Lecture 5:

Comb resonator design (1)

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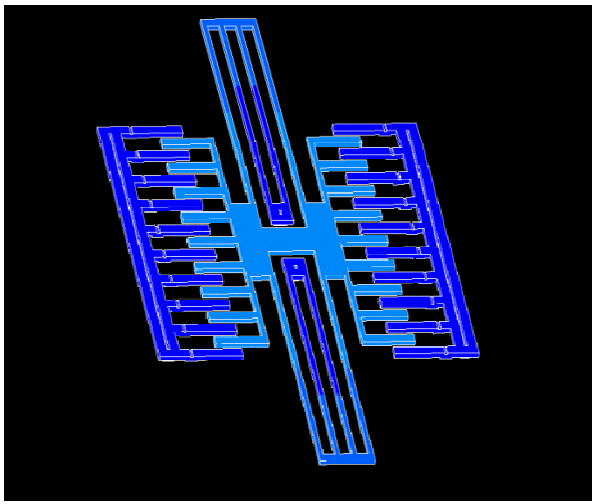
Why is comb drive type ?

- ***Principle :***
  Interlacing comb fingers create large capacitor area; electrostatically actuated suspended microstructures

- ***Features :***
  - Fairly linear relationship between capacitance and displacement
  - Higher surface area/ capacitance than parallel plate capacitor
  - Electrostatic actuation: low power (no DC current)



Electrostatic Comb Drive Type

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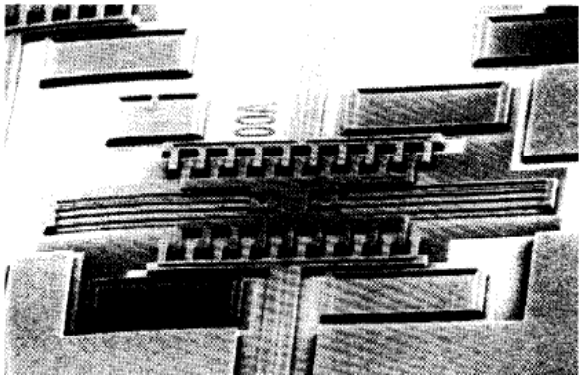
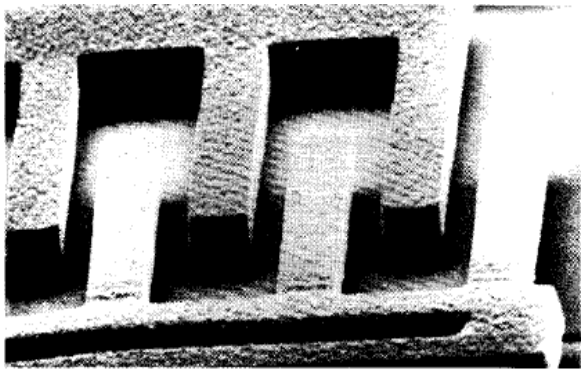
What is comb drive resonator ? (cont'd)

- ***Comb drives combine mechanical and electrostatic issues :***
  - Mechanical issues
    - Elasticity
    - Stress and strain
    - Resonance (natural frequency)
    - Damping
  - Electrical issues
    - Capacitance
    - Electrostatic forces
    - Electrostatic work and energy





Tang, Nguyen and Howe JMEMS 1989.

