HEAT EXCHANGE DEVICES

- **Heat Exchanger Types**
- **Overall Heat Transfer Coefficient**
- **Heat Exchanger Analysis:**
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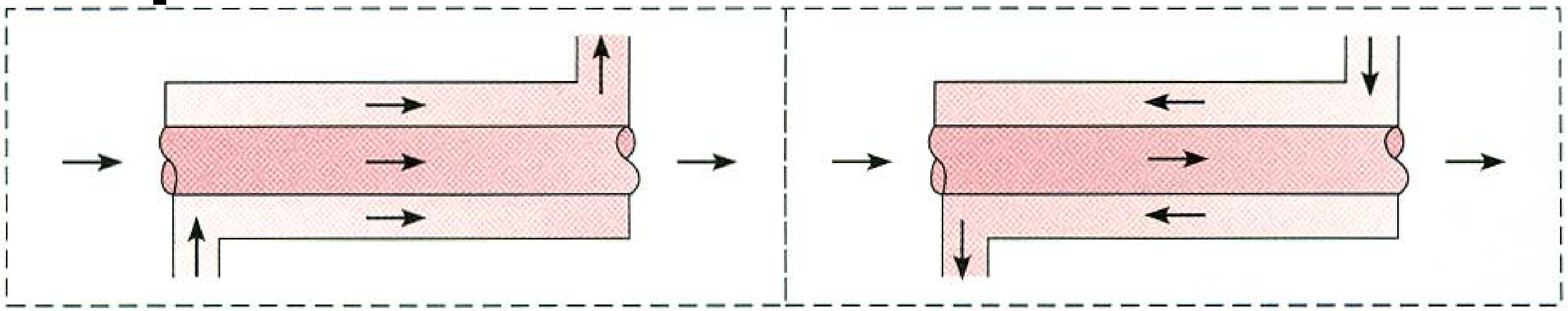
- **Heat Exchanger Analysis:**
  Effectiveness-NTU Method
- **Methodology of a Heat Exchanger Calculation**
- **Compact Heat Exchangers**

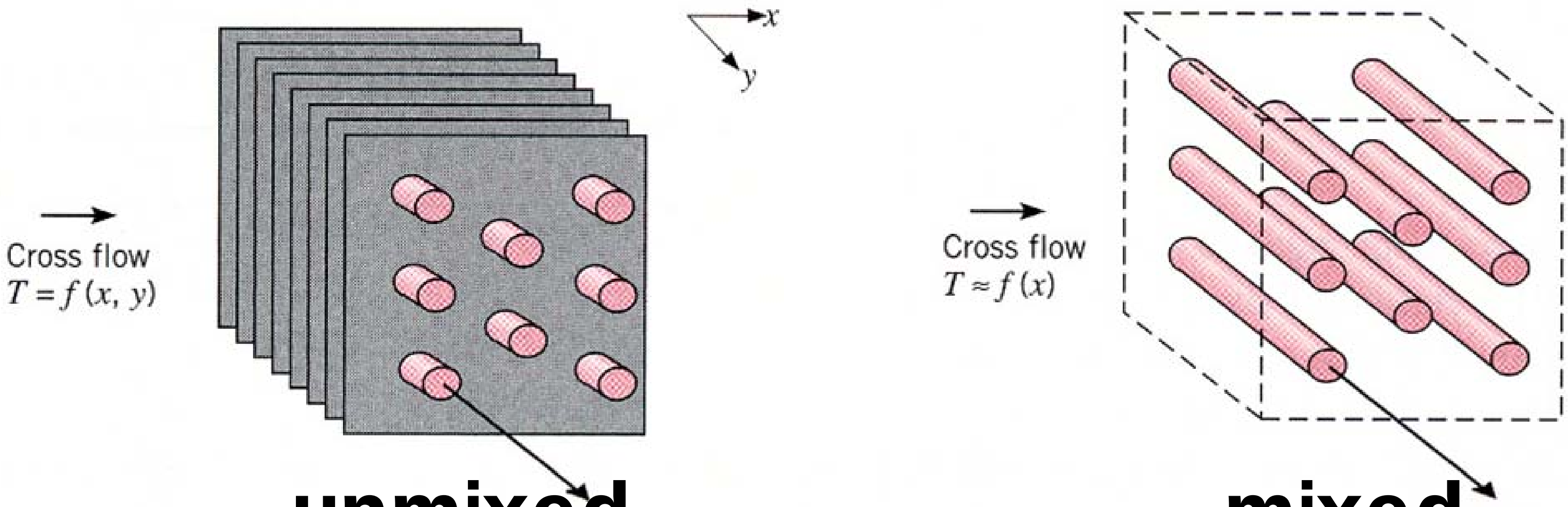
Heat Exchanger Types

Flow Arrangement

parallel-flow **counterflow**



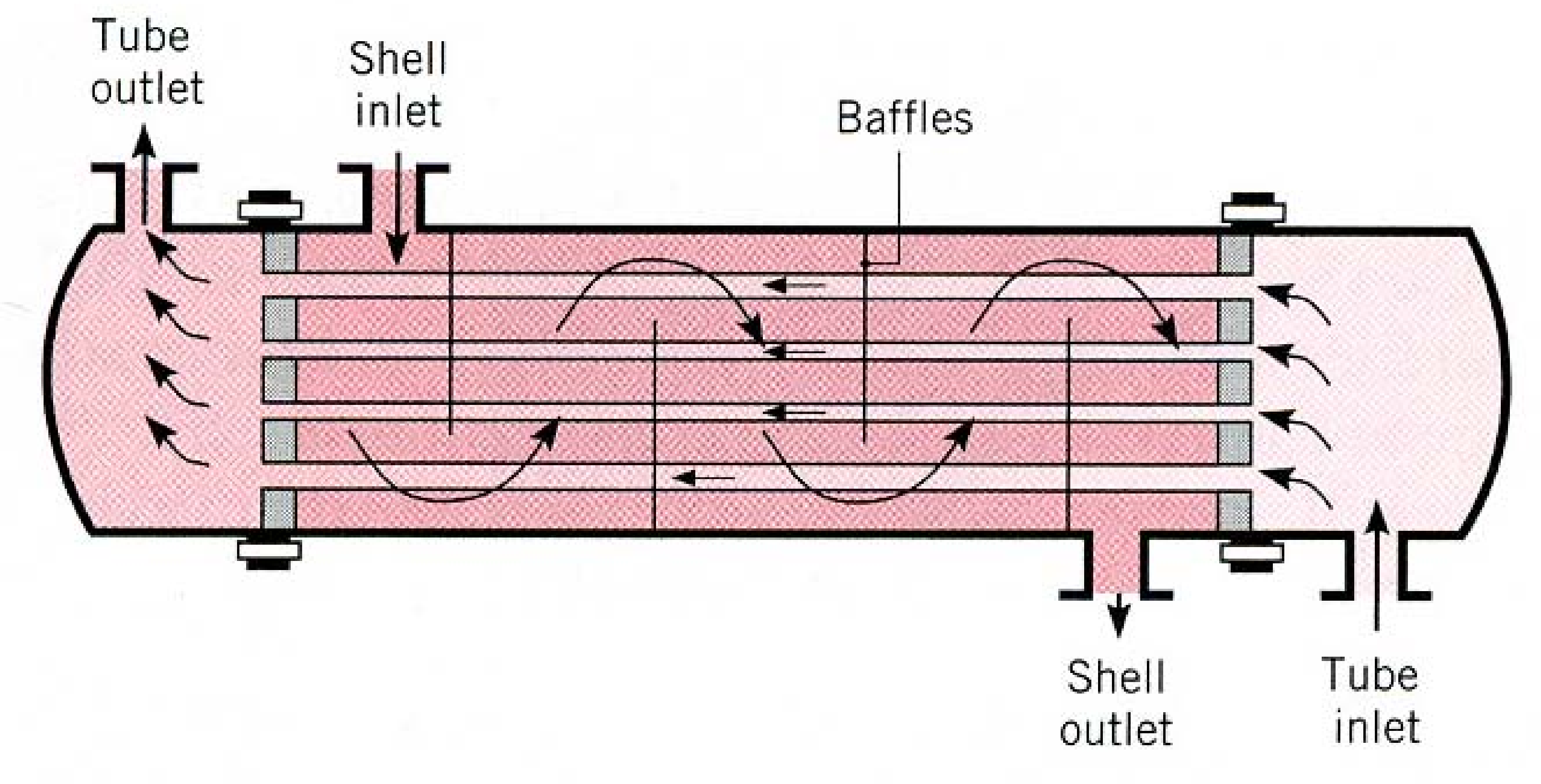
cross flow



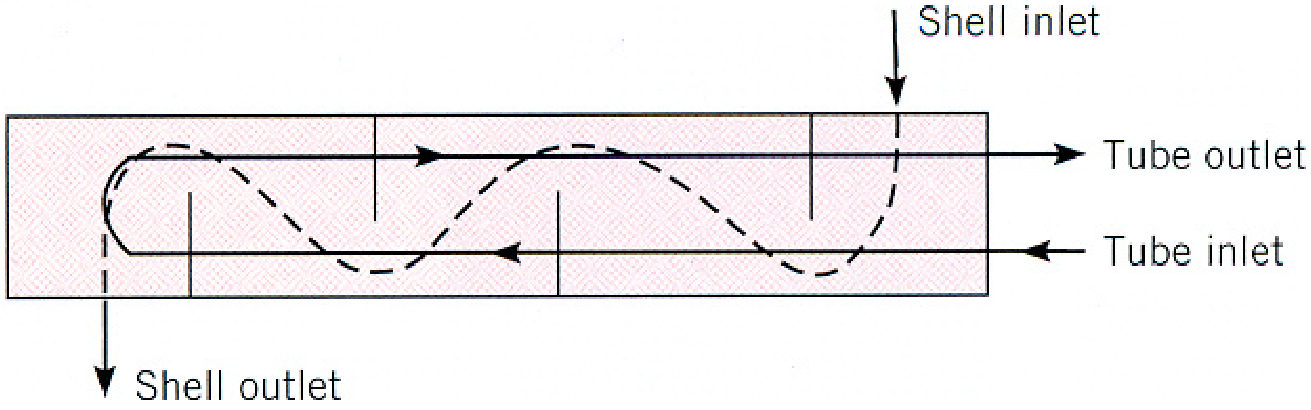
unmixed **mixed**

Type of Construction

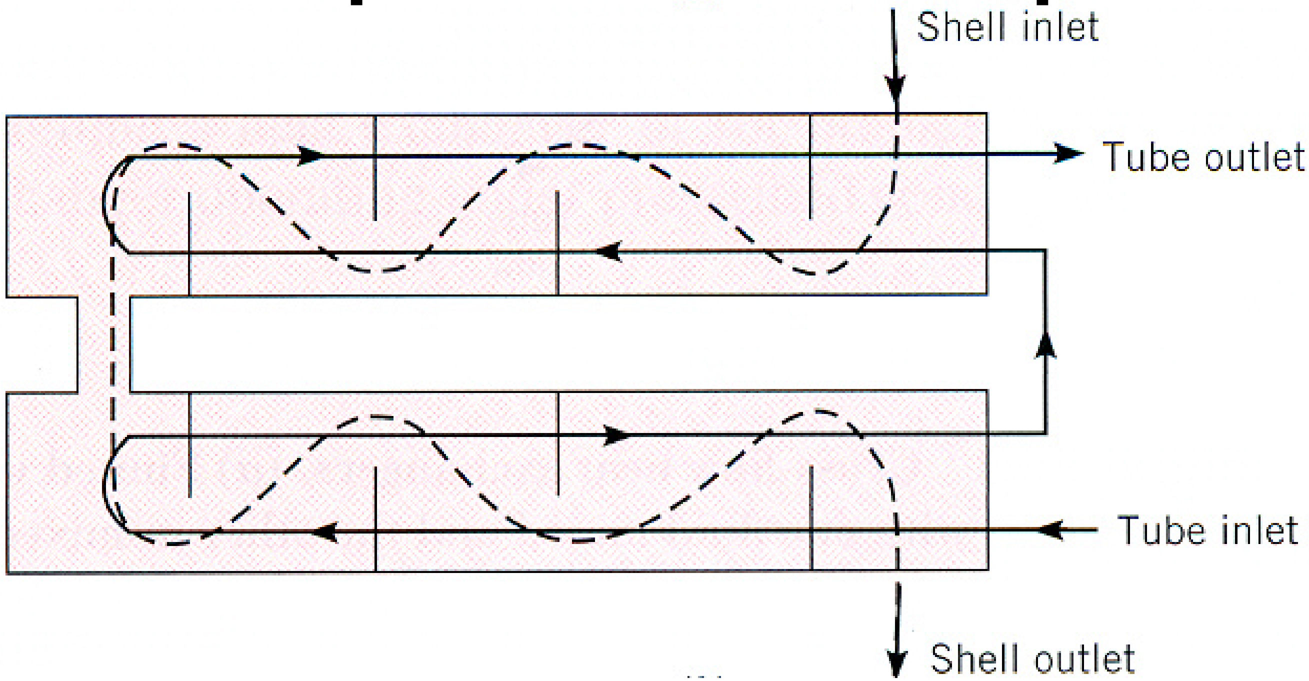
shell-and-tube heat exchanger



one shell pass and one tube pass
(cross-counter mode of operation)



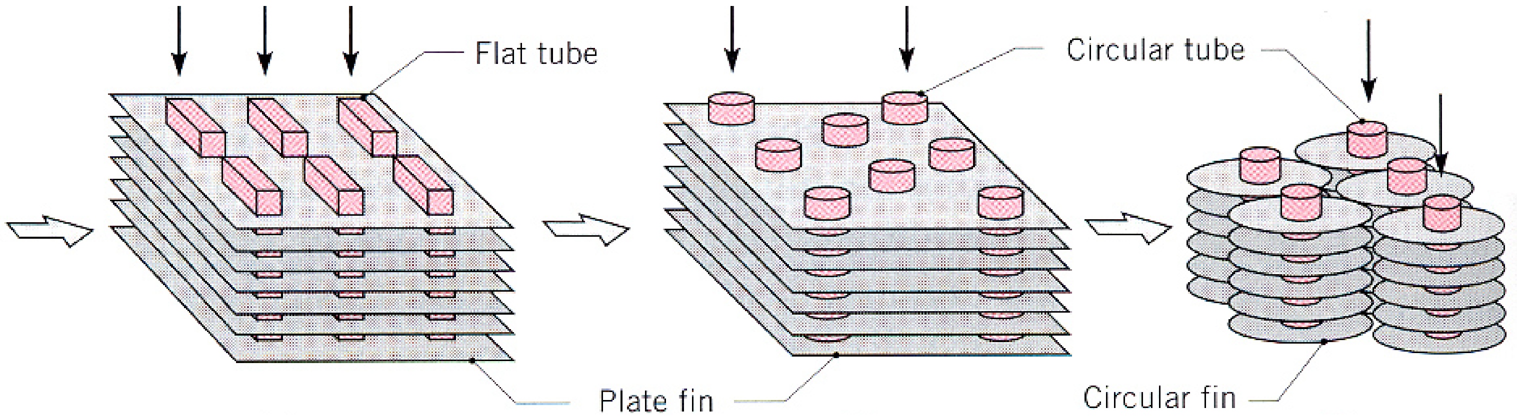
one shell pass and two tube passes



two shell passes and four tube passes

Compact Heat Exchanger

Fin-tube

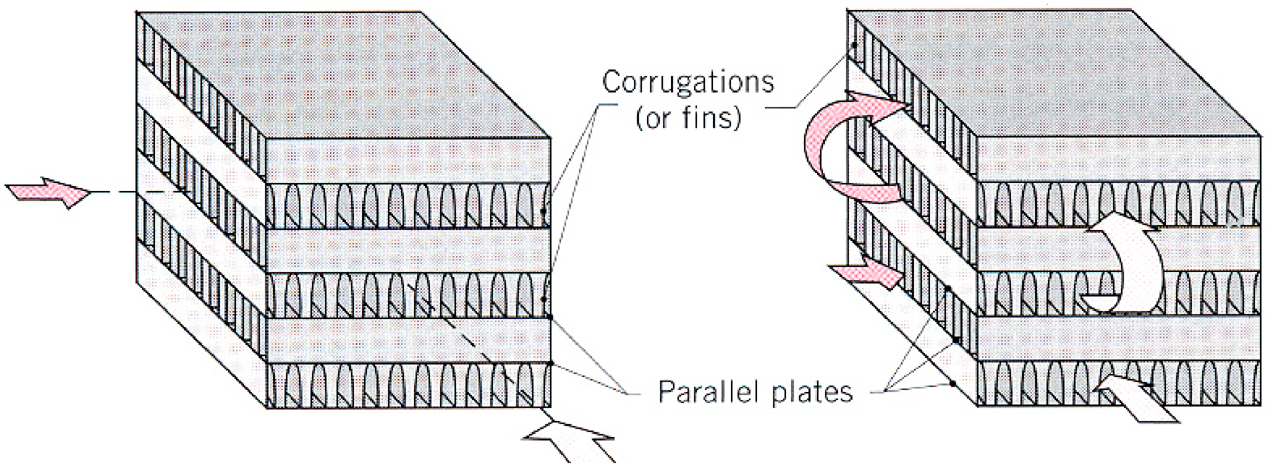


flat tubes, continuous plate fins

circular tubes, continuous plate fins

circular tubes, circular fins

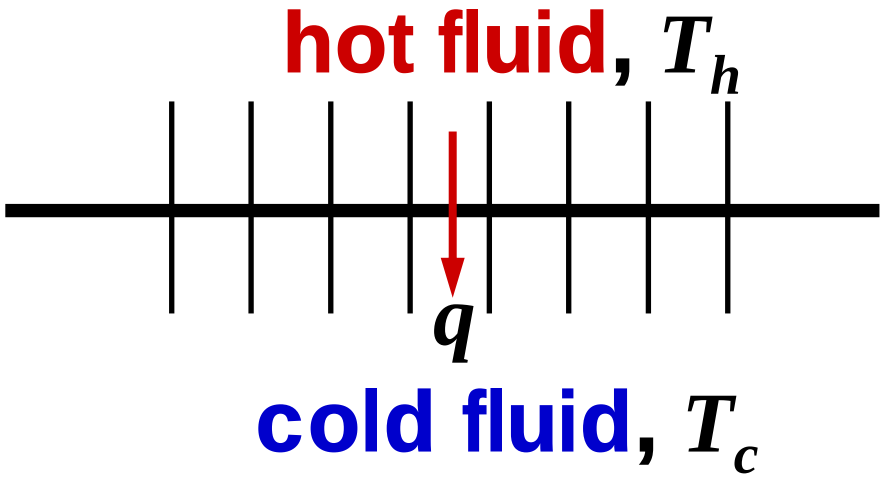
Plate-fin



single pass

multi-pass

Overall Heat Transfer Coefficient



$$q = UA\left(T_h - T_c\right) = \frac{T_h - T_c}{1/UA}$$

$$\frac{1}{UA} = \frac{1}{(UA)_c} = \frac{1}{(UA)_h}$$

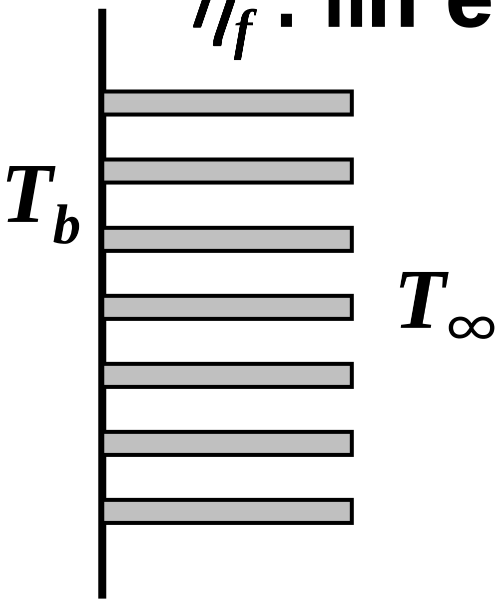
$$= \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}$$

R''_f : fouling factor (fluid impurities, rust formation, reaction)

η_o : temperature effectiveness or overall surface efficiency

temperature effectiveness or overall surface efficiency

η_f : fin efficiency



$$q = \eta_o h A \left(T_b - T_\infty\right), \quad A = A_f + A_b$$

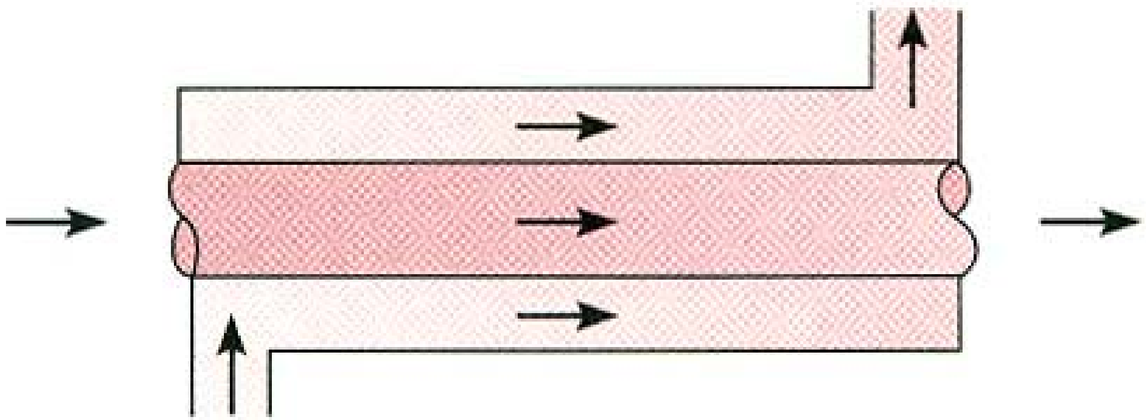
$$q = hA_b\left(T_b - T_\infty\right) + hA_f\eta_f\left(T_b - T_\infty\right)$$

$$= h\left(A_b + A_f\eta_f\right)\left(T_b - T_\infty\right)$$

$$= hA\left[1 - \frac{A_f}{A}\left(1 - \eta_f\right)\right]\left(T_b - T_\infty\right)$$

$$\eta_o = 1 - \frac{A_f}{A}\left(1 - \eta_f\right)$$

Overall heat transfer coefficient for the unfinned, tubular heat exchangers



$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

$$= \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln\left(D_o / D_i\right)}{2\pi k L} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

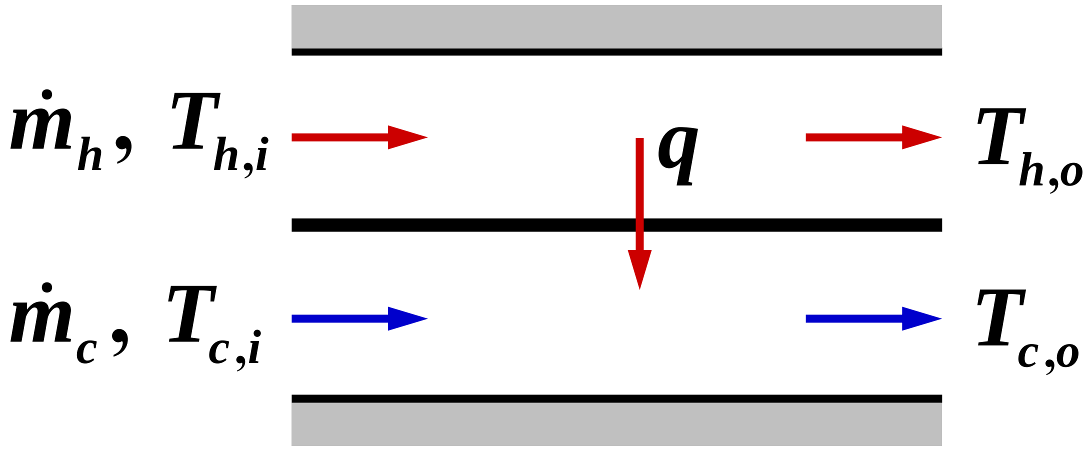
i : inner surface, *o* : outer surface

Heat Exchanger Analysis: LMTD Method

Assumptions:

- The heat exchanger is insulated from surroundings.
- Axial conduction along the tubes is negligible.
- The fluid specific heats are constant.
- The overall heat transfer coefficient is constant.

