Tokamak Transport

- Resistive plasma diffusion
- Pfirsch-Schluter current
- Pfirsch-Schluter diffusion
- Banana regime transport
- Plateau transport
- Ware pinch effect
- Bootstrap current
- Neoclassical resistivity
- Ripple transport

Resistive Plasma Diffusion

Resistive diffusion of plasma across a magnetic field

$$\left.\begin{aligned}\vec{j}\times\vec{B}&=\nabla p\\ \eta\vec{j}&=\vec{E}+\vec{v}\times\vec{B}\end{aligned}\right\}$$

$$\eta\nabla p=\eta\vec{j}\times\vec{B}=(\vec{E}+\vec{v}\times\vec{B})\times\vec{B}=\vec{E}\times\vec{B}-B^{2}\vec{v}_{\perp}$$

$$\vec{v}_{\perp}=\frac{\vec{E}\times\vec{B}}{B^{2}}-\eta_{\perp}\frac{\nabla p}{B^{2}}$$

Diffusion due to resistivity only, i.e. $\boldsymbol{E}=0$

$$n\vec{v}_{\perp}=-\eta_{\perp}n\frac{\nabla p}{B^{2}}$$

the rate of density change from the continuity equation

$$\frac{\partial n}{\partial t}=-\nabla\cdot(n\vec{v}_{\perp})=\nabla\cdot\left(\eta_{\perp}n\frac{\nabla p}{B^{2}}\right)=\nabla\cdot\left(\frac{\eta_{\perp}\nabla n}{2\mu_{o}}\frac{2\mu_{o}p}{B^{2}}\right)=\nabla\cdot\frac{\eta_{\perp}\beta}{2\mu_{o}}\nabla n=\nabla\cdot D\nabla n$$

$$\beta=\frac{p}{B^{2}/2\mu_{o}}\qquad D=\frac{\eta_{\perp}\beta}{2\mu_{o}}$$

Using collisional model,

$$\rho_{e}\sim\sqrt{2}m_{e}v_{Te}/eB$$

$$\left.\begin{aligned}\beta&\sim nm_{e}v_{Te}^{2}/(B^{2}/2\mu_{o})\\ \eta_{\perp}&\sim m_{e}/ne^{2}\tau_{e}\end{aligned}\right\}\longrightarrow\boxed{D\sim\frac{\rho_{e}^{2}}{\tau_{e}}}$$

Same as random walk model with step length ρ_e and time τ_e.

Diffusion in a Cylinder

Diffusion velocity in a circular cylinder

$$v_r = \frac{1}{B^2}\left(E_\theta B_z - E_z B_\theta - \eta_\perp \frac{dp}{dr} \right)$$

in a steady-state, $v_r = 0 \quad E_\theta = 0 \quad E_z = const.$ $\quad \frac{dp}{dr} = -\frac{E_z B_\theta}{\eta_\perp}$

Using parallel component of Ohm’s law, $E_z = \eta_{//} j_z$

$$\frac{dp}{dr} = -\frac{\eta_{//}}{\eta_\perp} j_z B_\theta$$

from poloidal beta, $\beta_p = \frac{2\int_0^a prdr}{(B_{\theta a}^2 / 2\mu_o)a^2}$ $\qquad 2\int_0^a prdr = -\int_0^a \frac{dp}{dr} r^2 dr$

Using Ampere’s law, $j_z = \frac{1}{\mu_o r}\frac{d}{dr}(rB_\theta)$

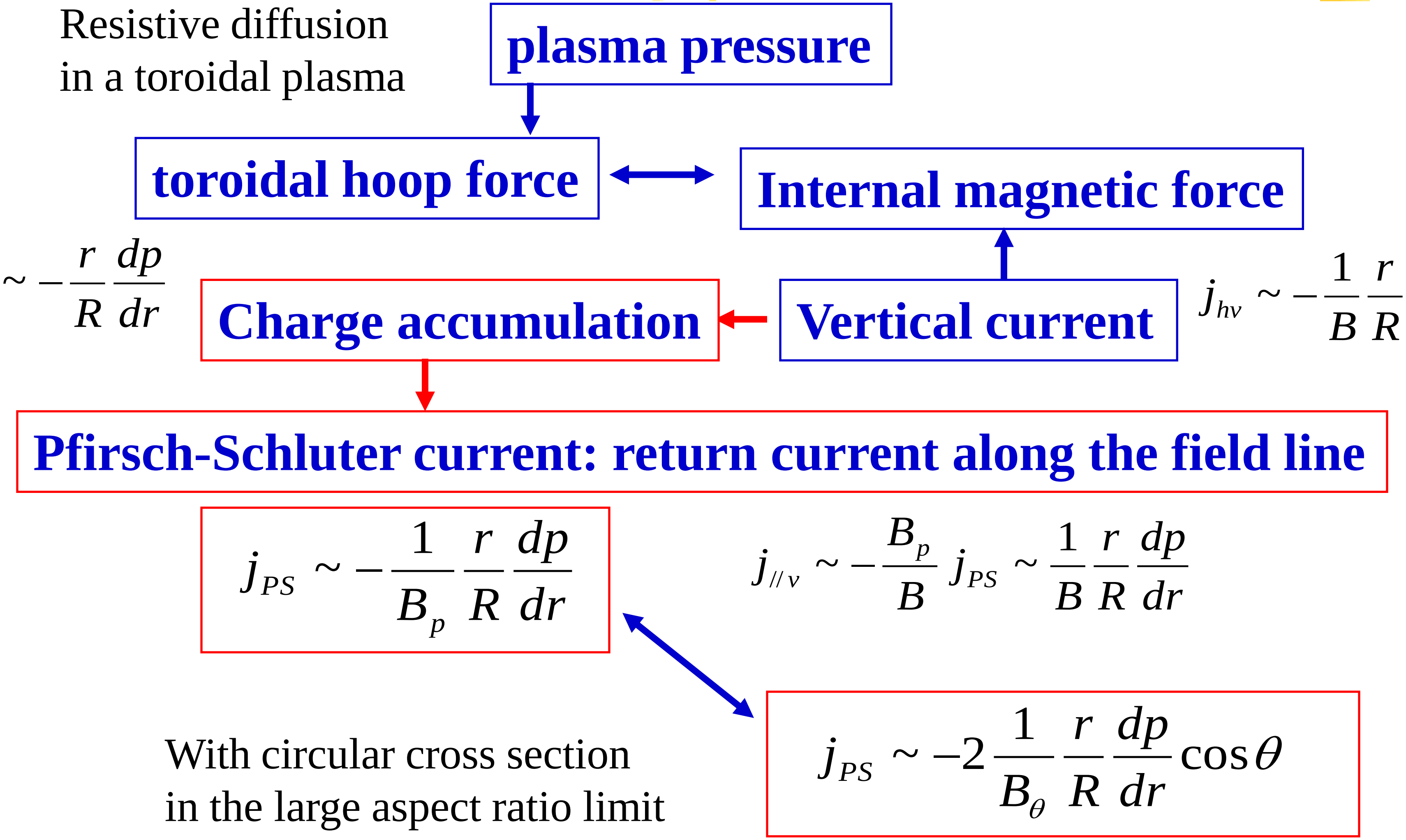
$$2\int_0^a prdr = \int_0^a \frac{\eta_{//}}{\eta_\perp} j_z B_\theta r^2 dr = \int_0^a \frac{\eta_{//}}{\eta_\perp} \frac{B_\theta r}{\mu_o} \frac{d}{dr}(rB_\theta) dr = \frac{\eta_{//}}{\eta_\perp} \frac{B_\theta^2}{2\mu_o} a^2$$

$\longrightarrow \boxed{\beta_p = \frac{\eta_{//}}{\eta_\perp}}$ $\qquad \beta_p = \frac{\eta_{//}}{\eta_\perp} = \frac{1}{2}$

paramagnetic effect, “deflected”
by anisotropic resistivity

Pfirsch-Schluter Current : heuristic approach

Resistive diffusion
in a toroidal plasma



plasma pressure

toroidal hoop force

Internal magnetic force

$$F \sim -\frac{r}{R}\frac{dp}{dr}$$

Charge accumulation

Vertical current

$$j_{hv} \sim -\frac{1}{B}\frac{r}{R}\frac{dp}{dr}$$

Pfirsch-Schluter current: return current along the field line

$$j_{PS} \sim -\frac{1}{B_p}\frac{r}{R}\frac{dp}{dr}$$

$$j_{//v} \sim -\frac{B_p}{B} j_{PS} \sim \frac{1}{B}\frac{r}{R}\frac{dp}{dr}$$

With circular cross section
in the large aspect ratio limit

$$j_{PS} \sim -2\frac{1}{B_\theta}\frac{r}{R}\frac{dp}{dr}\cos\theta$$

Pfirsch-Schluter Current : formal calculation

Poloidal current density

$$j_p = \frac{B_p}{B} j_{//} - \frac{B_\phi}{B} j_\perp$$

$$\vec{j}_\perp = -(\vec{j}\times\vec{B})\times\vec{B}/B^2$$
$$= -\nabla p\times\vec{B}/B^2$$

$$\left.\begin{aligned}\vec{j}_p &= \nabla f\times\nabla\phi \\ \vec{B}_p &= \nabla\psi\times\nabla\phi\end{aligned}\right\} \quad j_p = \frac{df}{d\psi}B_p$$

$$j_\perp = \frac{1}{B}|\nabla p| = -\frac{1}{B}\frac{dp}{d\psi}|\nabla\psi| = -\frac{RB_p}{B}\frac{dp}{d\psi}$$

$$\boxed{j_{//} = f'B - \frac{\mu_o fp'}{B}}$$

$$\mu_o f(\psi) = RB_\phi$$

In steady state,

$$\oint E_{ps}\,ds = 0 \longleftarrow \nabla\times\vec{E} = -\partial\vec{B}/\partial t = 0$$

Ohm's law

$$\eta_{//} j_{//} = \frac{B_p}{B}E_{ps} + \frac{B_\phi}{B}E_\phi$$

$$f' = \mu_o fp' \frac{<1/B_p>}{<B^2/B_p>} + \frac{<E_\phi B_\phi/B_p>}{\eta_{//}<B^2/B_p>}$$

$$j_{//} = -\mu_o fp'\left(\frac{1}{B} - \frac{<1/B_p>}{<B^2/B_p>}B\right) + \frac{<E_\phi B_\phi/B_p>}{\eta_{//}<B^2/B_p>}B$$

Pfirsch-Schluter Current

Pfirsch-Schluter Current :

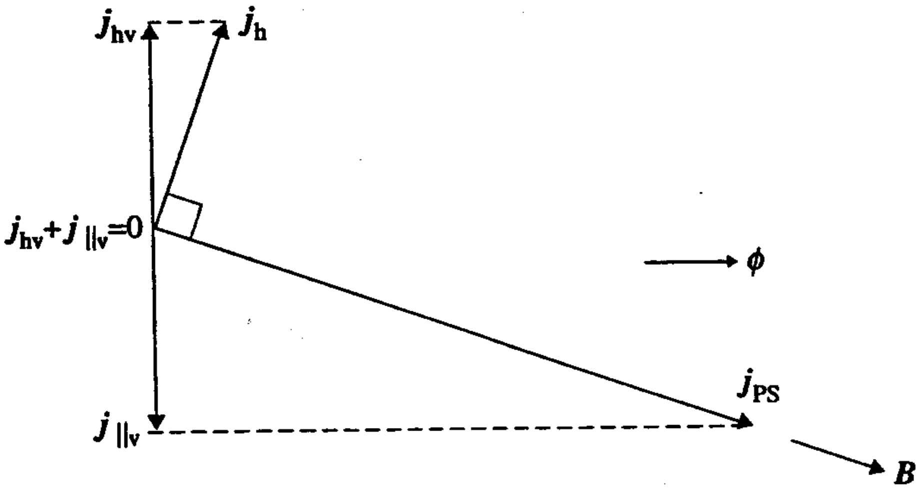
for circular cross section/large aspect-ratio case

Pfirsch-Schluter Current

$$j_{ps} = -\mu_o f p' \left(\frac{1}{B} - \frac{<1/B_p>}{<B^2/B_p>} B \right)$$

For circular cross section and large aspect-ratio case, taking $B_p / B_\phi \sim \varepsilon = r / R_o$

$$B_\phi = \frac{B_o}{1+\varepsilon\cos\theta} \qquad B_p = B_\theta(1+\varepsilon\Lambda(r)\cos\theta)$$



$$\left. \begin{array}{l} j_{ps} = -\mu_o f p' \dfrac{2\varepsilon}{B} \cos\theta + O(\varepsilon^2) \\ \mu_o f(\psi) = RB_\phi \qquad p' = \dfrac{1}{RB_\theta}\dfrac{dp}{dr} \end{array} \right\}$$

$$\longrightarrow \boxed{j_{PS} \sim -2\frac{1}{B_\theta}\frac{r}{R}\frac{dp}{dr}\cos\theta}$$

Pfirsch-Schluter Diffusion

Diffusion in a torus for a low temperature, collisional plasma

$$v_\perp = \frac{E_{ps}B_\phi - E_\phi B_p}{B^2} - \eta_\perp \frac{\nabla_\perp p}{B^2}$$

With parallel component of Ohm's law

$$\eta_{//} j_{//} = \frac{B_p}{B} E_{ps} + \frac{B_\phi}{B} E_\phi$$

$$v_\perp = \frac{B_\phi(\eta_{//} j_{//} B - E_\phi B_\phi)/B_p - E_\phi B_p}{B^2} - \eta_\perp \frac{\nabla_\perp p}{B^2}$$

$$= \frac{B_\phi}{BB_p}\eta_{//} j_{//} - \eta_\perp \frac{\nabla_\perp p}{B^2} - \frac{E_\phi}{B_p} = \boxed{\frac{B_\phi}{BB_p}\eta_{//} j_{ps}} \underline{- \eta_\perp \frac{\nabla_\perp p}{B^2} + \frac{1}{B_p}\left(\frac{< E_\phi B_\phi / B_p >}{< B^2 / B_p >} B_\phi - E_\phi\right)}$$

Pfirsch-Schluter Diffusion

Diffusion in a cylinder

Total plasma flux across a magnetic surface

$$\Gamma = 2\pi n \oint v_\perp R ds = 2\pi n < v_\perp R > \oint ds$$

$$< v_\perp R >_{ps} = -\eta_{//} \mu_o f p' \left\langle \frac{RB_\phi}{B_p}\left(\frac{1}{B^2} - \frac{< 1/B_p >}{< B^2/B_p >}\right)\right\rangle$$

$$\frac{< v_\perp R >_{ps}}{R_o} = -2\left(\frac{r}{R}\right)^2 \eta_{//} \frac{dp/dr}{B_\theta^2} \qquad \frac{< v_\perp R >}{R_o} = -\frac{dp/dr}{B^2}(\eta_\perp + \boxed{2q^2 \eta_{//}}) - \frac{E_\phi B_\theta}{B^2}$$

Banana Regime Transport : heuristic approach

In the absence of collisions, those particles with $v_{//} \le \varepsilon^{1/2} v_{\perp}$ **are trapped and these trapped particles dominate the transport. --> banana regime**

Collisions cause scattering out of the trapped region of velocity space with collisional diffusion in velocity space through an angle $\Delta\theta \sim \varepsilon^{1/2}$

Effective collision frequency for detrapping $\nu_{eff} = \nu / (\Delta\theta)^2 \sim \nu / \varepsilon$

Requirement for banana regime: effective collision frequency<bounce frequency

$$\nu < \frac{\varepsilon^{3/2} v_T}{qR} \quad \longleftarrow \quad \nu_{eff} \sim \nu / \varepsilon < \omega_b \sim \frac{\varepsilon^{1/2} v_T}{qR}$$

From the random walk model, banana width as a step length with the effective collision frequency for trapped particles alone gives diffusion coefficient *D*

$$w_b = \frac{q}{\varepsilon^{1/2}} \rho_e \quad \longrightarrow \quad D = \varepsilon^{1/2} w_b^2 \nu_{eff} \sim \varepsilon^{1/2} (\frac{q}{\varepsilon^{1/2}} \rho_e)^2 \frac{\nu_e}{\varepsilon} \sim \boxed{\frac{q^2}{\varepsilon^{3/2}}} \nu_e \rho_e{}^2$$