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Normal Stress and Strain

- ***Stress:*** force applied to surface

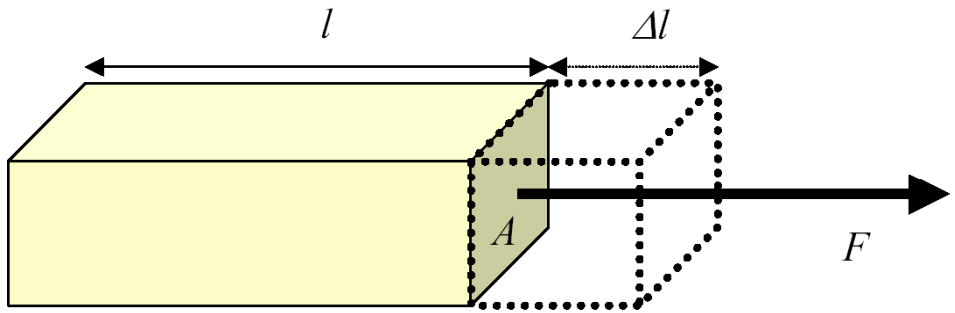
  $\sigma = F / A$

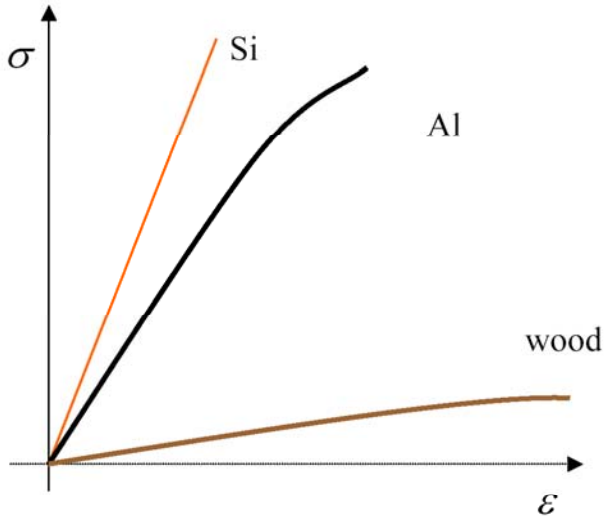
  measured in N/m^2 or Pa,
  compressive or tensile

- ***Strain:*** ratio of deformation to length

  $\varepsilon = \Delta l / l$

  measured in %, ppm,
  or microstrain





Young's Modulus:

$E = \sigma / \varepsilon$

Hooke's Law:

$K = F / \Delta l = EA / l$

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Shear Stress and Strain

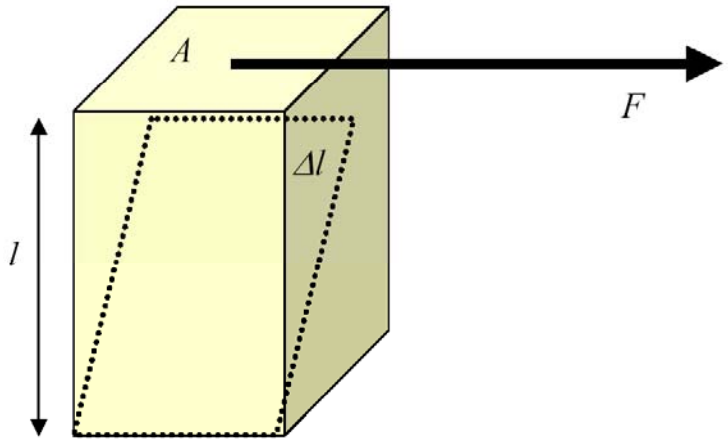
- ***Shear Stress:*** force applied parallel to surface

$$\tau = F / A$$

measured in N/m^2 or Pa,

- ***Strain:*** ratio of deformation to length

$$\gamma = \Delta l / l$$



Shear Modulus:

$$G = \tau / \gamma$$

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Poisson's Ratio

Tensile stress in x direction results in compressive stress in y and z direction (object becomes longer and thinner)

- ***Poisson's Ratio:***

$$\nu = \left| -\varepsilon_y / \varepsilon_x \right|$$

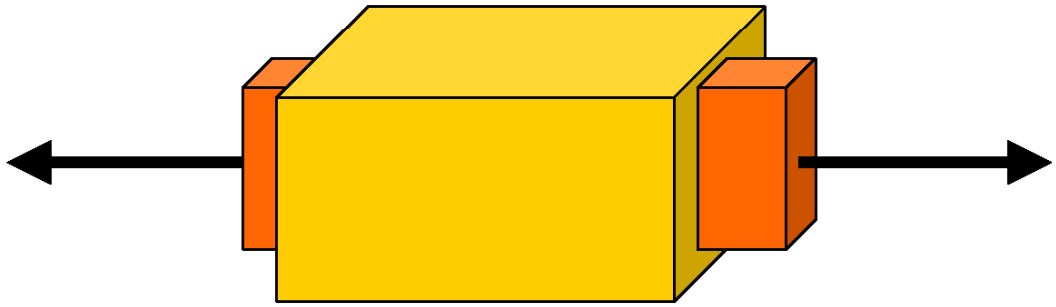
$$= \left| -\text{transverse strain / longitudinal strain} \right|$$

