RADIATIVE TRANSFER BETWEEN TWO OR MORE SURFACES

- View Factor
- Methods of View Factor Evaluation
- Blackbody Radiation Exchange
- Radiation Exchange between Diffuse, Gray Surfaces in an Enclosure
- Radiation in Participating Media

View Factor

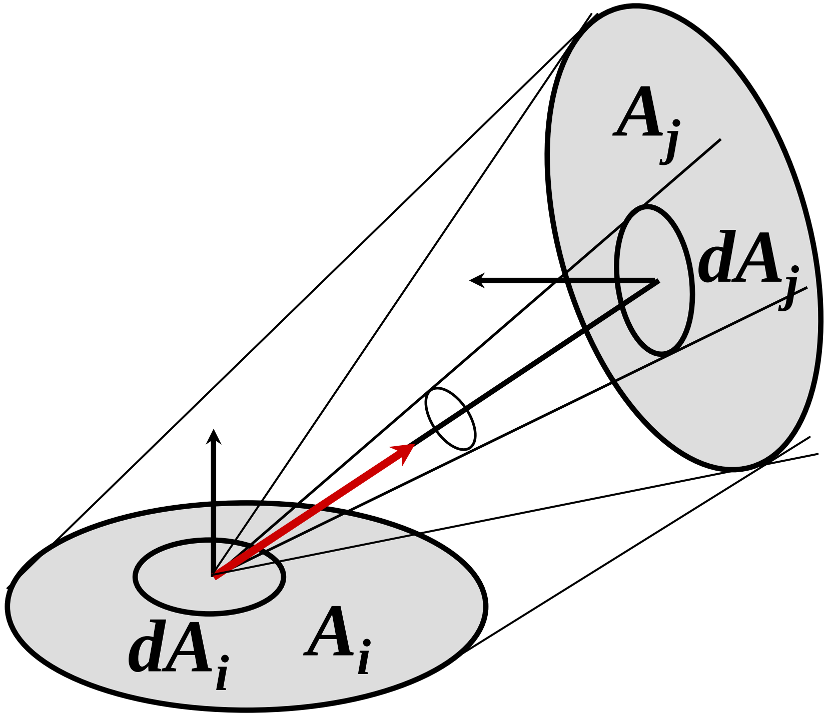
The (Diffuse) View Factor

configuration factor, angle factor, shape factor

View Factor Integral

$$F_{ij} = \frac{\text{radiation energy intercepted by } A_j}{\text{radiation energy leaving } A_i \text{ hemispherically}}$$

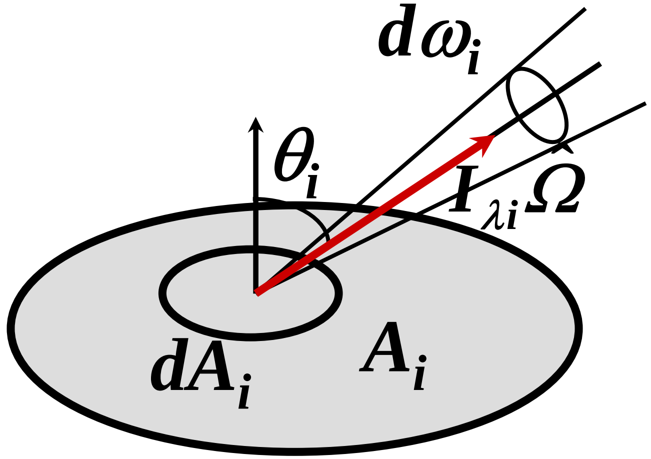
$$= \frac{q_{i\to j}}{q_i}$$



$$q_i = \int_{A_i} \int_{\cap} \int_0^{\infty} I_{\lambda i} \cos\theta_i dA_i d\omega_i d\lambda$$

$$= \int_{A_i} \left(\int_{\cap} I_i \cos\theta_i d\omega_i \right) dA_i$$

$$= \int_{A_i} J_i dA_i$$



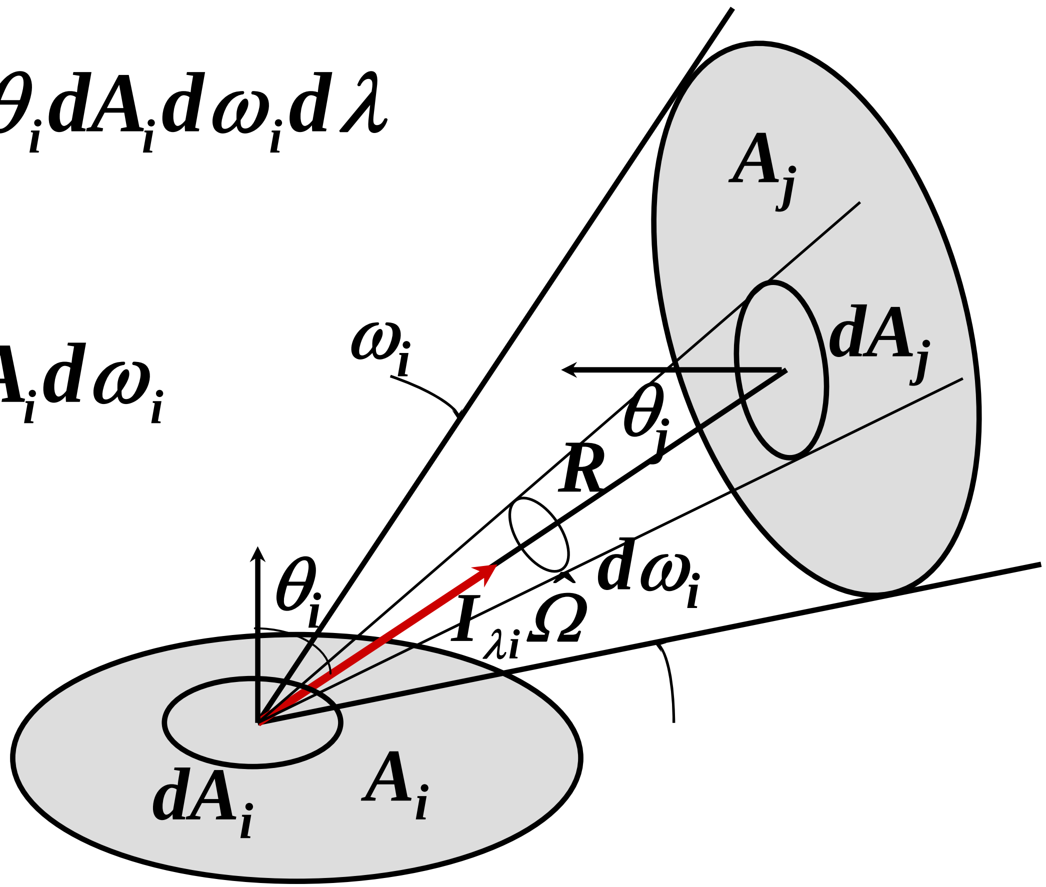
When J_i is unifrom over A_i,

$$q_i = J_i A_i$$

$$q_{i\rightarrow j} = \int_{A_i}\int_{\omega_i}\int_0^{\infty} I_{\lambda i}\cos\theta_i dA_i d\omega_i d\lambda$$

$$= \int_{A_i}\int_{\omega_i} I_i \cos\theta_i dA_i d\omega_i$$

$$d\omega_i = \frac{dA_j \cos\theta_j}{R^2}$$



$$q_{i\rightarrow j} = \int_{A_j}\int_{A_i} I_i \frac{\cos\theta_i \cos\theta_j}{R^2} dA_i dA_j$$

$$F_{ij} = \frac{q_{i\to j}}{q_i} = \frac{\int_{A_j}\int_{A_i} I_i \frac{\cos\theta_i \cos\theta_j}{R^2} dA_i dA_j}{\int_{A_i} J_i dA_i}$$

When I_i is independent of propagation direction (diffuse radiation) and J_i is uniform over A_i,

$$F_{ij} = \frac{\int_{A_j}\int_{A_i} \pi I_i \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j}{J_i A_i}$$

For diffuse radiation,

$$J = \int_{\cap}\int_0^{\infty} I_\lambda \cos\theta d\lambda d\omega = \int_{\cap} I \cos\theta d\omega = \pi I$$

$$F_{ij} = \frac{\int_{A_j}\int_{A_i} \pi I_i \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j}{J_i A_i}$$

$$= \frac{1}{A_i}\int_{A_j}\int_{A_i} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j$$

Similarly, $F_{ji} = \frac{1}{A_j}\int_{A_j}\int_{A_i} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j$

View Factor Relations

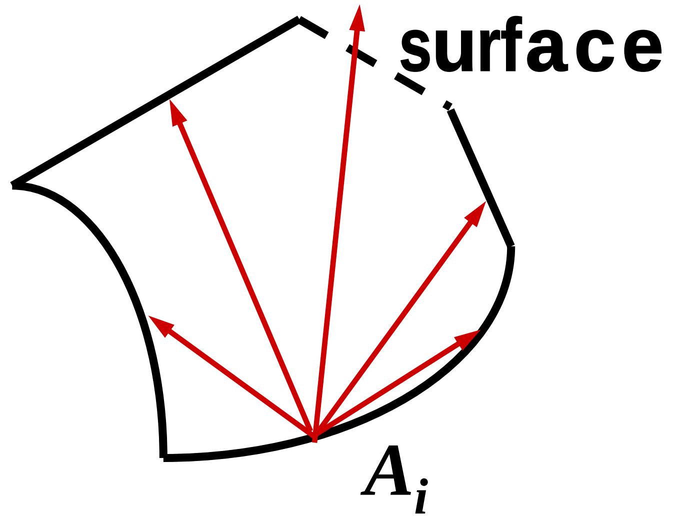
$$F_{ij} = \frac{1}{A_i}\int_{A_j}\int_{A_i}\frac{\cos\theta_i \cos\theta_j}{\pi R^2}dA_i dA_j$$

$$F_{ji} = \frac{1}{A_j}\int_{A_j}\int_{A_i}\frac{\cos\theta_i \cos\theta_j}{\pi R^2}dA_i dA_j$$

Reciprocity: $A_i F_{ij} = A_j F_{ji}$

Summation rule:

$$\sum_{j=1}^{N} F_{ij} = 1$$

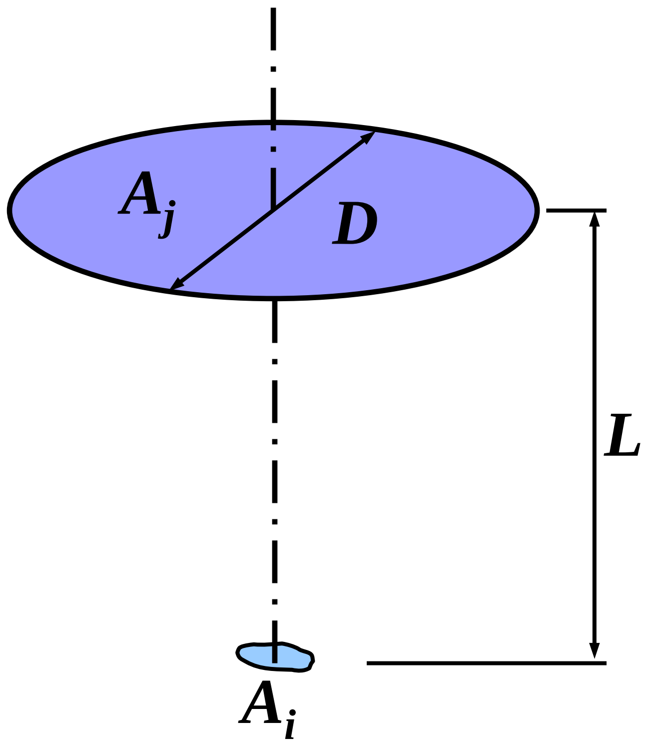


Methods of View Factor Evaluation

- **Direct integration**
  - Area integral
  - Contour integral
- **Flux algebra**
  - Cross-string method: 2D only
  - Decomposition of shapes
- **Sphere method**
  - Unit sphere method: only from a differential area
  - Inside sphere method

Area Integral

Example 13.1



Find:

View factor of small surface w.r.t. disk, F_{ij}

Assumptions:

1) **Diffuse surfaces.**

2) $A_i << A_j$

The view factor

$$F_{ij} = \frac{1}{A_i}\int_{A_j}\int_{A_i}\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_i dA_j$$

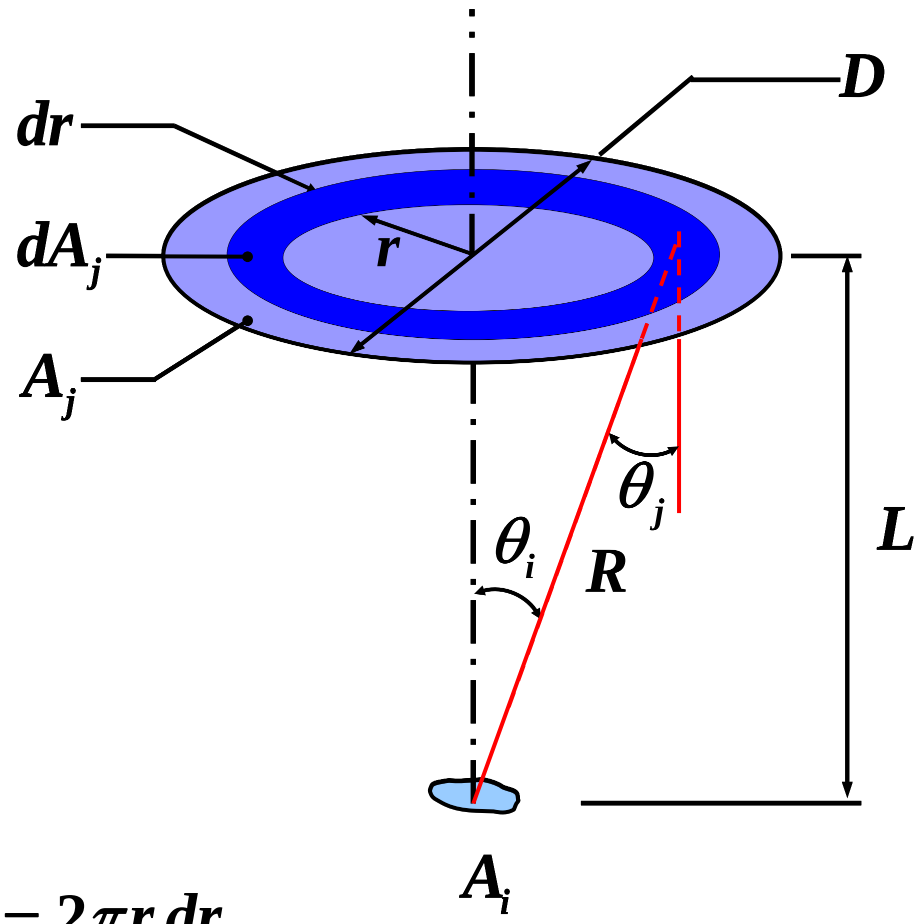
θ_i, θ_j and R independent of position on A_i

$$F_{ij} = \int_{A_j}\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_j$$

since $\theta_i = \theta_j \equiv \theta$

$$F_{ij} = \int_{A_j}\frac{\cos^2\theta}{\pi R^2}dA_j$$

$$R^2 = r^2 + L^2, \quad \cos\theta = \frac{L}{R}, \quad dA_j = 2\pi r\,dr$$



$$F_{ij} = \int_0^{D/2}\frac{(L/R)^2}{\pi R^2}(2\pi r dr) = 2L^2\int_0^{D/2}\frac{r\,dr}{(r^2+L^2)^2} = \frac{D^2}{D^2+4L^2}$$