Electron and ion thermal diffusivities

$$\chi_e \sim D \sim \frac{q^2}{\varepsilon^{3/2}} \nu_e \rho_e{}^2 \qquad \chi_i \sim \frac{q^2}{\varepsilon^{3/2}} \nu_i \rho_i{}^2 \qquad \chi_i \sim \left(\frac{m_i}{m_e} \right)^{1/2} \chi_e$$

Banana Regime Transport : kinetic approach

Fundamental kinetic equation in a steady state

$$\vec{v}_d = \frac{v_{//}^2 + v_\perp^2/2}{\omega_c}\frac{\vec{B}\times\nabla\vec{B}}{B^2}$$

$$\vec{v}\cdot\nabla f + \frac{Ze(\vec{E}+\vec{v}\times\vec{B})}{m}\cdot\frac{\partial f}{\partial\vec{v}} = C(f)$$

Drift kinetic equation $$\vec{v}_{//}\cdot\nabla f + \vec{v}_d\cdot\nabla f + \frac{ZeE_{//}}{m}\frac{\partial f}{\partial v_{//}} = C(f)$$

Fokker-Planck collision operator

Particle flux across a magnetic surface $$\Gamma = \iint f\vec{v}_d\cdot d\vec{S}d^3v$$

To solve DKE, two expansions are used

- small Larmor radius and inductive electric field $O(\rho/a)$ $O(ZeER/T)$
- small ratio of collision frequency to trapped particle bounce frequency $O(\nu_{eff}/\omega_b)$

$$f = f^{(0)} + f^{(1)} = f^{(0)} + f^{(1)^0} + f^{(1)^1}$$

$$W = Ze\phi + mv^2/2$$

$$\vec{v}_{//}\cdot\nabla f^{(0)} = C(f^{(0)}) \longrightarrow f^{(0)} = f_M(W) = N(m/2\pi T)^{3/2}\exp(-W/T)$$

$$\vec{v}_{//}\cdot\nabla f^{(1)} + \vec{v}_d\cdot\nabla f_M + \frac{ZeE_{//}v_{//}}{m}\frac{\partial f_M}{\partial W} = C(f^{(1)}) \qquad (*)$$

Banana Regime Transport : flux

To obtain an equation for the flux in terms of $f^{(1)0}$,

$$\Gamma = -\iint f\vec{v}_d\cdot\nabla\psi\frac{dS}{|\nabla\psi|}d^3v$$

$$\vec{v}_d\cdot\nabla\psi=\frac{v_{//}^2+\mu B/m}{\omega_c}\nabla\vec{B}\cdot\frac{\nabla\psi\times\vec{B}}{B^2}$$

From energy conservation, $0=\nabla(v_{//}^2+\mu B/m)=2v_{//}\nabla v_{//}+\mu\nabla B/m$

$$(v_{//}^2+\mu B/m)\nabla B=v_{//}^2\nabla B-2v_{//}\nabla v_{//}=-v_{//}B^2\nabla(v_{//}/B)$$

$$\vec{v}_d\cdot\nabla\psi=\frac{v_{//}}{\omega_c}(\vec{B}\times\nabla\psi)\cdot\nabla(v_{//}/B)$$

$$\vec{B}=\nabla\psi\times\nabla\phi+RB_\phi\nabla\phi$$

$$=\frac{v_{//}}{\omega_c}\left(\cancel{\frac{(\nabla\psi)^2}{R^2}}\nabla\phi-RB_\phi\nabla\psi\times\nabla\phi\right)\cdot\nabla(v_{//}/B)=-\frac{v_{//}}{\omega_c}I(\psi)\vec{B}\cdot\nabla(v_{//}/B)$$

$mv_{//}/B$ moment of eq(*)

$$\iint\frac{mv_{//}}{B}\vec{v}_{//}\cdot\nabla f^{(1)}\frac{dS}{|\nabla\psi|}d^3v=\iint\frac{mv_{//}}{B}\vec{v}_{//}\cdot\nabla f^{(1)}\frac{dS}{|\nabla\psi|}2\pi\sum_{\sigma=\pm1}Bd\mu dW/|v_{//}|$$

flux

$$\Gamma\boxed{=-\int\Bigg(\frac{m}{ZeB}\int v_{//}C(f^{(1)})}d^3v+n\frac{E_{//}}{B}\Bigg)I\frac{dS}{|\nabla\psi|}$$

collisional diffusion

Banana Regime Transport : F-P solution

$$\vec{v}_{//}\cdot\nabla f^{(1)}+\vec{v}_d\cdot\nabla f_M+\frac{ZeE_{//}v_{//}}{m}\frac{\partial f_M}{\partial W}=C(f^{(1)})$$

low collisionality expansion $f^{(1)}=f^{(1)^0}+f^{(1)^1}$

$$\vec{v}_d\cdot\nabla\psi=-\frac{v_{//}}{\omega_c}I(\psi)\vec{B}\cdot\nabla(v_{//}/B)$$

$$\frac{v_{//}}{B}\vec{B}\cdot\nabla f^{(1)^0}-\frac{m}{Ze}\frac{v_{//}}{B}I\vec{B}\cdot\nabla(\frac{v_{//}}{B})\frac{\partial f_M}{\partial\psi}=0$$

integrating

$$f^{(1)0}=\boxed{\frac{m}{Ze}\frac{v_{//}}{B}I\frac{\partial f_M}{\partial\psi}}+g(\psi,W,\mu,\sigma)$$

diamagnetic drift of Maxwellian distribution

Equation for $f^{(1)1}$ to determine g

$$\frac{v_{//}}{B}\vec{B}\cdot\nabla f^{(1)^1}+ZeE_{//}v_{//}\frac{\partial f_M}{\partial W}=C(f^{(1)^0})$$

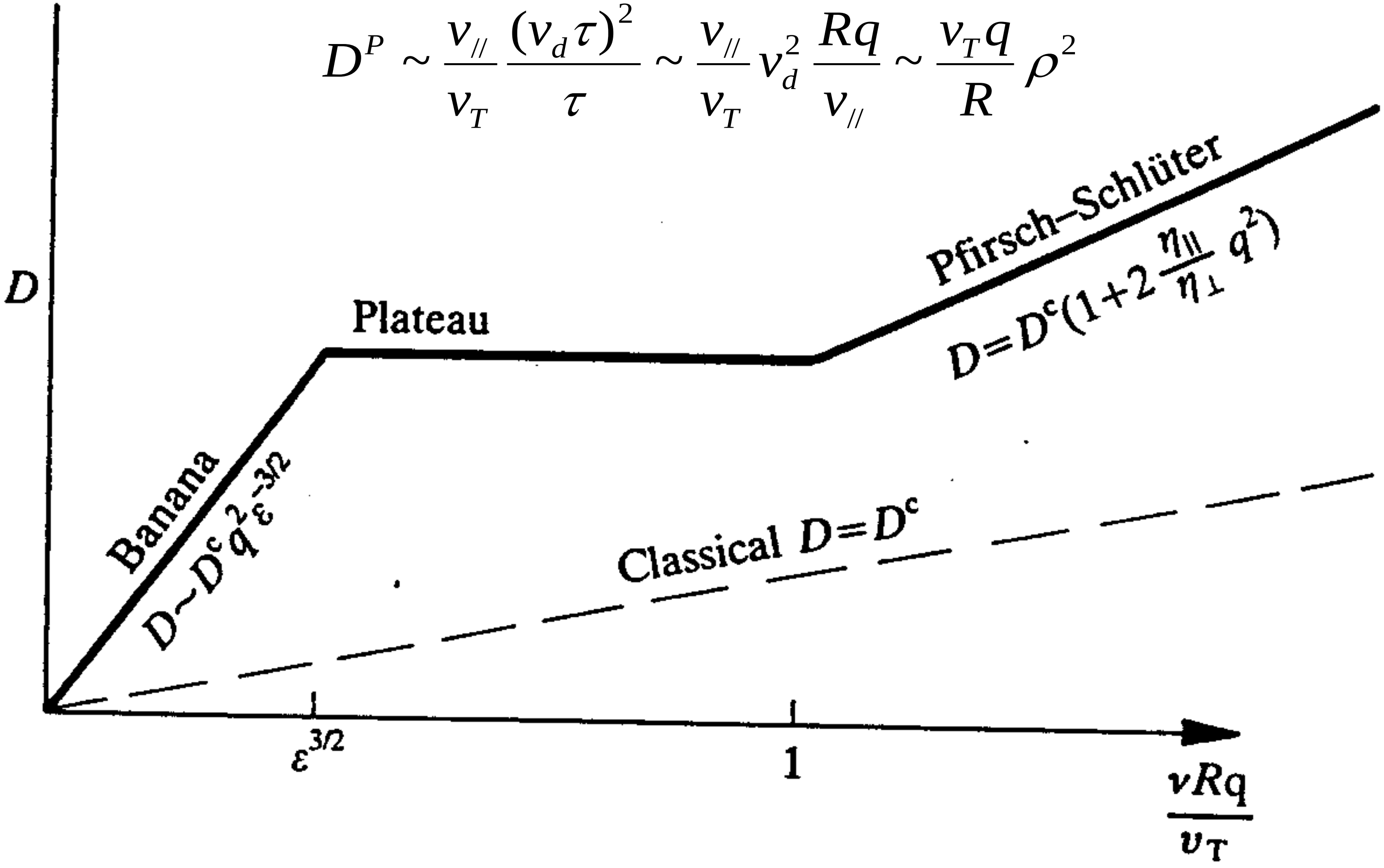
- integration for passing particles $$\oint\left\{C(\frac{mv_{//}}{ZeB}I\frac{\partial f_M}{\partial\psi}+g)-ZeE_{//}v_{//}\frac{\partial f_M}{\partial W}\right\}\frac{ds}{v_{//}}=0$$
- integration for trapped particles $$\int_{s(\theta_1)}^{s(\theta_2)}C(g)\frac{ds}{v_{//}}=0$$

Provide collisional constraint necessary to determine *g* and hence $f^{(1)0}$

Plateau Transport

Due to magnetic drift, particle drifts a radial distance $\delta \sim v_d \tau$
in a transit time $\tau \sim Rq / v_{//}$

$$D^P \sim \frac{v_{//}}{v_T} \frac{(v_d \tau)^2}{\tau} \sim \frac{v_{//}}{v_T} v_d^2 \frac{Rq}{v_{//}} \sim \frac{v_T q}{R} \rho^2$$



Ware Pinch Effect

Toroidal equation of motion in the banana regime, $\frac{d}{dt} m_j v_\phi = e_j (E_\phi + (\vec{v} \times \vec{B})_\phi)$

Bounce averaging gives $< (\vec{v} \times \vec{B})_\phi > = -E_\phi$ $\quad (\vec{v} \times \vec{B})_\phi = v_\perp B_\theta$

Time-averaged pinch velocity of the trapped particles $< v_\perp > = -\frac{E_\phi}{B_\theta}$

flux $\Gamma \sim \varepsilon^{1/2} n \frac{E_\phi}{B_\theta}$

$$\frac{d^2 s}{dt^2} = -\omega_b^2 s + \frac{e_j E_\phi}{m_j}$$

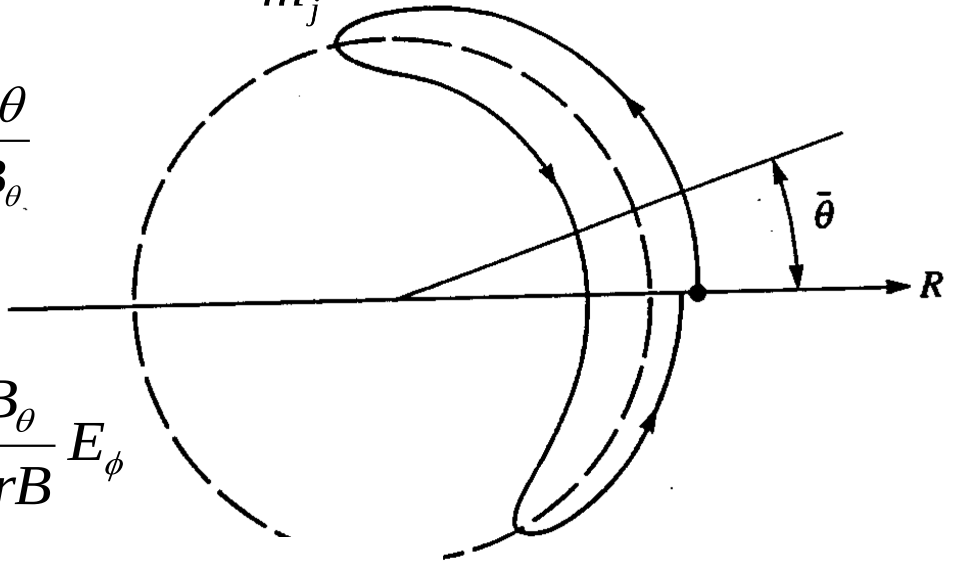
Modified trapped particle orbit

$$s = s_b \sin \omega_b t + \frac{e_j E_\phi}{m_j \omega_b^2} \qquad \frac{s}{B} = \frac{r\theta}{B_\theta}$$

$$\theta = \theta_b \sin \omega_b t + \frac{e_j B_\theta E_\phi}{m_j \omega_b^2 r B}$$

$$v_r = -v_{dj}\theta = -v_{dj}\theta_b \sin \omega_b t - \frac{e_j v_{dj} B_\theta}{m_j \omega_b^2 r B} E_\phi$$

$$\langle v_r \rangle = -\frac{e_j v_{dj} B_\theta}{m_j \omega_b^2 r B} E_\phi = -\frac{E_\phi}{B_\theta}$$



Time averaged velocity

Bootstrap Current

Onsager Symmetry

Ware Pinch $E_\phi \rightarrow$ $$<v_\perp> = -\frac{E_\phi}{B_\theta}$$

Bootstrap current $<v_\perp> \rightarrow E_\phi$

Trapped particle currents

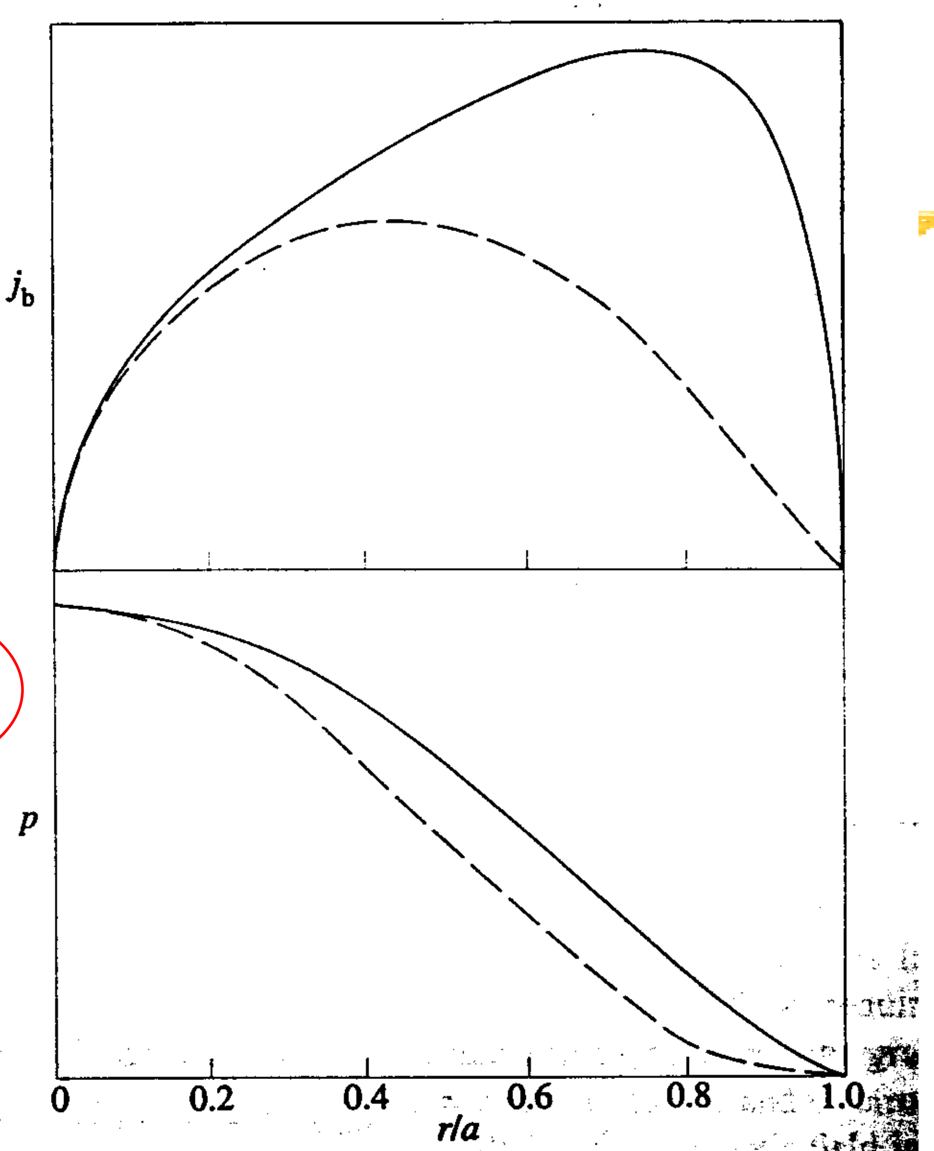
$$j_T \sim -e\varepsilon^{1/2}(\varepsilon^{1/2}v_T)w_b\frac{an}{dr} \sim -q\frac{\varepsilon^{1/2}}{B}T\frac{dn}{dr}$$

Momentum exchange between passing electrons and ions

$$\frac{m_e}{e}\nu_{ei}j_b \sim \boxed{\frac{\nu_{ee}}{\varepsilon}j_T\frac{m_e}{e}}$$

Momentum exchange between passing and trapped electrons

$$j_b \sim -\frac{\nu_{ee}}{\nu_{ei}}\frac{q}{\varepsilon^{1/2}}\frac{T}{B}\frac{dn}{dr} \sim -\frac{\varepsilon^{1/2}}{B_\theta}T\frac{dn}{dr}$$



$$j_b = -\frac{n\varepsilon^{1/2}}{B_\theta}[2.44(T_e+T_i)\frac{1}{n}\frac{dn}{dr}+0.69\frac{dT_e}{dr}-0.42\frac{dT_i}{dr}]$$

For ε --> 1, $j_b = -\frac{1}{B_\theta}\frac{dp}{dr}$

An interesting new effect is predicted for plasma in banana regime. Radial neoclassical diffusion can induce a toroidal current, called ***Bootstrap current*** which is very important for future steady-state tokamaks.

