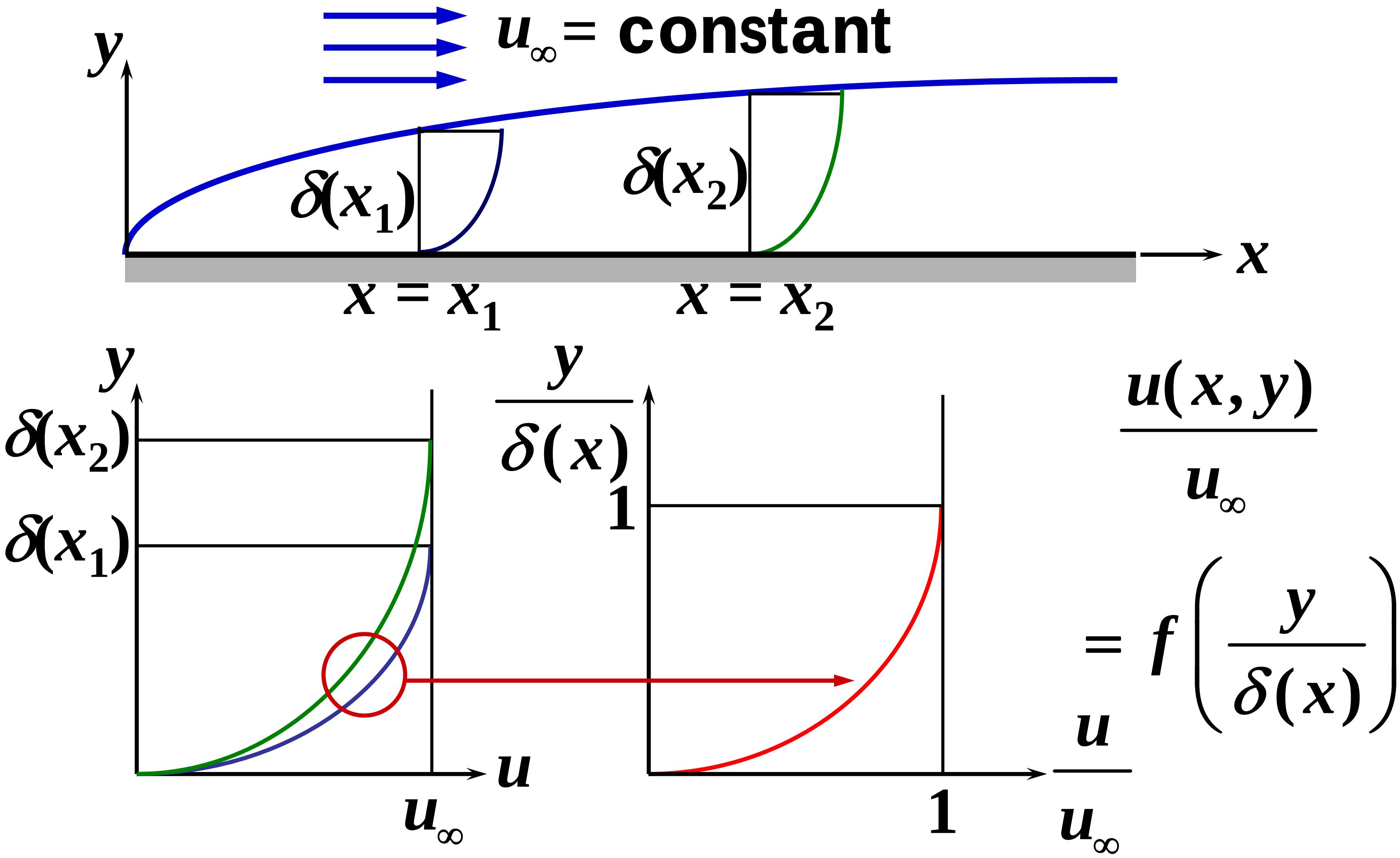
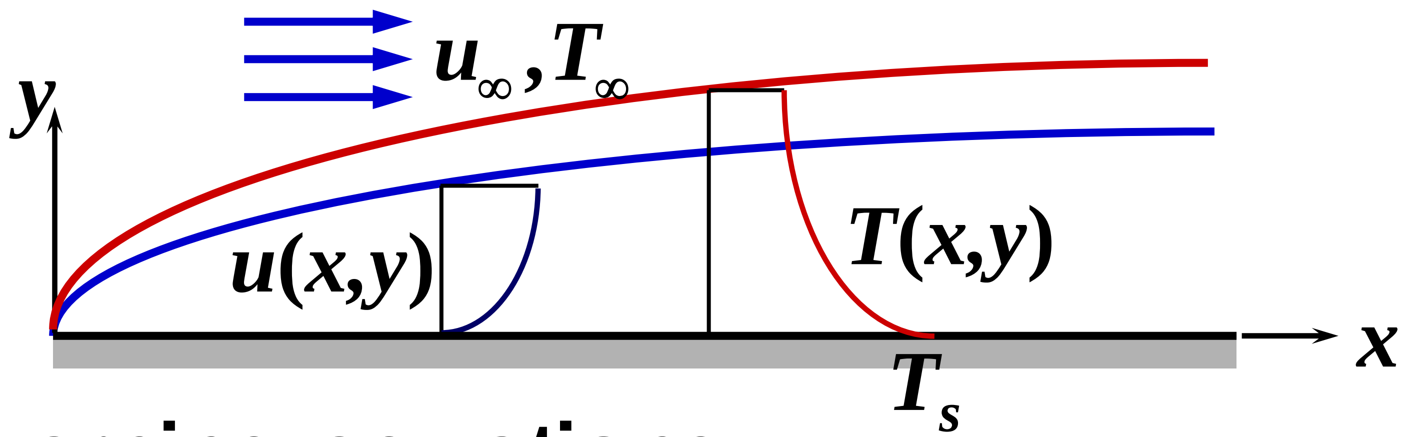
EXTERNAL FORCED CONVECTION

- Flat plate in Parallel Flow
- Cylinder & Sphere in Cross Flow
- Flow across Banks of Tubes
- Impinging Jets
- Packed Beds

Flat Plate in Parallel Flow

Laminar Flow: Similarity Solution





governing equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2},$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

boundary conditions

$$u(0,y)=u_\infty,\ u(x,0)=v(x,0)=0,\ u(x,\infty)=u_\infty$$

$$T(0,y)=T_\infty,\ T(x,0)=T_s,\ T(x,\infty)=T_\infty$$

Momentum equation

introducing stream function $u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x}$

similarity variable $\eta \equiv \frac{y}{\delta(x)}$

scaling analysis

$$u\frac{\partial u}{\partial x} \sim \nu \frac{\partial^2 u}{\partial y^2} \;\rightarrow u_\infty \frac{u_\infty}{x} \sim \nu \frac{u_\infty}{\delta^2} \;\rightarrow \delta^2 \sim \frac{\nu x}{u_\infty}$$

or $\delta \sim \sqrt{\frac{\nu x}{u_\infty}}$ **or** $\frac{\delta}{x} \sim \sqrt{\frac{\nu}{u_\infty x}} = \frac{1}{\sqrt{\mathrm{Re}_x}}$

Thus, let $\eta = \frac{y}{\delta(x)} = y\sqrt{\frac{u_\infty}{\nu x}} = \frac{y}{x}\sqrt{\mathrm{Re}_x}$

$$u = \frac{\partial \psi}{\partial y} \rightarrow u_\infty \sim \frac{\psi}{\delta} \rightarrow \psi \sim u_\infty \delta \sim u_\infty \sqrt{\frac{\nu x}{u_\infty}} = \sqrt{\nu u_\infty x}$$

Let $\dfrac{\psi}{\sqrt{\nu u_\infty x}} \equiv f(\eta)$ or $\psi(x, y) = \sqrt{\nu u_\infty x}\, f(\eta)$

then

$$u = \frac{\partial \psi}{\partial y} = u_\infty f', \quad v = -\frac{\partial \psi}{\partial x} = \frac{1}{2}\sqrt{\frac{\nu u_\infty}{x}}\left(\eta f' - f\right)$$

$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x}\eta f'', \quad \frac{\partial u}{\partial y} = u_\infty \sqrt{\frac{u_\infty}{\nu x}} f'', \quad \frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x} f'''$$

similarity equation

$$f''' + \frac{1}{2} f f'' = 0$$: Blasius solution

boundary conditions $\eta = y\sqrt{\frac{u_\infty}{\nu x}}$

$$u = \frac{\partial \psi}{\partial y} = u_\infty f', \quad v = \frac{1}{2}\sqrt{\frac{\nu u_\infty}{x}}(\eta f' - f)$$

$$u(x,0) = 0 \rightarrow \eta = 0 \rightarrow f'(0) = 0$$

$$v(x,0) = 0 \rightarrow \eta = 0 \rightarrow f(0) = 0$$

$$u(x,\infty) = u_\infty \rightarrow \eta \rightarrow \infty \rightarrow f'(\infty) = 1$$

$$u(0,y) = u_\infty \rightarrow \eta \rightarrow \infty \rightarrow f'(\infty) = 1$$

Flat plate laminar boundary layer functions

$\eta = y\sqrt{\frac{u_\infty}{\nu x}}$	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.4	0.027	0.133	0.331
0.8	0.106	0.265	0.327
1.2	0.238	0.394	0.317
1.6	0.420	0.517	0.297
2.0	0.650	0.630	0.267
2.4	0.922	0.729	0.228
2.8	1.231	0.812	0.184
3.2	1.569	0.876	0.139
3.6	1.930	0.923	0.098
4.0	2.306	0.956	0.064
4.4	2.692	0.976	0.039
4.8	3.085	0.988	0.022
5.2	3.482	0.994	0.011
5.6	3.880	0.997	0.005
6.0	4.280	0.999	0.002
6.4	4.679	1.000	0.001
6.8	5.079	1.000	0.000

local friction coefficient

$$\tau_s = 0.332 u_\infty \sqrt{\frac{\rho \mu u_\infty}{x}}$$

$$C_{f,x} \equiv \frac{\tau_s}{\rho u_\infty^2 / 2} = 0.664 \, \mathrm{Re}_x^{-1/2}$$

average friction coefficient

$$\bar{C}_{f,x} = \frac{1}{x}\int_0^x C_{f,x} dx = 0.664 \frac{1}{x}\int_0^x \sqrt{\frac{\nu}{u_\infty x}} dx$$

$$= 0.664 \frac{1}{x} 2 \sqrt{\frac{\nu x}{u_\infty}} = 1.328 \, \mathrm{Re}_x^{-1/2}$$

Energy equation

Let $\theta = \dfrac{T - T_s}{T_\infty - T_s}$, then $u\dfrac{\partial \theta}{\partial x} + v\dfrac{\partial \theta}{\partial y} = \alpha \dfrac{\partial^2 \theta}{\partial y^2}$

boundary conditions

$$\theta(0, y) = 1,\ \ \theta(x, 0) = 0,\ \ \theta(x, \infty) = 1$$

Assume $\theta(x, y) = \theta(\eta),\ \eta = y\sqrt{\dfrac{u_\infty}{\nu x}} = \dfrac{y}{x}\sqrt{\mathrm{Re}_x}$

$$\frac{\partial \theta}{\partial x} = -\frac{1}{2x}\eta\theta',\ \ \frac{\partial \theta}{\partial y} = \sqrt{\frac{u_\infty}{\nu x}}\theta',\ \ \frac{\partial^2 \theta}{\partial y^2} = \frac{u_\infty}{\nu x}\theta''$$

similarity equation

$$\theta'' + \frac{\Pr}{2} f \theta' = 0$$

boundary conditions

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}}$$

$$\theta(0, y) = 1 \rightarrow \eta \rightarrow \infty \rightarrow \theta(\infty) = 1$$

$$\theta(x, 0) = 0 \rightarrow \eta = 0 \rightarrow \theta(0) = 0$$

$$\theta(x, \infty) = 1 \rightarrow \eta \rightarrow \infty \rightarrow \theta(\infty) = 1$$

heat flux at the wall and heat transfer coefficient

$$\theta = \frac{T - T_s}{T_\infty - T_s}$$

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}}$$

$$q_s'' = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} = h_x\left(T_s - T_\infty\right)$$

$$= k\left(T_s - T_\infty\right)\sqrt{\frac{u_\infty}{\nu x}}\theta'(0)$$

$$h_x = k\sqrt{\frac{u_\infty}{\nu x}}\theta'(0)$$

but $\theta'(0) = 0.332\,\mathrm{Pr}^{1/3}$

thus, $h_x = 0.332k\sqrt{\frac{u_\infty}{\nu x}}\,\mathrm{Pr}^{1/3}$

local Nusselt number

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332k\sqrt{\frac{u_\infty}{\nu x}}\,\mathrm{Pr}^{1/3}\,\frac{x}{k} = 0.332\sqrt{\frac{u_\infty x}{\nu}}\,\mathrm{Pr}^{1/3}$$

$$= 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \quad (\mathrm{Pr} \geq 0.6)$$

average heat transfer coefficient

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = 0.332\frac{k}{x}\mathrm{Pr}^{1/3}\cdot 2\sqrt{\frac{u_\infty x}{\nu}}$$

$$= 0.664k\sqrt{\frac{u_\infty}{\nu x}}\,\mathrm{Pr}^{1/3}$$

average Nusselt number

$$\overline{\mathrm{Nu}}_x = \frac{\bar{h}_x x}{k} = 0.664\sqrt{\frac{u_\infty x}{\nu}}\,\mathrm{Pr}^{1/3} = 0.664\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

