NATURAL CONVECTION

- **Laminar Free Convection on a Vertical Surface**
- **Empirical Correlations: External Free Convection Flows**
- **Free Convection within Parallel Plate Channels**
- **Empirical Correlations: Enclosures**

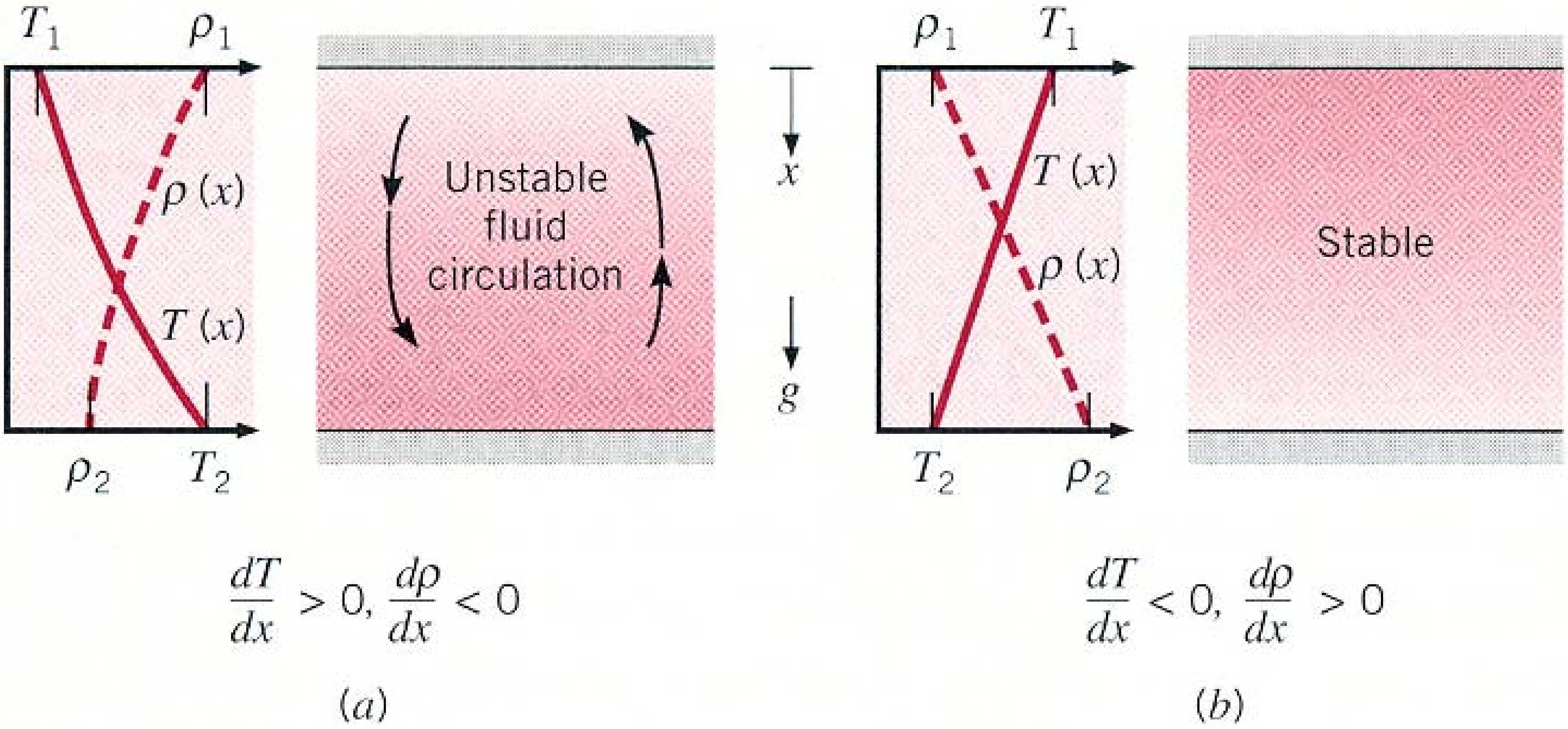


FIGURE 9.1 Conditions in a fluid between large horizontal plates at different temperatures. (*a*) Unstable temperature gradient. (*b*) Stable temperature gradient.

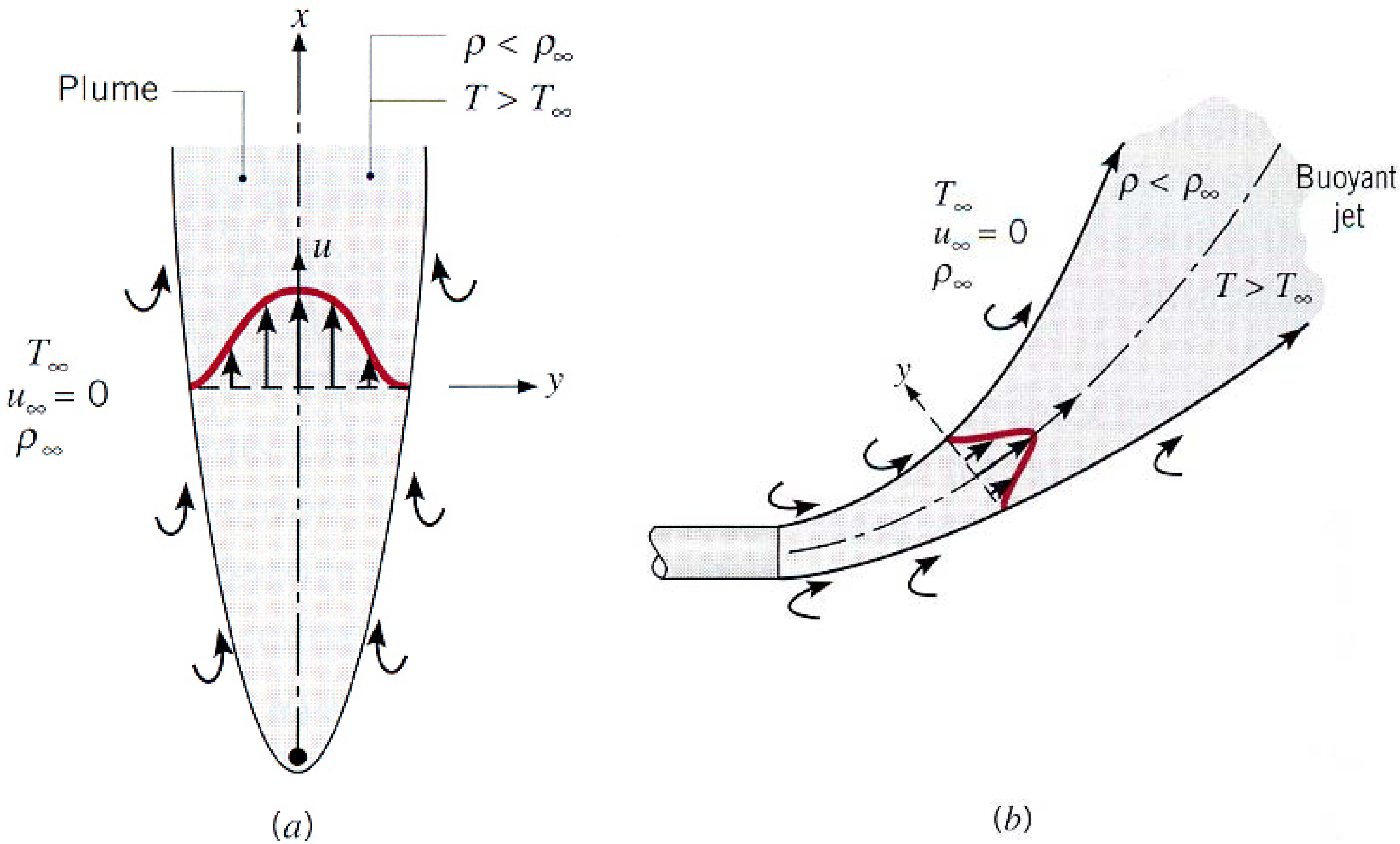
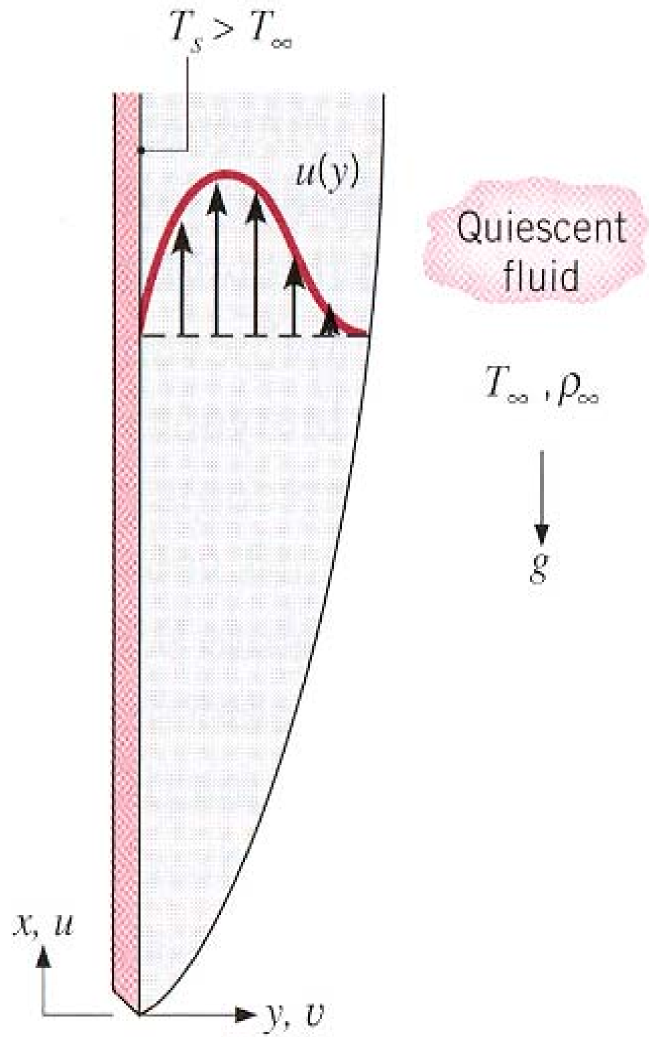


FIGURE 9.2 Buoyancy-driven free boundary layer flows in an extensive, quiescent medium. (*a*) Plume formation above a heated wire. (*b*) Buoyant jet associated with a heated discharge.