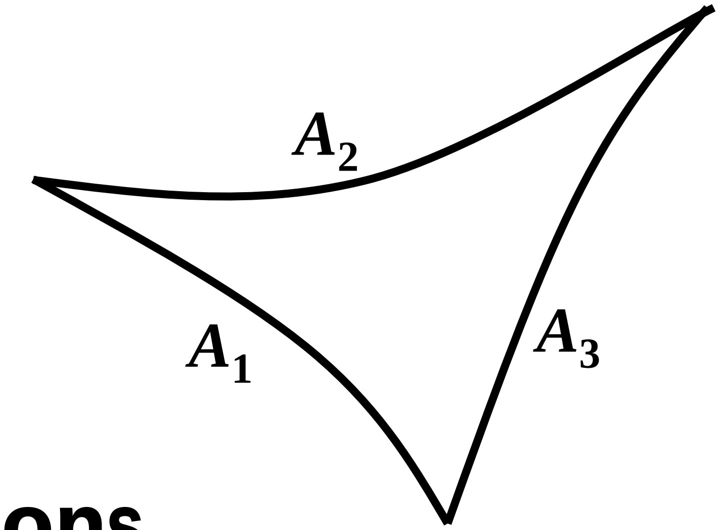
Cross-String Method : 2-D

For an enclosure with 3 planes or convex surfaces

$$A_1F_{12} = A_2F_{21}, \quad F_{12} + F_{13} = 1$$

$$A_1F_{13} = A_3F_{31}, \quad F_{21} + F_{23} = 1$$

$$A_2F_{23} = A_3F_{32}, \quad F_{31} + F_{32} = 1$$



6 unknowns and 6 equations

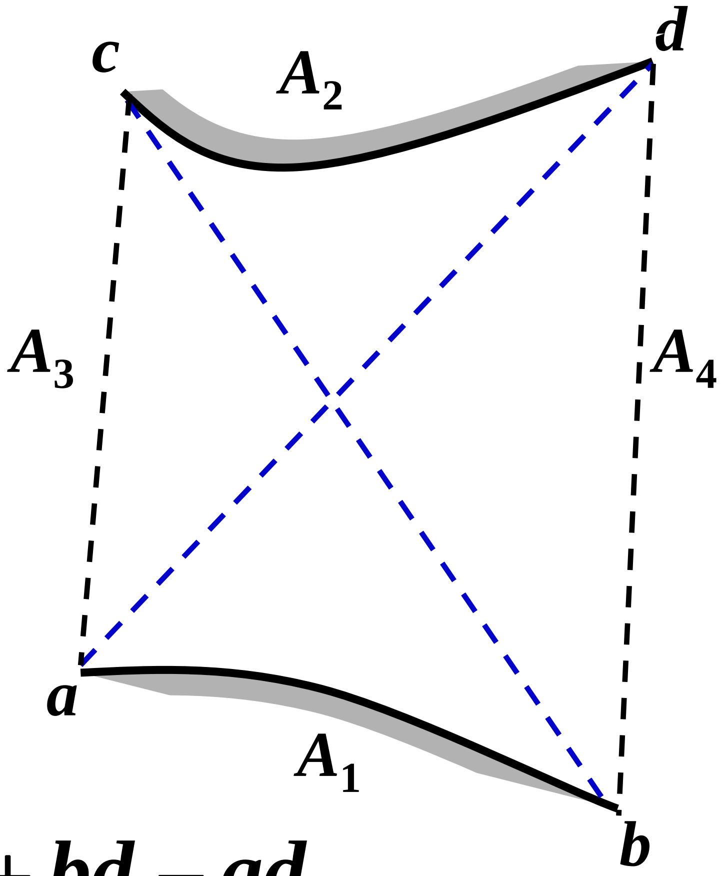
$$\rightarrow F_{12} = \frac{A_1 + A_2 - A_3}{2A_1}$$

cross-string method

$$F_{12} = 1 - F_{13} - F_{14}$$

$$F_{13} = \frac{ab + ac - bc}{2ab}$$

$$F_{14} = \frac{ab + bd - ad}{2ab}$$



$$F_{12} = 1 - \frac{ab + ac - bc + ab + bd - ad}{2ab}$$

$$= \frac{{\color{blue}(bc + ad)} - (ac + bd)}{2ab}$$

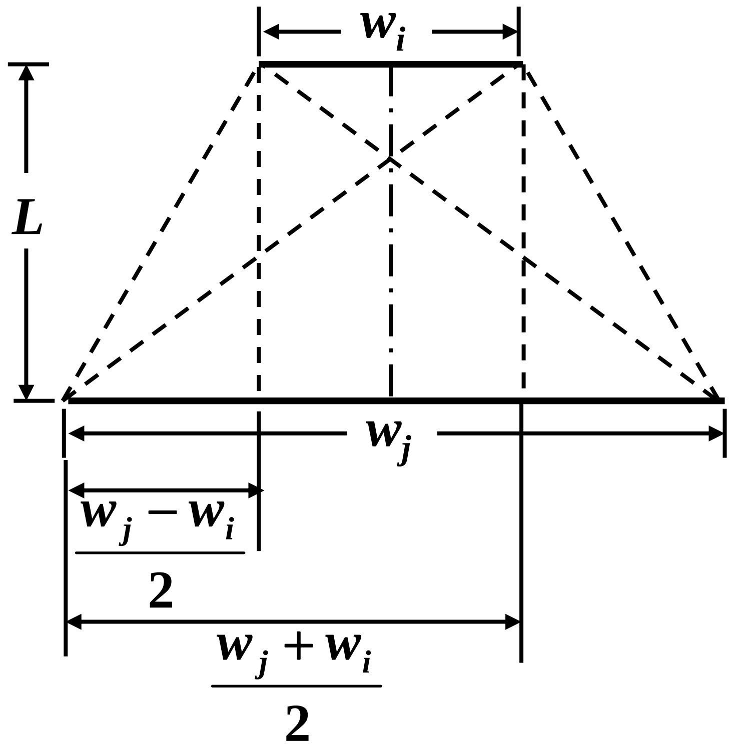
ex) view factor between parallel plates with midlines connected by perpendicular

$$F_{ij} = \frac{2\left[\left\{\left(w_j + w_i\right)/2\right\}^2 + L^2\right]^{1/2} - 2\left[\left\{\left(w_j - w_i\right)/2\right\}^2 + L^2\right]^{1/2}}{2w_i}$$

$$= \frac{\left[\left(w_j + w_i\right)^2 + 4L^2\right]^{1/2} - \left[\left(w_j - w_i\right)^2 + 4L^2\right]^{1/2}}{2w_i}$$

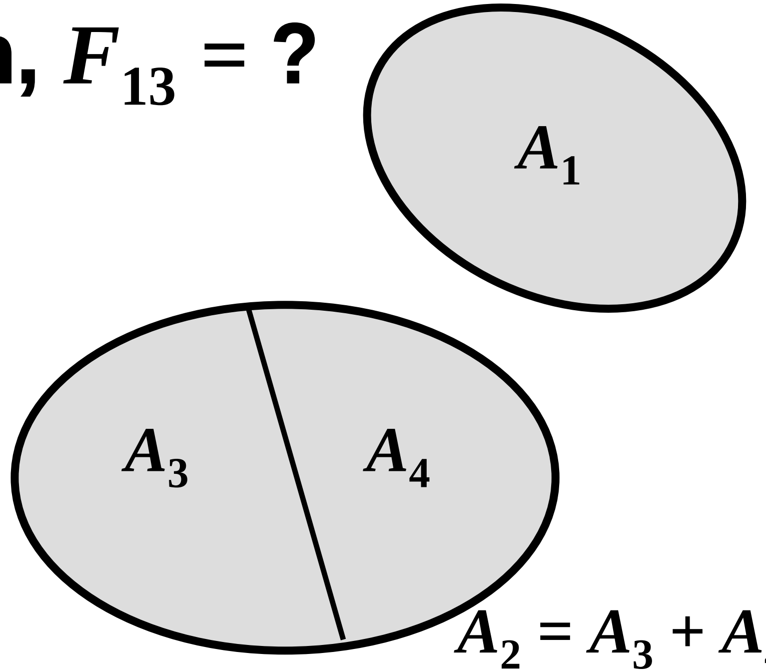
$$= \frac{\left[\left(W_j + W_i\right)^2 + 4\right]^{1/2} - \left[\left(W_j - W_i\right)^2 + 4\right]^{1/2}}{2W_i}$$

where $W_i = w_i / L, \;\; W_j = w_j / L$



Decomposition of Shapes

F_{12}: known, F_{14} : known, F_{13} = ?



$A_2 = A_3 + A_4$

$$F_{12} = F_{1-(3+4)} = F_{13} + F_{14}$$

$$F_{13} = F_{12} - F_{14}$$

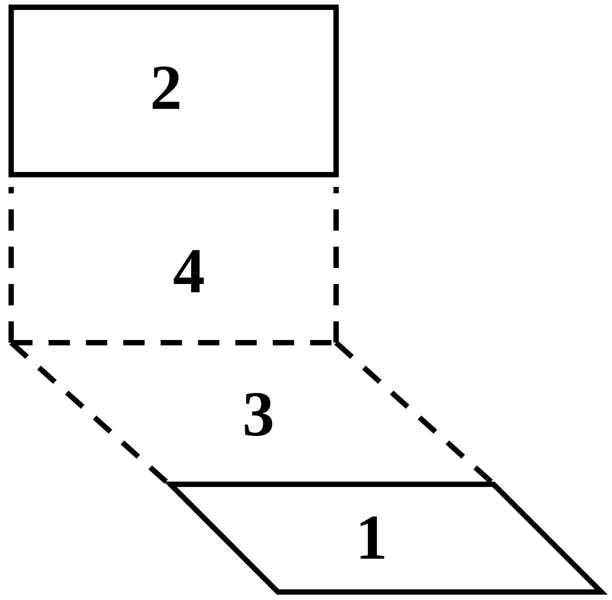
Remark: $F_{(3+4)-1} \neq F_{31} + F_{41}$

$$F_{(3+4)-1} = \frac{A_1}{A_{(3+4)}} F_{1-(3+4)}$$

$$= \frac{A_1}{A_2}\left[F_{13} + F_{14}\right]$$

$$= \frac{A_1}{A_2}\left[\frac{A_3}{A_1} F_{31} + \frac{A_4}{A_1} F_{41}\right] = \frac{A_3}{A_2} F_{31} + \frac{A_4}{A_2} F_{41}$$

ex) $F_{12} = ?$



known: $F_{(1+3)\text{-}(2+4)}, F_{(1+3)\text{-}4}, F_{3\text{-}(2+4)}, F_{34}$

$$F_{(1+3)-(2+4)} = F_{(1+3)-2} + F_{(1+3)-4}$$

$$= \frac{A_2}{A_{13}} F_{2-(1+3)} + F_{(1+3)-4}$$

$$= \frac{A_2}{A_{13}} \left(F_{21} + F_{23} \right) + F_{(1+3)-4}$$

$$F_{21} = \frac{A_1}{A_2} F_{12}$$

$$F_{3-(2+4)} = F_{32} + F_{34} = \frac{A_2}{A_3} F_{23} + F_{34} \quad \rightarrow F_{23} = \frac{A_3}{A_2} \left(F_{3-(2+4)} - F_{34} \right)$$

$$F_{(1+3)-(2+4)} = \frac{A_2}{A_{13}} \left[\frac{A_1}{A_2} F_{12} + \frac{A_3}{A_2} \left(F_{3-(2+4)} - F_{34} \right) \right] + F_{(1+3)-4}$$

$$F_{12} = \frac{A_{13} F_{(1+3)-(2+4)} + A_3 F_{34} - A_3 F_{3-(2+4)} - A_{13} F_{(1+3)-4}}{A_1}$$

Blackbody Radiation Exchange

Between two black surfaces

$$J_b = \int_0^\infty \int_\cap I_{\lambda b} \cos\theta d\omega d\lambda = \pi I_b = E_b = \sigma T^4$$

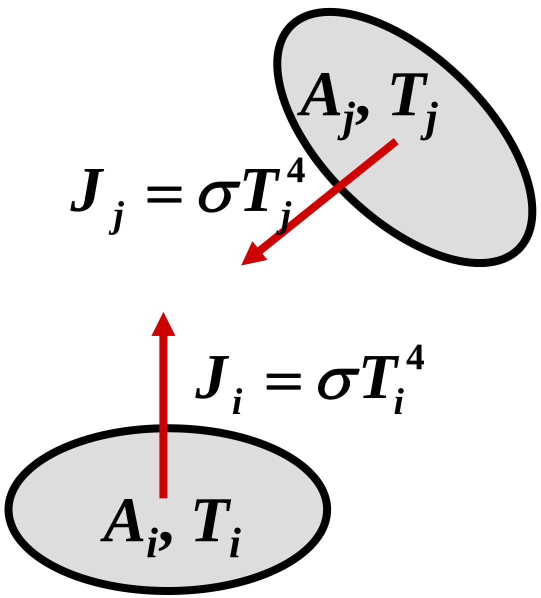
$$q_{i\to j} = \sigma T_i^4 A_i F_{ij}, \quad q_{j\to i} = \sigma T_j^4 A_j F_{ji}$$

$$q_i \equiv q_{ij} = q_{i\leftrightarrow j} = q_{i\to j} - q_{j\to i}$$

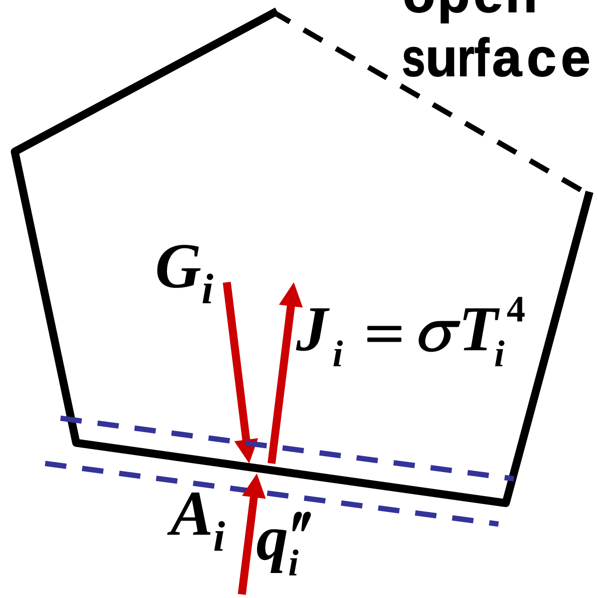
$$= \sigma T_i^4 A_i F_{ij} - \sigma T_j^4 A_j F_{ji}$$

$$= \sigma T_i^4 A_i F_{ij} - \sigma T_j^4 A_i F_{ij}$$

$$= \sigma A_i F_{ij} \left(T_i^4 - T_j^4\right)$$



enclosure with N surfaces

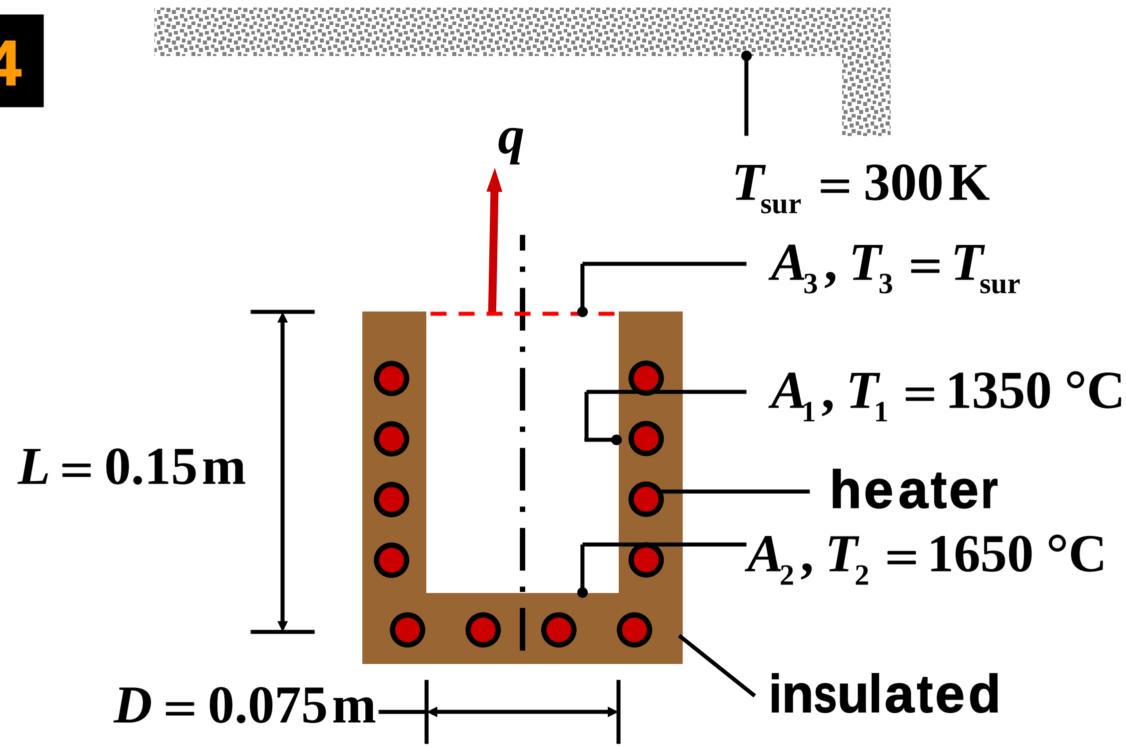


$$q_i = \left(J_i - G_i\right)A_i = J_i A_i - G_i A_i$$

$$G_i A_i = \sigma T_1^4 A_1 F_{1i} + \sigma T_2^4 A_2 F_{2i} + \cdots + \sigma T_N^4 A_N F_{Ni}$$

$$\begin{aligned} q_i &= \sigma T_i^4 A_i \\ &\quad - \left(\sigma T_1^4 A_1 F_{1i} + \sigma T_2^4 A_2 F_{2i} + \cdots + \sigma T_N^4 A_N F_{Ni}\right) \\ &= \sigma T_i^4 A_i - \sum_{j=1}^{N} \sigma T_j^4 A_j F_{ji} \\ &= \sigma T_i^4 A_i \sum_{j=1}^{N} F_{ij} - \sum_{j=1}^{N} \sigma T_j^4 A_i F_{ij} \\ &= \sum_{j=1}^{N} \sigma A_i F_{ij} \left(T_i^4 - T_j^4\right) \end{aligned}$$