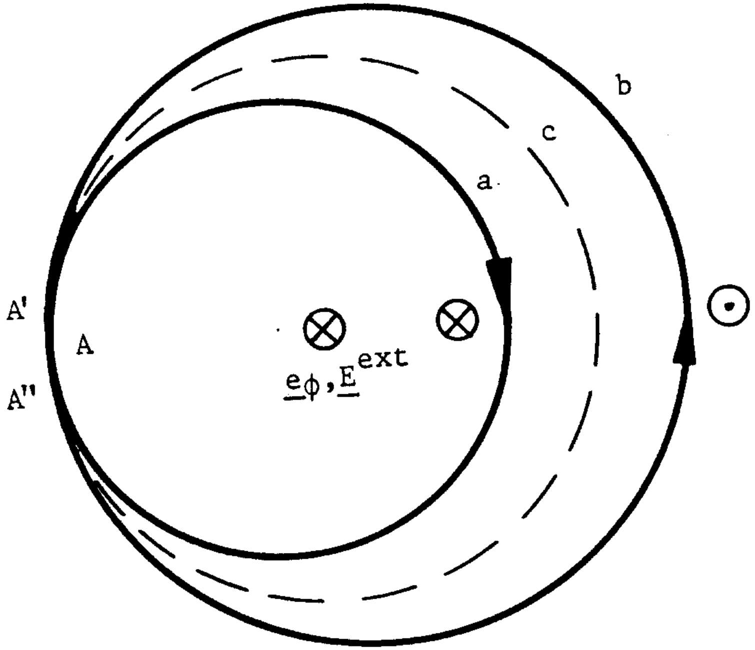


Fig. 3 Typical poloidal trajectories of a barely trapped electron (curve a+b) and of barely free electrons with parallel velocities in the direction of $\underline{E}$ (curve a) and in the direction opposite to $\underline{E}$ (curve b). The curve c is the trace of the magnetic surface. The axis of symmetry is at the left.

Total bootstrap currents

$$\frac{I_b}{I} = c\varepsilon^{1/2}\beta_p$$

Neoclassical Resistivity

In a cylindrical plasma, the resistivity along the field line is the Spitzer resistivity

$$\eta_{Sp} = \sigma_{Sp}^{-1} = \frac{1}{1.96 ne^2 \tau_e / m_e}$$

In a tokamak, the trapped electrons are unable to move freely along the magnetic field in response to an applied electric field. In the banana regime, the conductivity becomes

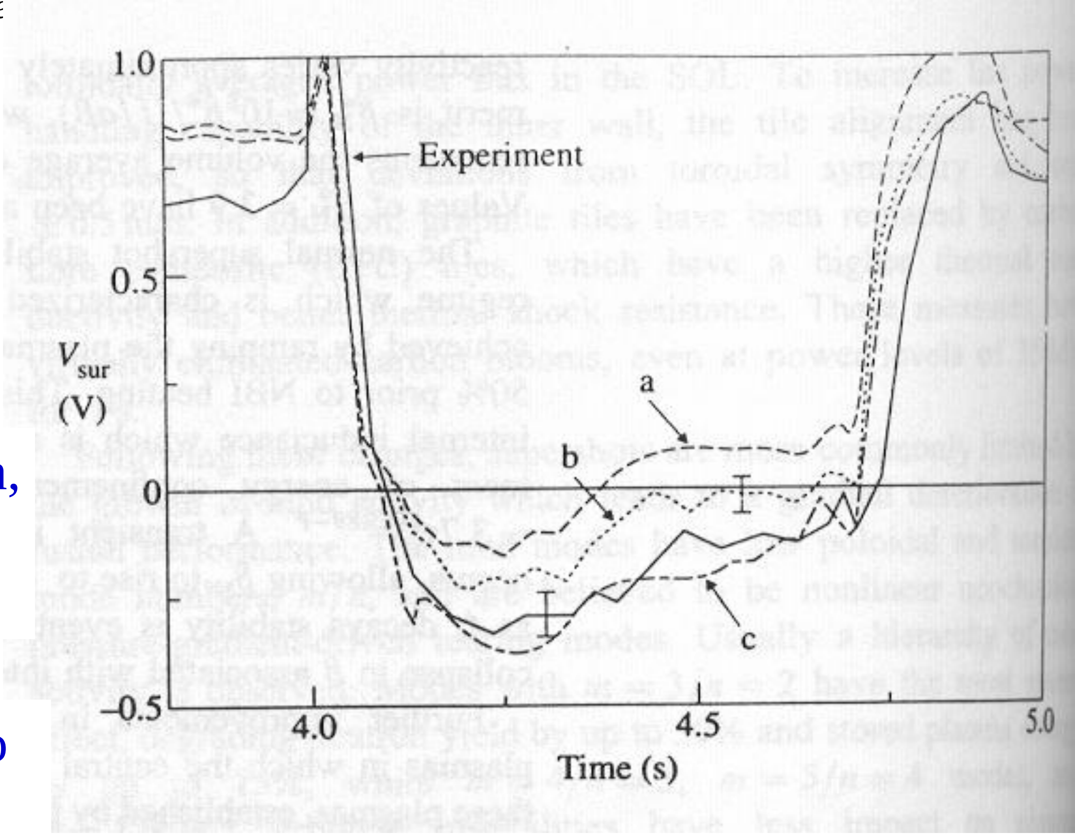
$$\sigma = \sigma_{Sp} f(\varepsilon)$$

In a large aspect-ratio approximation, the current density parallel to the magnetic field

$$j_{//} = \sigma_{Sp}(1 - 1.95\varepsilon^{1/2})E_{//} + j_b$$

A more accurate form of aspect-ratio dependence from the extended calculation is

$$\sigma = \sigma_{Sp}(1 - \varepsilon^{1/2})^2$$



Experimental evidence of bootstrap current on TFTR.

Ripple and Ripple Well Region

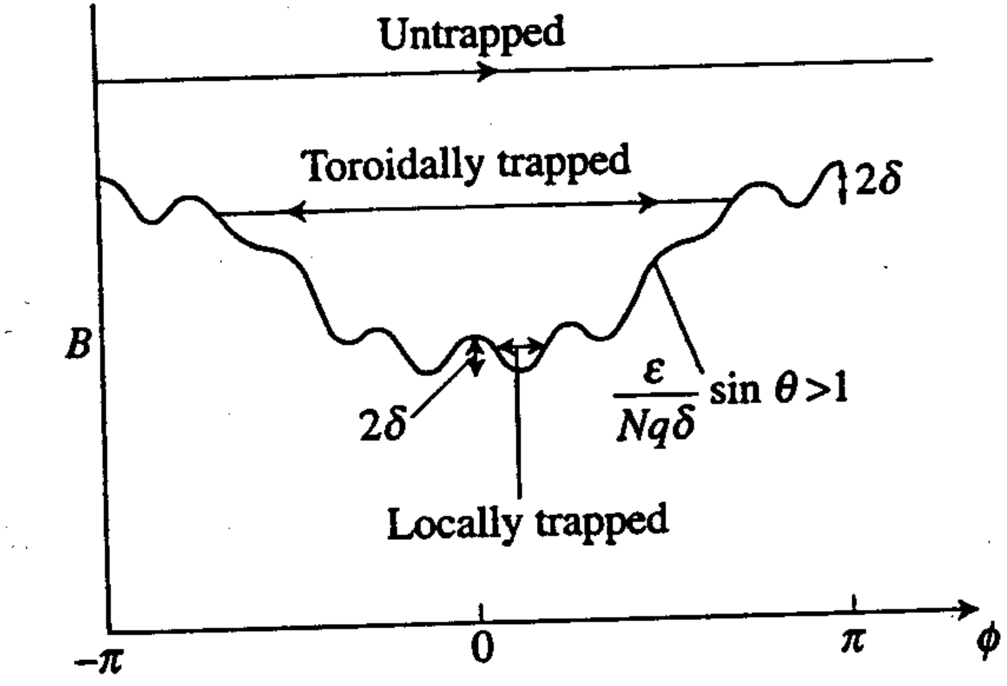
The finite number of toroidal field coils produce a short wavelength ‘ripple’ in the magnetic field strength as a field line is followed around the torus

$$B = B_o(1-\varepsilon\cos\theta)(1-\delta(r,\theta)\cos N\phi)$$

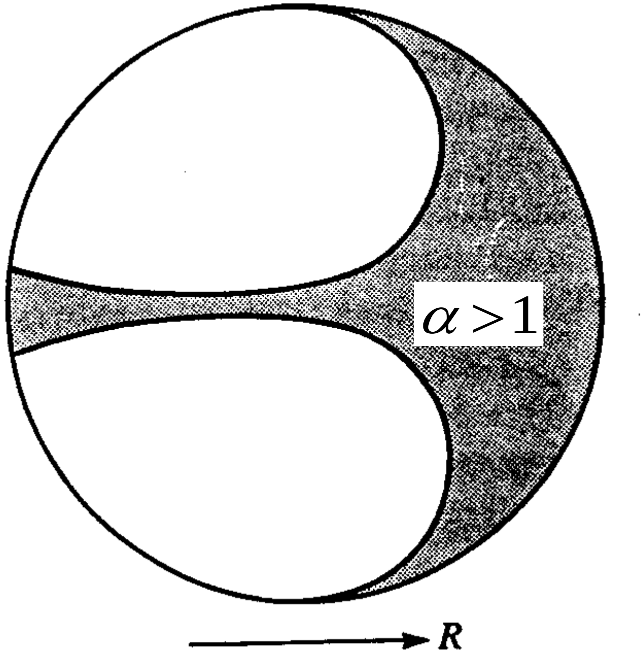
$$\delta = \left(\frac{R}{R_{outer}}\right)^N + \left(\frac{R_{inner}}{R}\right)^N$$

$$\phi = \phi_o + q\theta$$

The well vanish for angles θ such that $\alpha|\sin\theta| > 1$ $\alpha = \varepsilon/(Nq\delta)$



ripple well region



Ripple Transport

- **Ripple well transport :** in the ripple well region, banana trapped particles with tip positions at the edge of the ripple well region becoming trapped in the local toroidal wells and subsequently being lost via grad-B drifts vertically
- **Ripple banana transport :**ripple modifies the orbits of the banana trapped particles, leading to transport

- **for thermal particles :** collisional ripple well transport and collisional ripple diffusion
- **for fast particles :**
collisionless ripple well transport and collisionless stochastic diffusion

for a reactor

- loss of fast α- particles and the associated heat losses to the first wall
- loss of neutral beam injected fast particles, in particular near perpendicular beamlines
- the ripple amplitude at the plasma edge typically less than 1 to 2 percents in order to avoid excessive ripple well losses, while at the plasma center less than 0.01 percent to avoid stochastic diffusion --> need larger number of TF coils --> limit the accessibility of the tokamak

Ripple Transport

- **Collisional ripple well trapping transport :** particles are trapped into and de-trapped out of ripple well trapping by collisional processes, in particular by pitch-angle scattering

Residence time of ripple well trapped particles

Fraction of ripple well trapped particles

$$D \sim \delta^{1/2}\left(\frac{\rho v \delta}{R\nu}\right)^2 \frac{\nu}{\delta} \sim \frac{\delta^{3/2}\rho^2 v^2}{R^2 \nu}$$

Valid only for $\nu > v_d \delta / a$

Drift velocity : $v_d \sim \dfrac{\rho v}{R}$

- **Collisionless ripple trapping** : for fast particles the ripple well trapping processes represents a loss cone since trapped particles do not suffer a collision before being lost, i.e. $\nu < v_d \delta / a$

Different ripple value on approaching the turning point cause collisionless trapping

$$\Delta w(\partial\delta / \partial z) \sim \frac{v_d R}{N\nu\delta^{1/2}}(\partial\delta / \partial z)$$

The rate of collisionless trapping in terms of trapping probability *p* per bounce

$$p \sim \frac{\rho}{R}(Nq\varepsilon)^{1/2}$$

Ripple Transport

- **Collisional Ripple diffusion :**

Vertical step at one banana turning point

$$\Delta z = \Delta \cos(N\phi \pm \pi / 4)$$

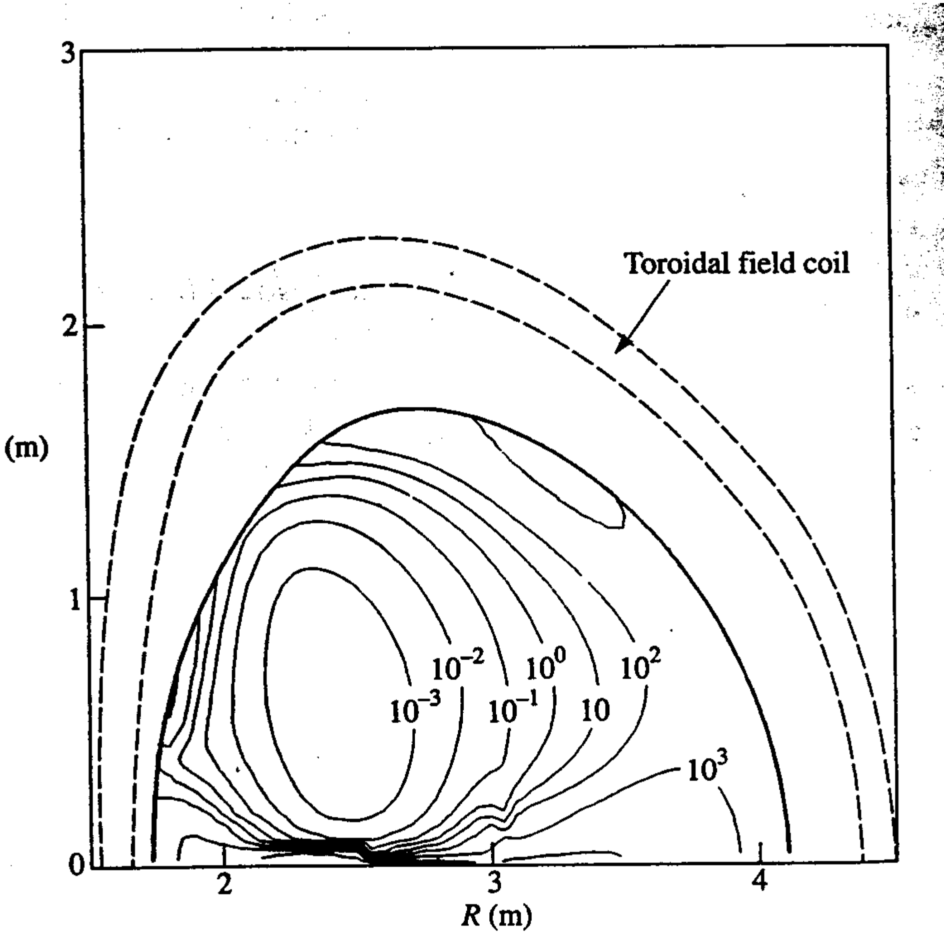
$$\Delta = \rho(\pi / N)^{0.5}(B_\phi / B_R)^{0.5}\alpha^{-1}$$

For small Δ, transports of particles can results from collisional de-correlation of the successive steps when $\nu > \dfrac{\varepsilon}{N^2 q^2}\dfrac{1}{\tau_b}$

When fully de-correlated, $D \sim \varepsilon^{1/2}\dfrac{\Delta^2}{\tau_b}$

→ Banana plateau regime

- **Stochastic diffusion :**



For large Δ, the step itself leads to de-correlation of the orbits when the change in toroidal bounce angle becomes $\Delta\phi = (\partial\phi / \partial z)\Delta \approx 2\pi / N$

Chirikov parameter $\gamma = N\Delta(\partial\phi / \partial z)$

For large gamma, full de-correlation leads to stochastic motion and to fast transport
Monte carlo codes gives the diffusion coefficient of

$$D = \frac{\Delta^2}{\tau_b}\frac{1}{1 + e^{(6.9-5.5\gamma)}}$$

Tokamak Energy Confinement

Tokamaks do not behave as predicted by neoclassical theory. The energy confinement, as measured by the confinement time, is found to be much shorter than the neoclassical value. As a result, an empirical representation of the confinement time has been widely used.