Laminar Free Convection on a Vertical Plate

Boussinesq approximation

Basic idea : In many problems, $\delta\rho/\rho_\infty << 1$. This makes the velocity field effectively solenoidal $\left(\nabla \cdot \vec{u} = 0\right)$ and also means that wherever the density appears in the momentum equation, it can be replaced by ρ_∞ except where it multiplies the body force



Free-convection boundary layers

$$\rho(T,p) = \rho_\infty + \left.\frac{\partial \rho}{\partial T}\right|_\infty (T - T_\infty) + \left.\frac{\partial \rho}{\partial p}\right|_\infty (p - p_\infty) + \cdots$$

$$\frac{\rho - \rho_\infty}{\rho_\infty} = \frac{1}{\rho_\infty}\left.\frac{\partial \rho}{\partial T}\right|_\infty (T - T_\infty) + \frac{1}{\rho_\infty}\left.\frac{\partial \rho}{\partial p}\right|_\infty (p - p_\infty) + \cdots$$

Ex) water at $15^\circ\mathrm{C}$ and 1 atm

thermal expansion coefficient : $\beta = -\frac{1}{\rho_\infty}\left.\frac{\partial \rho}{\partial T}\right|_\infty = 1.5 \times 10^{-4}\ \mathrm{K}^{-1}$

isothermal compressibility : $\kappa = \frac{1}{\rho_\infty}\left.\frac{\partial \rho}{\partial p}\right|_\infty = 5 \times 10^{-5}\ \mathrm{bar}^{-1}$

When $T - T_\infty = 10\ \mathrm{K}$, $p - p_\infty = 1\,\mathrm{bar}$,

the contribution of $T - T_\infty$ to $\delta\rho/\rho_\infty$ is 1.5×10^{-3},

whilst the contribution of $p - p_\infty$ to $\delta\rho/\rho_\infty$ is only 5×10^{-5}.

$$\frac{\rho-\rho_\infty}{\rho_\infty}=\frac{1}{\rho_\infty}\frac{\partial\rho}{\partial T}\bigg|_\infty\left(T-T_\infty\right)+\frac{1}{\rho_\infty}\frac{\partial\rho}{\partial p}\bigg|_\infty\left(p-p_\infty\right)+\cdots$$

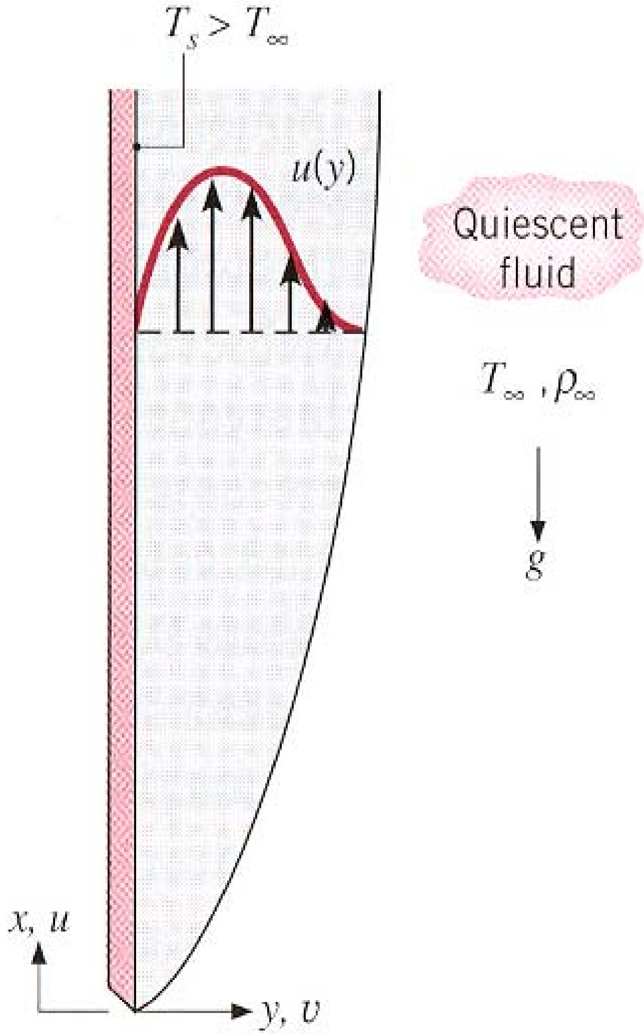
$$\approx-\beta\left(T-T_\infty\right)$$

Thus, $\rho=\rho_\infty\left[1-\beta\left(T-T_\infty\right)\right]$

$$p=-\rho_\infty gx+p_d$$

When the dynamic pressure is negligible,

$$\frac{dp}{dx}=-\rho_\infty g$$



$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{dp}{dx}+\mu\frac{\partial^2 u}{\partial y^2}-\rho g$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\rho_\infty g+\mu\frac{\partial^2 u}{\partial y^2}-\rho_\infty\left[1-\beta\left(T-T_\infty\right)\right]g$$

$$\rho_\infty\left[1-\beta\left(T-T_\infty\right)\right]u\frac{\partial u}{\partial x}+\rho_\infty\left[1-\beta\left(T-T_\infty\right)\right]v\frac{\partial u}{\partial y}$$

$$=\mu\frac{\partial^2 u}{\partial y^2}+\rho_\infty g\beta\left(T-T_\infty\right)$$

When $\beta(T-T_\infty)<<1,$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)$$

Similarity Solution to 2-D Boundary Layer Equations with Boussinesq Approximation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\left(T-T_\infty\right)+\nu\frac{\partial^2 u}{\partial y^2}$$

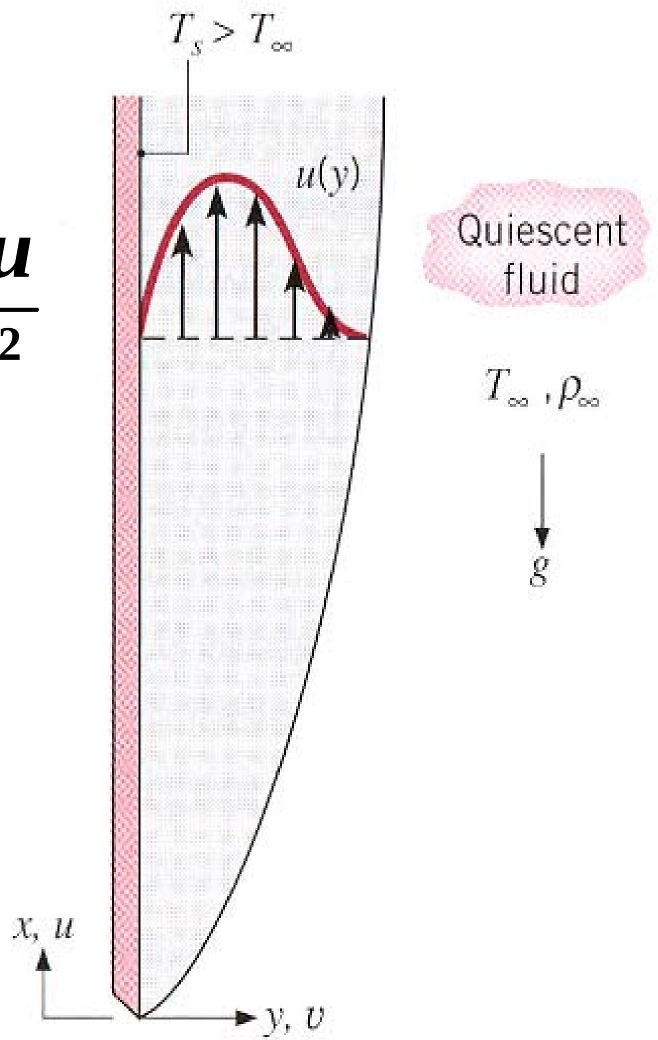
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

boundary conditions

$u,v=0,\ T=T_s$ **at** $y=0$

$u=0,\ T=T_\infty$ **at** $x=0$

$u\rightarrow 0,\ T\rightarrow T_\infty$ **as** $y\rightarrow\infty$



$$u = \frac{\partial \psi}{\partial y}, \;\; v = -\frac{\partial \psi}{\partial x}, \;\; \theta = \frac{T - T_\infty}{T_s - T_\infty}$$