Turbulent Flow Correlations

$$C_{f,x} = 0.0592\,\mathrm{Re}_x^{-1/5} \qquad \mathrm{Re}_x \le 10^7$$

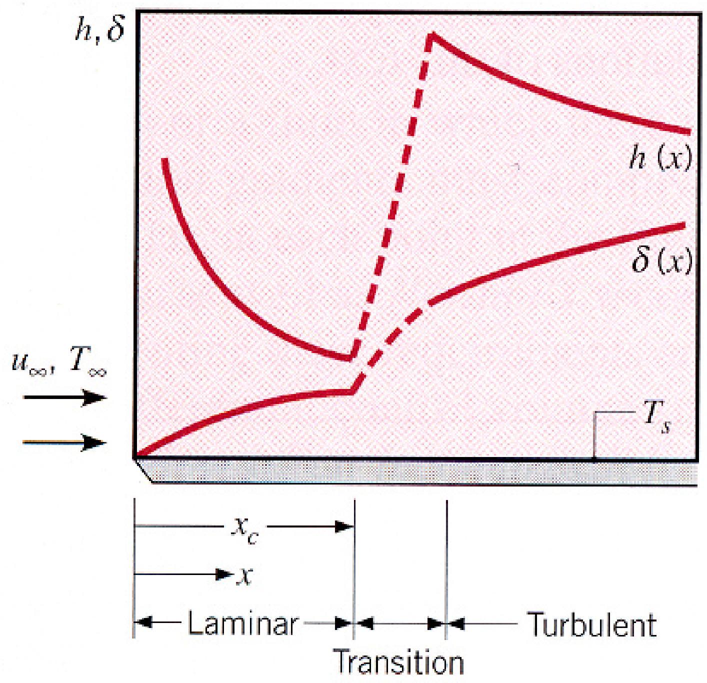
$$\delta = 0.37\,x\,\mathrm{Re}_x^{-1/5}$$

$$\mathrm{Nu}_x = \mathrm{St}\,\mathrm{Re}_x\,\mathrm{Pr}$$

$$= 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} \qquad \left(0.6 < \mathrm{Pr} < 60\right)$$

Mixed boundary layer conditions

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{\text{lam}}\,dx + \int_{x_c}^{L} h_{\text{turb}}\,dx\right)$$



$$\bar{h}_L = \left(\frac{k}{L}\right)\left[0.332\left(\frac{u_\infty}{v}\right)^{1/2}\int_0^{x_c}\frac{dx}{x^{1/2}}\right.$$

$$\left.+0.0296\left(\frac{u_\infty}{v}\right)^{4/5}\int_{x_c}^{L}\frac{dx}{x^{1/5}}\right]\text{Pr}^{1/3}$$

$$\overline{\text{Nu}}_L = \left[0.664\,\text{Re}_{x,c}^{1/2} + 0.037\left(\text{Re}_L^{4/5} - \text{Re}_{x,c}^{4/5}\right)\right]\text{Pr}^{1/3}$$

$$\overline{\text{Nu}}_L = \left(0.037\,\text{Re}_L^{4/5} - A\right)\text{Pr}^{1/3}$$

$$A = 0.037\,\text{Re}_{x,c}^{4/5} - 0.664\,\text{Re}_{x,c}^{1/2}$$

If $\mathrm{Re}_{x,c} = 5\times10^5$ is assumed,

$$\overline{\mathrm{Nu}}_L = \left(0.037\,\mathrm{Re}_L^{4/5} - 871\right)\mathrm{Pr}^{1/3} \quad \left[\begin{array}{c} 0.6 < \mathrm{Pr} < 60 \\ 5\times10^5 < \mathrm{Re}_L < 10^8 \\ \mathrm{Re}_{x,c} = 5\times10^5 \end{array}\right]$$

Similarly,

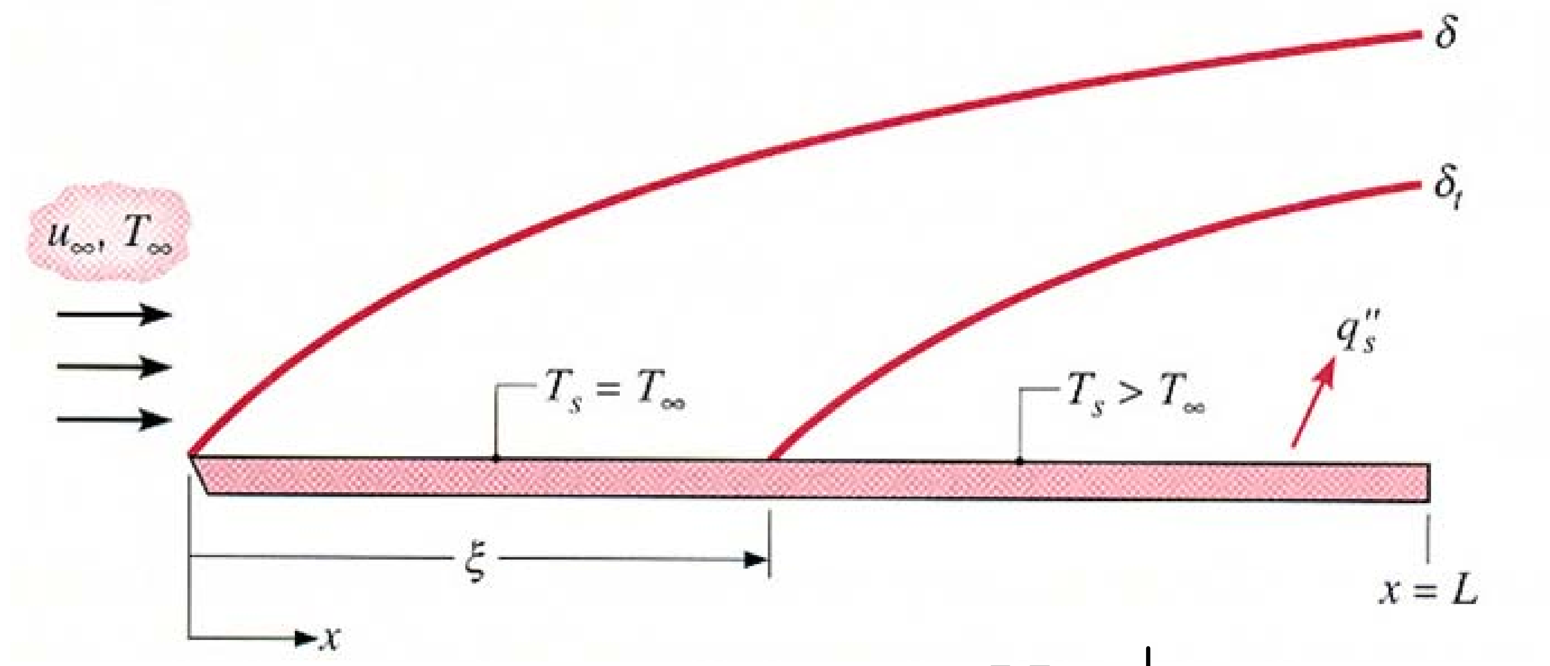
$$\bar{C}_{f,L} = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \quad \left[\begin{array}{c} 5\times10^5 < \mathrm{Re}_L < 10^8 \\ \mathrm{Re}_{x,c} = 5\times10^5 \end{array}\right]$$

In situations for which $L >> x_c$ $\left(\mathrm{Re}_L >> \mathrm{Re}_{x,c}\right)$,

$$A << 0.037\,\mathrm{Re}_L^{4/5}$$

$$\overline{\mathrm{Nu}}_L = 0.037\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3}, \quad \bar{C}_{f,L} = 0.074\,\mathrm{Re}_L^{-1/5}$$

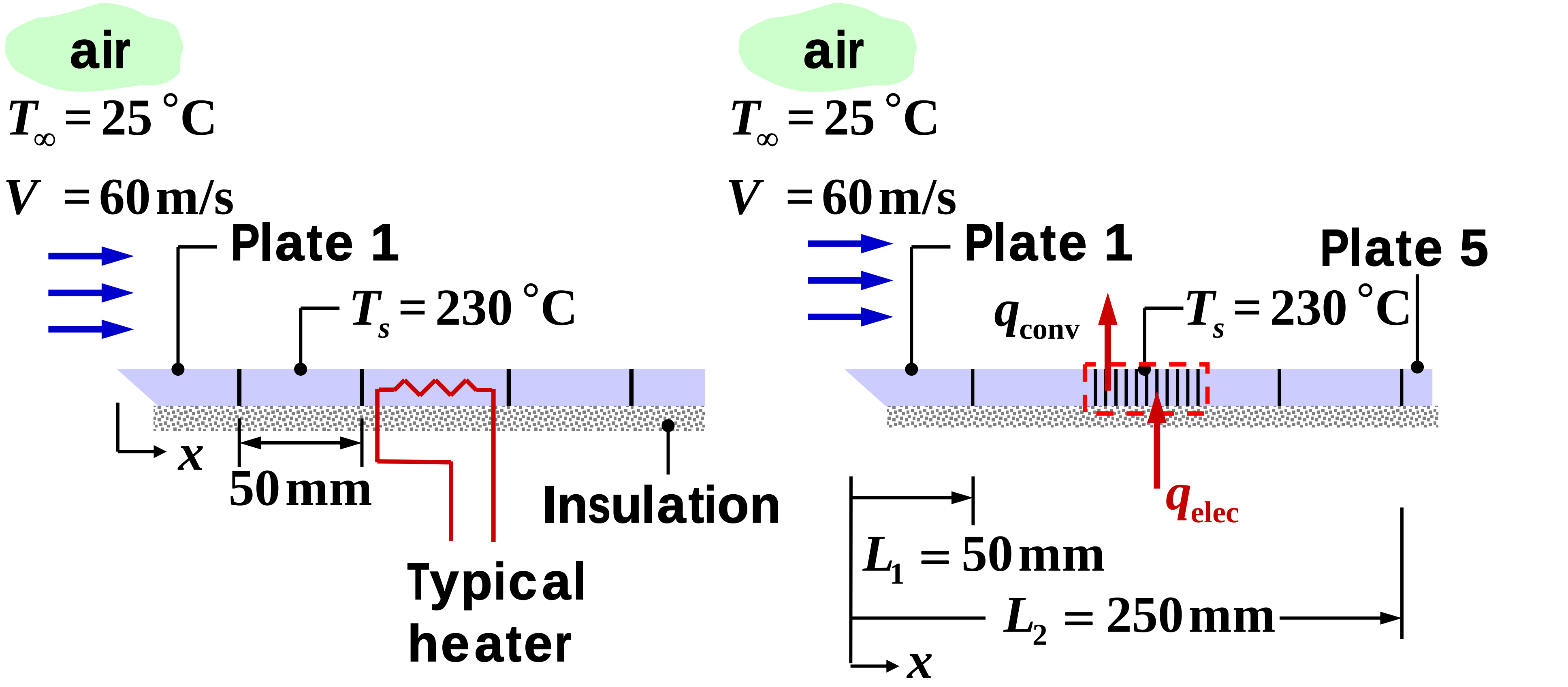
In case with unheated starting length



laminar: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_x\big|_{\xi=0}}{\left[1-\left(\xi/x\right)^{3/4}\right]^{1/3}}$$

turbulent: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_x\big|_{\xi=0}}{\left[1-\left(\xi/x\right)^{9/10}\right]^{1/9}}$$

Example 7.2



Find: Maximum heater power requirement

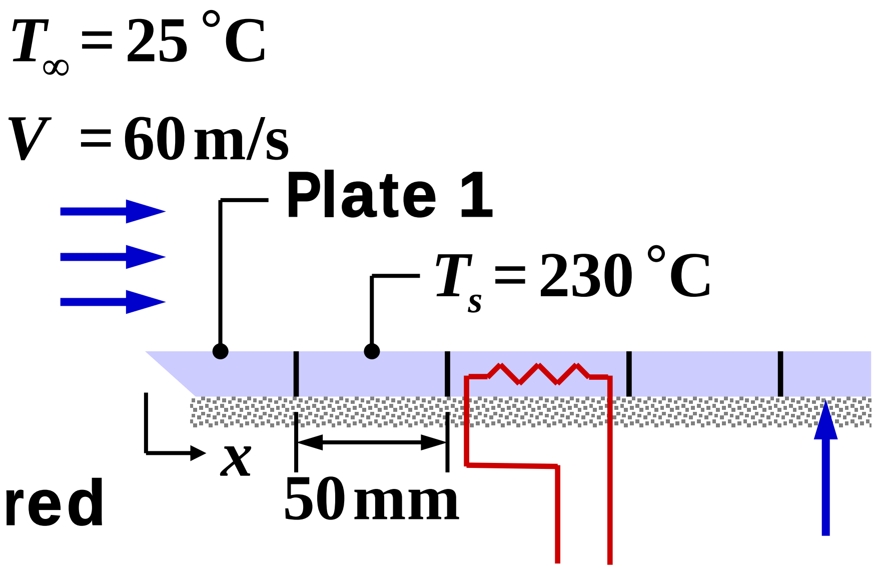
Assumption:

1. Negligible radiation effects
2. Bottom surface of plate adiabatic

critical Reynolds number

$$\mathrm{Re}_{x,c} = \frac{V x_c}{\nu} = 5\times10^5$$

$$x_c = \frac{\nu}{V}\mathrm{Re}_{x,c} = 0.22\ \mathrm{m}$$



Maximum power may be required at plate 1, 5, or 6.