- Global energy confinement time
  - Definition and significance
  - Various operation modes
  - Confinement scaling laws
- Transport and Energy loss mechanisms
  - Fluctuations and turbulence
  - Radiation losses

Global Energy Confinement Time

Definition $\tau_E = E_{total} /(P_{in} - dE_{total} /dt) \approx E_{total} / P_{in}$

- To predict the performance of future devices, the energy confinement time is one of the most important parameter
- Since tokamak transport is anomalous, empirical scaling laws for energy confinement are necessary
- ***Empirical scaling laws*** : regression analysis from available experimental database.

$$\tau_E = n^{\alpha} a^{\omega} R^{\varepsilon} B^{\rho} I_p^{\theta} q_a^{\psi} T^{\beta} P_{in}^{\iota}$$

Tokamak Operation Modes

- Ohmically heated plasmas
  - Ohmic energy confinement scaling
- Auxilliary heated plasma operation modes
  - L-mode
  - H-, VH-, CH-, CDH-modes
  - Super-shot, high-li mode, hot ion mode, PEP
  - ERS, NCS

Ohmic Energy Confinement Scaling

- Linear dependency on density
  - **Alcator (or INTOR) scaling :**

    $\tau_E[\mathrm{sec}]=0.5\, a^2\, (n/10^{20})$
  - **neo-Alcator (or Goldston)**

    $\tau_E[\mathrm{sec}]=0.071\, a^{1.04}\, R^{2.04}\, (n/10^{20})q^{0.5}$
  - **very promising, but conflict with neoclassical theory**
- Saturation at higher density range
  - $n_{sat}=0.06\mathrm{x}\ 10^{20}\, I_p R\, A^{0.5}\, a^{-2.5}\, \kappa^{-1}$
  - **partly due to increased radiative losses.**
  - **Improved confinement w/o density saturation observed at ASDEX (peaked density profile)**

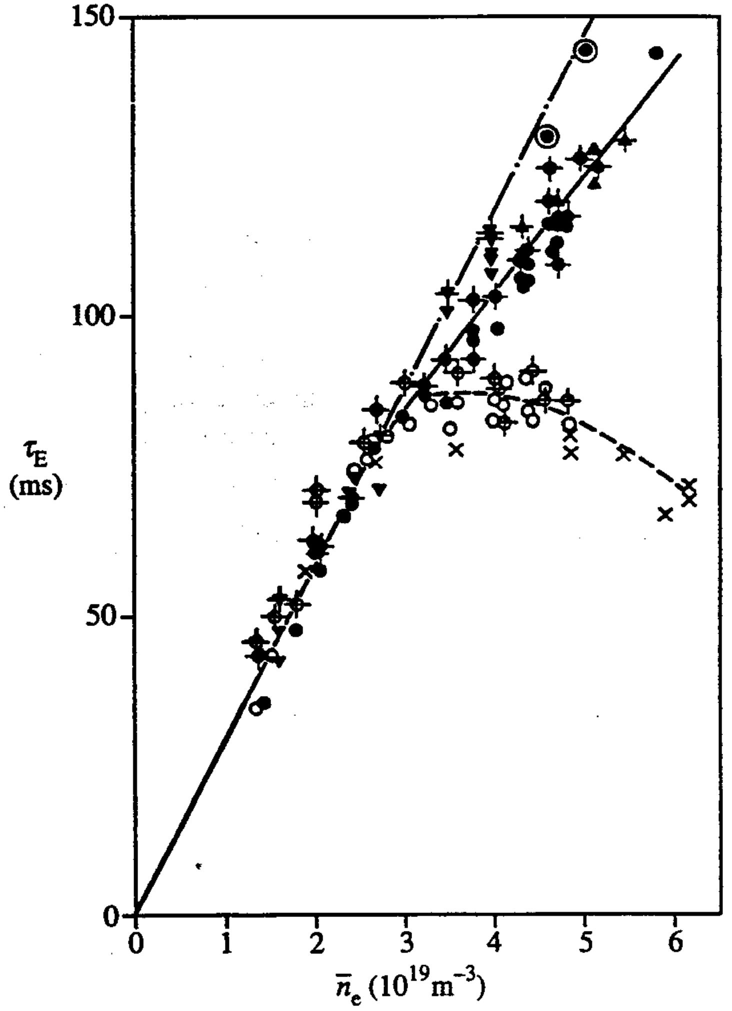
Ohmic Energy Confinement Scaling

- Improved confinement w/o density saturation observed at ASDEX (peaked density profile)
- New scaling law from C-Mod
  - **Similar scaling to L-mode :**

    $\tau_E \propto M^{0.3} I_p P_{tot}^{-0.5}$
  - **neo-Alcator scaling at low density**

    (n< 1.5x 10^{20} , k< 1.35)

$\tau_E = 0.07\ aqR^2 (n/10^{20})k^{0.5}$



Auxilliary-heated Plasmas : L-mode

- During the early phase of tokamak heating experiments, the plasma confinement was found to degrade very rapidly from the ohmic value with application of auxiliary heating. The typical observation is that the τ_E increases with the plasma current(note that it is opposite to the ohmic), decreases with the applied power, increases with R and a little dependence with density and minor radius.
- **Goldston scaling : $\tau_G[\mathrm{sec}]=0.037\ I\ R^{1.75}\ a^{-0.37}\ P^{-0.5}$**
- **ITER89-P scaling:**

$$\tau_E[\mathrm{sec}]=0.048\ I^{0.85}\ R^{1.2}\ a^{0.3}\ \kappa^{0.5}\ (n/10^{20})^{0.1}\ B^{0.2}\ A^{0.5}\ /P^{0.5}$$

- **Confinement scaling valid from ohmic to strongly additionally heated plasmas**
  - **Goldston : $\tau_{E,G}{}^{-2} = \tau_{OH}{}^{-2} + \tau_{AUX,G}{}^{-2}$**
  - **linear offset: $E=E_{OH} + E_{AUX}$ or $\tau_{E,} = E_{OH} / P_{tot} + \tau_{inc}$**
  - **transport model for anomalous electron (Rebut, Lallia and Watkins)**

L-mode Confinement

Using the power balance relation, Goldston confinement time takes the approximate form

$$\begin{cases} \tau_G \propto I_p / P^{0.5} \\ P \sim nTabR / \tau_E \end{cases}$$

$$\longrightarrow \quad \tau_G \sim \frac{I_p^2}{nT} g$$

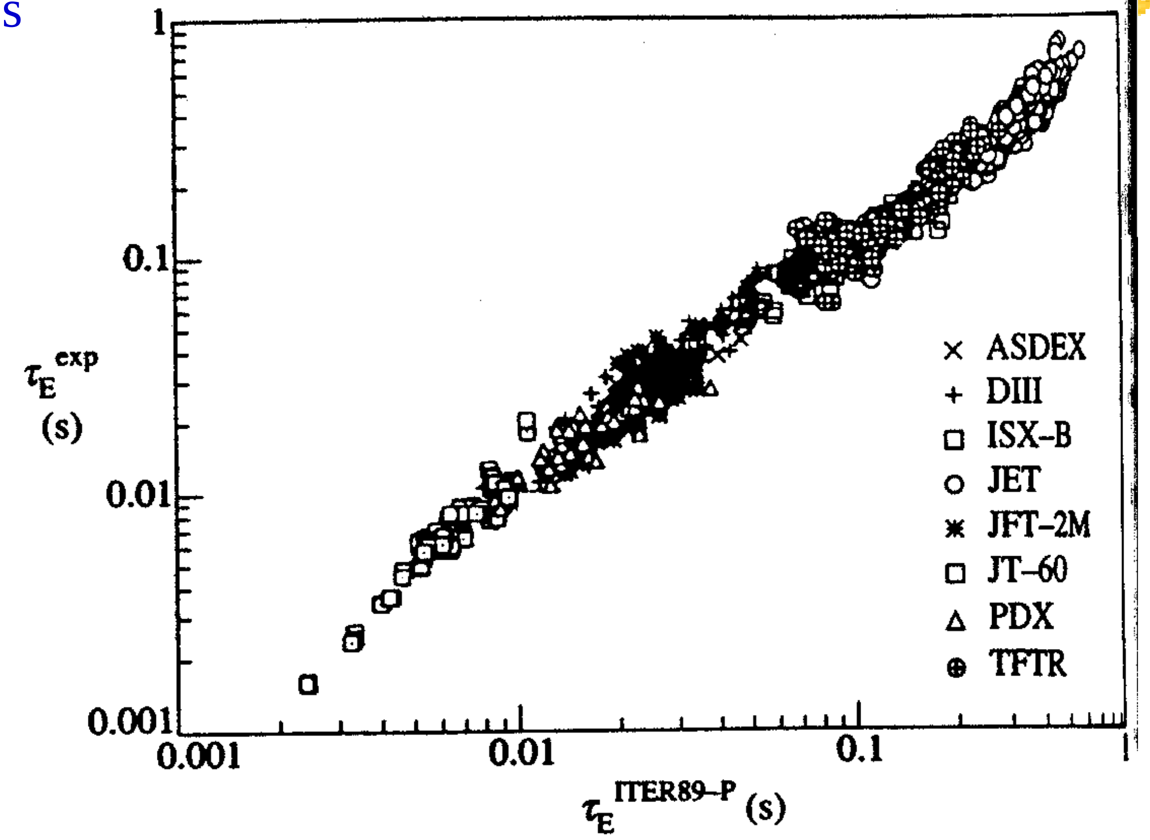


Fig. 4.12.2 Comparison of the experimental values of confinement time from a number of tokamaks with the L-mode scaling $\tau_E^{ITER89-P}$. (Yushmanov, P.N. *et al.* *Nuclear Fusion* **30**, 1999 (1990).)

ITER89-P scaling:

$$\tau_E[\text{sec}]=0.048\ I^{0.85}\, R^{1.2}\, a^{0.3}\, \kappa^{0.5}\, (n/10^{20})^{0.1}\, B^{0.2}\, A^{0.5}\, /P^{0.5}$$

Auxilliary-heated Plasmas : H-mode

- **In 1982 IAEA meeting,a new improved confinement regime in diverted ASDEX plasmas was reported (F. Wagner et al.) which was termed H-mode(high mode). Compared to the L-mode, the energy confinement similar to that of ohmic plasma was recovered. The reported H-mode confinement time typically had the twice that of the L-mode.**

The improvement in the energy confinement comes about mainly due to the increased density while keeping the central temperature relatively unchanged (note that since the L-mode has little density dependence, this is considered to be a breakthrough). The density and electron temperature profile during H-mode are broader and are characterized of having “pedestal” at the plasma edge.

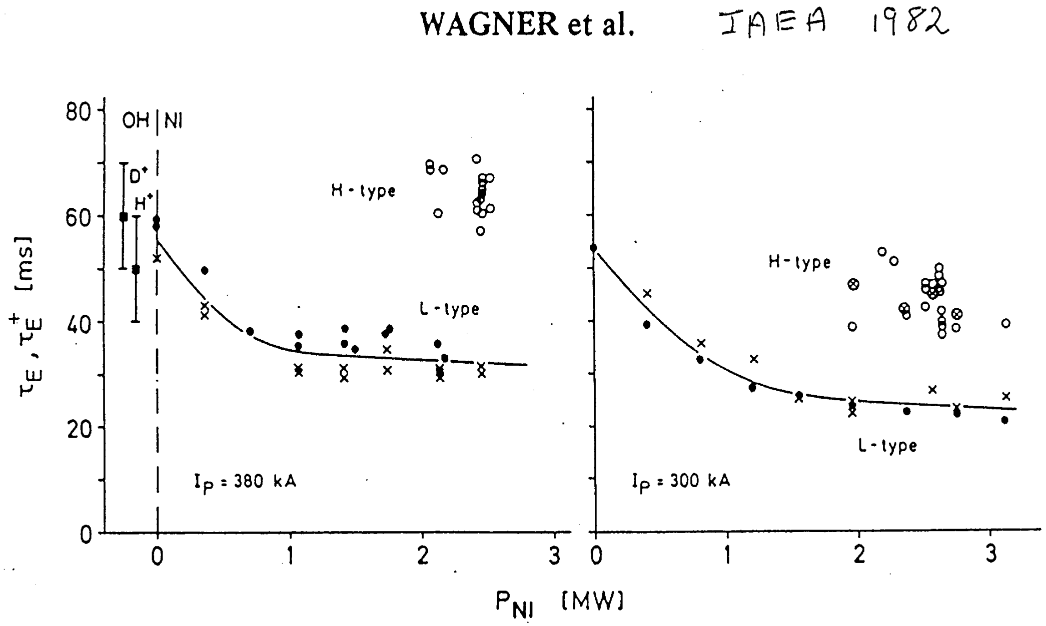
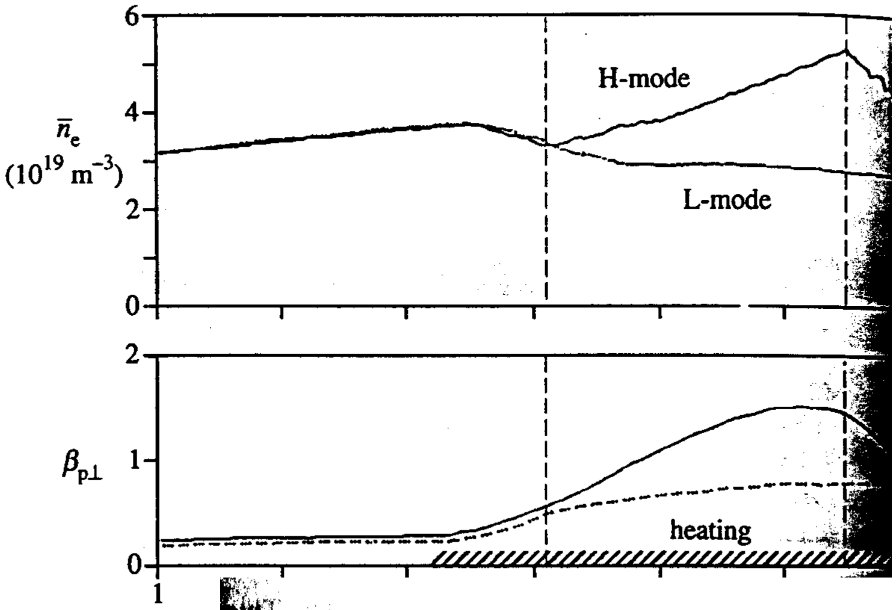


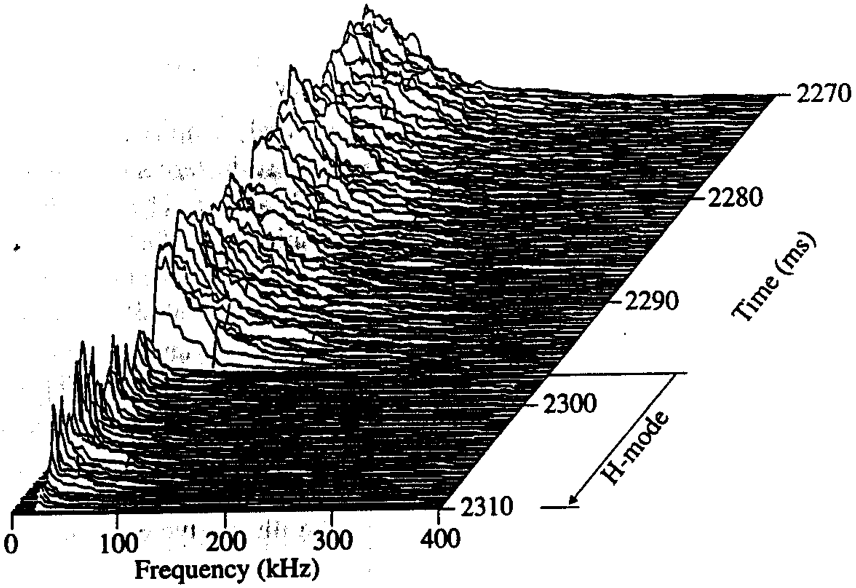
FIG.4. Thermally (τ_E, plotted as crosses)and magnetically (τ_E^+, plotted as circles) measured confinement times for two plasma currents and for L-type and H-type discharges.