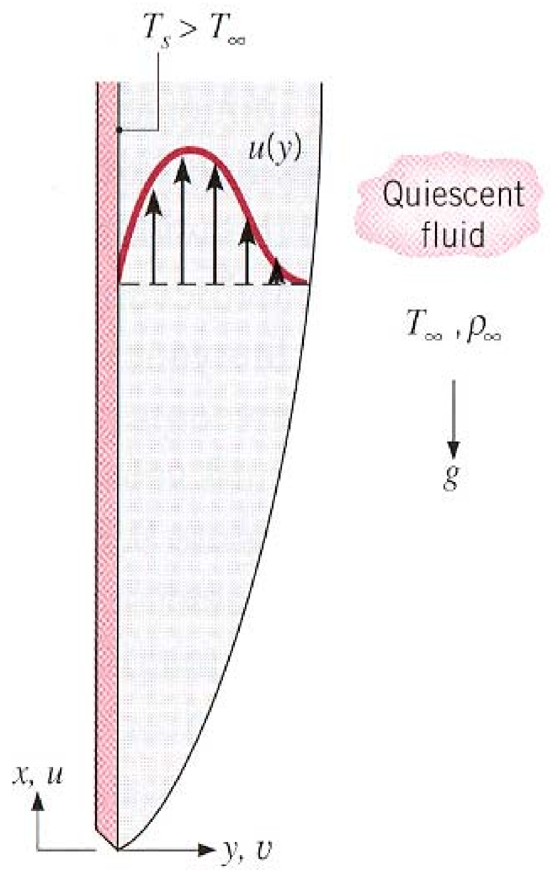
Scaling : $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta\left(T - T_\infty\right) + \nu\frac{\partial^2 u}{\partial y^2}$

$$u\frac{\partial u}{\partial x} \sim g\beta\left(T - T_\infty\right) \sim \nu\frac{\partial^2 u}{\partial y^2}$$

$$\rightarrow u_0\frac{u_0}{x} \sim g\beta\left(T - T_\infty\right) \sim \nu\frac{u_0}{\delta^2}$$

$$\delta^2 \sim \frac{\nu u_0}{g\beta\left(T_s - T_\infty\right)}, \;\; u_0 \sim \frac{\nu x}{\delta^2}$$

$$\rightarrow \delta^2 \sim \frac{\nu}{g\beta\left(T_s - T_\infty\right)}\frac{\nu x}{\delta^2}$$



$$\delta^4 \sim \frac{v^2 x}{g\beta\left(T_s - T_\infty\right)} \quad \text{or} \quad \delta = \left(\frac{v^2 x}{g\beta\left(T_s - T_\infty\right)}\right)^{1/4}$$

similarity variable :

$$\eta = \frac{y}{\delta} \sim y\left(\frac{g\beta\left(T_s - T_\infty\right)}{v^2 x}\right)^{1/4}$$

$$= \frac{y}{x}\left(\frac{g\beta\left(T_s - T_\infty\right)x^3}{v^2}\right)^{1/4} \equiv \frac{y}{x}\mathrm{Gr}_x^{1/4}$$

Grashof number : $\mathrm{Gr}_x = \dfrac{g\beta\left(T_s - T_\infty\right)x^3}{v^2}$

Let $\eta = \dfrac{y}{x}\left(\dfrac{\mathrm{Gr}_x}{4}\right)^{1/4} = y\left(\dfrac{g\beta\left(T_s - T_\infty\right)}{4v^2 x}\right)^{1/4}$

$$u = \frac{\partial \psi}{\partial y} \rightarrow \psi \sim u_0 \delta \qquad \left(u_0 \sim \frac{vx}{\delta^2} \right)$$

$$\sim \frac{vx}{\delta^2}\delta \sim \frac{vx}{\delta} \sim vx \left(\frac{g\beta\left(T_s - T_\infty\right)}{v^2 x} \right)^{1/4}$$

$$= v \left(\frac{g\beta\left(T_s - T_\infty\right) x^3}{v^2} \right)^{1/4} = v\mathrm{Gr}_x^{1/4}$$

Let $\dfrac{\psi}{4v\left(\mathrm{Gr}_x / 4\right)^{1/4}} \equiv f(\eta)$

or $\psi = 4v\left(\dfrac{\mathrm{Gr}_x}{4}\right)^{1/4} f(\eta)$

$$u = \frac{\partial \psi}{\partial y} = 4v\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}\frac{df}{d\eta}\frac{\partial \eta}{\partial y}$$

$$= 4v\left(\frac{Gr_x}{4}\right)^{1/4}\frac{1}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4} f' = \frac{2v}{x}\mathrm{Gr}_x^{1/2} f'$$

similarly for other terms

$$v,\ \frac{\partial u}{\partial x},\ \frac{\partial u}{\partial y},\ \frac{\partial^2 u}{\partial y^2},\ \theta,\ \frac{\partial \theta}{\partial x},\ \frac{\partial \theta}{\partial y},\ \frac{\partial^2 \theta}{\partial y^2}$$

similarity equations:

$$f''' + 3ff'' - 2f'^2 + \theta = 0,\quad \theta'' + 3\mathrm{Pr}\, f\theta' = 0$$

boundary conditions:

$$f'(0) = 0,\quad f(0) = 0,\quad f'(\infty) = 0,\quad \theta(0) = 1,\quad \theta(\infty) = 0$$

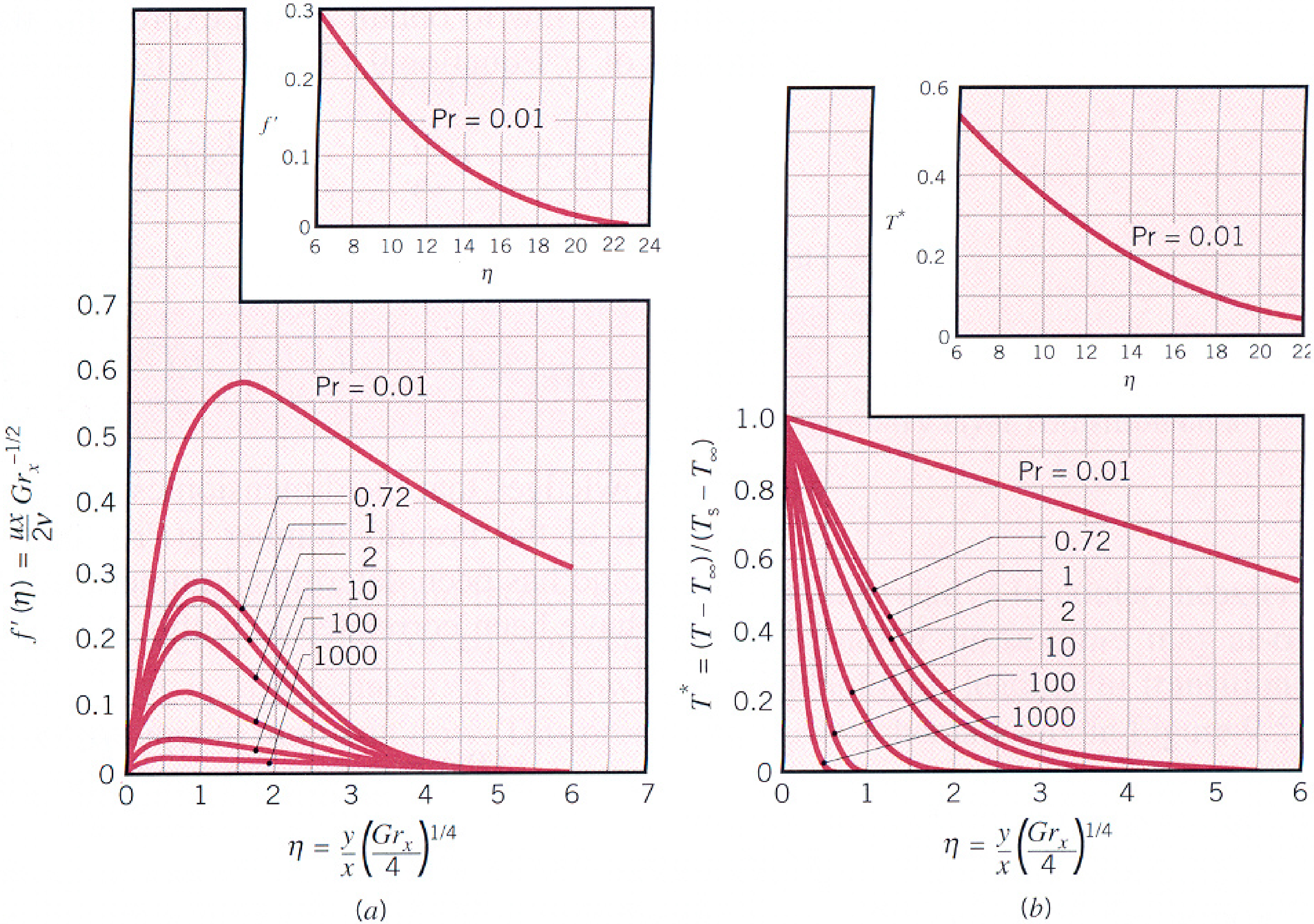


FIGURE 9.4 Laminar, free convection boundary layer conditions on an isothermal, vertical surface. (*a*) Velocity profiles. (*b*) Temperature profiles [3].

Nusselt number

$$\theta = \frac{T - T_\infty}{T_s - T_\infty},\ \ \eta = \frac{y}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}$$

$$q_s'' = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = -k\frac{\nu^2 x}{g\beta\left(T_s - T_\infty\right)}\frac{d\theta}{d\eta}\bigg|_{\eta=0}\frac{\partial \eta}{\partial y}$$

$$= -k\frac{\nu^2 x}{g\beta\left(T_s - T_\infty\right)}\theta'(0)\frac{1}{x}\left(\frac{1}{4}\mathrm{Gr}_x\right)^{1/4}$$

$$= h\left(T_s - T_\infty\right)$$

$$h = -\frac{k}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}\theta'(0) = -\frac{k}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4} g(\mathrm{Pr})$$

$$h = -\frac{k}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}\theta'(0) = -\frac{k}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4} g(\mathrm{Pr})$$

$$\mathrm{Nu}_x = \frac{hx}{k} = -\frac{\theta'(0)}{\sqrt{2}}\mathrm{Gr}_x^{1/4}$$

$$\mathrm{Nu}_x = \frac{\mathrm{Gr}_x^{1/4}}{\sqrt{2}} g(\mathrm{Pr})$$

$$g(\mathrm{Pr}) = \frac{0.75\,\mathrm{Pr}^{1/2}}{\left(0.609 + 1.221\,\mathrm{Pr}^{1/2} + 1.238\,\mathrm{Pr}\right)^{1/4}}$$

Average Nusselt number

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx = \frac{1}{L}\int_0^L \frac{k}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4} g(\mathrm{Pr})\,dx$$

$$= \frac{k}{L}\left(\frac{g\beta\left(T_s - T_\infty\right)}{4v^2}\right)^{1/4} g(\mathrm{Pr})\int_0^L \frac{dx}{x^{1/4}}$$

$$= \frac{4}{3}\frac{k}{L}\left(\frac{g\beta\left(T_s - T_\infty\right)L^3}{4v^2}\right)^{1/4} g(\mathrm{Pr})$$

$$= \frac{4}{3}\frac{k}{L} g(\mathrm{Pr})\left(\frac{\mathrm{Gr}_L}{4}\right)^{1/4}$$

$$\overline{\mathrm{Nu}_L} = \frac{\bar{h}L}{k} = \frac{4}{3} g(\mathrm{Pr})\left(\frac{\mathrm{Gr}_L}{4}\right)^{1/4} = \frac{4}{3}\mathrm{Nu}_L$$

limiting cases:

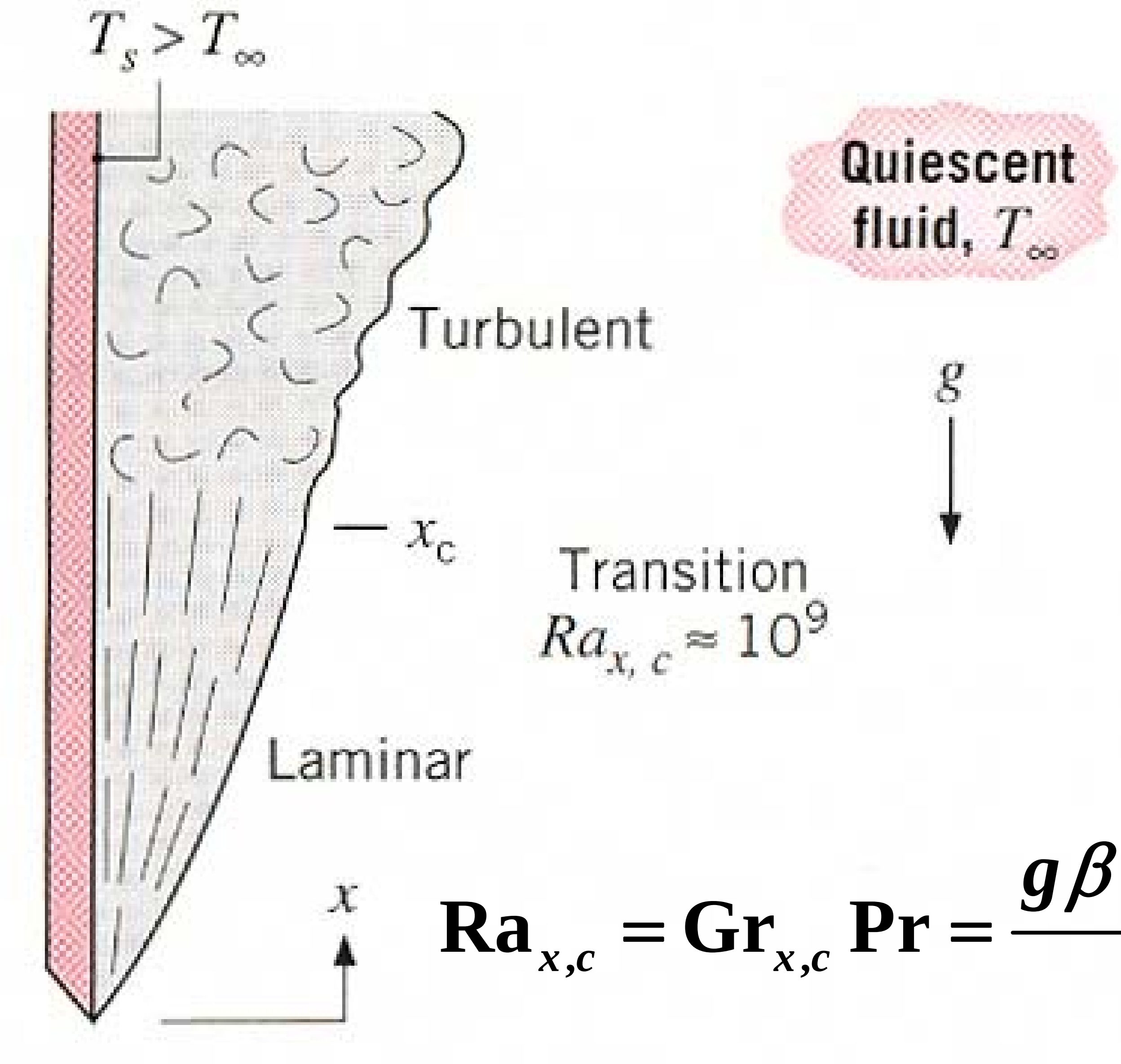
$$\mathrm{Nu}_x = \begin{cases} 0.600\left(\mathrm{Gr}_x \,\mathrm{Pr}\right)^{1/4} \mathrm{Pr}^{1/4} & \text{as } \mathrm{Pr} \to 0 \\ 0.503\left(\mathrm{Gr}_x \,\mathrm{Pr}\right)^{1/4} & \text{as } \mathrm{Pr} \to \infty \end{cases}$$

Rayleigh number :

$$\mathrm{Ra}_x = \mathrm{Gr}_x \,\mathrm{Pr}$$

$$= \frac{g\beta\left(T_s - T_\infty\right)x^3}{\nu^2}\frac{\nu}{\alpha} = \frac{g\beta\left(T_s - T_\infty\right)x^3}{\nu\alpha}$$

Critical Rayleigh number



$$\mathbf{Ra}_{x,c} = \mathbf{Gr}_{x,c}\,\mathbf{Pr} = \frac{g\beta\left(T_s - T_\infty\right)x^3}{\nu\alpha} \approx \mathbf{10^9}$$

Empirical Correlations: External Flows

Vertical Plate

isothermal plates

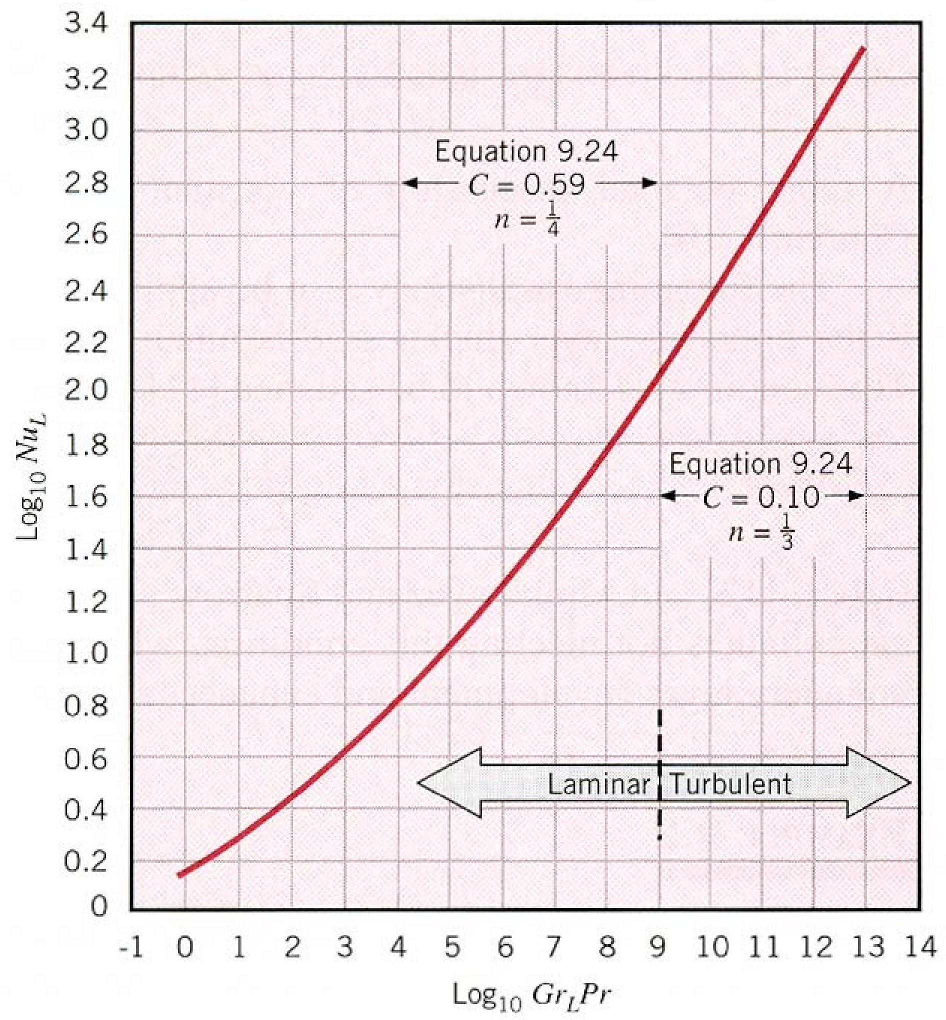
$$\overline{\mathrm{Nu}_L} = \frac{\bar{h}L}{k} = C\mathrm{Ra}_L^n$$

laminar:

$$C = 0.59,\ n = \frac{1}{2}$$

turbulent:

$$C = 0.10,\ n = \frac{1}{3}$$



Nusselt number

- Churchill and Chu (1975)

$$\overline{\mathrm{Nu}_L} = \left\{ 0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{\left[1+\left(0.429/\mathrm{Pr}\right)^{9/16}\right]^{8/27}} \right\}^2$$

for all Ra_L

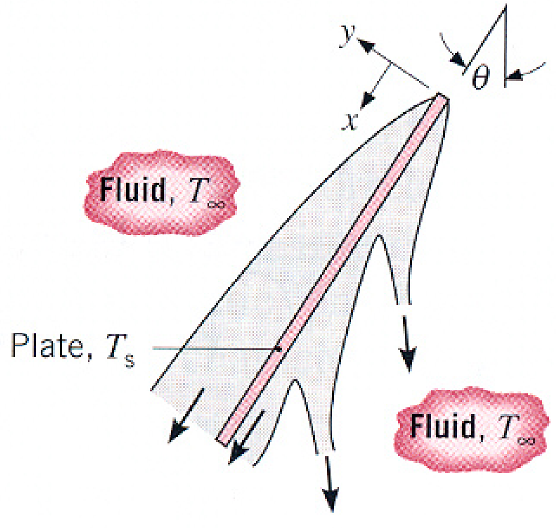
$$\overline{\mathrm{Nu}_L} = 0.68 + \frac{0.670\mathrm{Ra}_L^{1/4}}{\left[1+\left(0.429/\mathrm{Pr}\right)^{9/16}\right]^{4/9}}$$

$$\left(\mathrm{Ra}_L \le 10^9\right)$$

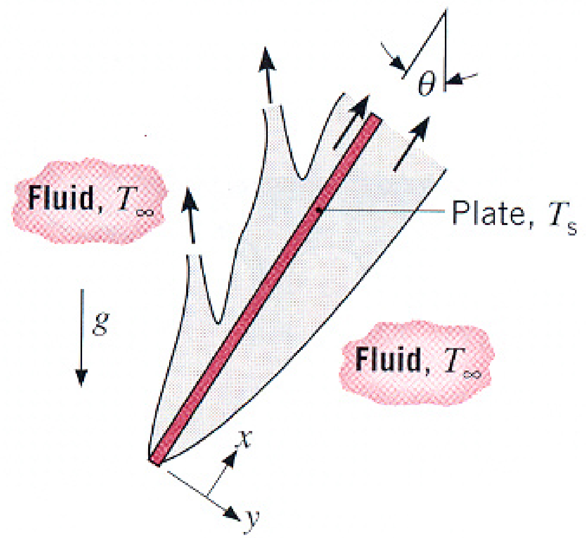
Inclined and Horizontal Plates

Inclined plates

Use correlations for the vertical plates by replacing g by $g\cos\theta$ for $0 \le \theta \le 60^\circ$



