6. Serviceability

INTRODUCTION

CRACKING IN FLEXURAL MEMBERS

REVISED KCI CODE PROVISIONS (2007)

DEFLECTIONS IN FLEXURAL MEMBER

DEFLECTIONS DUE TO LONG-TERM LOADS

KCI CODE PROVISIONS FOR DEFLECTION CONTROL

447.327
Theory of Reinforced Concrete and Lab. I
Spring 2008

6. Serviceability

INTRODUCTION

- Serviceability Limit state

  ; disrupt use of structures but do not cause collapse.

<u>Note</u>

1. Ultimate limit state leads to collapse.
2. Provision of adequate strength does not always guarantee the satisfactory performance in normal service.

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- Serviceability of a structure is determined by its

  - crack width

  - deflection

  - vibration

  - fatigue

  - extent of corrosion of its reinforcement

  - surface deterioration of its concrete

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CRACKING IN FLEXURAL MEMBERS

ALL reinforced concrete beams crack, generally starting at loads well BELOW service level, and possibly even PRIOR TO loading due to restrained shrinkage.

Flexural cracking due to loads in not only INEVITABLE, but actually NECESSARY for the reinforcement to be used effectively.

<u>Note</u>

Prior to flexural cracks, f_s is $n(E_s/E_x \approx 8)$ times the tensile stress in the adjacent concrete. For instance, when the concrete is close to its modulus of rupture of about 3.5MPa, f_s is only $8 * 3.5 = 28\text{MPa} \ll 400\text{MPa}$

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CRACKING IN FLEXURAL MEMBERS

Why is Crack Control needed?

- Cracking could have an effect on corrosion of the reinforcement.
- There is no clear correlation between corrosion and surface crack width in the usual range found in structures at service load level
- Rather than a small number of LARGE cracks, it is more desirable to have only HAIRLINE cracks and to accept more numerous cracks if necessary.

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Variables Affecting Width of Cracks

1. Shape of rebars

   Beams with undeformed round bars will show a relatively small number of rather wide cracks in service, while beams with deformed bars will show a larger number of very fine cracks.

2. Stress in the reinforcement (Gergely and Lutz, 1968)

   crack width is proportional to f_s^n, where an exponent n varies from 1.0 to 1.4. For steel stresses in the range of practical interest, 140~250MPa, it may be taken equal to 1.0, f_s can be calculated using elastic-cracked section and alternatively be taken as $0.6f_y$ (KCI 4.2.4)

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3. Concrete cover distance (Broms, 1965)

   increasing cover increases the spacing of cracks and also increases crack width.

4. Bar diameter and number.

   A larger number of small diameter reduces crack width rather than the minimum number of larger bars.

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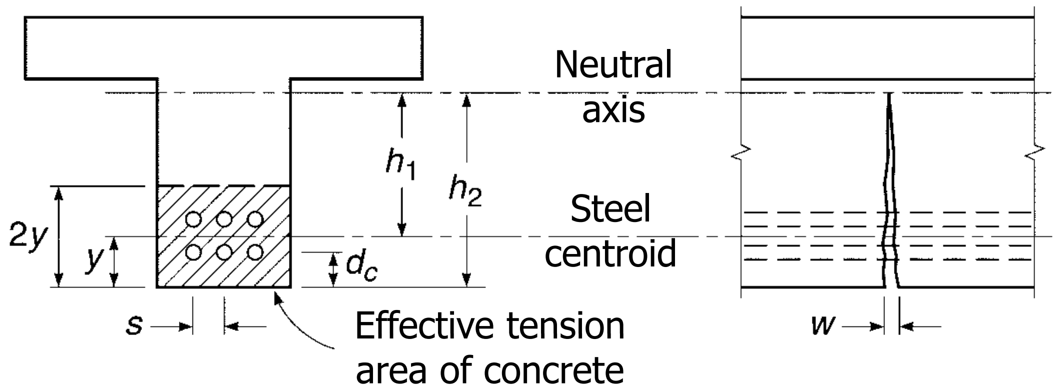
Equation for crack width

- Crack control provision in the KCI Code 4.2.4 is developed by Gergely and Lutz.(1968)

$$w = 1.08\beta_c f_s \sqrt[3]{d_c A} \times 10^{-5} \qquad (1)$$

, where w = maximum crack width

f_s = service load stress in reinforcement (mm)



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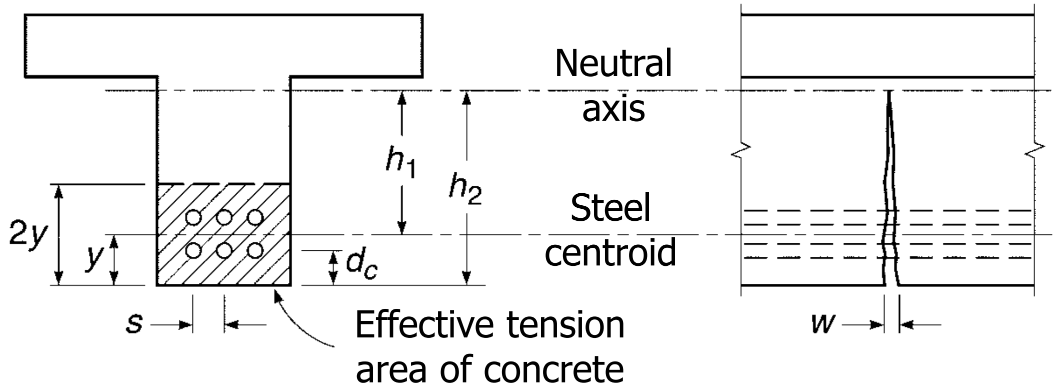
$$w = 1.08\beta_c f_s \sqrt[3]{d_c A} \times 10^{-5} \qquad (1)$$

, where β_c = ratio of distances from tension face and from steel centroid to neutral axis, h_2/h_1

d_c = thickness of concrete cover measured from tension face to center of bar closest to that face

A = concrete area surrounding one bar, equal to total effective tension area of concrete surrounding reinforcement and having same centroid, divided by number of bars(mm^2)

for 1 layer of bar $A=2d_c b/n$



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Equation for crack width

- In addition to Eq.(1), ACI Code provision for crack control is based on this equation as well.

$$w = 2,000 \frac{f_s}{E_s} \beta_c \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \quad \text{(mm)} \qquad (2)$$

, where s = maximum bar spacing

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KCI Code Provisions (2003) for Crack Control

- KCI Code 4.2.3 provides allowable crack width according to surrounding environmental condition. (*Handout 6-1*)
- Also, crack with of flexural member under service load is calculated using Eq.(1)

  - f_s can be taken as $0.6f_y$

  - β_c is 1.2 for beams
    1.35 for one-way slabs

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KCI Code Provisions (2003) for Crack Control

- In ACI Code, crack width is controlled by establishing a maximum center-to-center spacing s for the reinforcement closest to the surface.

$$s = \frac{96,000}{f_s} - 2.5t_c \le \frac{76,800}{f_s} \quad \text{(mm)} \qquad (3)$$

, where t_c is the nearest clear concrete cover but d_c can be used for design simplification.

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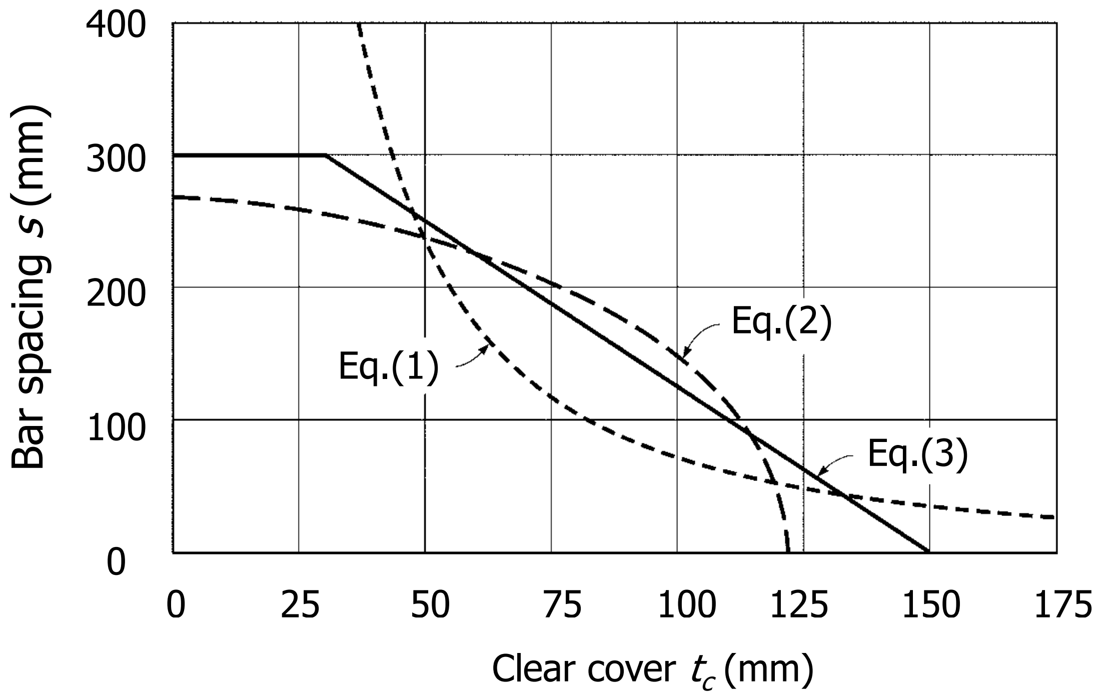

Maximum bar spacing vs. clear cover :

Comparison Eqs.(1), (2), and (3) for maximum crack width w=0.4mm, f_s=250MPa, β_c=1.2, bar size=D25

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REVISED KCI CODE PROVISIONS (2007)

Comparison of Code Provisions

- Limiting the crack width

$$w \leq w_a$$

: KCI 2003, 도로교설계기준(2005), KCI 2007 Appx.V

- Limiting the reinforcement spacing

$$s \leq s_a$$

: KCI 2007 (Ch. 4 & 6), posterior to ACI 318-99

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REVISED KCI CODE PROVISIONS (2007)

Comparison of Code Provisions

	KCI 2003	KCI 2007
Ch.4	$w \le w_a$ w_a = 환경조건 4가지에 대한 허용균열폭 w = Eq. (1)	- 환경조건 4가지와 해당 허용균열폭 삭제. - 수밀성이 요구되는 경우와 미관이 중요한 경우에는 허용균열폭을 설정하여 균열을 검토할 수 있음.
Ch.6	철근 항복강도 300 MPa 이상인 경우, $w \le w_a$	철근 간격 제한 $s < s_a$ s = Eq.(3)
Appx. V		- 환경조건 4가지에 대한 수정된 허용균열폭 - CEB-FIP 방식의 균열폭 계산법

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DEFLECTIONS IN FLEXURAL MEMBER

Why is Deflection Control Needed?

