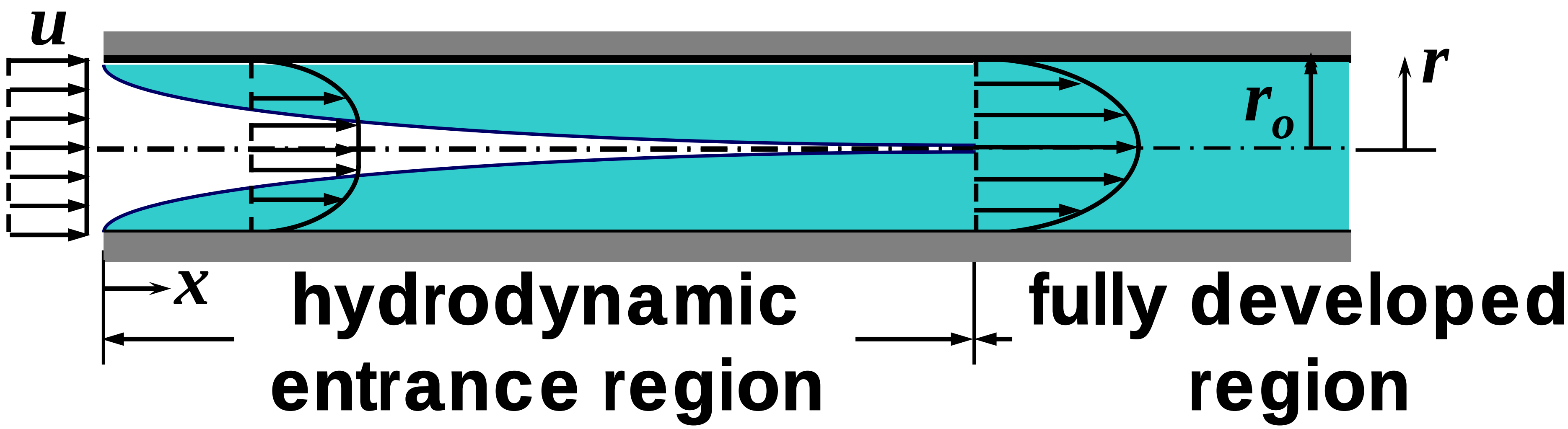
INTERNAL FORCED CONVECTION

- Hydrodynamic Considerations
- Thermal Considerations
- Overall Energy Balance
- Convection Correlations:
  Circular & Non-Circular Tubes
  Concentric Tube Annulus
- Heat Transfer Enhancement

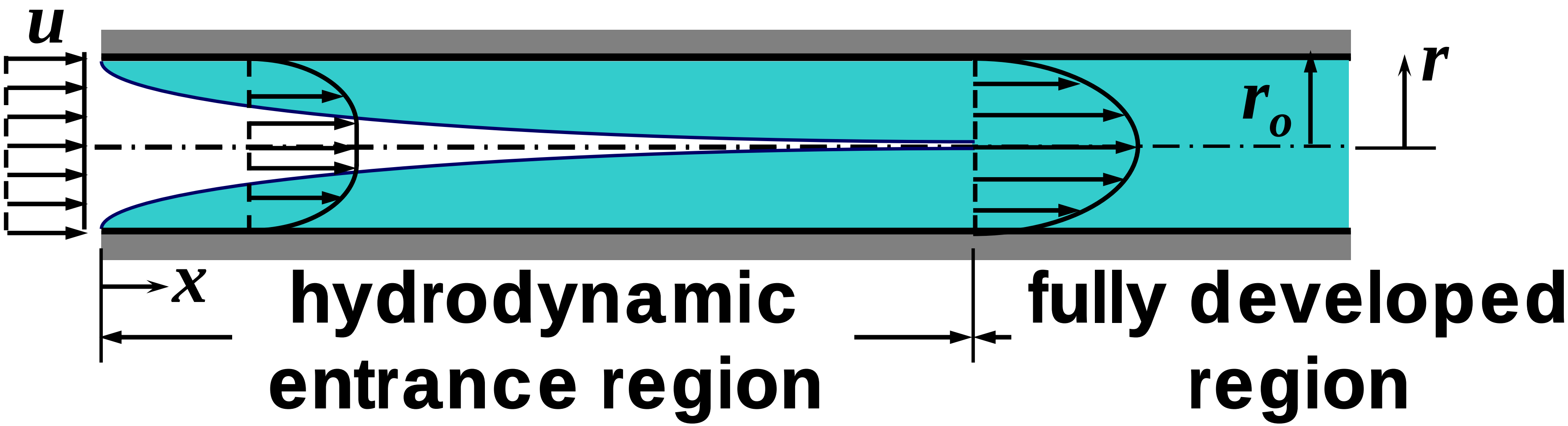
Hydrodynamic Considerations



hydrodynamic entrance region

flow acceleration

inertia force ~ pressure force
~ viscous force



hydrodynamically fully developed region

no acceleration (inertia force = 0)

pressure force ~ viscous force

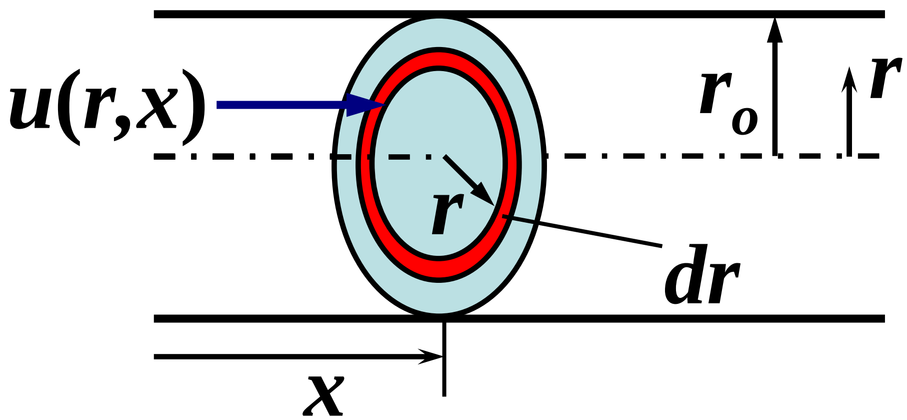
constant pressure gradient

fully developed condition :

$$v = 0,\ \frac{\partial u}{\partial x} = 0 \ \rightarrow \frac{du}{dt} = 0, \quad u = u(r)$$

Characteristic Velocity

mean velocity over the cross-sectional area



$$\dot{m} = \int_{A_c} \rho u(r,x) dA_c = \int_0^{r_o} \rho u(r,x) \cdot 2\pi r dr$$

$$\equiv \rho u_m \cdot \pi r_o^2 = \rho u_m A_c = \frac{\pi}{4} \rho u_m D^2$$

$$u_m = \frac{2}{r_o^2} \int_0^{r_o} u(r,x) \cdot r dr = \frac{4\dot{m}}{\pi \rho D^2}$$

Reynolds number : $$\mathrm{Re}_D = \frac{\rho u_m D}{\mu} = \frac{4\dot{m}}{\pi D \mu}$$

critical Reynolds number :

$$\mathrm{Re}_{D,c} \approx 2,300$$

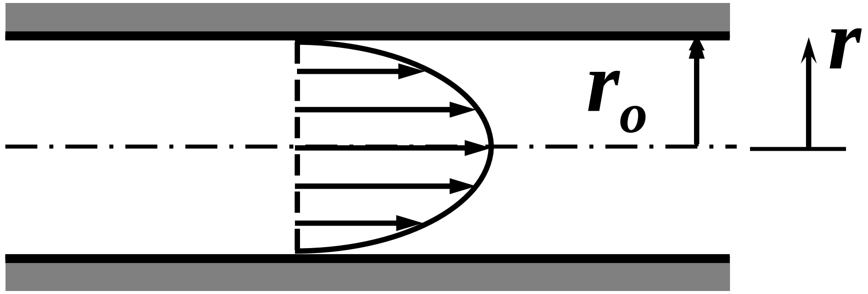
hydrodynamic entry length:

$$\left(\frac{x_{\mathrm{fd,h}}}{D}\right)_{\mathrm{lam}} \approx 0.05\,\mathrm{Re}_D, \quad 10 \leq \left(\frac{x_{\mathrm{fd,h}}}{D}\right)_{\mathrm{turb}} \leq 60$$

Velocity Profile in Fully Developed Region

momentum eq:

$$0 = -\frac{dp}{dx} + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)$$



or $$\frac{dp}{dx} = \frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \text{constant}$$

boundary conditions: $u(r_o) = 0, \quad \left.\frac{du}{dr}\right|_{r=0} = 0$

$$r\frac{du}{dr} = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r^2}{2} + C_1$$

$$u(r) = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r^2}{4} + C_1 \ln r + C_2$$

$$C_1 = 0, \quad C_2 = -\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r_o^2}{4}$$

$$u(r) = -\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r_o^2}{4}\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

$$u_m = \frac{2}{r_o^2}\int_0^{r_o} u(r,x)\cdot r dr = -\frac{r_o^2}{8\mu}\frac{dp}{dx}$$

$$u(r) = 2u_m\left(1-\frac{r^2}{r_o^2}\right)$$

$$u(0) = 2u_m \equiv u_c, \quad u(r) = u_c\left(1-\frac{r^2}{r_o^2}\right)$$

Friction Coefficient

$$\tau_s = -\mu \left.\frac{\partial u}{\partial r}\right|_{r=r_o} = -\mu \cdot 2u_m\left(-\frac{2}{r_o}\right) = \frac{4\mu u_m}{r_o}$$

friction coefficient

$$C_f = \frac{\tau_s}{\rho u_m^2/2} = \frac{4\mu u_m}{r_o}\cdot\frac{2}{\rho u_m^2} = \frac{16}{\mathrm{Re}_D}$$

Moody friction factor

$$f \equiv \frac{-(dp/dx)D}{\rho u_m^2/2} = \frac{8\mu u_m D}{r_o^2 \rho u_m^2/2} = \frac{64\mu}{D\rho u_m} = \frac{64}{\mathrm{Re}_D} = 4C_f$$

