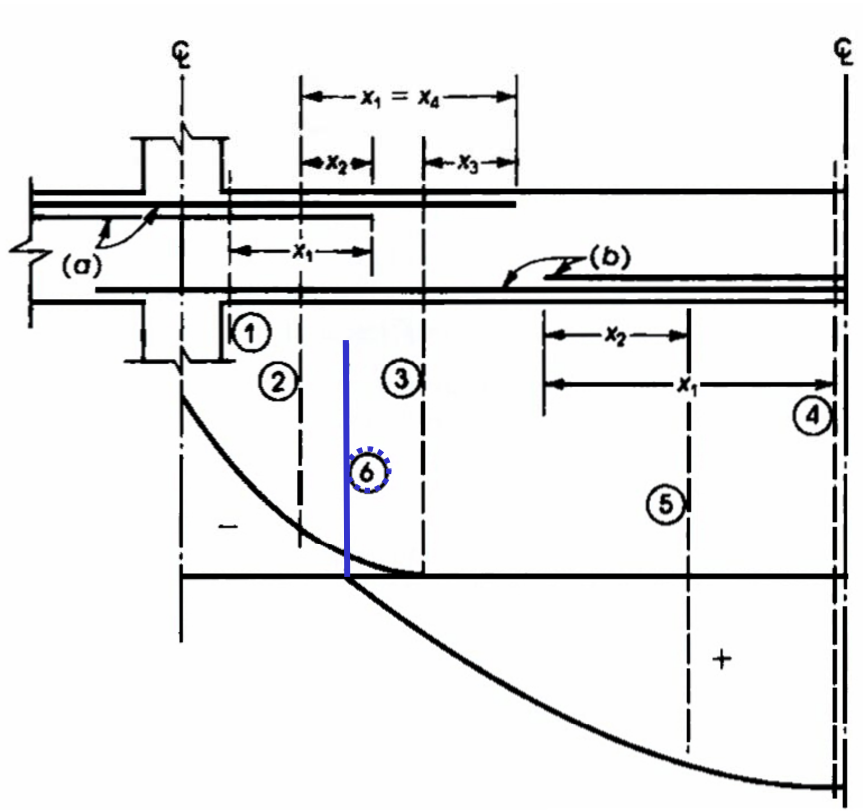


5. Bond / Anchorage / Develop. Length

Critical Sections in Flexural Member

Critical sections in POSITIVE moment reinforcement

Section 6 is at the points of inflection limits are according to KCI Code 8.5.2(3)



General Procedure and Rules for Bar Cutoff

Bar cutoff general Procedure

1. Determine theoretical flexural cutoff *points* for envelope of bending moment diagram.
2. Extract the bars to satisfy detailing rules (according to KCI Code provisions)
3. Design extra stirrups for points where bars are cutoff in zone of flexural tension

5. Bond / Anchorage / Develop. Length

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for both positive & negative moment bars

Rule1 Bars must extend the longer of d or 12d_b past the flexural cutoff points except at supports or the ends of cantilevers (KCI 8.5.1)

Rule2 Bars must extend at least l_d from the point of maximum bar stress or from the flexural cutoff points of adjacent bars (KCI 8.5.1)

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for positive moment bars

Rule3 **Structural Integrity**

- ***Simple Supports*** At least one-third of the positive moment reinforcement must be extend 150mm into the supports (KCI 8.5.2).

- ***Continuous interior beams with closed stirrups***. At least one-fourth of the positive moment reinforcement must extend 150mm into the support (KCI 8.5.2)

5. Bond / Anchorage / Develop. Length

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for positive moment bars

Rule3 ***Structural Integrity***

- ***Continuous interior beams without closed stirrups.*** At least one-fourth of the positive moment reinforcement must be continuous or shall be spliced near the support with a class A tension splice and at non-continuous supports be terminated with a standard hook. (KCI 5.8.1).

5. Bond / Anchorage / Develop. Length

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for positive moment bars

Rule3 ***Structural Integrity***

- ***Continuous perimeter beams*** At least one-fourth of the positive moment reinforcement required at midspan shall be made continuous around the perimeter of the building and must be enclosed within closed stirrups or stirrups with 135° hooks around top bars. (*to be continued at next page*)

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for positive moment bars

Rule3 **Structural Integrity**

- ***Continuous perimeter beams*** The required continuity of reinforcement may be provided by splicing the bottom reinforcement at or near the support with class A tension splices (KCI 5.8.1).

5. Bond / Anchorage / Develop. Length

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for positive moment bars

Rule3 **Structural Integrity**

- ***Beams forming part of a frame that is the primary lateral load resisting system for the building*.**

  This reinforcement must be anchored to develop the specified yield strength, f_y, at the face of the support (KCI 8.5.2)

5. Bond / Anchorage / Develop. Length

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for positive moment bars

Rule4 **Stirrups** At the positive moment point of inflection and at simple supports, the positive moment reinforcement must be satisfy the following equation for KCI 8.5.2.

$$l_d \le \frac{M_n}{V_u} + l_a$$

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for positive moment bars

Rule4 **Stirrups** An increase of 30 % in value of M_n / V_u shall be permitted when the ends of reinforcement are confined by compressive reaction (generally true for simply supports).

$$l_d \leq 1.3 \frac{M_n}{V_u} + l_a$$

5. Bond / Anchorage / Develop. Length

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for negative moment bars

Rule5 Negative moment reinforcement must be anchored into or through supporting columns or members (KCI 8.5.3).