1) Visual appearance

$\Delta > \dfrac{l}{250}$ are generally visible.

e.g.) 10m span ⇨ 40mm

2) Damage to non-structural elements.

- cracking of supported walls and partitions
- ill-fitting door and windows.

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Why is Deflection Control Needed?

3) Malfunction

- poor roof drainage
- misalignment of sensitive machinery

4) Damage to structural element

- contact with other members ; modify load paths

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Two Approaches to Deflection Control

1) Indirect method

setting suitable upper limits on the span-depth ratio

2) Direct method

calculating deflections and to compare those predicted values with specific limitations by codes

- immediate deflection : can be calculated based on the properties of the uncracked elastic member, the cracked elastic member or some combination of these.

- long term deflection : due to creep and shrinkage.

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Immediate deflections

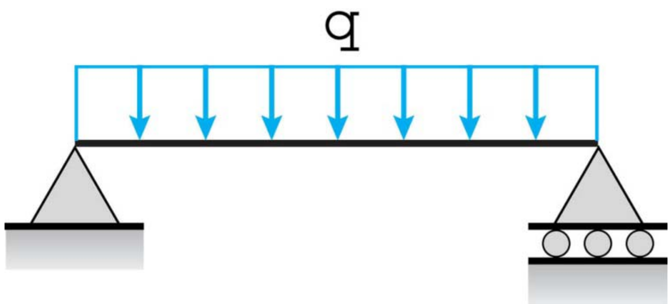
- Elastic deflections

$$\Delta = \frac{f\,(loads, spans, supports)}{EI}$$

, where *EI* is flexural rigidity

⇨ *f* can easily be computed using classic structural theories.

e.g.)



$$⇨ \; \Delta_{\max} = \frac{5ql^4}{384EI}$$

⇨ Therefore, the particular problem is the determination of appropriate *EI* for a member consisting of two materials.

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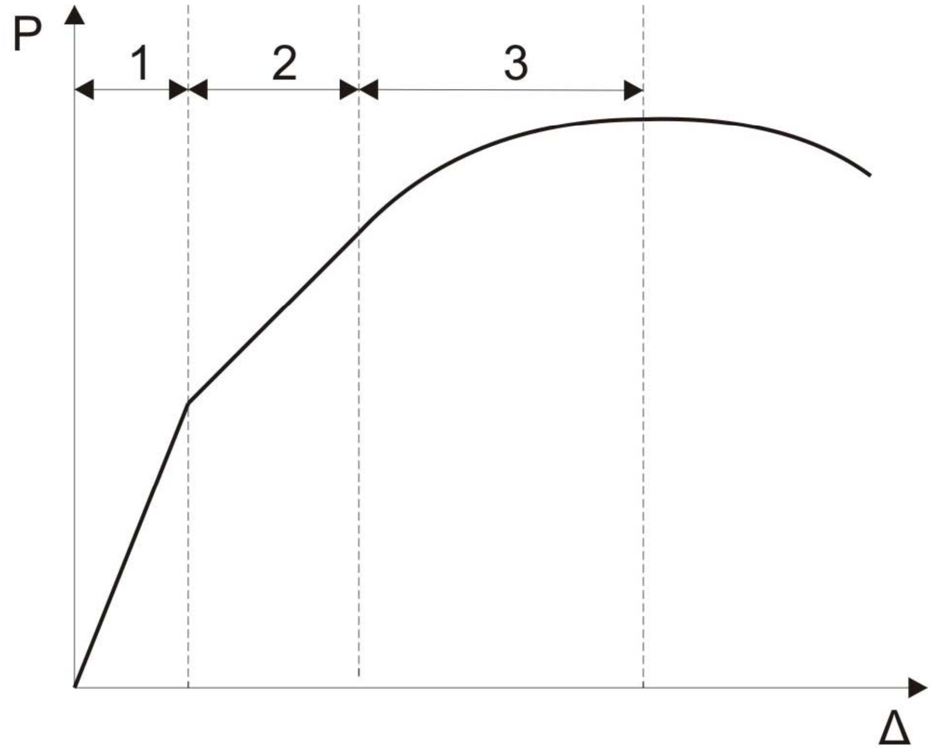
Deflection Behavior of Beams (*immediate* deflection)

- The load-deflection relationship of reinforced concrete beam is basically trilinear.



Region I :
Precracking stage

Region II :
Postcracking stage

Region III :
Postserviceability stage

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Deflection Behavior of Beams (*immediate* deflection)

– Region I : Precracking stage

  where a structural member is crack-free

– Region II : Postcracking stage

  where the structural member develops acceptable controlled cracking both in distribution and width

– Region III : Postserviceability stage

  where the stress in the tension reinforcement reaches the limit state of yielding

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Region I : precracking stage

- The maximum tensile stress of concrete is less than its tensile strength in flexure, that is less than the modulus of rupture f_r of the concrete.
- The flexural stiffness *EI* can be estimated using the Young's Modulus of concrete, E_c, and the uncracked reinforced concrete cross-section. (i.e. gross section is effective.)
- The load deflection behavior depends on the stress-strain relationship of the concrete. (i.e. follows the Hook's law)

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Region I : precracking stage (cont.)

- The value of E_c can be obtained from the KCI empirical expression.

  - unit weight w_c=1,450~2,500kg/m^3

$$E_c = 0.043 w_c^{1.5} \sqrt{f_{ck}} \quad \text{(MPa)} \quad \text{for } f_{ck} \leq 30\text{MPa} \quad (4)$$

$$E_c = 0.3 w_c^{1.5} \sqrt{f_{ck}} + 7,700 \quad \text{(MPa)} \quad \text{for } f_{ck} > 30\text{MPa} \quad (5)$$

  - for normal weight concrete w_c=2,300kg/m^3

$$E_c = 4,700 \sqrt{f_{ck}} \quad \text{(MPa)} \quad \text{for } f_{ck} \leq 30\text{MPa} \quad (6)$$

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Region I : precracking stage (cont.)

- The precracking region stops at the initiation of the first flexural crack when the concrete stress reaches its modulus of rupture f_r
- The value of f_r for normal-weight concrete is

$$f_r = 0.63\sqrt{f_{ck}} \tag{7}$$

; for all lightweight concrete, multiply the value of f_r by 0.75

; for sand-light weight concrete, multiply the value of f_r by 0.85

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Region I : precracking stage *(cont.)*

- There are two approaches in estimation of the moment of inertia I in this region.

  i) Gross section : where I_g is calculated neglecting the presence of any reinforcing steel

  ii) Transformed section : where I_{gt} is calculated taking into account the additional stiffness contributed by the steel reinforcement.

Note 1) method ii) is more accurate than i)

2) I_{gt} is often expressed as I_{ut}

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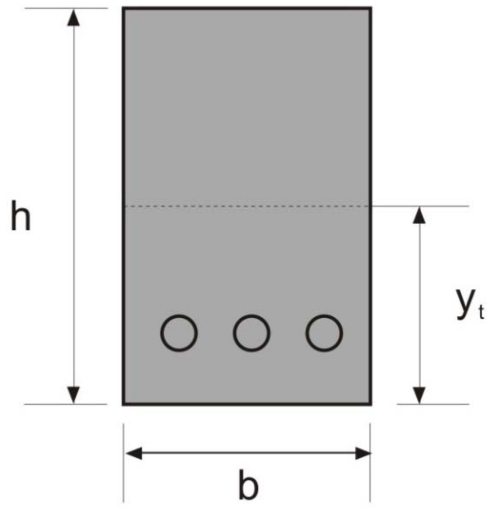
Review chapter 3

- Calculation of I_g and M_{cr}



Neglecting the stiffness by steel

$$I_g = \frac{bh^3}{12} \tag{8}$$

$$M_{cr} = \frac{I_g f_r}{y_t} = \frac{\left(\frac{bh^3}{12}\right) f_r}{\left(\frac{h}{2}\right)} = \frac{bh^2 f_r}{6} \tag{9}$$

Note that $y_t = h/2$ for rectangular beam

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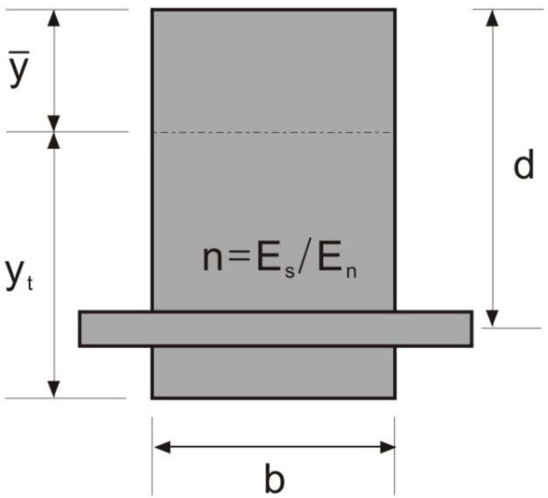
Review chapter 3 (cont.)

- Calculation of I_g and M_{cr}



$$\bar{y} = \frac{Q_x}{A} = \frac{\int y\,dA}{\int dA} = \frac{\dfrac{bh^2}{2} + (n-1)A_s d}{bh + (n-1)A_s} \tag{10}$$

$$I_{ut} = \int y^2 dA$$

$$= \frac{bh^3}{12} + \left(\frac{h}{2} - \bar{y}\right) bh + (d - \bar{y})^2 (n-1) A_s \tag{11}$$

$$M_{cr} = \frac{I_{ut} f_r}{y_t} \tag{12}$$

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<u>Region II : postcracking stage</u>

- The precracking region ends at the initiation of the first crack and moves into region II.

  ; most beams lie in this region at service loads.

- At limit state of service load cracking, the contribution of tension-zone concrete to flexural stiffness is neglection.

  ; The moment of inertia of the cracked section is designated I_{cr}

<u>Note</u> stresses are STILL elastic in spite of section cracked.