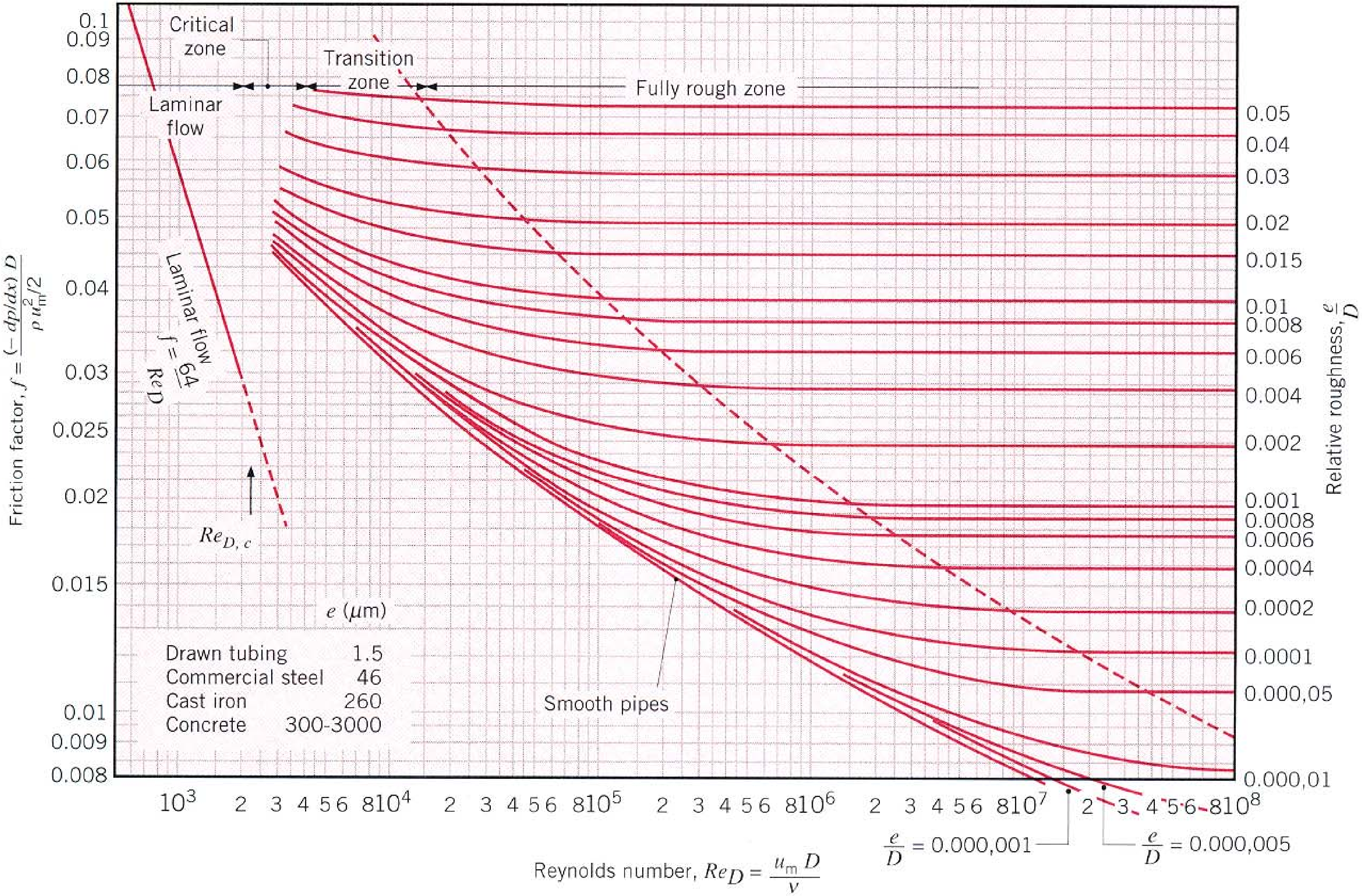
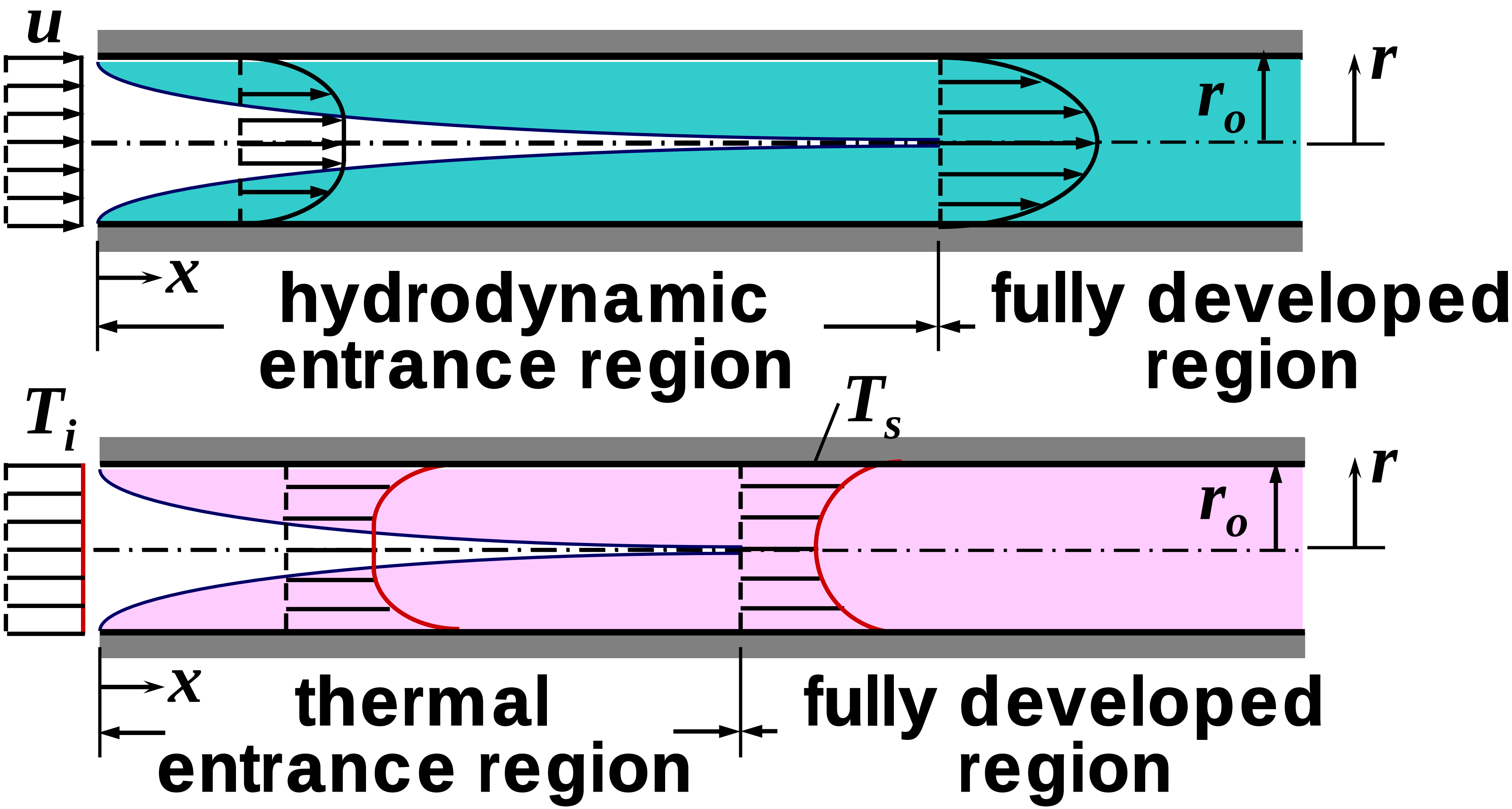


FIGURE 8.3 Friction factor for fully developed flow in a circular tube [3]. Used with permission.

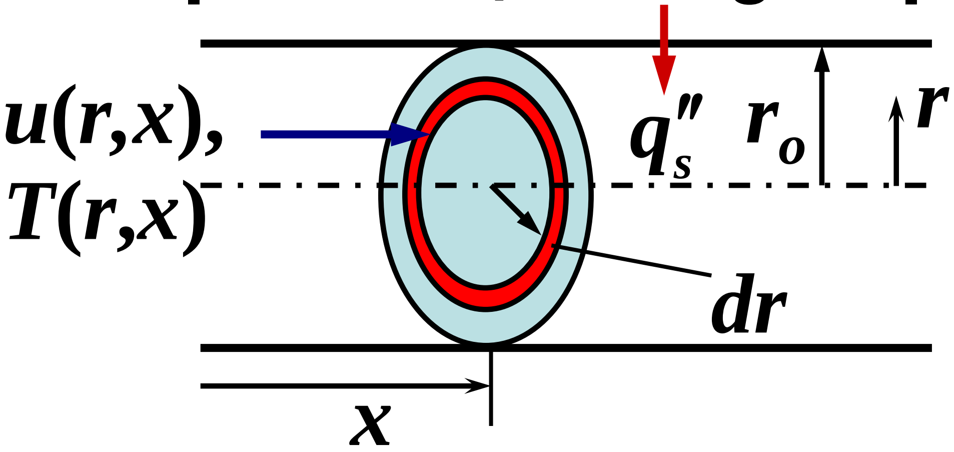
Thermal Considerations



Thermal entry length : $\left(\dfrac{x_{\mathrm{fd,t}}}{D}\right)_{\mathrm{lam}} \approx 0.05\,\mathrm{Re}_D\,\mathrm{Pr}$