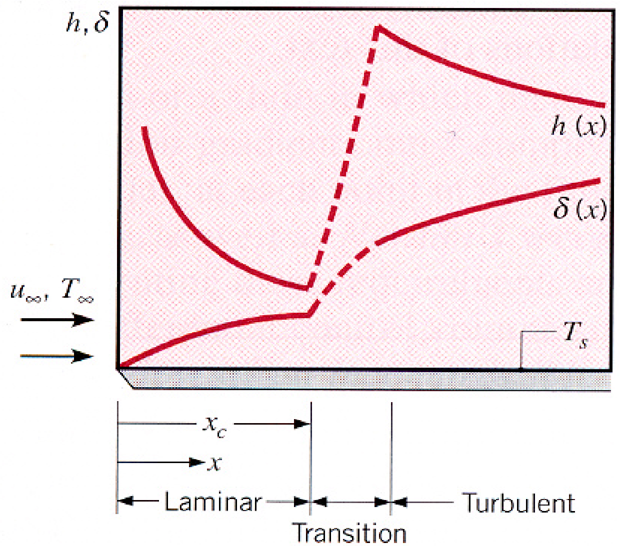
Heater 1 : laminar convection coefficient

$$q_1 = \bar{h}_1 L_1 w\left(T_s - T_\infty\right)$$

$$\overline{\mathrm{Nu}_1} = 0.664\,\mathrm{Re}_1^{1/2}\,\mathrm{Pr}^{1/3} = 198$$

$$\bar{h}_1 = \frac{\overline{\mathrm{Nu}_1} k}{L_1} = 134\ \mathrm{W/m^2\cdot K}$$

$$q_1 = 1370\ \mathrm{W}$$



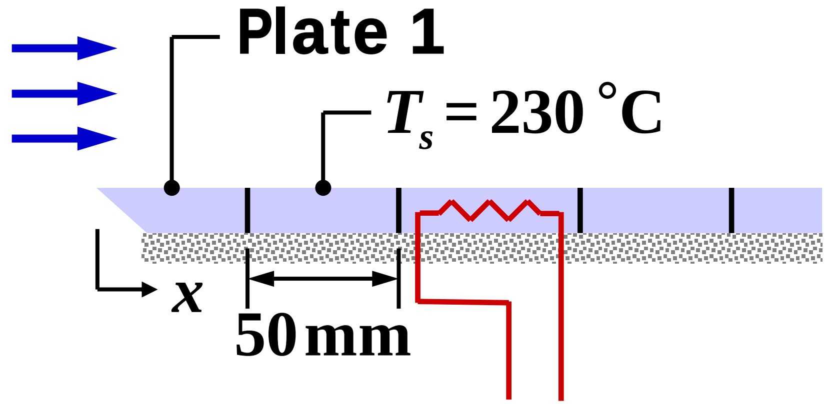
Heater 5 : from plate 1 to plate 4: laminar
from plate 1 to plate 5: mixed laminar and turbulent

$$q_5 = \bar{h}_{1-5} L_5 w (T_s - T_\infty) - \bar{h}_{1-4} L_4 w (T_s - T_\infty)$$

$$= (\bar{h}_{1-5} L_5 - \bar{h}_{1-4} L_4) w (T_s - T_\infty)$$

$T_\infty = 25\ ^\circ\mathrm{C}$

$V = 60\,\mathrm{m/s}$



$$\overline{\mathrm{Nu}}_4 = 0.664\,\mathrm{Re}_4^{1/2}\,\mathrm{Pr}^{1/3} = 396$$

$$\bar{h}_{1-4} = \frac{\overline{\mathrm{Nu}_4} k}{L_4} = 67\ \mathrm{W/m^2 \cdot K}$$

$$\overline{\mathrm{Nu}}_5 = (0.037\,\mathrm{Re}_5^{4/5} - 871)\,\mathrm{Pr}^{1/3} = 546$$

$$\bar{h}_{1-5} = \frac{\overline{\mathrm{Nu}_5} k}{L_5} = 74\ \mathrm{W/m^2 \cdot K}$$

$$q_5 = 1050\ \mathrm{W}$$

Heater 6 : from plate 1 to plate 5 or plate 6: mixed laminar and turbulent condition

$$q_6 = \left(\bar{h}_{1-6} L_6 - \bar{h}_{1-5} L_5\right) w \left(T_s - T_\infty\right)$$

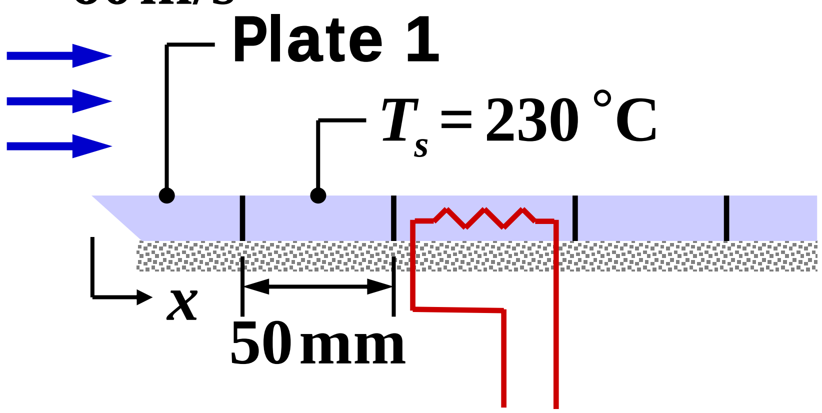
$$\bar{h}_{1-5} = 74\ \text{W/m}^2 \cdot \text{K}$$

$$\overline{\text{Nu}}_6 = \left(0.037\,\text{Re}_6^{4/5} - 871\right)\text{Pr}^{1/3} = 753$$

$$\bar{h}_{1-6} = \frac{\overline{\text{Nu}}_6 k}{L_6} = 85\ \text{W/m}^2 \cdot \text{K}$$

$T_\infty = 25\ ^\circ\text{C}$

$V = 60\,\text{m/s}$



$$q_6 = 1440\ \text{W}$$

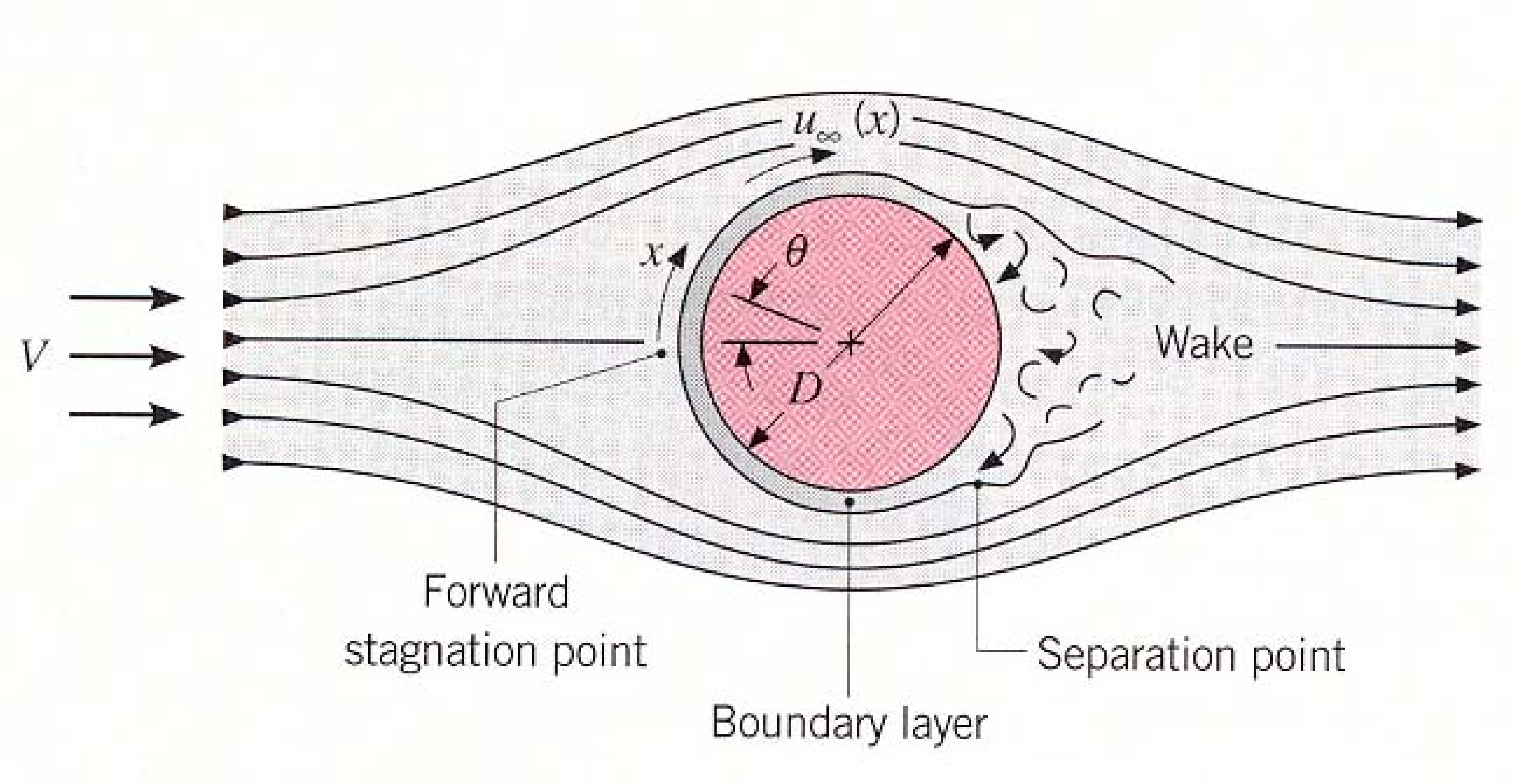
$$q_1 = 1370\ \text{W}$$

$$q_5 = 1050\ \text{W}$$

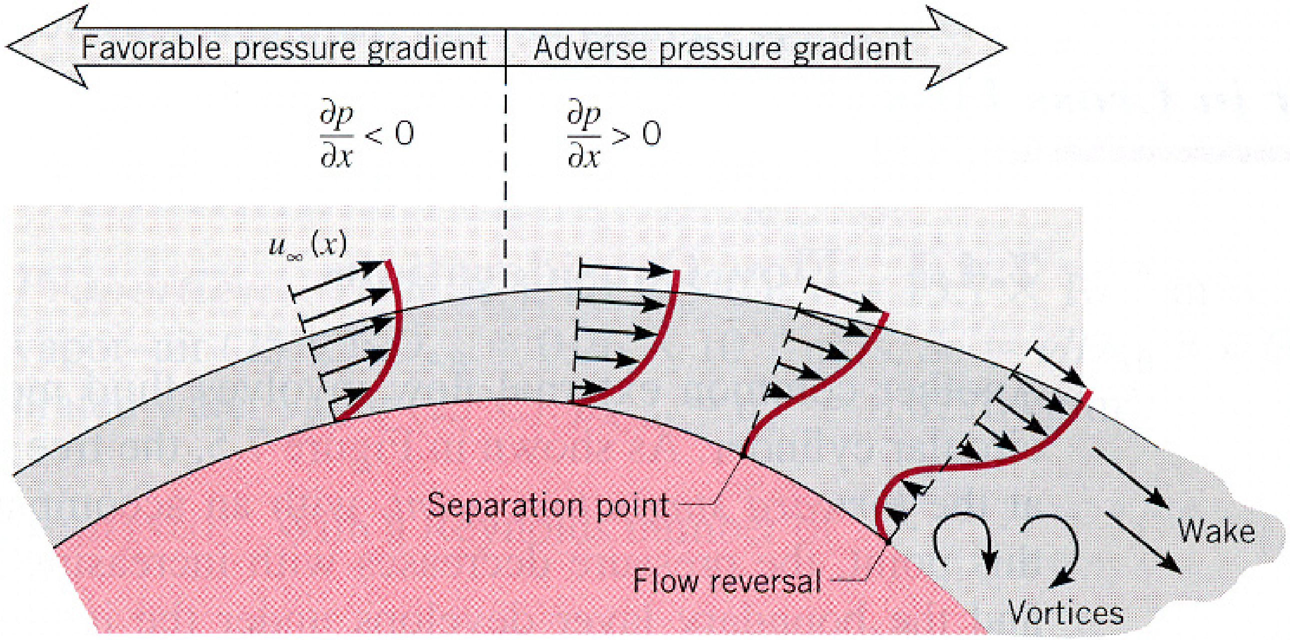
$$q_{\text{conv},6} > q_{\text{conv},1} > q_{\text{conv},5}$$