(a)

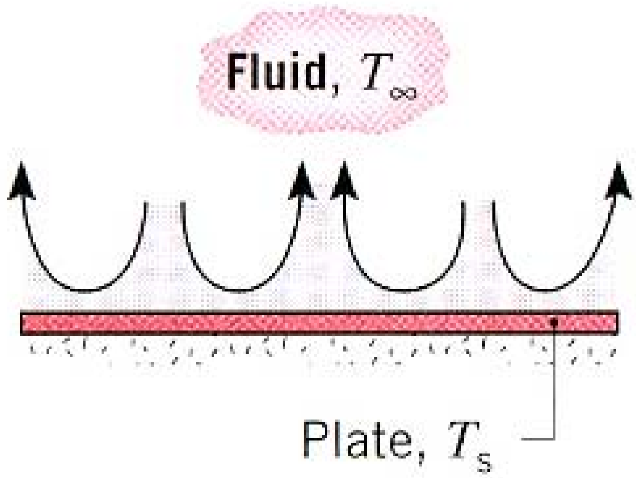


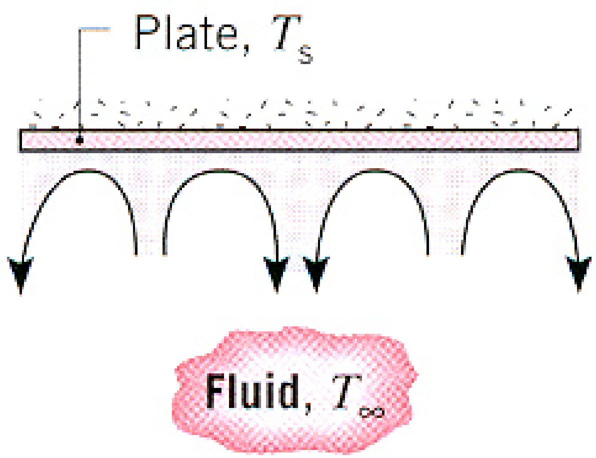
(c)

Horizontal plates

$$L \equiv \frac{A_s}{P}$$ (A_s: plate surface area, P: perimeter)

upper surface of heated plate or lower surface of cooled plate





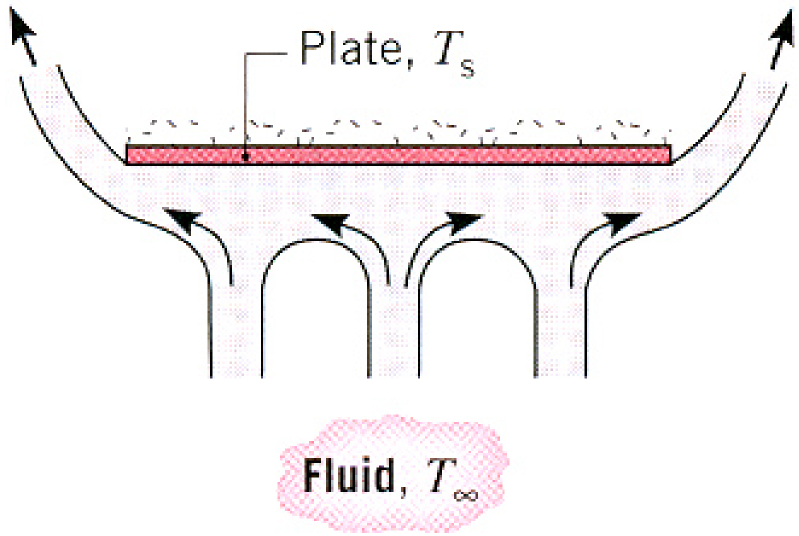
$$\overline{\mathrm{Nu}_L} = 0.54\mathrm{Ra}_L^{1/4}$$

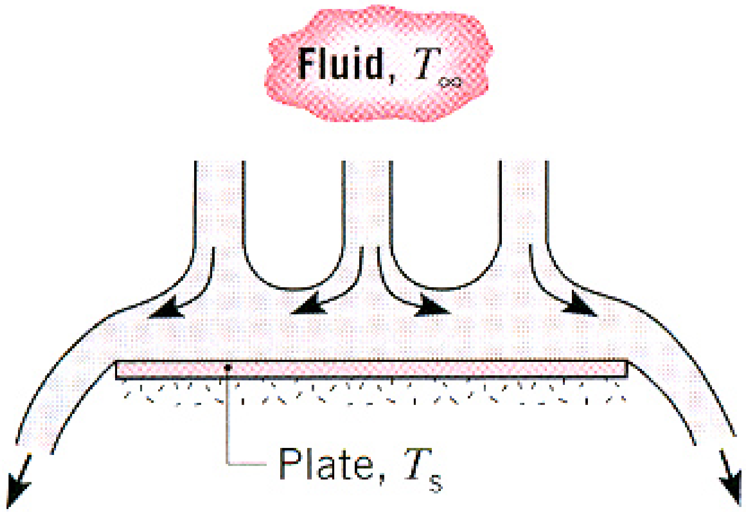
$$\left(10^4 \lesssim \mathrm{Ra}_L \lesssim 10^7\right)$$

$$\overline{\mathrm{Nu}_L} = 0.15\mathrm{Ra}_L^{1/3}$$

$$\left(10^7 \lesssim \mathrm{Ra}_L \lesssim 10^{11}\right)$$

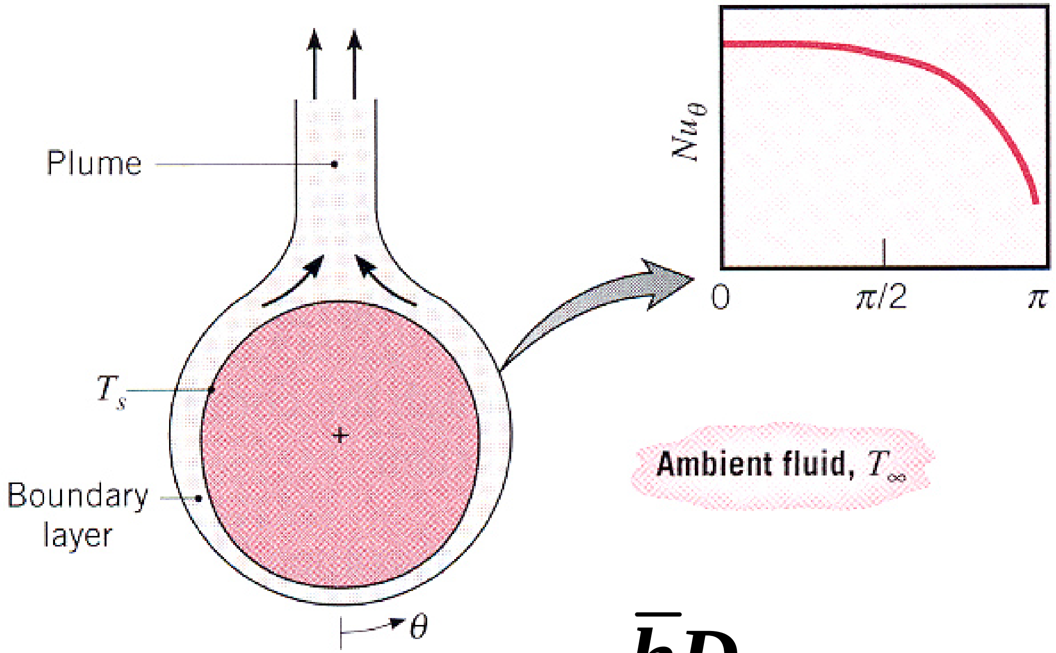
lower surface of heated plate or upper surface of cooled plate





$$\overline{\mathrm{Nu}_L} = 0.27\mathrm{Ra}_L^{1/4} \qquad \left(10^5 \lesssim \mathrm{Ra}_L \lesssim 10^{10}\right)$$

Long Horizontal Cylinder



- **Morgan (1975)** $\overline{\mathrm{Nu}_D} = \dfrac{\bar{h}D}{k} = C\mathrm{Ra}_D^n$, C, n **Table 9.1**
- **Churchil and Chu (1975)**

$$\overline{\mathrm{Nu}_D} = \left\{ 0.60 + \frac{0.387\mathrm{Ra}_D^{1/6}}{\left[1+\left(0.559/\mathrm{Pr}\right)^{9/16}\right]^{8/27}} \right\}^2, \quad \mathrm{Ra}_D \lesssim 10^{12}$$