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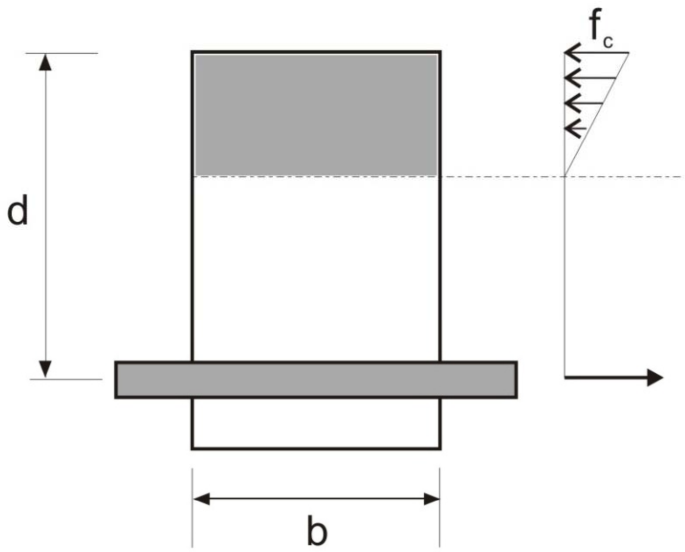
Region II : postcracking stage



- Calculation of I_{cr}

$$I_{cr} = \frac{bc^3}{3} + nA_s(d-c)^2 \qquad (13)$$

, where the depth of neutral axis c can be calculated from the following,

$$bc\frac{c}{2} - nA_s(d-c) = 0$$

$$\Rightarrow \frac{bc^2}{2} - nA_s d + nA_s c = 0 \qquad (14)$$

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Region III : postserviceability cracking stage

- The load-deflection diagram is considerably flatten in this region due to substantial loss in stiffness of the section.

  ; extensive cracking takes place.

- The beam is considered at this stage to have structurally failed by yielding of the tension steel, and continues to deflect without additional loading.

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Region III : postserviceability cracking stage (cont.)

- The cracks continue to open, and the neutral axis continues to rise upward until total crushing of the concrete when rupture occurs.

Note

Postyield deflection and limit deflection at failure are not of major significant in design and hence are not being discussed in any detail, however, it is IMPORTANT to recognize the reserve deflection capacity as a measure of ductility in structures.

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Concept of effective moment of inertia I_e

- In case of Region II, in realty only of the beam cross-section is cracked.

  ; the uncracked segments below the neutral axis along the beam span possess SOME degree of stiffness, which contributes to over all beam rigidity.

- Generally, as the load approaches the steel yield load level, the stiffness value approaches $E_c I_{cr}$
- Therefore, actual stiffness lies between $E_c I_g$ and $E_c I_{cr}$ at Region II.

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Concept of effective moment of inertia I_e

- The KCI Code recommends that deflection be calculated using and effective moment of Inertia, I_e, where

$$I_{cr} < I_e < I_{ut} \tag{15}$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_{ut} + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_{ut} \tag{16}$$

where, I_{cr} is of the cracked transformed section and M_a is the applied moment.

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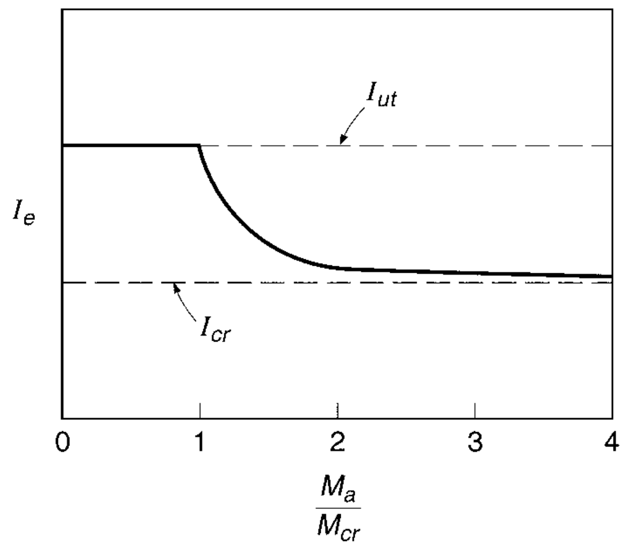
Concept of effective moment of inertia I_e



Observation

1. $M_a/M_{cr} < 1.0 \Rightarrow I_e = I_{ut}$

   $M_a/M_{cr} > 3.0 \Rightarrow I_e = I_{cr}$

2. Typical values of M_a/M_{cr} at full service load range from 1.5 to 3.0

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DEFLECTIONS DUE TO LONG-TERM LOADS

- In addition to deflections that occur IMMEDIATELY, reinforced concrete members are subjected to added deflections that occur gradually over long periods.
- These additional deflection are due mainly to CREEP and SHRINKAGE and may eventually become excessive.
- Creep generally dominates, but for some types of members, shrinkage deflections are large and should be considered separately.

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DEFLECTIONS DUE TO LONG-TERM LOADS

Effect of Concrete Creep on curvature

- Creep deformations of concrete are directly proportional to the compressive stress up to and beyond the usual service load range.
- For a reinforced concrete beam, the long-term deformation is much more complicated than for an axially loaded cylinder

⇦ while the concrete creeps, the steel does not.

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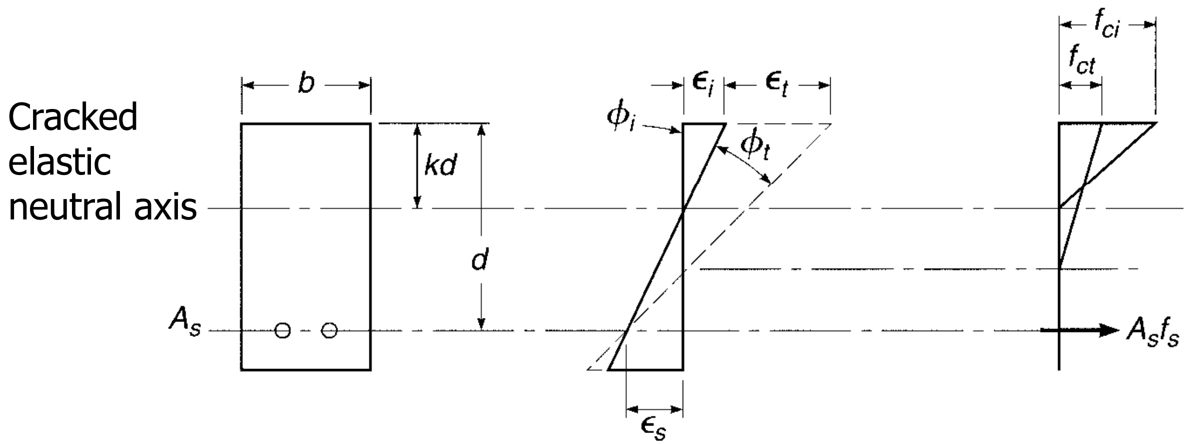
Effect of Concrete Creep on curvature



- The initial strain ε_i increases, due to creep, by the amount ε_t, while the strain ε_s in the steel is essentially unchanged.
- Because the rotation of the strain distribution diagram, the neutral axis moves down

$$\frac{\phi_t}{\phi_i} < \frac{\varepsilon_t}{\varepsilon_i} \qquad (17)$$

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Effect of Concrete Creep on curvature

- Due to the lowering of the neutral axis associated with creep, and the resulting increase in compression area, the compressive stress required to meet equilibrium is less than before.

  ; In contrast to cylinder creep test, the beam creep occurs at a gradually diminishing stress.

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Effect of Concrete Creep on curvature

- With the new neutral axis, the internal lever arm is LESS, requiring an increase in both compressive and tensile resultant forces.
- In fact, this will require a small increase in stress and strain in the steel.

⇨ ε_s in not constant as assumed originally.

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Calculation of Long-Term deflection

- Due to the complexities mentioned previously, in practice, a simplified empirical approach by a factor λ to obtain the additional long term deflection is employed.

$$\Delta_t = \lambda \Delta_i \tag{18}$$

, where Δ_i is the initial elastic deflection (immediate deflection) and Δt is the additional long-term deflection due to the combined effect of creep and shrinkage.

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Calculation of Long-Term deflection

- If a beam carries a certain sustained load w (e.g. the dead load plus the average traffic load on a bridge) and is subject to a short-term HEAVY live load P (e.g. the weight of an unusually heavy vehicle), the maximum to total deflection under this combined loading is obtained as follows.

  (1) Calculate the instantaneous deflection Δ_{iw} caused by the sustained load w

  (2) Calculate the additional long-term deflection Δ_{tw} caused by the sustained load w

$$\Delta_{tw} = \lambda \Delta_{iw} \qquad (19)$$

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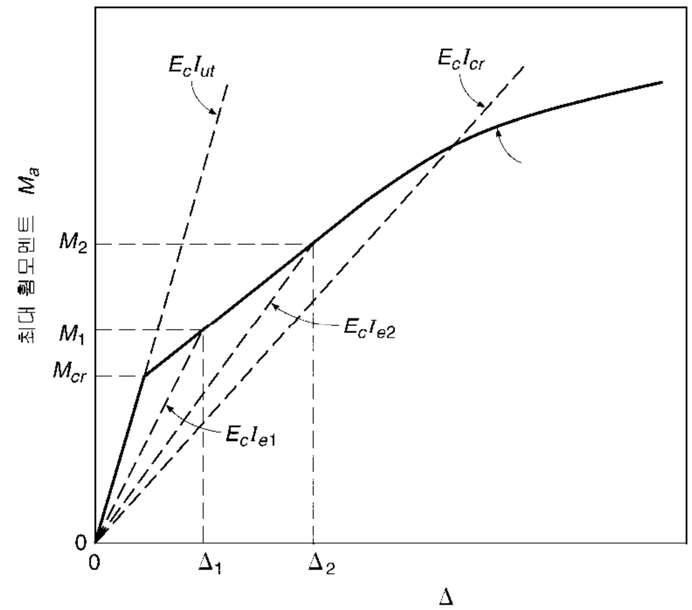
Calculation of Long-Term deflection

(3) The total deflection caused by the sustained part of the load is

$$\Delta = \Delta_w + \Delta_{iP} \qquad (20)$$

(4) In calculating the additional instantaneous deflection caused by the short-term load *P*, we should consider the fact that the load-deflection relation after cracking is NONLINEAR



$$\Delta_{ip} = \Delta_{i(w+P)} - \Delta_{iw} \qquad (21)$$

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Calculation of Long-Term deflection

Note

$\Delta_{i(w+P)}$ is the total instantaneous deflection that would be obtained if w and P were applied simultaneously, calculated by using I_e determined for the moment causes by $w + P$.

(5) The total deflection under the sustained load plus heavy short-term load is

$$\Delta = \Delta_w + \Delta_{iP} \tag{22}$$

Note

In calculations of deflections, careful attention must be paid to the LOAD HISTORY.