Cylinder & Sphere in Cross Flow

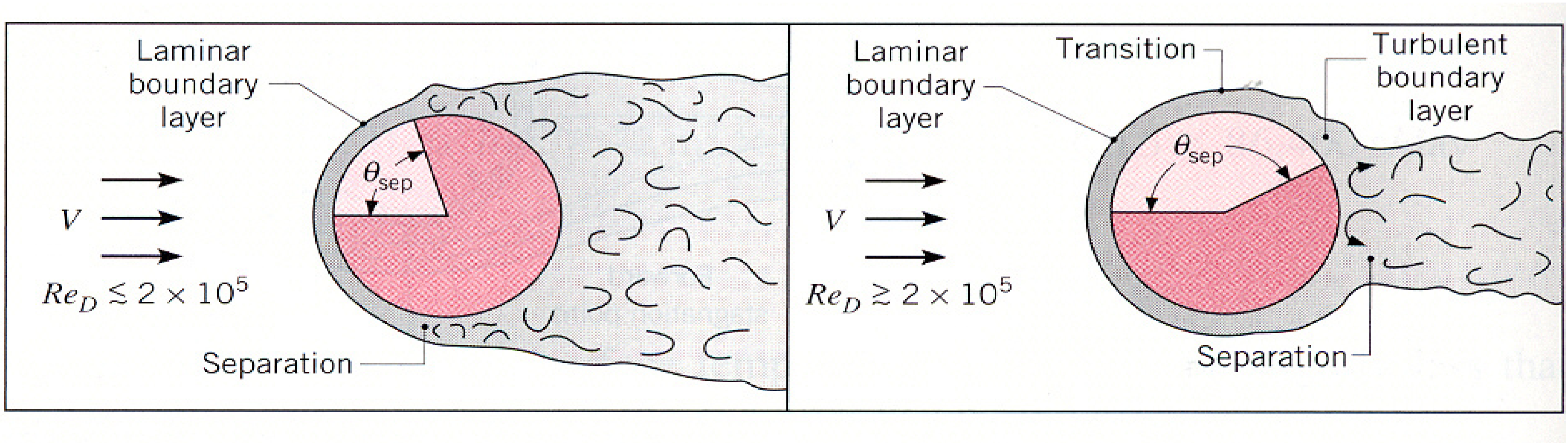
Circular Cylinder in Cross Flow



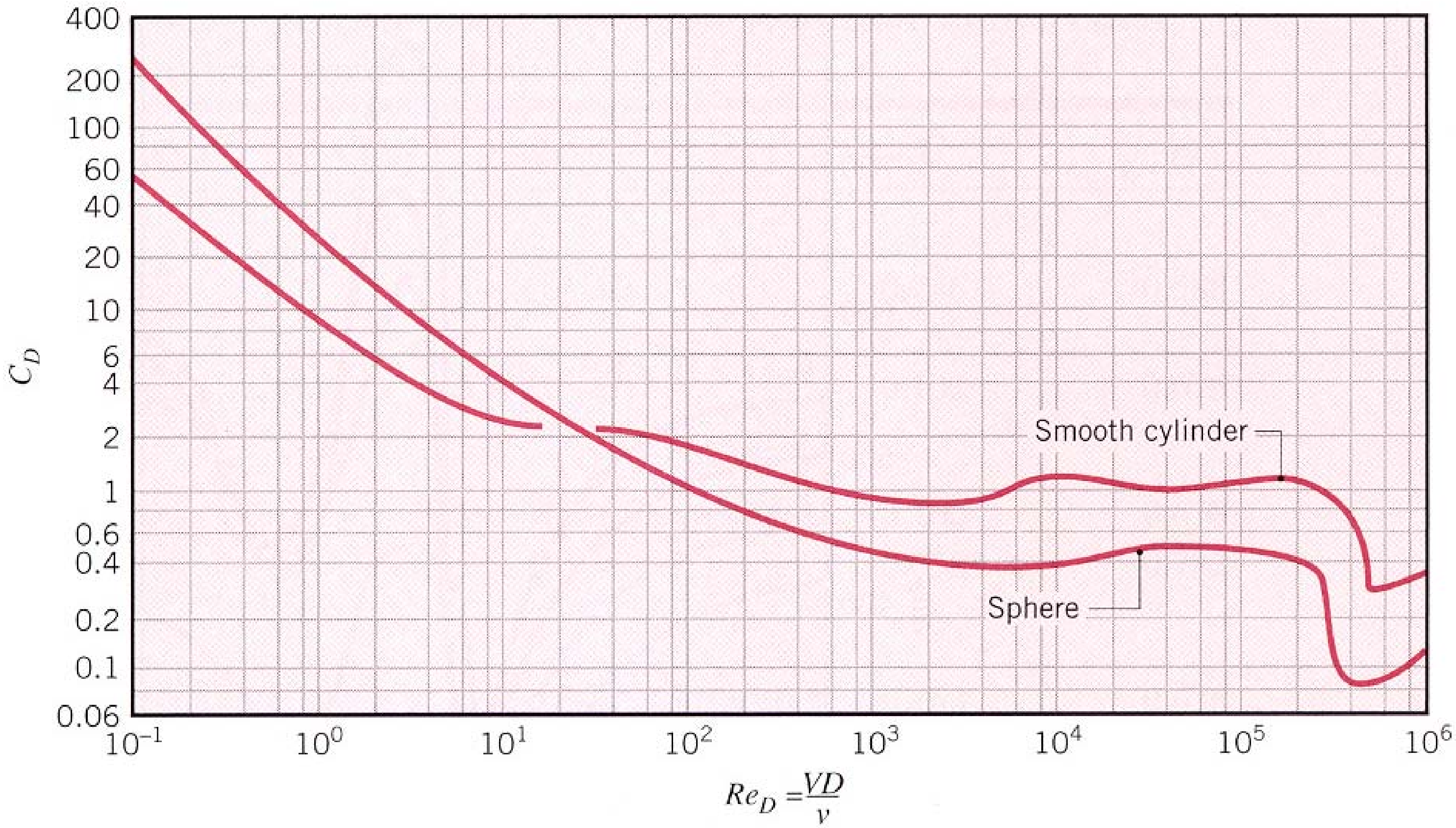
Boundary layer and separation on a circular cylinder in cross flow



Boundary layer separation



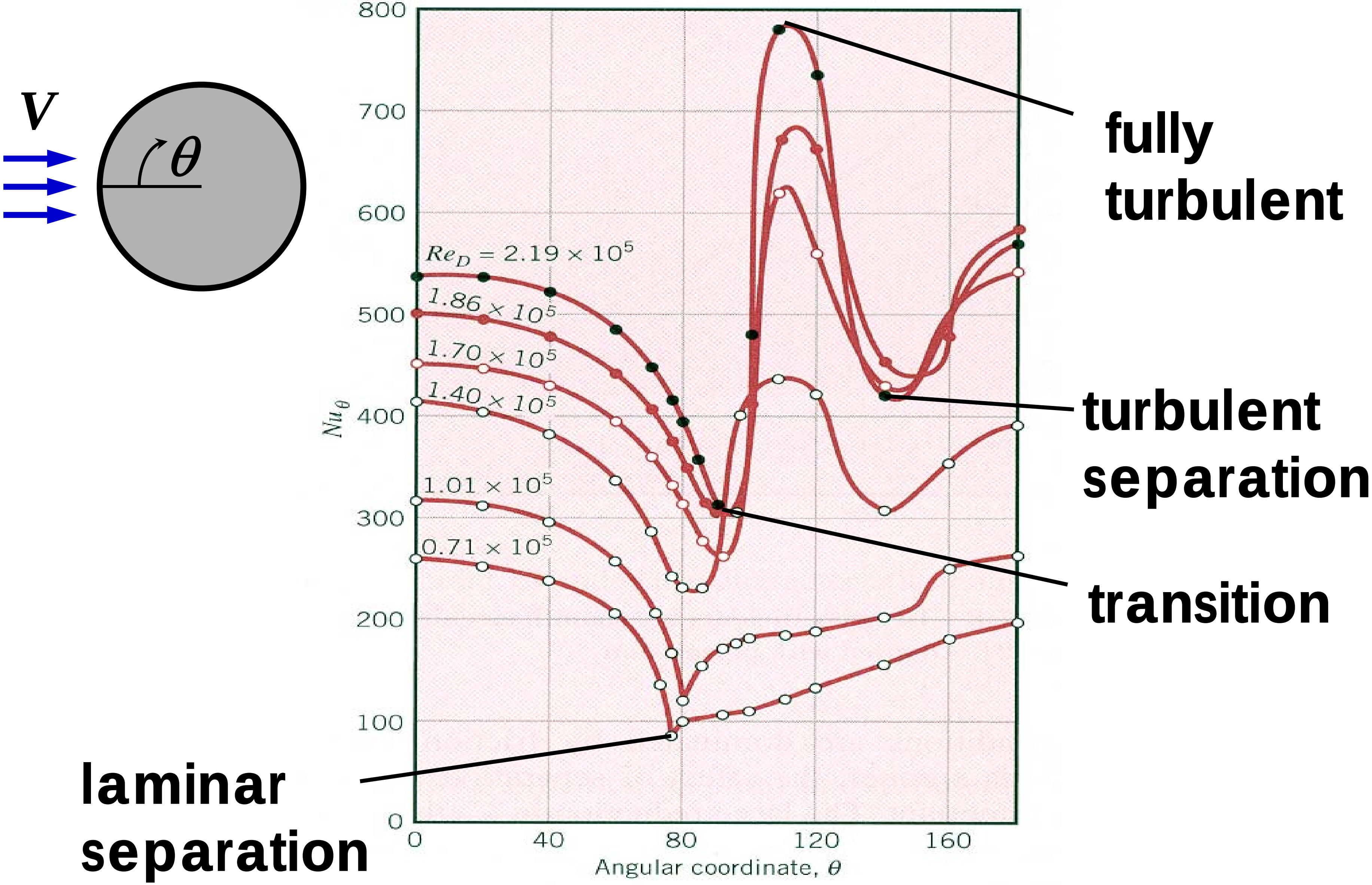
Laminar and turbulent boundary layer separation



Drag coefficients for a smooth circular cylinder and for a sphere

drag coefficient: $C_D \equiv \dfrac{F_D}{A_f\left(\rho V^2/2\right)}$

drag: friction drag + form (pressure) drag



Local Nusselt number variation

- **Stagnation Nusselt number**

$$\mathbf{Nu}_D(\theta = 0^\circ) = 1.15\,\mathbf{Re}_D^{1/2}\,\mathbf{Pr}^{1/3}$$

- **Hilpert (1933)**

$$\overline{\mathbf{Nu}}_D \equiv \frac{\bar{h}D}{k} = C\,\mathbf{Re}_D^m\,\mathbf{Pr}^{1/3}$$

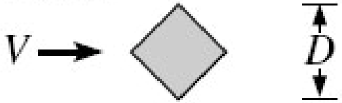
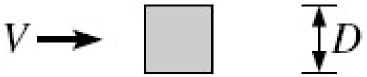
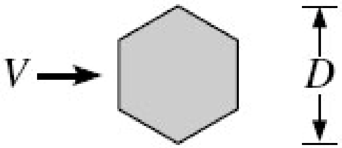
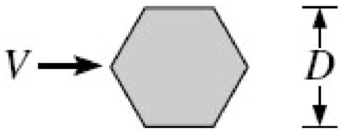
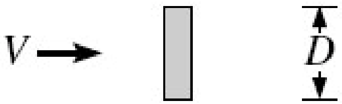
properties at $T_f = \left(T_\infty + T_s\right)/2$

for C **and** m**, Table 7.2 and Table 7.3**

TABLE 7.2 Constants of Equation 7.44 for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

TABLE 7.3 Constants of Equation 7.44 for noncircular cylinders in cross flow of a gas [13]

Geometry	Re_D	C	m
Square			
	5×10^3–10^5	0.246	0.588
	5×10^3–10^5	0.102	0.675
Hexagon			
	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
	5×10^3–10^5	0.153	0.638
Vertical plate			
	4×10^3–1.5×10^4	0.228	0.731

- **Zhukauskas (1972)**

$$\overline{\mathrm{Nu}}_D = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n\left(\mathrm{Pr}/\mathrm{Pr}_s\right)^{1/4} \quad \begin{bmatrix} 0.7 < \mathrm{Pr} < 500 \\ 1 < \mathrm{Re}_D < 10^6 \end{bmatrix}$$

$n = 0.37$ **for** $\mathrm{Pr} \le 10$**,** $n = 0.35$ **for** $\mathrm{Pr} > 10$

for C **and** m**, see Table 7.4**

properties at T_∞ **except** Pr_s

- **Churchill & Bernstein (1977)**

$$\overline{\mathrm{Nu}}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(0.4/\mathrm{Pr}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

properties at T_f $\quad \mathrm{Pr} \le 10,\ \mathrm{Re}_D\mathrm{Pr} > 0.2$

Sphere in Cross Flow

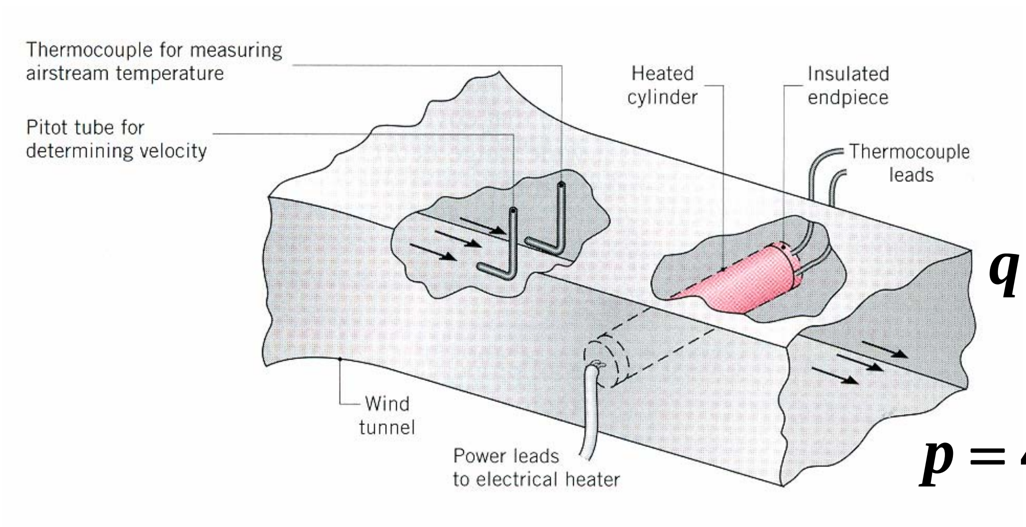
- **Whitaker (1972)**

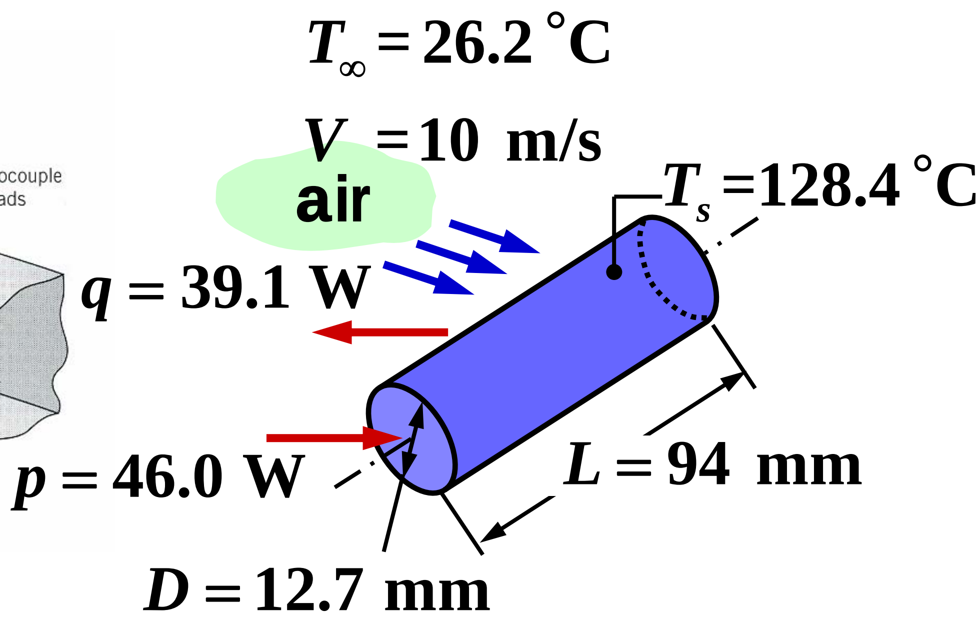
$$\overline{\mathrm{Nu}}_D = 2 + \left(0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right)\mathrm{Pr}^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