Mixed Mean Temperature

(bulk fluid temperature, mixing-cup temperature)



$$\int_0^{r_o} u\rho c T \cdot 2\pi r dr \equiv T_m \int_0^{r_o} u\rho c \cdot 2\pi r dr = \rho c T_m u_m \pi r_o^2$$

$$T_m(x) = \frac{2}{u_m r_o^2}\int_0^{r_o} uTrdr \qquad \left(u_m = \frac{2}{r_o^2}\int_0^{r_o} u(r,x)\cdot rdr \right)$$

heat transfer coefficient :

$$q_s'' = k\frac{\partial T}{\partial r}\bigg|_{r=r_o} = h\left(T_s - T_m\right)$$

Thermally Fully Developed Condition

experimental observation: in the far downstream region, h =const. for a given fluid and tube diameter

Suppose $$\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}=\phi(r)$$ **alone**

$$\left[\frac{\partial}{\partial r}\left(\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right)\right]_{r=r_o}=-\frac{\left.\partial T/\partial r\right)_{r=r_o}}{T_s(x)-T_m(x)}$$

= constant (not function of x)

$$q_s''=h\left(T_s-T_m\right)=k\left.\frac{\partial T}{\partial r}\right|_{r=r_o}\rightarrow\frac{h}{k}=-\frac{\left.\partial T/\partial r\right)_{r=r_o}}{T_s(x)-T_m(x)}$$

= constant

thermally fully developed condition

$$\frac{\partial}{\partial x}\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]=0$$

$$\left(\frac{dT_s}{dx}-\frac{\partial T}{\partial x}\right)\left(T_s-T_m\right)-\left(T_s-T\right)\left(\frac{dT_s}{dx}-\frac{dT_m}{dx}\right)=0$$

$$\frac{\partial T}{\partial x}=\frac{dT_s}{dx}-\frac{T_s-T}{T_s-T_m}\frac{dT_s}{dx}+\frac{T_s-T}{T_s-T_m}\frac{dT_m}{dx}$$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} - \frac{T_s - T}{T_s - T_m}\frac{dT_s}{dx} + \frac{T_s - T}{T_s - T_m}\frac{dT_m}{dx}$$

Constant surface heat flux condition

$$q_s'' = h\left(T_s - T_m\right) = \text{const.} \rightarrow T_s - T_m = \text{const.}$$

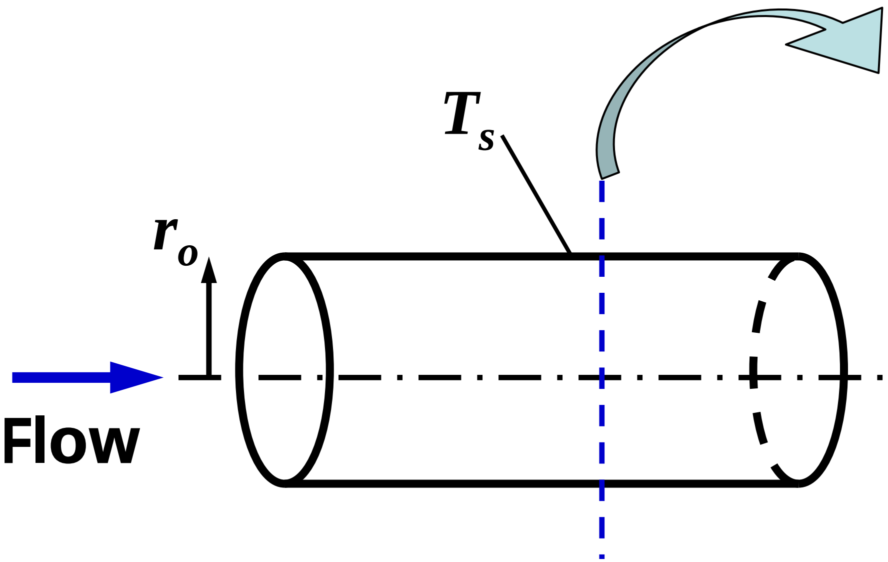
$$\frac{dT_s}{dx} - \frac{dT_m}{dx} = 0 \rightarrow \frac{dT_s}{dx} = \frac{dT_m}{dx}$$

$$\rightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx}$$

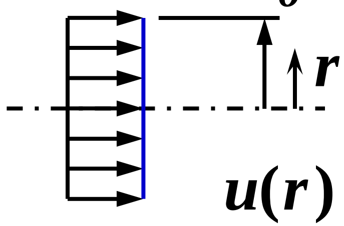
Constant surface temperature condition

$$T_s = \text{const.} \rightarrow \frac{\partial T}{\partial x} = \frac{T_s - T}{T_s - T_m}\frac{dT_m}{dx}$$

Example 8.1

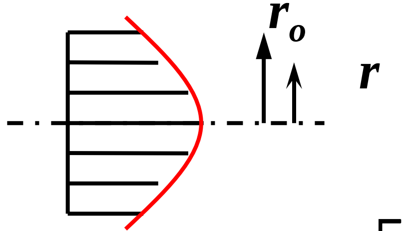


Velocity profile



$$u(r) = C_1$$

Temperature profile



$$T(r) - T_s = C_2\left[1 - \left(r / r_o\right)^2\right]$$

slug flow: liquid metal, $\mathrm{Pr} << 1$

Find:

Nusselt number at the prescribed location

Assumptions:

Incompressible, constant property flow

$$u(r) = C_1, \quad T(r) - T_s = C_2\left[1 - \left(r/r_o\right)^2\right]$$

$$\mathrm{Nu}_D = \frac{hD}{k}, \quad h = \frac{q_s''}{T_s - T_m}, \quad q_s'' = k\left.\frac{\partial T}{\partial r}\right|_{r=r_o}$$

$$T_m = \frac{2}{u_m r_o^2}\int_0^{r_o} uTrdr = \frac{2}{u_m r_o^2}\int_0^{r_o} C_1\left\{T_s + C_2\left[1 - \left(r/r_o\right)^2\right]\right\}rdr$$

$$= \frac{2}{r_o^2}\int_0^{r_o}\left\{T_s + C_2\left[1 - \left(r/r_o\right)^2\right]\right\}rdr$$

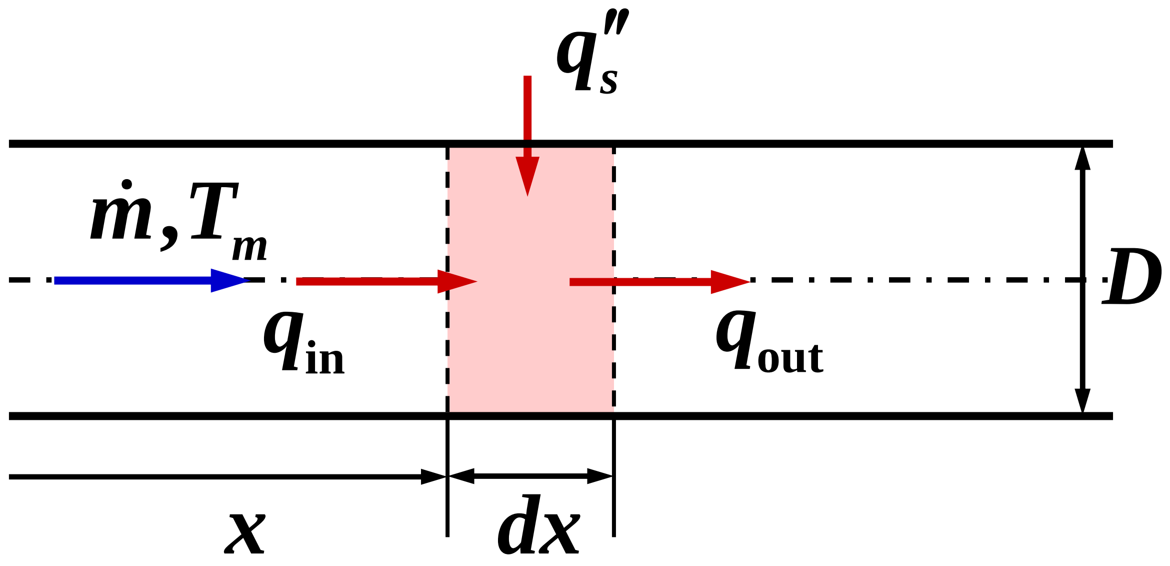
$$= \frac{2}{r_o^2}\left(T_s\frac{r_o^2}{2} + \frac{C_2}{2}r_o^2 - \frac{C_2}{4}r_o^2\right) = T_s + \frac{C_2}{2}$$

$$q_s'' = k\left.\frac{\partial T}{\partial r}\right|_{r=r_o} = -kC_2 2\left.\frac{r}{r_o^2}\right|_{r=r_o} = -2C_2\frac{k}{r_o}$$

$$h = \frac{q_s''}{T_s - T_m} = \frac{-2C_2 k/r_o}{-C_2/2} = \frac{4k}{r_o}$$

Thus $\mathrm{Nu}_D = \frac{hD}{k} = 8$

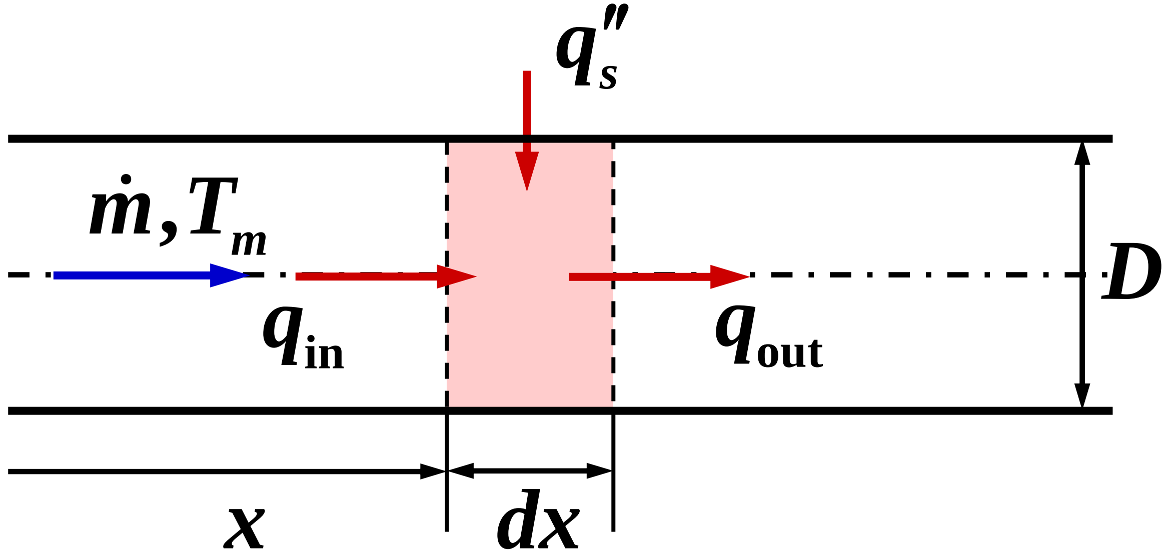
Overall Energy Balance



perimeter: $P = \pi D$

$$\dot{m} c_p T_m + q_s'' P dx = \dot{m} c_p T_m + \frac{d}{dx}\left(\dot{m} c_p T_m\right) dx$$

$$q_s'' P = \frac{d}{dx}\left(\dot{m} c_p T_m\right) \equiv \frac{dq_{\text{conv}}}{dx}$$



$$\int_i^o dq_{\text{conv}} = \int_i^o d\left(\dot{m} c_p T_m\right)$$

or $\quad q_{\text{conv}} = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right)$