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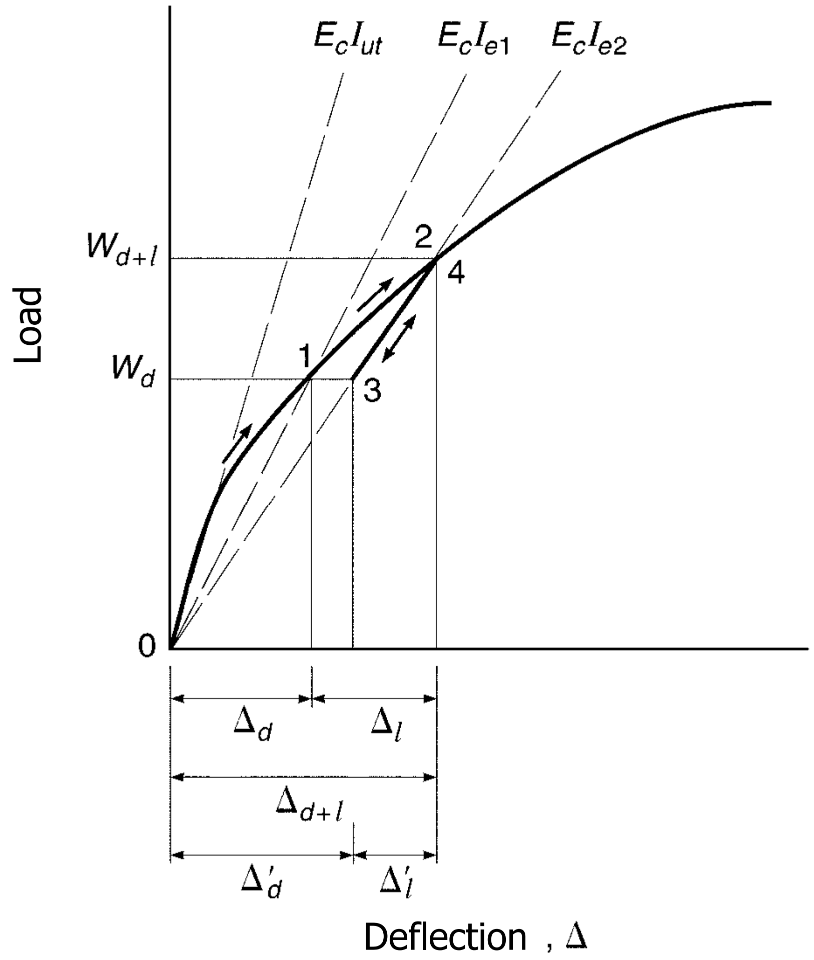
Calculation of Long-Term deflection



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KCI CODE PROVISIONS FOR DEFLECTION CONTROL

Minimum Depth-Span Ratio

- A simple indirect method for deflection control

  ; impose restrictions on the minimum member depth *h*, relative to the span *l*

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Minimum Depth-Span Ratio

Table 6.1 Minimum thickness of nonprestressed beams or one-way slabs unless deflections are computed

Member	Simply Supported	One End Continuous	Both Ends Continous	Cantilever
	Minimum Thickness, *h*			
	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	$l/20$	$l/24$	$l/28$	$l/10$
Beams or ribbed one-way slabs	$l/16$	$l/18.5$	$l/21$	$l/8$

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Minimum Depth-Span Ratio

- Values given in Table 6.1 are to be used directly for normal-weight concrete with w_c=2,300kg/m^3 and reinforcement with f_y=400MPa

  Modification factors for other cases

  - for light-weight concrete with 1,500~2,000kg/m^3

$$1.65 - 0.00031 w_c \geq 1.09 \tag{23}$$

  - for other reinforcement

$$0.43 + f_y / 700 \tag{24}$$

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Minimum Depth-Span Ratio

Table 6.3 Minimum Thickness of Superstructure of Bridges

Superstructure Type	Minimum thickness, h	
	Simple Span	Continuous span
Slab of which longitudinal bars are parallel to vehicle direction	$\frac{1.2(l+3,000)}{30}$	$\frac{(l+3,000)}{30}$
T-shape Girder	$0.070l$	$0.065l$
Box Girder	$0.060l$	$0.055l$
Footpath Bridge Girder	$0.033l$	$0.033l$

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KCI CODE PROVISIONS FOR DEFLECTION CONTROL

Calculation of Immediate Deflection

- Deflections must be calculated and compared with limiting values (KCI Code Table 4.3.3) for the following cases.

  (1) When there is needed to use member depth shallower than one permitted by Table 6.1 and 6.2

  (2) When members support construction that is likely to be damaged by large deflections.

  (3) For prestressed concrete members.

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KCI CODE PROVISIONS FOR DEFLECTION CONTROL

Continuous Spans

- KCI Code 4.3.1 calls for a simple average of values obtained from Eq.(25) for the critical positive and negative moment sections.

$$I_e = 0.5I_{em} + 0.25(I_{e1}+I_{e2}) \qquad (25)$$

,where I_{em} is the effective moment of inertia

for midspan section

I_{e1} and I_{e2} are those for the negative moment section at the respective beam ends.

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Note

ACI Code 9.5.2, as an option, alike permits use of I_e for continuous prismatic beams to be taken equal to the value obtained from Eq.(25) at midspan ; for cantilever, I_e calculated at the support may be used.

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KCI CODE PROVISIONS FOR DEFLECTION CONTROL

Long-Term Deflection Multipliers

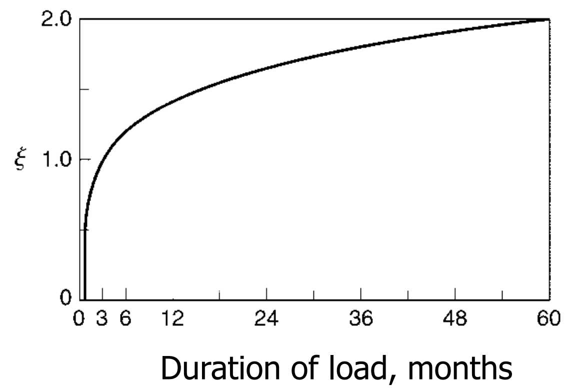
- KCI Code 4.3.1 specifies that additional long term deflection Δt can be calculated by multiplying the immediate deflection Δi by the factor

$$\lambda = \frac{\xi}{1+50\rho'} \qquad (26)$$

Note

(1) ρ' should be that at the midspan section for simple and continuous span or that at the support for cantilever.

(2) ξ and Eq.(26) are valid for both normal and light-weight concrete.



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KCI CODE PROVISIONS FOR DEFLECTION CONTROL

Permissible Deflections

Table 6.2 Maximum allowable computed deflections

Types of Member	Deflection to Be Considered	Deflection Limitation
Flat roofs no supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to the live load L	$\frac{l}{180}$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to the live load L	$\frac{l}{360}$
Roof of floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of the nonstructural elements (sum of the long-time deflection due to all sustained loads and the immediate deflection due to any additional live load	$\frac{l}{480}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\frac{l}{240}$

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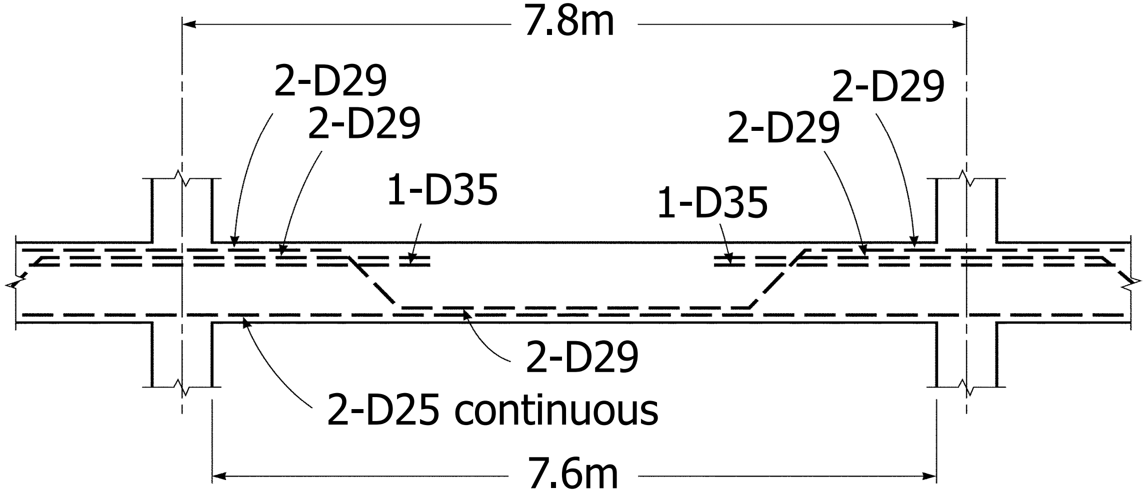
Example 6.2 Deflection Calculation

- The beam is a part of the floor system.
- Dead load w_d=16kN. Live load w_l= 33kN/m
  Of the total live load, 20% (0.2w_l= 6.6kN/m) is sustained in nature and 80% (0.8w_l= 26.4kN/m) is applied intermittently over the life.



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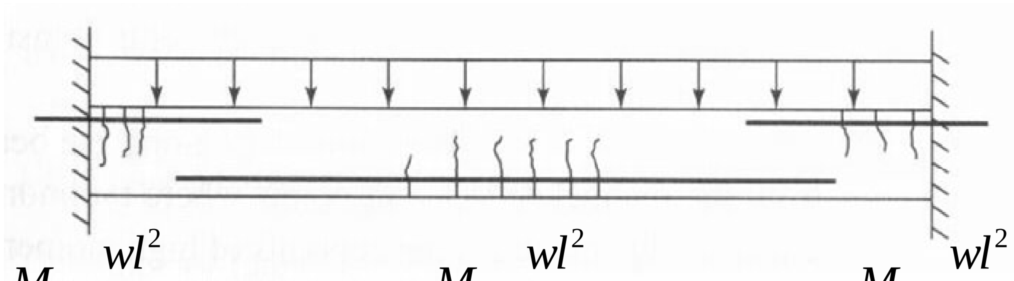
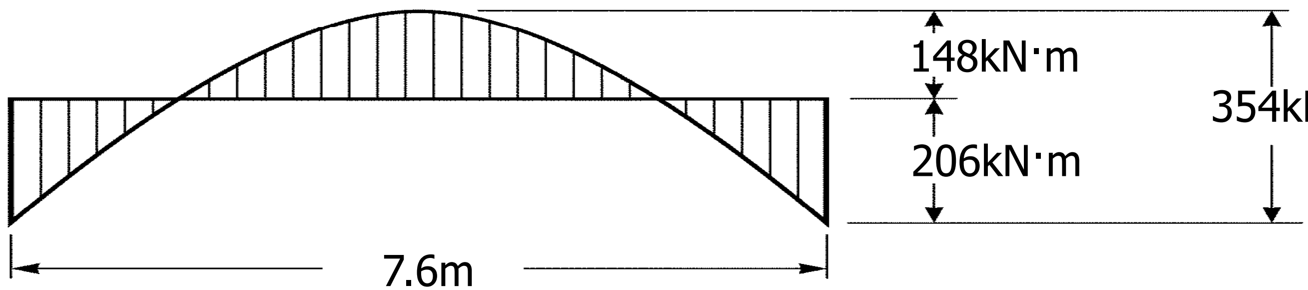
Example 6.2 Deflection Calculation

- Under the full dead load and live load, the moment diagram is,



$$M = \frac{wl^2}{12} \qquad M = \frac{wl^2}{24} \qquad M = \frac{wl^2}{12}$$



Example 6.2 Deflection Calculation

- This beam will support nonstructural partitions that would be damaged if large deflections were to occur.
- These nonstructural partitions will be installed shortly after construction shoring is removed and dead loads take effect, but before significant creep occurs.
- Calculate the part of the total deflection that would adversely affect partitions, i.e., the sum of long-term deflection due to dead and partial live load plus immediate deflection due to the nonsustained part of the live load.