$$\left[\begin{array}{c} 0.71 < \mathrm{Pr} < 380 \\ 3.5 < \mathrm{Re}_D < 7.6 \times 10^4 \\ 1.0 < (\mu/\mu_s) < 3.2 \end{array}\right]$$

properties at T_∞ except μ_s

Example 7.3





15% of the power dissipation is lost by radiation and conduction through the endpieces.

Find:

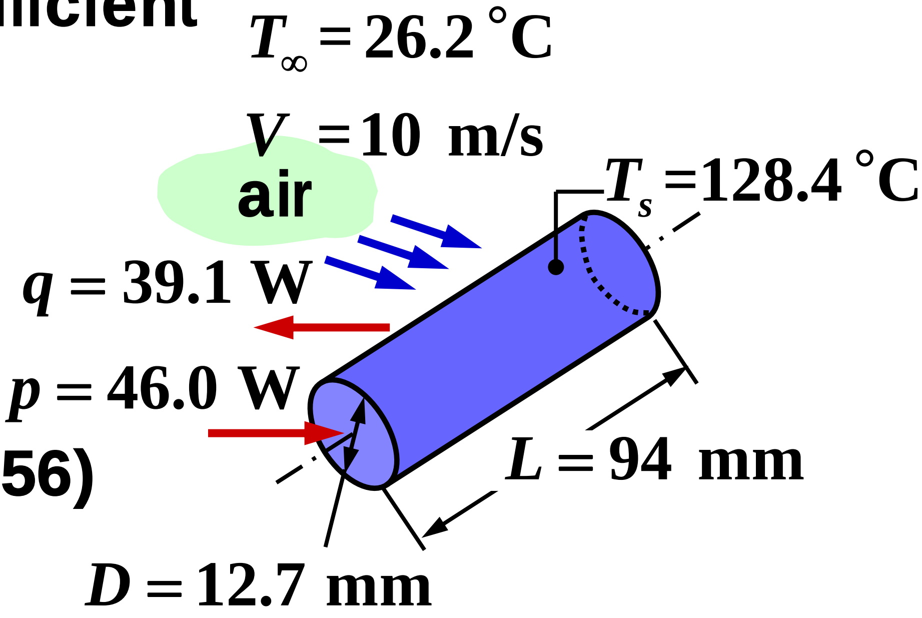
1. **Convection coefficient associated with the operating conditions**
2. **Convection coefficient from an appropriate correlation**

Assumption: Uniform cylinder surface temperature

1) average heat transfer coefficient

$$\bar{h} = \frac{q}{A(T_s - T_\infty)} = 102\ \mathrm{W/m^2 \cdot K}$$

with $q = 0.85P,\ A = \pi DL$



2) Zhukauskas relation (Eq. 7.56)

$$\overline{\mathrm{Nu}_D} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n \left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4}$$

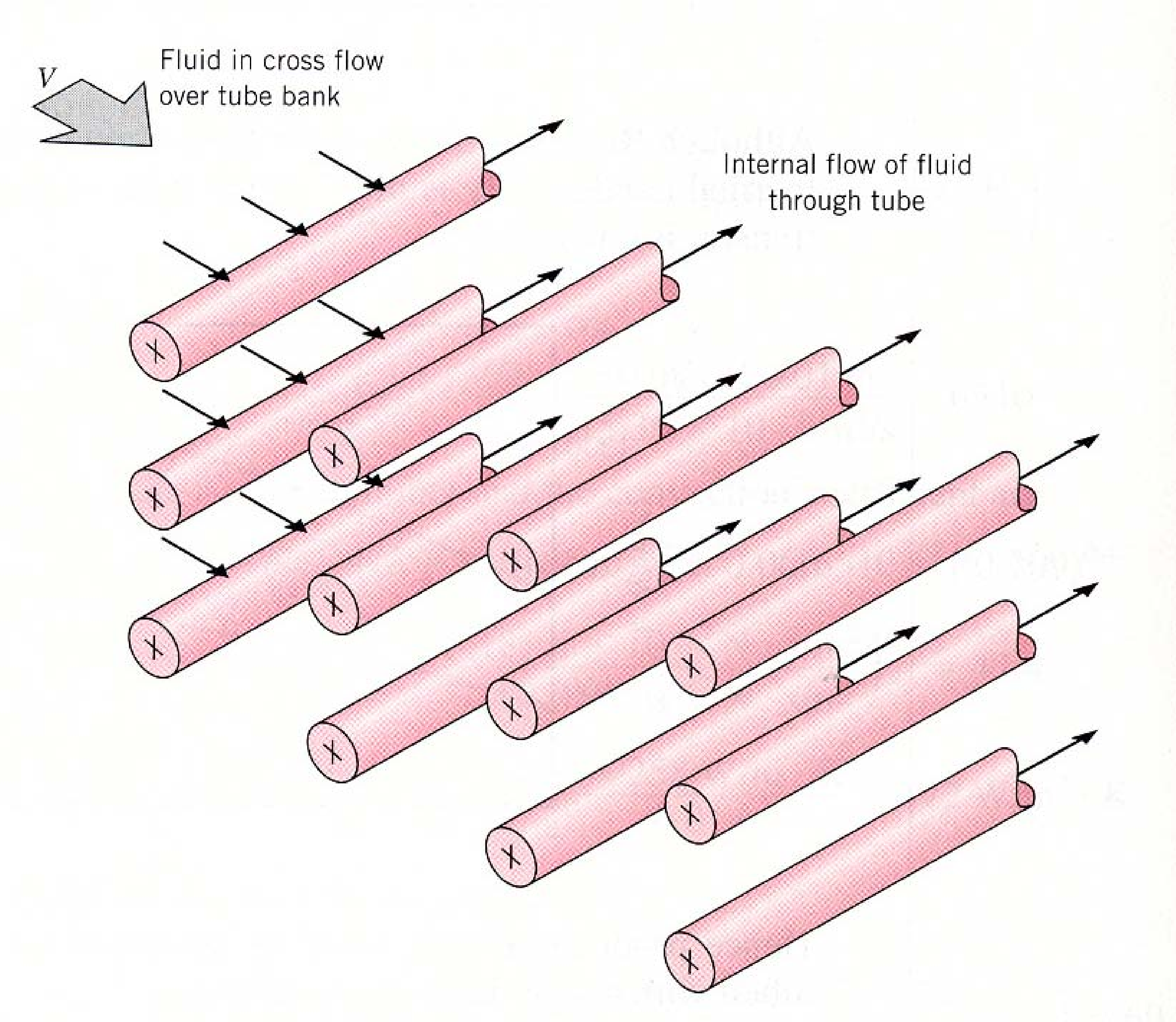
All properties, except Pr_s, are evaluated at T_∞.

$$\mathrm{Re}_D = \frac{VD}{\nu} = 7992$$

From Table 7.4, $C = 0.26,\ m = 0.6.$ Since $\mathrm{Pr} < 10,\ n = 0.37$

$$\overline{\mathrm{Nu}_D} = 50.5, \quad \bar{h} = \overline{\mathrm{Nu}_D}\frac{k}{D} = 105\ \mathrm{W/m^2 \cdot K}$$

Flow across Banks of Tubes



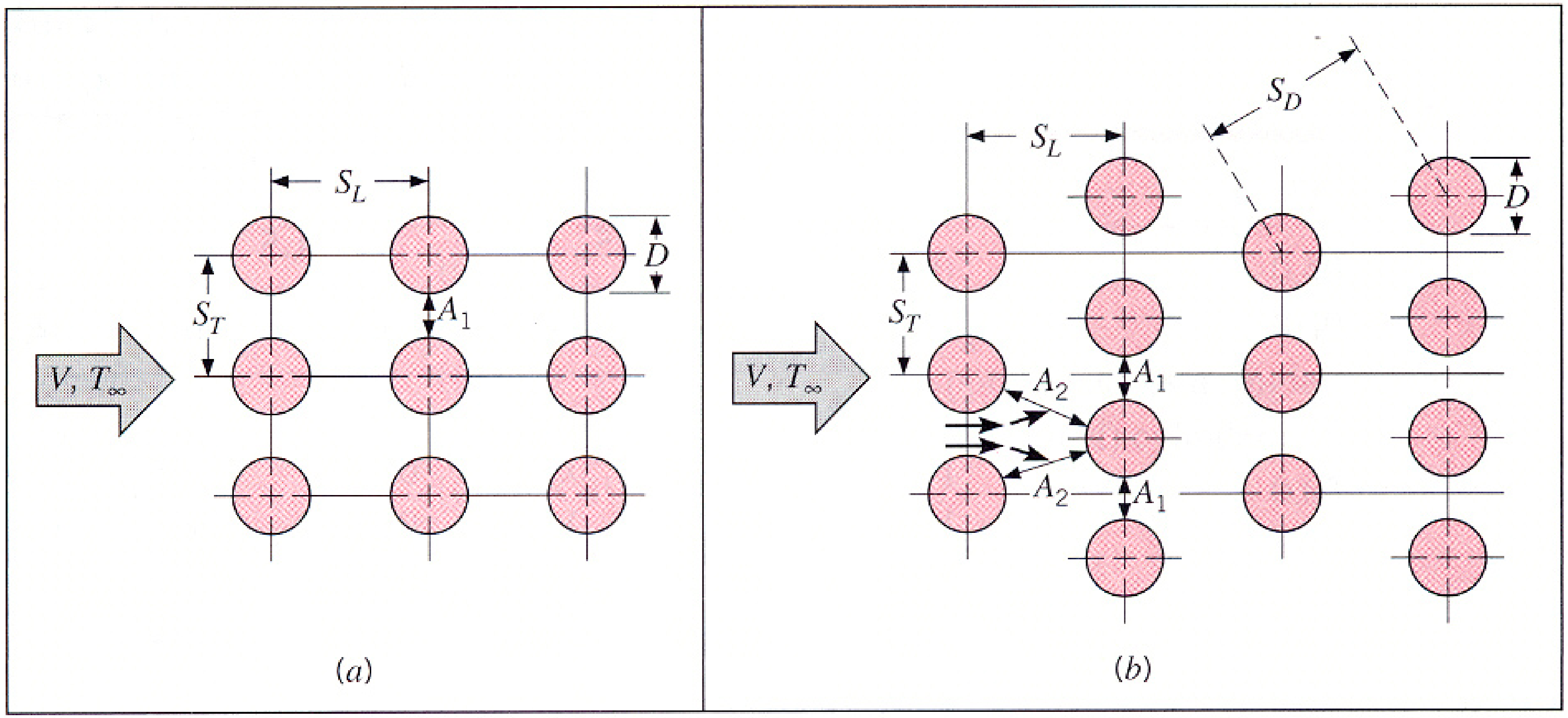


FIGURE 7.11 Tube arrangements in a bank. (*a*) Aligned. (*b*) Staggered.