H-mode Characteristics

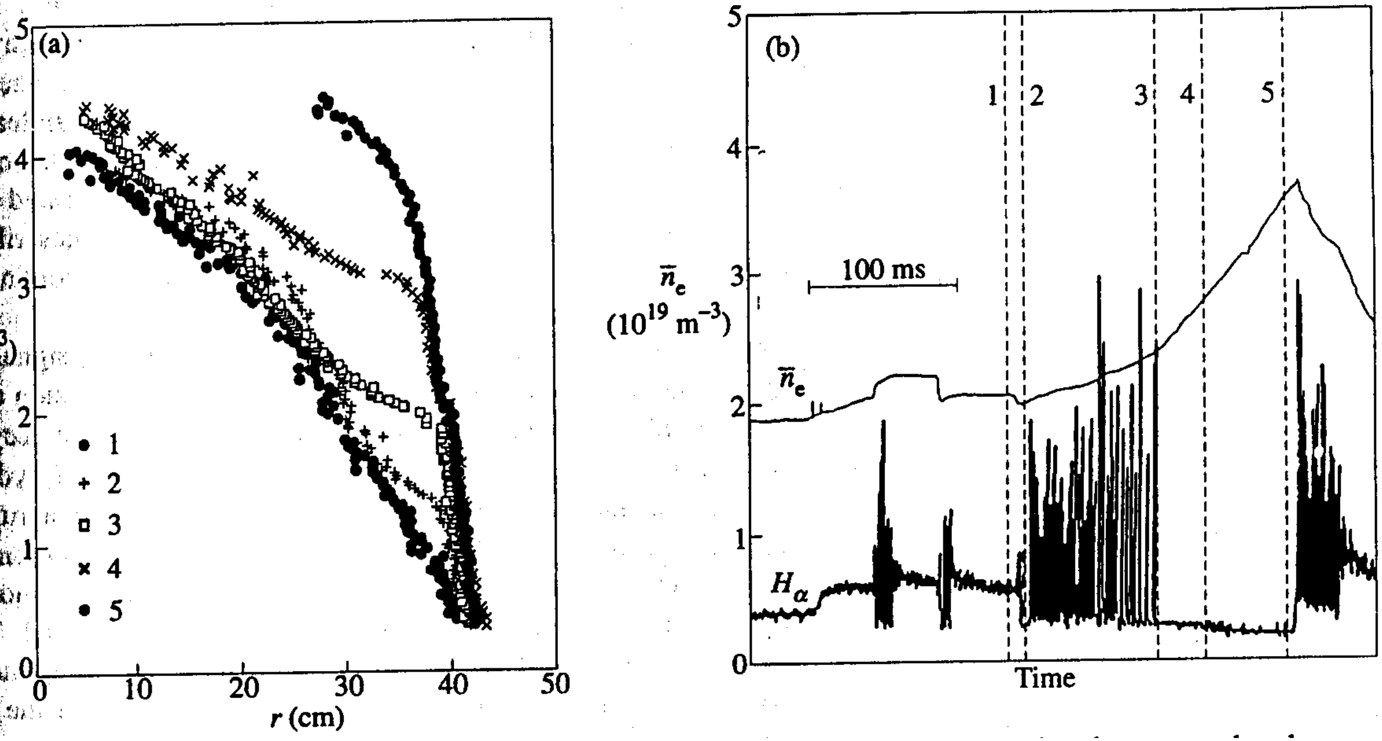
- H-mode has a similar scaling law, but with 1-2 times improvement over L-mode : $\tau_H = H\,\tau_E{}^{\mathrm{ITER\ 89\text{-}P}}$ or $\tau_{Th}{}^{\mathrm{ITER\ H93\text{-}P}}[\mathrm{sec}]=0.053\ I^{1.06}\ R^{1.9}\ a^{-0.11}\ \kappa^{0.66}\ (n/10^{20})^{0.17}\ B^{0.32}\ A^{0.41}\ /P^{0.67}$
- Reduced edge recycling and often with ELM (edge localized mode) activities.
- Broader density and temperature profiles give a larger stored energy and high beta discharges.
- However, not much central temperature and density increase shows less favorable for producing neutrons.
- H-mode transition seems to be strongly related to the formation of radial electric field (or poloidal rotation).
- Power threshold for H-mode increases with the product of density$^{(0.5\text{-}1.0)}$, magnetic field, and surface area$^{(0.5\text{-}1.0)}$.
- Various H-modes : VH (DIIID), CH(PBX-M), CDH(ASDEX-U)



$\bar{n}_e$
(10^{19} m^{-3})
6
4
2
0
H-mode
L-mode
$\beta_{p\perp}$
2
1
0
heating
1



2270
2280
2290
2300
2310
Time (ms)
H-mode
0
100
200
300
400
Frequency (kHz)



(a)
5
4
3
2
1
0
0
10
20
30
40
50
r (cm)
● 1
+ 2
□ 3
× 4
● 5
(b)
5
4
3
2
1
0
$\bar{n}_e$
(10^{19} m^{-3})
100 ms
1
2
3
4
5
$\bar{n}_e$
H_α
Time

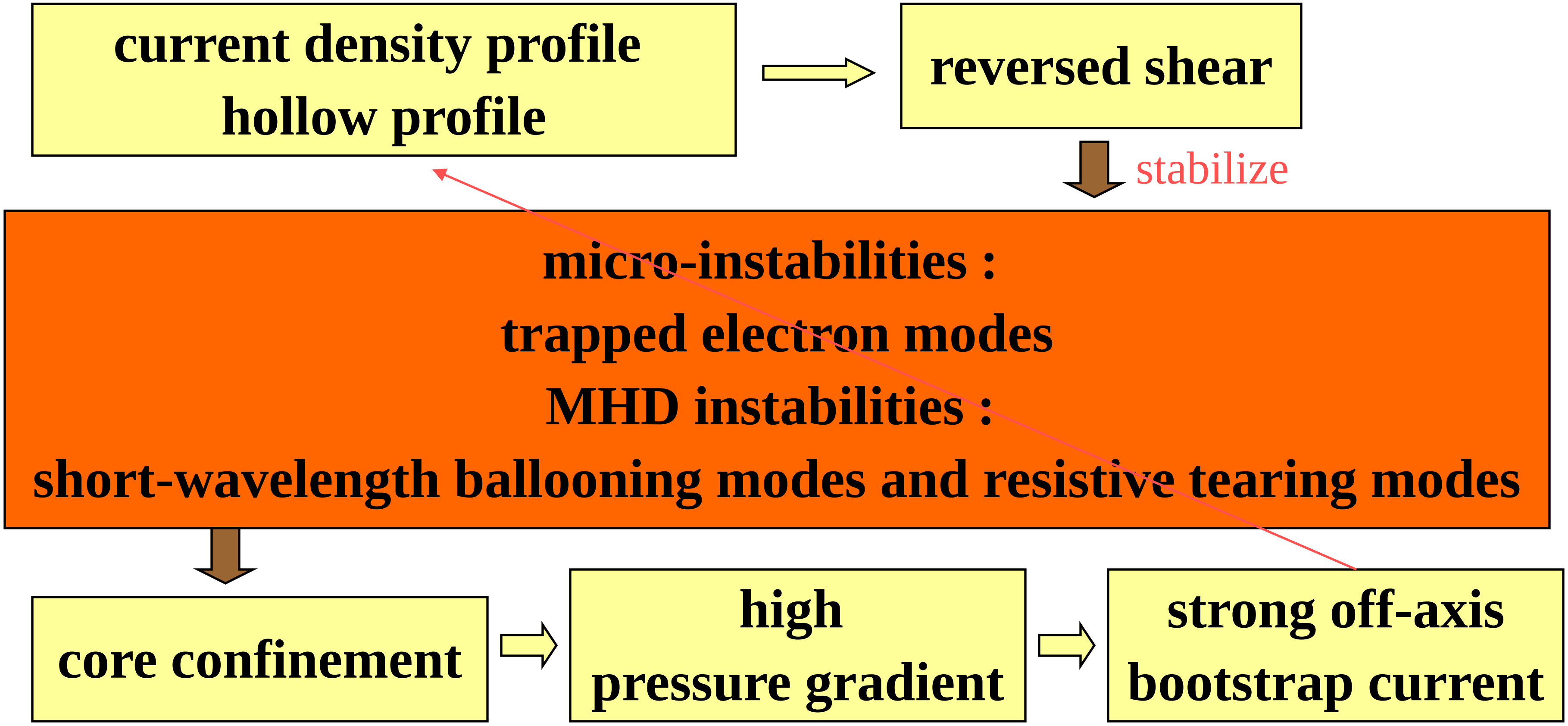
Other Improved Confinement Modes

- **Super-Shot(TFTR):** neutral beam injection into a low density plasma, two oppositely injected balanced beams, low edge recycling, peaked density profile, H=3
- **Hot Ion Mode(JET) :** similar mode to TFTR super shot, high power NBI in low density target plasmas, beryllium-conditioned wall, centrally peaked density and temperature profiles, highest fusion triple product in JET, Q_{DT}~1
- **High-li Mode :** peaked current density profile $\tau_E \propto l_i^{\alpha} \quad 0.67 < \alpha < 0.8$
- **PEP(Pellet Enhanced Performance) or 'High β_p' H-mode :** H=3.8, peaked pressure profiles by the injection of hydrogen pellets(JET, JT60-U)
- **VH Mode :** H=3.6, boronized wall(DIII-D), beryllium surfaces(JET), edge temperature pedestal and a high edge bootstrap current
- **Enhanced Reversed Shear or Negative Central Shear :** TFTR (PRL **75**(1995)4417) and DIII-D (PRL **75**(1995)4421)

Enhanced Reversed Shear(ERS:TFTR)
Negative Central Shear(NCS:DIII-D)

- **Current density profile can be optimized to be desirable for confinement, stability, and bootstrap current.**

--> reversed magnetic shear i.e. hollow current density profile



current density profile
hollow profile

reversed shear

stabilize

micro-instabilities :
trapped electron modes
MHD instabilities :
short-wavelength ballooning modes and resistive tearing modes

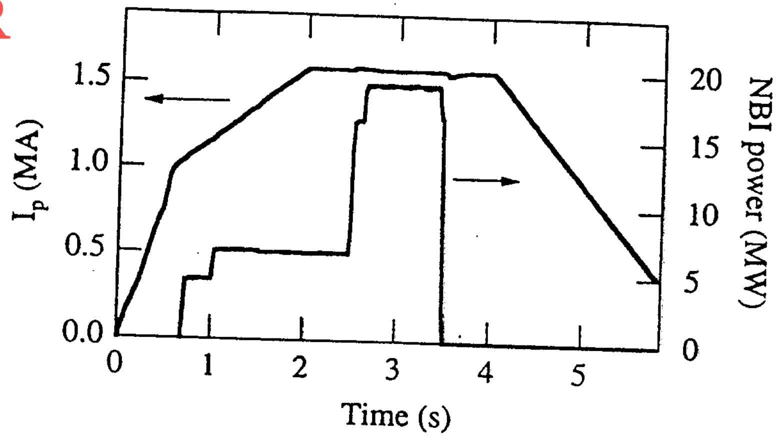
core confinement

high
pressure gradient

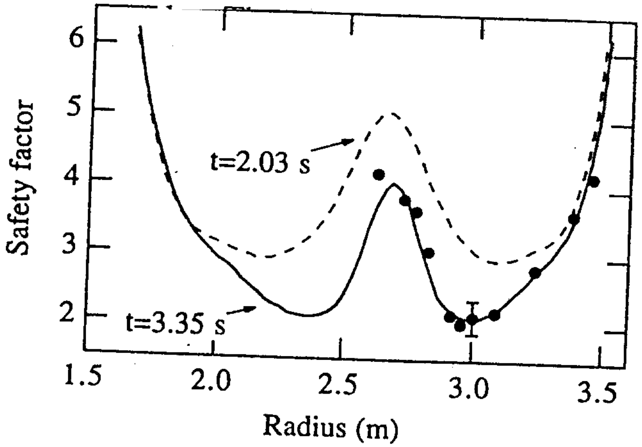
strong off-axis
bootstrap current

TFTR

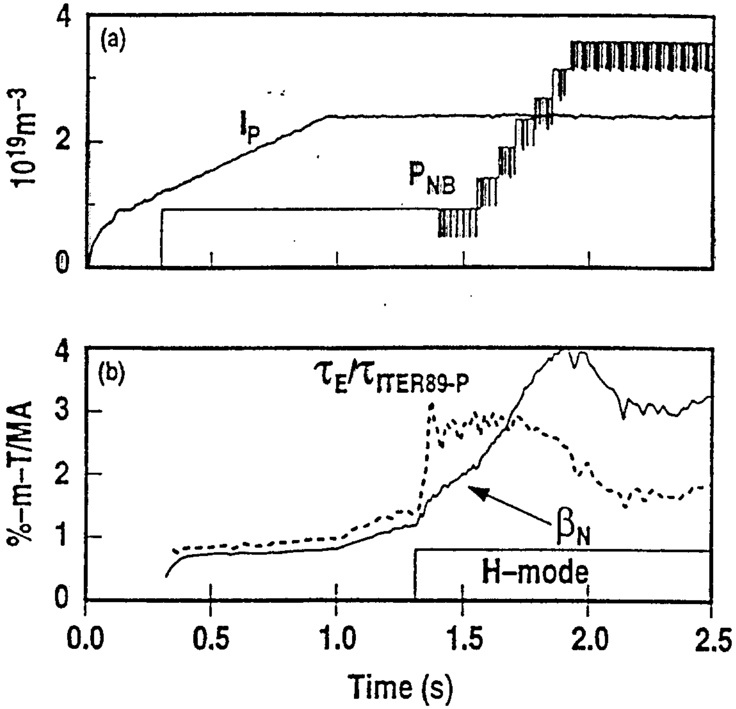
DIII-D



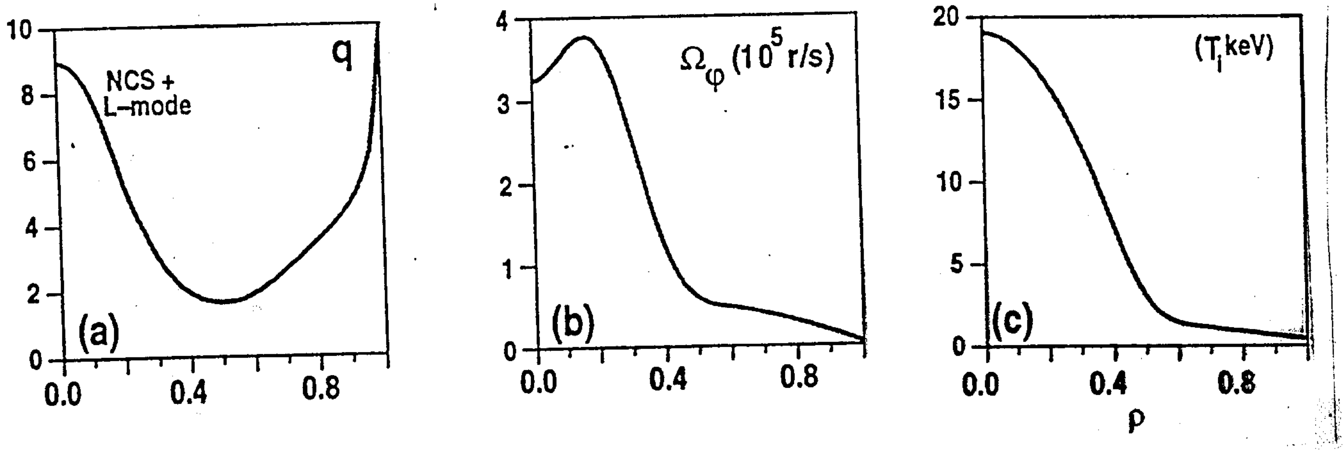
1.5
1.0
0.5
0.0
I_p (MA)
20
15
10
5
0
NBI power (MW)
0
1
2
3
4
5
Time (s)