Parallel-flow Heat Exchanger

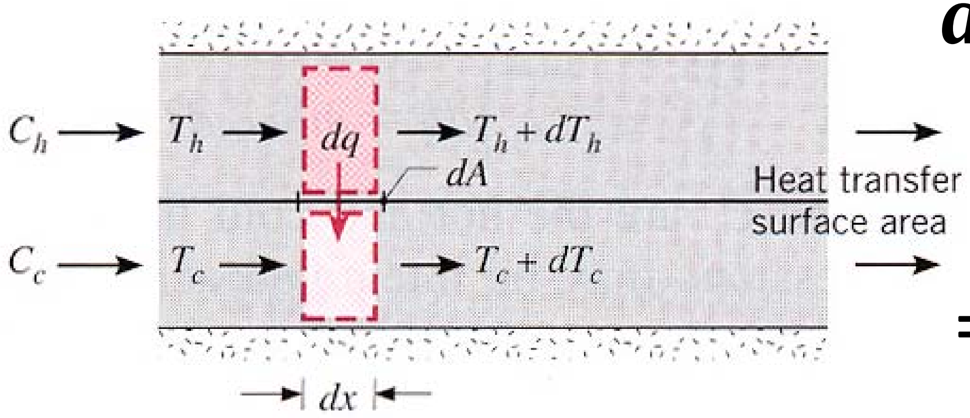


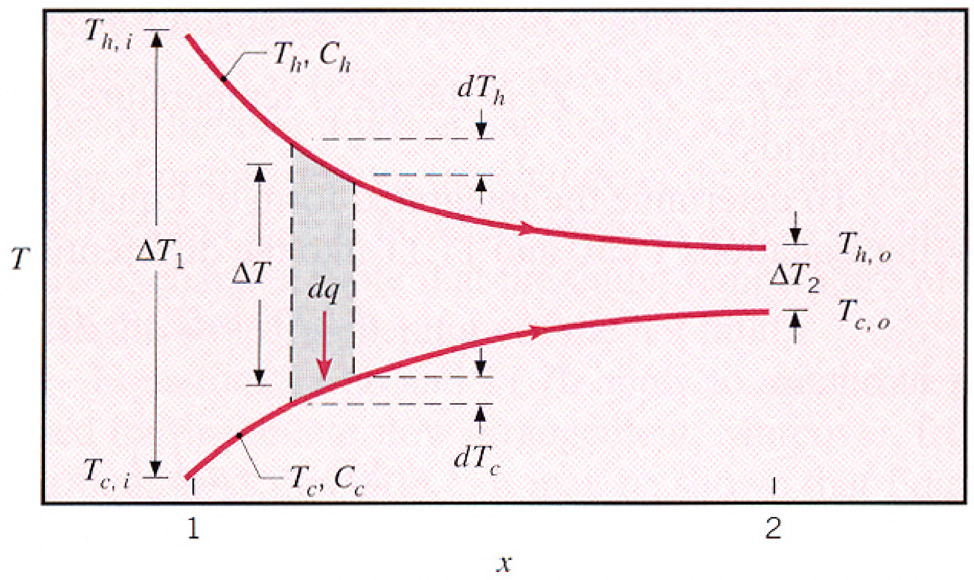
$$q = (\dot{m}c_p)_h \left(T_{h,i} - T_{h,o}\right) = C_h \left(T_{h,i} - T_{h,o}\right)$$

$$= (\dot{m}c_p)_c \left(T_{c,o} - T_{c,i}\right) = C_c \left(T_{c,o} - T_{c,i}\right)$$

$$\equiv UA\Delta T_m$$

$\dot{m}c_p = C$: **heat capacity rate**





$$dq = -\dot{m}_h c_{p,h} dT_h = -C_h dT_h$$

$$= \dot{m}_c c_{p,c} dT_c = C_c dT_c$$

$$= UdA\left(T_h - T_c\right) = UdA\Delta T$$

$$dT_h = -\frac{dq}{C_h}, \quad dT_c = \frac{dq}{C_c}$$

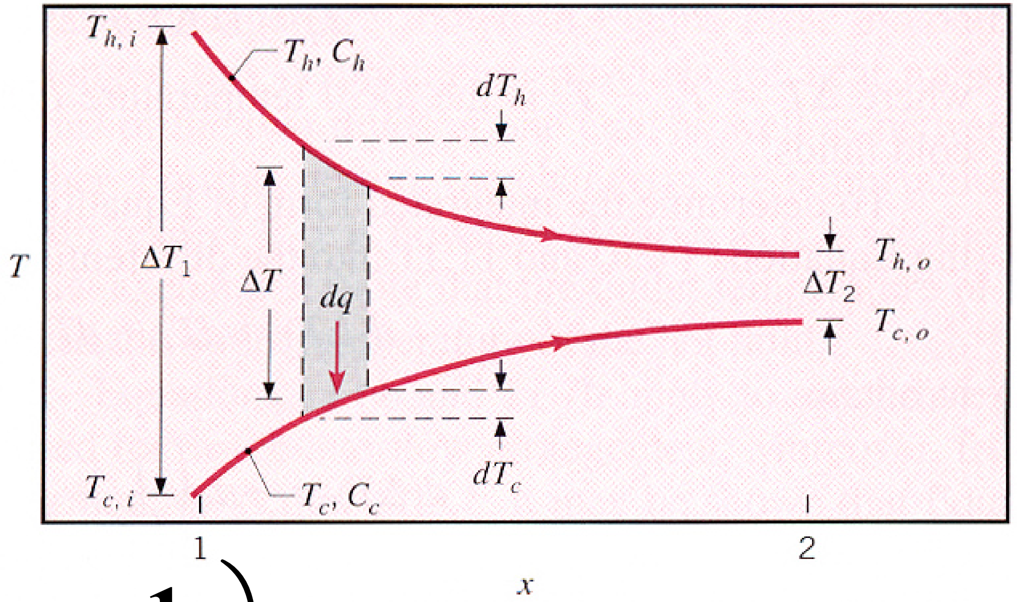
$$d(\Delta T) = d\left(T_h - T_c\right)$$

$$= dT_h - dT_c$$

$$= -dq\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$dq = -\frac{d(\Delta T)}{\left(\frac{1}{C_h} + \frac{1}{C_c}\right)}$$

$$= U dA \Delta T$$



$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = \int_1^2 -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right) dA$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

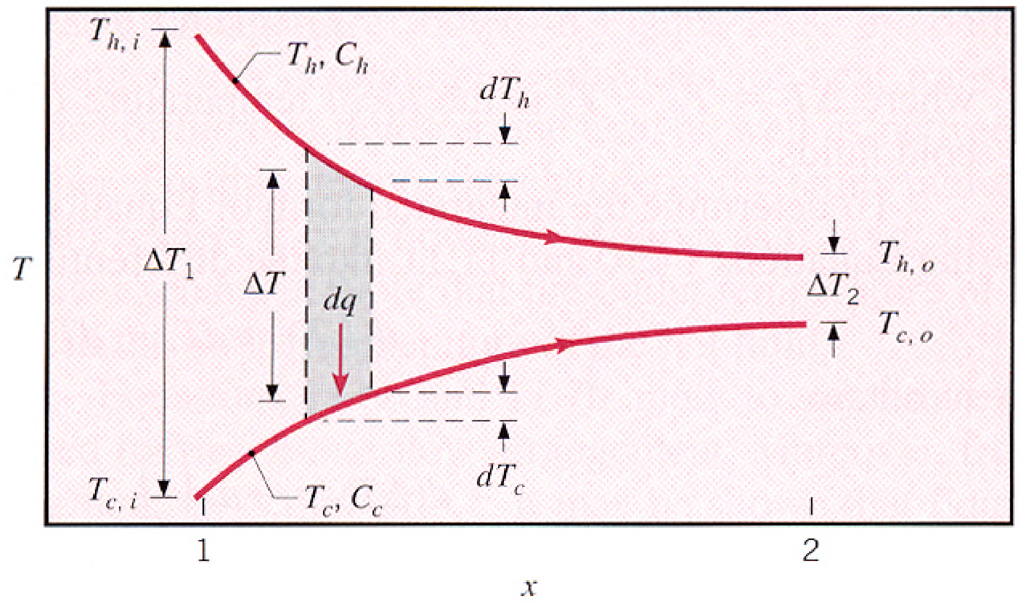
$$q = C_h\left(T_{h,i} - T_{h,o}\right) = C_c\left(T_{c,o} - T_{c,i}\right)$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{T_{h,i}-T_{h,o}}{q}+\frac{T_{c,o}-T_{c,i}}{q}\right)$$

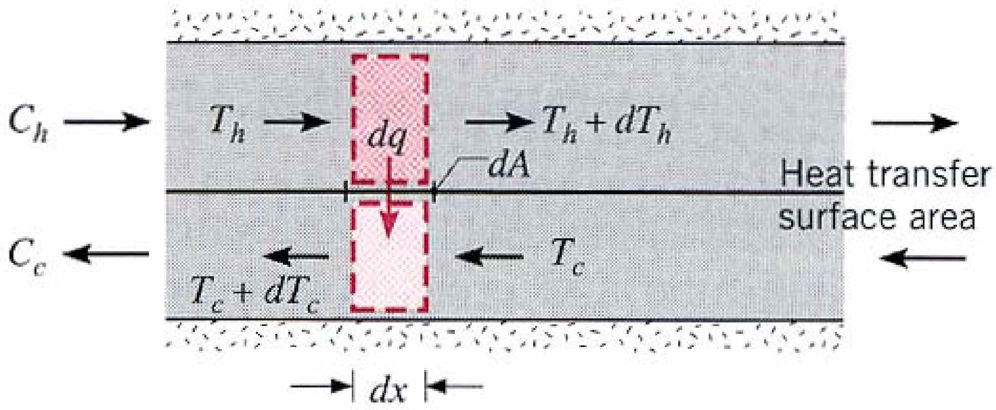
$$= -\frac{UA}{q}\left[\left(T_{h,i}-T_{c,i}\right)-\left(T_{h,o}-T_{c,o}\right)\right] = -\frac{UA}{q}\left[\Delta T_1-\Delta T_2\right]$$

$$q = UA\frac{\Delta T_2-\Delta T_1}{\ln\left(\Delta T_2/\Delta T_1\right)} \equiv UA\Delta T_{\mathrm{lm}}$$

$$\Delta T_{\mathrm{lm}} = \frac{\Delta T_2-\Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

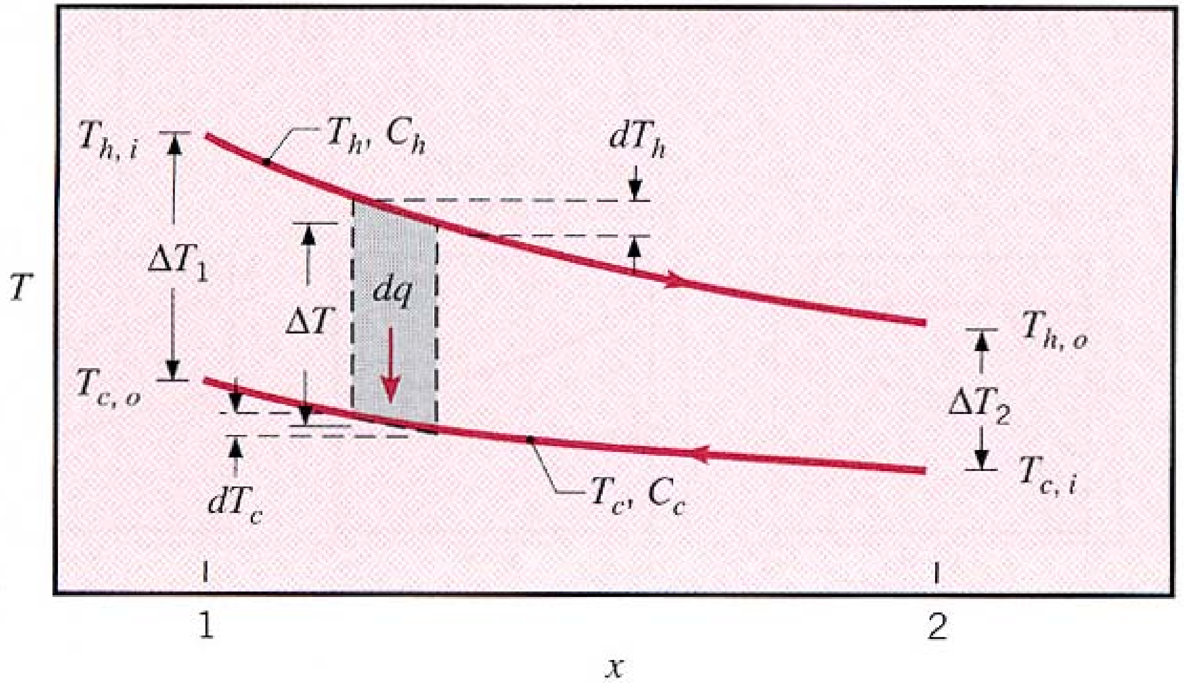


Counterflow Heat Exchanger



$$q = UA\Delta T_{\text{lm}}$$

$$\Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\dfrac{\Delta T_2}{\Delta T_1}\right)}$$



$$\Delta T_1 = T_{h1} - T_{c1}$$

$$= T_{h,i} - T_{c,o}$$

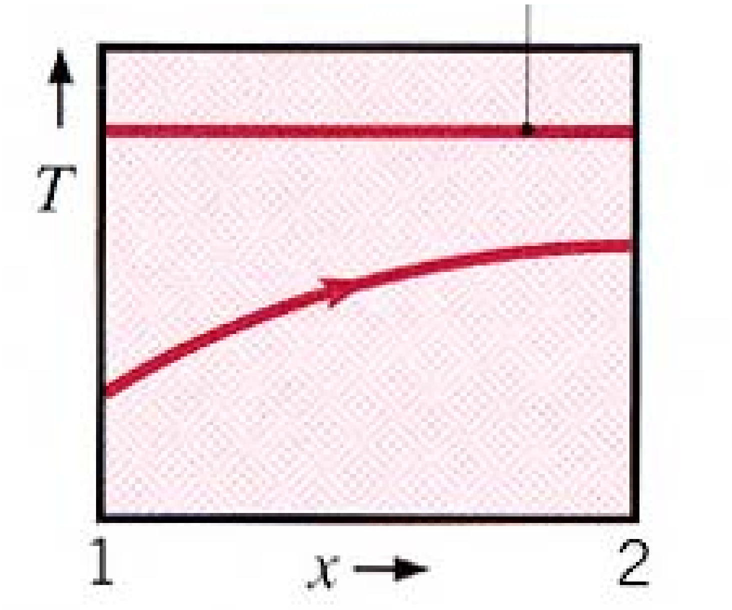
$$\Delta T_2 = T_{h2} - T_{c2}$$

$$= T_{h,o} - T_{c,i}$$

Special Operating Conditions

$$q = C_h\left(T_{h,i} - T_{h,o}\right) = C_c\left(T_{c,o} - T_{c,i}\right)$$

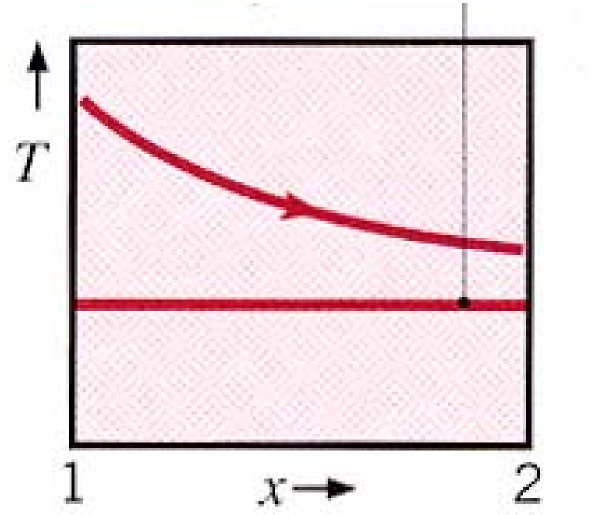
a) $C_h >> C_c$ **or a condensing vapor (**$C_h \to \infty$ **)**



$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right)$$

$$= \dot{m}_h h_{fg}$$

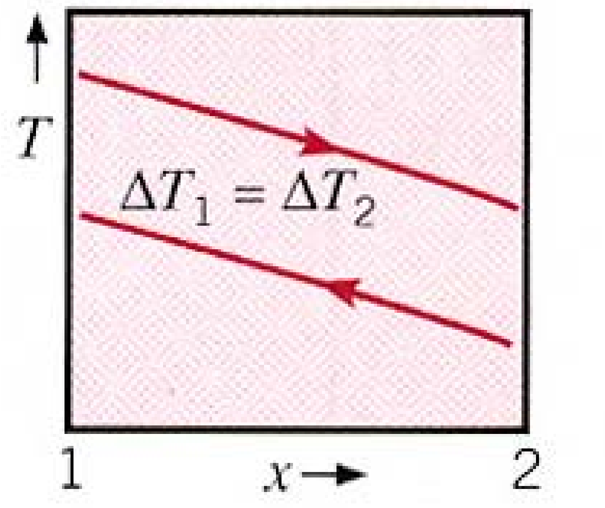
b) $C_h << C_c$ or an evaporating liquid ($C_c \to \infty$)



$$q = \dot{m}_c c_{p,c} \left(T_{c,o} - T_{c,i} \right)$$

$$= \dot{m}_c h_{fg}$$

c) $C_h = C_c$



$$\Delta T_1 = \Delta T_2$$

$$q = C_c \Delta T_1 = C_h \Delta T_2$$

$$\Delta T_1 = \Delta T_2 = \Delta T_{\text{lm}}$$

$$\Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2 / \Delta T_1 \right)}$$