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = \frac{P}{\dot{m} c_p} h\left(T_s - T_m\right)$$

Constant surface heat flux condition

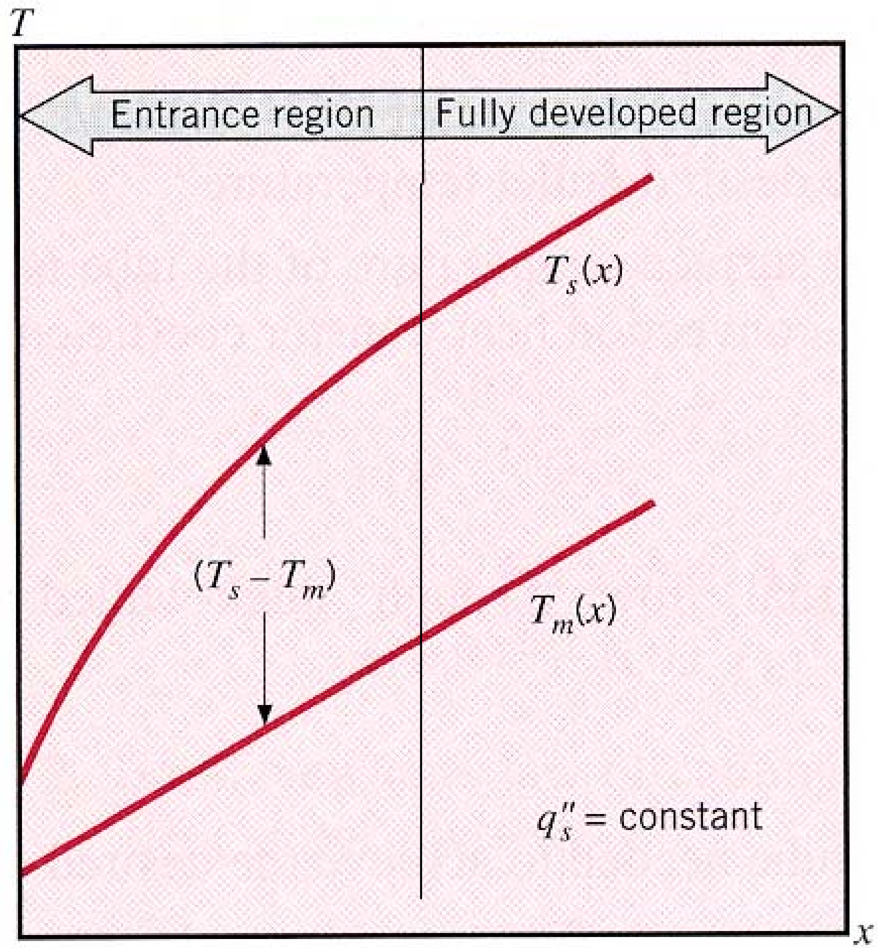
$$\frac{dT_m}{dx}=\frac{q_s''P}{\dot{m}c_p}=\frac{P}{\dot{m}c_p}h\left(T_s-T_m\right),\ q_s''=\text{const.}$$

$$dT_m=\frac{q_s''P}{\dot{m}c_p}dx$$

$$\int_0^x dT_m=\int_0^x\frac{q_s''P}{\dot{m}c_p}dx$$

$$T_m(x)=T_{m,i}+\frac{q_s''P}{\dot{m}c_p}x$$

$$T_s(x)=T_m(x)+\frac{q_s''}{h}$$



$$T_s(x)-T_m(x)=\frac{q_s''}{h}$$ = **constant in fully developed region**

Constant surface temperature condition

$$\frac{dT_m}{dx} = \frac{P}{\dot{m}c_p} h\left(T_s - T_m\right) \quad \text{or} \quad \frac{dT_m}{T_m - T_s} = -\frac{P}{\dot{m}c_p} h dx$$

$$\int_0^x \frac{dT_m}{T_m - T_s} = -\frac{P}{\dot{m}c_p} \int_0^x h dx$$

$$\left[\ln\left(T_m - T_s\right)\right]_0^x = -\frac{P}{\dot{m}c_p} \int_0^x h dx$$

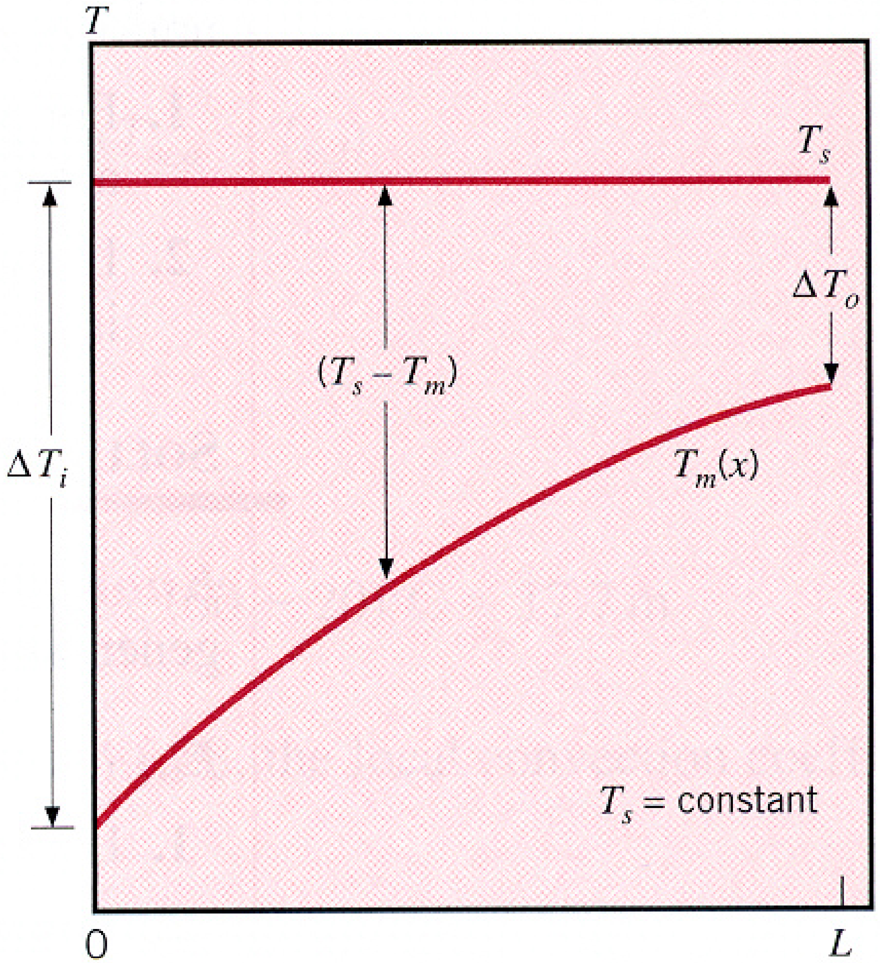
$$= -\frac{Px}{\dot{m}c_p} \frac{1}{x} \int_0^x h dx = -\frac{Px}{\dot{m}c_p} \bar{h}_x$$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \frac{\Delta T}{\Delta T_i} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}_x \right)$$

For a tube of length L

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \frac{\Delta T_o}{\Delta T_i}$$

$$= \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}_L \right)$$



Log Mean Temperature Difference (LMTD)

$$q_{\text{conv}} = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right)$$

$$= \dot{m}c_p\left[\left(T_s - T_{m,i}\right) - \left(T_s - T_{m,o}\right)\right]$$