6. Serviceability

Solution

1) The given and preliminary calculations are,

$$\underline{f_{ck} = 21MPa} \qquad \underline{f_{ck} = 300MPa}$$

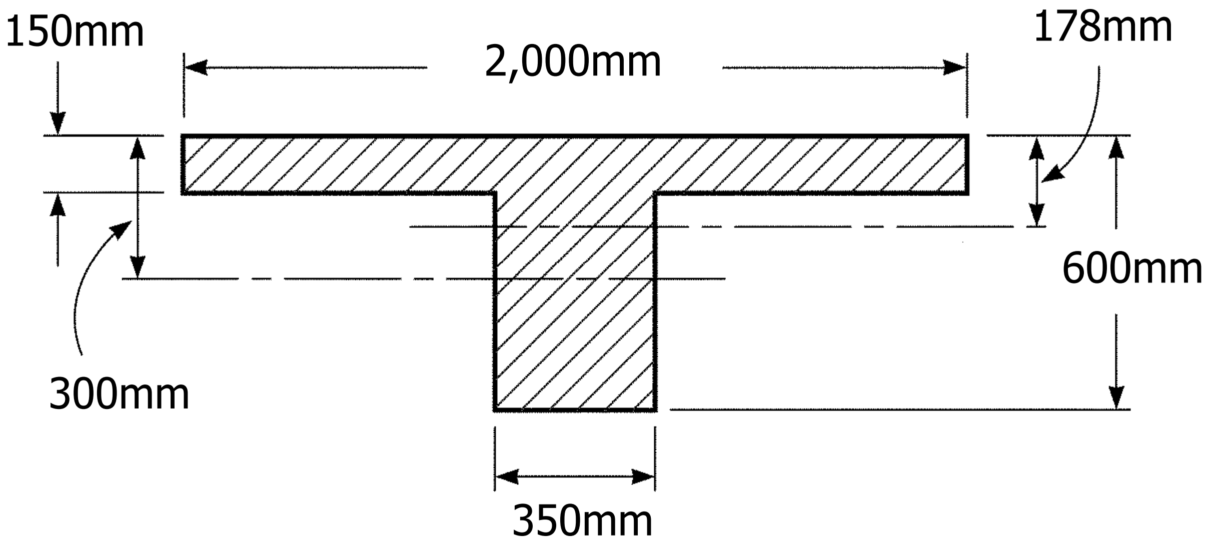
$$E_c = 4,700\sqrt{21} = 21,538N/mm^2$$

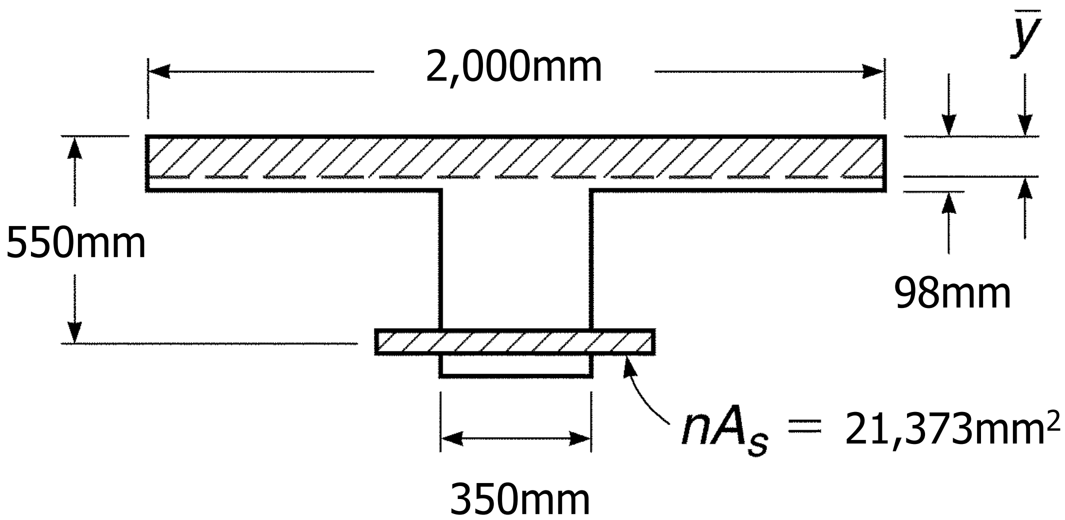
$$E_s = 200,000N/mm^2$$

$$n = \frac{E_s}{E_c} = \frac{200,000}{21,538} = 9.286$$

$$f_r = 0.63\sqrt{f_{ck}} = 0.63\sqrt{21} = 2.89N/mm^2$$

6. Serviceability





6. Serviceability

2) The centroid axis and I_g in the POSITIVE moment region

$$\bar{y} = \frac{Q_x}{A} = \frac{\int y dA}{\int dA} = \frac{(2,000)(150)\left(\frac{150}{2}\right) + (350)(600-150)\left(150 + \frac{450}{2}\right)}{(350)(600-150) + (2,000)(150)} = 178.28 \approx \underline{178mm}$$

$$I_g = \frac{(2,000)(150)^3}{12} + (2,000)(150)\left(178 - \frac{150}{2}\right)^2 + \frac{(350)(600-150)^3}{12}$$

$$+ (350)(600-150)\left[\left(150 + \frac{600-150}{2}\right) - 178\right]^2$$

$$= 1.251543 \times 10^{10} \approx \underline{1.25 \times 10^{10} mm^4}$$

6. Serviceability

3) The centroid axis and I_{cr} in the POSITIVE moment region

$$(2,000)(\bar{y})\left(\frac{\bar{y}}{2}\right)-(21,373)(550-\bar{y})=0 \quad \Rightarrow \quad \bar{y}=\underline{98mm}$$

$$I_{cr}=\frac{(2,000)(98)^3}{12}+(2,000)(98)\left(\frac{98}{2}\right)^2+(21,373)(550-98)^2$$

$$=4,994,050,725\approx\underline{4.99\times10^9\,mm^4}$$

6. Serviceability

4) The cracking moment

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(2.89)(1.25\times10^{10})}{(600-178)} = 85,604,265 \approx \underline{8.56 kN\cdot m}$$

$$\Rightarrow \quad \frac{M_{cr}}{M_a} = \frac{85.6}{148} = 0.578$$

5) The effective moment of inertia in the POSITIVE region

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g$$

$$= (0.578)^3(1.25\times10^{10}) + (1-0.578^3)(4.99\times10^9) = \underline{6.44\times10^9 mm^4}$$

6. Serviceability

6) The centroid axis and I_g in the NEGATIVE moment region

$$\bar{y} = \underline{300mm}$$

$$I_g = \frac{(350)(600)^3}{12} = 6,300,000,000 = \underline{6.3 \times 10^9\, mm^4}$$

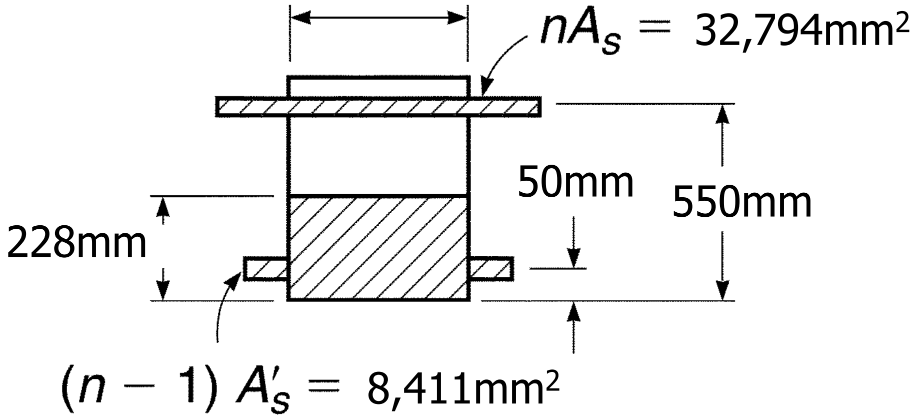
Why is this so simple?

6. Serviceability

7) The centroid axis and I_{cr} in the NEGATIVE moment region

$$(350)(\bar{y})\left(\frac{\bar{y}}{2}\right)+(8,411)(\bar{y}-50)-(32,794)(550-\bar{y})=0$$

$$\Rightarrow \quad \bar{y}=227.7\approx \underline{228mm}$$



$$I_{cr}=\frac{(350)(228)^3}{12}+(350)(228)\left(\frac{228}{2}\right)^2+(8,411)(228-50)^2$$
$$+(32,794)(550-228)^2=5,049,481,620\approx \underline{5.08\times 10^9\,mm^4}$$

6. Serviceability

8) The cracking moment

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(2.89)(6.3\times 10^9)}{300} = 60,690,000 \approx \underline{60.7 kN\cdot m}$$

$$\Rightarrow \quad \frac{M_{cr}}{M_a} = \frac{60.7}{206} = 0.295$$

9) The effective moment of inertia in the NEGATIVE region

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g$$

$$= (0.295)^3(6.3\times 10^9) + (1-0.295^3)(5.05\times 10^9) = \underline{5.08\times 10^9 mm^4}$$

6. Serviceability

10) The average value of Ie to be used in calculation of deflection according to KCI 4.3.1 and Eq.(25) in lecture note,

$$I_{e,av} = \frac{1}{2}(6.44 + 5.08) \times 10^9 = \underline{5.76 \times 10^9 mm^4}$$

<u>Note</u>

For the positive bending zone, with no compression zone, λ=2.0.