< Materials >
- Homogeneous
  - Isotropic
  - Anisotropic
- Heterogeneous

Metals: $\nu \approx 0.3$

Rubbers: $\nu \approx 0.5$

Cork: $\nu \approx 0$



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Mechanical Property for Poly Crystalline Silicon

- ***Young's modulus for different deposition processes***

Doping Conditions / Deposition temp		PSG diffusion doping			$POCl_3$ diffusion doping							
		1000℃ 60min	1050℃ 30min	1000 ℃ 120min	850℃ 180min	850℃ 210min	850 ℃ 240min	950℃ 60min	950℃ 90min	950 ℃ 120min	1000℃ 60min	1000 ℃ 90min
Young's modulus (GPa)	625 ℃	154.3 ±2.3	155.1 ±1.8	151.1 ±3	149.6 ±2.6	150.7 ±2.5	153.1 ±2.1	149 ±1.7	151.1 ±1.1	150.4 ±1.1	152.6 ±1.4	148.6 ±2
	605 ℃	159.3 ±3.4	160.5 ±1.7	161.8 ±1.4	162.6 ±0.6	161.9 ±2.3	163.1 ±0.4	161.2 ±2	155.6 ±5.6	161.8 ±1.4	157.1 ±3.7	156 ±5.9
	585 ℃	170.1 ±2.2	169.4 ±6.2	162.4 ±1.6	168.5 ±1.9	162.2 ±2.6	163.5 ±3.1	159 ±3.3	165.6 ±1.4	160.6 ±1	159.8 ±1	156.6 ±2

[Ref] S. Lee et al., IOP Journal of Micromechanics and Microengineering, vol. 8, no. 4, pp. 330-337, Dec. 1998.

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Mechanical Property for Single Crystal Silicon

- ***Silicon Crystallography***

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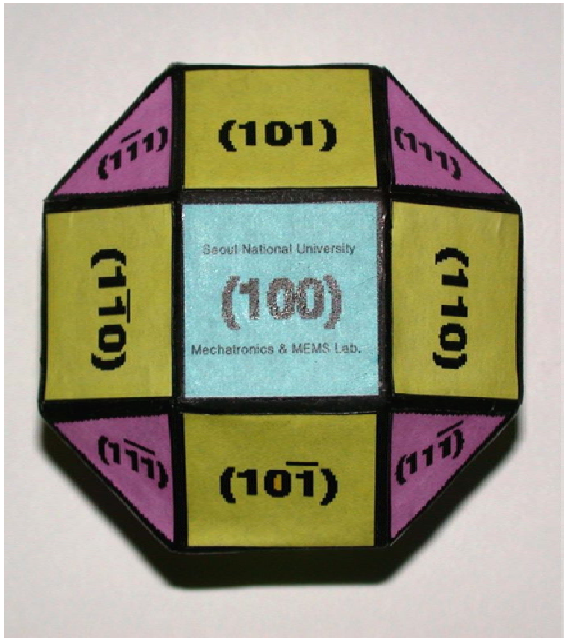
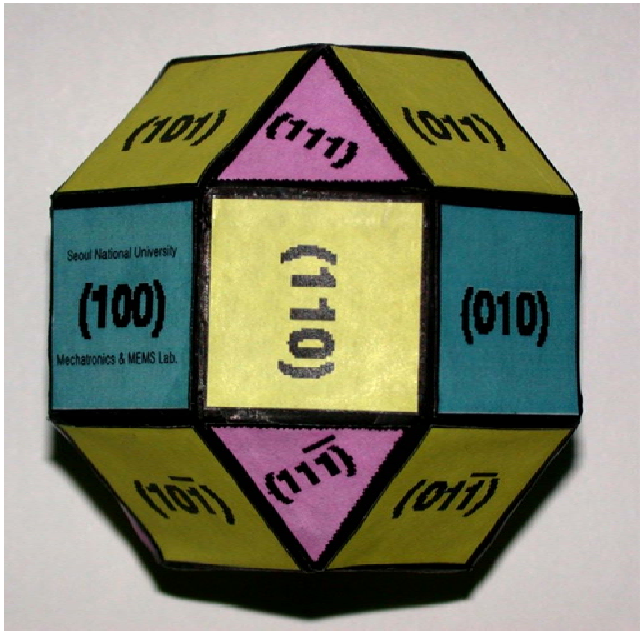
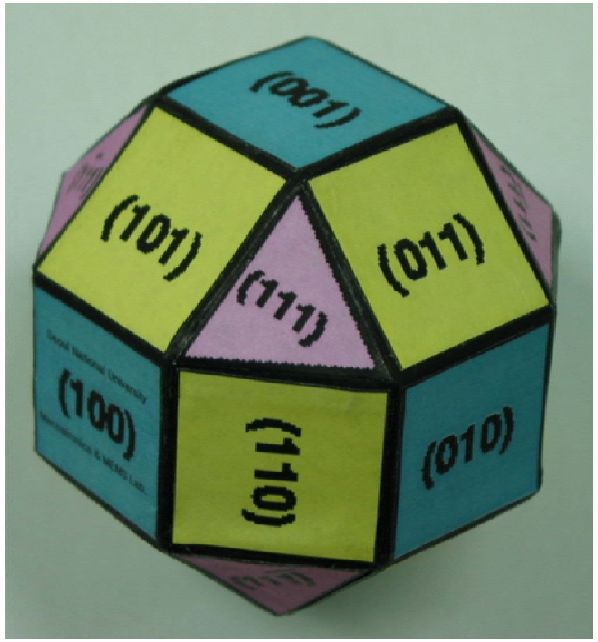