Multipass and Cross-flow Heat Exchangers

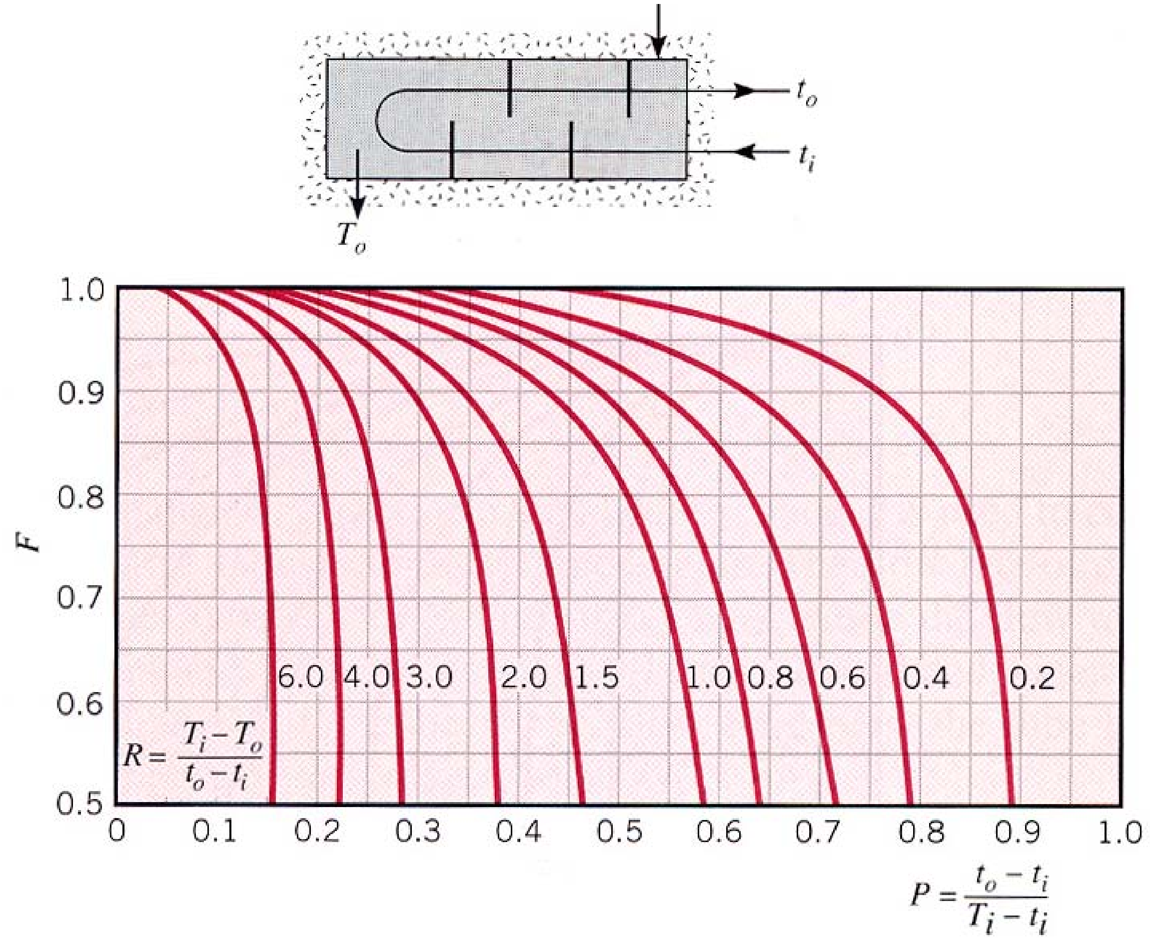
$$\Delta T_{\text{lm}} = F\,\Delta T_{\text{lm,CF}}$$

F: correction factor

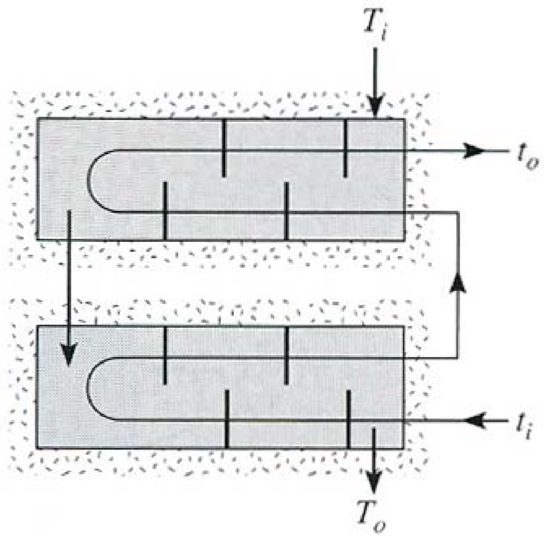
CF: counterflow condition

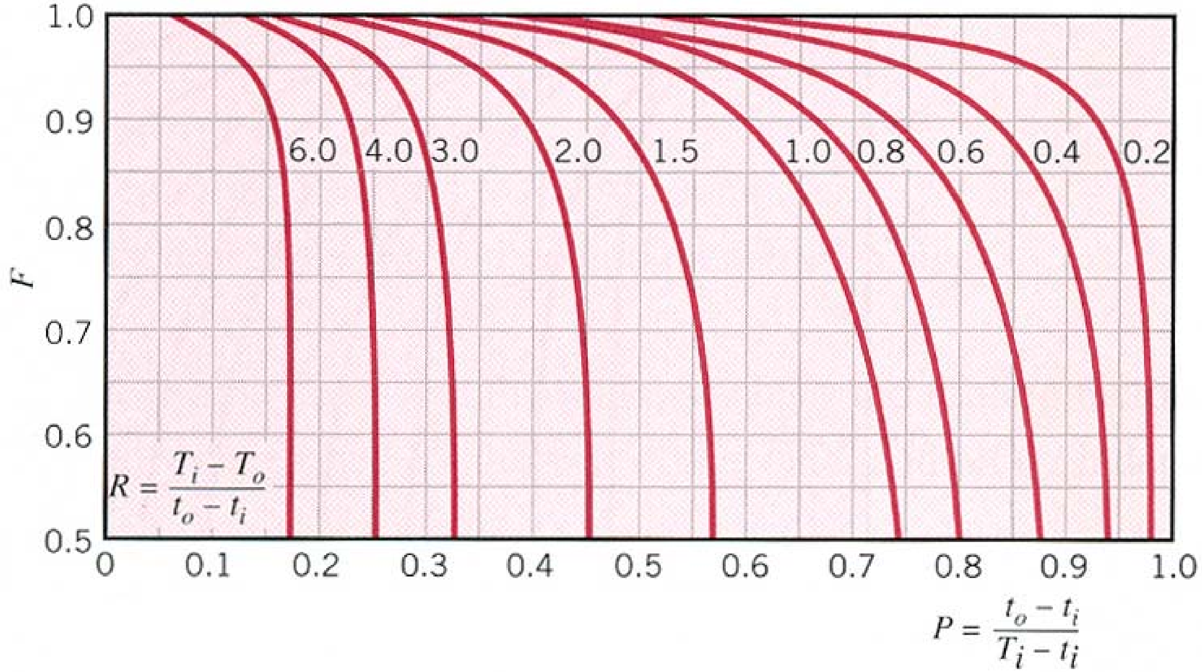
$$\Delta T_1 = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,o} - T_{c,i}$$

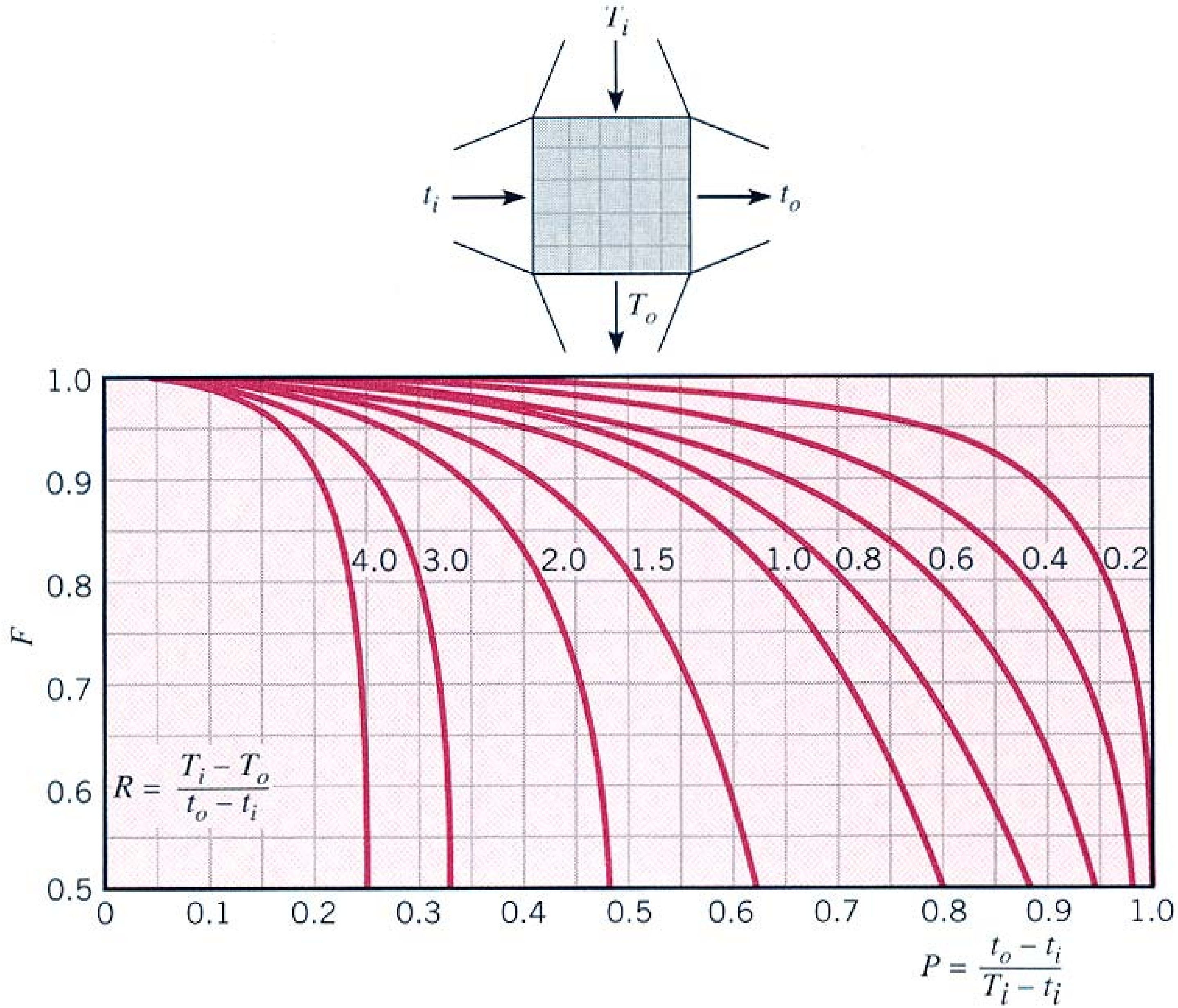


Correction factor for a shell-and-tube heat exchanger with one shell and any multiple of two tube passes

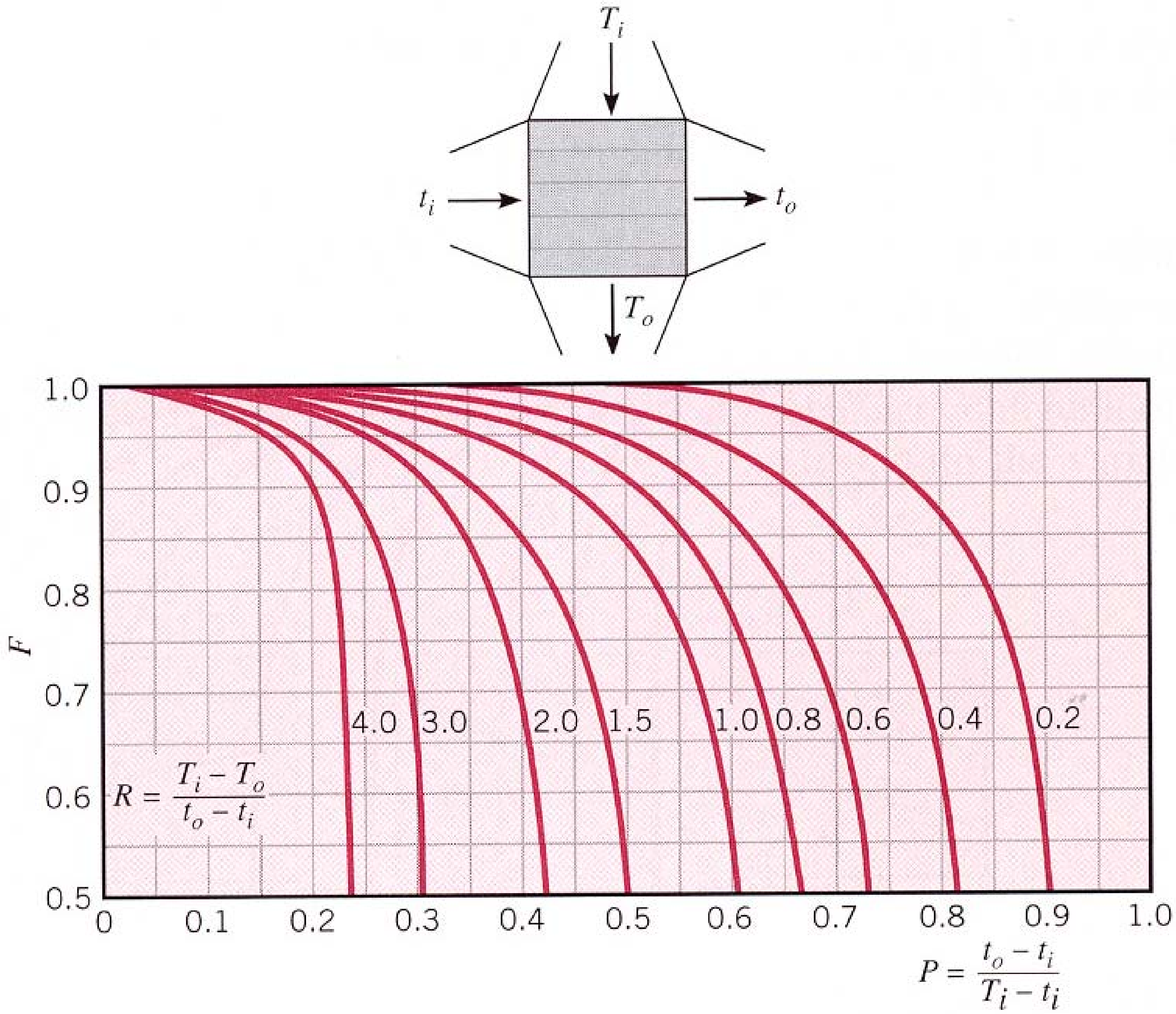




Correction factor for a shell-and-tube heat exchanger with two shell passes and any multiple of four tube passes

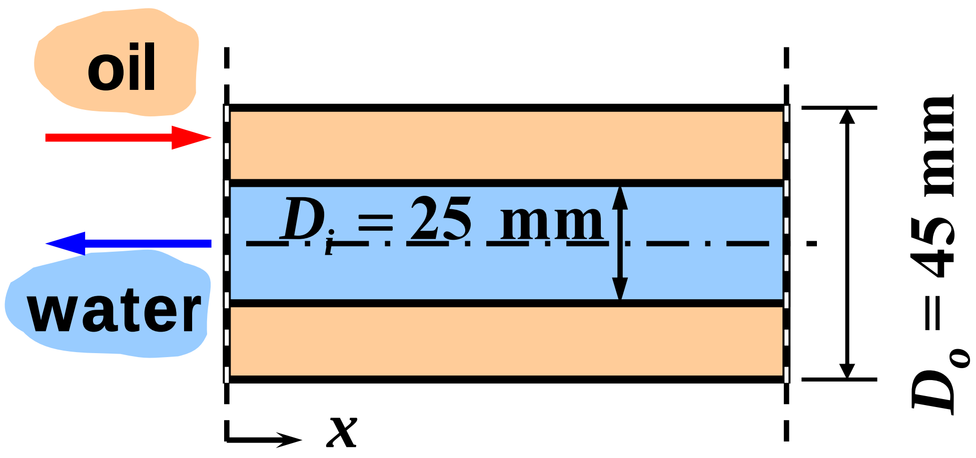


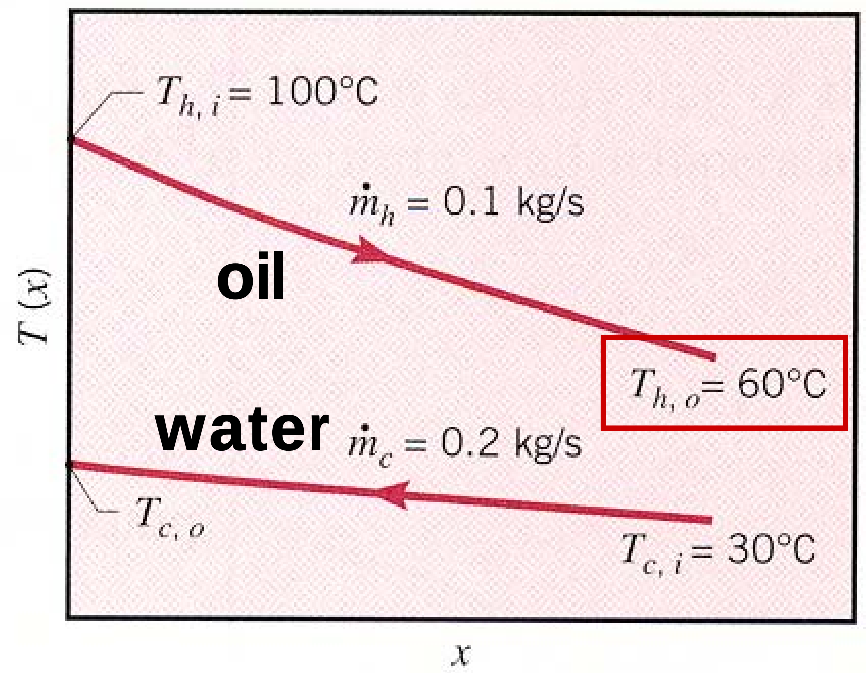
Correction factor for a single-pass, cross-flow heat exchangers with both fluid mixed



Correction factor for a single-pass, cross-flow heat exchangers with one fluid mixed and the other unmixed

Example 11.1





Find:

Tube length to achieve a desired hot fluid outlet temperature

Assumption:

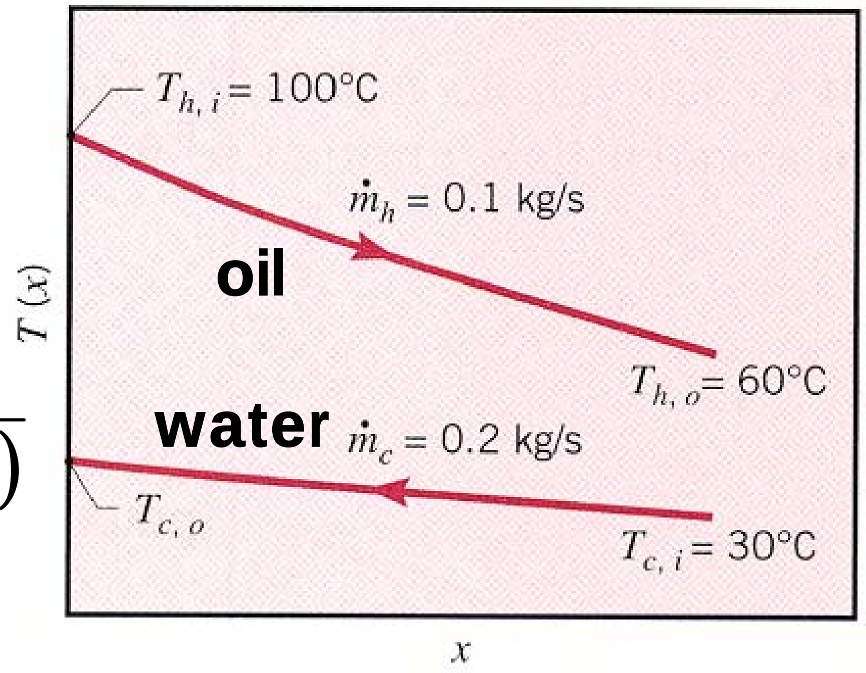
Negligible tube wall thermal resistance and fouling factors

$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right)$$

$$= \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)$$

$$\equiv UA\Delta T_{\mathrm{lm}}, \quad A = \pi D_i L$$

$$L = \frac{q}{U\pi D_i \Delta T_{\mathrm{lm}}}, \quad U = \frac{1}{\left(1/h_i\right)+\left(1/h_o\right)}$$



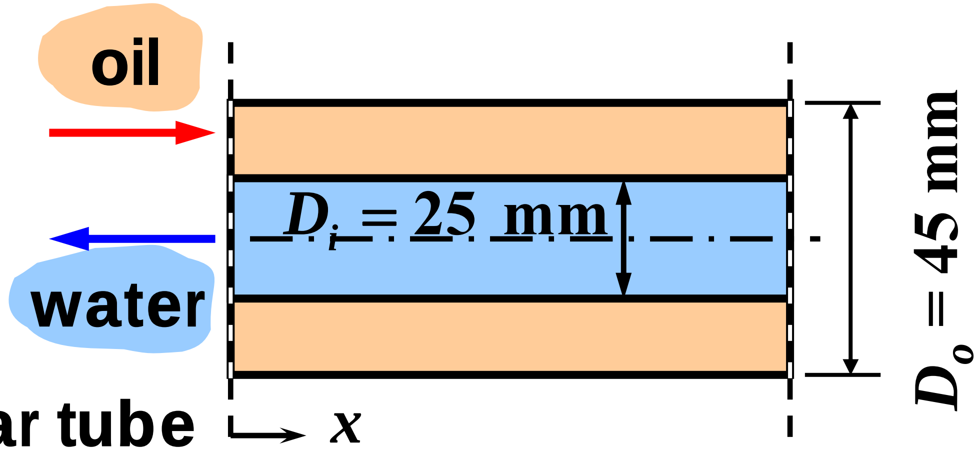
$$\Delta T_{\mathrm{lm}} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ln\left[\left(T_{h,i} - T_{c,o}\right)/\left(T_{h,o} - T_{c,i}\right)\right]}$$