6. Serviceability

11) The deflection under full dead plus live load of 49kN/m can be calculated using the Moment-Area Method.

$$\Delta_{d+l} = \frac{2}{EI}\left[\int_0^{3.8}(-24.5x^2 + 186.2x - 206)(\frac{1}{2}x)dx\right]$$

$$= \frac{1}{EI}\left[(\frac{-24.5}{4})(3.8^4) + (\frac{186.2}{3})(3.8^3) - (\frac{206}{2})(3.8^2)\right] = \frac{641.2}{EI}$$

$$= \frac{641.2\times10^{12}}{(2.15\times10^4)(5.76\times10^9)} = \underline{5.18mm}$$

6. Serviceability

12) The time dependent portion of dead load

$$\Delta_d = 5.18 \times \frac{16}{49} \times 2.0 = \underline{3.38mm}$$

13) Sum of immediate and time dependent deflectio due to the sustained portion of live load

$$\Delta_{0.20l} = 5.18 \times \frac{33}{49} \times 0.20 \times 3.0 = \underline{2.09mm}$$

14) The instantaneous deflection due to application of short-term portion of live load

$$\Delta_{0.80l} = 5.18 \times \frac{33}{49} \times 0.80 = \underline{2.79mm}$$

6. Serviceability

15) The total deflection

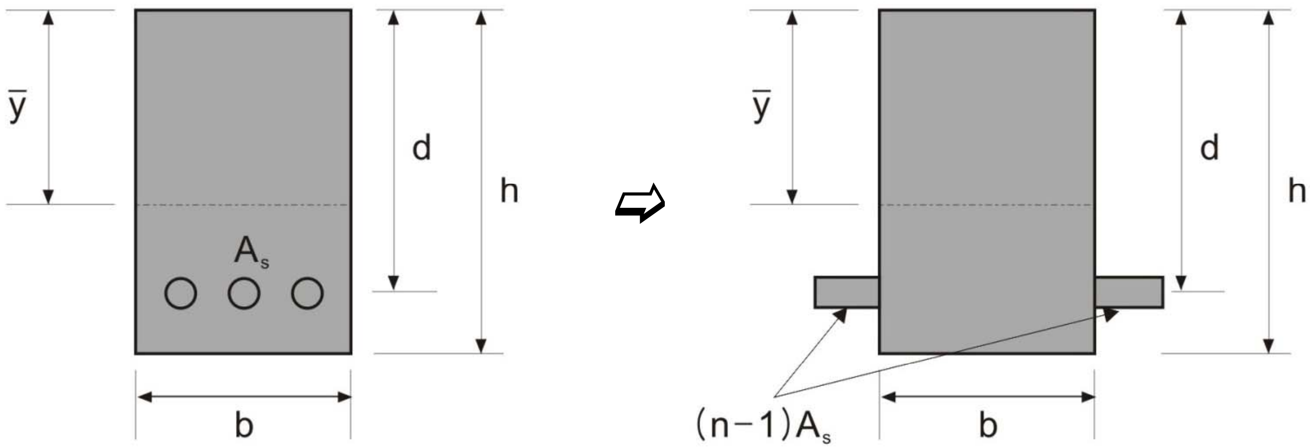
$$\Delta = 3.38 + 2.09 + 2.79 = \underline{8.26mm}$$

16) Check KCI Code Provision

$$\frac{l}{480} = \frac{7,800}{480} = 16.25$$

$$\Delta = 8.26 < 16.25 \qquad \text{O.K.}$$

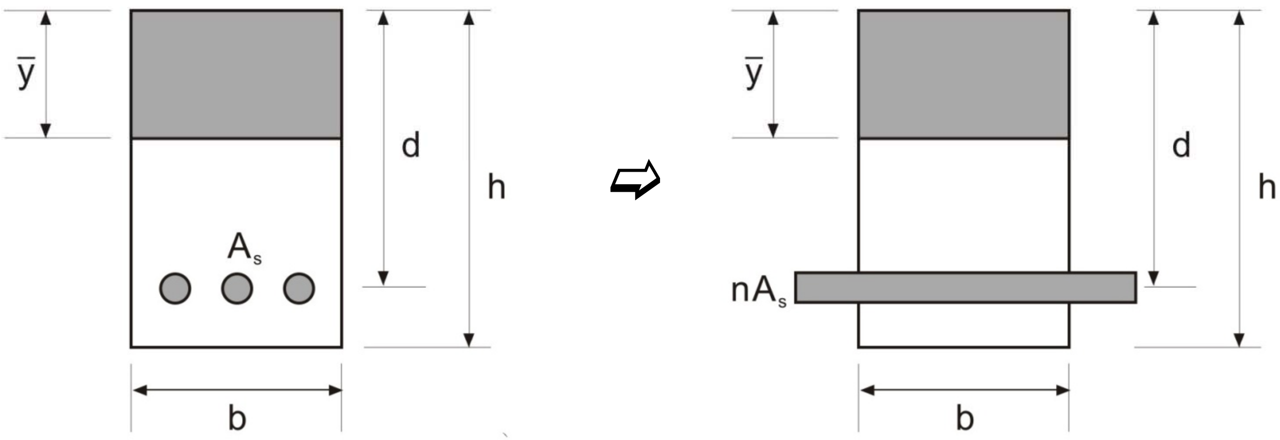
Service……



$$\bar{y} = \frac{Q_x}{A} = \frac{\int y dA}{\int dA} = \frac{bh\frac{h}{2} + (n-1)A_s d}{bh + (n-1)A_s}$$

$$I_{gt} = \frac{bh^3}{12} + bh(\frac{h}{2} - \bar{y})^2 + (n-1)A_s(d - \bar{y})^2$$

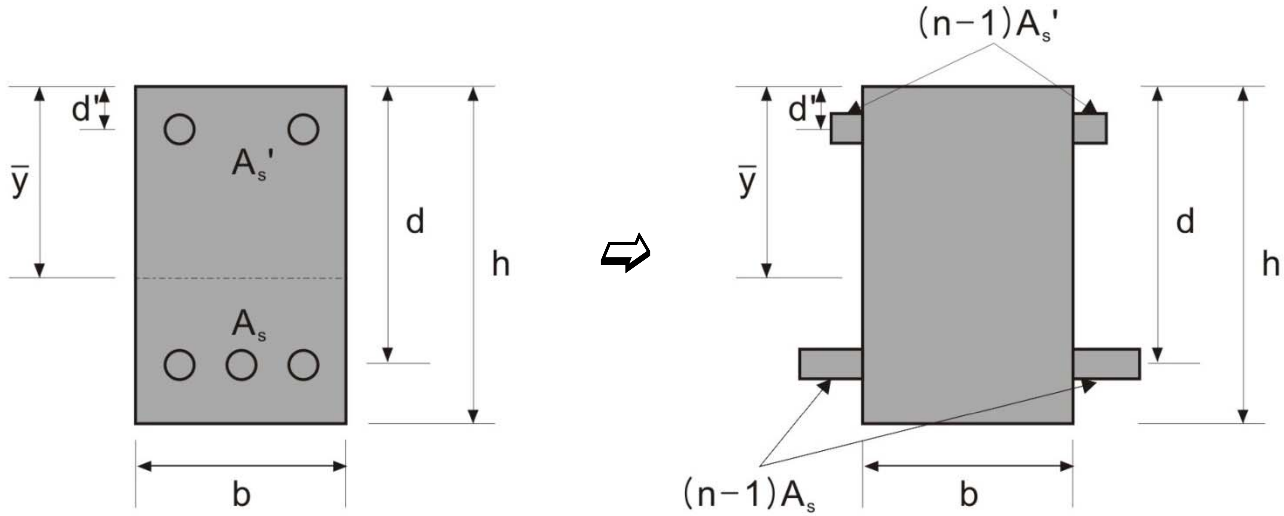
Service……



$$\bar{y} = \frac{Q_x}{A} = \frac{\int y dA}{\int dA} = \frac{b\bar{y}\frac{\bar{y}}{2} + nA_s d}{nA_s + b\bar{y}} \quad \Rightarrow \quad b\bar{y}\frac{\bar{y}}{2} - nA_s(d-\bar{y}) = 0$$

$$I_{cr} = \frac{b\bar{y}^3}{12} + b\bar{y}\left(\frac{\bar{y}}{2}\right)^2 + nA_s(d-\bar{y})^2 = \frac{b\bar{y}^3}{3} + nA_s(d-\bar{y})^2$$

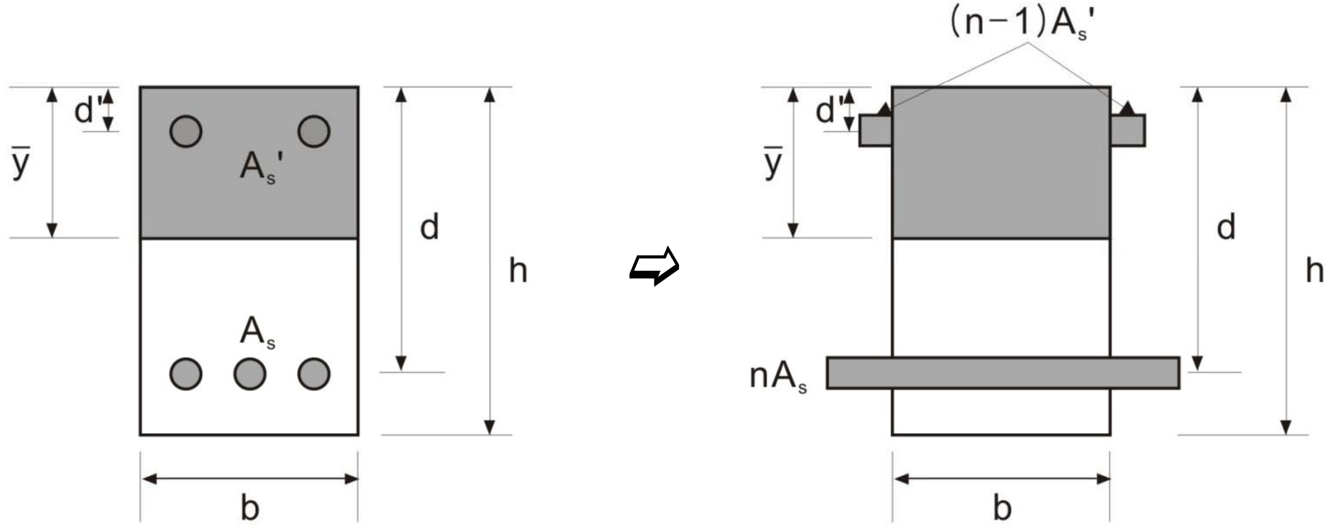
Service……



$$\bar{y} = \frac{Q_x}{A} = \frac{\int y dA}{\int dA} = \frac{bh\frac{h}{2} + (n-1)A_s d + (n-1)A_s' d'}{bh + (n-1)A_s + (n-1)A_s'}$$

$$I_{gt} = \frac{bh^3}{12} + bh(\frac{h}{2} - \bar{y})^2 + (n-1)A_s(d - \bar{y})^2 + (n-1)A_s'(d' - \bar{y})^2$$