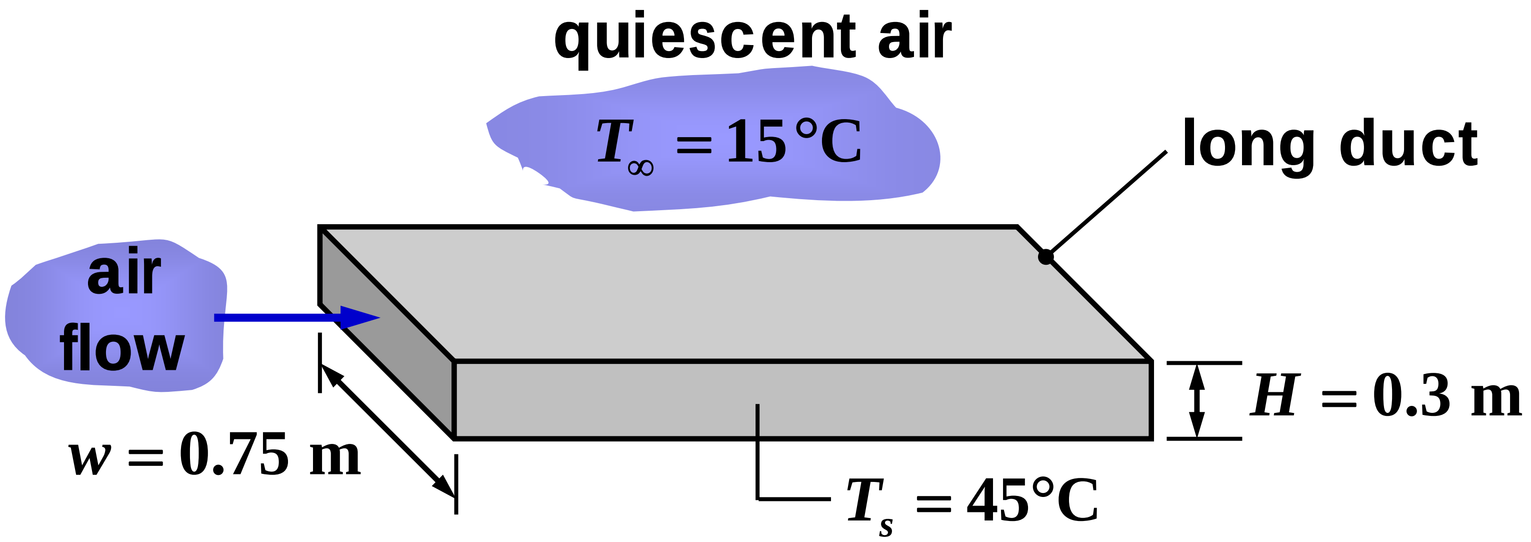
Spheres

- **Churchil (1983)**

$$\overline{\mathrm{Nu}_D} = 2 + \frac{0.589\mathrm{Ra}_D^{1/4}}{\left[1+\left(0.469/\mathrm{Pr}\right)^{9/16}\right]^{4/9}}$$

$$\mathrm{Pr} \geq 0.7,\ \ \mathrm{Ra}_D \lesssim 10^{11}$$

Example 9.3

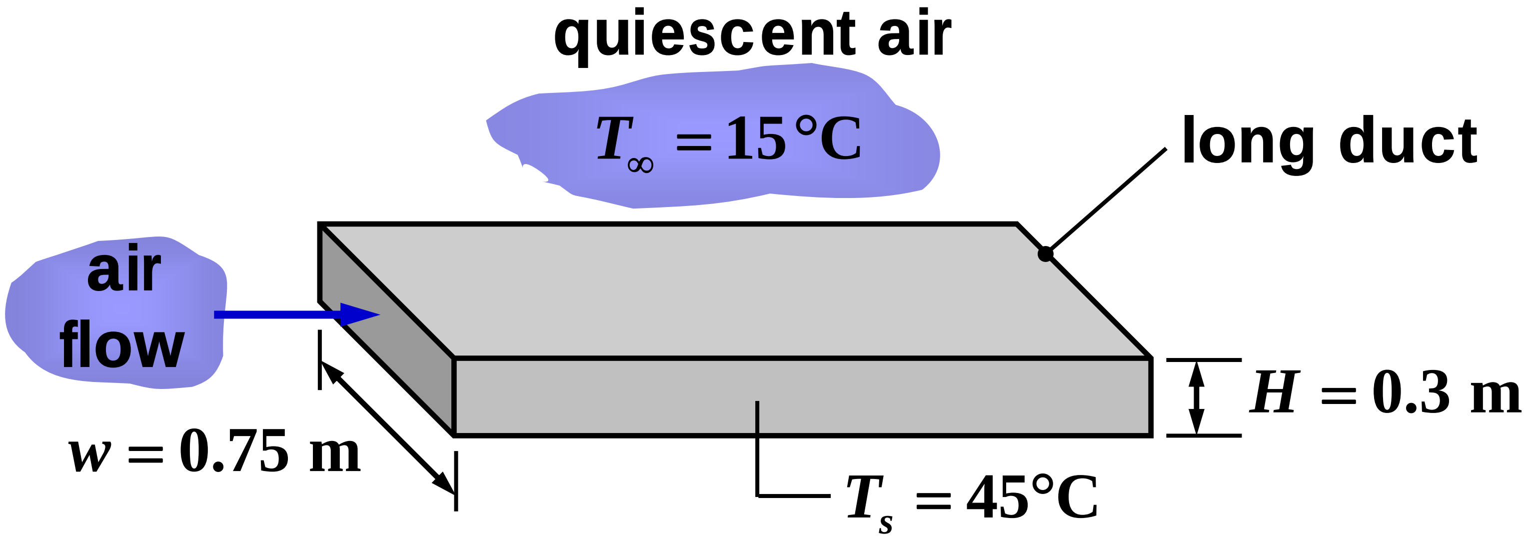


Find:

Heat loss from duct per meter of length

Assumption:

Radiative effects are negligible.



heat loss from two side walls, top wall and bottom wall

side wall: vertical plate

top wall: upper surface of heated plate

bottom wall: lower surface of heated plate

$$q' = 2q'_s + q'_t + q'_b$$

$$= 2\bar{h}_s H\left(T_s - T_\infty\right) + \bar{h}_t w\left(T_s - T_\infty\right) + \bar{h}_b w\left(T_s - T_\infty\right)$$

side wall: vertical plate

Churchill and Chu (1975)

$$\overline{\mathrm{Nu}_L} = \left\{ 0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{\left[1+\left(0.429/\mathrm{Pr}\right)^{9/16}\right]^{8/27}} \right\}^2 \quad \text{for all } \mathrm{Ra}_L$$

$$\overline{\mathrm{Nu}_L} = 0.68 + \frac{0.670\mathrm{Ra}_L^{1/4}}{\left[1+\left(0.429/\mathrm{Pr}\right)^{9/16}\right]^{4/9}} \quad \left(\mathrm{Ra}_L \le 10^9\right)$$

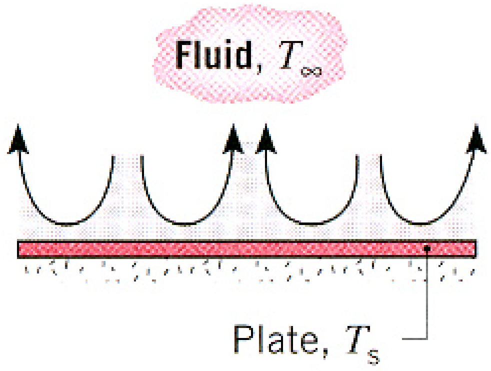
air: $\nu = 16.2\times10^{-6}\ \mathrm{m^2/s}$, $\alpha = 22.9\times10^{-6}\ \mathrm{m^2/s}$

$k = 0.0265\ \mathrm{W/m\cdot K}$, $\beta = 0.0033\ \mathrm{K^{-1}}$, $\mathrm{Pr} = 0.71$

$$\mathrm{Ra}_L = \frac{g\beta\left(T_s - T_\infty\right)H^3}{\nu\alpha} = 7.07\times10^7$$

$$\overline{h}_s = \frac{k}{H}\overline{\mathrm{Nu}_L} = 4.23\ \mathrm{W/m^2\cdot K}$$

top wall: upper surface of heated plate



$$\overline{\mathrm{Nu}_L} = 0.54\mathrm{Ra}_L^{1/4} \quad (10^4 \lesssim \mathrm{Ra}_L \lesssim 10^7),$$

$$\overline{\mathrm{Nu}_L} = 0.15\mathrm{Ra}_L^{1/3} \quad (10^7 \lesssim \mathrm{Ra}_L \lesssim 10^{11})$$

$L \equiv \dfrac{A_s}{P}$ **(A_s: plate surface area, P : perimeter)**

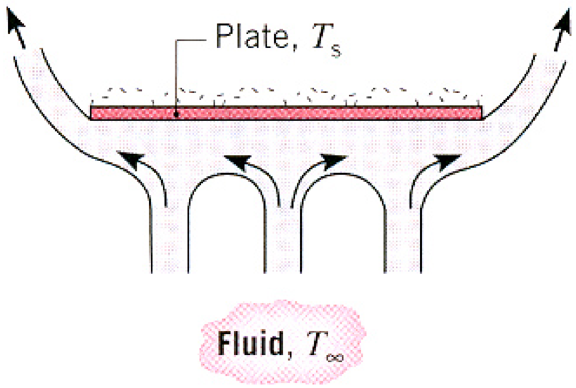
$$L = \frac{wL}{2(w+L)} \approx \frac{wL}{2L} = \frac{w}{2} = 0.375\,\mathrm{m}$$

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)(w/2)^3}{\nu\alpha} = 1.38 \times 10^8$$

$$\frac{\bar{h}_t(w/2)}{k} = 0.15\mathrm{Ra}_L^{1/3}$$

$$\bar{h}_t = [k/(w/2)] \times 0.15\mathrm{Ra}_L^{1/3} = 5.47\ \mathrm{W/m^2 \cdot K}$$

bottom wall: lower surface of heated plate



$$\overline{\mathrm{Nu}_L} = 0.27\mathrm{Ra}_L^{1/4} \qquad (10^5 \lesssim \mathrm{Ra}_L \lesssim 10^{10})$$