$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = 8524\ \mathrm{W} \quad c_{p,h} = 2131\,\mathrm{J/kg\cdot K}\ \textbf{(oil at } T = 80^\circ\mathrm{C)}$$

$$T_{c,o} = \frac{q}{\dot{m}_c c_{p,c}} + T_{c,i} = 40.2^\circ\mathrm{C} \quad c_{p,c} = 4178\ \mathrm{J/kg\cdot K}\ \textbf{(water at } T = 35^\circ\mathrm{C)}$$

$$\Delta T_{\mathrm{lm}} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ln\left[\left(T_{h,i} - T_{c,o}\right)/\left(T_{h,o} - T_{c,i}\right)\right]} = 43.2^\circ\mathrm{C}$$

$$U = \frac{1}{(1/h_i)+(1/h_o)}$$



h_i : water flow in a circular tube

water: $\left(\bar{T}_c \approx 35°\mathrm{C}\right)$

$$\mu = 725\times10^{-6}\,\mathrm{N\cdot s/m^2},\; k = 0.625\ \mathrm{W/m\cdot K},\ \mathrm{Pr} = 4.85$$

$$\mathrm{Re}_D = \frac{4\dot{m}_c}{\pi D_i \mu} = 14,050$$

Dittus-Boelter equation

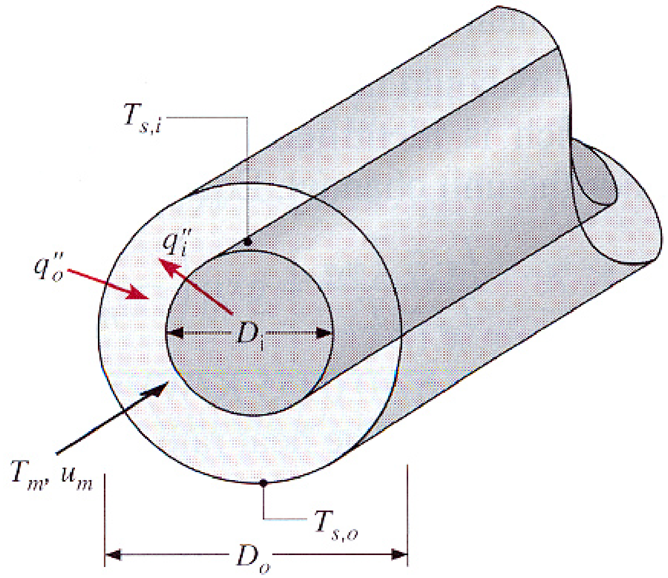
$$\mathrm{Nu}_D = 0.023\mathrm{Re}_D^{4/5}\mathrm{Pr}^{0.4} = 0.023(14,050)^{4/5}(4.85)^{0.4} = 90$$

$$h_i = \mathrm{Nu}_D \frac{k}{D_i} = 2250\ \mathrm{W/m^2\cdot K}$$

h_o : oil flow through an annulus

oil: $\left(\overline{T}_h = 80°\text{C}\right)$

$$\mu = 3.25\times10^{-2}\,\text{N}\cdot\text{s}/\text{m}^2,\; k = 0.138\ \text{W/m}\cdot\text{K}$$



$$\text{Re}_D = \frac{\rho u_m D_h}{\mu}$$

$$D_h = \frac{4(\pi/4)\left(D_o^2 - D_i^2\right)}{\pi D_o + \pi D_i} = D_o - D_i$$

$$\text{Re}_D = \frac{\rho\left(D_o - D_i\right)}{\mu}\frac{\dot{m}_h}{\rho\pi\left(D_o^2 - D_i^2\right)/4} = \frac{4\dot{m}_h}{\pi\left(D_o + D_i\right)\mu} = 56.0$$

The annular flow is therefore laminar. The convection coefficient at the inner surface may be obtained from Table 8.2 with $D_i/D_o = 0.56$.

$$\text{Nu}_{D_h} = \frac{h_o D_h}{k} = 5.56, \quad h_o = \text{Nu}_{D_h}\frac{k}{D_h} = 38.4\,\text{W/m}^2\cdot\text{K}$$

The overall convection coefficient is then

$$U = \frac{1}{(1/h_i) + (1/h_o)} = 37.8\,\mathrm{W/m^2 \cdot K}$$

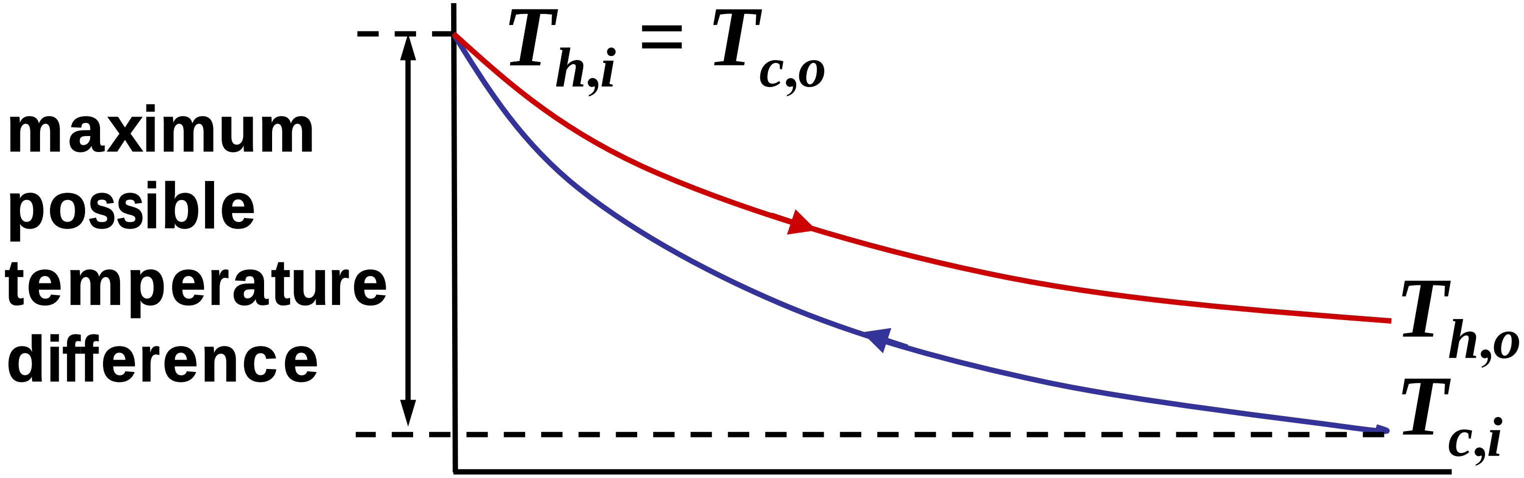
$$L = \frac{q}{U \pi D_i \Delta T_{\mathrm{lm}}} = 66.5\,\mathrm{m}$$

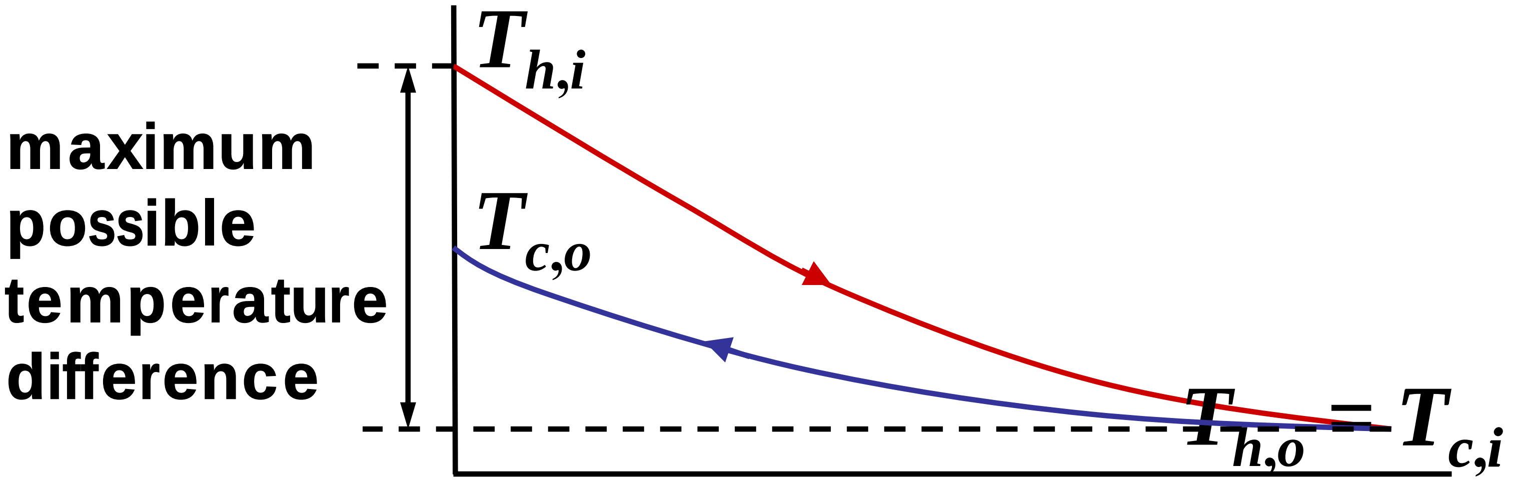
Heat Exchanger Analysis: ε-NTU Method

Useful when only the inlet fluid temperatures are known

Effectiveness of a Heat Exchanger

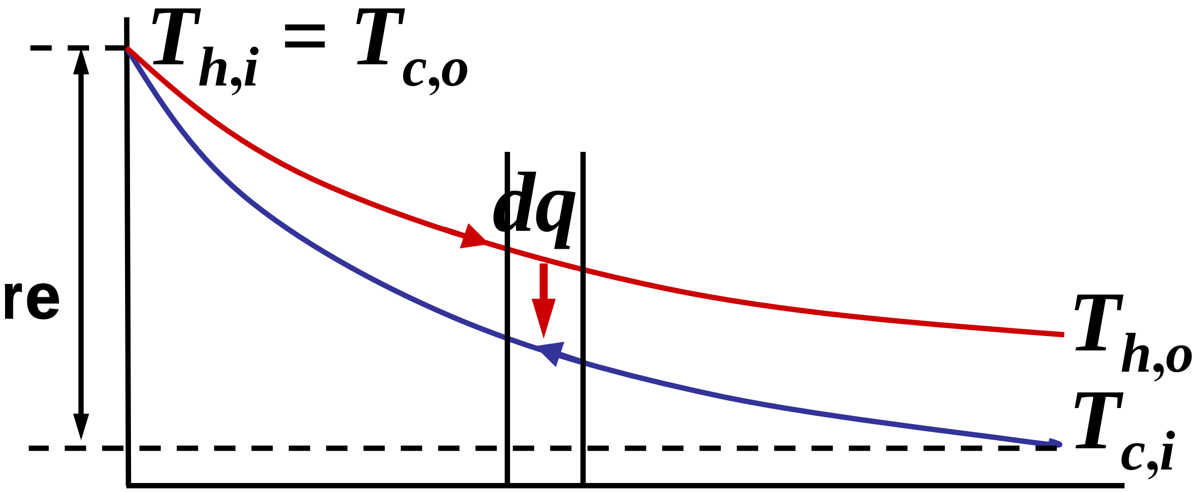
counterflow heat exchanger with infinite length





when $C_c < C_h$

Temperature variation in the low temperature fluid is large.



$$dq = -C_h dT_h = C_c dT_c, \qquad |dT_c| > |dT_h|$$

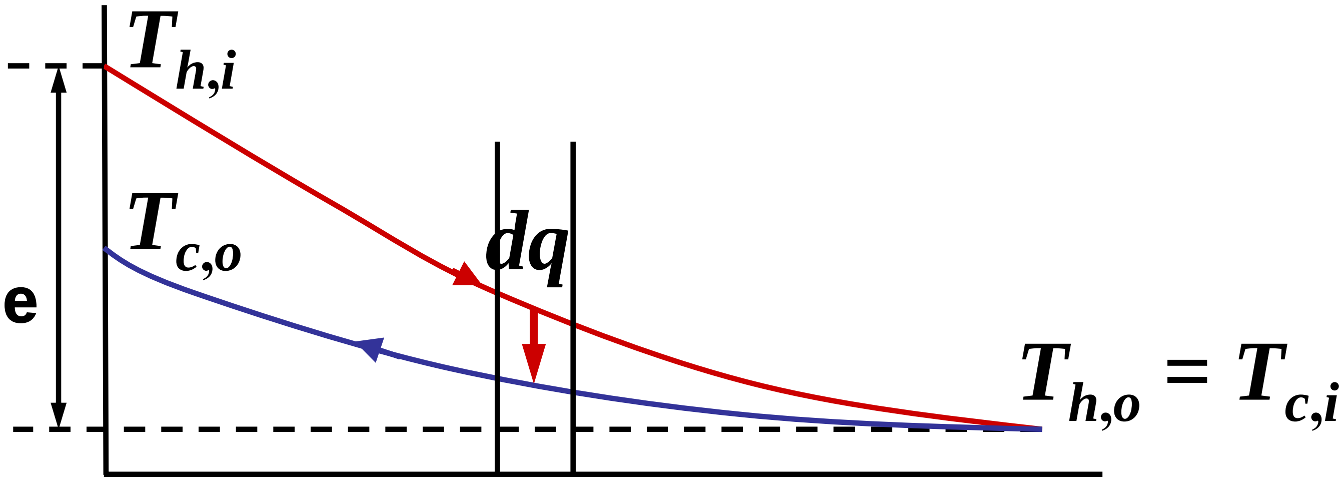
$$A \to \infty \qquad T_{c,o} = T_{h,i}$$

$$q_{\max} = C_c\left(T_{c,o} - T_{c,i}\right) = C_c\left(T_{h,i} - T_{c,i}\right) = C_{\min}\left(T_{h,i} - T_{c,i}\right)$$

when $C_h < C_c$

Temperature variation in the high temperature fluid is large.

maximum possible temperature difference



$$dq = -C_h dT_h = C_c dT_c, \quad |dT_h| > |dT_c|$$

$$q_{\max} = C_h\left(T_{h,i} - T_{h,o}\right) = C_h\left(T_{h,i} - T_{c,i}\right) = C_{\min}\left(T_{h,i} - T_{c,i}\right)$$

in either case $q_{\max} = C_{\min}\left(T_{h,i} - T_{c,i}\right)$

effectiveness:

$$\varepsilon \equiv \frac{q}{q_{\max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)}$$

$$q = \varepsilon q_{\max} = \varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right)$$

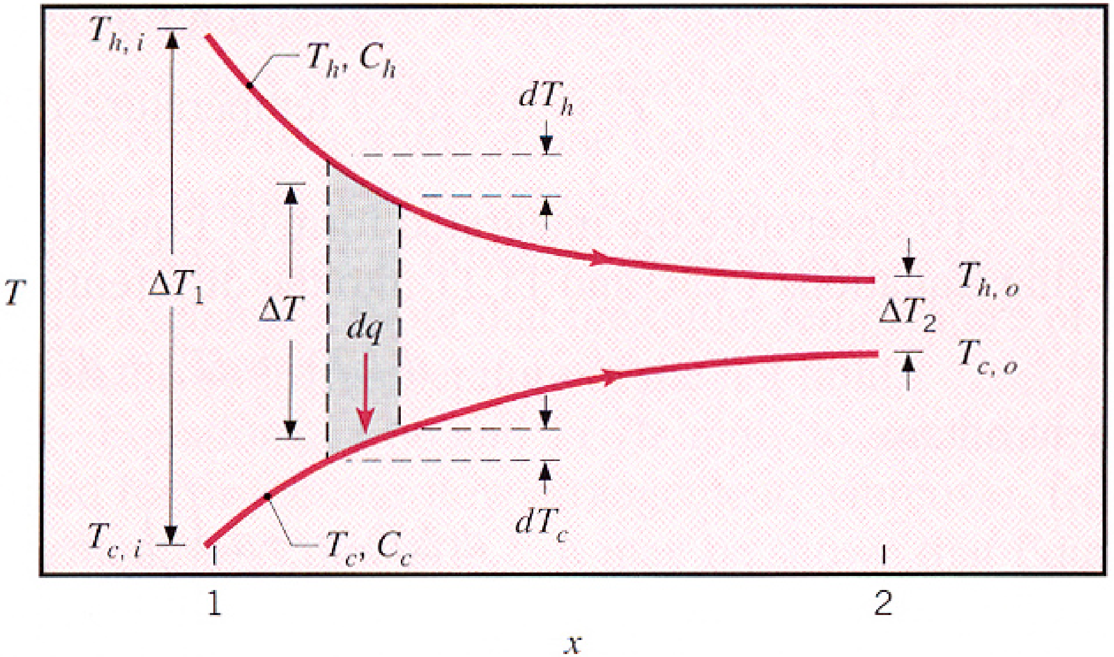
Number of Transfer Unit:

$$\mathrm{NTU} \equiv \frac{UA}{C_{\min}}$$

Effectiveness-NTU Relations

Ex) parallel-flow heat exchanger for which

$$C_{\min} = C_h$$



$$\varepsilon = \frac{q}{C_{\min}\left(T_{h,i} - T_{c,i}\right)}$$

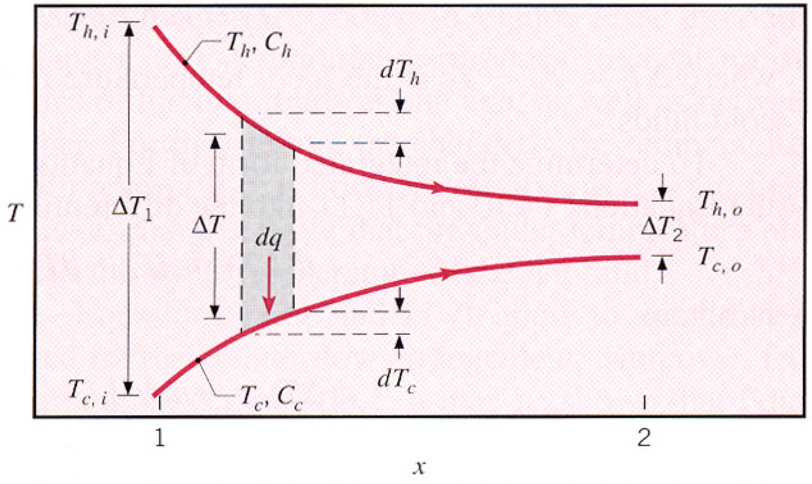
$$C_{\min} = C_h$$

$$C_{\max} = C_c$$

$$q = UA\Delta T_{\mathrm{lm}} = C_{\min}\left(T_{h,i} - T_{h,o}\right) = C_{\max}\left(T_{c,o} - T_{c,i}\right)$$

$$= \varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right)$$

$$UA = \frac{\varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right)}{\Delta T_{\mathrm{lm}}}$$



$$\mathrm{NTU} = \frac{UA}{C_{\min}} = \frac{\varepsilon\left(T_{h,i} - T_{c,i}\right)}{\Delta T_{\mathrm{lm}}}$$

$$= \frac{\varepsilon\left(T_{h,i} - T_{c,i}\right)\ln\dfrac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}}}{\left(T_{h,o} - T_{c,o}\right) - \left(T_{h,i} - T_{c,i}\right)}$$

$$\Delta T_{\mathrm{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\dfrac{\Delta T_2}{\Delta T_1}\right)}$$

$$\mathrm{NTU} = \frac{UA}{C_{\min}} = \frac{\varepsilon\left(T_{h,i} - T_{c,i}\right)}{\Delta T_{\mathrm{lm}}}$$

$$= \frac{\varepsilon\left(T_{h,i} - T_{c,i}\right)\ln\dfrac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}}}{\left(T_{h,o} - T_{c,o}\right) - \left(T_{h,i} - T_{c,i}\right)}$$

$$\left(T_{h,o} - T_{c,o}\right) - \left(T_{h,i} - T_{c,i}\right) = \left(T_{h,o} - T_{h,i}\right) - \left(T_{c,o} - T_{c,i}\right)$$

$$q = C_{\min}\left(T_{h,i} - T_{h,o}\right) = C_{\max}\left(T_{c,o} - T_{c,i}\right)$$

$$= \varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right)$$

$$T_{h,i} - T_{h,o} = \varepsilon\left(T_{h,i} - T_{c,i}\right),\quad T_{c,o} - T_{c,i} = \varepsilon\frac{C_{\min}}{C_{\max}}\left(T_{h,i} - T_{c,i}\right)$$

$$\left(T_{h,o}-T_{c,o}\right)-\left(T_{h,i}-T_{c,i}\right)=\left(T_{h,o}-T_{h,i}\right)-\left(T_{c,o}-T_{c,i}\right)$$

$$=-\varepsilon\left(T_{h,i}-T_{c,i}\right)-\varepsilon\frac{C_{\min}}{C_{\max}}\left(T_{h,i}-T_{c,i}\right)$$

$$=-\varepsilon\left(T_{h,i}-T_{c,i}\right)\left(1+\frac{C_{\min}}{C_{\max}}\right)$$

$$\mathrm{NTU}=\frac{\varepsilon\left(T_{h,i}-T_{c,i}\right)\ln\dfrac{T_{h,o}-T_{c,o}}{T_{h,i}-T_{c,i}}}{-\varepsilon\left(1+\dfrac{C_{\min}}{C_{\max}}\right)\left(T_{h,i}-T_{c,i}\right)}=\frac{\ln\dfrac{T_{h,o}-T_{c,o}}{T_{h,i}-T_{c,i}}}{-\left(1+\dfrac{C_{\min}}{C_{\max}}\right)}$$

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp\left[-\mathrm{NTU}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]$$