Example 13.4



Find:

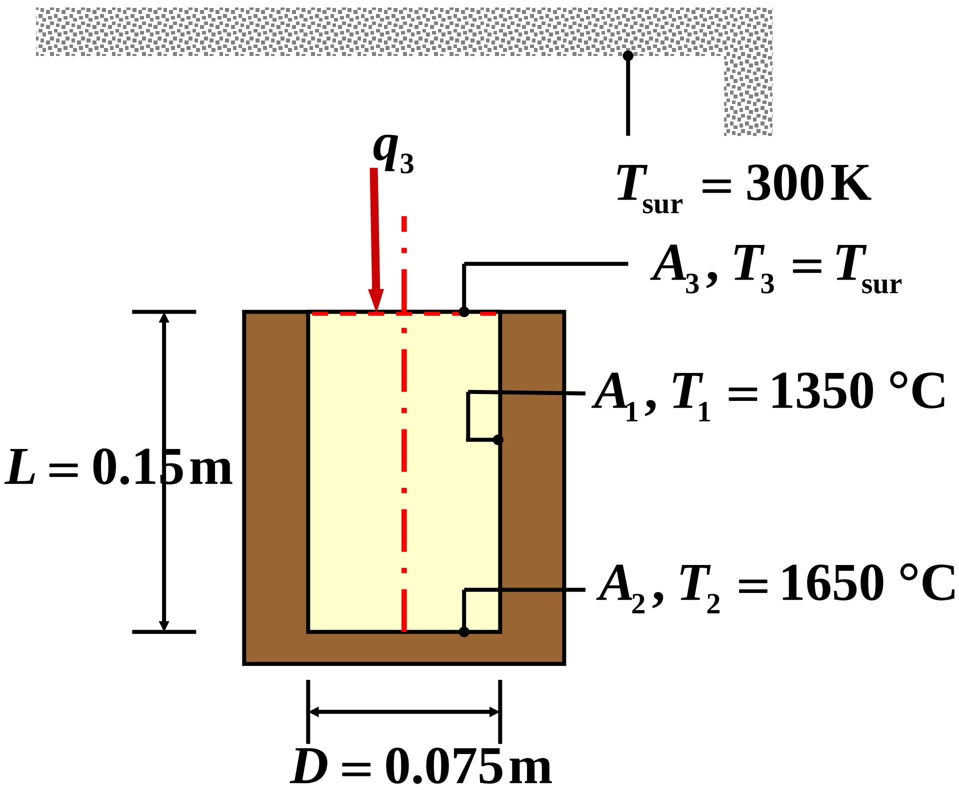
Power required to maintain prescribed temperatures

Assumptions:

1) Interior surfaces behave as blackbodies.
2) Heat transfer by convection is negligible.
3) Outer surface of furnace is adiabatic.

Heat loss

$$q_3 = (J_3 - G_3)A_3$$
$$= \sigma T_3^4 A_3$$
$$-(\sigma T_1^4 A_1 F_{13} + \sigma T_2^4 A_2 F_{23})$$



F_{23}: **From Fig. 13.5,**

$$r_j / L = 0.0375\ \mathrm{m} / 0.15\ \mathrm{m} = 0.25$$

$$L / r_i = 0.15\ \mathrm{m} / 0.0375\ \mathrm{m} = 4$$

$$F_{23} = 0.06$$

F_{13}: $F_{21} = 1 - F_{23} = 0.94$

0.06

$$F_{13} = F_{12} = \frac{A_2}{A_1} F_{21} = \frac{\pi D^2 / 4}{\pi D L} F_{21} = \frac{0.075}{4 \times 0.15} \times 0.94 = 0.118$$

$$q_3 = -1844\ \mathrm{W}$$

Radiation Exchange between Diffuse, Gray Surfaces in an Enclosure

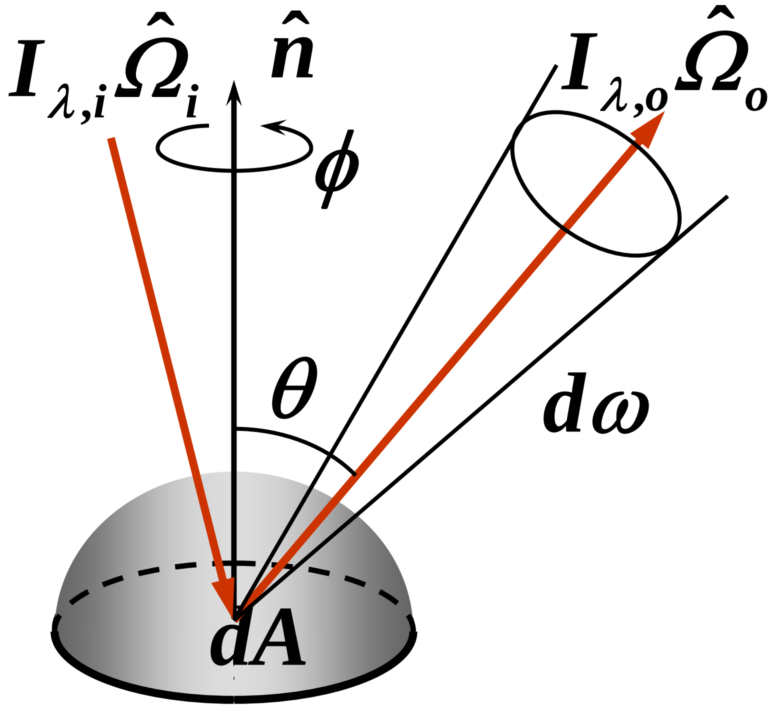
- **Ray tracing method**
- **Net-radiation method**

Irradiation and radiosity

G: **irradiation,** $\mathrm{W/m^2}$
J: **radiosity,** $\mathrm{W/m^2}$

$$J = \int_0^\infty \int_\cap I_{\lambda,o} \cos\theta \, d\omega \, d\lambda$$

$$G = \int_0^\infty \int_\cap I_{\lambda,i} \cos\theta \, d\omega \, d\lambda$$



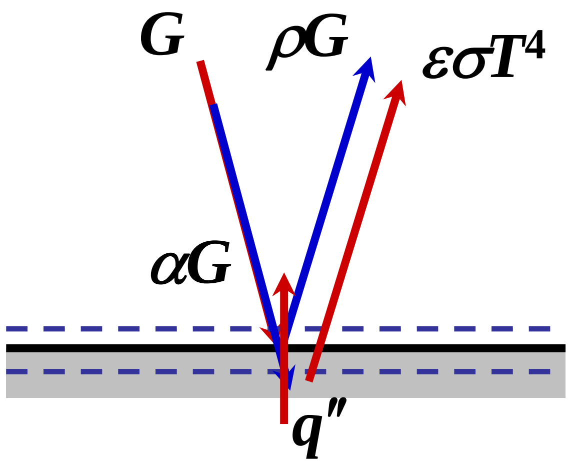
$$J = \varepsilon\sigma T^4 + \rho G$$

$$q'' = J - G = \left(\varepsilon\sigma T^4 + \rho G\right) - G$$

$$= \varepsilon\sigma T^4 - \left(1 - \rho\right) G$$

$$= \varepsilon\sigma T^4 - \alpha G$$

$$= \varepsilon\left(\sigma T^4 - G\right)$$



$$q'' = J - \frac{1}{\rho}\left(J - \varepsilon\sigma T^4\right) = \frac{\varepsilon}{\rho}\left(\sigma T^4 - J\right)$$

$$= \frac{\varepsilon}{1 - \varepsilon}\left(\sigma T^4 - J\right)$$

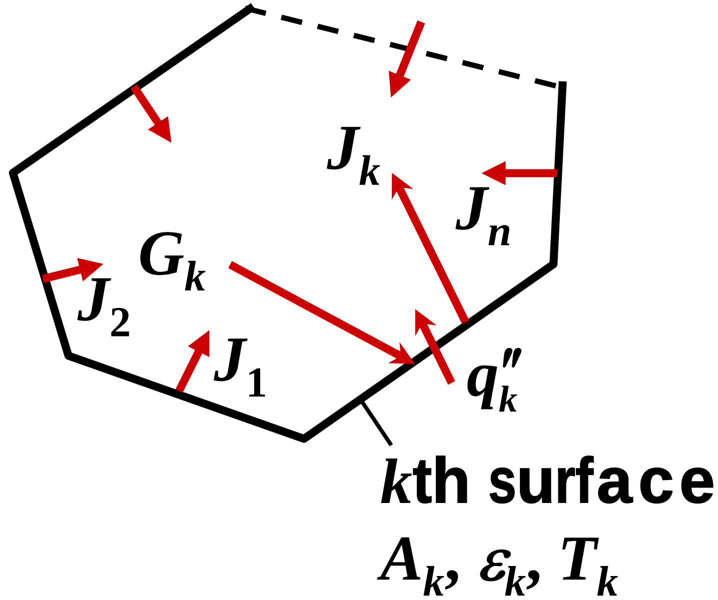
Radiation Exchange in an Enclosure

enclosure with n surfaces

$$q''_k = J_k - G_k$$

$$J_k = \varepsilon_k \sigma T_k^4 + (1-\varepsilon_k) G_k$$

$$q''_k = \frac{\varepsilon_k}{1-\varepsilon_k}\left(\sigma T_k^4 - J_k\right)$$



irradiation

$$G_k A_k = J_1 A_1 F_{1k} + J_2 A_2 F_{2k} + \cdots + J_n A_n F_{nk}$$

$$= J_1 A_k F_{k1} + J_2 A_k F_{k2} + \cdots + J_n A_k F_{kn}$$

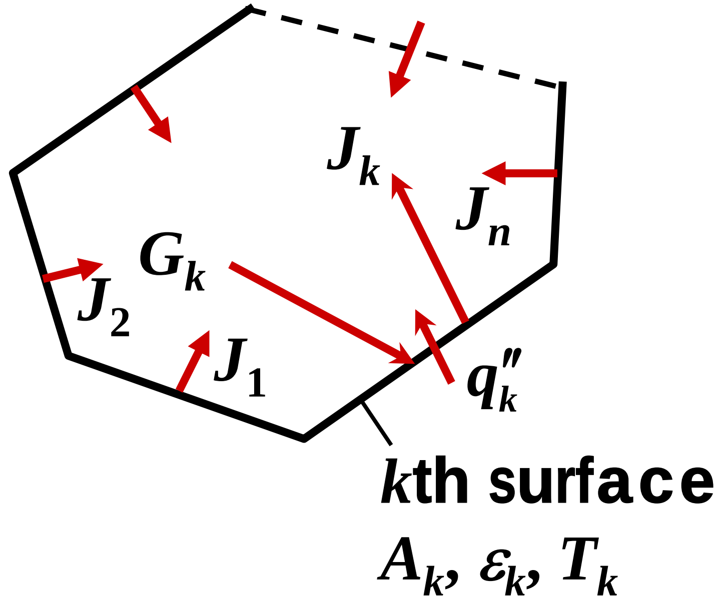
$$= \sum_{i=1}^{n} J_i A_k F_{ki} = A_k \sum_{i=1}^{n} J_i F_{ki} \quad \text{or} \quad G_k = \sum_{i=1}^{n} J_i F_{ki}$$

Summary

$$q''_k = \frac{\varepsilon_k}{1-\varepsilon_k}\left(\sigma T_k^4 - J_k\right)$$

$$J_k = \varepsilon_k \sigma T_k^4 + \left(1-\varepsilon_k\right)\sum_{i=1}^{n} J_i F_{ki}$$

$$k = 1, 2, 3, \cdots, n$$



at the boundary T_k or q_k specified

$2n$ unknowns: J_k and q_k or T_k

When all T_k's are specified, the two equations are decoupled.

n unknowns: J_k

Electric Network Analogy

$$q''_k = \frac{\varepsilon_k}{1-\varepsilon_k}\left(\sigma T_k^4 - J_k\right), \quad G_k = \sum_{i=1}^{n} J_i F_{ki}$$

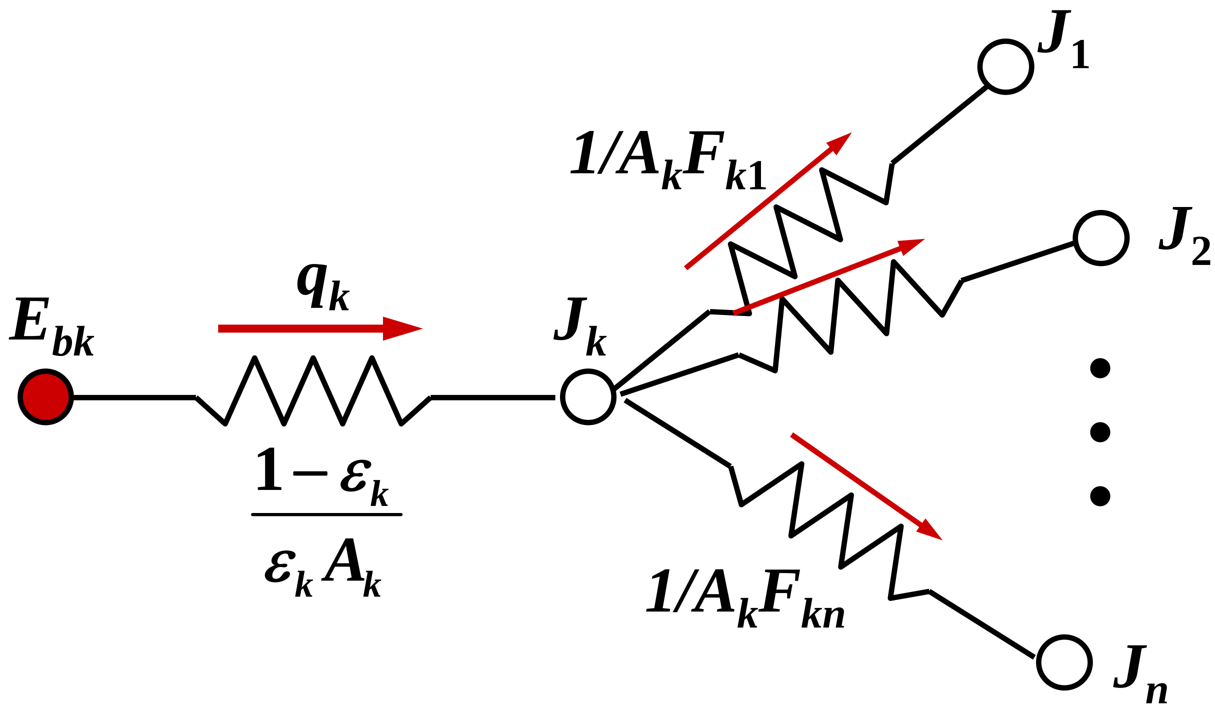
$$q_k = q''_k A_k = \frac{\sigma T_k^4 - J_k}{\left(1-\varepsilon_k\right)/\varepsilon_k A_k} \equiv \frac{E_{bk} - J_k}{R}$$

$$= A_k\left(J_k - G_k\right) = A_k\left(J_k \sum_{i=1}^{n} F_{ki} - \sum_{i=1}^{n} J_i F_{ki}\right)$$