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

for negative moment bars

Rule6 **Structural Integrity**

- ***Interior beams*** At least one-third of the negative moment reinforcement must be extended by the greatest of d, 12 d_b or (l_n / 16) past the negative moment point of inflection (KCI 8.5.3).

5. Bond / Anchorage / Develop. Length

General Procedure and Rules for Bar Cutoff

Bar cutoff general Rules

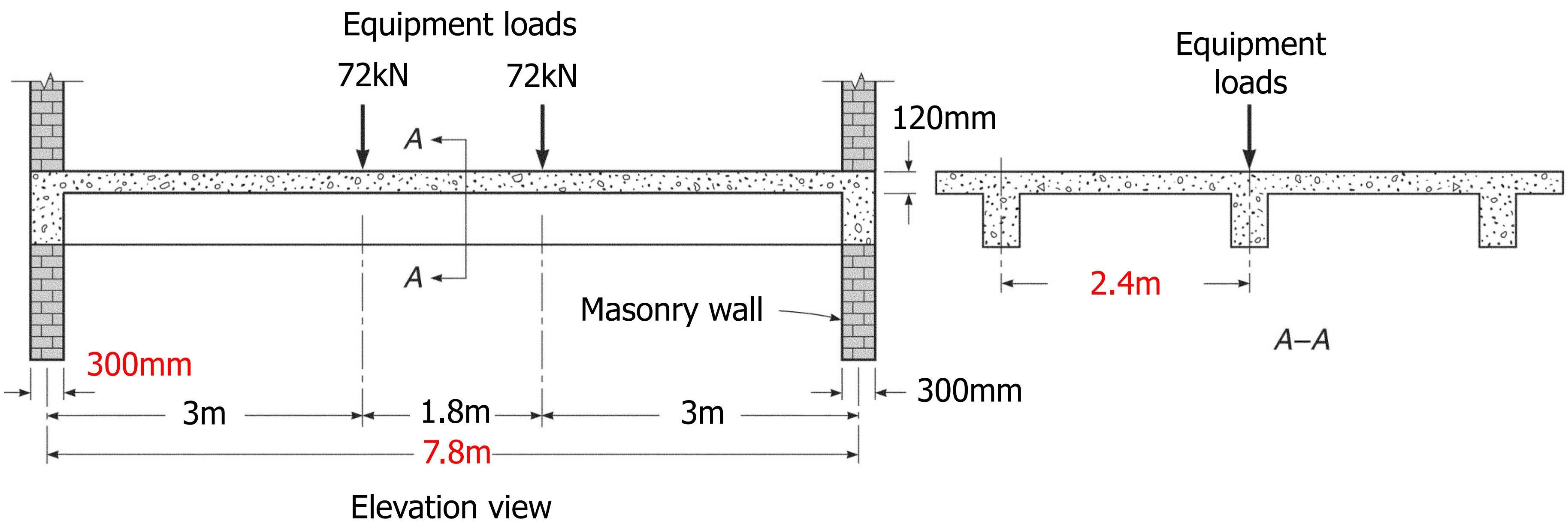
for negative moment bars

Rule6 **Structural Integrity**

- ***Perimeter beams***. In addition to satisfying rule 6a, one-sixth of the negative reinforcement required at the support must be made continuous at mid-span. This can be achieved by means of a class A tension splice at mid-span (KCI 5.8.1).