Let $\Delta T_i = T_s - T_{m,i}$, $\Delta T_o = T_s - T_{m,o}$,

then $q_{\text{conv}} = \dot{m}c_p\left(\Delta T_i - \Delta T_o\right)$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \frac{\Delta T_o}{\Delta T_i} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}_L\right)$$

$$-\frac{PL}{\dot{m}c_p}\bar{h}_L = \ln\frac{\Delta T_o}{\Delta T_i} \rightarrow \dot{m}c_p = \frac{PL\bar{h}_L}{\ln(\Delta T_i / \Delta T_o)}$$

$$q_{\text{conv}} = \dot{m}c_p\left(\Delta T_i - \Delta T_o\right)$$

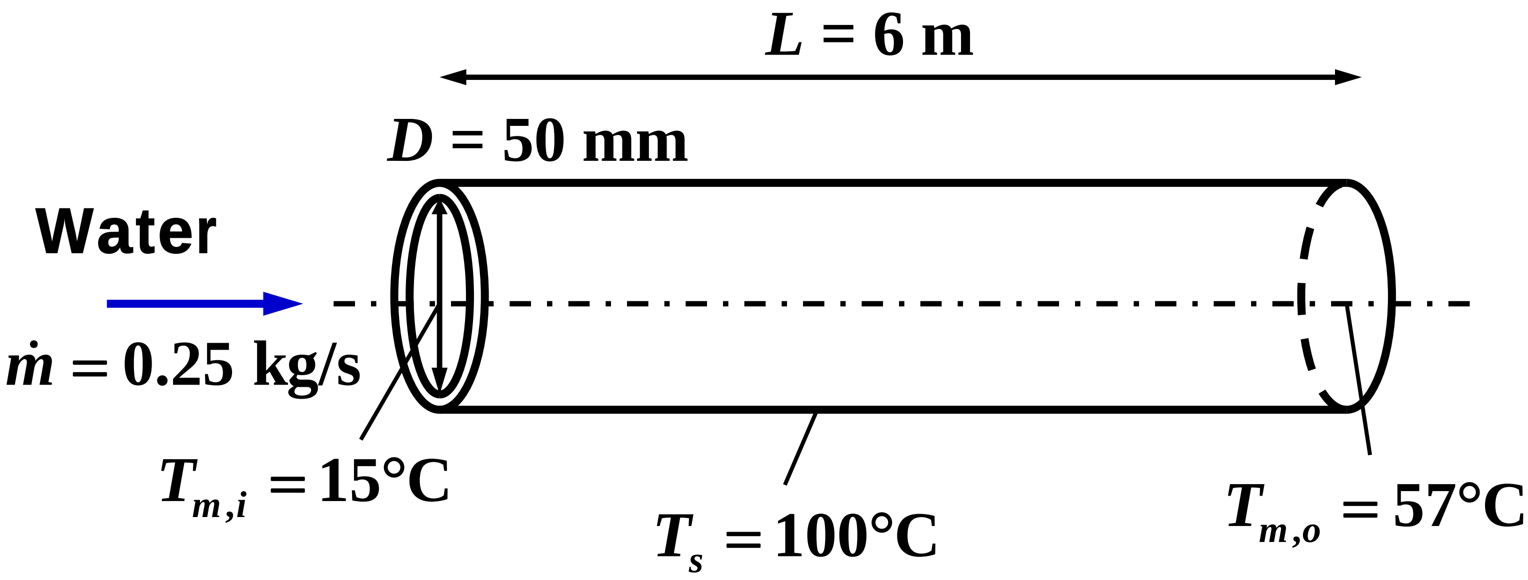
$$\dot{m}c_p = \frac{PL\bar{h}_L}{\ln(\Delta T_i / \Delta T_o)}$$

$$q_{\text{conv}} = \frac{PL\bar{h}_L}{\ln\left(\Delta T_i / \Delta T_o\right)}\left(\Delta T_i - \Delta T_o\right)$$

$$= PL\bar{h}_L \frac{\Delta T_o - \Delta T_i}{\ln\left(\Delta T_o / \Delta T_i\right)} \equiv \bar{h}_L A_s \Delta T_{\text{lm}}$$

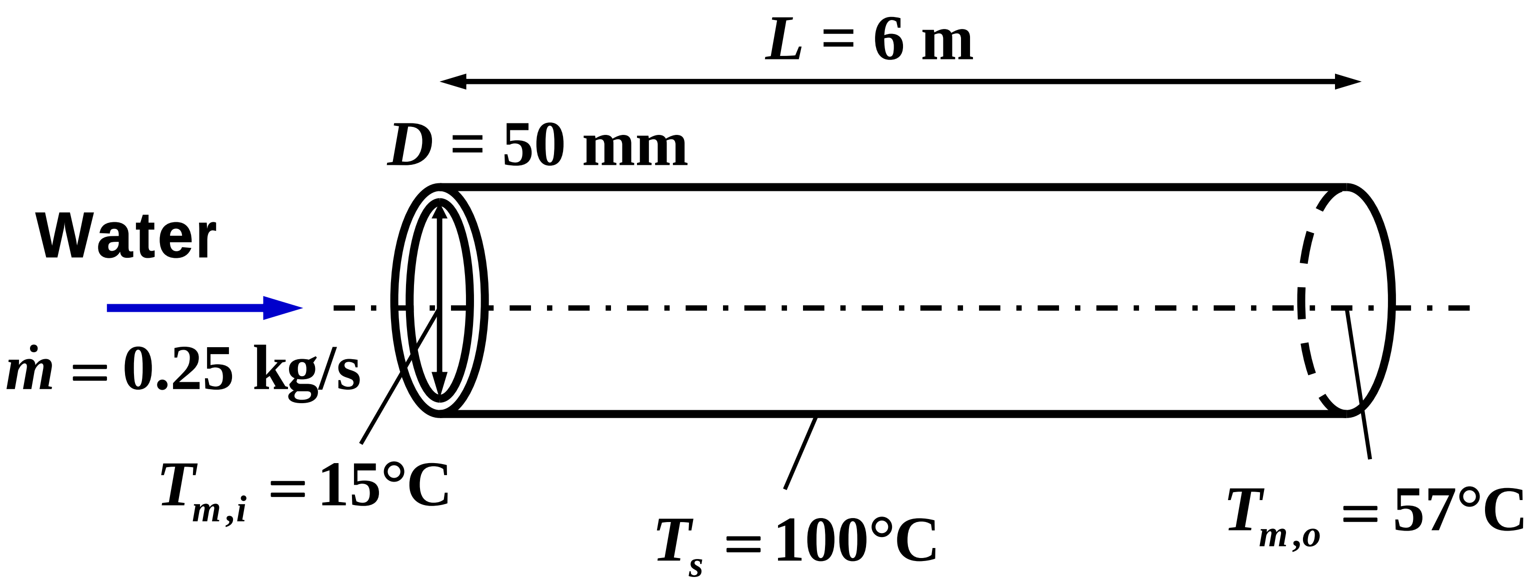
$$\Delta T_{\text{lm}} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}$$

Example 8.3



Find:

Average convection heat transfer coefficient



$$q_{\text{conv}} = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = \bar{h}A_s\Delta T_{\text{lm}} \rightarrow \bar{h} = \frac{\dot{m}c_p\left(T_{m,o} - T_{m,i}\right)}{A_s\Delta T_{\text{lm}}}$$

$$\Delta T_{\text{lm}} = \frac{\left(T_s - T_{m,o}\right) - \left(T_s - T_{m,i}\right)}{\ln\left[\left(T_s - T_{m,o}\right)/\left(T_s - T_{m,i}\right)\right]} = 61.6°\text{C}$$

$$\bar{h} = \frac{0.25 \times 4178 \times \left(57 - 15\right)}{\pi \times 0.05 \times 6 \times 61.6} = 756\ \text{W/m}^2\text{K}$$

Convection Correlations

Laminar Flow in Fully Developed Region

in the cylindrical coordinate system (r,x)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial x^2}\right]$$

for fully developed flow

$$v=0\rightarrow u\frac{\partial T}{\partial x}=\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial x^2}\right]$$

$$u(r)=2u_m\left(1-\frac{r^2}{r_o^2}\right)$$

Scaling u, T, r, x

$$u\frac{\partial T}{\partial x} \sim \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \quad u_m \frac{\Delta T}{x} \sim \alpha \frac{\Delta T}{r_o^2}, \quad x \sim \frac{u_m r_o^2}{\alpha}$$

$$x^* = \frac{x\alpha}{u_m r_o^2} = \frac{x}{r_o}\frac{\alpha}{\nu}\frac{\nu}{u_m r_o} = \frac{x/r_o}{\mathrm{Re}_{r_o}\,\mathrm{Pr}},$$

$$\theta = \frac{T_s - T}{T_s - T_{m,i}}, \quad u^* = \frac{u}{u_m}, \quad r^* = \frac{r}{r_o}$$

Then, $$\frac{u^*}{2}\frac{\partial \theta}{\partial x^*} = \frac{1}{\mathrm{Re}_{r_o}^2\,\mathrm{Pr}^2}\frac{\partial^2\theta}{\partial x^{*2}} + \frac{\partial^2\theta}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial \theta}{\partial r^*}$$

When $\mathrm{Re}_{r_o}\,\mathrm{Pr} = \mathrm{Pe} > 100,$ $$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Constant surface heat flux condition

$$u\frac{\partial T}{\partial x}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \quad \frac{\partial T}{\partial x}=\frac{dT_s}{dx}=\frac{dT_m}{dx}$$

Thus, $u\dfrac{dT_m}{dx}=\dfrac{\alpha}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right)$

or $2u_m\left(1-\dfrac{r^2}{r_o^2}\right)\dfrac{dT_m}{dx}=\dfrac{\alpha}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right)$

boundary conditions

$$T(r_o,x)=T_s(x), \quad \left.\frac{\partial T}{\partial r}\right|_{r=0}=0 \qquad \left(q_s''=k\left.\frac{\partial T}{\partial r}\right|_{r=r_o}\right)$$

from overall energy balance :

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = \frac{4 q_s''}{\rho u_m D c_p} = \text{const.}$$

Integrating energy equation with boundary conditions

$$T(r,x) = T_s(x) - \frac{2u_m}{\alpha}\frac{dT_m}{dx}\left(\frac{3}{16}r_s^2 + \frac{1}{16}\frac{r^4}{r_s^2} - \frac{r^2}{4}\right)$$

$$T_m(x) = \frac{2}{r_o^2 u_m}\int_0^{r_o} uTrdr = T_s(x) - \frac{11}{96}\frac{2u_m}{\alpha}\frac{dT_m}{dx}r_o^2$$

$$= T_s(x) - \frac{11}{24}\frac{q_s'' r_o}{k}$$

$$q_s'' = h\left(T_s - T_m\right) = h \cdot \frac{11}{24}\frac{q_s'' r_o}{k}$$

$$h = \frac{24}{11}\frac{k}{r_o} = \frac{48}{11}\frac{k}{D}$$

$$\mathrm{Nu}_D = \frac{hD}{k} = \frac{48}{11} = 4.364$$

Constant surface temperature condition

$$\frac{\partial T}{\partial x}=\frac{T_s-T}{T_s-T_m}\frac{dT_m}{dx},\quad u\frac{T_s-T}{T_s-T_m}\frac{dT_m}{dx}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