Silicon (100)



Silicon (110)



Silicon (111)

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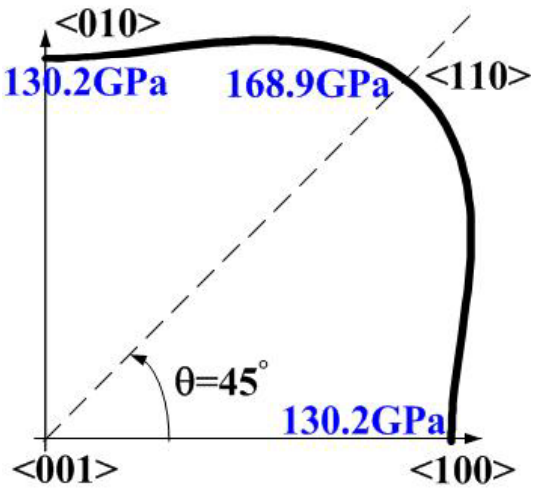
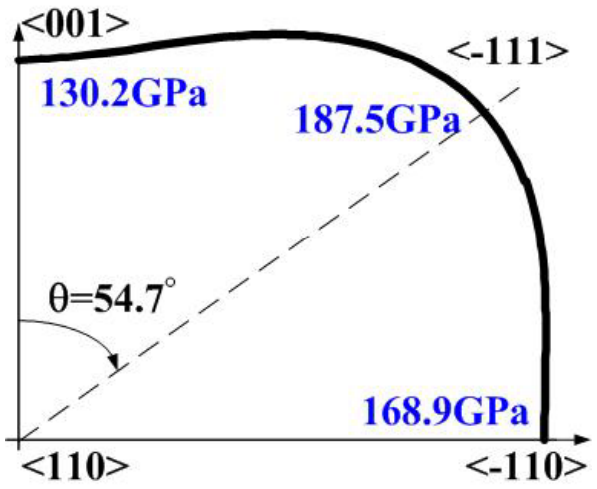
Mechanical Property for Single Crystal Silicon (cont'd)