Geometric characteristics

staggered or aligned

tube diameter D

transverse pitch S_T and longitudinal pitch S_L

Average Nusselt number for the entire tube bundle

- Grimison (1937)

$$\overline{\mathrm{Nu}}_D = 1.13 C_1 \mathrm{Re}_{D,\max}^m \mathrm{Pr}^{1/3}, \quad \begin{bmatrix} N_L \geq 10 \\ 200 < \mathrm{Re}_{D,\max} < 40{,}000 \\ \mathrm{Pr} \geq 0.7 \end{bmatrix}$$

see Table 7.5 for C_1 and m

N_L: number of rows

$$\mathrm{Re}_{D,\max}^m \equiv \rho V_{\max} D / \mu$$

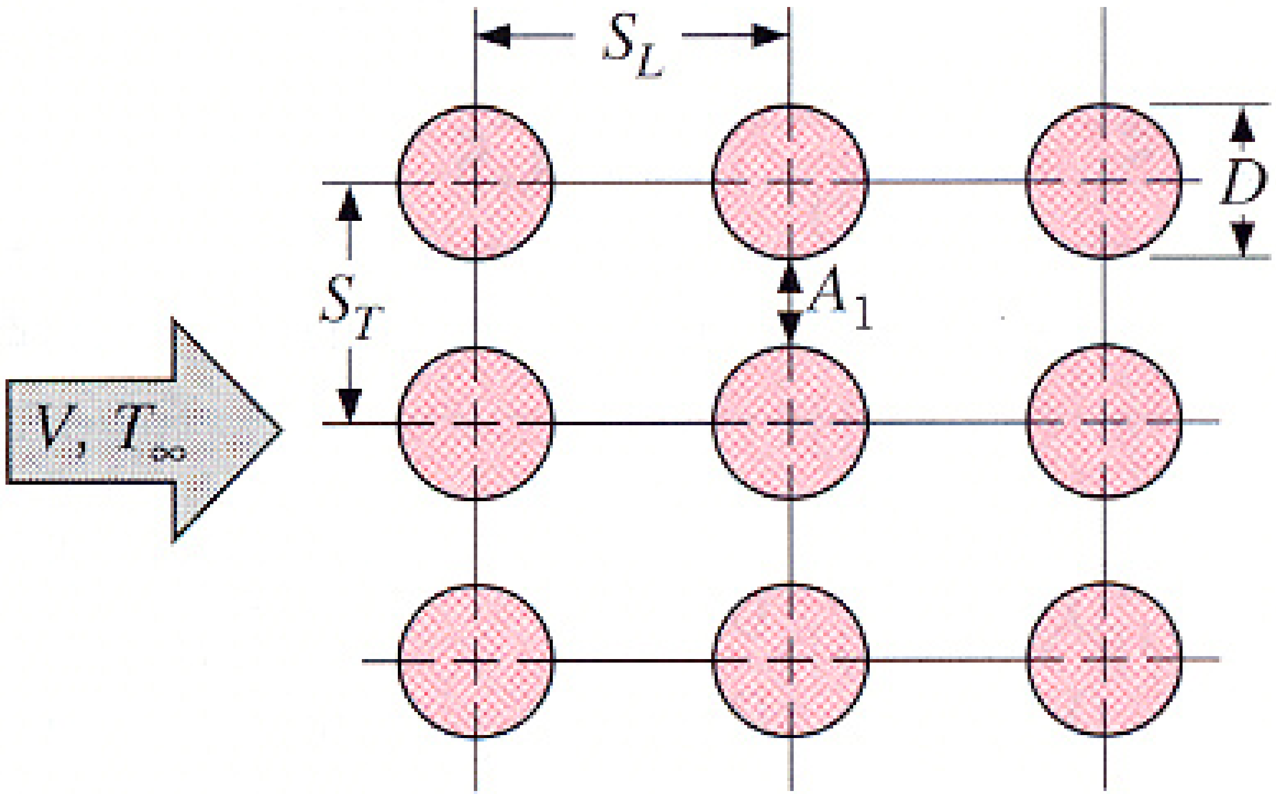
$V_{\max}$: maximum velocity within tube bank

When $N_L < 10$, $\overline{\mathrm{Nu}}_D \Big|_{(N_L<10)} = C_2 \, \overline{\mathrm{Nu}}_D \Big|_{(N_L \geq 10)}$

see Table 7.6 for C_2

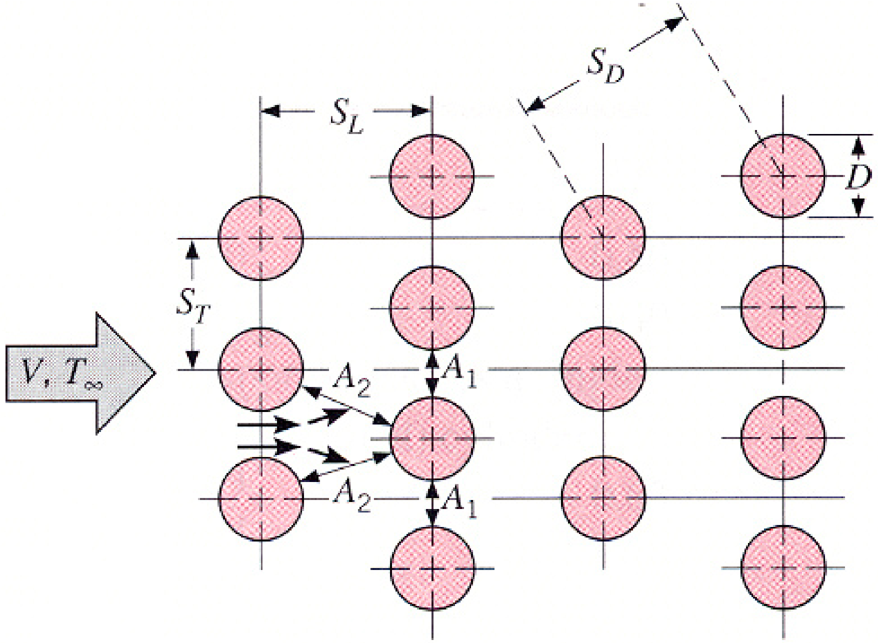
Maximum fluid velocity

aligned arrangement



$$V_{max} = \frac{S_T}{S_T - D} V$$

staggered arrangement



1) maximum at A_2 if

$$S_D = \left[S_L^2 + \left(\frac{S_T}{2} \right)^2 \right]^{1/2} < \frac{S_T + D}{2} : V_{\max} = \frac{S_T}{2(S_D - D)} V$$

2) maximum at A_1: $V_{\max} = \dfrac{S_T}{S_T - D} V$

- **Zhukauskas (1972)**

$$\overline{\mathrm{Nu}}_D = C\,\mathrm{Re}_{D,\max}^m\,\mathrm{Pr}^{0.36}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4}$$

$$\left[\begin{array}{c} N_L \geq 20 \\ 0.7 < \mathrm{Pr} < 500 \\ 1000 < \mathrm{Re}_{D,\max} < 2\times 10^6 \end{array}\right]$$

for C and m, see Table 7.7

properties at $T_m = \left(T_i + T_o\right)/2$ except Pr_s

When $N_L < 20,\ \overline{\mathrm{Nu}}_D\Big|_{(N_L<20)} = C_2\,\overline{\mathrm{Nu}}_D\Big|_{(N_L\geq 20)}$

see Table 7.8 for C_2

Log Mean Temperature Difference (LMTD)

When T_∞ is constant,

$$q = hA\left(T_s - T_\infty\right), \ \ \Delta T = T_s - T_\infty$$

When the fluid temperature varies in the flow direction, $q = hA\Delta T_{\text{lm}}$

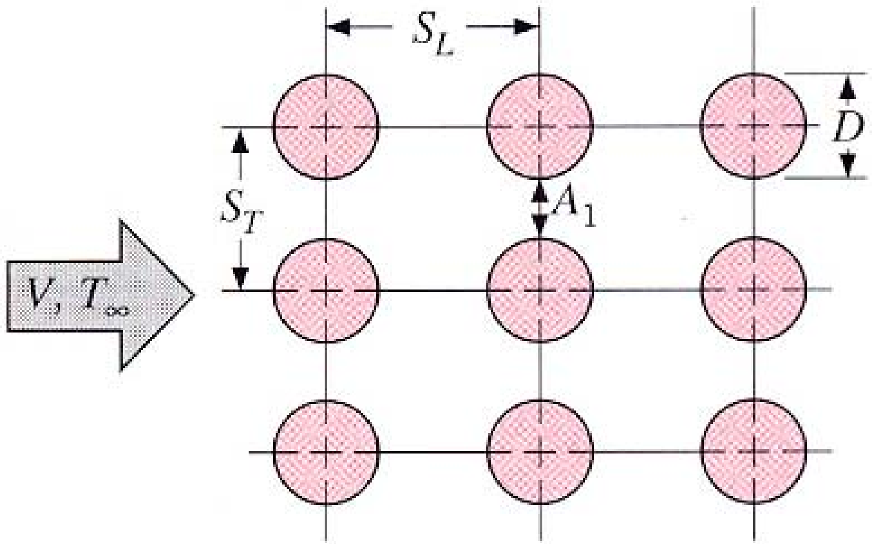
$$\Delta T_{\text{lm}} = \frac{\left(T_s - T_i\right) - \left(T_s - T_o\right)}{\ln\left[\left(T_s - T_i\right)/\left(T_s - T_o\right)\right]}$$

outlet temperature : $\dfrac{T_s - T_o}{T_s - T_i} = \exp\left(-\dfrac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right)$

N: total number of tubes

N_T: number of tubes in the transverse plane

heat transfer rate per unit length of tubes

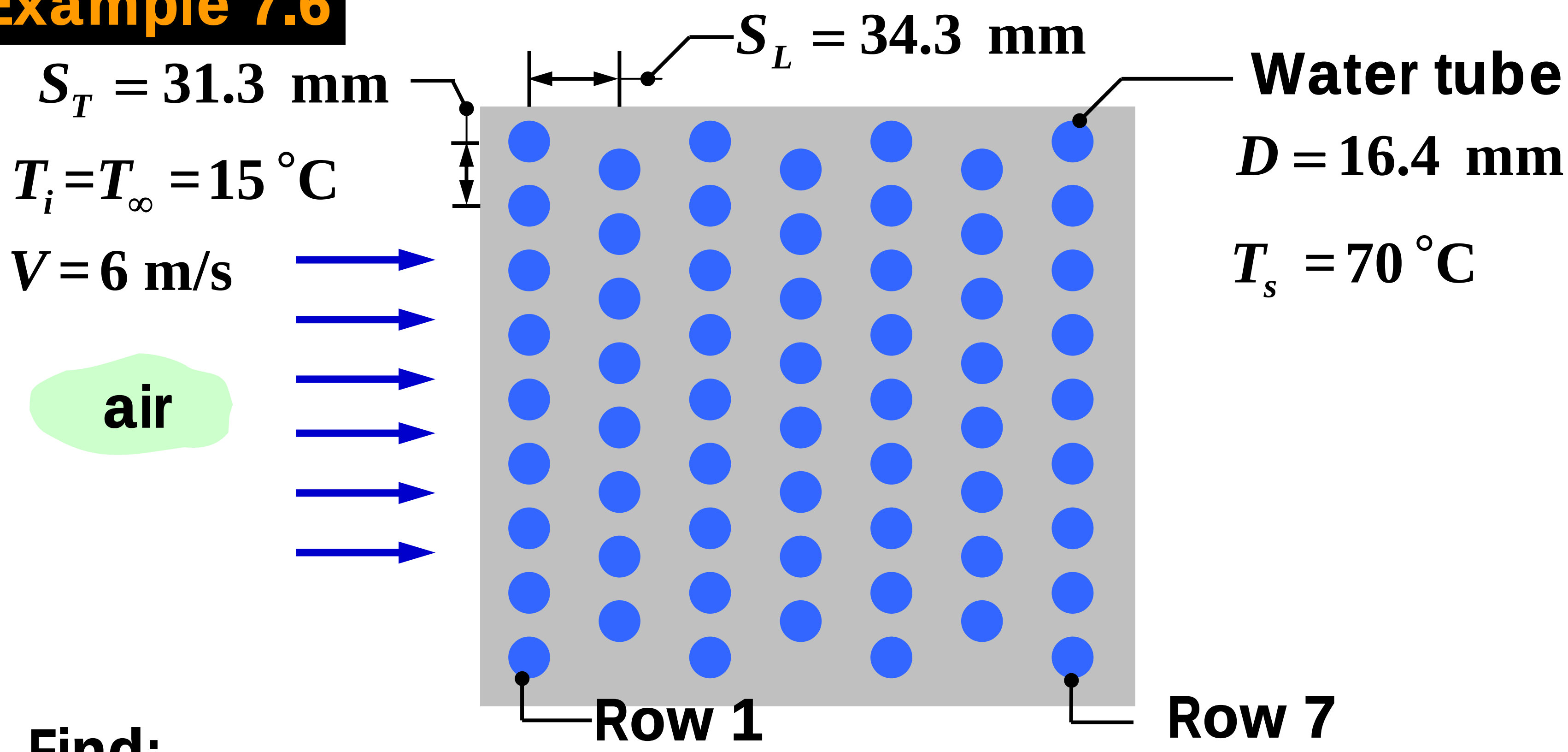


$$q' = N\left(\bar{h}\pi D\right)\Delta T_{\text{lm}}$$

pressure drop : $\Delta p = N_L \chi \left(\dfrac{\rho V_{\max}^2}{2}\right) f$

see Figure 7.13 and 7.14 for χ **and** f

Example 7.6



Find:

1) Air-side convection coefficient $\bar{h}$ and heat rate q'

2) Pressure drop Δp

Assumption:

1. Negligible radiation effects
2. Negligible effect of change in air temperature across tube bank on air properties.

1) heat transfer coefficient and heat transfer rate

$$q' = N\left(\bar{h}\pi D\right)\Delta T_{\mathrm{lm}}$$

Zhukauskas (1972)

$$\overline{\mathrm{Nu}}_D = C\,\mathrm{Re}_{D,\max}^m\,\mathrm{Pr}^{0.36}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4}, \quad \left[\begin{array}{c} N_L \ge 20 \\ 0.7 < \mathrm{Pr} < 500 \\ 1000 < \mathrm{Re}_{D,\max} < 2\times10^6 \end{array}\right]$$

properties at $T_m = \left(T_i + T_o\right)/2$ **except** Pr_s

When $N_L < 20,\ \overline{\mathrm{Nu}}_D\Big|_{(N_L<20)} = C_2\,\overline{\mathrm{Nu}}_D\Big|_{(N_L\ge20)}$

$$\overline{\mathrm{Nu}}_D = C_2 C\,\mathrm{Re}_{D,\max}^m\,\mathrm{Pr}^{0.36}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4}$$

$$S_D = \left[S_L^2 + \left(S_T/2\right)^2\right]^{1/2} = 37.7\,\mathrm{mm} > \left(S_T + D\right)/2$$

$$V_{\max} = \frac{S_T}{S_T - D}V = 12.6\,\mathrm{m/s}$$

$$\overline{\mathrm{Nu}}_D = C_2 C\,\mathrm{Re}_{D,\max}^{m}\,\mathrm{Pr}^{0.36}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4}$$

$$\mathrm{Re}_{D,\max} = \frac{V_{\max} D}{\nu} = 13,943$$

From Tables 7.7 and 7.8

$$\frac{S_T}{S_L} = 0.91 < 2,\quad C = 0.35\left(\frac{S_T}{S_L}\right)^{1/5} = 0.34,\ m = 0.6 \ \text{and}\ C_2 = 0.95$$

$$\bar{h} = \overline{\mathrm{Nu}}_D \frac{k}{D} = 135.6\,\mathrm{W/m^2 \cdot K}$$

$$q' = N\left(\bar{h}\pi D\right)\Delta T_{\mathrm{lm}}$$

$$\Delta T_{\mathrm{lm}} = \frac{\left(T_s - T_i\right) - \left(T_s - T_o\right)}{\ln\left[\left(T_s - T_i\right)/\left(T_s - T_o\right)\right]}$$

outlet temperature : $\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right)$

N: total number of tubes

N_T: number of tubes in the transverse plane

$$\Delta T_{\mathrm{lm}} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left[(T_s - T_i)/(T_s - T_o)\right]} = 49.6^\circ \mathrm{C}$$

$$q' = N\left(\bar{h}\pi D\right)\Delta T_{\mathrm{lm}} = 19.4 \ \mathrm{kW/m}$$

2) pressure drop

$$\Delta p = N_L \chi \left(\frac{\rho V_{\max}^2}{2}\right) f$$

$$= 2.46 \times 10^{-3} \ \mathrm{bars}$$

$V_{\max} = 12.6\,\mathrm{m/s}$

$\mathrm{Re}_{D,\max} = 13{,}943$

$P_T = (S_T / D) = 1.91$

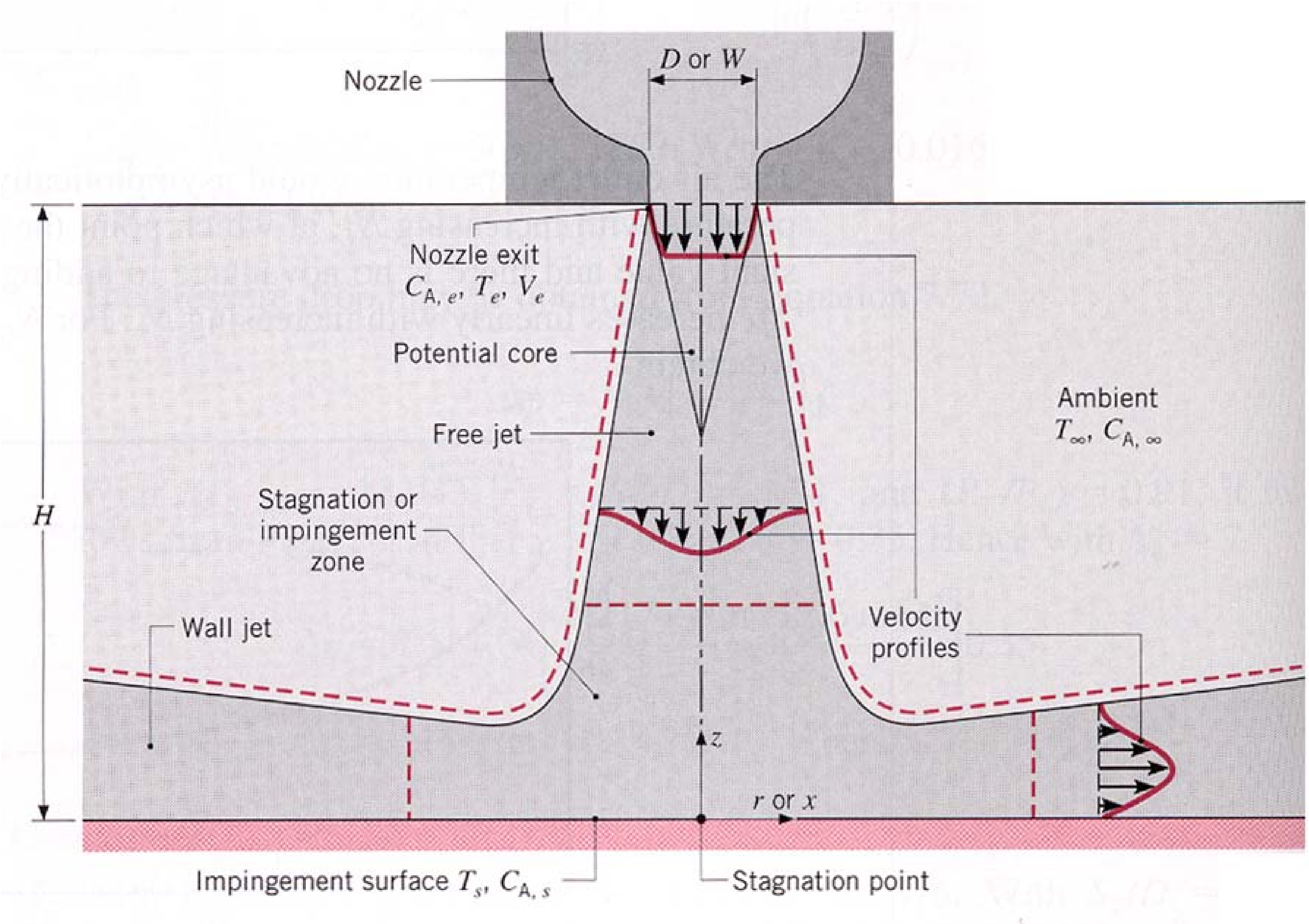
$(P_T / P_L) = 0.91$

From Figure 7.14

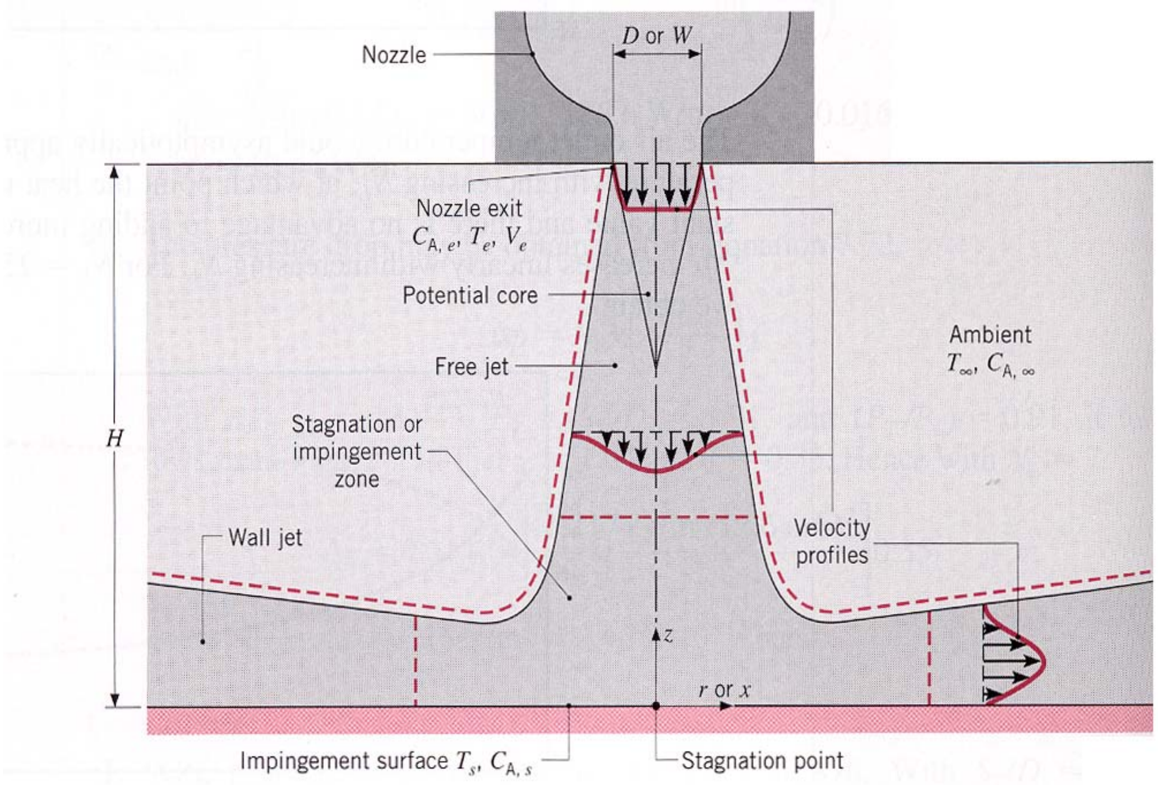
$\chi \approx 1.04 \qquad f \approx 0.35$

$N_L = 7$

Impinging Jets



Single round or slot impinging jet

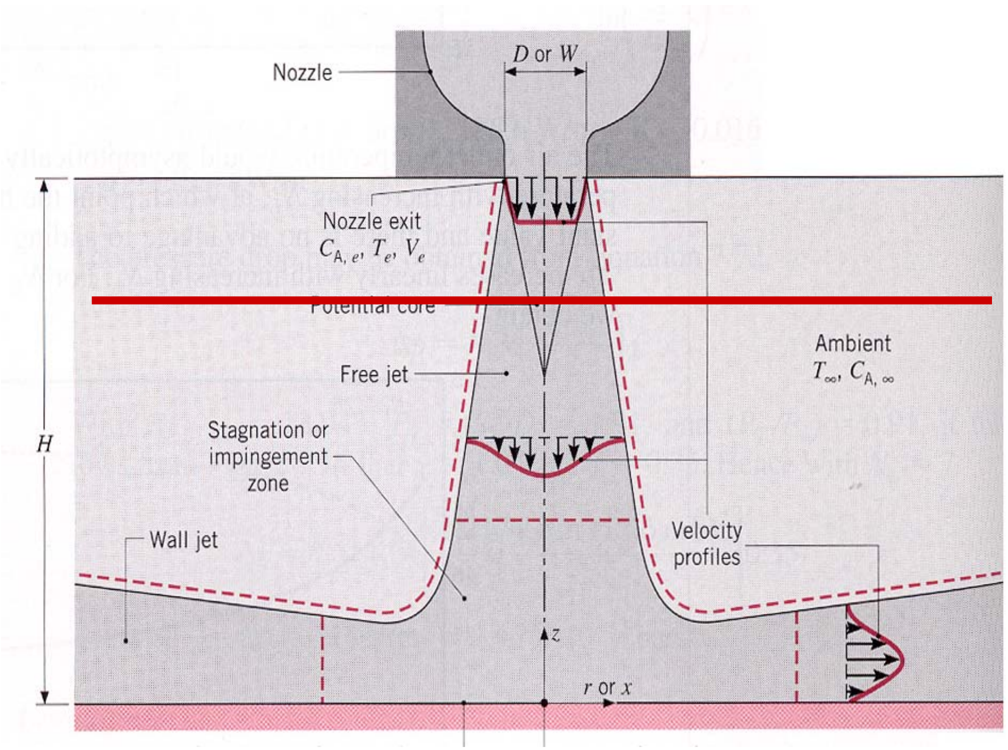


$$\overline{\mathrm{Nu}} = f(\mathrm{Re}, \mathrm{Pr}, r/D_h \text{ or } x/D_h, H/D_h)$$

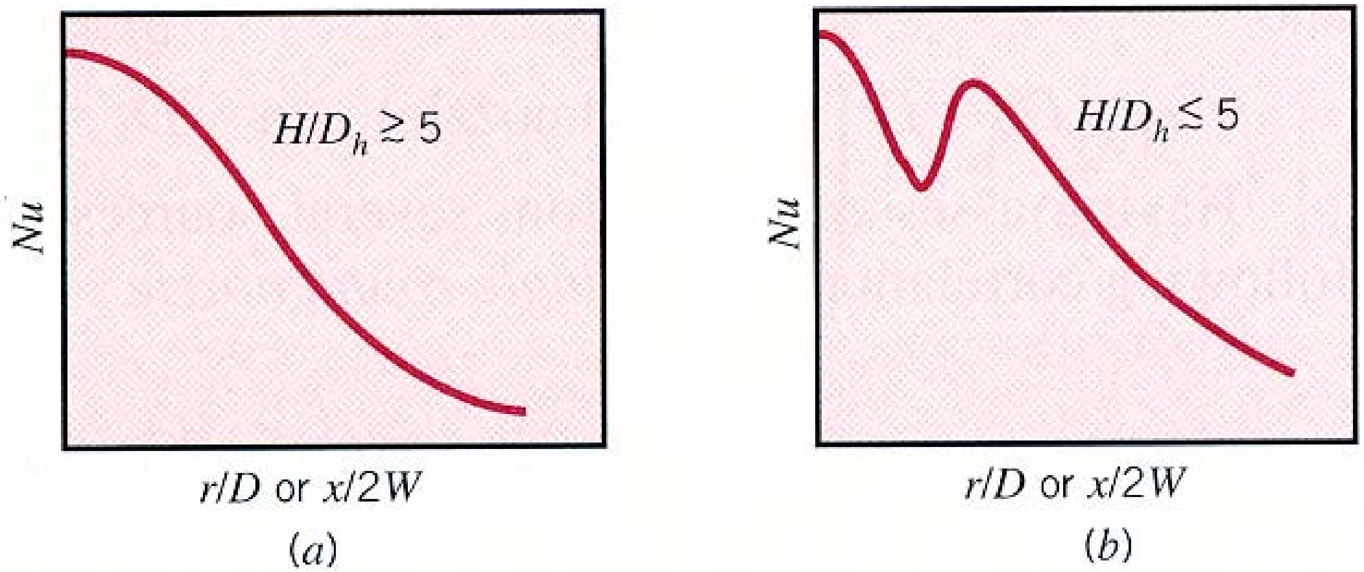
$$\overline{\mathrm{Nu}} = \frac{\bar{h}D_h}{k}, \ \mathrm{Re} = \frac{V_e D_h}{\nu}$$