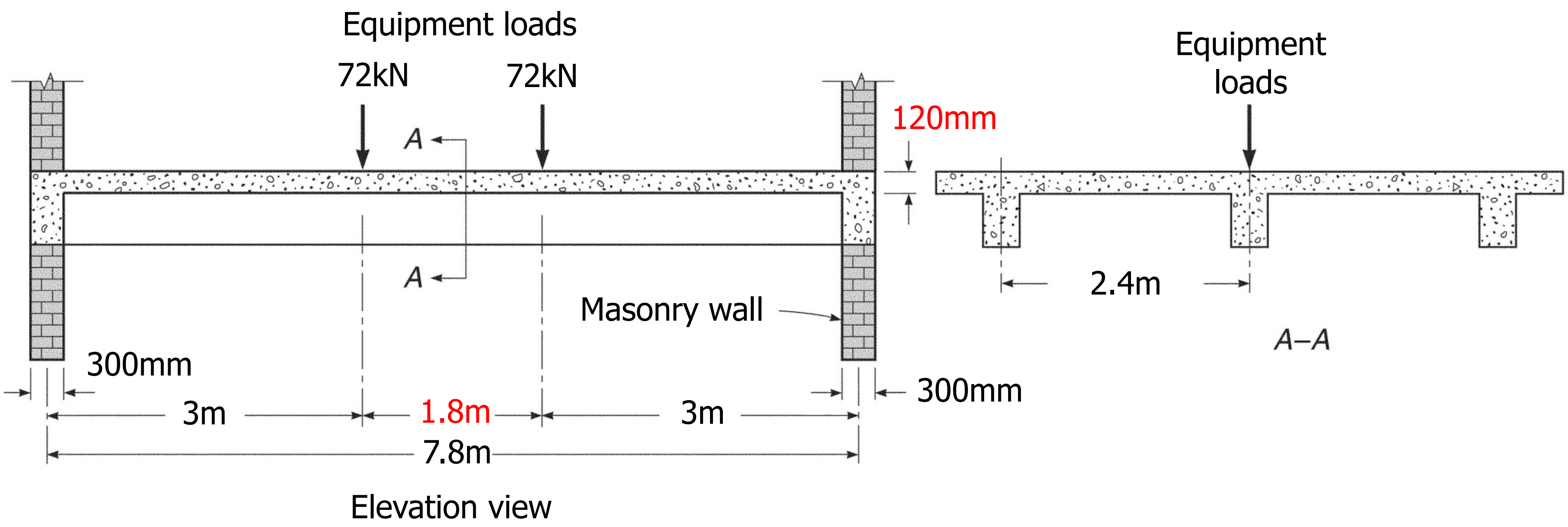
Example 5.3 Integrated Beam Design

- A floor system consists of a single span T beams 2.4m on centers supported by 300mm masonry walls spaced at 7.5m between inside faces.



5. Bond / Anchorage / Develop. Length

- A 120mm monolithic slab carries a uniformly distributed *service* live load of 8kN/m^2
- Also carries two 72kN equipment loads applied over the stem of the T beam 900mm from the span centerline. f_{ck}=30MPa, f_y=400MPa



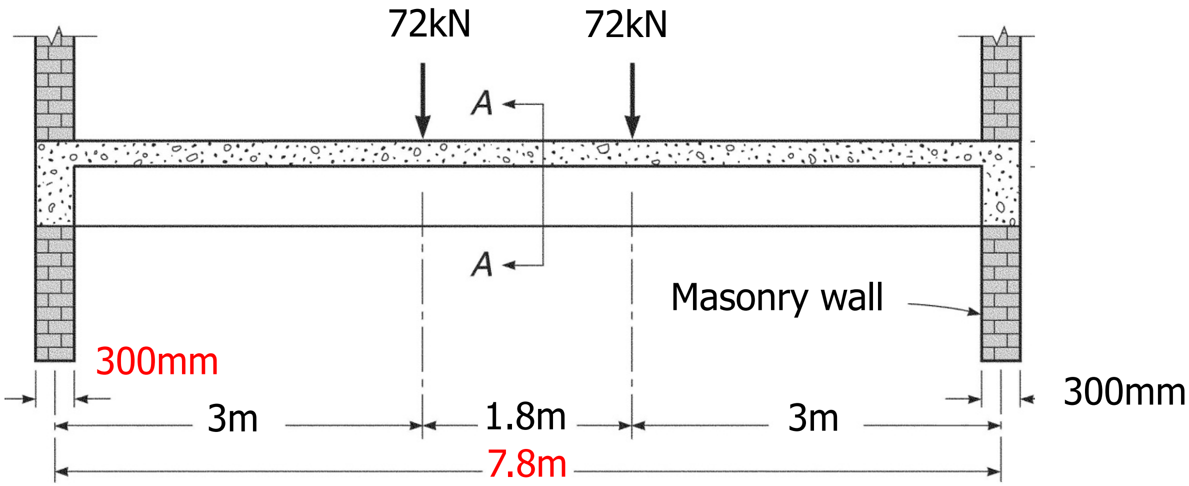
5. Bond / Anchorage / Develop. Length

Solution

1) According to KCI Code, the span length is to be taken as the clear span plus the beam depth, but need not exceed the distance between the centers of supports

   In this case, the effective span is 7.5+0.3=7.8m, because we are going to assume the beam WEB dimensions to be 300 by 600mm.

   Letting the unit weight of concrete be $24kN/m^3$



Solution

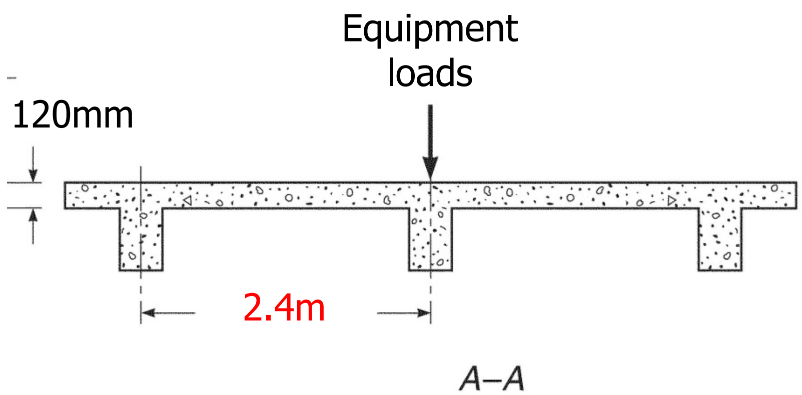
2) The calculated and factored dead load are