6
5
4
3
2
Safety factor
t=2.03 s
t=3.35 s
1.5
2.0
2.5
3.0
3.5
Radius (m)



4
2
0
10^{19}m^{-3}
(a)
I_P
P_{NB}
4
3
2
1
0
%-m-T/MA
(b)
$\tau_E/\tau_{ITER89-P}$
β_N
H-mode
0.0
0.5
1.0
1.5
2.0
2.5
Time (s)



10
8
6
4
2
0
q
NCS +
L-mode
(a)
0.0
0.4
0.8
4
3
2
1
0
Ω_φ (10^5 r/s)
(b)
0.0
0.4
0.8
20
15
10
5
0
(T_i keV)
(c)
0.0
0.4
0.8
ρ

Scaling Laws

- Theory
  - **scale invariance technique**

    (Connor-Taylor or Kadomtsev constraints)
  - **exploits the invariance of the governing equations under scale transformations**
  - **no information on geometrical ratios and the safety factor**
- Experimental scaling relations
  - **Bohm diffusion** $\tau_E = \tau_B F(\beta, \nu_*)$
  - **gyro-Bohm scaling** $\tau_E = \frac{\tau_B}{\rho_*} F(\beta, \nu_*)$
  - **ITER89-P scaling law gives Bohm scaling with** $\tau_E \sim \tau_B \beta^{-1/2} \nu_*^{-1/4}$
- Simple physics model
  - **beam fueling profile**

Scaling Laws Based on Theory

Vlasov (or F-P) equation with quasi-neutrality condition (or Poisson's eqn)

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial \vec{x}}+\frac{e_j(\vec{E}+\vec{v}\times\vec{B})}{m_j}\cdot\frac{\partial f}{\partial \vec{v}}=0 \qquad \sum_j e_j\int f_j d^3v=0$$

Three scale transformations $T1:\ f_j \to \alpha f_j$ $\quad T2: v\to\beta v, t\to\beta^{-1}t, E\to\beta^2E, B\to\beta B$

$T3: t\to\gamma t, x\to\gamma x, E\to\gamma^{-1}E, B\to\gamma^{-1}B$

Confinement time

$$\tau_E \propto n^p B^q T^r a^s$$

three constraints

$C1:\ p=0$

$C2: q+2r=-1$

$C3: s-q=1_j$

Scaling relation

$$\tau_E \propto \frac{1}{B}\left(\frac{T}{a^2B^2}\right)^r$$

$$\tau_E=\frac{1}{B}F\left(\frac{T}{a^2B^2}\right)$$

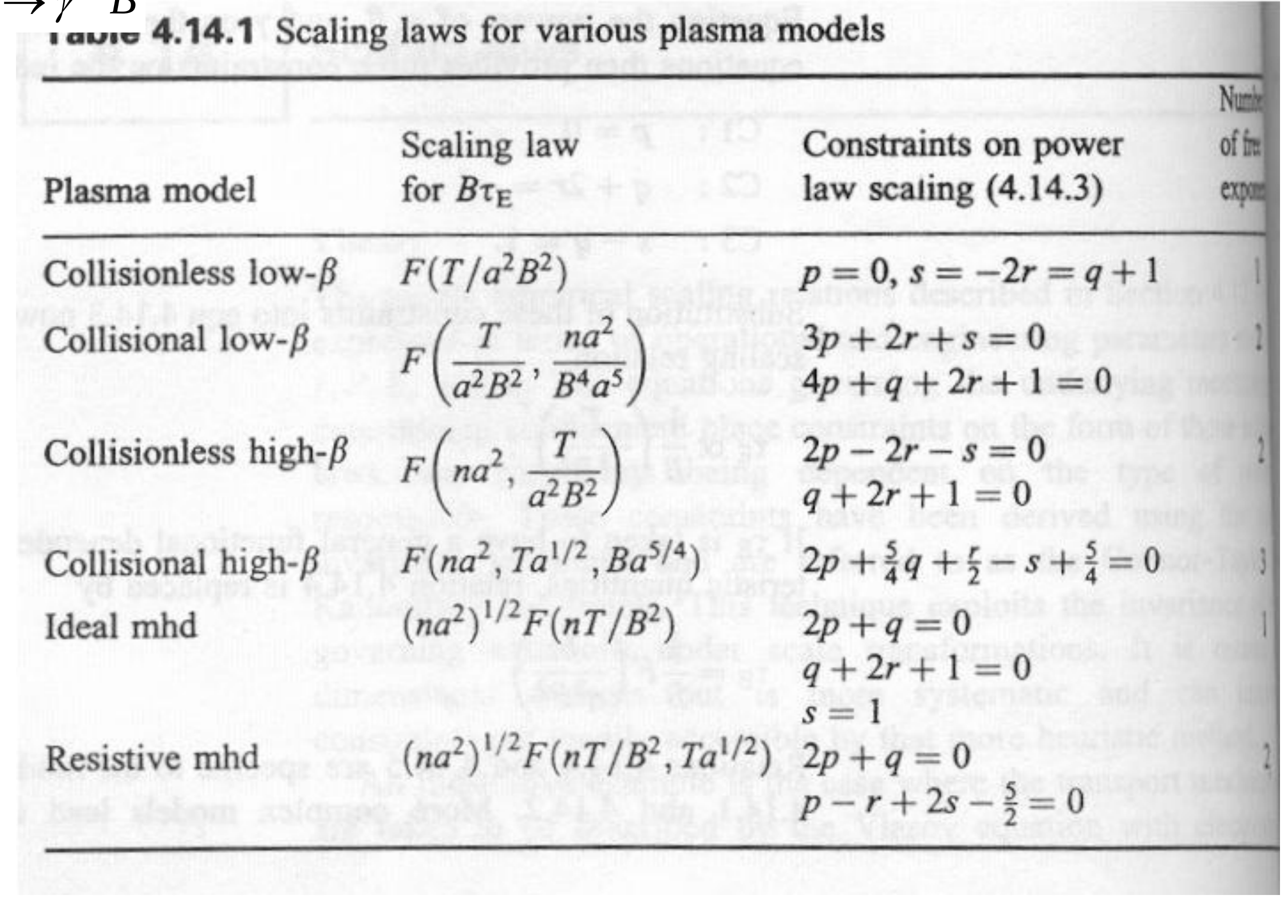


Table 4.14.1 Scaling laws for various plasma models

Plasma model	Scaling law for $B\tau_E$	Constraints on power law scaling (4.14.3)	Numb of fre expon
Collisionless low-β	$F(T/a^2B^2)$	$p=0,\ s=-2r=q+1$	1
Collisional low-β	$F\left(\frac{T}{a^2B^2},\frac{na^2}{B^4a^5}\right)$	$3p+2r+s=0$ $4p+q+2r+1=0$	2
Collisionless high-β	$F\left(na^2,\frac{T}{a^2B^2}\right)$	$2p-2r-s=0$ $q+2r+1=0$	2
Collisional high-β	$F(na^2, Ta^{1/2}, Ba^{5/4})$	$2p+\frac{5}{4}q+\frac{r}{2}-s+\frac{5}{4}=0$	3
Ideal mhd	$(na^2)^{1/2}F(nT/B^2)$	$2p+q=0$ $q+2r+1=0$ $s=1$	1
Resistive mhd	$(na^2)^{1/2}F(nT/B^2, Ta^{1/2})$	$2p+q=0$ $p-r+2s-\frac{5}{2}=0$	2

Experimental Scaling Relations

Bohm diffusion time $\tau_B = a^2 / D_B \quad D_B = T / eB \quad D_B \sim \omega_c \rho^2$

$$\tau_E = \frac{1}{B} F\left(\frac{T}{a^2 B^2}\right) \qquad \tau_B \sim \omega_c^{-1} (a/\rho)^2 \longrightarrow \qquad \tau_E = \tau_B F(\rho_*) \qquad \rho_* = \frac{\rho}{a}$$

More generally

$$\boxed{\tau_E = \tau_B F(\rho_*, \beta, \nu_*, \lambda_*)}$$

$$\nu_* = \frac{\nu}{\varepsilon^{3/2} v_T / Rq}$$

$$\lambda_* = \frac{\lambda_D}{a}$$

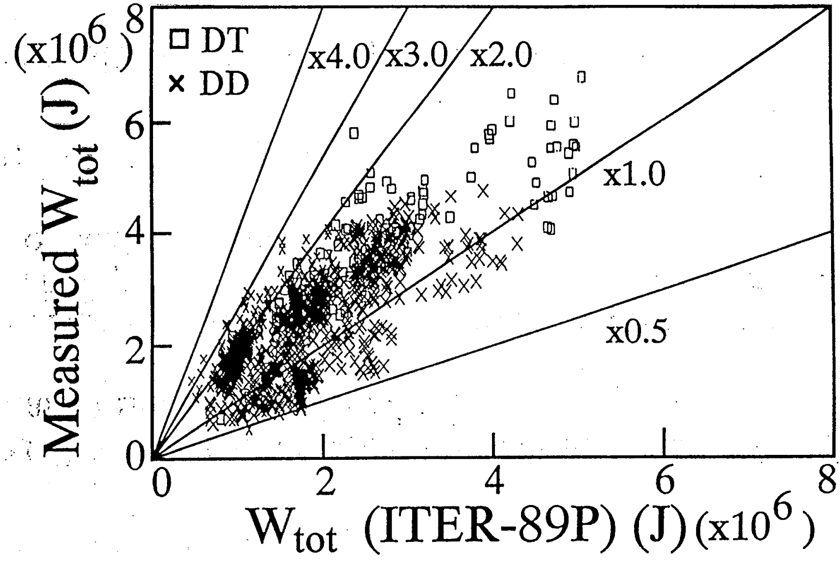
- Gyro-Bohm scaling : confinement governed by small scale turbulence on the scale of Larmor radius, the collisionless skin depth, or the layer width of resistive instabilities

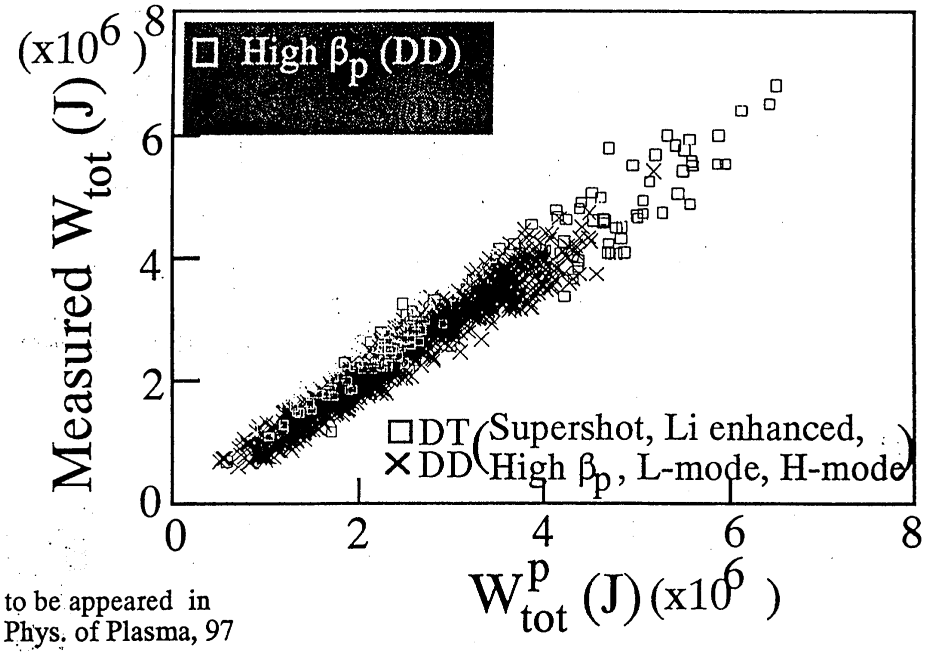
$$\tau_E = \frac{\tau_B}{\rho_*} F(\beta, \nu_*)$$

- Bohm scaling : turbulence on the scale of a $\tau_E = \tau_B F(\beta, \nu_*)$

- ITER89-P scaling law gives Bohm scaling with $\tau_E \sim \tau_B \beta^{-1/2} \nu_*^{-1/4}$

Scaling Law Based on Physics Model

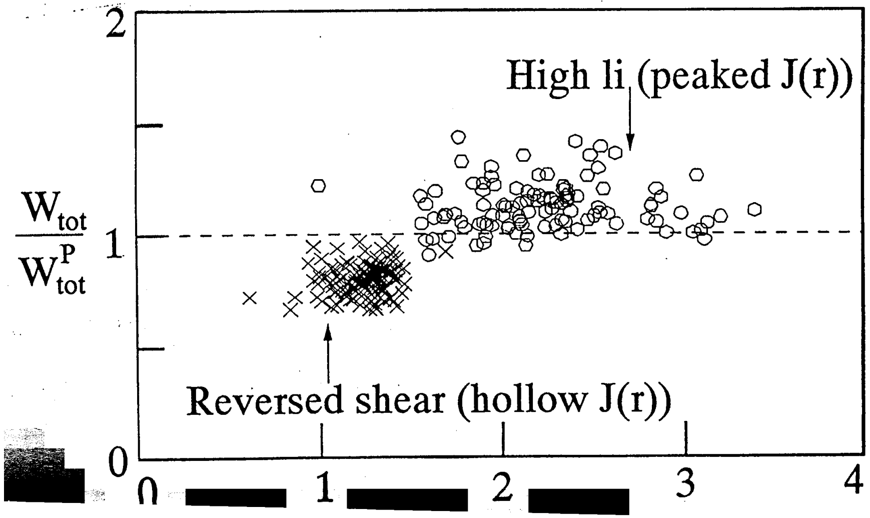




to be appeared in
Phys. of Plasma, 97

$$\left(2.04 \times 10^4 P_B^{1.3} H_{ne}^{0.8} + 310 P_B^{0.7} I_P^{0.4} \right)$$

(Ions) (Electrons)



Transport and Energy Loss Mechanisms

Energy confinement <-- particle diffusion and convection, radiation losses

- Anomalous transport
  - Transport coefficients
  - Fluctuations
  - Turbulence-induced transport
  - Candidate modes for anomalous transport
- Radiation losses
  - Impurity transport
  - Radiation losses
  - Impurity radiation

Transport Coefficients

Equations for fluxes

$$\Gamma_j = -\alpha_{11}\frac{dn_j}{dr} - \alpha_{12}\frac{dT_j}{dr} \qquad \alpha_{11} = D_j$$

$$q_j = -\alpha_{21}\frac{dn_j}{dr} - \alpha_{22}\frac{dT_j}{dr} \qquad \alpha_{22} = n\chi_j$$

Other expression for the particle flux

$$\Gamma_e = -D\frac{dn_e}{dr} - Vn_e$$

For transient perturbations from pellet injection or gas puff

Transport coefficients : $\chi_e, \chi_i \sim 1m^2s^{-1}$ $\quad D \sim D_z \sim \chi_e/4$ $\quad V \sim 10ms^{-1}$

neoclassical $\chi_i^{\ n} \sim 0.3m^2s^{-1}$ $\quad \chi_e^{\ n} \sim D^n \sim \left(\frac{m_e}{m_i}\right)^{1/2}\chi_i^{\ n}$ $\quad D_z^n \sim 0.1m^2s^{-1}$