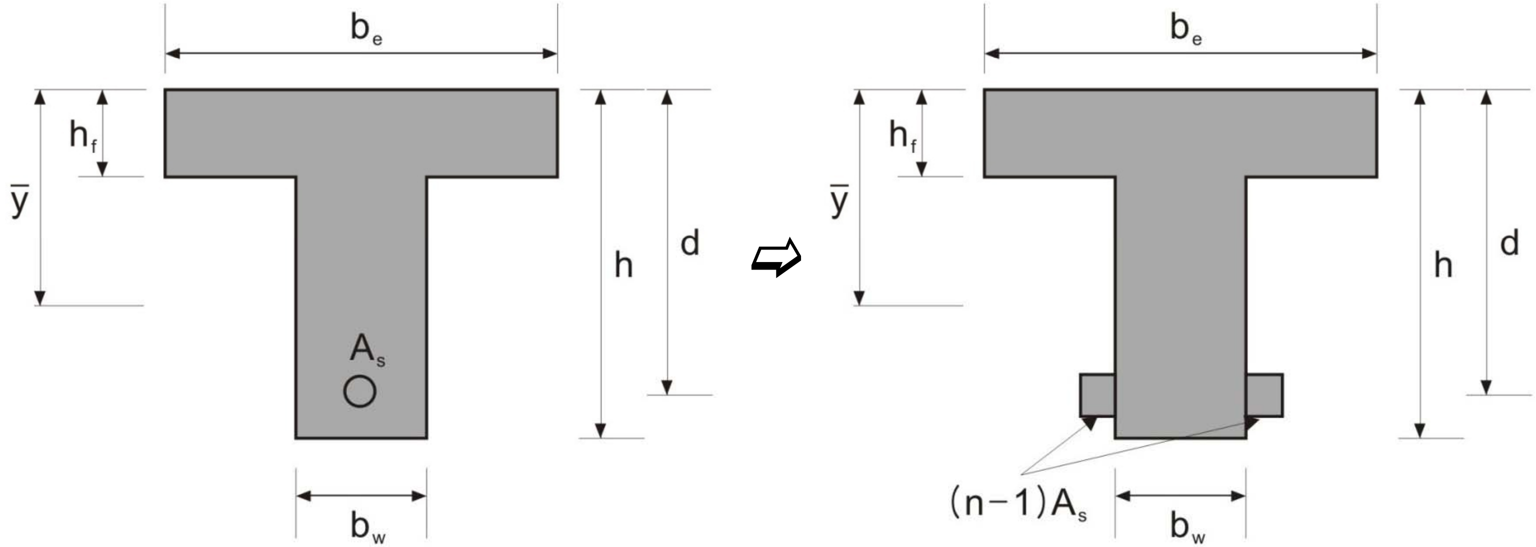
Service……



$$\bar{y} = \frac{Q_x}{A} = \frac{\int y\,dA}{\int dA} = \frac{nA_s d + (n-1)A_s' d' + b\bar{y}\frac{\bar{y}}{2}}{nA_s + (n-1)A_s' + b\bar{y}} \Rightarrow b\bar{y}\frac{\bar{y}}{2} + (n-1)A_s'(\bar{y} - d') - nA_s(d - \bar{y}) = 0$$

$$I_{cr} = \frac{b\bar{y}^3}{12} + b\bar{y}\left(\frac{\bar{y}}{2}\right)^2 + (n-1)A_s'\left(\frac{\bar{y}}{2} - d'\right)^2 + nA_s(d - \bar{y})^2$$

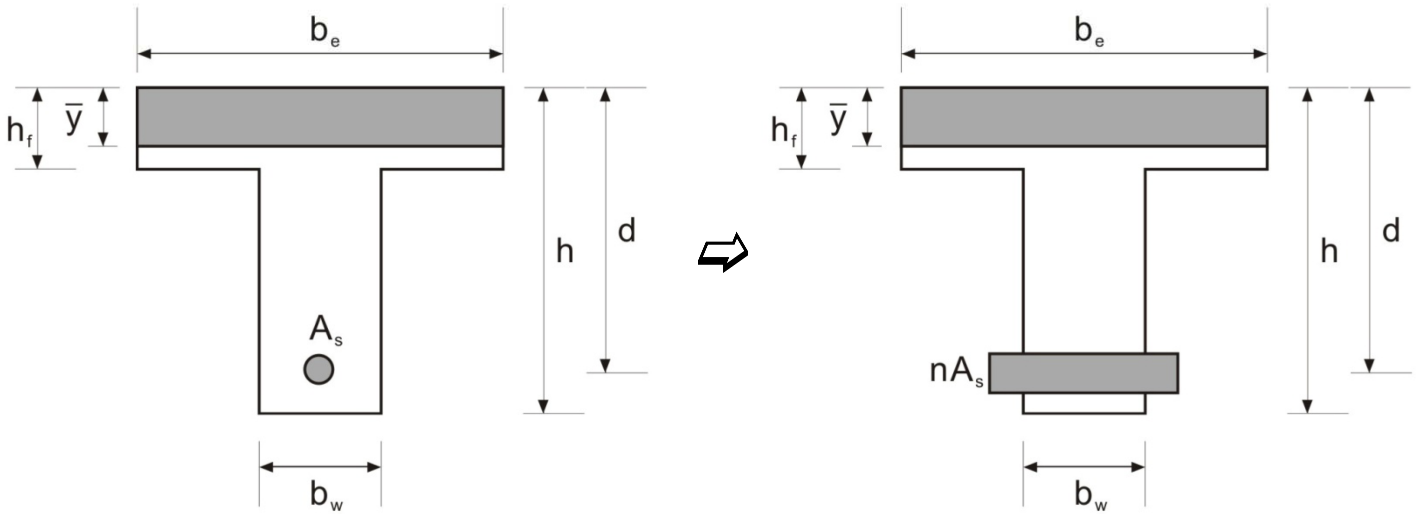
Service……



$$\bar{y} = \frac{Q_x}{A} = \frac{\int y\,dA}{\int dA} = \frac{b_e h_f \frac{h_f}{2} + b_w (h - h_f)(h_f + \frac{h - h_f}{2}) + (n-1) A_s d}{b_e h_f + b_w (h - h_f) + (n-1) A_s}$$

$$I_{gt} = \frac{b_e h_f^3}{12} + b_e h_f (\bar{y} - \frac{h_f}{2})^2 + \frac{b_w (h - h_f)^3}{12} + b_w (h - h_f) \left[\bar{y} - \left(h_f + \frac{h - h_f}{2} \right) \right]^2 + (n-1) A_s (d - \bar{y})^2$$

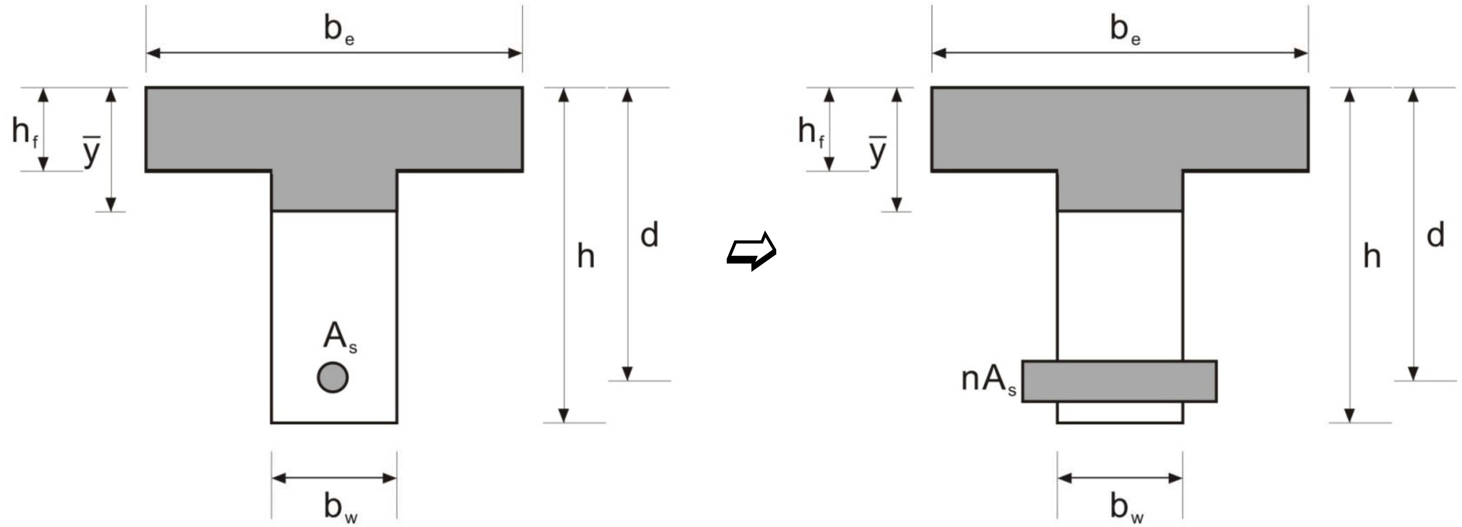
Service……



$$\bar{y} = \frac{Q_x}{A} = \frac{\int y\,dA}{\int dA} = \frac{b_e \bar{y}\,\dfrac{\bar{y}}{2} + nA_s d}{b_e \bar{y} + nA_s} \Rightarrow b_e \bar{y}\,\frac{\bar{y}}{2} - nA_s (d - \bar{y}) = 0$$

$$I_{cr} = \frac{b_e \bar{y}^3}{12} + b_e \bar{y}\left(\frac{\bar{y}}{2}\right)^2 + nA_s (d - \bar{y})^2$$