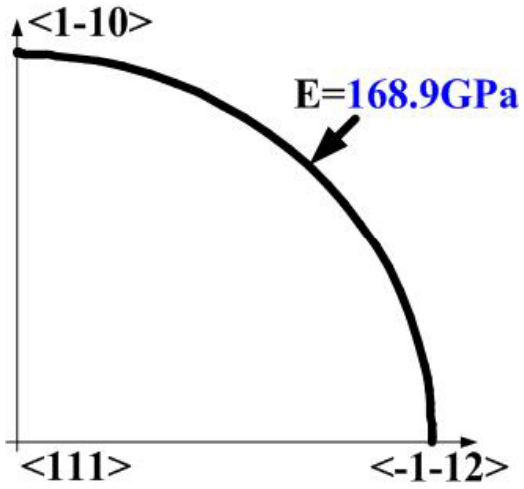
- ***Young's modulus***



Silicon (100)



Silicon (110)



Silicon (111)

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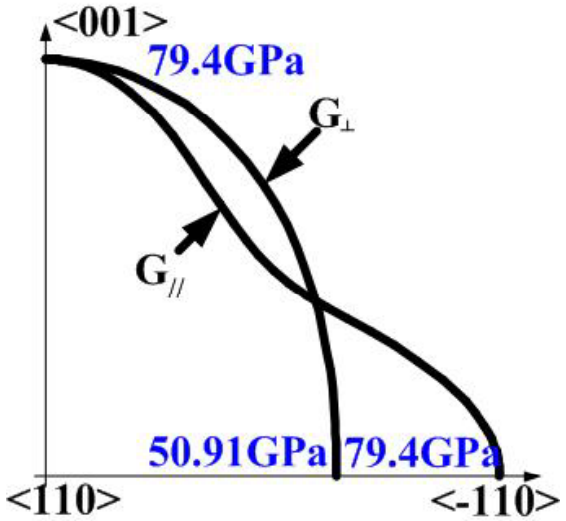
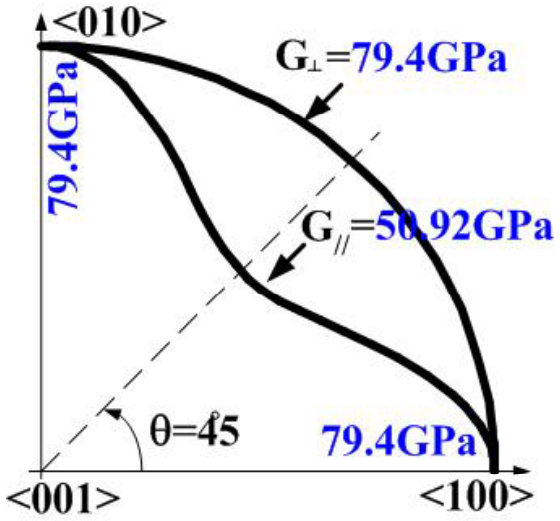
Mechanical Property for Single Crystal Silicon (cont'd)

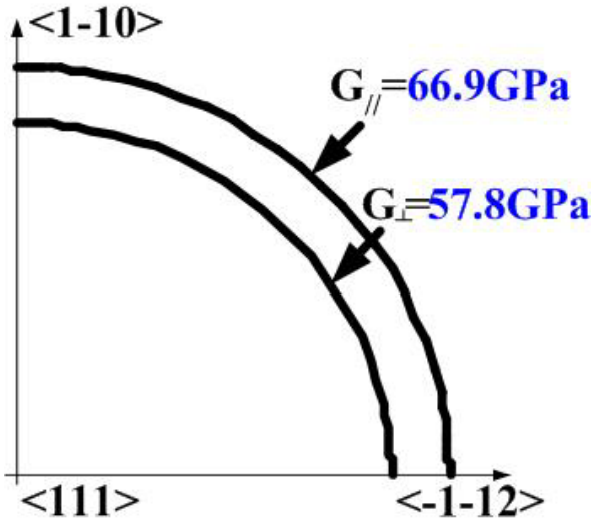
- ***Shear modulus***



Silicon (100)



Silicon (110)



Silicon (111)