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \frac{\left(T_{h,o} - T_{h,i}\right) + \left(T_{h,i} - T_{c,o}\right)}{T_{h,i} - T_{c,i}}$$

$$= \frac{\left(T_{h,o} - T_{h,i}\right) + \left(T_{h,i} - T_{c,i}\right) + \left(T_{c,i} - T_{c,o}\right)}{T_{h,i} - T_{c,i}}$$

$$q = C_{\min}\left(T_{h,i} - T_{h,o}\right) = C_{\max}\left(T_{c,o} - T_{c,i}\right)$$

$$= \varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right)$$

$$\frac{T_{h,o}-T_{c,o}}{T_{h,i}-T_{c,i}}=\frac{-\frac{q}{C_{\min}}+\frac{q}{\varepsilon C_{\min}}-\frac{q}{C_{\max}}}{\frac{q}{\varepsilon C_{\min}}}$$

$$=-\varepsilon+1-\varepsilon\frac{C_{\min}}{C_{\max}}=1-\varepsilon\left(1+\frac{C_{\min}}{C_{\max}}\right)$$

$$\frac{T_{h,o}-T_{c,o}}{T_{h,i}-T_{c,i}}=\exp\left[-\mathrm{NTU}\left(1+\frac{C_{\min}}{C_{\max}}\right)\right]$$

$$1-\varepsilon\left(1+\frac{C_{\min}}{C_{\max}}\right)=\exp\left[-\mathrm{NTU}\left(1+\frac{C_{\min}}{C_{\max}}\right)\right]$$

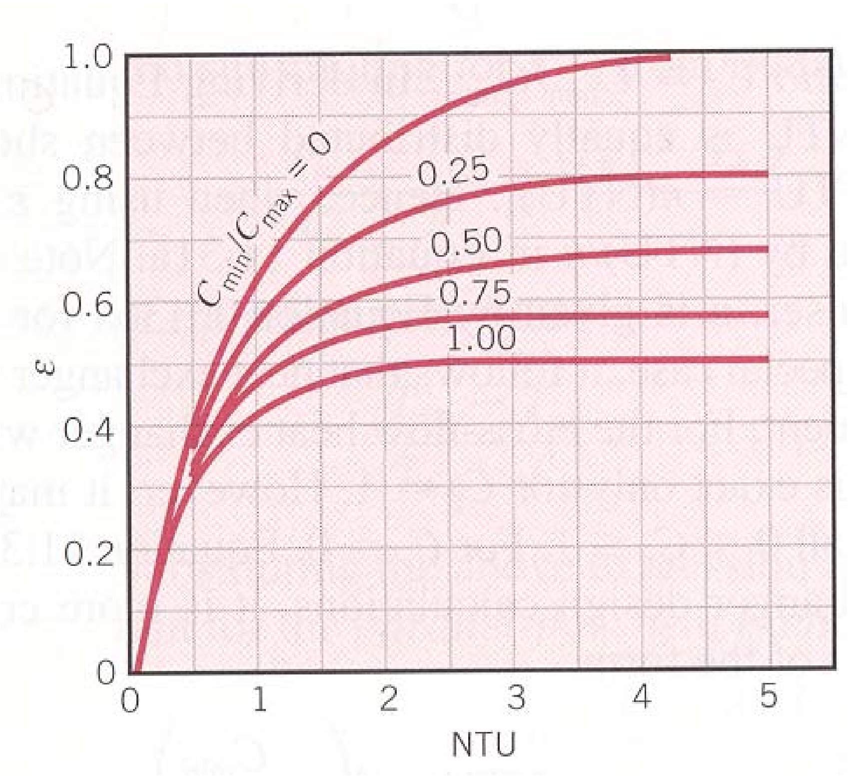
$$\varepsilon\left(1+\frac{C_{\min}}{C_{\max}}\right)=1-\exp\left[-\mathrm{NTU}\left(1+\frac{C_{\min}}{C_{\max}}\right)\right]$$

$$\varepsilon=\frac{1-\exp\left[-\mathrm{NTU}\left(1+C_r\right)\right]}{1+C_r},\quad C_r=\frac{C_{\min}}{C_{\max}}$$

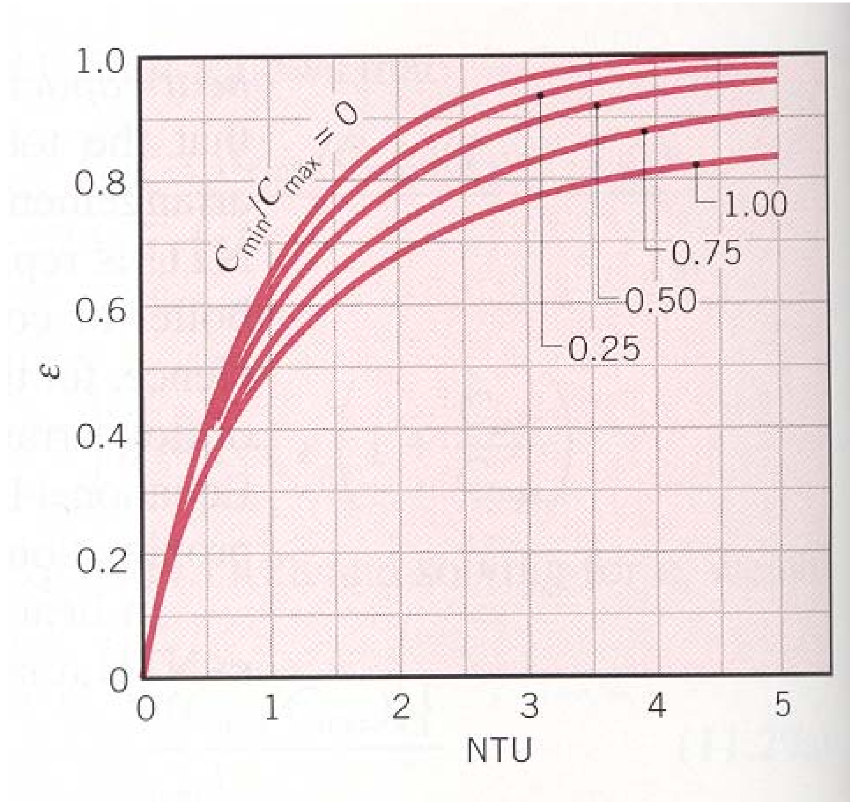
In general, $\varepsilon = f(\mathrm{NTU}, C_r)$

Heat Exchanger Effectiveness Relations

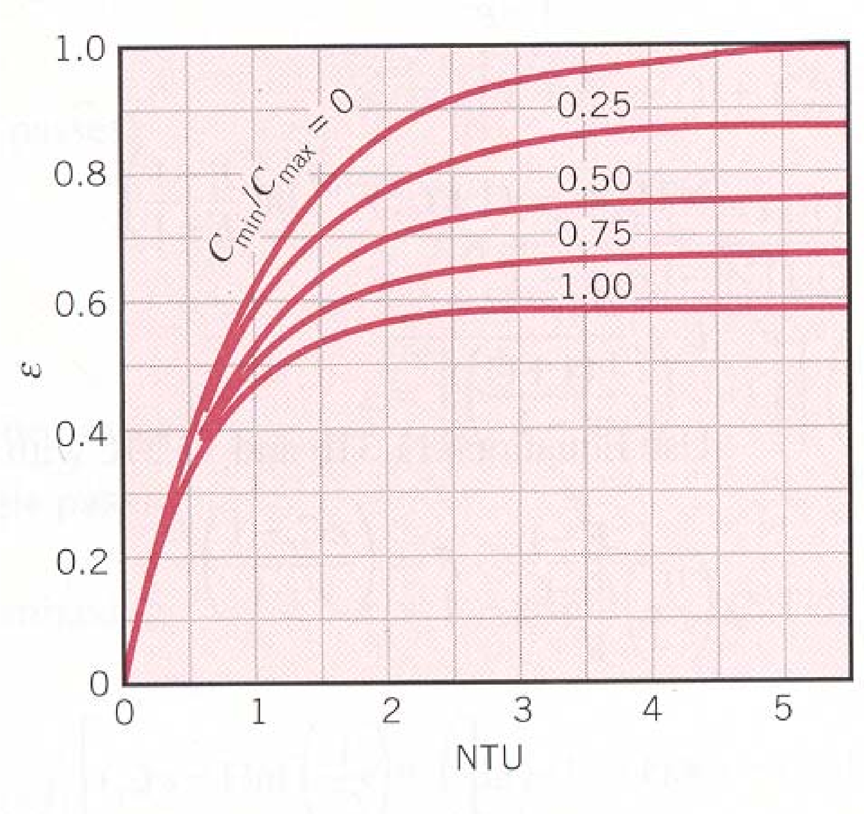
Flow Arrangement	Relation		
Concentric tube			
Parallel flow	$\varepsilon = \dfrac{1 - \exp\,[-\text{NTU}(1 + C_r)]}{1 + C_r}$		(11.28a)
Counterflow	$\varepsilon = \dfrac{1 - \exp\,[-\text{NTU}(1 - C_r)]}{1 - C_r \exp\,[-\text{NTU}(1 - C_r)]}$	$(C_r < 1)$	
	$\varepsilon = \dfrac{\text{NTU}}{1 + \text{NTU}}$	$(C_r = 1)$	(11.29a)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp\,[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp\,[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$		(11.30a)
n Shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$		(11.31a)
Cross-flow (single pass)			
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\text{NTU})^{0.22}\{\exp\,[-C_r(\text{NTU})^{0.78}] - 1\}\right]$		(11.32)
C_{max} (mixed), C_{min} (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\,\{-C_r[1 - \exp\,(-\text{NTU})]\})$		(11.33a)
C_{min} (mixed), C_{max} (unmixed)	$\varepsilon = 1 - \exp\,(-C_r^{-1}\{1 - \exp\,[-C_r(\text{NTU})]\})$		(11.34a)
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp\,(-\text{NTU})$		(11.35a)



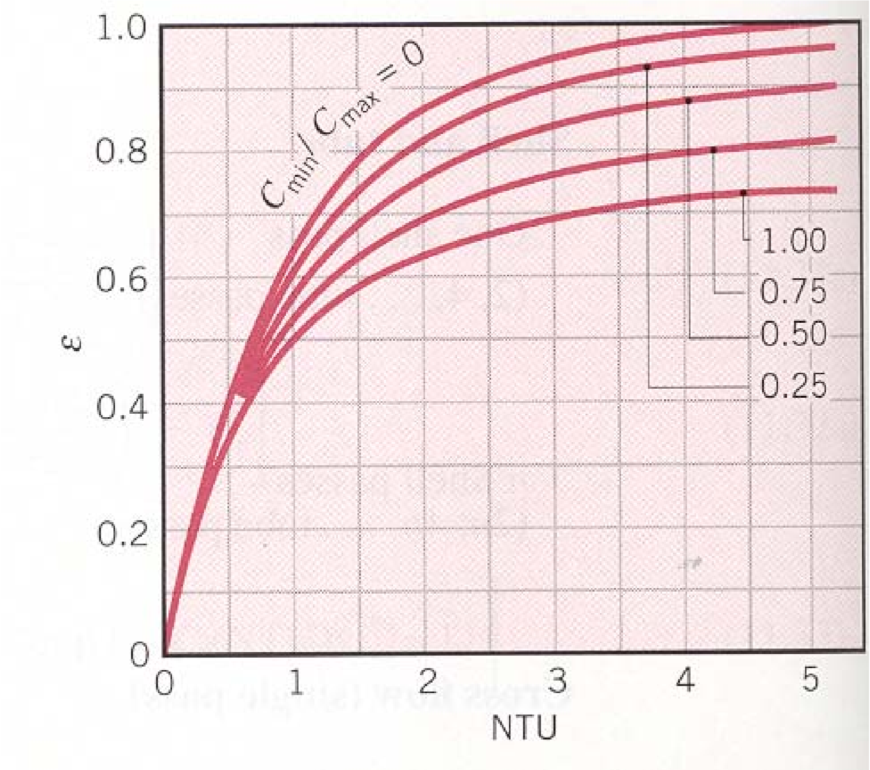
Effectiveness of a parallel flow heat exchanger



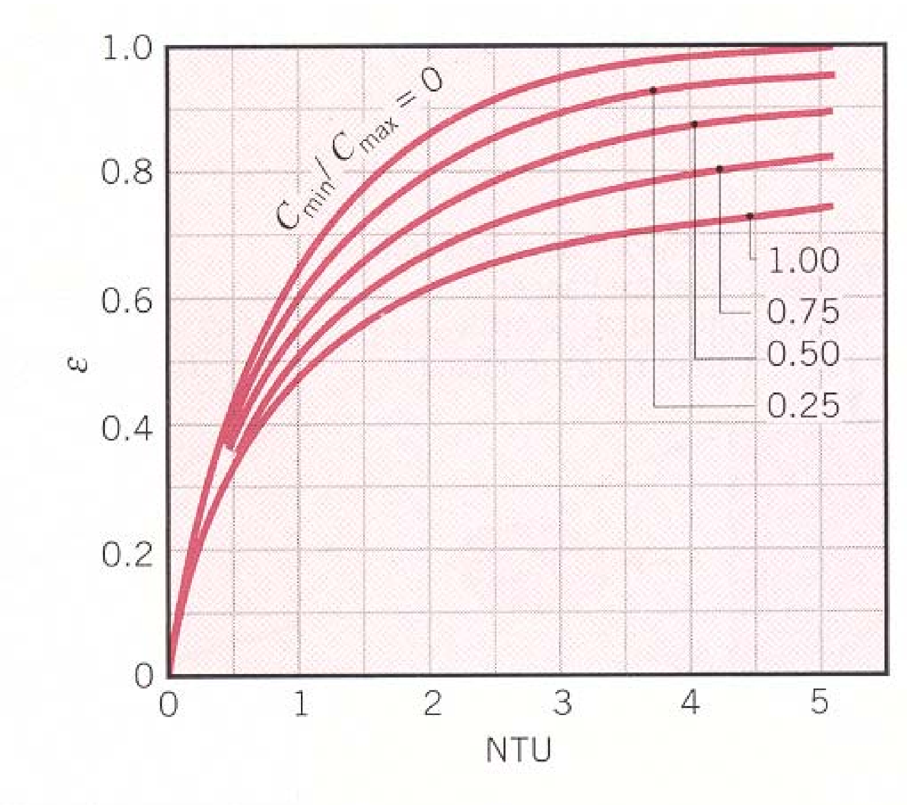
Effectiveness of a counter-flow heat exchanger



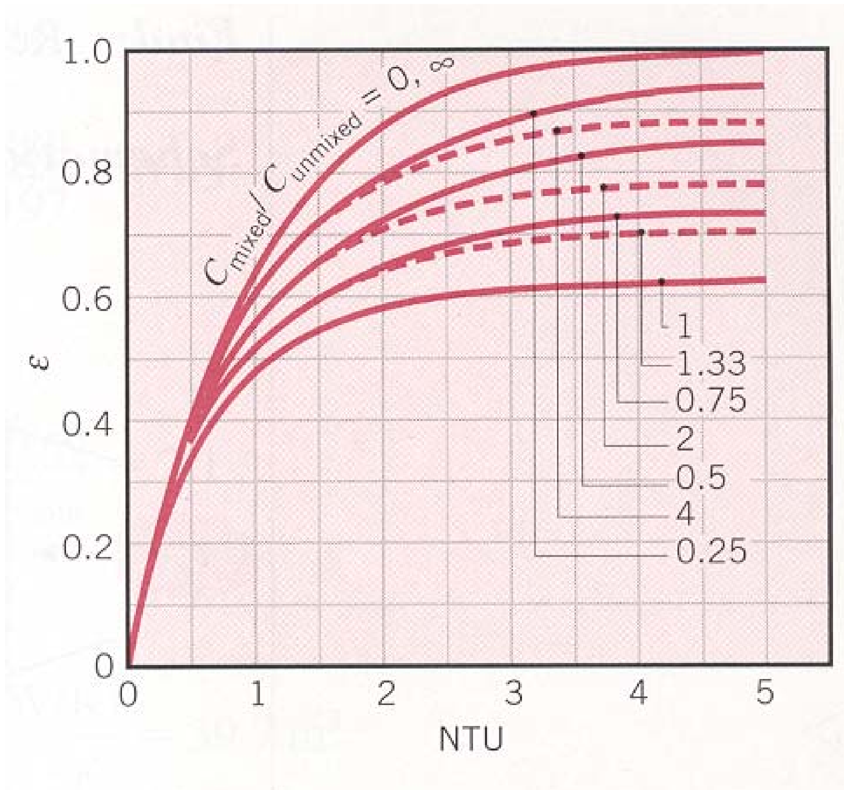
Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes



Effectiveness of a shell-and-tube heat exchanger with two shell passes and any multiple of four tube passes

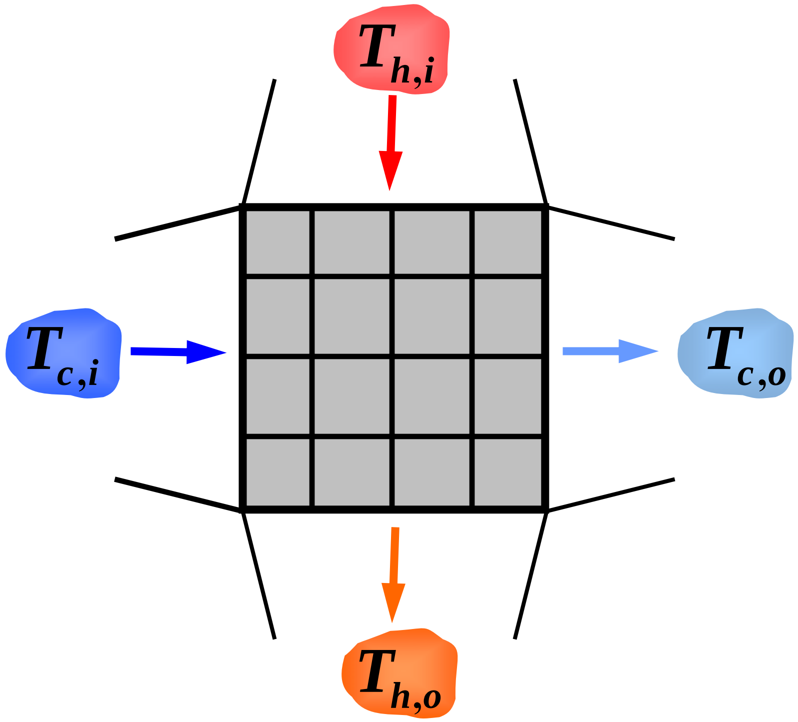


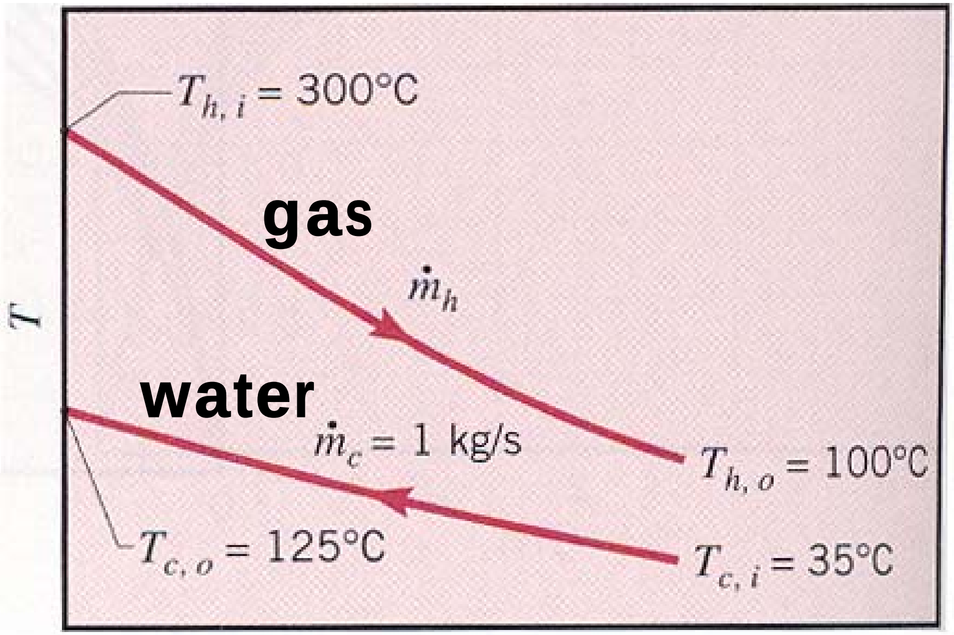
Effectiveness of a single-pass, cross-flow heat exchanger with both fluids unmixed



Effectiveness of a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed

Example 11.3





finned-tube, cross-flow heat exchanger

$U_h = 100\ \text{W/m}^2 \cdot \text{K}$

both fluids unmixed

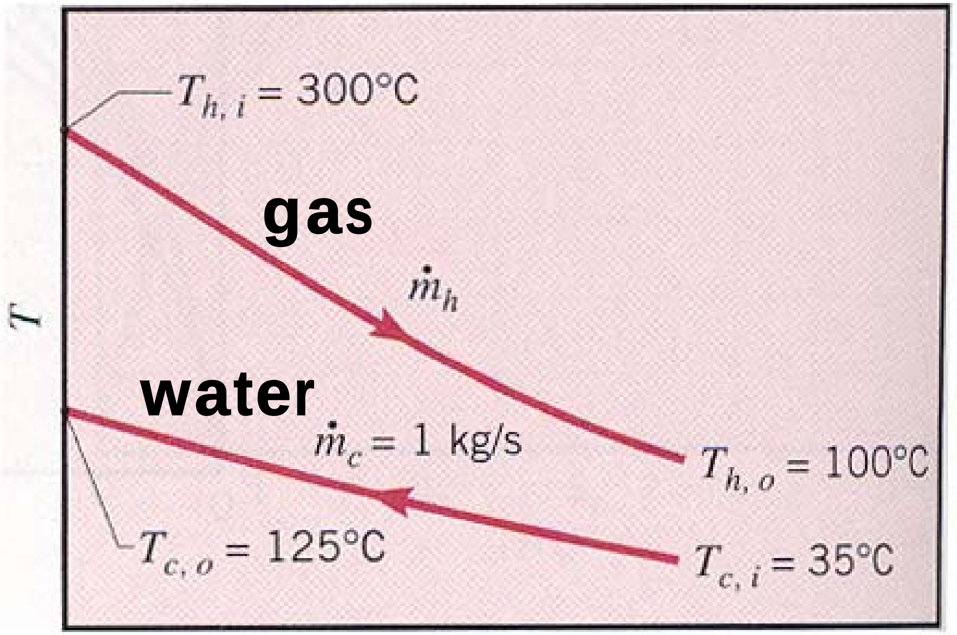
Find:

Required gas side surface area

$$\mathrm{NTU} = \frac{U_h A_h}{C_{\min}}$$

$$\rightarrow A_h = \frac{C_{\min}\mathrm{NTU}}{U_h}$$

$$\varepsilon = \frac{q}{C_{\min}\left(T_{h,i} - T_{c,i}\right)}$$



$$\varepsilon = 1 - \exp\left[\left(\frac{1}{C_r}\right)(\mathrm{NTU})^{0.22}\left\{\exp\left[-C_r(\mathrm{NTU})^{0.78}\right] - 1\right\}\right]$$

or Fig. 11.18

$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)$$

water: $\left(\bar{T}_c = 80°\mathrm{C}\right)$ $c_p = 4197\ \mathrm{J/kg\cdot K}$

gas: $c_p = 1000\ \mathrm{J/kg\cdot K}$

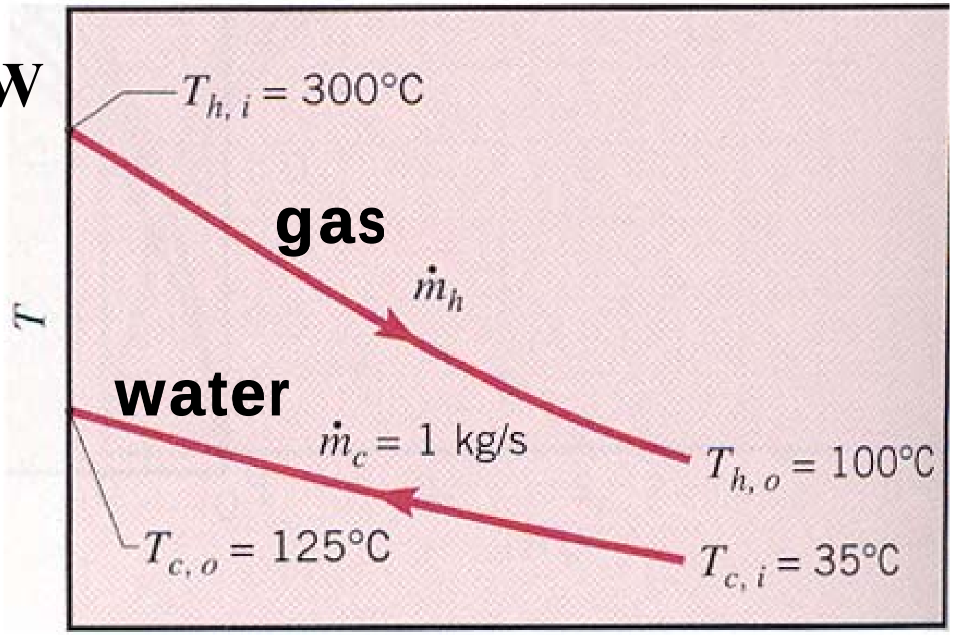
$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)$$

$$C_c = \dot{m}_c c_{p,c} = 4197\ \mathrm{W/K}$$

$$C_h = \dot{m}_h c_{p,h} = C_c \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}} = 1889\ \mathrm{W/K} = C_{\min}$$

$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = 3.77 \times 10^5\ \mathrm{W}$$

$$\varepsilon = \frac{q}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} = 0.75$$



from ε-NTU relation or from Fig. 11.18 with

$$C_r = \frac{C_{\min}}{C_{\max}} = \frac{1889}{4197} = 0.45$$

$$\mathrm{NTU} = \frac{U_h A_h}{C_{\min}} \approx 2.1, \quad A_h = \frac{C_{\min}\mathrm{NTU}}{U_h} = 39.7\ \mathrm{m}^2$$

This problem can also be solved by LMTD method.

Methodology of HX Calculation

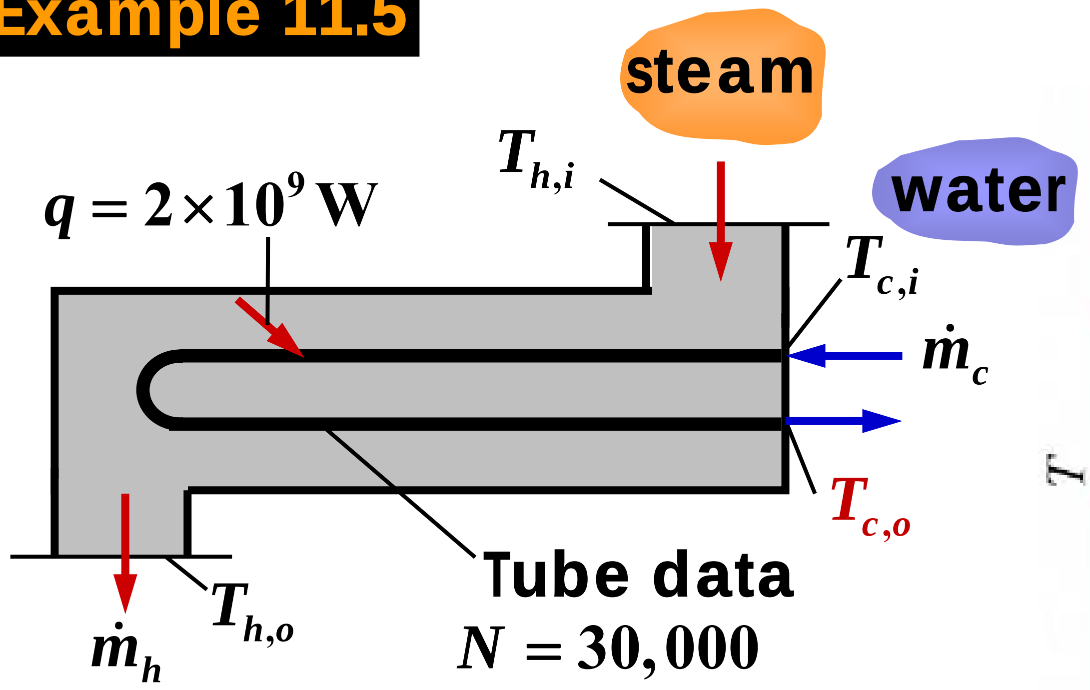
- **heat exchanger design**

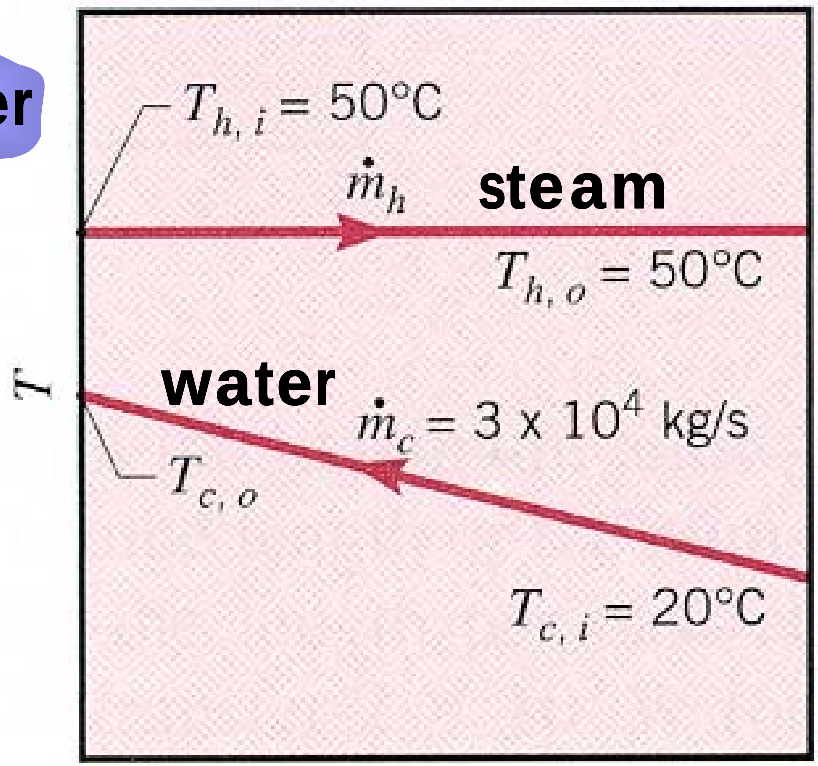
  LMTD, ε-NTU

- **heat exchanger performance analysis**

  mainly by ε-NTU

Example 11.5





Tube data

$N = 30{,}000$

$D = 0.025\,\text{m}$

$h_o = 11{,}000\ \text{W/m}^2\cdot\text{K}$

L = length/pass

$\dot{m} = 1\,\text{kg/s}$ per tube

Find:

1) Outlet temperature of the cooling water
2) Tube length per pass to achieve required heat transfer

Using the LMTD method

$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = \dot{m}_h h_{fg}$$

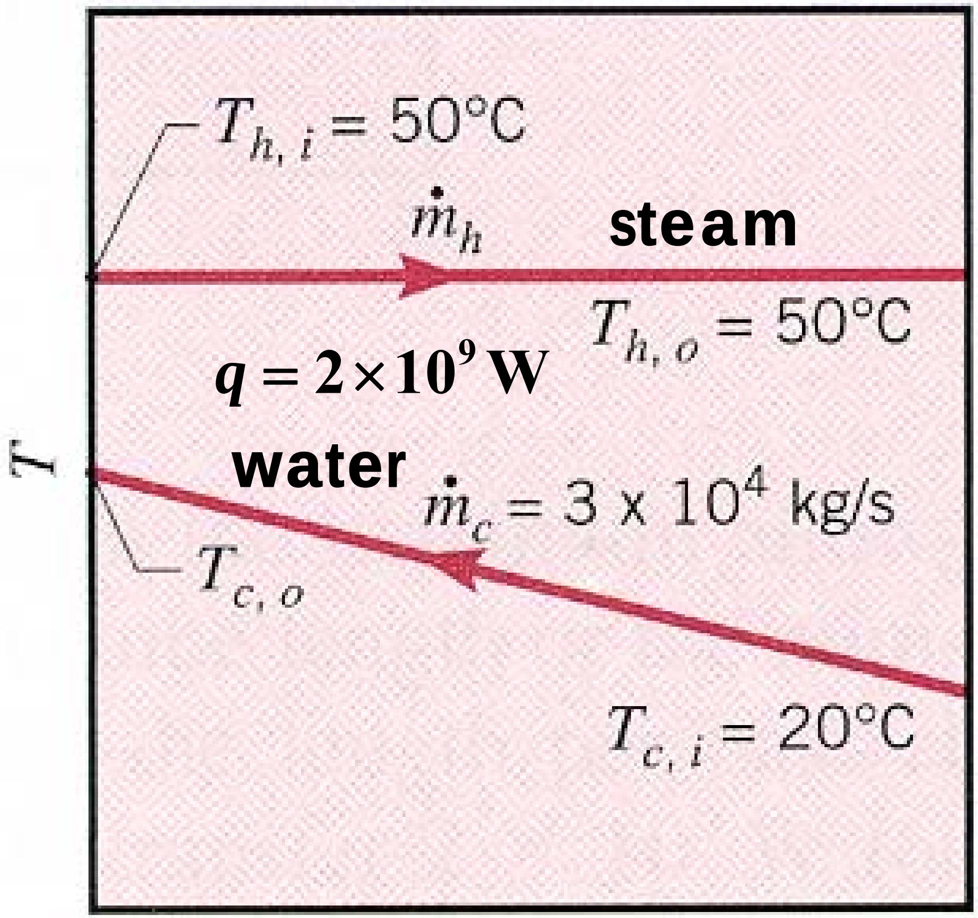
$$= \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)$$

$$\equiv UA\Delta T_{\text{lm}}$$

$$A = \frac{q}{U\Delta T_{\text{lm}}} = 2N\pi DL$$

$$L = \frac{q}{2UN\pi D_i F\,\Delta T_{\text{lm,CF}}}$$

$$U = \frac{1}{\left(1/h_i\right)+\left(1/h_o\right)}$$



$$\Delta T_{\text{lm}} = F\,\Delta T_{\text{lm,CF}},\quad \Delta T_{\text{lm,CF}} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ln\left[\left(T_{h,i} - T_{c,o}\right)/\left(T_{h,o} - T_{c,i}\right)\right]}$$

water: $\left(\bar{T}_c \approx 27°\text{C}\right)$

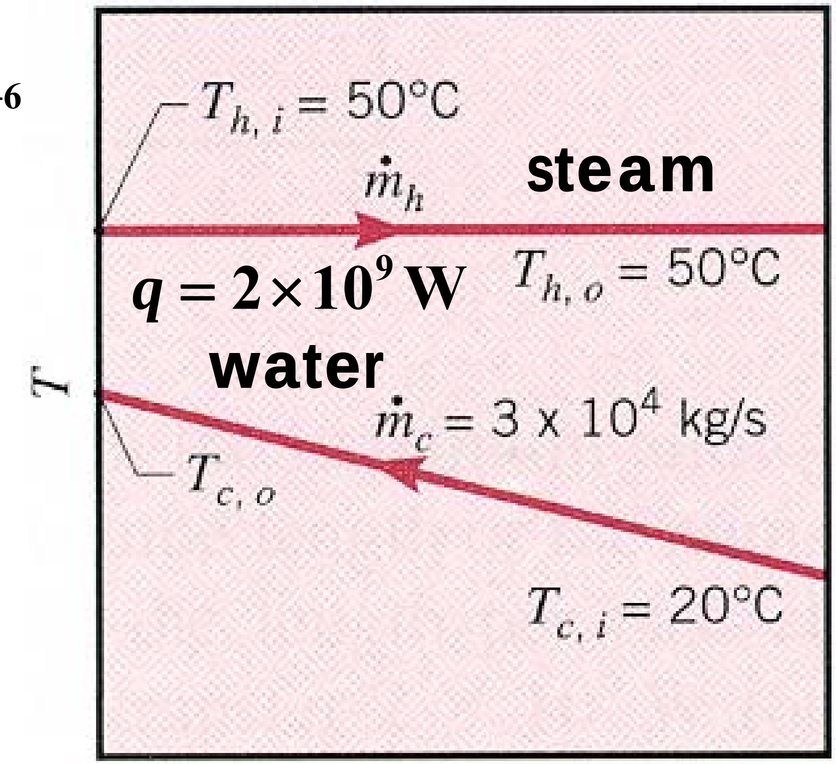
$$c_p = 4179\,\text{J/kg}\cdot\text{K}\ ,\ \ \mu = 855\times10^{-6}$$

$$k = 0.613\ \text{W/m}\cdot\text{K}\ ,\ \ \text{Pr} = 5.83$$

$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)$$

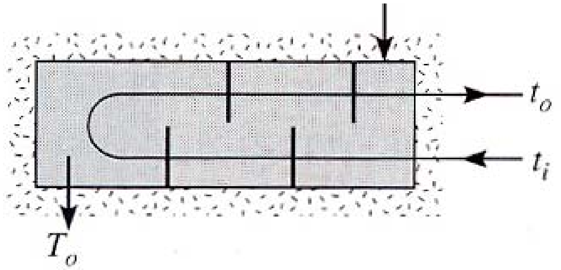
$$T_{c,o} = T_{c,i} + \frac{q}{\dot{m}_c c_{p,c}}$$

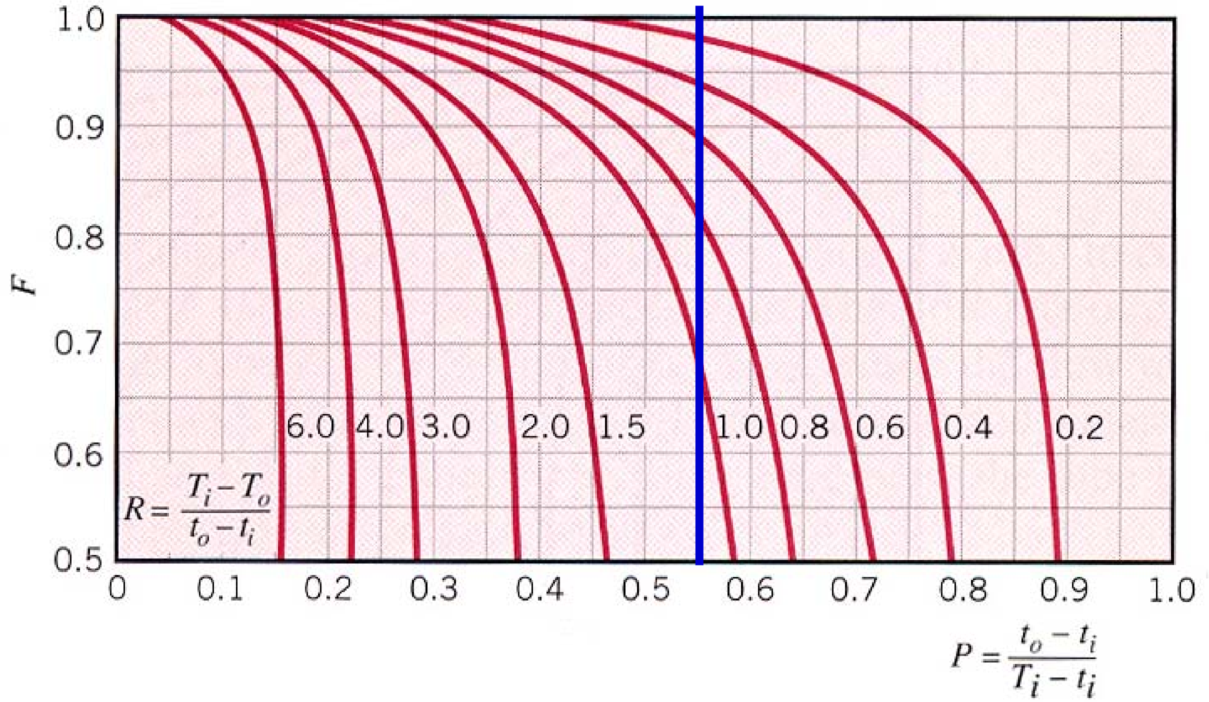
$$= 20\,°\text{C} + 16\,°\text{C} = 36\,°\text{C}$$



$$\Delta T_{\text{lm,CF}} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ln\left[\left(T_{h,i} - T_{c,o}\right)/\left(T_{h,o} - T_{c,i}\right)\right]} = 21\,°\text{C}$$

***F*:**





$$R = \frac{T_i - T_o}{t_o - t_i} = 0, \quad P = \frac{t_o - t_i}{T_i - t_i} = \frac{36.5 - 20}{50 - 20} = 0.55$$

$$F = 1$$

water: $\left(\bar{T}_c \approx 27°\text{C}\right)$

$$c_p = 4179\,\text{J/kg}\cdot\text{K}\ ,\ \ \mu = 855\times10^{-6}\,\text{N}\cdot\text{s}/\text{m}^2$$

$$k = 0.613\ \text{W/m}\cdot\text{K}\ ,\ \ \text{Pr} = 5.83$$

$N = 30,000$
$D = 0.025\,\text{m}$
$h_o = 11,000\,\text{W/m}^2\cdot\text{K}$
L = length/pass
$\dot{m} = 1\,\text{kg/s}$ per tube

$$h_i:\ \text{Re}_D = \frac{4\dot{m}}{\pi D\mu} = 59,567$$

$$\text{Nu}_D = 0.023\,\text{Re}_D^{4/5}\,\text{Pr}^{0.4}$$

$$= 0.023(59,567)^{4/5}(5.83)^{0.4} = 308$$

$$h_i = \text{Nu}_D\frac{k}{D} = 7552\,\text{W/m}^2\cdot\text{K}$$

$$U = \frac{1}{\left(1/h_i\right)+\left(1/h_o\right)} = \frac{1}{\left(1/7552\right)+\left(1/11,000\right)} = 4478\,\text{W/m}^2\cdot\text{K}$$

$$L = \frac{q}{U\left(N\,2\pi D\right)F\Delta T_{\text{lm,CF}}} = 4.51\,\text{m}$$