Slab $(0.12)(2.4-0.3)(24) = 6.05kN/m$

Beam $(0.3)(0.6)(24) = 4.32kN/m$

⇨ $w_d = 6.05 + 4.32 = 10.37kN/m$

factored $w_d = 1.4w_d = \underline{14.5kN/m}$



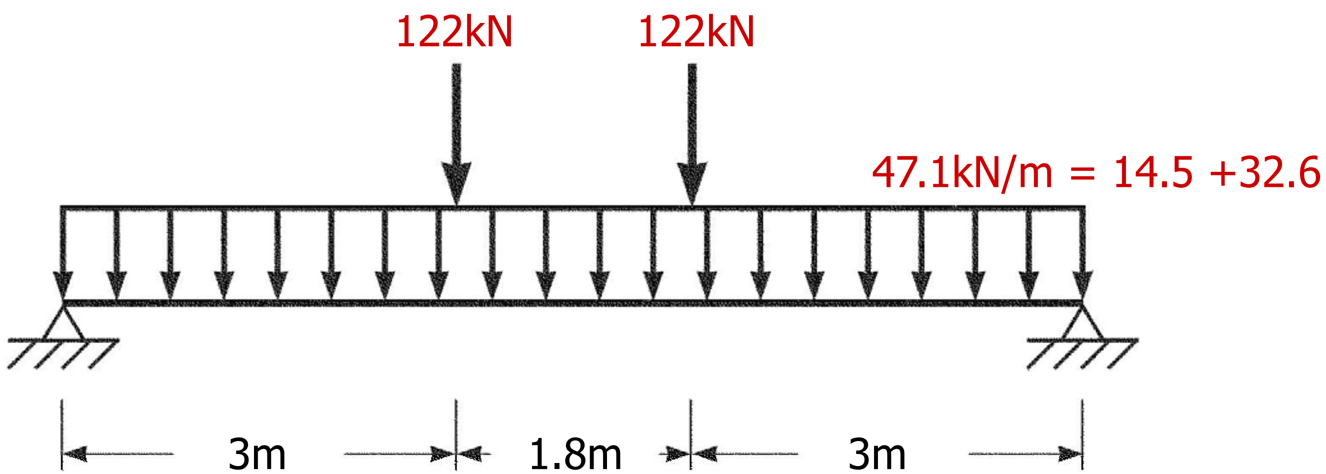
5. Bond / Anchorage / Develop. Length

3) The applied and factored live loads are

uniform load $w_l = 8 \times 2.4 = 19.2 kN/m$

factored $w_l = 1.7 \times 19.2 = \underline{32.6 kN/m}$

concentrated load $P_u = 1.7 \times 72 = \underline{122 kN}$



5. Bond / Anchorage / Develop. Length

4) In lieu of other controlling criteria, the beam WEB dimension will be selected on the basis of *SHEAR*.

   The left and right reactions are,

$$122 + (14.5 + 32.6)\left(\frac{7.8}{2}\right) = \underline{306kN}$$

5) With the effective beam depth estimated to be 500mm, the maximum shear that need be considered in design is,

$$306 - (47.1)(0.15 + 0.5) = \underline{275kN}$$

at shear critical section

5. Bond / Anchorage / Develop. Length

6) Although the KCI Code permit V_s as high as $0.67\sqrt{f_{ck}} b_w d$, this would require very heavy web reinforcement.

Therefore, *conventional lower limit $0.33\sqrt{f_{ck}} b_w d$ is adopted*

$$\Rightarrow V_n = V_s + V_c$$

$$= 0.33\sqrt{f_{ck}}\, b_w d + \frac{1}{6}\sqrt{f_{ck}}\, b_w d$$

$$= 0.5\sqrt{f_{ck}}\, b_w d$$

5. Bond / Anchorage / Develop. Length

7) Check the beam dimension assumed

$$b_w d = \frac{V_u}{\phi(0.5\sqrt{f_{ck}})} = \frac{275\times10^3}{(0.8)(0.5)(\sqrt{30})} = 125,520mm^2$$

⇨ let the beam dimensions b_w=300mm and d=450mm (exact value=418mm), providing a total beam depth h=550mm.

⇨ Therefore, beam dimensions are changed from 300mm by 600mm to 300mm by 550mm

<u>Note</u> The assumed dead load of the beam *need not* be revised due to small change.

8) Determination of the effective flange width,

i) $\dfrac{l}{4} = \dfrac{7,800}{4} = 1,950mm$

ii) $16h_f + b_w = (16)(120) + 300 = 2,220mm$

iii) distance between the center of adjacent slab $= 2,400\text{mm}$

⇨ 1,950mm controls

9) The maximum moment at midspan

$$M_u = \frac{1}{8} w_u l^2 + P_u a = \left(\frac{1}{8}\right)(47.1)(7.8)^2 + (122)(3) = 724 kN \cdot m$$

5. Bond / Anchorage / Develop. Length

10) Assuming that the stress-block depth a is equal to the slab thickness,

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2} \right)} = \frac{724 \times 10^6}{(0.85)(400)(450 - 120/2)} = 5,460 mm^2$$

then

$$a = \frac{A_s f_y}{0.85 f_{ck} b} = \frac{(5,460)(400)}{(0.85)(30)(1,950)} = 43.9 mm < 120 mm$$

⇨ rectangular beam equations are valid for this T beam.