Generally, $\chi_i \sim 1to10\chi_i^{\ n}$ $\quad \chi_e \sim 10^2\chi_e^{\ n}$

- D, Dz and χ_i can approach neoclassical levels in the plasma core and in high confinement regime, but χ_e almost always anomalous.
- Neoclassical value in the core and very anomalous in the outer region
- various confinement regimes show different characteristics of χ_i and χ_e

Anomalous Transport

* *Bohm diffusion.*

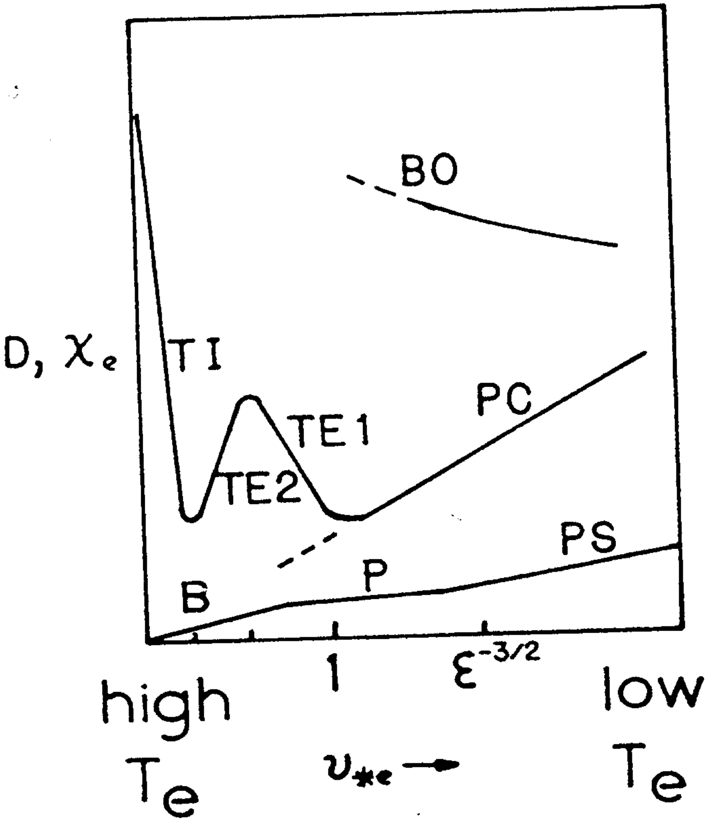
$D = T_e/16eB \quad (m^2/s)$

* *pseudoclassical.*
considerations of drift wave turbulence.
diffusion coefficient are given by

$D \sim 2\rho_\theta^2/\tau_e \quad (m^2/s)$

$\chi_e \sim 6n\rho_\theta^2/\tau_e \quad (m^{-1}s^{-1})$

* *trapped particle instabilities.*
$\nu_{*e} < 1$, the trapped electron instability
At lower collision frequencies,
the trapped ion mode may occur.



Fluctuations

Anomalous transport is believed to arise from turbulent diffusion caused by fluctuations in the plasma either electrostatic or electromagnetic.

ExB drift velocity produced by turbulent fluctuations, $\delta v_\perp = \frac{\delta E_\perp}{B}$

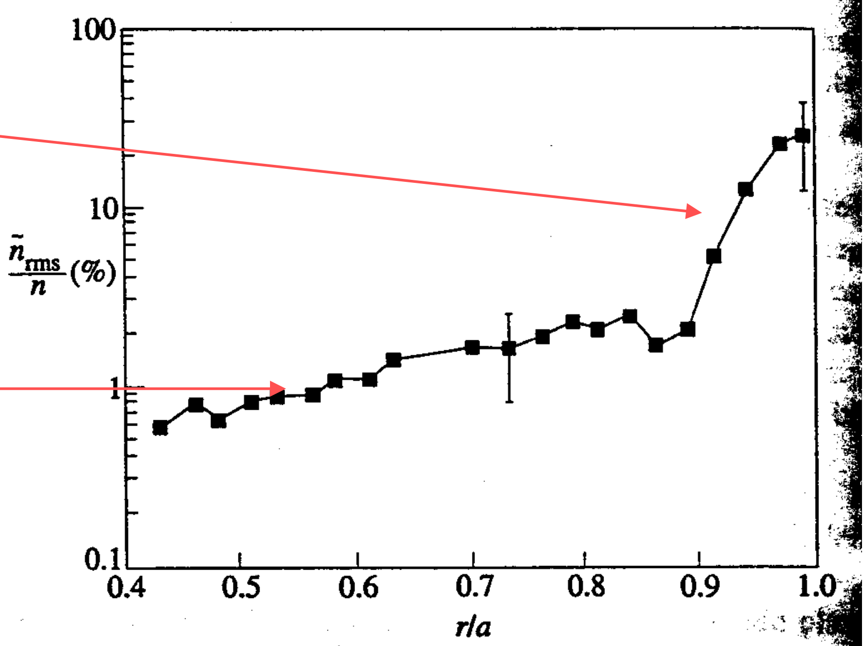
Convective particle flow combined with density fluctuations $\Gamma =< \delta v_\perp \delta n >$

Heat flux combined with temperature fluctuations $q_j = \frac{3}{2} n_j < \delta v_\perp \delta T_j >$

Flux induced by magnetic fluctuations $\Gamma = \frac{n}{B} < \delta v_{// j} \delta B_r >$

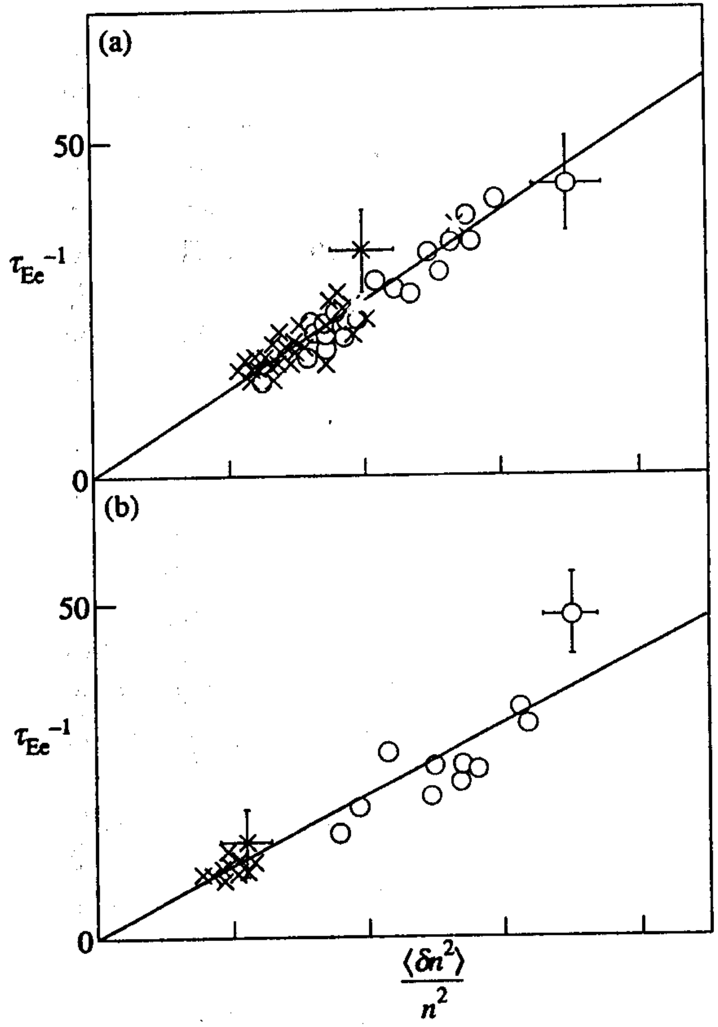
- Measured by Langmuir probe
- electrostatic fluctuations ~ 50%
- $\delta B/B \sim 10^{-4}$

- Measured by FIR scattering, HIBP BES, and microwave reflectometry
- density fluctuations ~ 1%



Fluctuations: Correlations with Transport

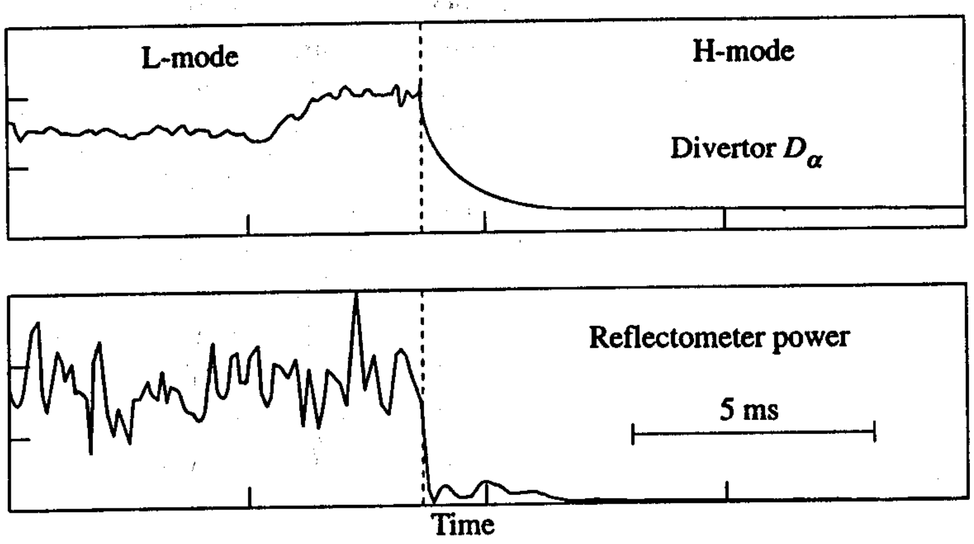
Comparison of the radial dependence of the fluctuation *amplitude* and $\chi(r)$.



Globally, the correlation between the level of fluctuations and the confinement time

Clear correlation found in ohmic, ICRF, NBI heated plasmas in TFR

Correlation between fluctuations and L-H transition in DIII-D and ASDEX



Turbulence-Induced Transport

Theoretical picture : Free energy released by an instability drives a steady level of fluctuations in the associated perturbed quantities, resulting in a radial transport of particles and energy.

- level 1: assume the behavior of the underlying instability as given
  - calculate the transport arising from the electrostatic and electromagnetic fluctuations, δE and δB, ***by random walk estimates for turbulent transport*** arising ***from the fluctuating ExB drift velocity and from the parallel motion*** along stochastic magnetic field lines
  - quasi-linear theory : turbulent fluxes such as particle flux are calculated using the linear plasma response to δE and δB
- level 2: attempt to calculate the nonlinear saturated state of the micro-instability
  - scale invariance approach to discuss local rather than global transport
  - strong turbulence theory : saturation is taken to occur when the perturbed gradient ∇ δ n is comparable with the equilibrium gradient ∇ n
  - weak turbulence theory : the linear growth of a particular wavelength fluctuation is balanced against its decay due to nonlinear scattering to other wavelengths

Candidate Modes for Anomalous Transport

• Several micro-instabilities have been invoked to explain the observed fluctuations and the associated transport, especially for anomalous transport
• A commonly used model for ohmic and L-mode plasmas assumes electrostatic trapped electron drift waves and ion temperature gradient modes with a contribution from resistive ballooning modes near the plasma edge

- Electron drift wave instabilities :

  diffusivities have gyro-Bohm form

- Ion temperature gradient modes(η_i modes):

  $\eta_i = \mathrm{d}\ln T_i / \mathrm{d}\ln n$, the η_i mode with $k_\perp \rho_s < 1$ become unstable

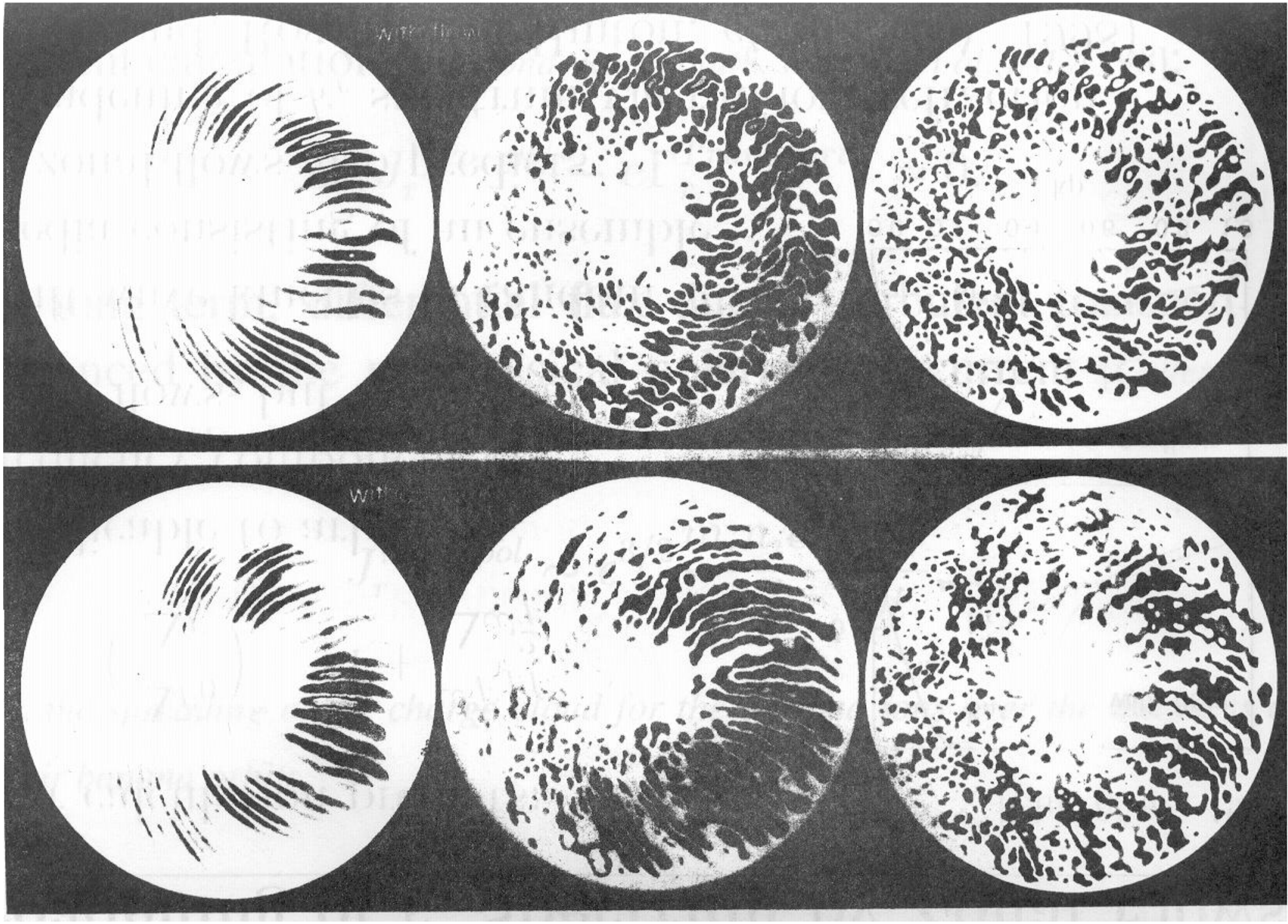
  possible explanation of the confinement scalings for both the saturated ohmic and the L-mode confinements