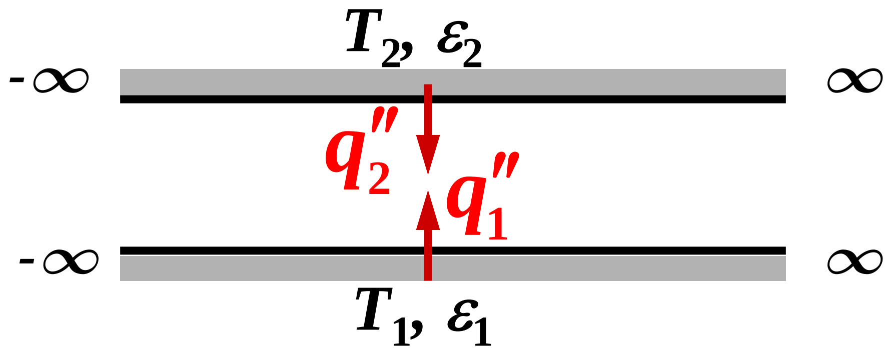
$$= A_k \sum_{i=1}^{n}\left(J_k F_{ki} - J_i F_{ki}\right)$$

$$= \sum_{i=1}^{n} A_k F_{ki}\left(J_k - J_i\right) = \sum_{i=1}^{n} \frac{J_k - J_i}{1/A_k F_{ki}}$$

$$q_k = \frac{E_{bk} - J_k}{\dfrac{1-\varepsilon_k}{\varepsilon_k A_k}} = \sum_{i=1}^{n} \frac{J_k - J_i}{\dfrac{1}{A_k F_{ki}}}$$



ex) two infinite parallel plates



$$q_k'' = \frac{\varepsilon_k}{1-\varepsilon_k}\left(\sigma T_k^4 - J_k\right)$$

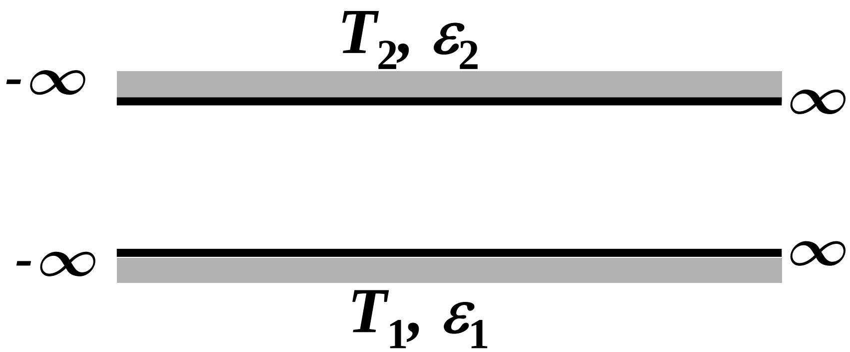
$$q_1'' = \frac{\varepsilon_1}{1-\varepsilon_1}\left(\sigma T_1^4 - J_1\right), \quad q_2'' = \frac{\varepsilon_2}{1-\varepsilon_2}\left(\sigma T_2^4 - J_2\right)$$

$$J_k = \varepsilon_k \sigma T_k^4 + \left(1-\varepsilon_k\right)\sum_{i=1}^{n} J_i F_{ki}$$

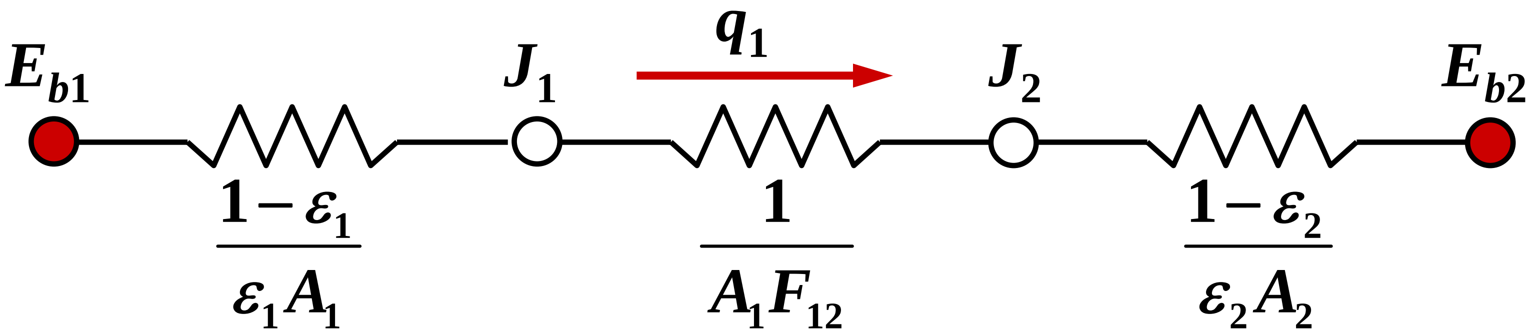
$$J_1 = \varepsilon_1 \sigma T_1^4 + \left(1-\varepsilon_1\right) J_2, \quad J_2 = \varepsilon_2 \sigma T_2^4 + \left(1-\varepsilon_2\right) J_1$$

$$q_1'' = -q_2'' = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

Using network analogy

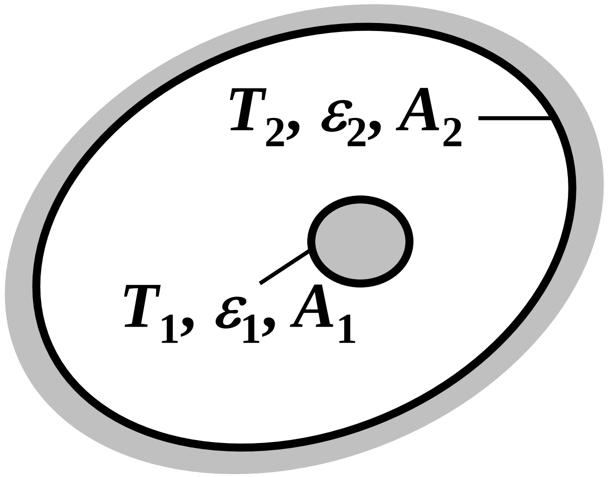


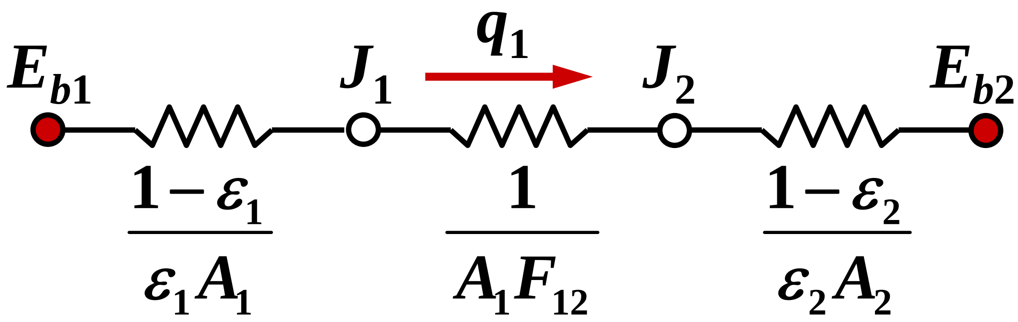
$$q_k = \frac{E_{bk} - J_k}{\dfrac{1-\varepsilon_k}{\varepsilon_k A_k}} = \sum_{i=1}^{n} \frac{J_k - J_i}{\dfrac{1}{A_k F_{ki}}}$$



$$q_1'' = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + 1 + \dfrac{1-\varepsilon_2}{\varepsilon_2}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

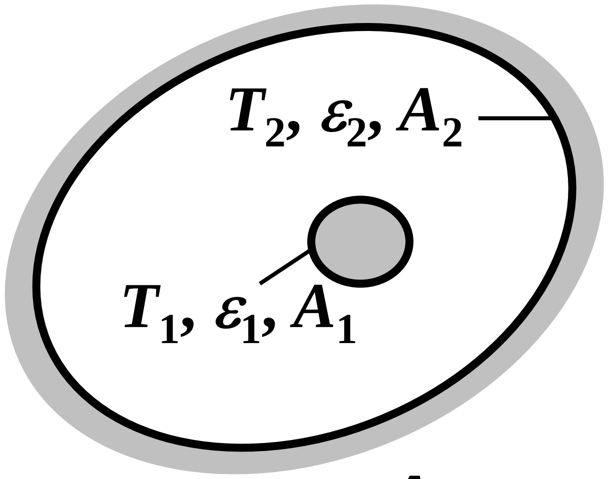
ex) a body in an enclosure





$$q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$



$$q_1 = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

When $\frac{A_1}{A_2} << 1, \quad q_1 = \varepsilon_1 \sigma A_1 \left(T_1^4 - T_2^4\right)$

The enclosure acts like a black cavity.

Remark: when A_2 **is a black enclosure**

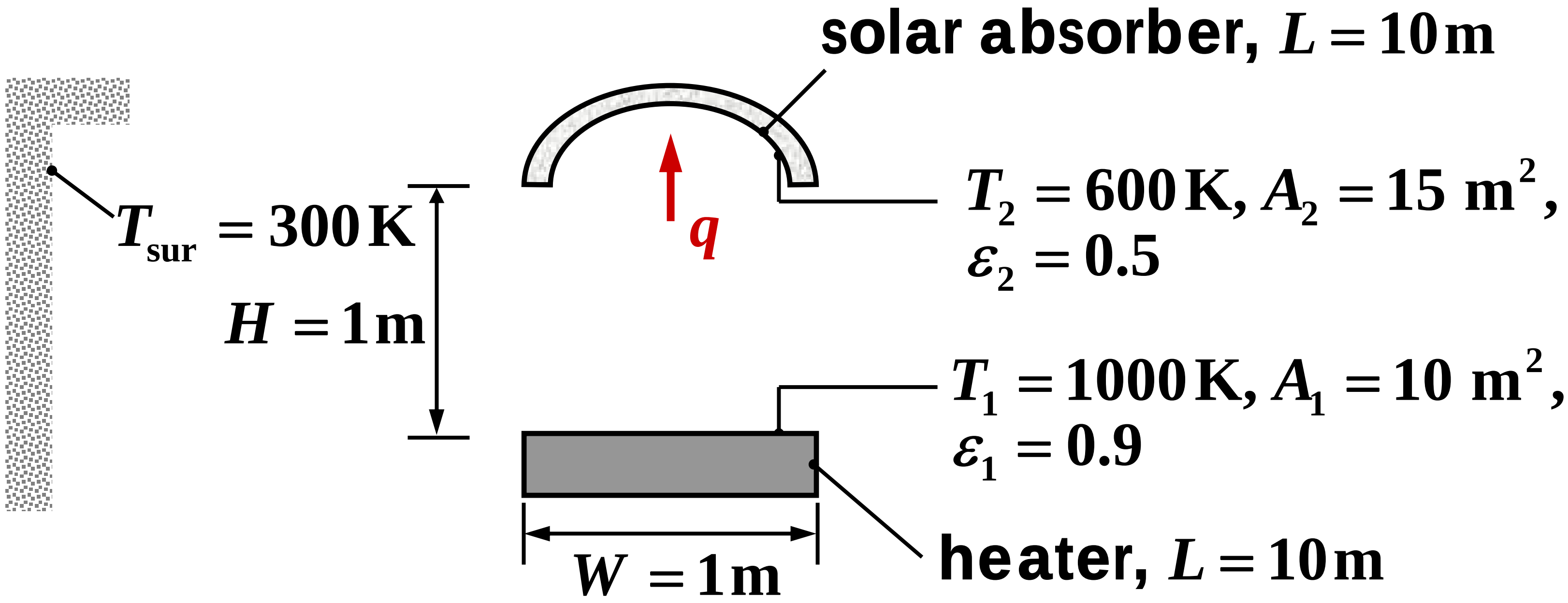
$$q_1 = \varepsilon_1 \sigma T_1^4 A_1 - \alpha_1 G_1 A_1$$

$$G_1 A_1 = \sigma T_2^4 A_2 F_{21} = \sigma T_2^4 A_1 F_{12} = \sigma T_2^4 A_1$$

$$q_1 = \varepsilon_1 \sigma T_1^4 A_1 - \varepsilon_1 \sigma T_2^4 A_1 = \varepsilon_1 \sigma A_1 \left(T_1^4 - T_2^4\right)$$

Example 13.3

Coating on a curved solar absorber surface

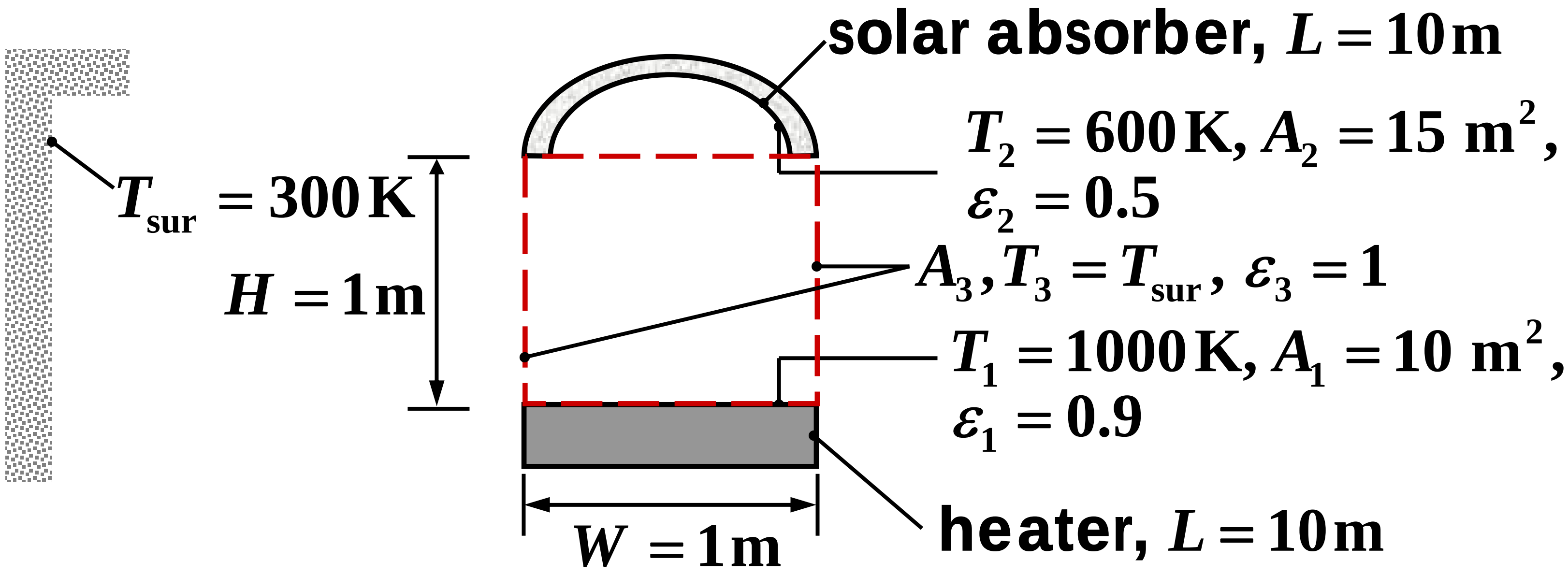


Find:

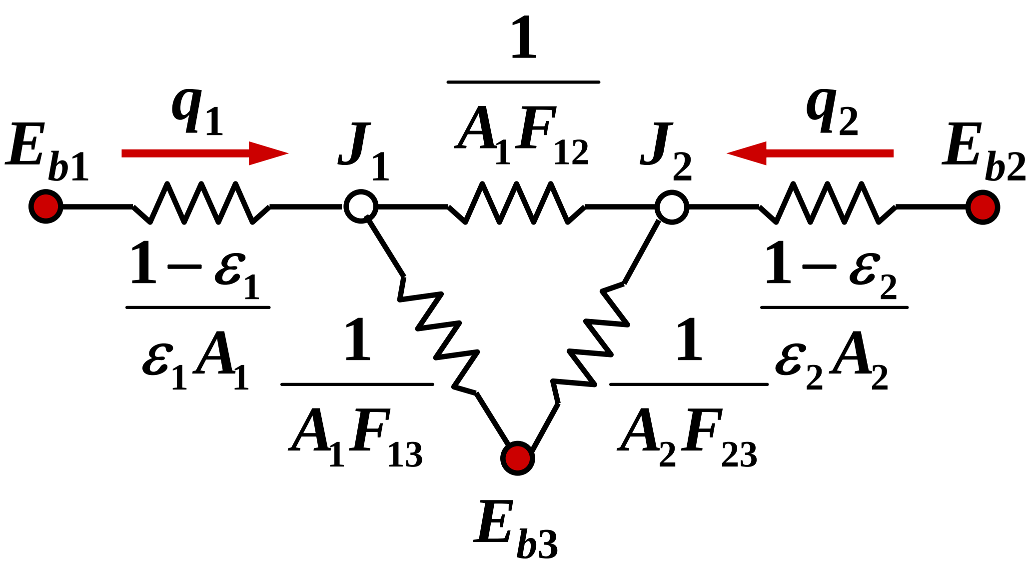
Net rate of heat transfer to the absorber surface

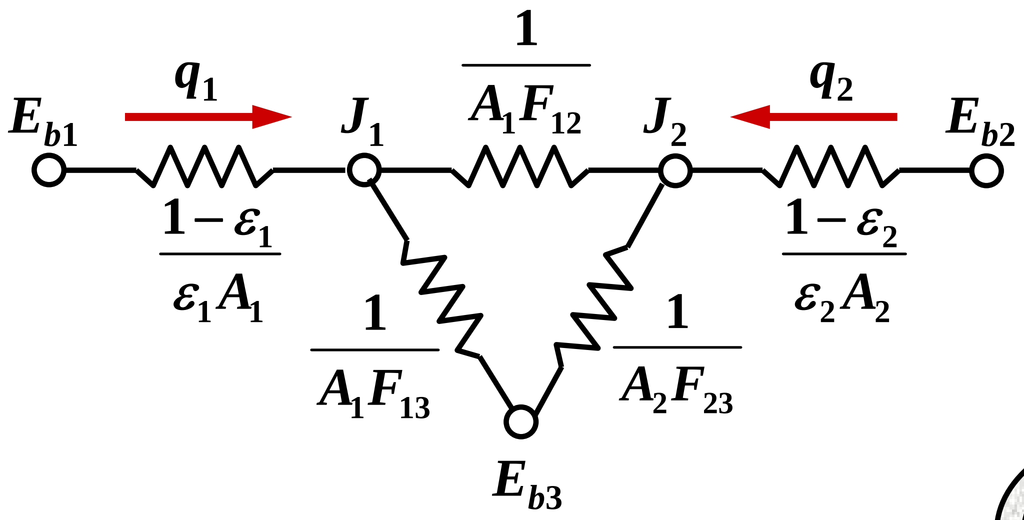
Assumptions:

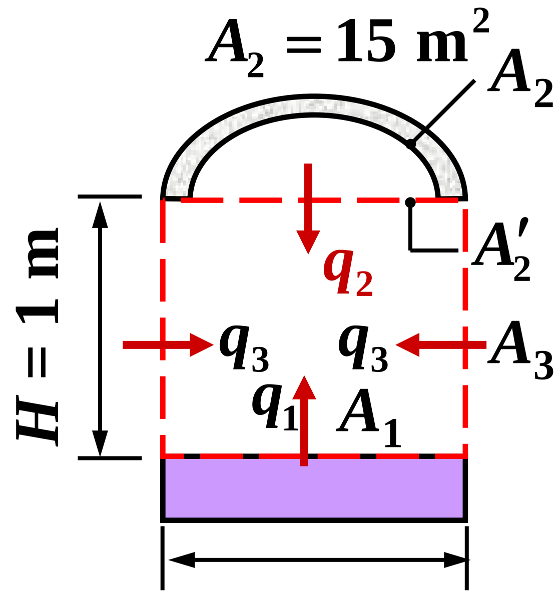
1) Convection effects are negligible.
2) Absorber and heater surfaces are diffuse and gray.



Electric network: 3-surface enclosure system







$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - E_{b3}}{1/A_1F_{13}}$$

$$q_2 = \frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - E_{b3}}{1/A_2F_{23}}$$

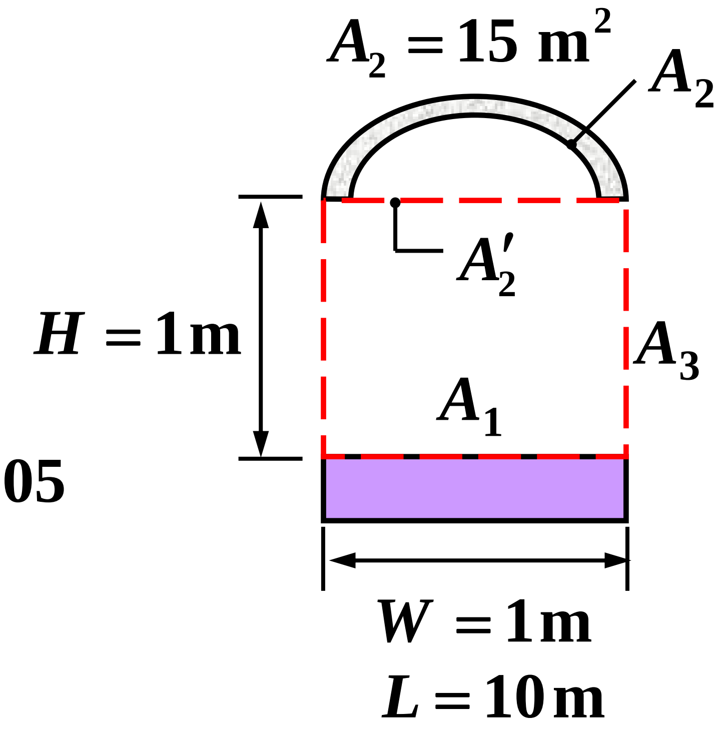
F_{12}: $F_{12} = F_{12'} \quad \left(F_{12} + F_{13} = 1,\ F_{12'} + F_{13} = 1\right)$

From Fig. 13.4: $Y/L = 10,\ X/L = 1 \rightarrow F_{12} = 0.39$

F_{21}: $A_2F_{21} = A_1F_{12},\ F_{21} = \frac{A_1}{A_2}F_{12} = \frac{1\times 10}{15}\times 0.39 = 0.26$

F_{13}: $F_{13} = 1 - F_{12} = 0.61$

F_{23}: $F_{32} = F_{32'} = F_{31} = \dfrac{A_1}{A_3} F_{13}$



$$F_{32} = \frac{A_1}{A_3} F_{13} = \frac{1 \times 10}{2(10 \times 1)} \times 0.61 = 0.305$$

$$F_{23} = \frac{A_3}{A_2} F_{32} = \frac{20}{15} \times 0.305 = 0.41$$

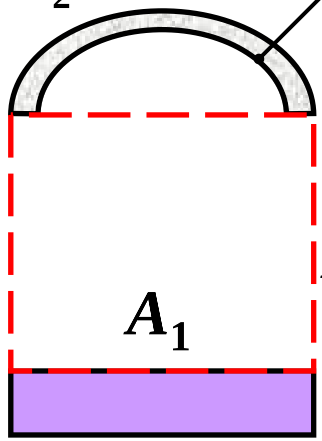
$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1 F_{12}} + \frac{J_1 - E_{b3}}{1/A_1 F_{13}}$$

$$q_2 = \frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2 - J_1}{1/A_2 F_{21}} + \frac{J_2 - E_{b3}}{1/A_2 F_{23}}$$

$$E_{b1} = \sigma T_1^4 = 56,700\ \text{W/m}^2$$

$$E_{b2} = \sigma T_2^4 = 7,348\ \text{W/m}^2$$

$$E_{b3} = \sigma T_3^4 = 459\ \text{W/m}^2$$



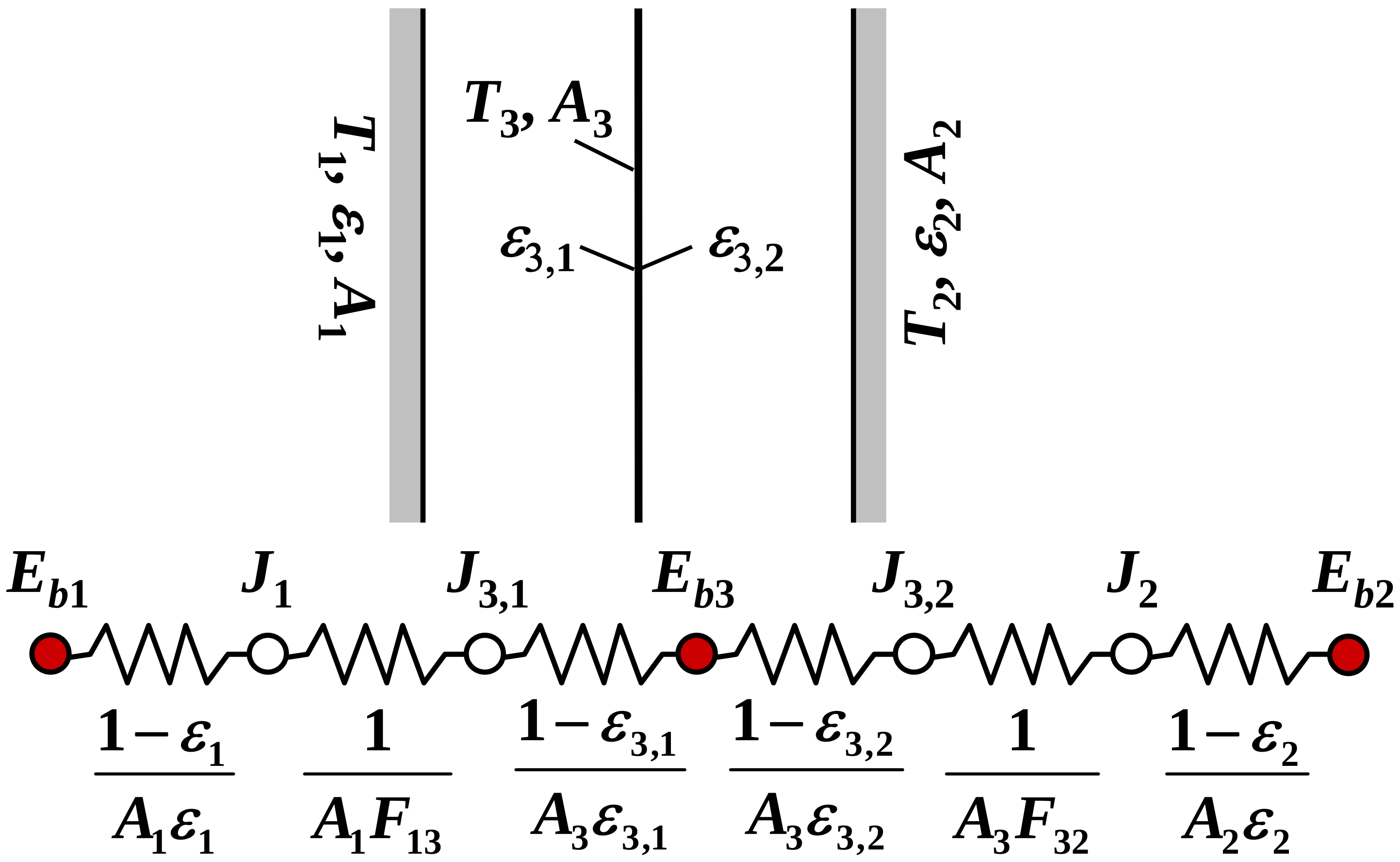
$$\frac{56,700 - J_1}{(1-0.9)/0.9} = \frac{J_1 - J_2}{1/0.39} + \frac{J_1 - 459}{1/0.61} \rightarrow -10J_1 + 0.39J_2 = -510,002$$

$$\frac{7,348 - J_2}{(1-0.5)/0.5} = \frac{J_2 - J_1}{1/0.26} + \frac{J_2 - 459}{1/0.41} \rightarrow 0.26J_1 - 1.67J_2 = -7536$$

$$\therefore J_2 = 12,528\ \text{W/m}^2$$

$$q_2 = \frac{E_{b2} - J_2}{(1-\varepsilon_2)/A_2\varepsilon_2} = \frac{7,348 - 12,528}{(1-0.5)/0.5\times 15} = -77.7\ \text{kW}$$