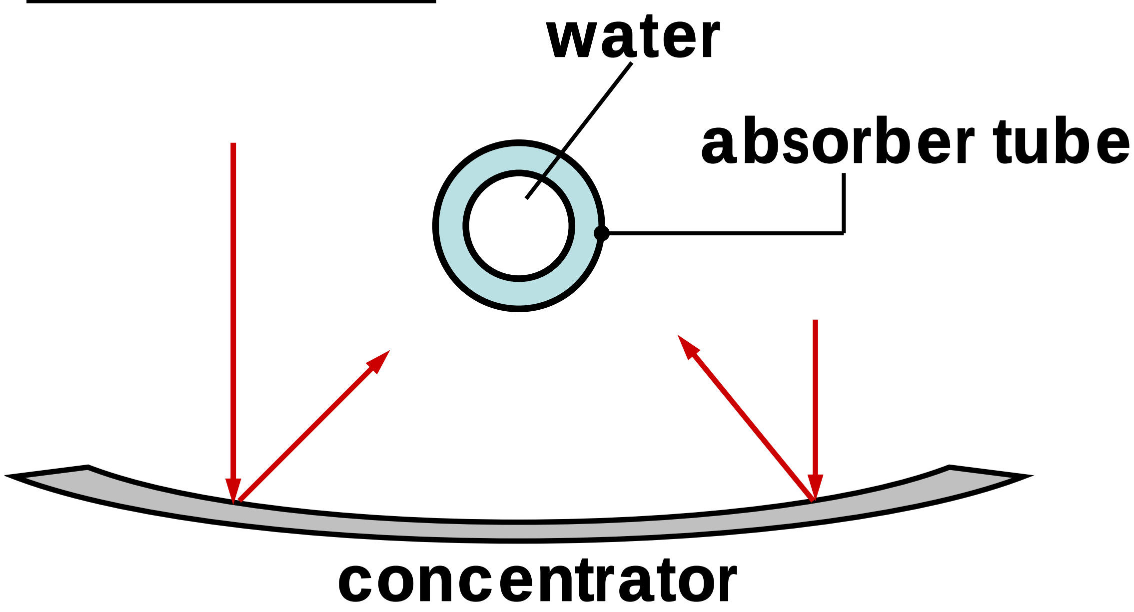
$$2u_m\left(1-\frac{r^2}{r_o^2}\right)\frac{T_s-T}{T_s-T_m}\frac{dT_m}{dx}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Assume $$\frac{T_s-T}{T_s-T_m}=\sum_{n=0}^{\infty}C_{2n}\left(\frac{r}{r_o}\right)^{2n}$$

$$C_0=1,\ C_2=-\frac{\lambda_0^2}{4}=-1.828397,\ C_{2n}=\frac{\lambda_0^2}{(2n)^2}\left(C_{2n-4}-C_{2n-2}\right)$$

$$\lambda_0=2.704364 \qquad \mathrm{Nu}_D=\frac{hD}{k}=\frac{\lambda_0^2}{2}=3.657$$

Example 8.4



Conditions:

$q_s'' = 2000\ \mathrm{W/m^2}$

$\dot{m} = 0.01\ \mathrm{kg/s}$

$D = 60\ \mathrm{mm}$

$T_{m,i} = 20°\mathrm{C}$

$T_{m,o} = 80°\mathrm{C}$

Find:

1) Length of tube L to achieve required heating
2) Surface temperature $T_{s,o}$ at the outlet section, $x = L$

Assumptions:

1) Incompressible flow with constant properties
2) Fully developed conditions at tube outlet

1) From energy balance

$$q_{\text{conv}} = \dot{m} c_p \left(T_{m,o} - T_{m,i} \right) = q_s'' \left(\pi D L \right)$$

$$L = \frac{\dot{m} c_p \left(T_{m,o} - T_{m,i} \right)}{\pi D q_s''} = 6.65 \text{ m}$$

Conditions:

$$q_s'' = 2000 \text{ W/m}^2$$

$$\dot{m} = 0.01 \text{ kg/s}$$

$$D = 60 \text{ mm}$$

$$T_{m,i} = 20°\text{C}$$

$$T_{m,o} = 80°\text{C}$$

2) surface temperature at the outlet

$$q_s'' = h \left(T_{s,o} - T_{m,o} \right) \rightarrow T_{s,o} = \frac{q_s''}{h} + T_{m,o}$$

$$\text{Re}_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.01}{\pi \times 0.06 \times 352 \times 10^{-6}} = 603 \rightarrow$$ **laminar flow**

fully developed condition $\text{Nu}_D = \frac{hD}{k} = 4.36$

$$h = 4.36 \frac{D}{k} = 48.7 \text{ W/m}^2\text{K}$$

$$T_{s,o} = \frac{q_s''}{h} + T_{m,o} = \frac{2000}{48.7} + 80 = 121°\text{C}$$

Laminar Flow in the Entry Region

Thermal entry length

$$\overline{\mathrm{Nu}_D} = 3.66 + \frac{0.0668(D/L)\mathrm{Re}_D\,\mathrm{Pr}}{1+0.04\left[(D/L)\mathrm{Re}_D\,\mathrm{Pr}\right]^{2/3}}$$

T_s = constant,

$$\overline{\mathrm{Nu}_D} = \frac{\bar{h}D}{k}, \quad q = \bar{h}A_s\Delta T_{\mathrm{lm}}$$

properties at $\bar{T}_m = \left(T_{m,i} + T_{m,o}\right)/2$

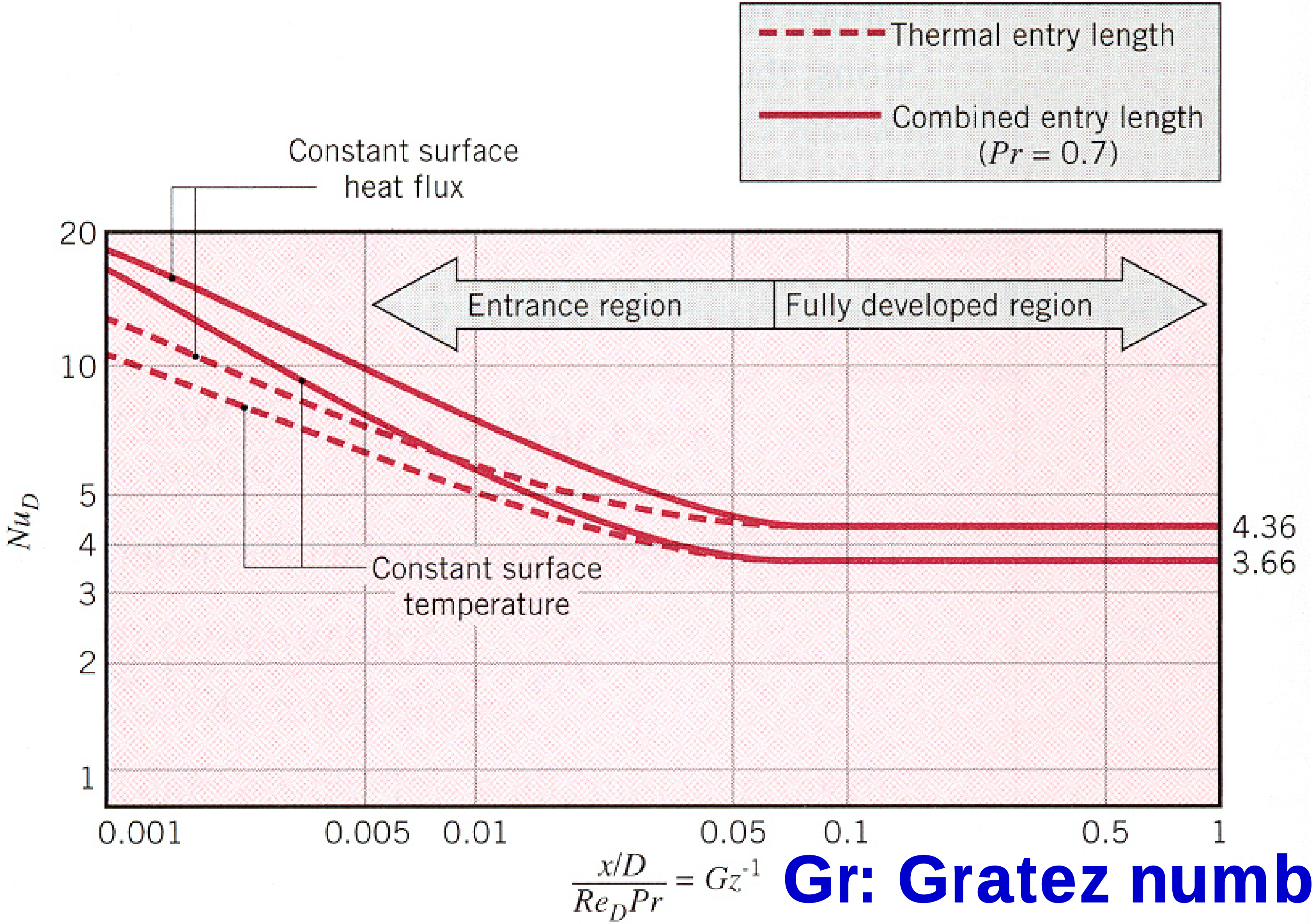
Combined entry length
(hydrodynamic + thermal)

$$\overline{\mathrm{Nu}_D} = 1.86\left(\frac{\mathrm{Re}_D\,\mathrm{Pr}}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$\left[\begin{array}{c} 0.48 < \mathrm{Pr} < 16{,}700 \\ 0.0044 < \left(\dfrac{\mu}{\mu_s}\right) < 9.75 \end{array}\right]$$

T_s = constant

properties at $\bar{T}_m = \left(T_{m,i} + T_{m,o}\right)/2$



Gr: Gratez number

FIGURE 8.9 Local Nusselt number obtained from entry length solutions for laminar flow in a circular tube [2]. Adapted with permission.