Using ε-NTU method

$$\text{NTU} = \frac{UA}{C_{\min}}$$

$$A = 2N\pi DL \rightarrow L = \frac{C_{\min}\text{NTU}}{2N\pi DU}$$

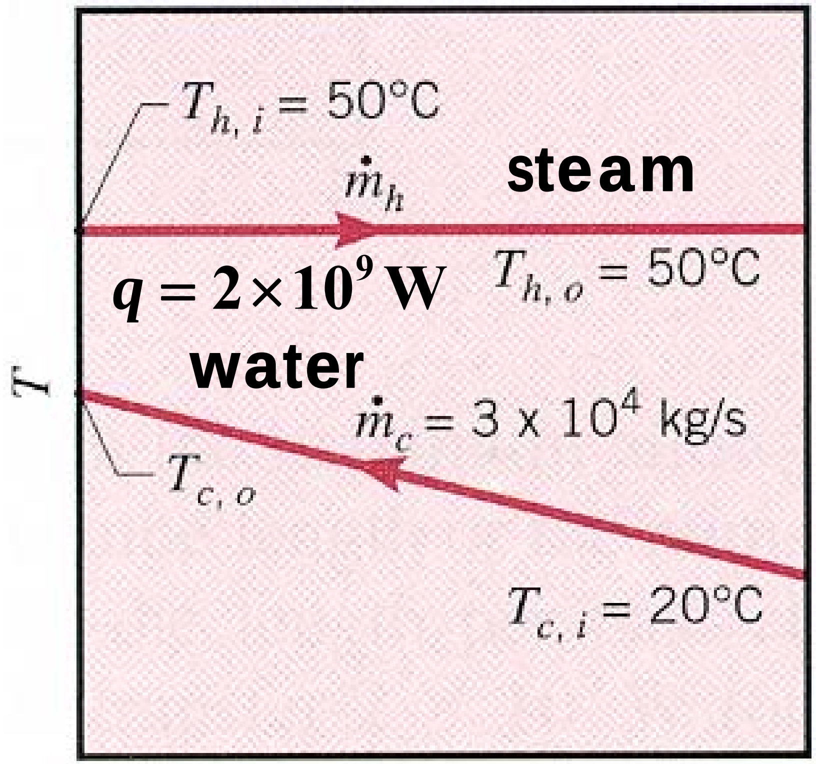
$$\varepsilon = \frac{q}{C_{\min}\left(T_{h,i} - T_{c,i}\right)}$$

$$C_h = \infty = C_{\max}$$

$$C_{\min} = \dot{m}_c c_{p,c} = 1.25\times10^8 \text{ W/K}$$

$$\varepsilon = \frac{q}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} = 0.53$$

$$C_r = \frac{C_{\min}}{C_{\max}} = 0$$



$$\varepsilon = 0.53$$

$$\frac{C_{min}}{C_{max}} = 0$$

$$L = \frac{C_{min}\mathrm{NTU}}{2N\pi DU} = 4.46\ \mathrm{m}$$

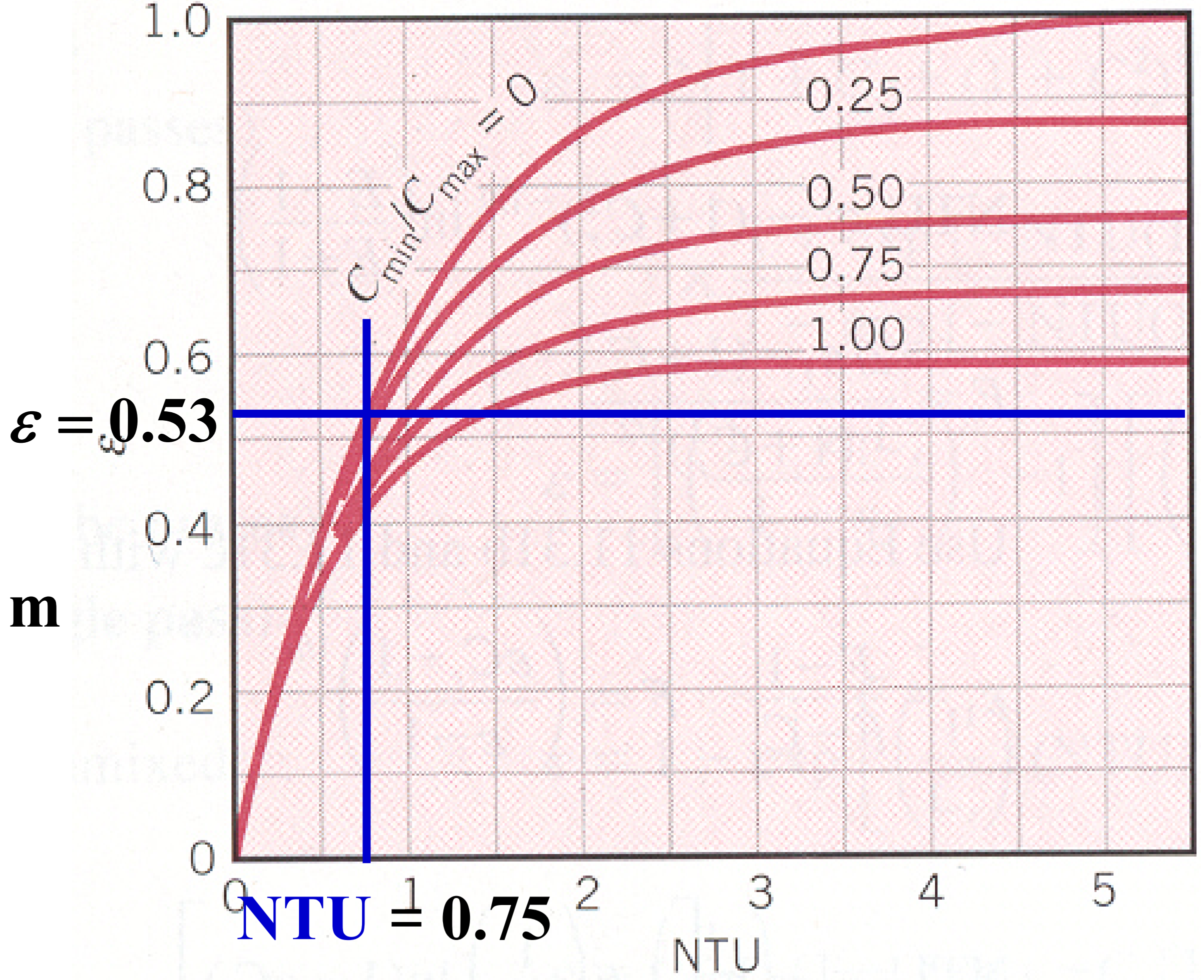


Figure 11.16 Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc. tube passes) (Equation 11.31).

Compact Heat Exchangers

A large heat transfer area per unit volume

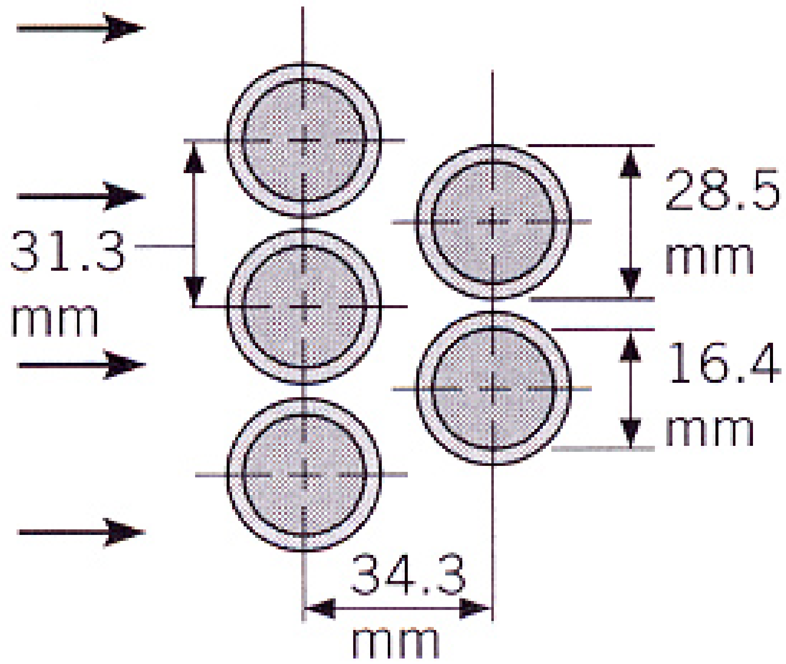
One of the fluid: gas

Geometric parameters

A_{ff}: **minimum free-flow area of the finned passage**

A_{fr}: **frontal area**

$\sigma = A_{ff}/A_{fr}$



D_h: **hydraulic diameter of the flow passage**

α: **heat transfer surface area per total heat exchanger volume**

A_f/A: **the ratio of fin to total heat transfer surface area**

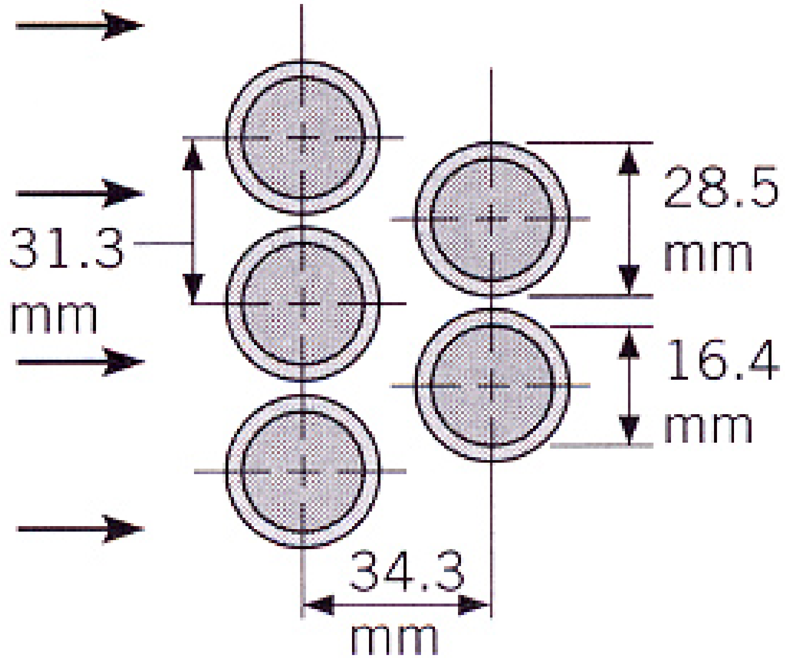
Colburn *j* factor:

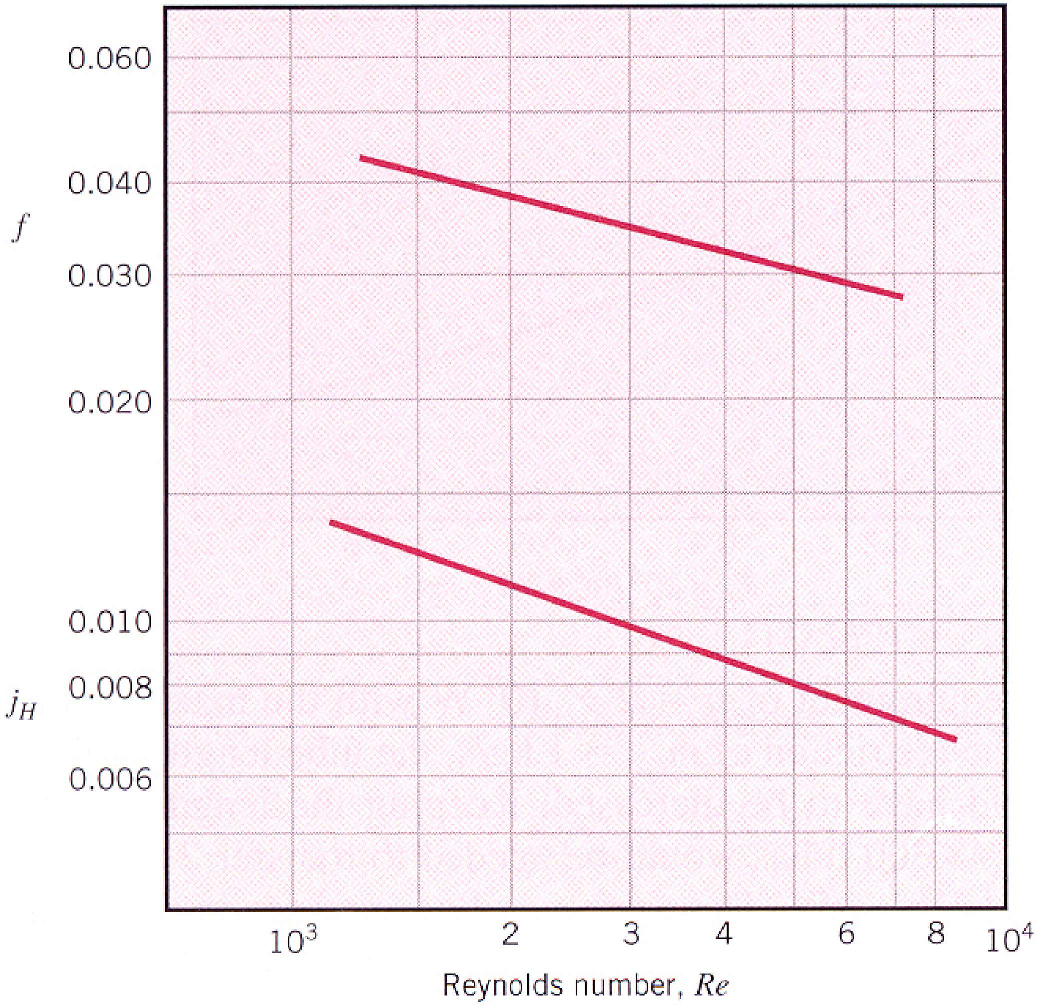
$$j_H = \mathrm{St}\,\mathrm{Pr}^{2/3}, \ \mathrm{St} = h / G c_p$$

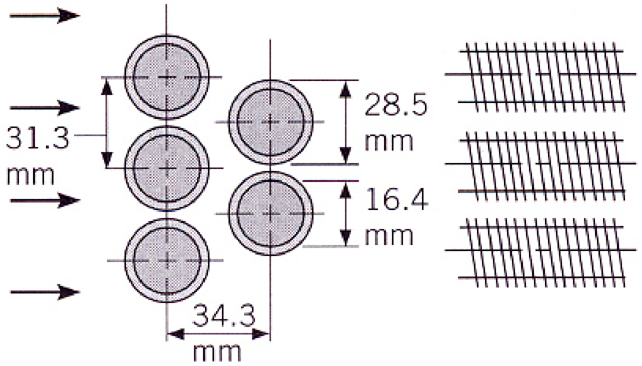
$$G \equiv \rho V_{\max} = \frac{\rho V A_{\mathrm{fr}}}{A_{\mathrm{ff}}}$$

$$= \frac{\dot{m}}{A_{\mathrm{ff}}} = \frac{\dot{m}}{\sigma A_{\mathrm{fr}}}$$

$$\mathrm{Re} = \frac{G D_h}{\mu}$$

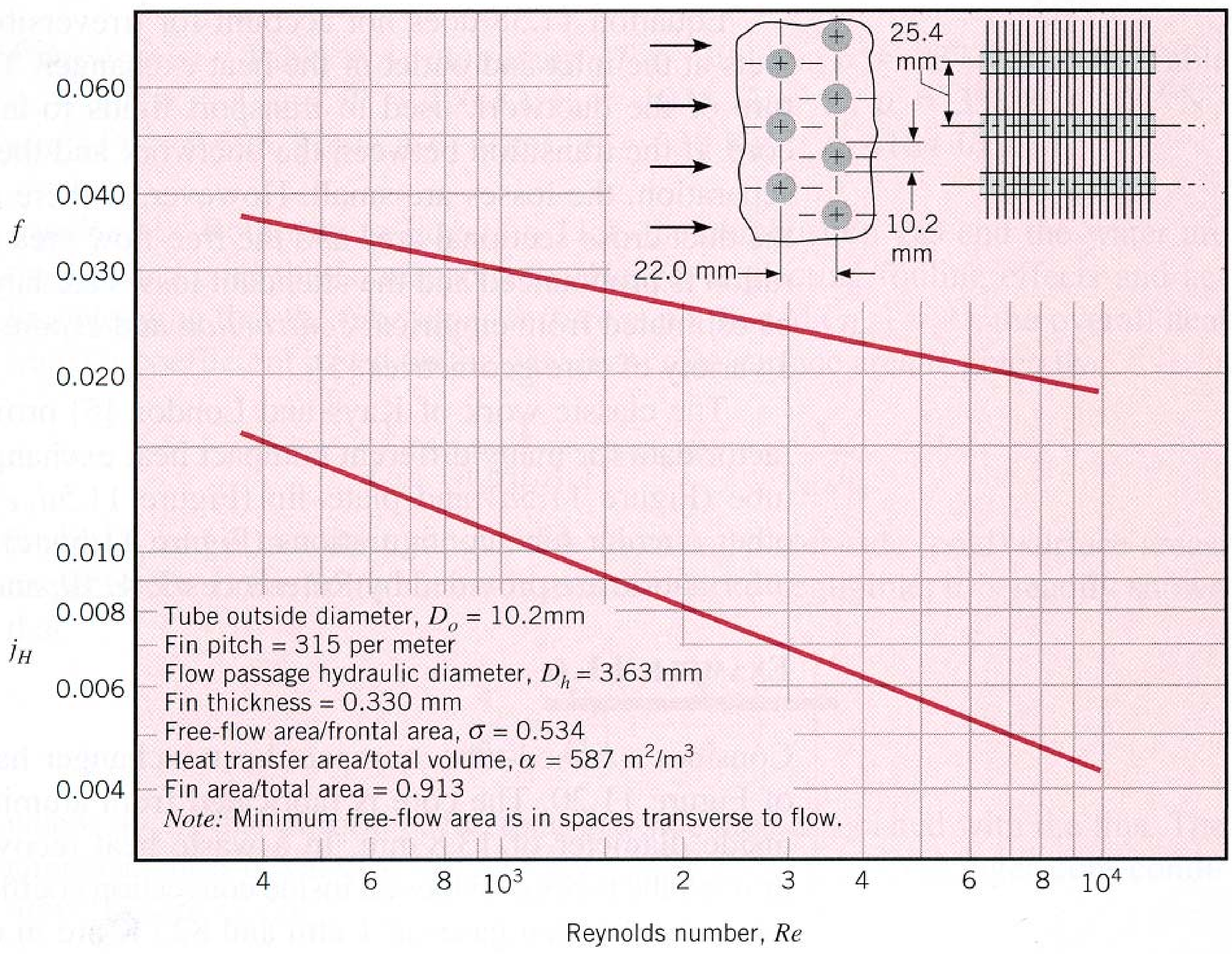






Tube outside diameter, D_o = 16.4 mm
Fin pitch = 275 per meter
Flow passage hydraulic diameter, D_h = 6.68 mm
Fin thickness, t = 0.254 mm
Free-flow area/frontal area, σ = 0.449
Heat transfer area/total volume, α = 269 m^2/m^3
Fin area/total area, A_f/A = 0.830
Note: Minimum free-flow area is in spaces transverse to flow.

Heat transfer and friction factor for a circular tube-circular fin heat exchanger, surface CF-7.0-5/8J



Heat transfer and friction factor for a circular tube-continuous fin heat exchanger, surface 8.0-3/8T

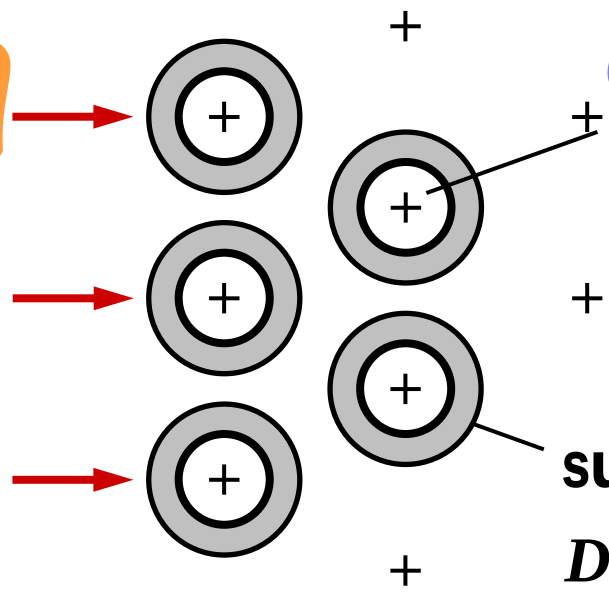
Example 11.6

combustion gas

$\dot{m}_h = 1.25\,\mathrm{kg/s}$

$T_{h,i} = 825\,\mathrm{K}$

$A_{fr} = 0.20\,\mathrm{m^2}$



water

$h_i = h_c = 1500\,\mathrm{W/m^2 \cdot K}$

$\dot{m}_c = 1\ \mathrm{kg/s}$

$T_{c,i} = 290\,\mathrm{K},\ T_{c,o} = 370\,\mathrm{K}$

surface CF-7.0-5/8J

$D_i = 13.8\,\mathrm{mm}$

Find:

1) Gas-side overall heat transfer coefficient

2) Heat exchanger volume

Assumption:

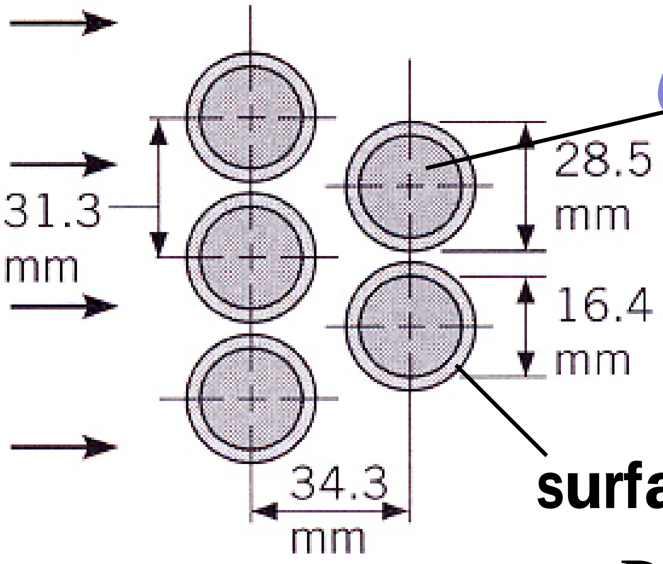
Gas has properties of atmospheric air at an assumed mean temperature of 700 K.

combustion gas

$\dot{m}_h = 1.25\,\mathrm{kg/s}$

$T_{h,i} = 825\,\mathrm{K}$

$A_{fr} = 0.20\,\mathrm{m}^2$



water

$h_i = h_c = 1500\,\mathrm{W/m^2 \cdot K}$

$\dot{m}_c = 1\ \mathrm{kg/s}$

$T_{c,i} = 290\,\mathrm{K},\ T_{c,o} = 370\,\mathrm{K}$

surface CF-7.0--5/8J

$D_i = 13.8\,\mathrm{mm}$

Tube outside diameter, D_o = 16.4 mm
Fin pitch = 275 per meter
Flow passage hydraulic diameter, D_h = 6.68 mm
Fin thickness, t = 0.254 mm
Free-flow area/frontal area, σ = 0.449
Heat transfer area/total volume, α = 269 m^2/m^3
Fin area/total area, A_f/A = 0.830
Note: Minimum free-flow area is in spaces transverse to flow.