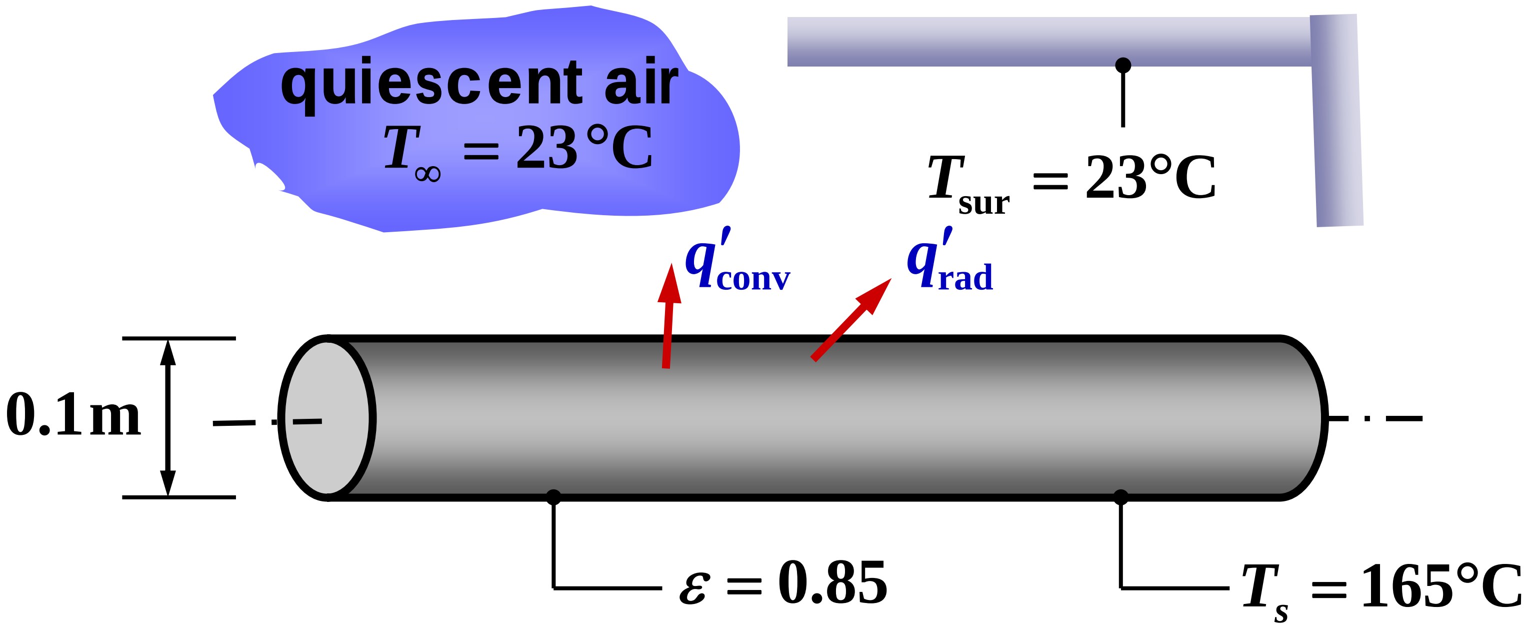
$$\mathrm{Ra}_L = \frac{g\beta\left(T_s - T_\infty\right)\left(w/2\right)^3}{\nu\alpha} = 1.38\times10^8$$

$$\bar{h}_b = \left[k/\left(w/2\right)\right]\times 0.27\mathrm{Ra}_L^{1/4} = 2.07\ \mathrm{W/m^2\cdot K}$$

$$q' = 2q'_s + q'_t + q'_b = \left(2\bar{h}_s\cdot H + \bar{h}_t\cdot w + \bar{h}_b\cdot w\right)\left(T_s - T_\infty\right)$$

$$= 246\ \mathrm{W/m}$$

Example 9.4



Find:

Heat loss from the pipe per unit length q' [W/m]

total heat loss per unit length of pipe

$$q' = q'_{conv} + q'_{rad} = \bar{h}\pi D\left(T_s - T_\infty\right) + \varepsilon\pi D\sigma\left(T_s^4 - T_{sur}^4\right)$$

long horizontal cylinder

air : $\nu = 22.8\times10^{-6}\,\mathrm{m^2/s}$, $\alpha = 32.8\times10^{-6}\ \mathrm{m^2/s}$, $\mathrm{Pr} = 0.697$

$k = 0.0313\ \mathrm{W/m\cdot K}$, $\beta = 2.725\times10^{-3}\,\mathrm{K^{-1}}$

$$\mathrm{Ra}_D = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\nu\alpha} = 5.073\times10^6$$

Churchil and Chu (1975)

$$\overline{\mathrm{Nu}_D} = \left\{0.60 + \frac{0.387\mathrm{Ra}_D^{1/6}}{\left[1+(0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2 \qquad \mathrm{Ra}_D \lesssim 10^{12}$$

$$\overline{\mathrm{Nu}_D} = 23.3$$

$$\bar{h} = \frac{k}{D}\overline{\mathrm{Nu}_D} = 7.29\ \mathrm{W/m^2\cdot K}$$

$$q' = \bar{h}\pi D\left(T_s - T_\infty\right) + \varepsilon\pi D\sigma\left(T_s^{\,4} - T_{\mathrm{sur}}^4\right) = 325 + 441 = 766\ \mathrm{W/m}$$

Free Convection in Parallel Plate Channels

Vertical Channels

- **Elenbaas (1942)**

symmetrically heated isothermal plates

$$\overline{\mathrm{Nu}}_S = \frac{1}{24}\mathrm{Ra}_S\left(\frac{S}{L}\right)\left\{1-\exp\left[-\frac{35}{\mathrm{Ra}_S\left(S/L\right)}\right]\right\}^{3/4}$$

$$\overline{\mathrm{Nu}}_S = \left(\frac{q/A}{T_s - T_\infty}\right)\frac{S}{k}, \quad \mathrm{Ra}_S = \frac{g\beta\left(T_s - T_\infty\right)S^3}{\alpha\nu}$$

fully developed limit $(S/L \rightarrow 0)$

$$\overline{\mathrm{Nu}}_{S\,(\mathrm{fd})} = \frac{\mathrm{Ra}_S\left(S/L\right)}{24}$$