D_h: hydrauic diameter, $D_h = \dfrac{4A}{P}$ P : wetted perimeter

$D_h = D$: round nozzle, $D_h = 2W$: slot nozzle



H: nozzle-to-plate spacing



For a single round nozzle

$$\frac{\overline{\mathrm{Nu}}}{\mathrm{Pr}^{0.42}} = G\left(\frac{r}{D}, \frac{H}{D}\right) F_1(\mathrm{Re}), \quad \left[\begin{array}{c} 2{,}000 \leq \mathrm{Re} \leq 400{,}000 \\ 2 \leq H/D \leq 12 \\ 2.5 \leq r/D \leq 7.5 \\ \text{or } \; 0.04 \geq A_r \geq 0.004 \end{array}\right]$$

$$F_1 = 2\,\mathrm{Re}^{1/2}\left(1 + 0.005\,\mathrm{Re}^{0.55}\right)^{1/2}$$

$$G = \frac{D}{r}\,\frac{1 - 1.1D/r}{1 + 0.1(H/D - 6)D/r}$$

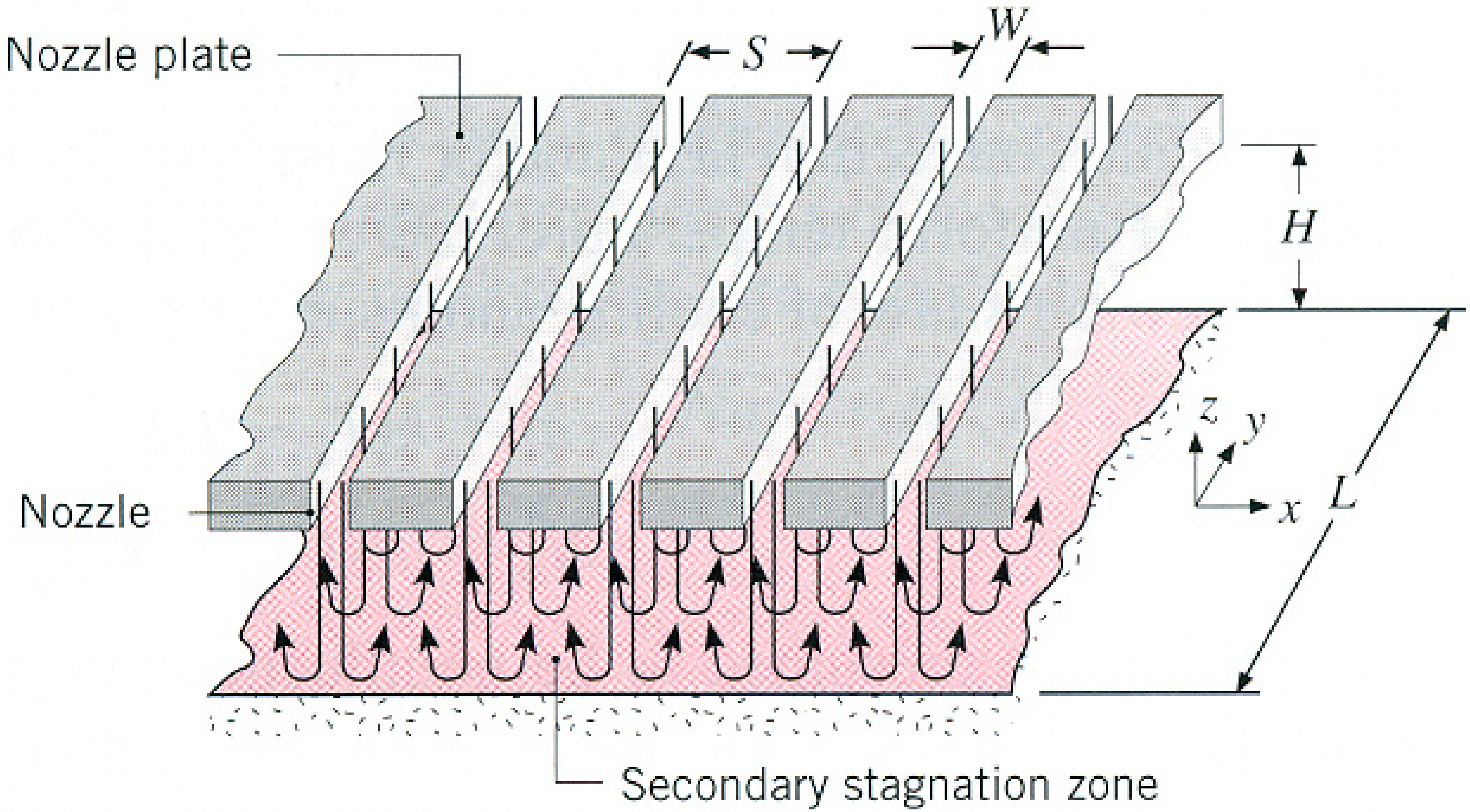
or $$G = 2A_r^{1/2}\,\frac{1 - 2.2A_r^{1/2}}{1 + 0.2(H/D - 6)A_r^{1/2}}$$

For a single slot nozzle

$$\frac{\overline{\mathrm{Nu}}}{\mathrm{Pr}^{0.42}} = \frac{3.06}{x/W + H/W + 2.78}\mathrm{Re}^{m}$$

$$\begin{bmatrix} 3{,}000 \le \mathrm{Re} \le 90{,}000 \\ 2 \le H/W \le 10 \\ 4 \le x/W \le 20 \end{bmatrix}$$

$$m = 0.695 - \left[\left(\frac{x}{2W}\right) + \left(\frac{H}{2W}\right)^{1.33} + 3.06\right]^{-1}$$



An array of slot jets

For an array of round nozzles

$$\frac{\overline{\mathrm{Nu}}}{\mathrm{Pr}^{0.42}} = K\left(A_r, \frac{H}{D}\right) G\left(A_r, \frac{H}{D}\right) F_2(\mathrm{Re})$$

$$\begin{bmatrix} 2{,}000 \le \mathrm{Re} \le 100{,}000 \\ 2 \le H/D \le 12 \\ 0.004 \le A_r \le 0.04 \end{bmatrix}$$

$$K = \left[1 + \left(\frac{H/D}{0.6/A_r^{1/2}}\right)^6\right]^{-0.05}, \quad F_2 = 0.5\,\mathrm{Re}^{2/3}$$

For an array of slot nozzles

$$\frac{\overline{\mathrm{Nu}}}{\mathrm{Pr}^{0.42}} = \frac{2}{3} A_{r,o}^{3/4} \left(\frac{2\,\mathrm{Re}}{A_r / A_{r,o} + A_{r,o} / A_r} \right)^{2/3}$$

$$\begin{bmatrix} 1{,}500 \le \mathrm{Re} \le 40{,}000 \\ 2 \le H/W \le 80 \\ 0.008 \le A_r \le 2.5 A_{r,o} \end{bmatrix}$$

$$A_{r,o} = \left\{ 60 + 4 \left[\left(\frac{H}{2W} \right) - 2 \right]^2 \right\}^{-1/2}$$

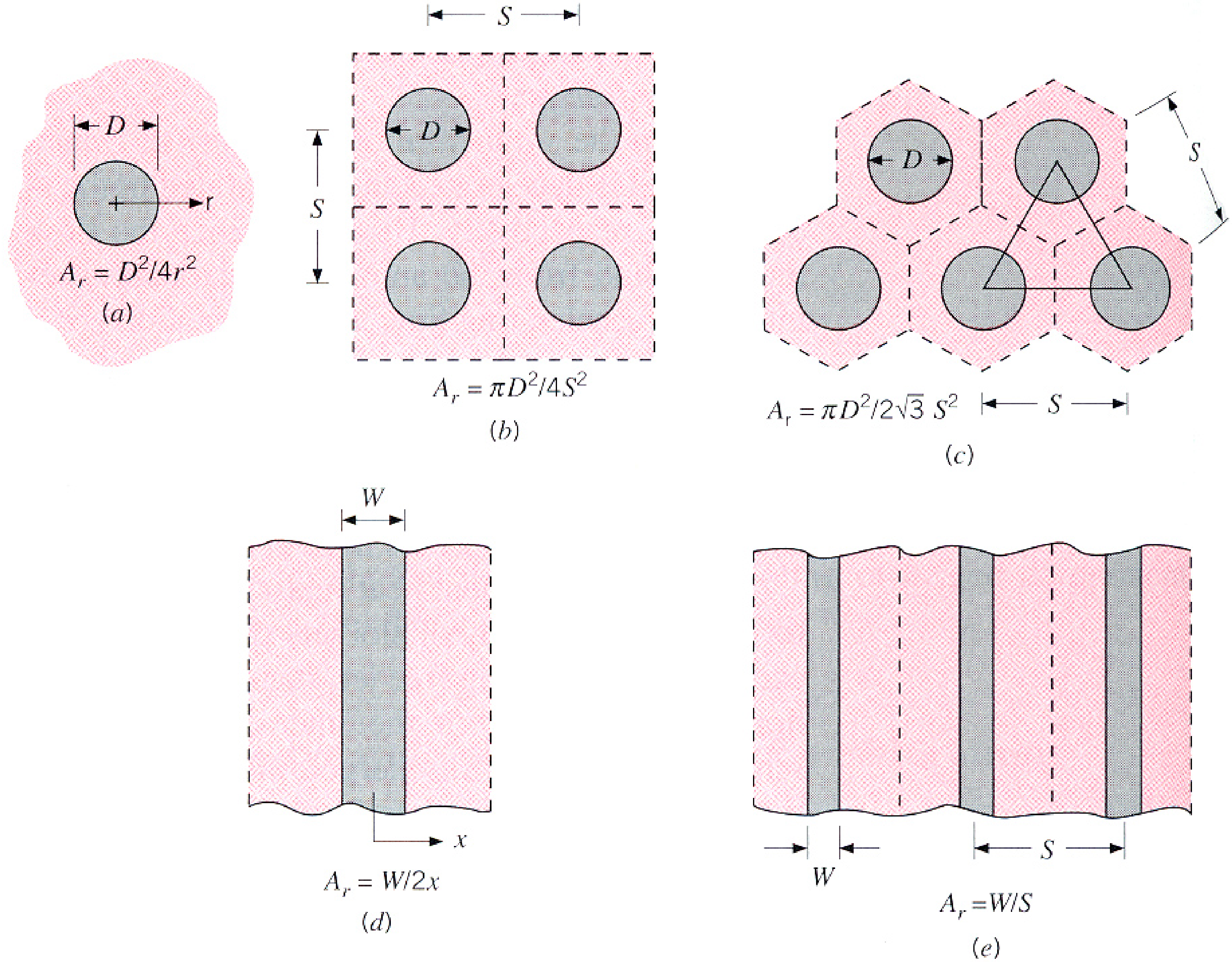
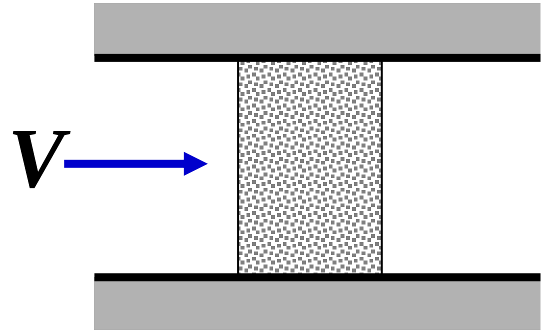


FIGURE 7.17 Plan view of pertinent geometrical features for (*a*) single round jet, (*b*) in-line array of round jets, (*c*) staggered array of round jets, (*d*) single slot jet, and (*e*) array of slot jets.

Packed Beds

Gas flow in a bed of spheres



$$\varepsilon \bar{j}_H = 2.06\,\mathrm{Re}_D^{-0.575}, \quad \begin{bmatrix} \mathrm{Pr} \approx 0.7 \\ 90 \le \mathrm{Re}_D \le 4,000 \end{bmatrix}$$

$\bar{j}_H$: Colburn factor $\left(= \dfrac{C_f}{2} = \mathrm{St}\,\mathrm{Pr}^{2/3} \right)$

ε : porosity or void fraction $(0.3 \sim 0.5)$

$$q = \bar{h} A_{p,t} \Delta T_{\mathrm{lm}}, \quad \frac{T_s - T_o}{T_s - T_i} = \exp\left(- \frac{\bar{h} A_{p,t}}{\rho V A_{c,b} c_p} \right)$$

$A_{p,t}$: total surface area of the particles

$A_{c,b}$: bed cross-sectional area