1) Gas-side overall heat transfer coefficient

$$\frac{1}{UA} = \frac{1}{(UA)_c} = \frac{1}{(UA)_h}$$

$$= \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}$$

$$\frac{1}{U_h} = \frac{1}{h_c \left(A_c / A_h \right)} + A_h R_w + \frac{1}{\eta_{o,h} h_h}$$

$$j_H = \mathrm{St}\,\mathrm{Pr}^{2/3}, \ \mathrm{St} = h_h / G c_p$$

$$\frac{A_c}{A_h}: \; A_c = \pi D_i L,$$

$$A_h = A_b + A_{f,h}$$

$$= \left(\pi D_o L - \cancel{\pi D_o t n}\right) + A_{f,h}$$

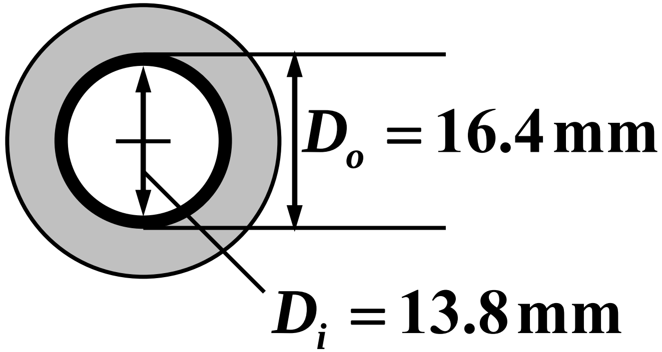
$$\frac{A_h}{A_c} \approx \frac{\pi D_o L + A_{f,h}}{\pi D_i L} = \frac{D_o}{D_i} + \frac{A_{f,h}}{A_c}$$

$$1 \approx \frac{D_o}{D_i}\frac{A_c}{A_h} + \frac{A_{f,h}}{A_h}$$

$$\frac{A_c}{A_h} \approx \frac{D_i}{D_o}\left(1 - \frac{A_{f,h}}{A_h}\right) \rightarrow \frac{A_c}{A_h} \approx \frac{13.8}{16.4}\left(1 - 0.830\right) = 0.143$$

Fin area/total area,

$$A_f / A = 0.830$$



$$A_h R_w = \frac{\ln\left(D_o / D_i\right)}{2\pi L k / A_h}, \quad A_c = \pi D_i L \rightarrow L = \frac{A_c}{\pi D_i}$$

aluminum $\left(T \approx 300\,\mathrm{K}\right) \quad k = 237\,\mathrm{W/m \cdot K}$

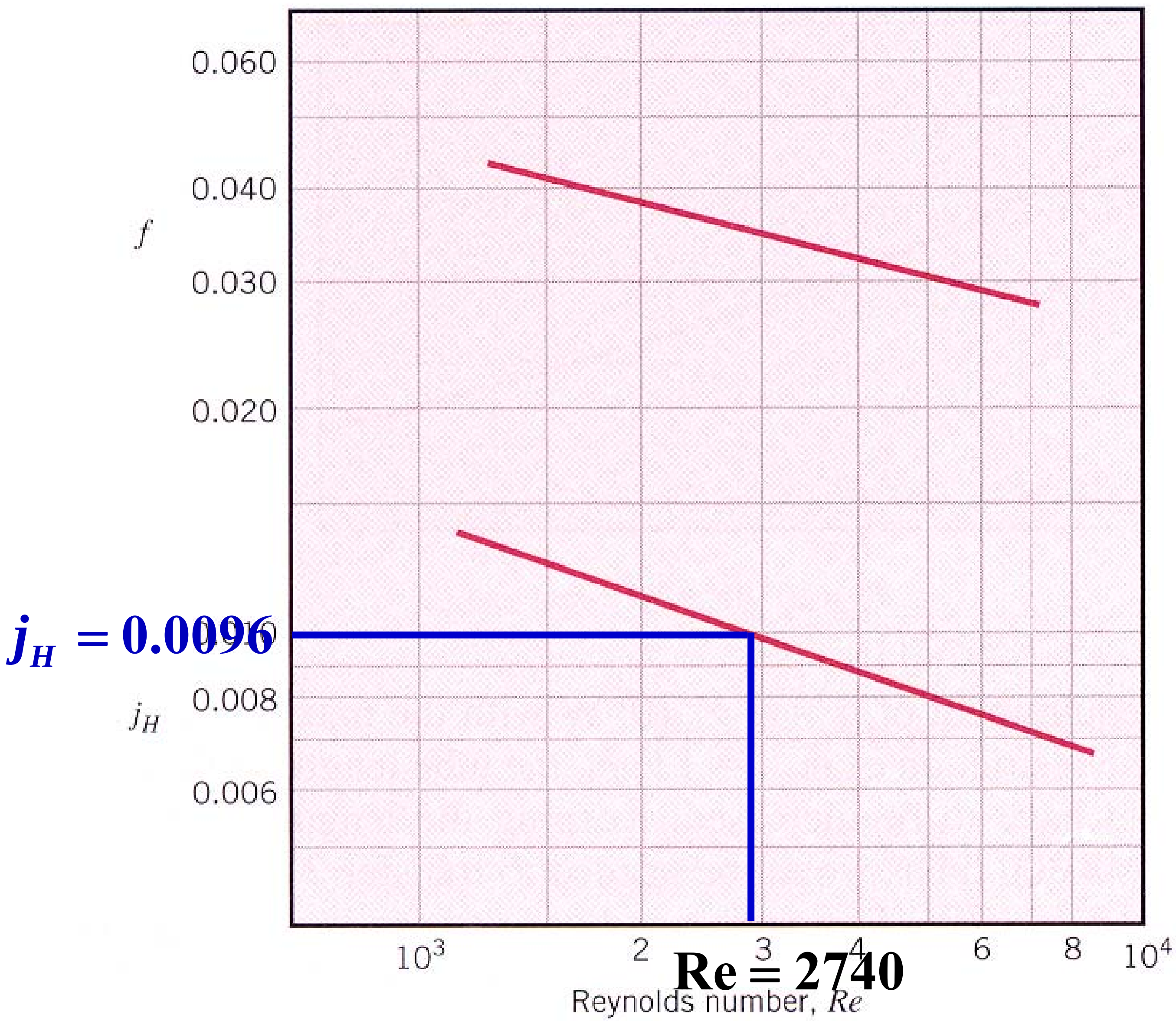
$$= \frac{D_i \ln\left(D_o / D_i\right)}{2k\left(A_c / A_h\right)} = 3.51 \times 10^{-5}\,\mathrm{m^2 \cdot K/W}$$

h_h: $\mathrm{Re} = \dfrac{G D_h}{\mu}, \quad D_h = 6.68\ \mathrm{mm}, \quad G = \dfrac{\dot{m}_h}{A_{\mathrm{ff}}} = \dfrac{\dot{m}_h}{\sigma A_{\mathrm{fr}}} = 13.9\,\mathrm{kg/s \cdot m^2}$

$\dot{m}_h = 1.25\,\mathrm{kg/s}, \ A_{\mathrm{fr}} = 0.20\,\mathrm{m^2}, \ \sigma = 0.449$

air $\left(p = 1\,\mathrm{atm}, \bar{T} = 700\,\mathrm{K}\right) \ \mu = 338.8 \times 10^{-7}\,\mathrm{N \cdot s / m^2}$

$$\mathrm{Re} = \frac{G D_h}{\mu} = \frac{13.9\,\mathrm{kg/s \cdot m^2} \times 6.68 \times 10^{-3}\,\mathrm{m}}{338.8 \times 10^{-7}\,\mathrm{kg/s \cdot m}} = 2740$$



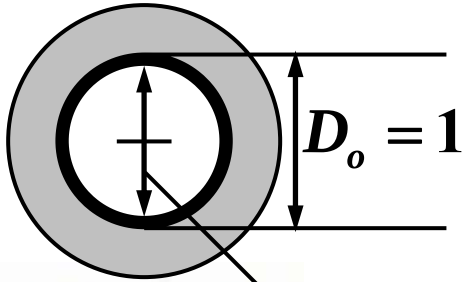
0.060
0.040
f
0.030
0.020
j_H = 0.0096
0.008
j_H
0.006
10^3
2
3
4
6
8
10^4
Re = 2740
Reynolds number, Re

$$j_H = 0.0096, \quad j_H = \mathrm{St}\,\mathrm{Pr}^{2/3}, \ \mathrm{St} = h_h / Gc_p$$

$$\text{air}\left(p = 1\,\text{atm}, \bar{T} = 700\,\text{K}\right) \ \ \mathrm{Pr} = 0.695$$

$$h_h = 0.0096 \frac{Gc_p}{\mathrm{Pr}^{2/3}} = 183\ \mathrm{W/m \cdot K}$$

$t = 0.254\,\mathrm{mm}$



$D_o = 16.4\,\mathrm{mm}$

$D_i = 13.8\,\mathrm{mm}$

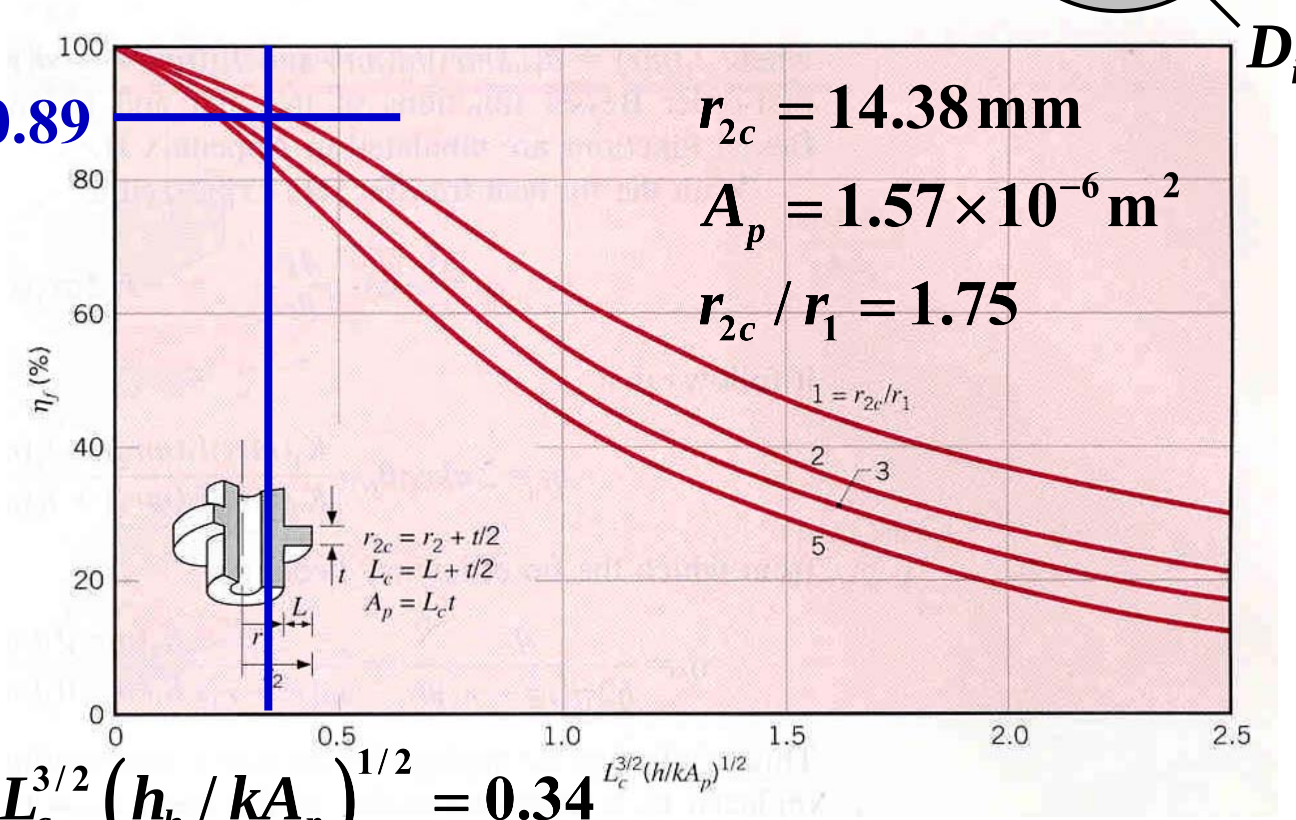
$\eta_{o,h}$:

$\eta_f \approx 0.89$

$r_{2c} = 14.38\,\mathrm{mm}$

$A_p = 1.57 \times 10^{-6}\,\mathrm{m}^2$

$r_{2c} / r_1 = 1.75$



$$L_c^{3/2}\left(h_h / kA_p\right)^{1/2} = 0.34$$

$$\eta_{o,h} = 1 - \frac{A_f}{A}\left(1 - \eta_f\right) = 0.91$$

$$\frac{1}{U_h} = \frac{1}{h_c\left(A_c / A_h\right)} + A_h R_w + \frac{1}{\eta_{o,h} h_h} = 0.0107\,\mathrm{m^2 \cdot K / W}$$

$$U_h = 93.4\,\mathrm{W/m^2 \cdot K}$$

2) Heat exchanger volume

$$\alpha = \frac{A_h}{V}$$

$$V = \frac{A_h}{\alpha}$$

$$\mathrm{NTU} = \frac{U_h A_h}{C_{\min}}$$

$$\varepsilon = \frac{q}{q_{\max}} = \frac{q}{C_{\min}\left(T_{h,i} - T_{c,i}\right)}$$

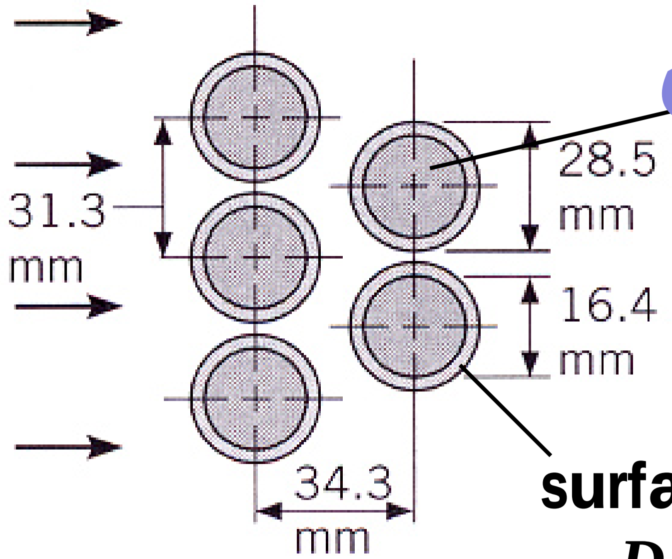
Tube outside diameter, D_o = 16.4 mm
Fin pitch = 275 per meter
Flow passage hydraulic diameter, D_h = 6.68 mm
Fin thickness, t = 0.254 mm
Free-flow area/frontal area, σ = 0.449
Heat transfer area/total volume, α = 269 $\mathrm{m^2/m^3}$
Fin area/total area, A_f/A = 0.830
Note: Minimum free-flow area is in spaces transverse to flow.

combustion gas

$\dot{m}_h = 1.25\,\text{kg/s}$

$T_{h,i} = 825\,\text{K}$

$A_{fr} = 0.20\,\text{m}^2$



water

$h_i = h_c = 1500\,\text{W/m}^2\cdot\text{K}$

$\dot{m}_c = 1\,\text{kg/s}$

$T_{c,i} = 290\,\text{K},\ T_{c,o} = 370\,\text{K}$

surface CF-7.0--5/8J

$D_i = 13.8\,\text{mm}$

$$C_c = \dot{m}_c c_{p,c} = 1\,\text{kg/s} \times 4184\,\text{J/kg}\cdot\text{K} = 4184\,\text{W/K}$$

$$C_h = \dot{m}_h c_{p,h} = 1.25\,\text{kg/s} \times 1075\,\text{J/kg}\cdot\text{K} = 1344\,\text{W/K}$$

$$C_{\min} = C_h = 1344\,\text{W/K}$$

$$q = C_c\left(T_{c,o} - T_{c,i}\right) = 3.35\times10^5\ \text{W}$$

$$q_{\max} = C_{\min}\left(T_{h,i} - T_{c,i}\right) = 7.19\times10^5\ \text{W}$$

$$\varepsilon = \frac{q}{q_{\max}} = 0.466,\quad C_r = \frac{C_{\min}}{C_{\max}} = 0.321$$

$$\varepsilon = \frac{q}{q_{max}} = 0.466,$$

$$C_r = \frac{C_{min}}{C_{max}} = 0.321$$

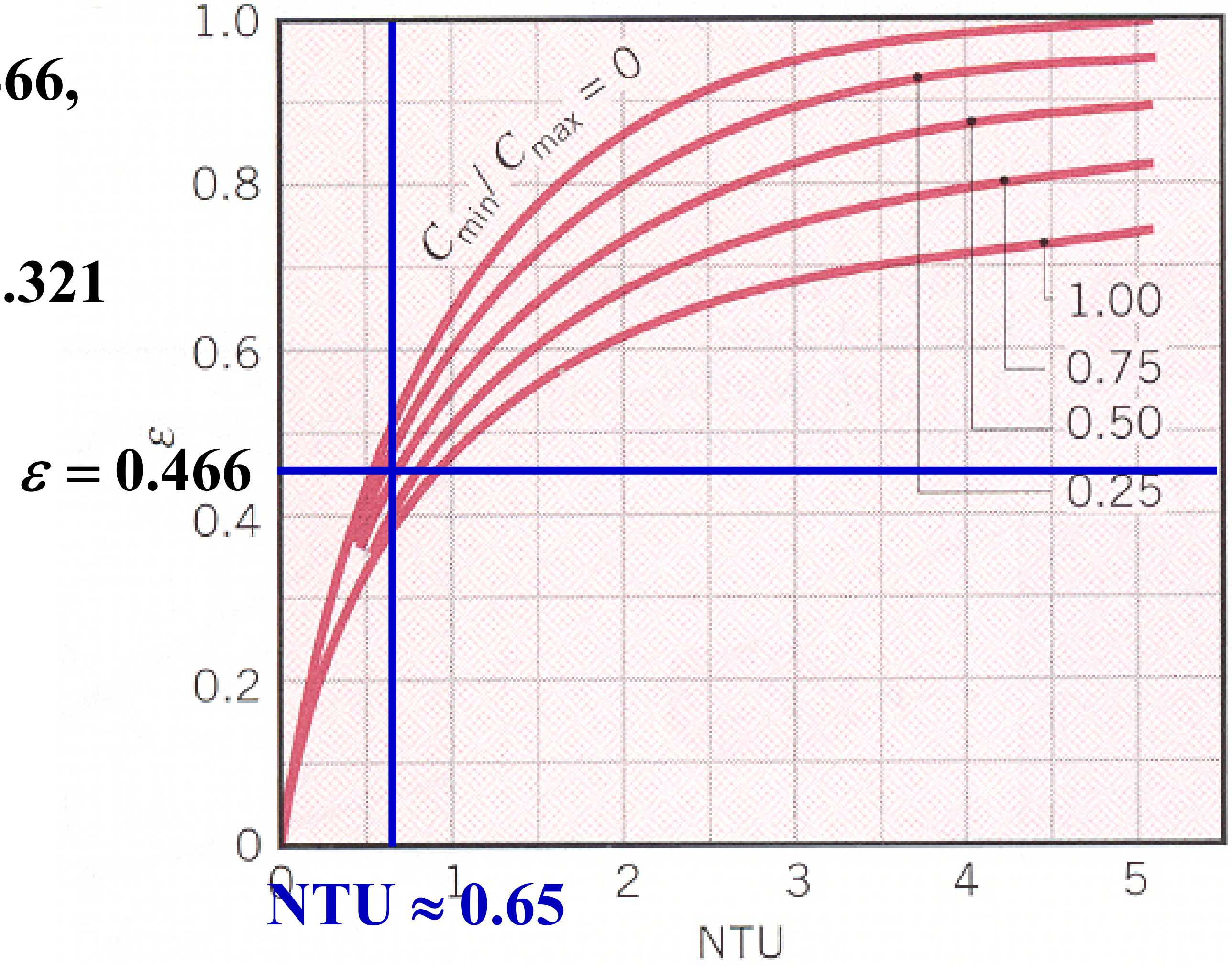


FIGURE 11.18 Effectiveness of a single-pass, cross-flow heat exchanger with both fluids unmixed (Equation 11.33).

$$\mathrm{NTU} = \frac{U_h A_h}{C_{\min}} \approx 0.65$$

$$A_h = \frac{C_{\min}\mathrm{NTU}}{U_h} = \frac{1344\ \mathrm{W/K} \times 0.65}{93.4\ \mathrm{W/m^2 \cdot K}} = 9.35\ \mathrm{m^2}$$

$$V = \frac{A_h}{\alpha}, \quad \alpha = 269\ \mathrm{m^2/m^3}$$

$$V = \frac{A_h}{\alpha} = \frac{9.35\ \mathrm{m^2}}{269\ \mathrm{m^2/m^3}} = 0.0348\ \mathrm{m^3}$$