Radiation Shield



$$q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{13}} + \frac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \frac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \frac{1}{A_3F_{32}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

when $A_1 = A_2 = A_3$ **and** $F_{13} = F_{32} = 1$

$$q_1 = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_{3,1}}{\varepsilon_{3,1}} + \frac{1-\varepsilon_{3,2}}{\varepsilon_{3,2}} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2}}$$

$$= \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 2}$$

when $\varepsilon_1 = \varepsilon_2 = \varepsilon_{3,1} = \varepsilon_{3,2} \equiv \varepsilon$, $q_1 = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{2\left(\frac{2}{\varepsilon} - 1\right)}$

for N **shields,** $\left(q_1\right)_N = \frac{1}{N+1}\left(q_1\right)_0$

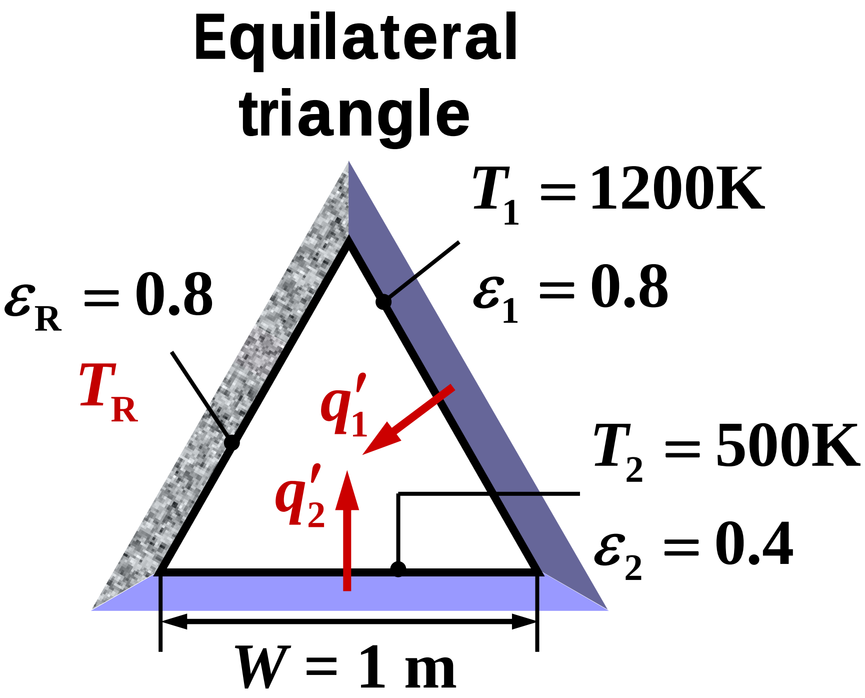
Reradiating Surface

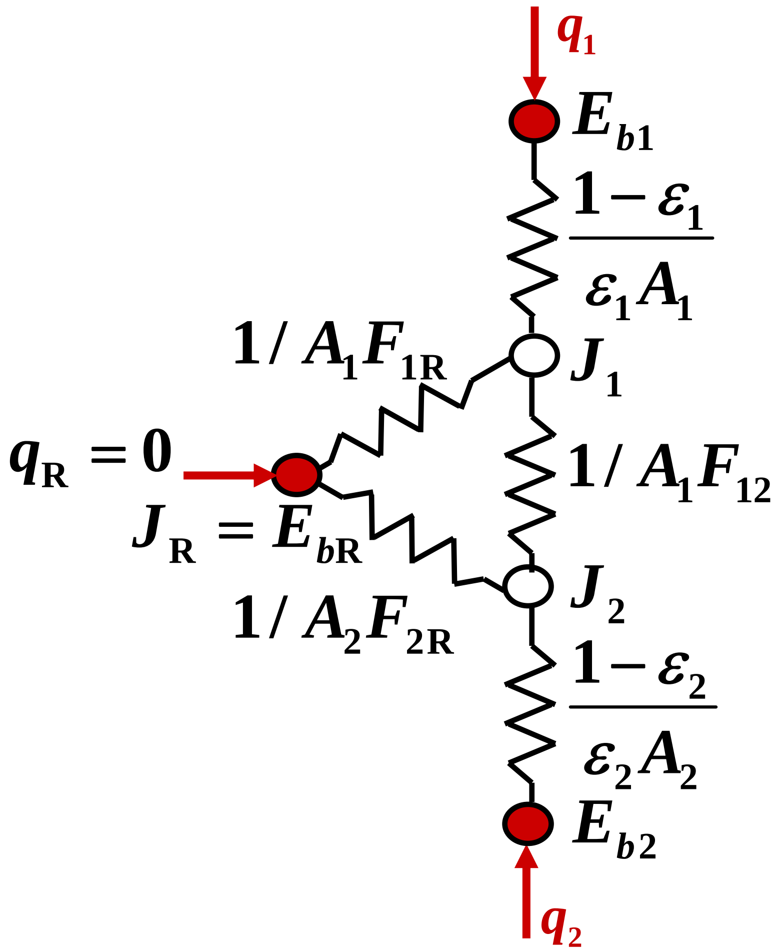
a surface with zero net radiation transfer

$$q_i = A_i\left(J_i - G_i\right) = \frac{A_i\varepsilon_i}{1-\varepsilon_i}\left(\sigma T_i^4 - J_i\right) = 0$$

$$J_i = G_i = \sigma T_i^4$$

Example 13.6





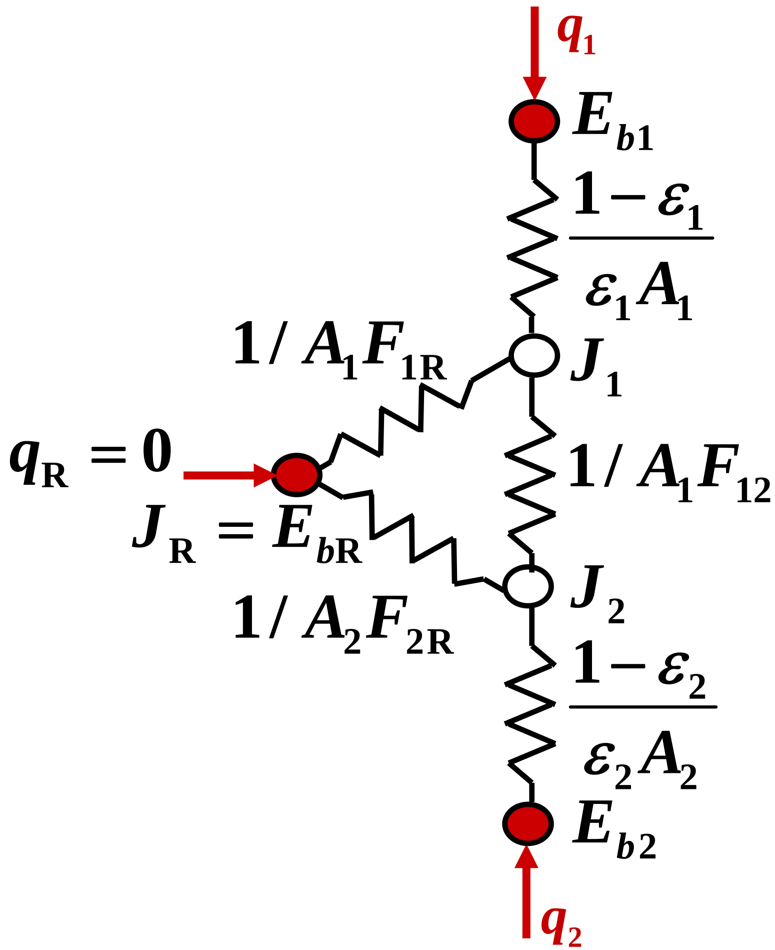
Find:

1) **Rate at which heat must be supplied per unit length of duct,** q'_1, q'_2
2) **Temperature of the insulated surface,** T_R

Assumption: All surface are opaque, diffuse, gray, and of uniform radiosity.

1) q_1, q_2

$$q_1 = \frac{E_{b1} - J_1}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1}} = \frac{J_1 - J_2}{R} = \frac{J_2 - E_{b2}}{\dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$



$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + R + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

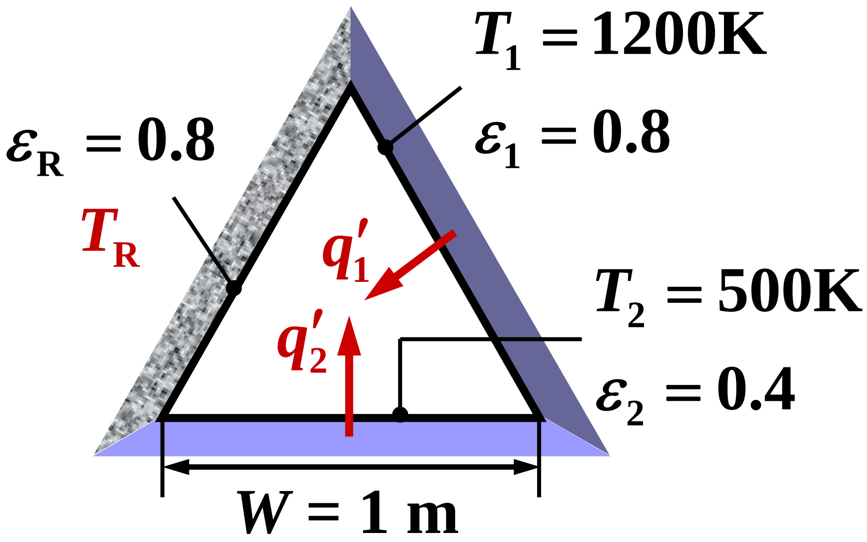
$$\frac{1}{R} = A_1 F_{12} + \frac{1}{1/A_1F_{1R} + 1/A_2F_{2R}}$$

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + \left[\left(1/A_1F_{1R}\right) + \left(1/A_2F_{2R}\right)\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

From symmetry,

$$F_{12} = F_{1R} = F_{2R} = 0.5$$

$$A_1 = A_2 = W \cdot L$$



$$q_1' = \frac{q_1}{L} = \frac{5.67\times10^{-8}\left(1200^4 - 500^4\right)}{\dfrac{1-0.8}{0.8\times1} + \dfrac{1}{1\times0.5+\left(2+2\right)^{-1}} + \dfrac{1-0.4}{0.4\times1}}$$

$$= 37\text{ kW/m} = -q_2'$$

2) T_R

$$\frac{J_1 - E_{bR}}{1/A_1F_{1R}} = \frac{E_{bR} - J_2}{1/A_2F_{2R}}$$

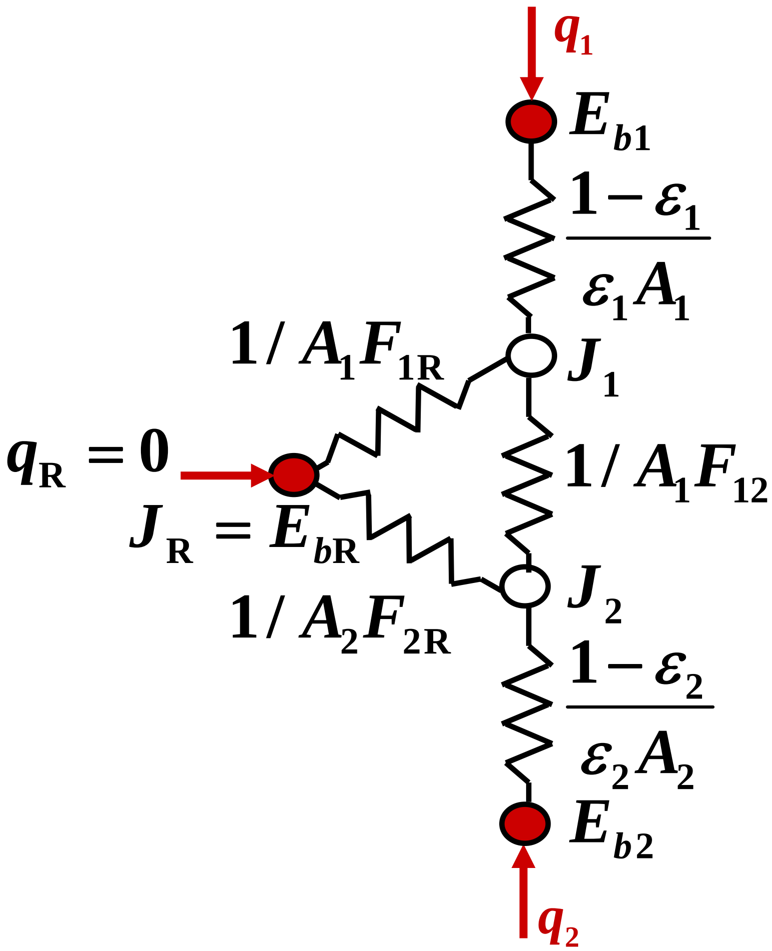
$$q_1 = \frac{E_{b1} - J_1}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1}} = \frac{J_1 - J_2}{R} = \frac{J_2 - E_{b2}}{\dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

$$J_1 = E_{b1} - \frac{1-\varepsilon_1}{\varepsilon_1 A_1} q_1 = 108{,}323\ \mathrm{W/m^2}$$

$$J_2 = E_{b2} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2} q_1 = 59{,}043\ \mathrm{W/m^2}$$

$$E_{bR} = \sigma T_R^4 = 83{,}683\ \mathrm{W/m^2}$$

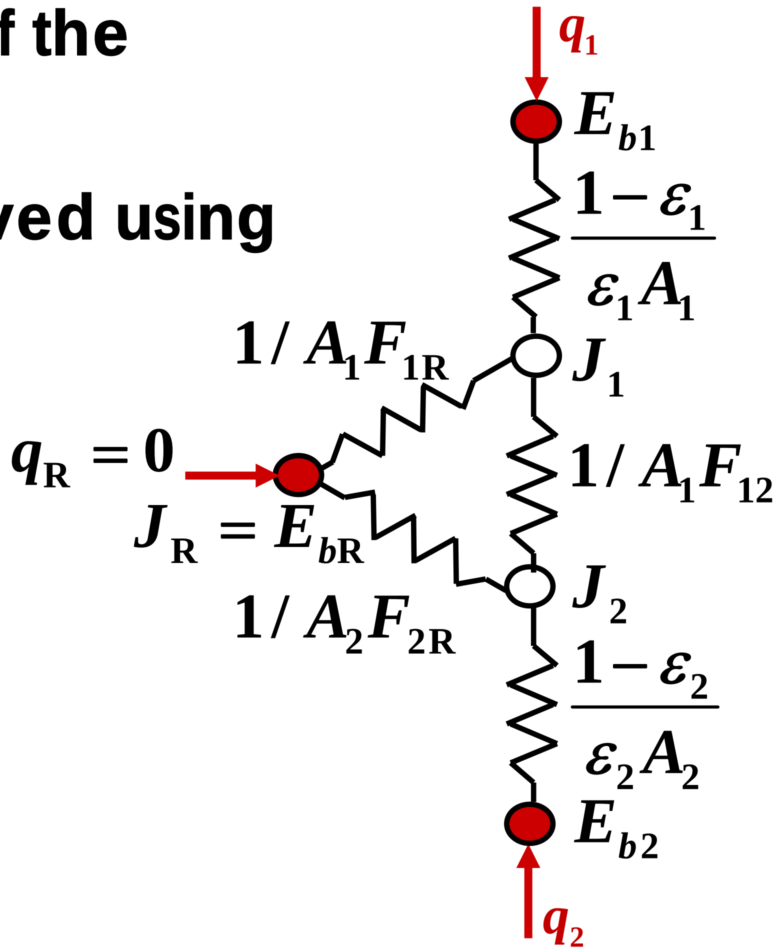
$$T_R = \left(\frac{83{,}683}{5.67 \times 10^{-8}} \right)^{1/4} = 1102\ \mathrm{K}$$



Comments:

1) The results are independent of the value of ε_R.