S: **channel width**

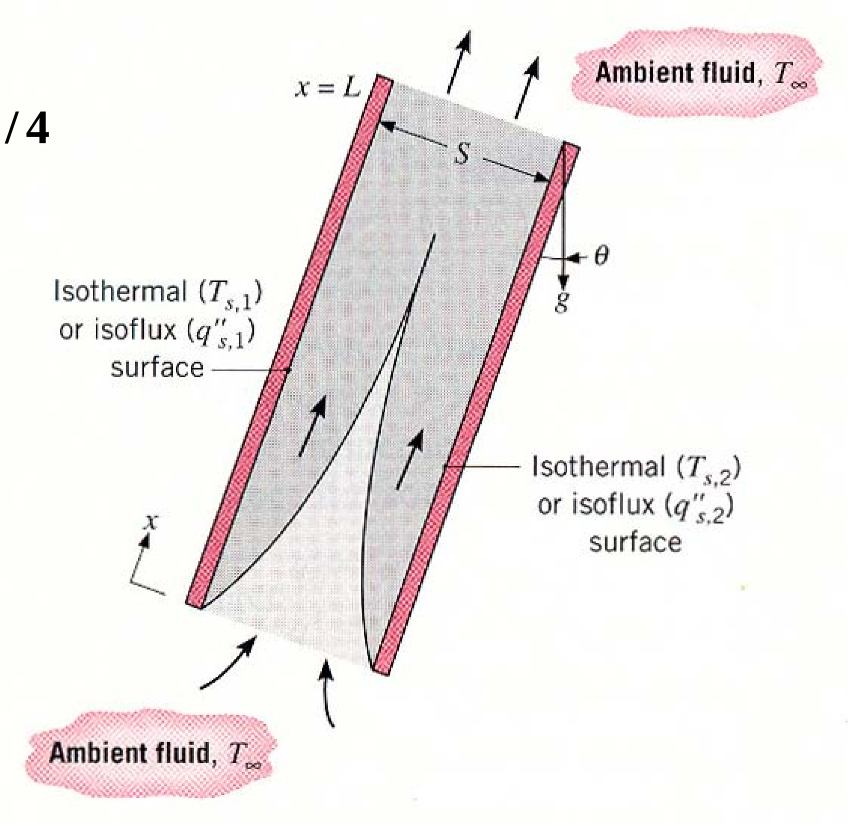
L: **channel length**

Inclined Channels

- **Azevedo and Sparrow (1985)**

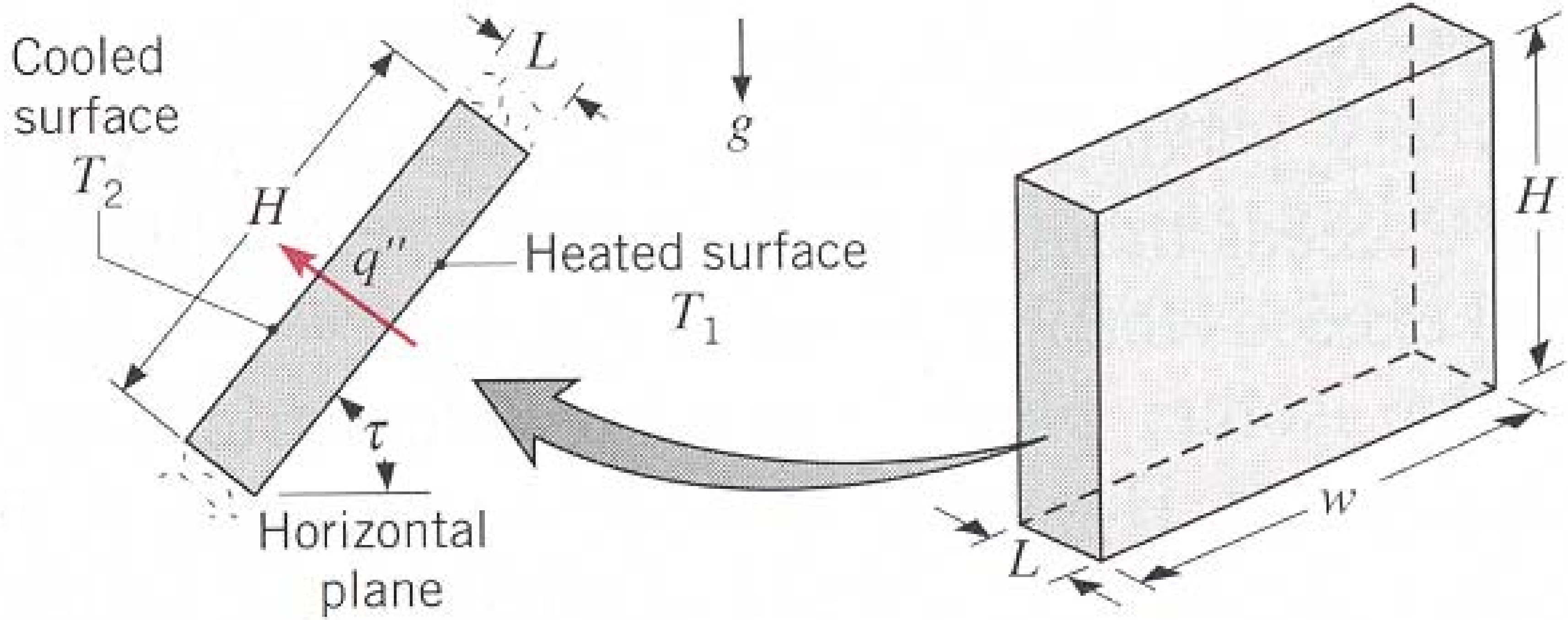
$$0 \le \theta \le 45^\circ$$

$$\overline{\mathbf{Nu}_S} = 0.645\left[\mathbf{Ra}_S\left(S/L\right)\right]^{1/4}$$



Empirical Correlations: Enclosures

Rectangular Cavities



$$q'' = h\left(T_1 - T_2\right)$$

Horizontal cavity heated from below

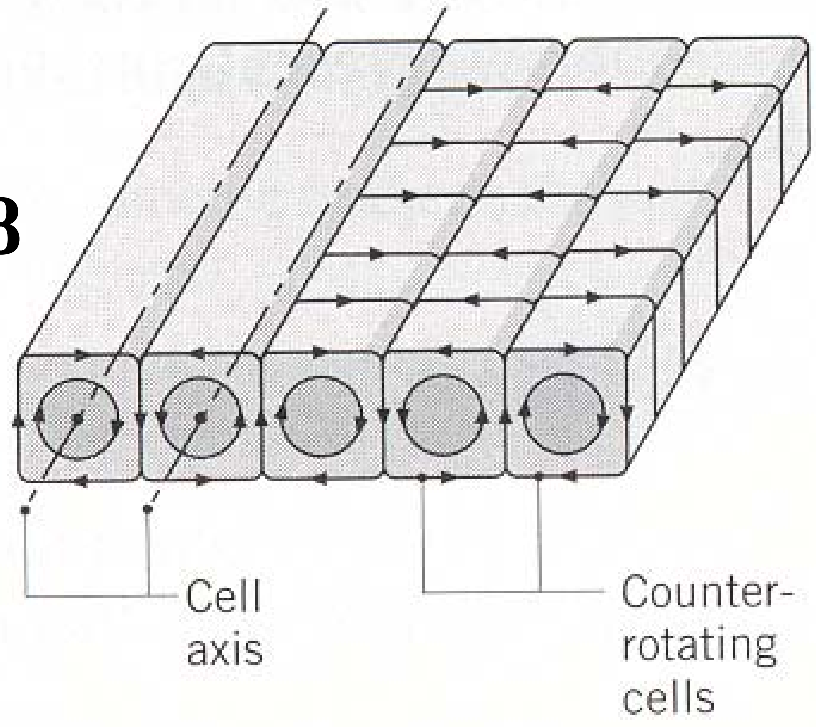
conduction regime

$$\mathrm{Ra}_L = \frac{g\beta\left(T_1 - T_2\right)L^3}{\alpha\nu} < 1708$$

$$\mathrm{Nu}_L = 1$$

longitudinal roll cells

$$1708 < \mathrm{Ra}_L \le 5\times10^4$$

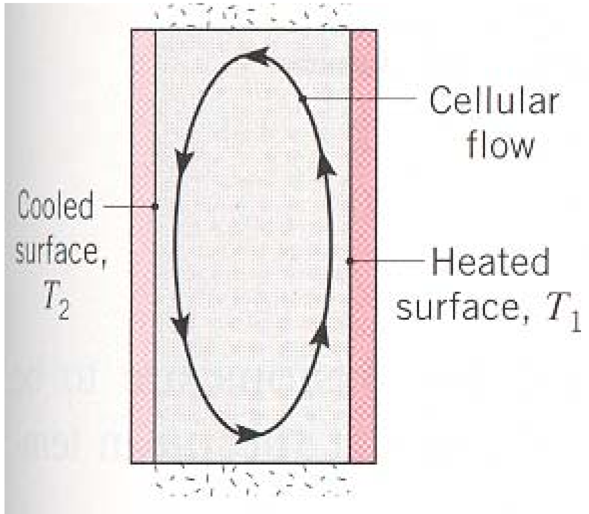


- **Globe and Dropkin (1959)**

$$\overline{\mathrm{Nu}_L} = \frac{\bar{h}L}{k} = 0.069\mathrm{Ra}_L^{1/3}\,\mathrm{Pr}^{0.074}$$

$$3\times10^5 < \mathrm{Ra}_L < 7\times10^9$$

Vertical rectangular cavity



conduction regime

$$\mathrm{Ra}_L = \frac{g\beta\left(T_1 - T_2\right)L^3}{\alpha\nu} \le 10^3$$

$$\mathrm{Nu}_L = 1$$

- **Catton (1978)**

  for aspect ratio $1 < \left(H/L\right) < 10$

$$\overline{\mathrm{Nu}_L} = 0.18\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.29} \qquad \overline{\mathrm{Nu}_L} = 0.22\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4}$$

$$\left[\begin{array}{l} 1 < \dfrac{H}{L} < 2 \\ 10^{-3} < \mathrm{Pr} < 10^5 \\ 10^3 < \dfrac{\mathrm{Ra}_L\,\mathrm{Pr}}{0.2+\mathrm{Pr}} \end{array}\right] \qquad \left[\begin{array}{l} 2 < \dfrac{H}{L} < 10 \\ \mathrm{Pr} < 10^5 \\ 10^3 < \mathrm{Ra}_L < 10^{10} \end{array}\right]$$

for large aspect ratio

- **MacGregor and Emery (1969)**

$$\overline{\mathrm{Nu}_L} = 0.42\mathrm{Ra}_L^{1/4}\,\mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} \qquad \left[\begin{array}{l} 10 < \dfrac{H}{L} < 40 \\ 1 < \mathrm{Pr} < 2\times 10^4 \\ 10^4 < \mathrm{Ra}_L < 10^7 \end{array}\right]$$

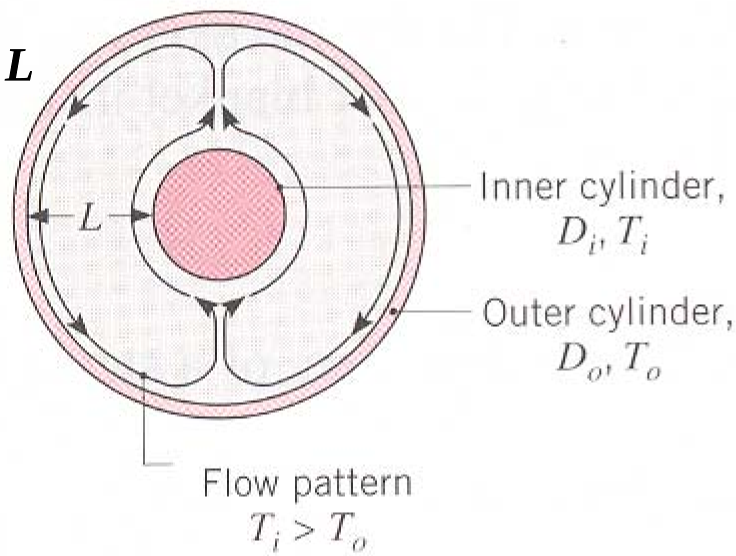
$$\overline{\mathrm{Nu}_L} = 0.046\mathrm{Ra}_L^{1/3} \qquad \left[\begin{array}{l} 1 < \dfrac{H}{L} < 40 \\ 1 < \mathrm{Pr} < 20 \\ 10^6 < \mathrm{Ra}_L < 10^9 \end{array}\right]$$

Concentric Cylinders

$$q' = \frac{2\pi k_{\text{eff}}}{\ln\left(D_o / D_i\right)}\left(T_i - T_o\right)$$

k_{eff}: **effective thermal conductivity**

$$\text{Ra}_c^* = \frac{\left[\ln(D_o / D_i)\right]^4}{L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5}\text{Ra}_L$$



for $\text{Ra}_c^* < 100$

$$k_{\text{eff}} = k$$

for $10^2 \le \text{Ra}_c^* \le 10^7$

$$\frac{k_{\text{eff}}}{k} = 0.386\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}\left(\text{Ra}_c^*\right)^{1/4}$$

Concentric Spheres

- **Raithby and Hollands (1975)**

$$q = k_{\text{eff}}\pi\left(\frac{D_i D_o}{L}\right)(T_i - T_o)$$

for $10^2 \le \text{Ra}_c^* \le 10^4$

$$\frac{k_{\text{eff}}}{k} = 0.74\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}\left(\text{Ra}_c^*\right)^{1/4}$$

$$\text{Ra}_s^* = \frac{L}{\left(D_o / D_i\right)^4}\frac{\text{Ra}_L}{\left(D_i^{-7/5} + D_o^{-7/5}\right)^5}$$