Turbulent flow in circular tubes

Fully developed region

- **Dittus-Boelter equation (smooth wall)**

$$\mathrm{Nu}_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n$$

$$n = 0.4\ \left(T_s > T_m\right),\ n = 0.3\ \left(T_s < T_m\right)$$

$$0.7 \le \mathrm{Pr} \le 160,\ \ \mathrm{Re}_D \ge 10{,}000,\ \ \frac{L}{D} \ge 10$$

moderate temperature difference

properties at T_m

- **Sieder & Tate (1936)**

large temperature difference

$$\mathrm{Nu}_D = 0.027\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$0.7 \le \mathrm{Pr} \le 16{,}700,\ \ \mathrm{Re}_D \ge 10{,}000,\ \ \frac{L}{D} \ge 10$$

properties at T_m

- **Petukhov (1970)**

$$\mathrm{Nu}_D = \frac{(f/8)\mathrm{Re}_D\,\mathrm{Pr}}{1.07 + 12.7(f/8)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}$$

f **from Moody diagram**

$$0.5 < \mathrm{Pr} \le 2{,}000,\ \ 10^4 < \mathrm{Re}_D < 5 \times 10^6$$

properties at T_m

- **Gnielinski (1976)**

$$\mathrm{Nu}_D = \frac{(f/8)(\mathrm{Re}_D - 1000)\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}$$

$$0.5 < \mathrm{Pr} \le 2{,}000,\ \ 3{,}000 < \mathrm{Re}_D < 5 \times 10^6$$

properties at T_m

Entry region

$$\overline{\mathrm{Nu}}_D = \mathrm{Nu}_{D,\mathrm{fd}}$$ since $10 \le (x_{\mathrm{fd}}/D) \le 60$

For short tubes : $$\frac{\overline{\mathrm{Nu}}_D}{\mathrm{Nu}_{D,\mathrm{fd}}} = 1 + \frac{C}{(x/D)^m}$$

Liquid metals

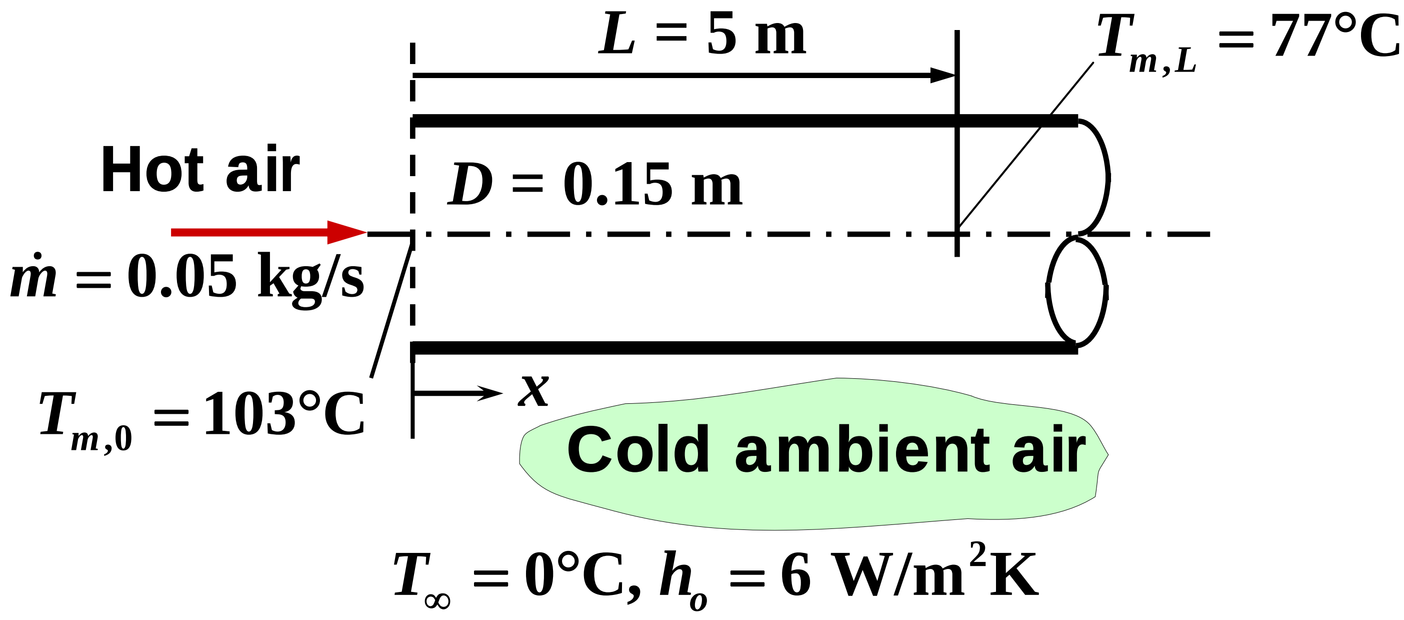
fully developed turbulent flow

$$\mathrm{Nu}_D = 4.82 + 0.0185\,\mathrm{Pe}_D^{0.827}, \quad q_s'' = \text{constant}$$

$$\left[\begin{array}{l} 3.6\times10^3 < \mathrm{Re}_D < 9.05\times10^5 \\ 10^2 < \mathrm{Pe}_D < 10^4 \end{array}\right]$$

$$\mathrm{Nu}_D = 5.0 + 0.025\,\mathrm{Pe}_D^{0.8}, \quad T_s = \text{constant}$$

Example 8.6



Find:

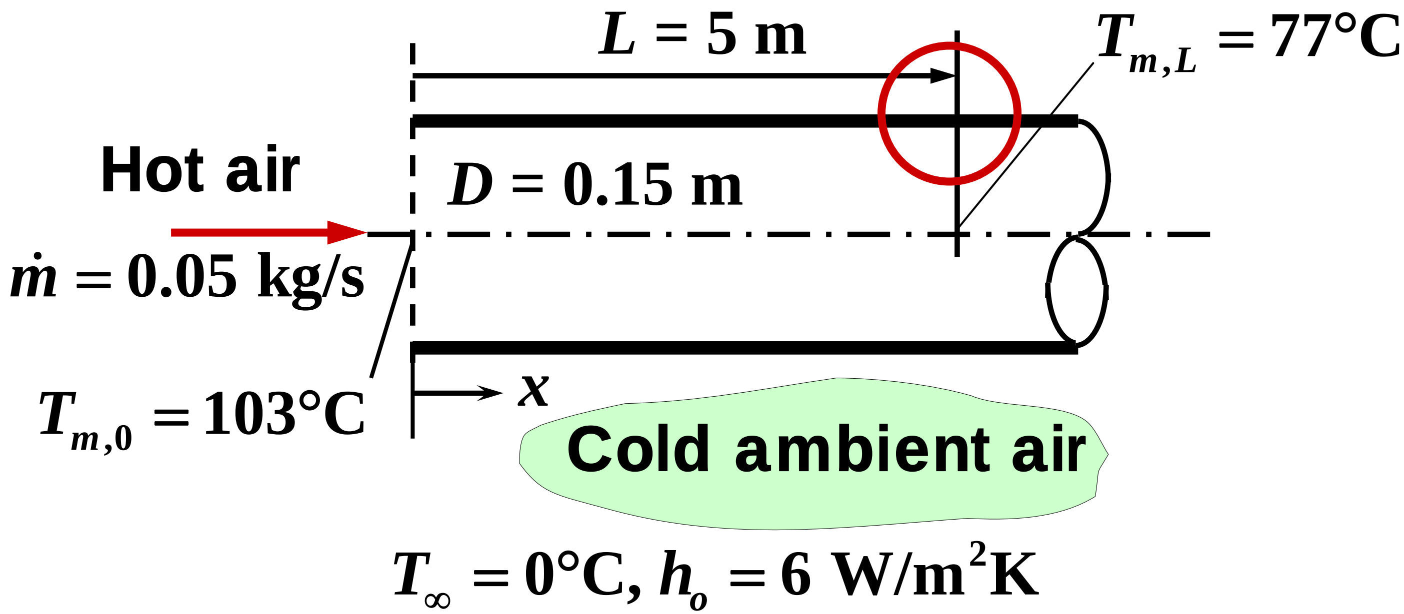
1) **Heat loss from the duct over the length** L, q[W]

2) **Heat flux** $q_s''(L)$ **and surface temperature** $T_{s,L}$ **at** $x = L$

Assumptions:

1) **Steady-state, constant properties, ideal gas behavior**

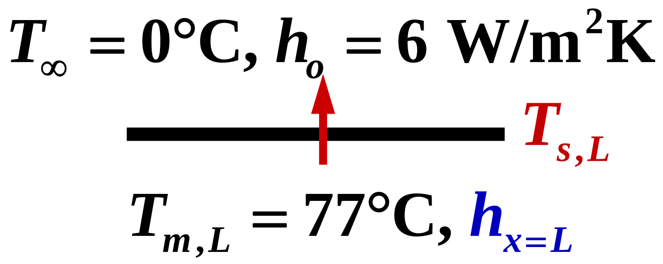
2) **Uniform convection coefficient at outer surface of duct**

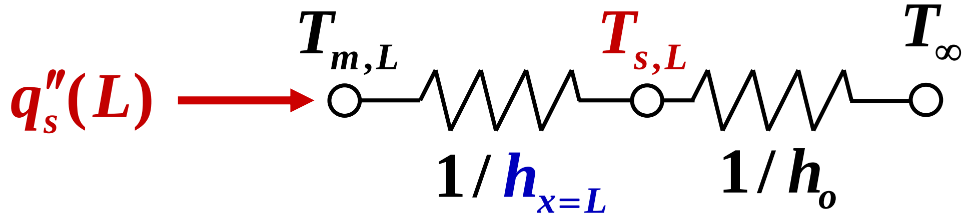


1) Energy balance for the entire duct :

$$q = \dot{m} c_p \left(T_{m,L} - T_{m,0}\right) = 0.05 \times 1010 \times \left(77 - 103\right) = -1313 \;\; \mathrm{W}$$