- Resistive ballooning modes:

  driven by the pressure gradient at bad curvature region

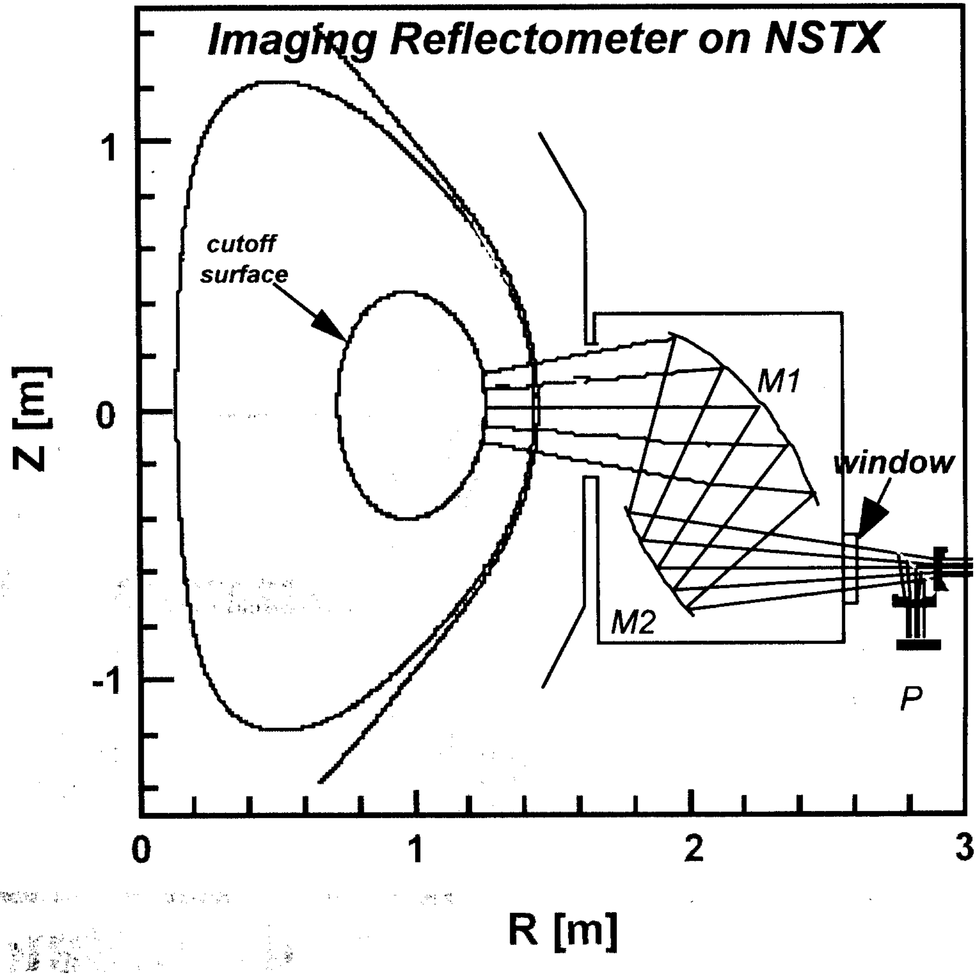
- Micro-tearing modes

Effect of Zonal Flow :gyrokinetic simulation



3-D Imaging of Turbulence

: microwave imaging reflectometry

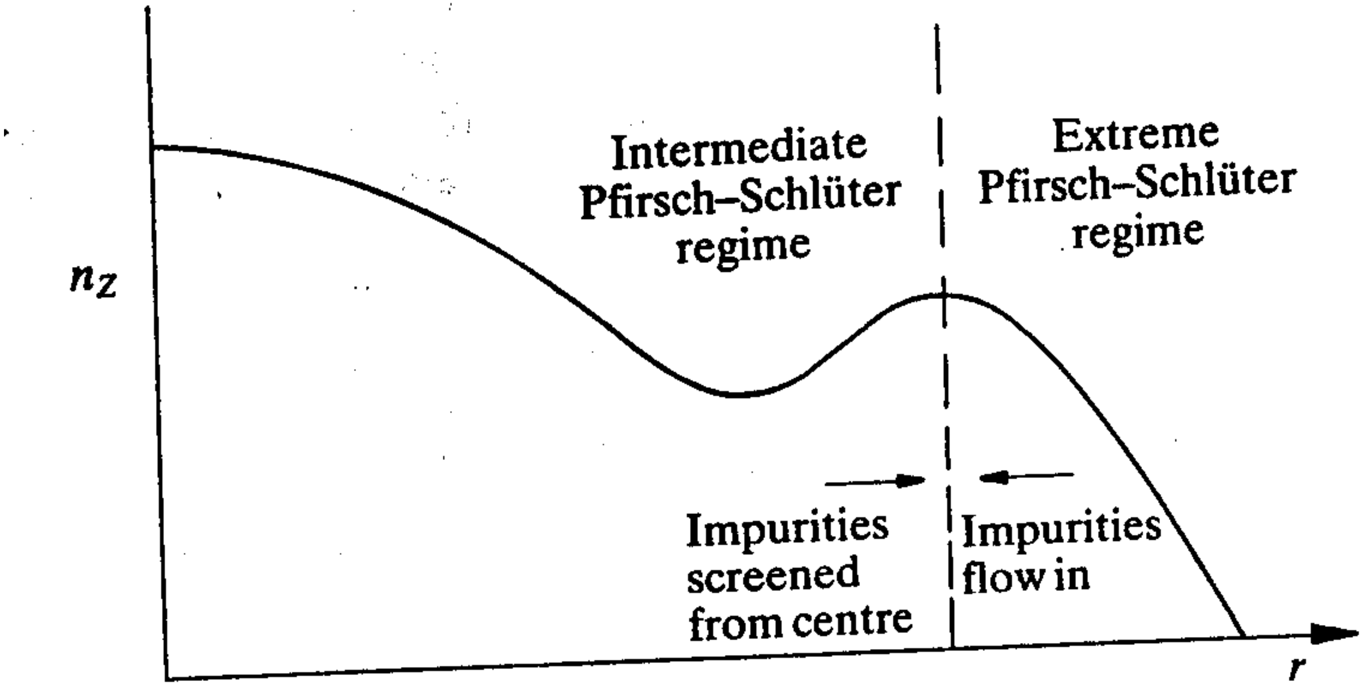


Impurity Transport

- Neoclassical impurity transport

$$\frac{1}{n_Z}\frac{dn_Z}{dr}=\frac{Z}{n_i}\frac{dn_i}{dr}+\frac{\alpha}{T}\frac{dT}{dr}$$

$$\frac{n_Z(r)}{n_Z(0)}=\left(\frac{n_i(r)}{n_i(0)}\right)^Z$$



Impurity Transport

- Anomalous impurity transport
  - simple universal empirical formula

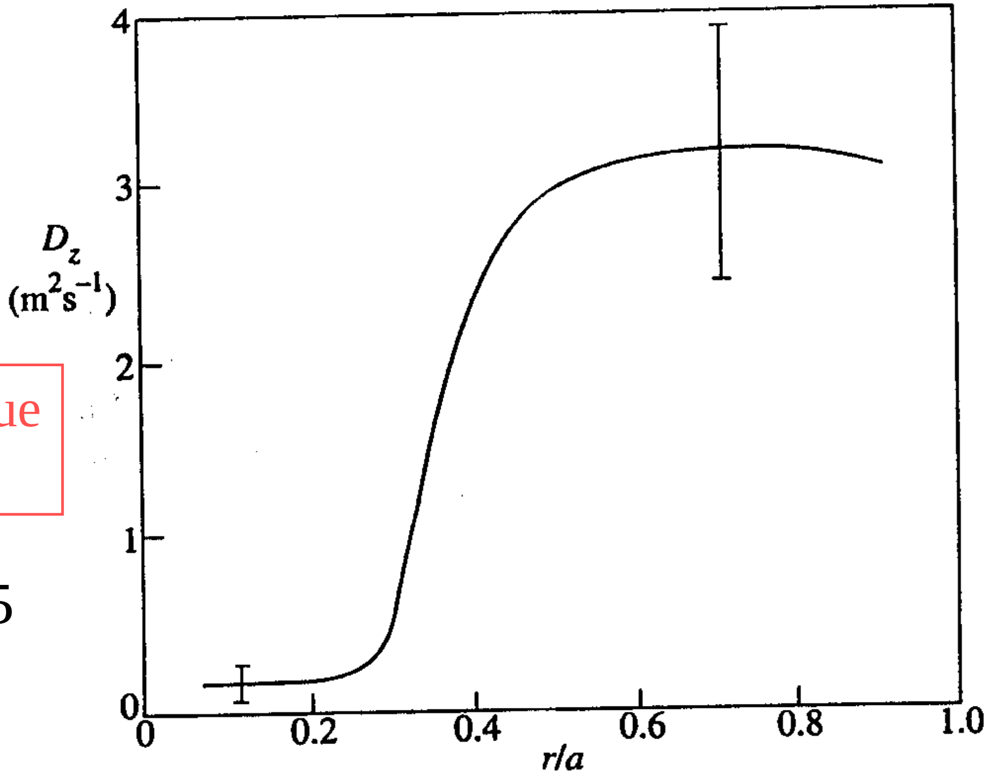
$$\Gamma_Z = -D_Z\left(\frac{dn_Z}{dr} + 2S\,\frac{r}{a^2}\,n_Z\right) \qquad D_Z = 0.25 - 6m^2s^{-1} \qquad S = 0.5 - 2$$

  - experiments in JET

$$D_Z = 0.1m^2s^{-1}$$

- 2-10 times of neoclassical value
- rapid transition near s = 0.5

$$s = \frac{r}{q}\frac{dq}{dr} \approx 0.5$$



Radiation Losses

- Bremsstrahlung

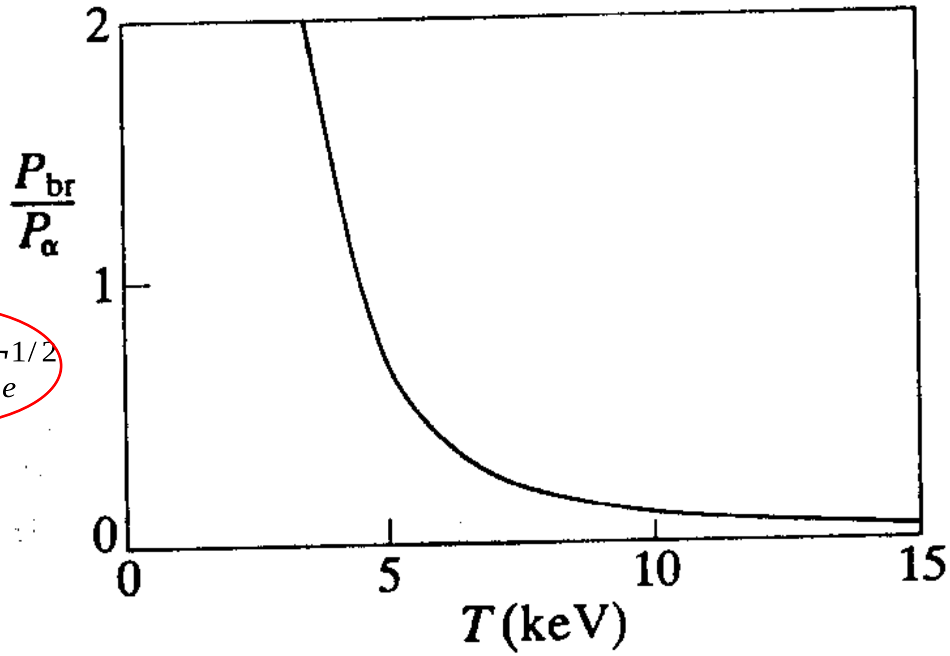
$$P = \frac{e^2}{6\pi\varepsilon_o c^3} a^2 \qquad a = \frac{Ze^2}{4\pi\varepsilon_o m_e r^2}$$

$$\delta E = \frac{Z^2 e^6}{6(2\pi\varepsilon_o cr)^3 m_e^2 v}$$

$$P_{br} = n_e n_Z \int \delta E v 2\pi r dr$$

$$= \frac{n_e n_Z Z^2 e^6}{24\pi^2 \varepsilon_o^3 c^3 m_e^2} \int \frac{1}{r^2} dr$$

$$= g \frac{e^6}{6(3/2)^{1/2} \pi^{3/2} \varepsilon_o^3 c^3 h m_e^{3/2}} n_e n_Z Z^2 T_e^{1/2}$$



- Cyclotron radiations

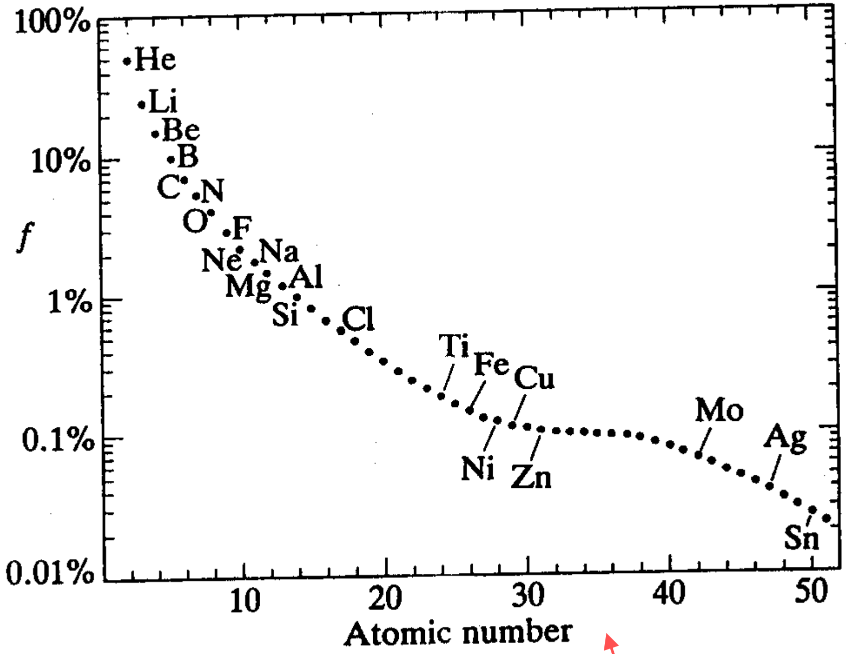
$$a = \omega_{ce}^2 \rho_e \qquad \rho_e = (2T_e / m_e)^{1/2} / \omega_{ce}$$

$$P_c = \frac{e^4}{3\pi\varepsilon_o c^3 m_e^3} n_e B^2 T_e$$

- optically thick --> black body radiation
- power loss: 1MW/m^3 --> 10^{-2} MW/m^3

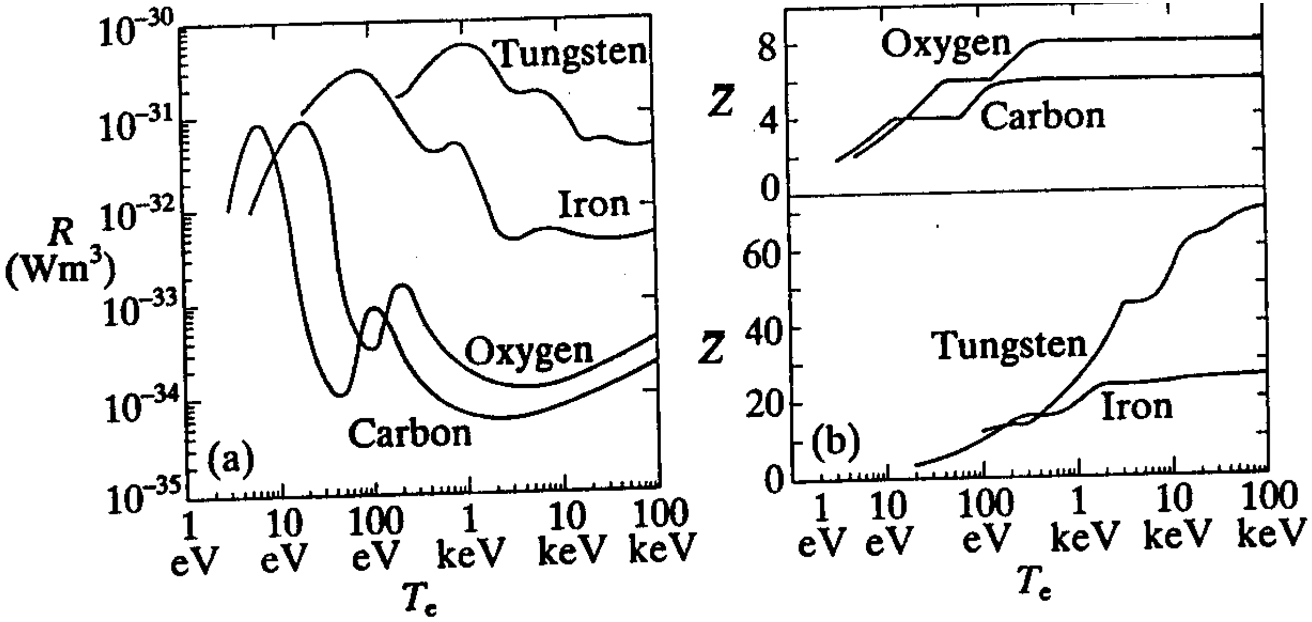
Impurity Radiation

$$P_R = n_e n_I R(T_e)$$



Radiated power fraction

$$F = \frac{n_e n_I R}{\frac{1}{4} n_H^2 < \sigma v > E_f} = \frac{(1 + f\bar{Z}) f R}{\frac{1}{4} < \sigma v > E_f}$$



F=0.1 at T=10keV

$$f = n_I / n_H$$

$$\bar{Z} = \sum n_Z Z / n_I$$

$$E_f = 17.6 MeV$$