5. Bond / Anchorage / Develop. Length

11) Calculation of improved reinforcement amount with calculated stress-block depth *a*

$$A_s = \frac{724 \times 10^6}{(0.85)(400)(450 - 43.9/2)} = 4,970 mm^2$$

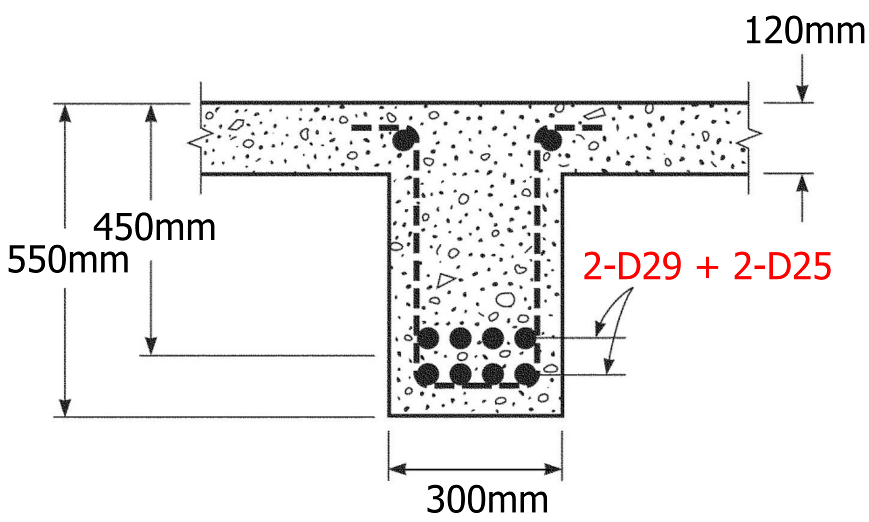
12) Check the maximum reinforcement ratio

$$\begin{aligned}
\rho_{\max} &= 0.75\rho_b \\
&= 0.75\left(0.85\beta_1 \frac{f_{ck}}{f_y} \frac{600}{600 + f_y}\right) \\
&= (0.75)(0.85)(0.85)\frac{30}{400}\frac{600}{600+400} \\
&= 0.0244 > \rho = \frac{A_s}{bd} = \frac{4,970}{(1,950)(450)} = 0.00566 \quad \text{O.K.}
\end{aligned}$$

1st trial

13) Provide four D29 and four D25 bars with a total area of 4,597mm^2.

but this is smaller than A_s = 4,970mm^2 (N.G.)



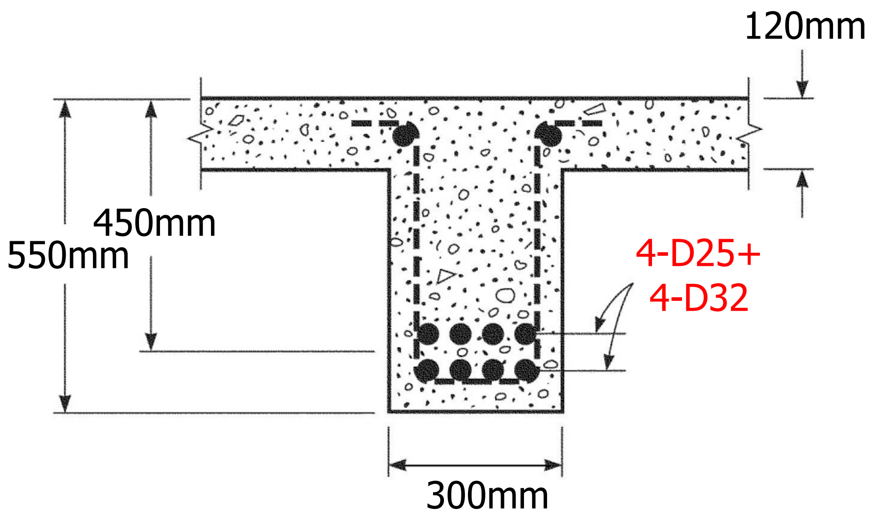
5. Bond / Anchorage / Develop. Length

2nd trial

13) Provide four D32 and four D25 bars with a total area of 5,204mm^2.

They will be arranged in two rows, with D25 bars at the upper row and D32 bars at the lower row.

⇦ Of course, spacing limitation according to KCI Code should be satisfied.



5. Bond / Anchorage / Develop. Length

14) While KCI Code permit discontinuation of one-third of the longitudinal rebars for simple span, in this case, it is convenient to discontinue the upper layer.

15) The moment capacity of the member after the upper layer of bars has been discontinued is then found. (A_s for 4D32=3,177mm^2)

$$a = \frac{A_s f_y}{0.85 f_{ck} b} = \frac{(3,177)(400)}{(0.85)(30)(1,950)} = 25.5mm$$

$$\Rightarrow \quad \phi M_n = \phi A_s f_y (d - \frac{a}{2})$$

$$= (0.85)(3,177)(400)(450 - \frac{25.5}{2}) = 472.2kN \cdot m$$

5. Bond / Anchorage / Develop. Length

16) If x is the distance from the support centerline to the point where the moment is 472.2kN·m, then

$$306x - \frac{47.1x^2}{2} = 472.2$$

$$\Rightarrow \quad x = 1.78m$$

17) The upper bar must be continued beyond this theoretical cutoff point at least d or $12d_b$

d=450mm, $12d_b$=(12)(25)=300mm

5. Bond / Anchorage / Develop. Length

Note

The full development length l_d must be provided PAST the maximum-moment section at which the stress in bars to be cut is assumed to be f_y.

Because of the heavy concentrated load near the midspan, the point of peak stress will be assumed to be at the concentrated loads rather than the midspan.

18) Calculation of development length.

Assuming the cover to the outside of the D10 stirrups, side cover is 5+40=45mm, or $1.4d_b \geq 1.0d_b$

Assuming equal clear spacing between all four bars, the clear spacing is [300-2×(40+10+32+32)]/3=24mm, or $0.75d_b \leq 1.0d_b$ (N.G.)