2) Heat flux and surface temperature at $x = L$





$$q_s''(L) = \frac{T_{m,L} - T_\infty}{1/h_{x=L} + 1/h_o}$$

or $$q_s''(L) = h_{x=L}\left(T_{m,L} - T_{s,L}\right) = h_o\left(T_{s,L} - T_\infty\right)$$

$$\mathrm{Re}_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.05}{\pi\times 0.15\times 208\times 10^{-7}} = 20{,}404$$ **turbulent flow**

$$L/D = 5/0.15 = 33.3 > 10$$ **fully developed region**

$$\mathrm{Nu}_D = \frac{h_{x=L}D}{k} = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{0.3} = 57.9$$

$$h_{x=L} = \mathrm{Nu}_D\frac{k}{D} = 57.9\times\frac{0.03}{0.15} = 11.6\ \mathrm{W/m^2K}$$

Hence, $$q_s''(L) = \frac{T_{m,L}-T_\infty}{1/h_{x=L}+1/h_o} = \frac{77-0}{1/11.6+1/6.0} = 304.5\ \mathrm{W/m^2}$$

$$q_s''(L) = h_{x=L}\left(T_{m,L}-T_{s,L}\right) = h_o\left(T_{s,L}-T_\infty\right)$$

$$T_{s,L} = T_{m,L} - \frac{q_s''(L)}{h_{x=L}} = 77 - \frac{304.5}{11.6} = 50.7°\mathrm{C}$$

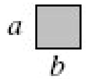
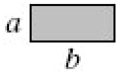
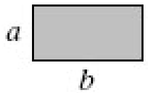
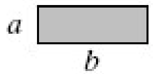
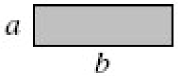
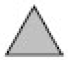
Convection Correlations: Noncircular Tube

hydraulic diameter : $D_h \equiv \dfrac{4A_c}{P}$

A_c **: flow cross-sectional area**

P **: wetted perimeter**

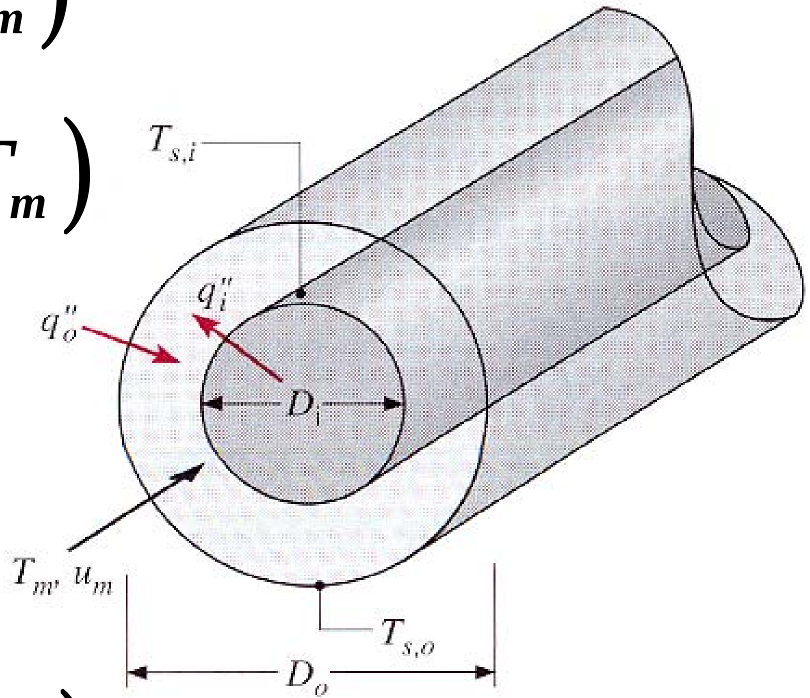
Nusselt numbers and friction factors for fully developed laminar flow in tubes of different cross section

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q''_s)	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	8.0	6.49	5.60	82
	∞	8.23	7.54	96
 Heated / Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

Concentric Tube Annulus

inner wall : $q_i'' = h_i\left(T_{s,i} - T_m\right)$

outer wall : $q_o'' = h_o\left(T_{s,o} - T_m\right)$



Nusselt numbers

$$\mathrm{Nu}_i \equiv \frac{h_i D_h}{k}, \quad \mathrm{Nu}_o \equiv \frac{h_o D_h}{k}$$

$$D_h = \frac{4A_c}{P} = \frac{4(\pi/4)\left(D_o^2 - D_i^2\right)}{\pi D_o + \pi D_i} = D_o - D_i$$

$$\mathrm{Nu}_i = \frac{\mathrm{Nu}_{ii}}{1-\left(q_o''/q_i''\right)\theta_i^*}, \quad \mathrm{Nu}_o = \frac{\mathrm{Nu}_{oo}}{1-\left(q_i''/q_o''\right)\theta_o^*}$$

For fully developed laminar flow

1) One surface insulated and the other at constant temperature

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
≈1.00	4.86	4.86

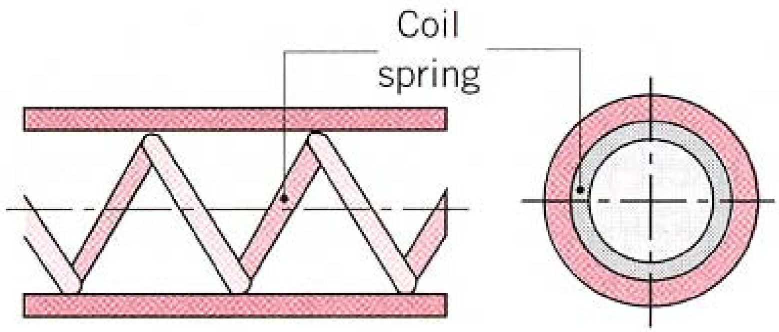
2) Uniform heat flux maintained at both surfaces

D_i/D_o	Nu_{ii}	Nu_{oo}	θ_i^*	θ_o^*
0	—	4.364	∞	0
0.05	17.81	4.792	2.18	0.0294
0.10	11.91	4.834	1.383	0.0562
0.20	8.499	4.833	0.905	0.1041
0.40	6.583	4.979	0.603	0.1823
0.60	5.912	5.099	0.473	0.2455
0.80	5.58	5.24	0.401	0.299
1.00	5.385	5.385	0.346	0.346

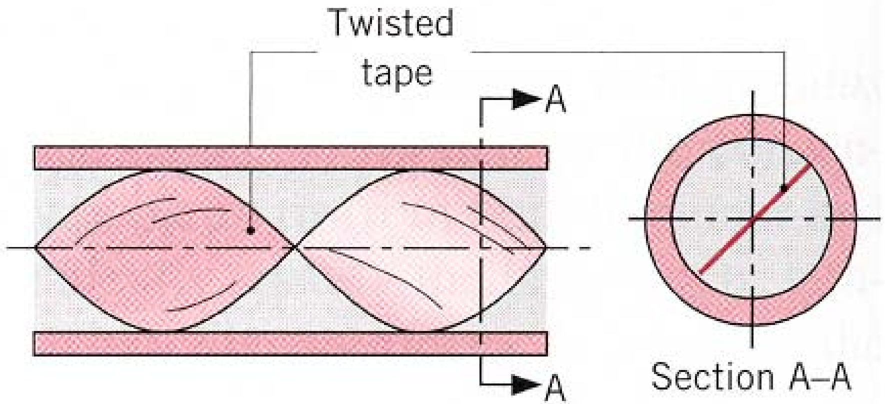
$$\mathrm{Nu}_i = \frac{\mathrm{Nu}_{ii}}{1-\left(q_o''/q_i''\right)\theta_i^*},\quad \mathrm{Nu}_o = \frac{\mathrm{Nu}_{oo}}{1-\left(q_i''/q_o''\right)\theta_o^*}$$

Heat Transfer Enhancement

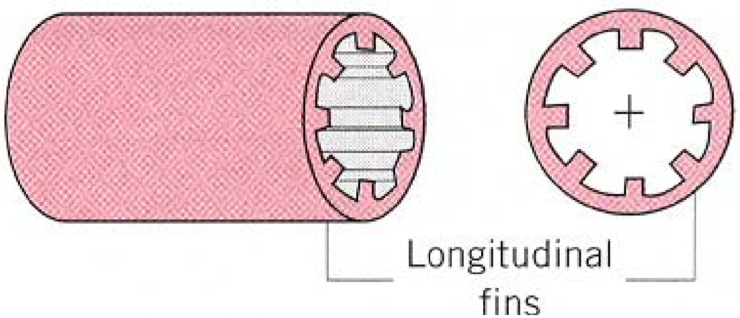
Internal flow heat transfer enhancement schemes



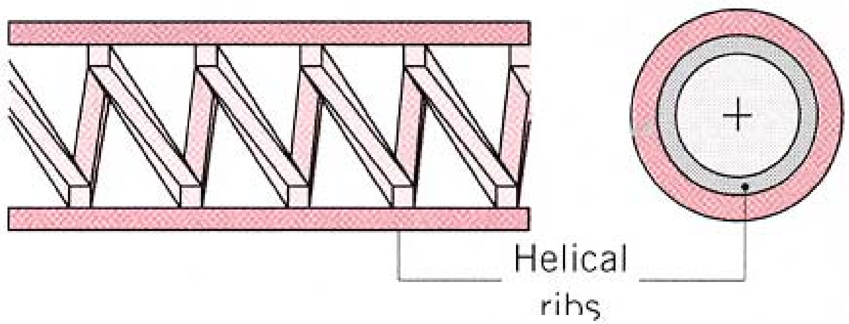
Coil-spring wire insert



Twisted tape insert



Longitudinal fins



Helical ribs

Helically coiled tube and secondary flow