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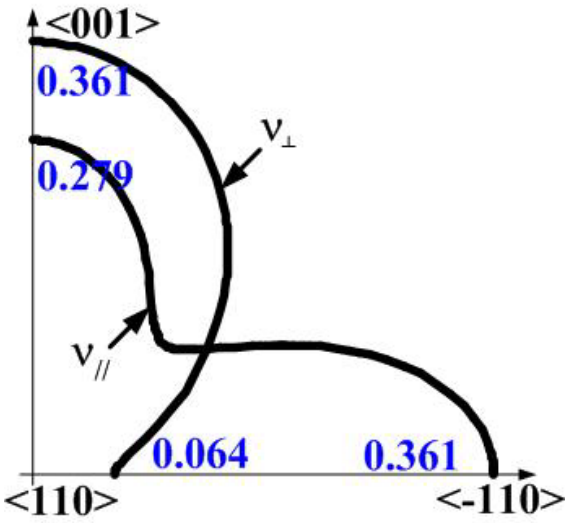
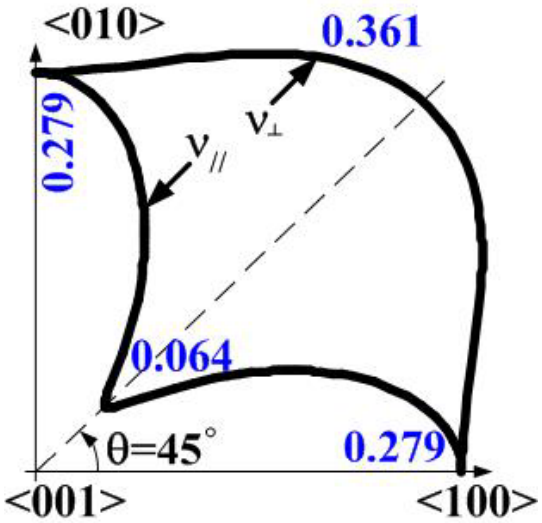
Mechanical Property for Single Crystal Silicon (cont'd)

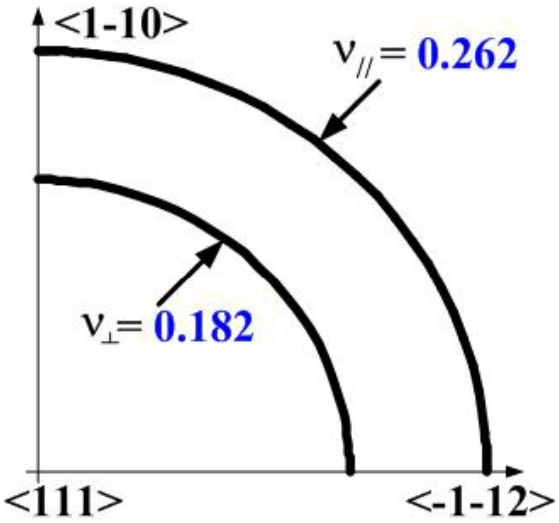
- ***Poisson's ratio***



Silicon (100)



Silicon (110)



Silicon (111)

[Ref] J. Kim et al., Proceedings of Transducers 2001, pp. 662-665, Munich, Germany, June 2001.

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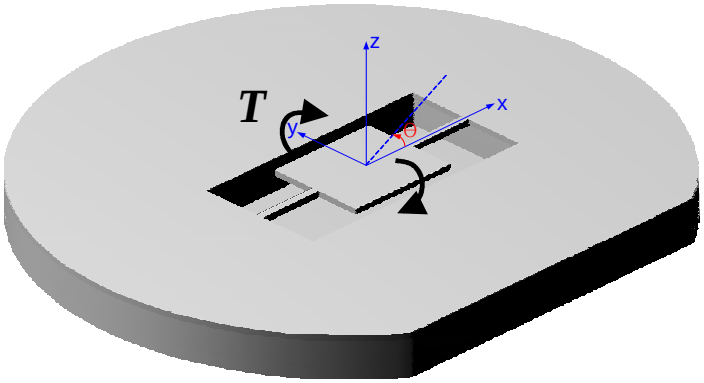
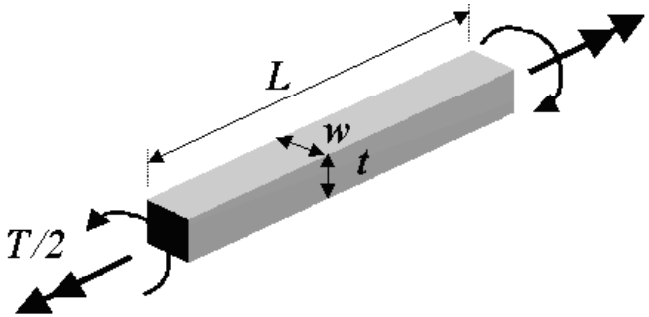
Mechanical Property for Single Crystal Silicon (cont'd)