Back to 13)

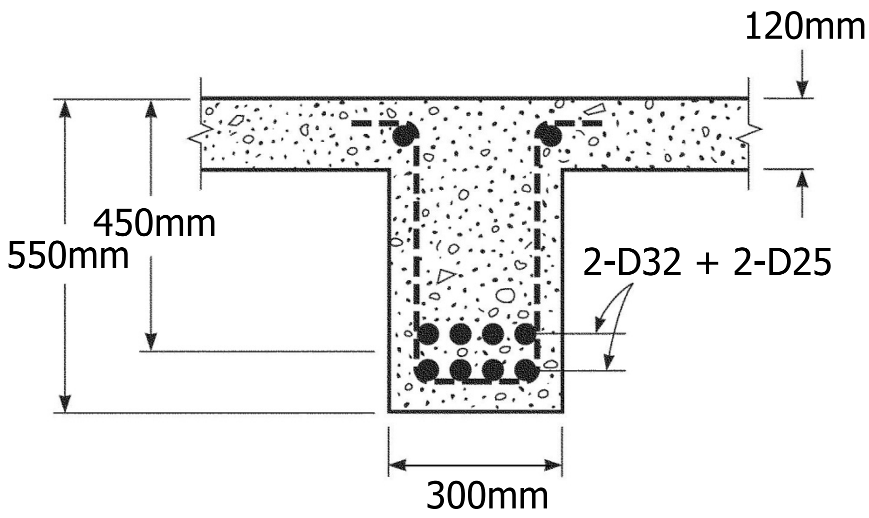
5. Bond / Anchorage / Develop. Length

3rd trial

13) Provide four D32 and four D25 bars with a total area of 5,204mm^2.

They will be arranged in two rows, with D32 bars at the outer end of each rows.

⇦ Of course, spacing limitation according to KCI Code should be satisfied.



5. Bond / Anchorage / Develop. Length

14) While KCI Code permit discontinuation of one-third of the longitudinal rebars for simple span, in this case, it is convenient to discontinue the upper layer, consisting of one-half of the total area.

15) The moment capacity of the member after the upper layer of bars has been discontinued is then found. (A_s for 2D32 and 2D25=2,602mm^2)

$$a = \frac{A_s f_y}{0.85 f_{ck} b} = \frac{(2,602)(400)}{(0.85)(30)(1,950)} = 20.9mm$$

$$\Rightarrow \quad \phi M_n = \phi A_s f_y (d - \frac{a}{2})$$

$$= (0.85)(2,602)(400)(450 - \frac{20.9}{2}) = 388.8kN \cdot m$$

5. Bond / Anchorage / Develop. Length

16) If x is the distance from the support centerline to the point where the moment is 388.8kN·m, then

$$306x - \frac{47.1x^2}{2} = 388.8$$

$$\Rightarrow \quad x = 1.43m$$

17) The upper bar must be continued beyond this theoretical cutoff point at least d or $12d_b$

d=450mm, $12d_b$=(12)(32)=384mm

5. Bond / Anchorage / Develop. Length

Note

The full development length l_d must be provided PAST the maximum-moment section at which the stress in bars to be cut is assumed to be f_y.

Because of the heavy concentrated load near the midspan, the point of peak stress will be assumed to be at the concentrated loads rather than the midspan.

18) Calculation of development length.

Assuming the cover to the outside of the D10 stirrups, side cover is 5+40=45mm, or $1.4d_b \geq 1.0d_b$

Assuming equal clear spacing between all four bars, the clear spacing is [300-2×(40+10+32+25)]/3=28.7mm, or $0.9d_b \leq 1.0d_b$ (N.G.)

Back to 13)

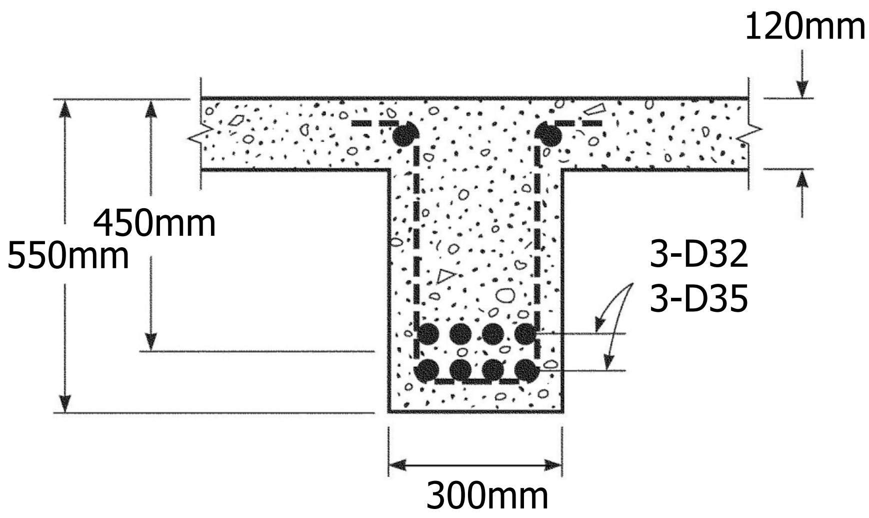
5. Bond / Anchorage / Develop. Length

4th trial

13) Provide three D35 and three D32 bars with a total area of 5,253mm^2.

They will be arranged in two rows, with D32 bars at the upper row and D35 bars at the lower row.