2) This problem may also be solved using the matrix inversion method.



$$\frac{E_{b1}-J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1-J_2}{1/A_1F_{12}} + \frac{J_1-J_R}{1/A_1F_{1R}}$$

$$\frac{E_{b2}-J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{J_2-J_1}{1/A_2F_{21}} + \frac{J_2-J_R}{1/A_2F_{2R}}$$

$$0 = \frac{J_R-J_1}{1/A_RF_{R1}} + \frac{J_R-J_2}{1/A_RF_{R2}}$$

$$10J_1 - J_2 - J_R = 940,584$$

$$-J_1 + 3.33J_2 - J_R = 4725$$

$$-J_1 - J_2 + 2J_R = 0$$

From these equations, the matrices are

$$\begin{bmatrix} 10 & -1 & -1 \\ -1 & 3.33 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_R \end{bmatrix} = \begin{bmatrix} 940,584 \\ 4725 \\ 0 \end{bmatrix}$$

Solving the equation, it follows that

$$J_1 = 108,328\ \mathrm{W/m^2}$$

$$J_2 = 59,018\ \mathrm{W/m^2}$$

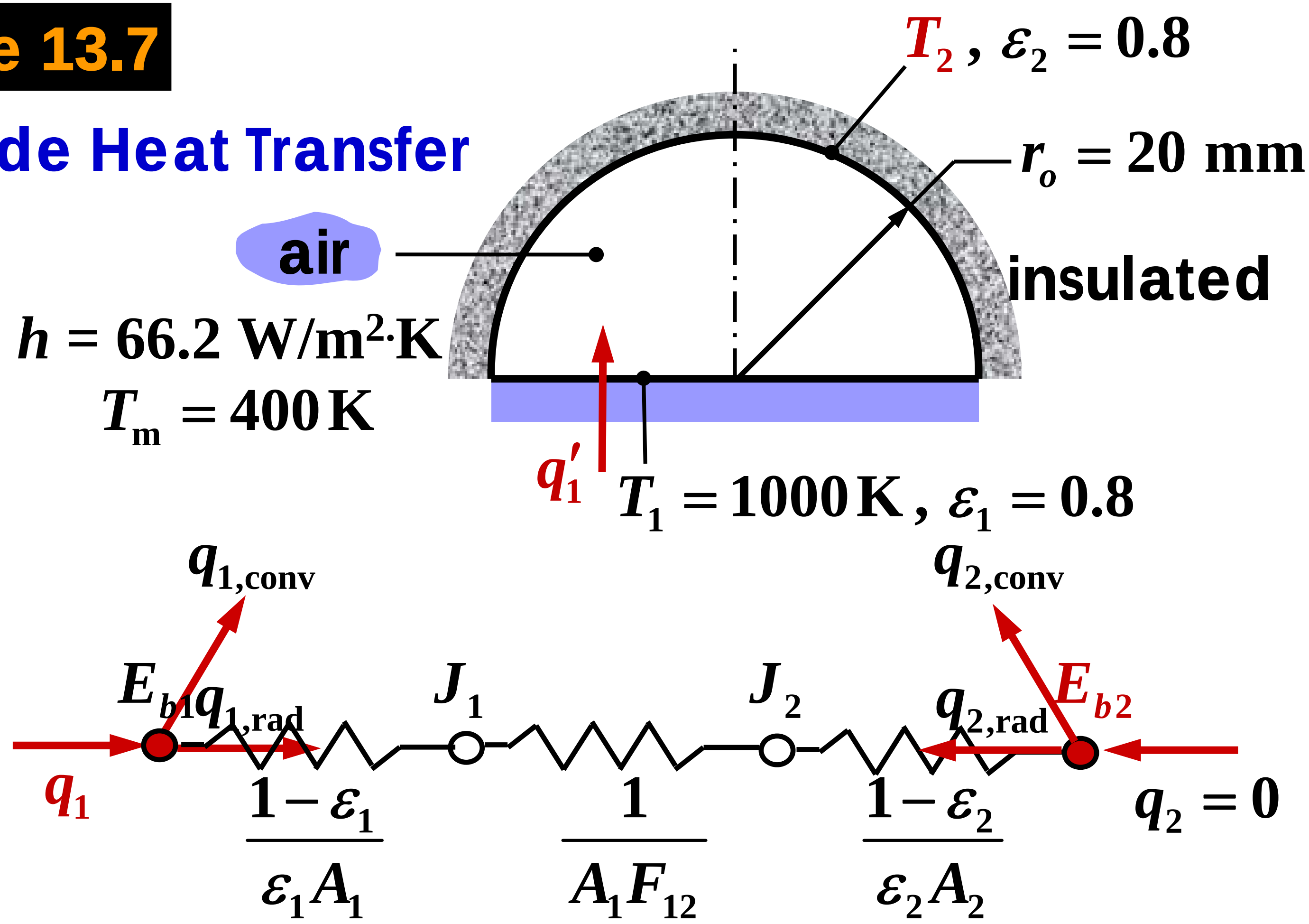
$$J_\mathrm{R} = 83,673\ \mathrm{W/m^2}$$

Recognizing that $J_\mathrm{R} = \sigma T_\mathrm{R}^4$,

$$T_\mathrm{R} = \left(\frac{J_\mathrm{R}}{\sigma}\right)^{1/4} = \left(\frac{83,673}{5.67\times10^{-8}}\right)^{1/4} = 1102\ \mathrm{K}$$

Example 13.7

Multimode Heat Transfer

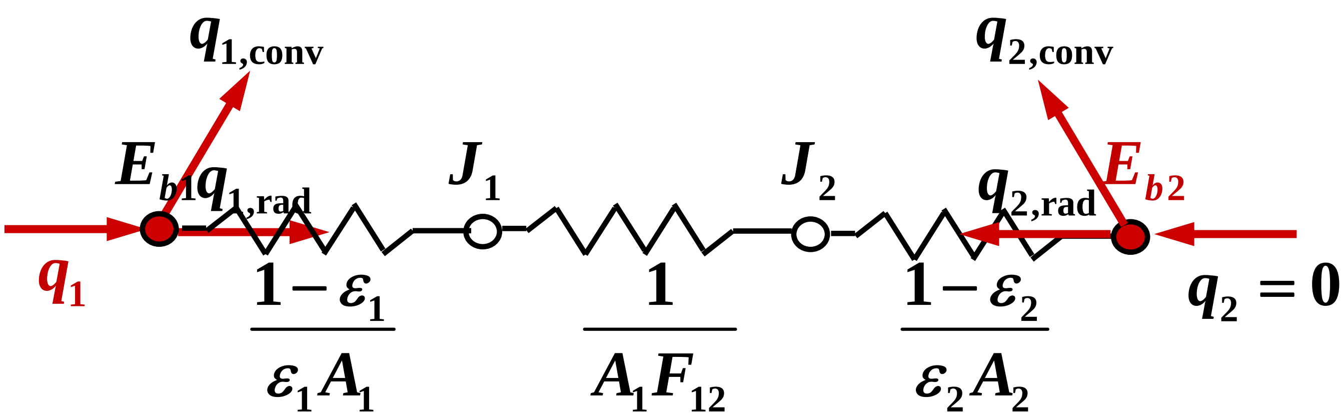


Find:

Rate at which heat must be supplied and temperature of insulated surface.

Assumption:

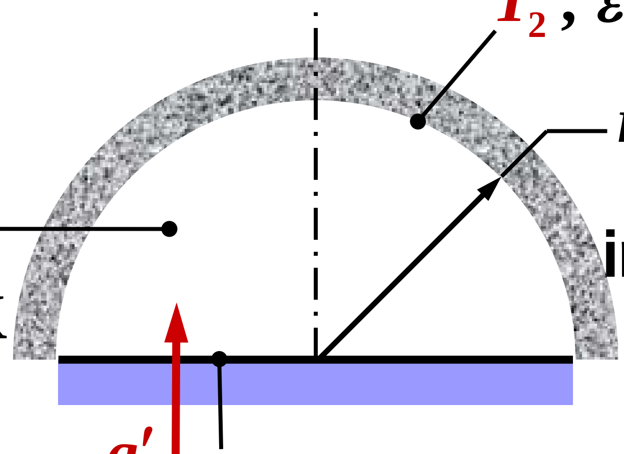
Diffuse, gray surfaces



$$q_1 = q_{1,\text{rad}} + q_{1,\text{conv}}, \quad q_2 = q_{2,\text{rad}} + q_{2,\text{conv}} = 0$$

$$q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} + hA_1\left(T_1 - T_\text{m}\right)$$

$$q_2 = \frac{\sigma\left(T_2^4 - T_1^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} + hA_2\left(T_2 - T_\text{m}\right) = 0$$



$$\frac{\sigma\left(T_2^4 - T_1^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} + hA_2\left(T_2 - T_m\right) = 0, \quad F_{12} = 1$$

$$5.67\times10^{-8} T_2^4 + 146.5 T_2 - 115{,}313 = 0 \quad \rightarrow T_2 = 696 \text{ K}$$

$$q_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} + hA_1\left(T_1 - T_m\right) = 2820 \text{ W/m}$$

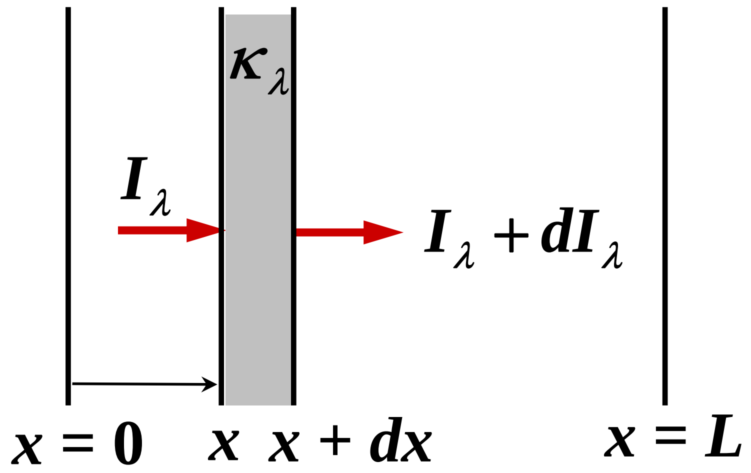
Radiation in Participating Media

participating:
absorbing, emitting and scattering

Attenuation by Absorption and Scattering

$$dI_\lambda = -\kappa_\lambda I_\lambda dx \quad \text{(experimental observation)}$$

or $$\frac{dI_\lambda}{dx} = -\kappa_\lambda I_\lambda$$



κ_λ: **extinction coefficient** [cm^{-1}]

$$\kappa_\lambda = \kappa_\lambda(\lambda, T, P, C_i)$$

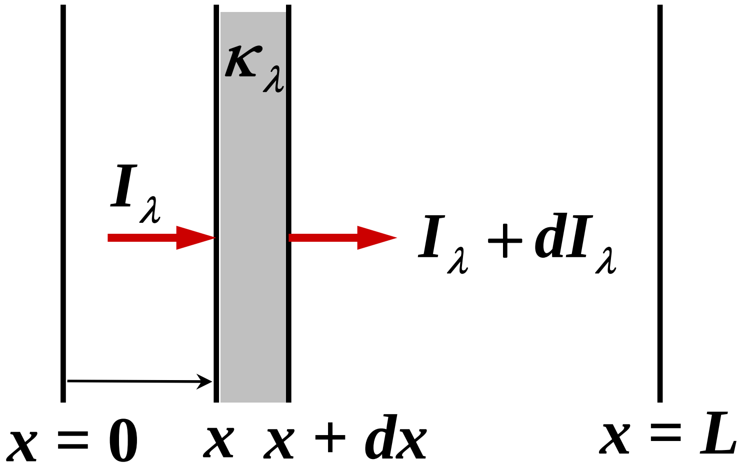
$$\kappa_\lambda = a_\lambda + \sigma_\lambda$$

a_λ: **absorption coefficient**

σ_λ: **scattering coefficient**

$$dI_\lambda = -\kappa_\lambda I_\lambda dx$$

$$\int_{i_\lambda(0)}^{i_\lambda(L)} \frac{dI_\lambda}{I_\lambda} = -\int_0^L \kappa_\lambda(x)dx$$



$$\ln\frac{I_\lambda(L)}{I_\lambda(0)} = -\int_0^L \kappa_\lambda(x)dx$$

$I_\lambda(L) = I_\lambda(0)\exp\left[-\int_0^L \kappa_\lambda(x)dx\right]$: **Bouguer's law**

When κ_λ **= constant,** $I_\lambda(L) = I_\lambda(0)e^{-\kappa_\lambda L}$

Transmittance, Absorptance and Emittance

$$I_\lambda(L) = I_\lambda(0)e^{-\kappa_\lambda L}$$

transmittance

$$\tau_\lambda = \frac{I_\lambda(L)}{I_\lambda(0)} = e^{-\kappa_\lambda L}$$

absorptance

$$\alpha_\lambda = \frac{I_\lambda(0) - I_\lambda(L)}{I_\lambda(0)} = 1 - e^{-\kappa_\lambda L} = 1 - \tau_\lambda$$

emittance: when Kirchhoff's law is assumed

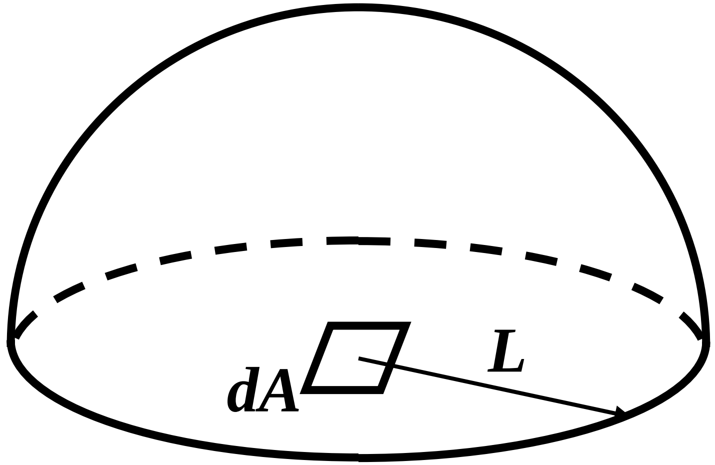
$$\alpha_\lambda = \varepsilon_\lambda$$

Gaseous Emission and Absorption

radiation emission from a hemispherical gas mass of temperature T_g to a surface element dA, which is located at the center of the hemisphere's base

$$E_g = \varepsilon_g \sigma T_g^4$$

$$\varepsilon_g = \varepsilon_g(T_g, P_g L)$$

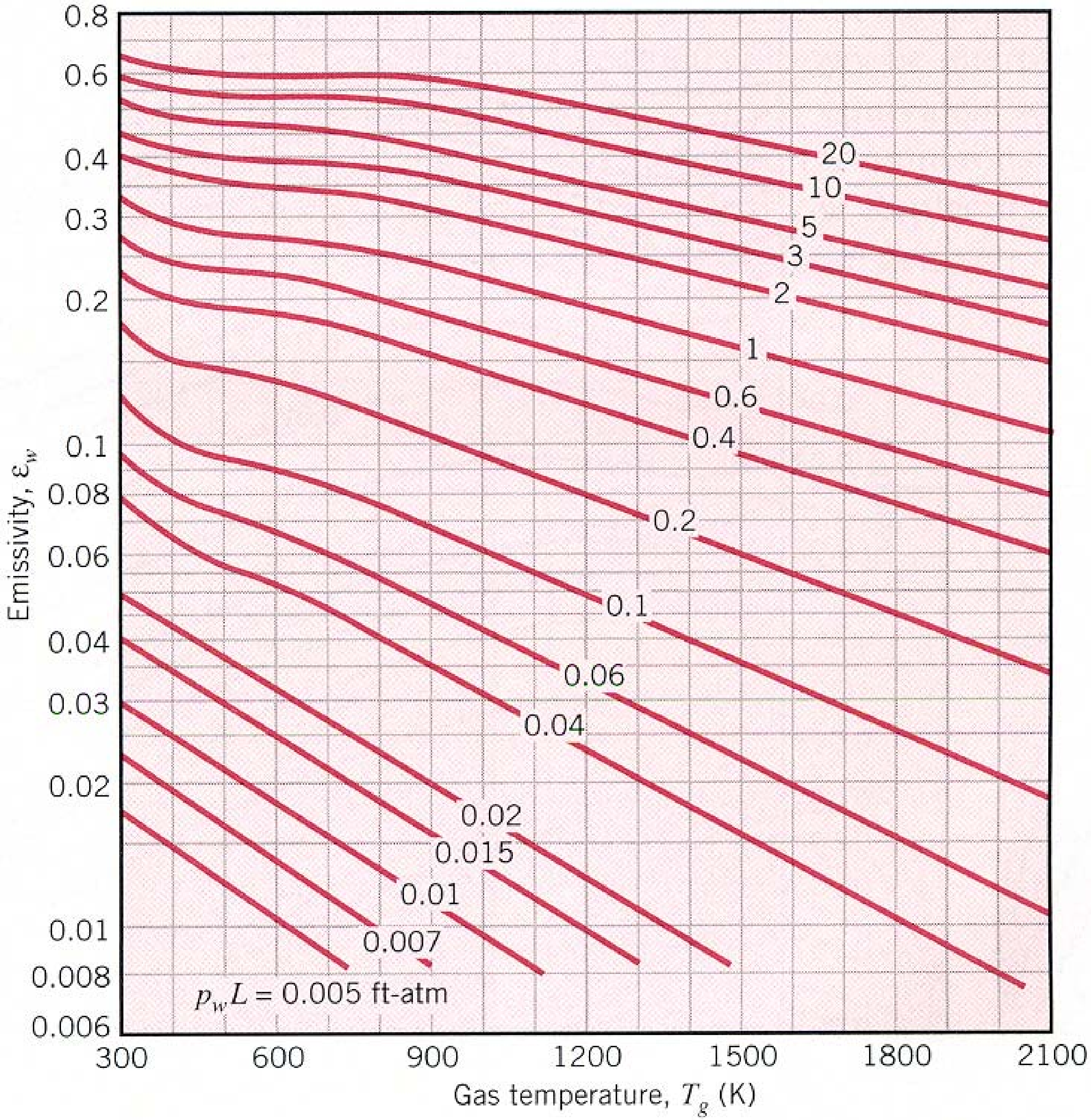


mixtures of water and carbon dioxide:

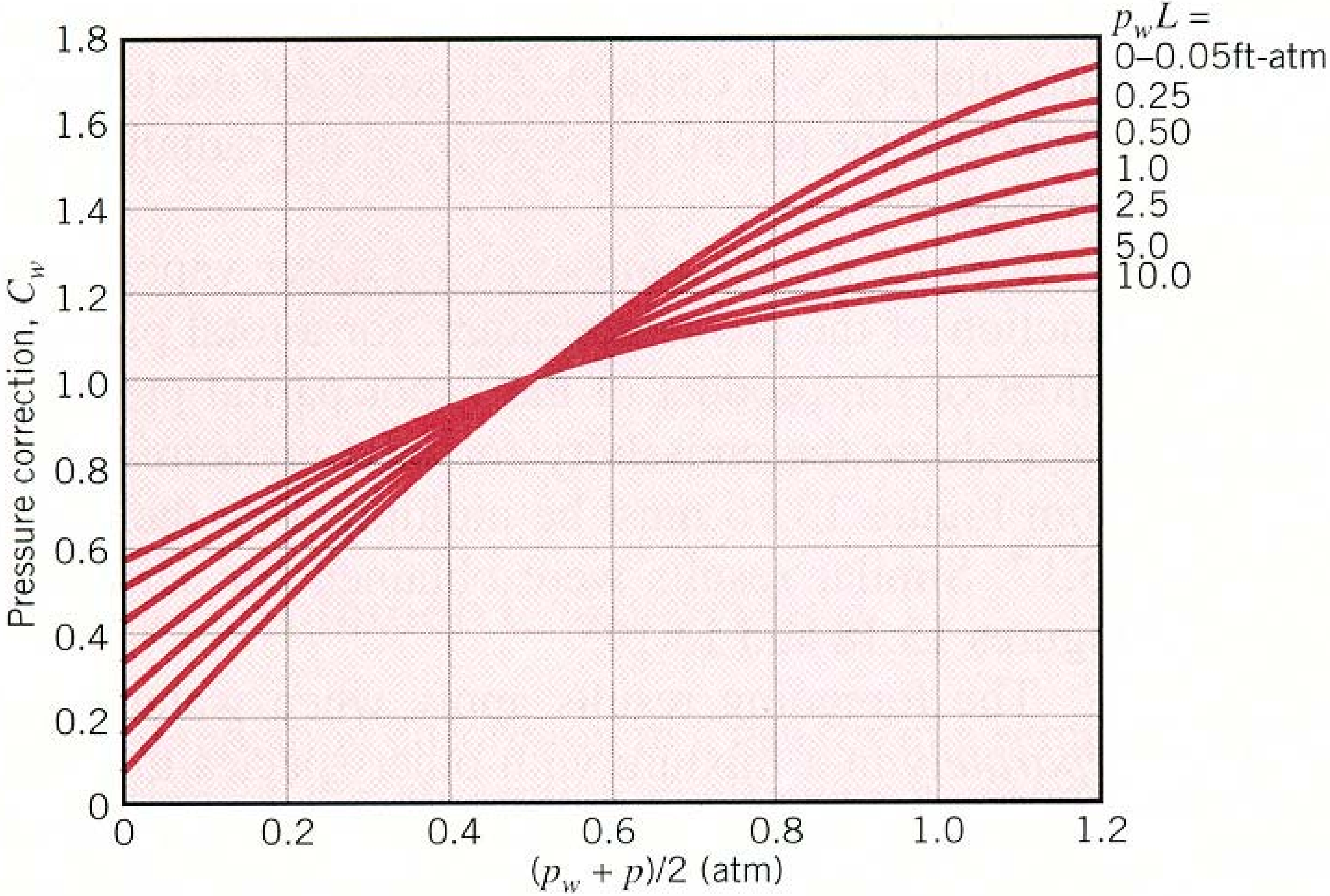
$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon$$

For geometries other than hemisphere: use mean beam length L_e

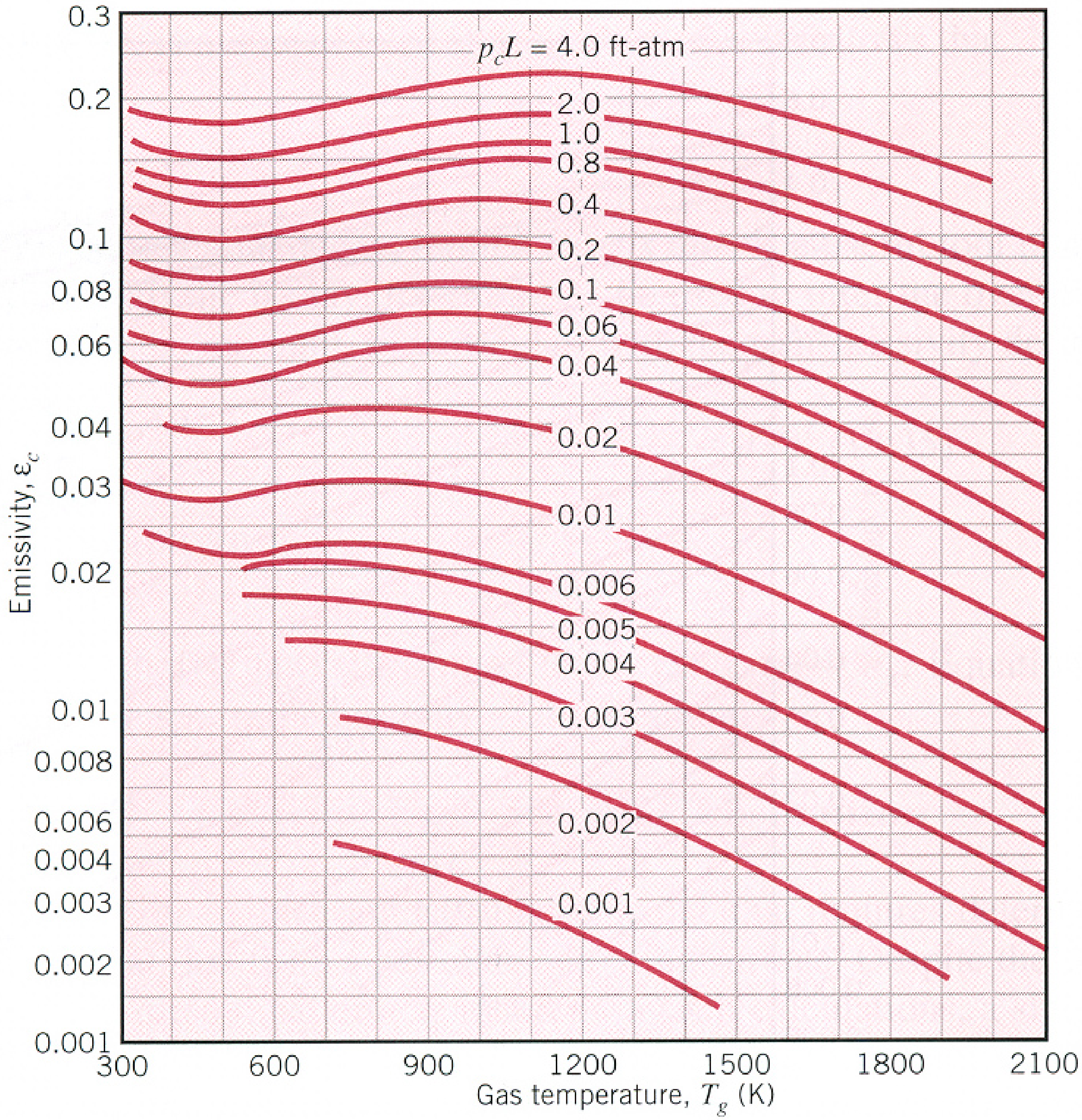
Emissivity of water vapor in a mixture with nonradiating gases at 1-atm total pressure and of hemispherical shape



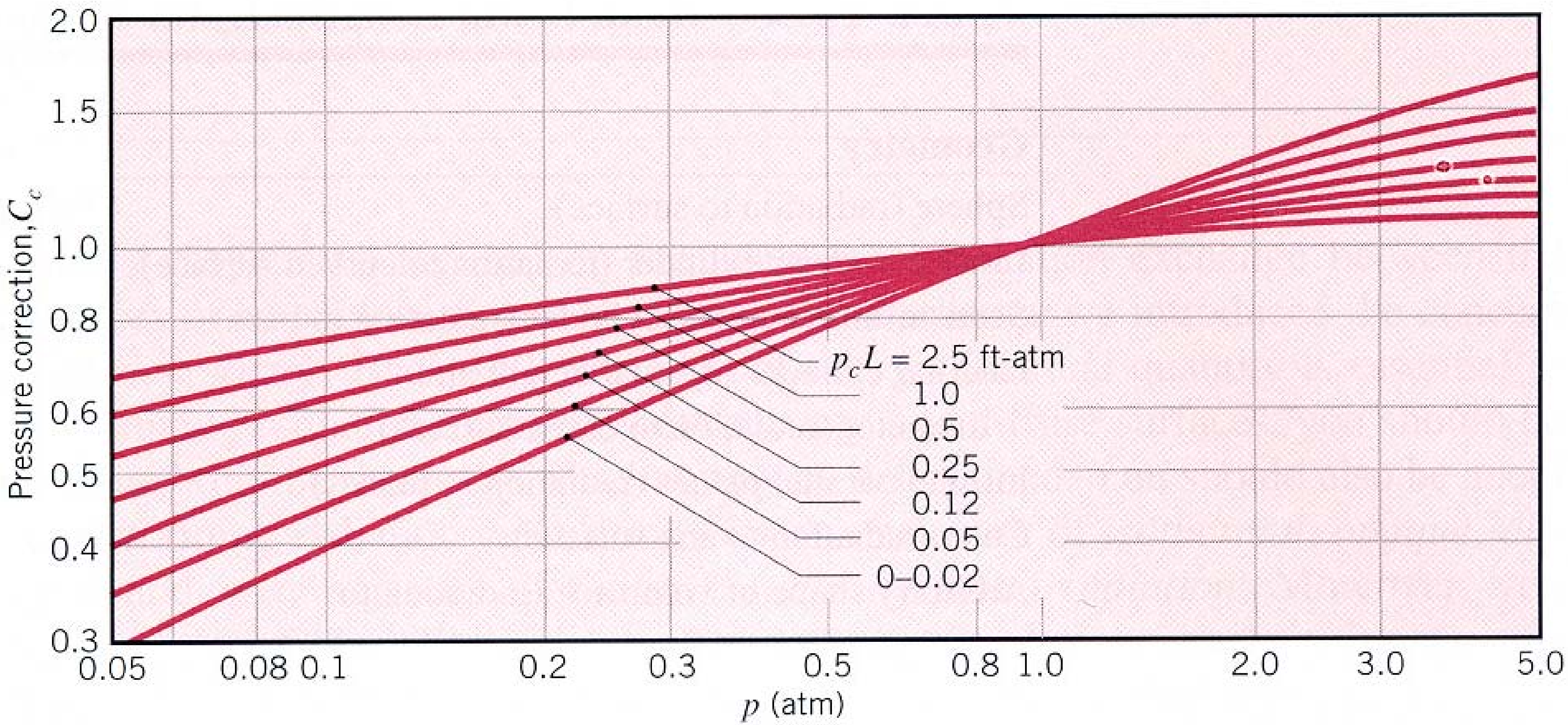
Correction factor for obtaining water vapor emissivities at pressure other than 1 atm



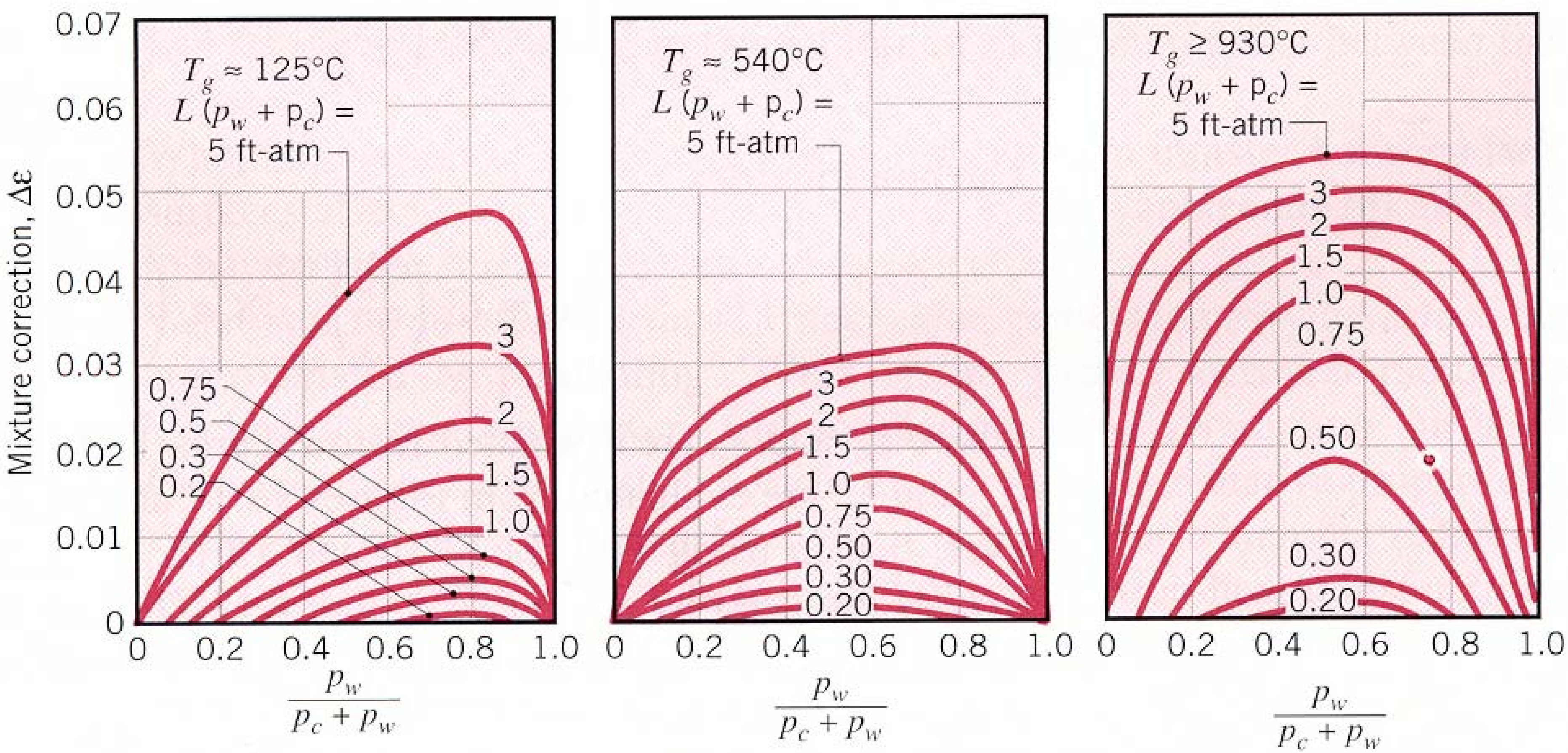
Emissivity of carbon dioxide in a mixture with nonradiating gases at 1-atm total pressure and of hemispherical shape



Correction factor for obtaining carbon dioxide emissivities at pressure other than 1 atm



Correction factors associated with mixtures of water vapor and carbon dioxide



Net Radiation Transfer from Gas Mass to Black Surfaces

$$q_{\text{net}} = A_s \sigma \left(\varepsilon_g T_g^4 - \alpha_g T_s^4 \right)$$

water:

$$\alpha_w = C_w \left(\frac{T_g}{T_s} \right)^{0.45} \times \varepsilon_w (T_s, P_w L_e \frac{T_s}{T_g})$$

carbon dioxide:

$$\alpha_c = C_c \left(\frac{T_g}{T_s} \right)^{0.65} \times \varepsilon_c (T_s, P_c L_e \frac{T_s}{T_g})$$