- ***Torsional stiffness (1)***
  - Schematic of torsional spring





✓ The dimension splits of torsional springs

- thickness(t) : 10 ㎛, 20 ㎛
- length(l) : 40 ㎛
- width(w) : 2 ㎛, 4 ㎛

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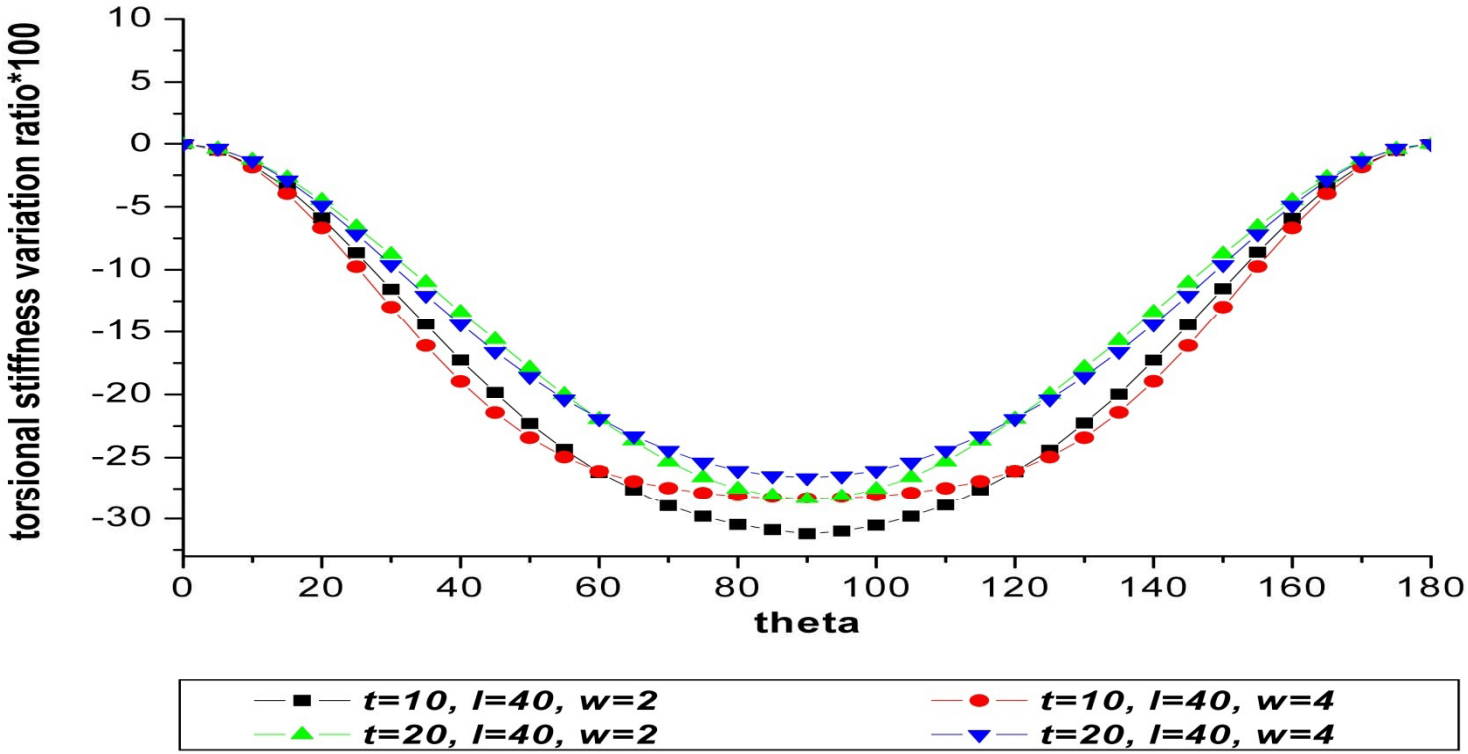
Mechanical Property for Single Crystal Silicon (cont'd)