⇦ Of course, spacing limitation according to KCI Code should be satisfied.



5. Bond / Anchorage / Develop. Length

14) While KCI Code permit discontinuation of one-third of the longitudinal rebars for simple span, in this case, it is convenient to discontinue the upper layer.

15) The moment capacity of the member after the upper layer of bars has been discontinued is then found. (A_s for 3D35=2,870mm^2)

$$a = \frac{A_s f_y}{0.85 f_{ck} b} = \frac{(2,870)(400)}{(0.85)(30)(1,950)} = 23.08mm$$

$$\Rightarrow \quad \phi M_n = \phi A_s f_y (d - \frac{a}{2})$$

$$= (0.85)(2,870)(400)(450 - \frac{23.08}{2}) = 427.8kN \cdot m$$

5. Bond / Anchorage / Develop. Length

16) If x is the distance from the support centerline to the point where the moment is 427.8kN·m, then

$$306x - \frac{47.1x^2}{2} = 427.8$$

$$\Rightarrow \quad x = 1.59m$$

17) The upper bar must be continued beyond this theoretical cutoff point at least d or $12d_b$

d=450mm, $12d_b$=(12)(32)=384mm

5. Bond / Anchorage / Develop. Length

Note

The full development length l_d must be provided PAST the maximum-moment section at which the stress in bars to be cut is assumed to be f_y.

Because of the heavy concentrated load near the midspan, the point of peak stress will be assumed to be at the concentrated loads rather than the midspan.

18) Calculation of development length.

Assuming 40mm the cover to the outside of the D10 stirrups, side cover is 10+40=50mm, or $1.43d_b \geq 1.0d_b$

Assuming equal clear spacing between all three bars, the clear spacing is [300-2×(40+10)-3×35]/2=47.5mm, or $1.36d_b \geq 1.0d_b$

5. Bond / Anchorage / Develop. Length

Noting that the KCI Code requirements for minimum stirrups are met, it is clear that all restrictions for the use of the simplified equation for development length are met. From the Table 5.1 (slide 27page)

$$l_d = \left(\frac{0.6 f_y \alpha \beta \lambda}{\sqrt{f_{ck}}} \right) d_b = \left(\frac{(0.6)(400)(1)(1)(1)}{\sqrt{30}} \right) (32)$$

$$= 1,402mm = 1.4m$$

19) Thus, (1) the bar must be continued at least 0.9+1.4=2.3m past the midspan point. (3.9-2.3=1.6m from the support centerline)

But, in addition (2) they must continue to a point 1.59-0.45=1.14m from the support centerline.

⇦ KCI Code requirement ; d=0.45 > $12d_b$=0.384

5. Bond / Anchorage / Develop. Length

20) Requirement (2) controls, so upper layer will be terminated 1.14-0.15 = 0.99m from the support face.

21) The bottom layer of bars will be extended to a point 75mm from the end of the beam, providing 1.59+0.075=1.665m embedment past the critical section for cutoff of the upper bars.

This exceeds the development length, l_d=1.402m of the lower set of bars.

Note

A simpler design, using very little extra steel, would result from extending all six positive bars into the support. Whether or not the more elaborate calculations and more complicate placement are justified would depend largely on the number of repetitions of the design in the total structure.

5. Bond / Anchorage / Develop. Length

23) Checking the bar cutoff general rule 4 (slide 67, KCI 8.5.2) to ensure that the continued steel is sufficiently small diameter determines that

$$l_d \leq 1.3\frac{M_n}{V_u} + l_a = (1.3)\frac{\left(\frac{427.8}{0.85}\right)(1,000)}{306} + 75 = 2,213mm$$

The actual l_d of 1,402mm satisfies this restriction.

<u>Note</u>

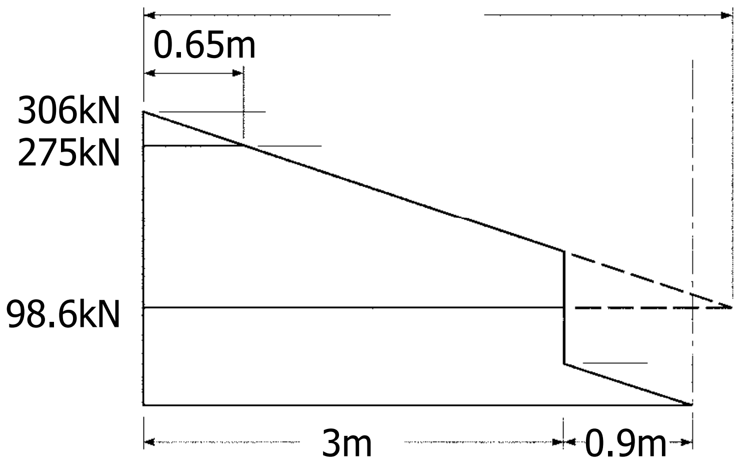
Since the cut bars are located in the tension zone, special binding stirrups will be used to control cracking; these will be selected after the normal shear reinforcement has been determined.

5. Bond / Anchorage / Develop. Length

24) The shear contribution of concrete is

$$\phi V_c = \phi \frac{1}{6}\sqrt{f_{ck}}\, b_w d = (0.8)(\frac{1}{6})(\sqrt{30})(300)(450)$$

$$= 98,590N = 98.6kN$$



Therefore, web reinforcement must be provided for the shaded part of the shear diagram.