Service……

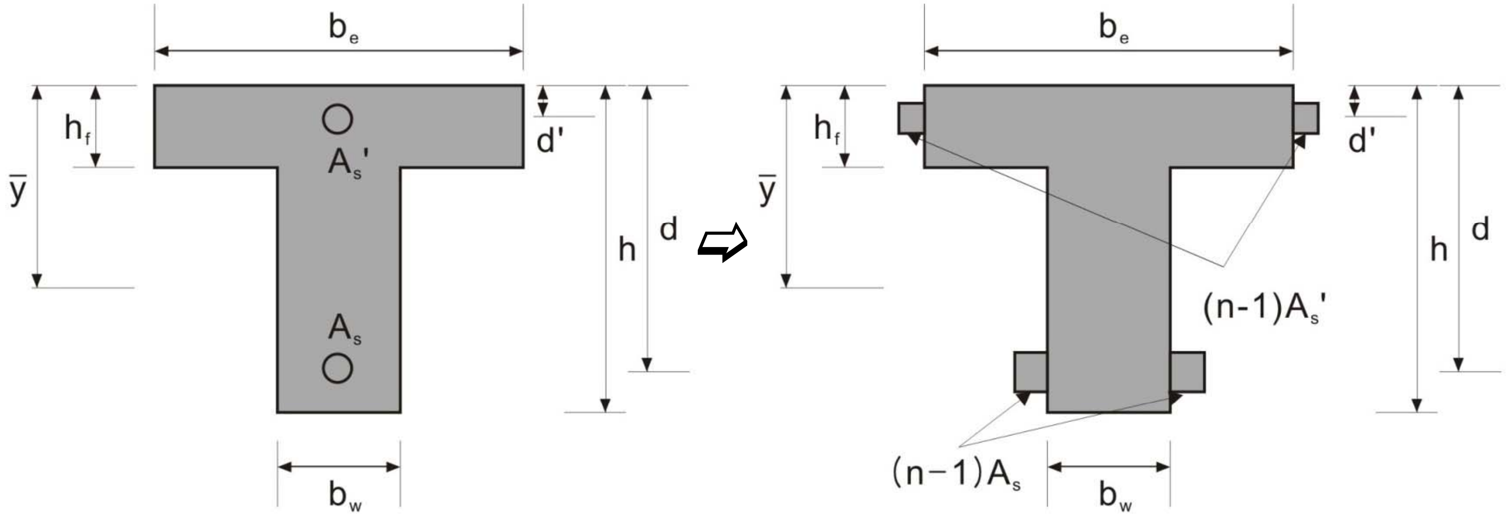


$$\bar{y} = \frac{Q_x}{A} = \frac{\int y dA}{\int dA} = \frac{b_e h_f \dfrac{h_f}{2} + (\bar{y} - h_f) b_w \left(h_f + \dfrac{\bar{y} - h_f}{2} \right) + nA_s d}{b_e h_f + (\bar{y} - h_f) b_w + nA_s}$$

$$\Rightarrow \quad b_e h_f (\bar{y} - \frac{h_f}{2}) + b_w (\bar{y} - h_f) \frac{(\bar{y} - h_f)}{2} - nA_s (d - \bar{y}) = 0$$

$$I_{cr} = \frac{b_e h_f^3}{12} + b_e h_f \left(\bar{y} - \frac{h_f}{2} \right)^2 + \frac{b_w (\bar{y} - h_f)^3}{12} + b_w (\bar{y} - h_f) \left(\frac{\bar{y} - h_f}{2} \right)^2 + nA_s (d - \bar{y})^2$$

Service……

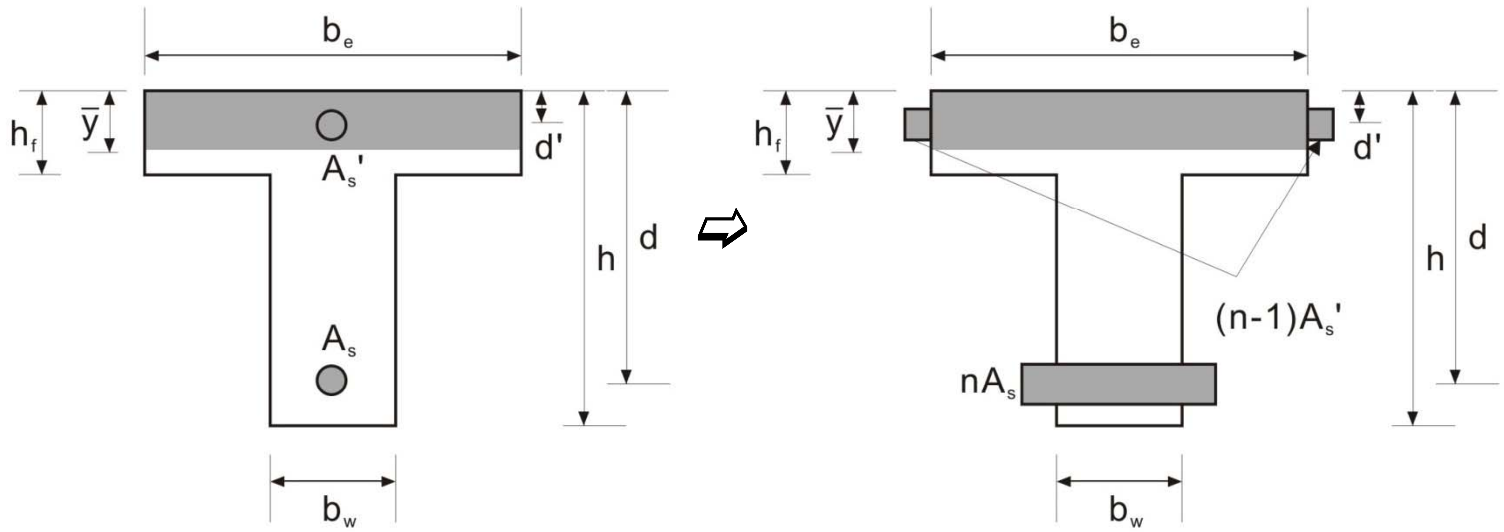


$$\bar{y} = \frac{Q_x}{A} = \frac{\int y dA}{\int dA} = \frac{b_e h_f \frac{h_f}{2} + b_w (h - h_f)(h_f + \frac{h - h_f}{2}) + (n-1)A_s' d' + (n-1)A_s d}{b_e h_f + b_w (h - h_f) + (n-1)A_s' + (n-1)A_s}$$

$$I_{gt} = \frac{b_e h_f^3}{12} + b_e h_f (\bar{y} - \frac{h_f}{2})^2 + \frac{b_w (h - h_f)^3}{12} + b_w (h - h_f)\left[\bar{y} - \left(h_f + \frac{h - h_f}{2}\right)\right]^2$$

$$+ (n-1)A_s' (\bar{y} - d')^2 + (n-1)A_s (d - \bar{y})^2$$

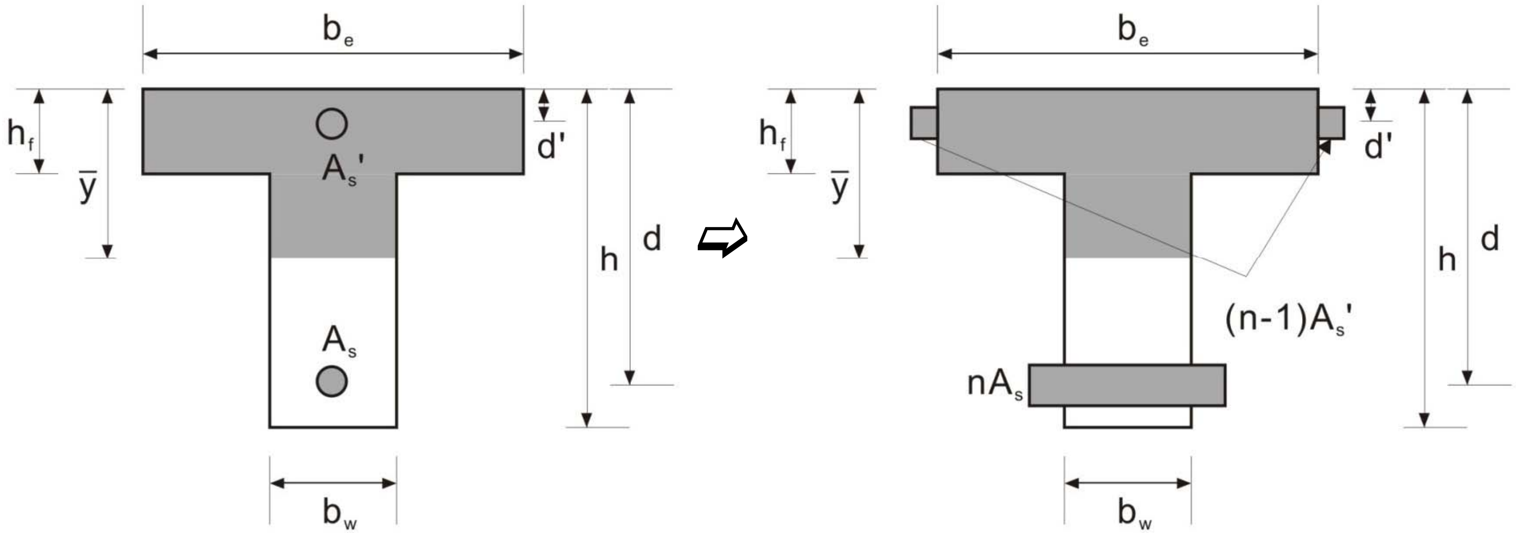
Service......



$$\bar{y} = \frac{Q_x}{A} = \frac{\int y\,dA}{\int dA} = \frac{b_e \bar{y}\dfrac{\bar{y}}{2} + nA_s d + (n-1)A_s' d'}{b_e \bar{y} + nA_s + (n-1)A_s'} \Rightarrow b_e \bar{y}\frac{\bar{y}}{2} + (n-1)A_s'(\bar{y} - d') - nA_s(d - \bar{y}) = 0$$

$$I_{cr} = \frac{b_e \bar{y}^3}{12} + b_e \bar{y}\left(\frac{\bar{y}}{2}\right)^2 + (n-1)A_s'\left(\bar{y} - d'\right)^2 + nA_s(d - \bar{y})^2$$

Service……



$$\bar{y} = \frac{Q_x}{A} = \frac{\int y\,dA}{\int dA} = \frac{b_e h_f \dfrac{h_f}{2} + b_w(\bar{y}-h_f)\left(h_f + \dfrac{\bar{y}-h_f}{2}\right) + nA_s d + (n-1)A_s' d'}{b_e h_f + b_w(\bar{y}-h_f) + nA_s + (n-1)A_s'}$$

$$\Rightarrow \quad b_e h_f\left(\bar{y}-\frac{h_f}{2}\right) + b_w(\bar{y}-h_f)\frac{(\bar{y}-h_f)}{2} + (n-1)A_s'(\bar{y}-d') - nA_s(d-\bar{y}) = 0$$

$$I_{cr} = \frac{b_e h_f^3}{12} + b_e h_f\left(\bar{y}-\frac{h_f}{2}\right)^2 + \frac{b_w(\bar{y}-h_f)^3}{12} + b_w(\bar{y}-h_f)\left(\frac{\bar{y}-h_f}{2}\right)^2 + (n-1)A_s'(\bar{y}-d')^2 + nA_s(d-\bar{y})^2$$