5. Bond / Anchorage / Develop. Length

25) Select D10 stirrups and check the maximum spacing,

i) $\frac{d}{2} = \frac{450}{2} = \underline{225mm}$ *controls*

ii) $600mm$

iii) $\frac{A_v f_y}{0.35 b_w} = \frac{(2 \times 71)(400)}{(0.35)(300)} = 541mm$

26) Your share………………. Steel portioning, etc.

5. Bond / Anchorage / Develop. Length

We skipped the following topics in this class.

- ANCHORAGE of TENSION BARS by HOOKS
- ANCHORAGE REQUIREMENTS for WEB REINFORCEMENT
- WELDED WIRE REINFORCEMENT
- DEVELOPMENT of BARS in CONPRESSION
- BUNDLED BARS

But, those are very important issues in practice. At least, you all have to keep it mind that such requirements are provided by KCI Code.

5. Bond / Anchorage / Develop. Length

BAR SPLICES

- The need to SPLICE reinforcing bars is a reality due to the limited lengths of steel available.
- All bars are readily available in lengths from 6m to 12m due to shipping purpose.
- The most effective means of continuity in reinforcement is to WELD the cut pieces without reducing the mechanical properties of bars.
- However, COST considerations require alternative methods

5. Bond / Anchorage / Develop. Length

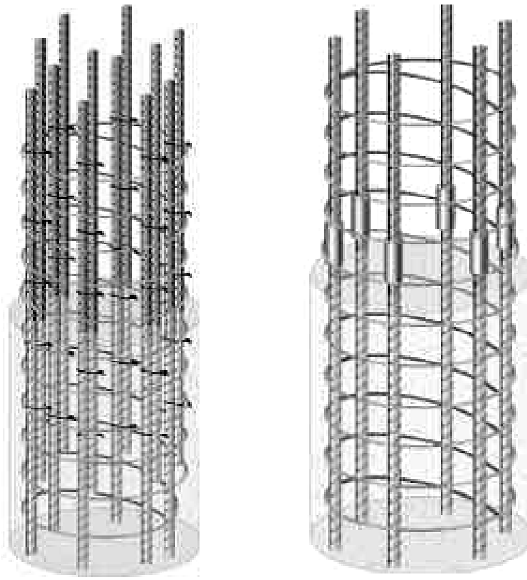
Three Types of Splicing

1. Lap splicing

   depends on full bond development of the two bars at the lap for bars not larger than D35

2. Mechanical connecting

   can be achieved by mechanical sleeves threaded on the ends of bars to be connected. – economical/effective for larger-diameter bars.



5. Bond / Anchorage / Develop. Length

Three Types of Splicing

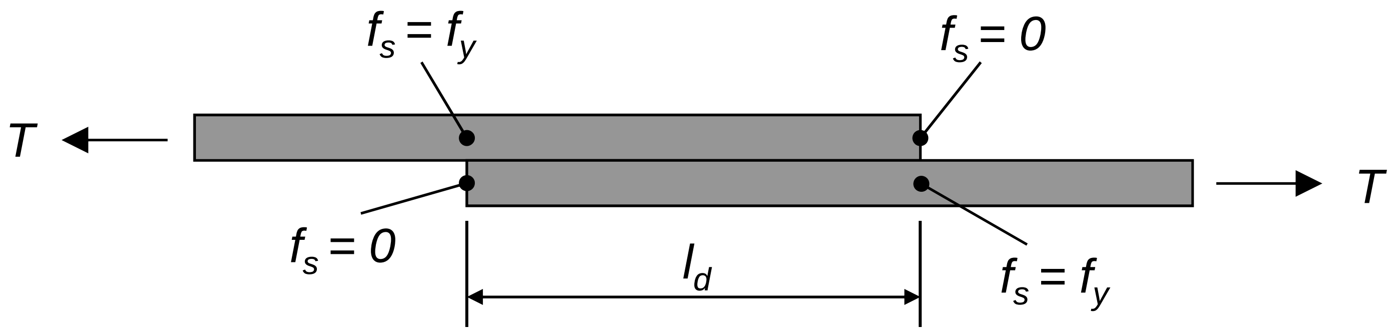
3. Welding

   can become economically justifiable for bar sizes larger than No.11 bars

5. Bond / Anchorage / Develop. Length

Concept of Lap Splicing

- The idealized tensile stress distribution in the bars along the splice length l_d has a maximum values f_y at the splice end and $0.5f_y$ at $l_d/2$



At failure, the expected of slip is approximately $(0.5f_y/E_s)(0.5l_d)$

5. Bond / Anchorage / Develop. Length

Lap Splices in Tension

Two classifications of lap splices corresponding to the minimum length of lap required. (KCI 8.6.2)

The minimum length l_d, but not less than 300mm is,

class A : $1.0 l_d$

class B : $1.3 l_d$

Note

Class A splices are allowed when the area of reinforcement that required by analysis over the entire length of the splice and one-half or less of the total reinforcement is spliced within the required lap length.

5. Bond / Anchorage / Develop. Length

Lap Splices in Compression

The minimum length of lap for compression splices is, (KCI 8.6.3)

For bars with $f_y \leq$ 400MPa $0.072 f_y d_b$

For bars with $f_y <$ 400MPa $(0.13 f_y - 24) d_b$

But not less than 300mm.

For, f_{ck}<21MPa, the required lap is increased by one-third.