- ***Torsional stiffness (2)***
  - Silicon (110)



The torsional stiffness on silicon (110) varies from -31.2 % to -26.6 %

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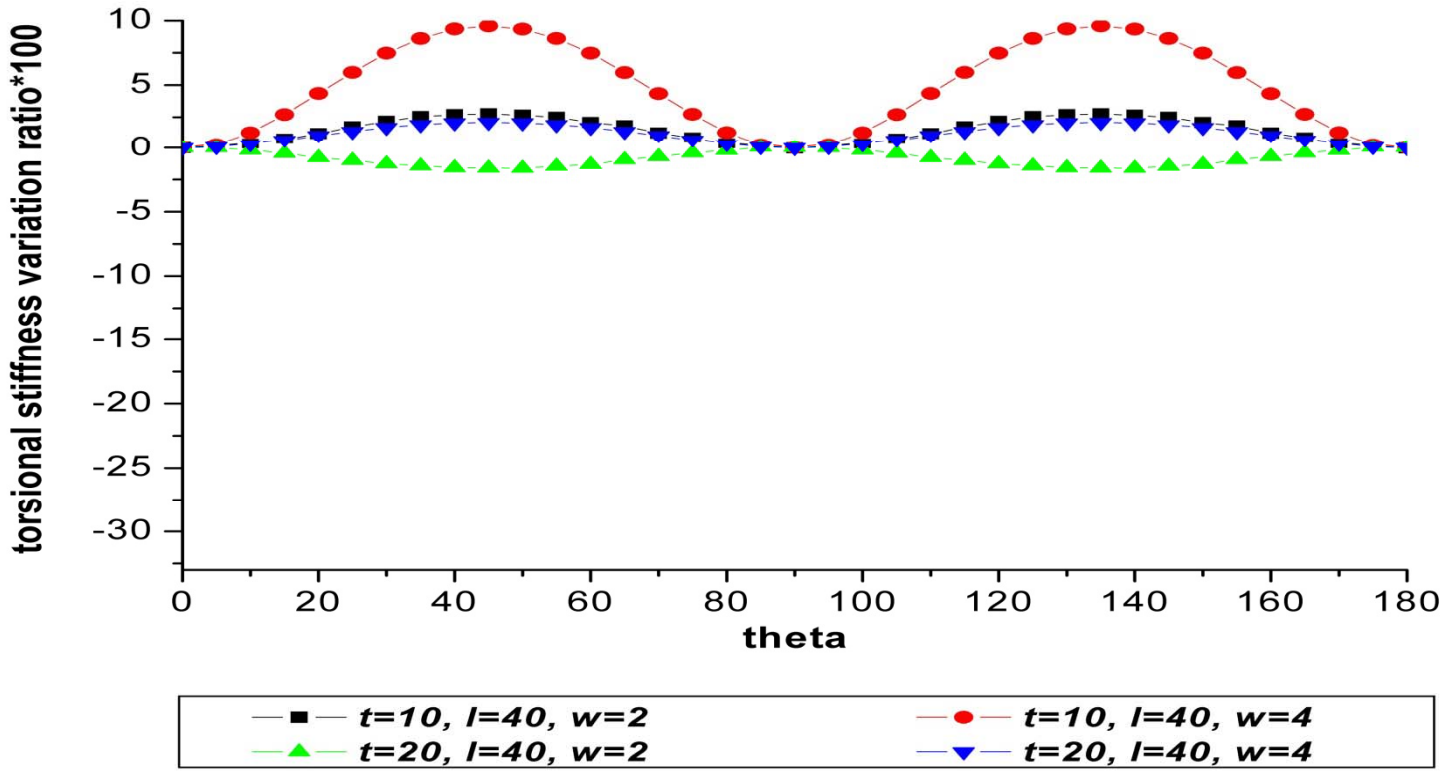
Mechanical Property for Single Crystal Silicon (cont'd)

- ***Torsional stiffness (3)***
  - Silicon (100)



The torsional stiffness on silicon (100) varies from -1.6 % to 9.6 %.

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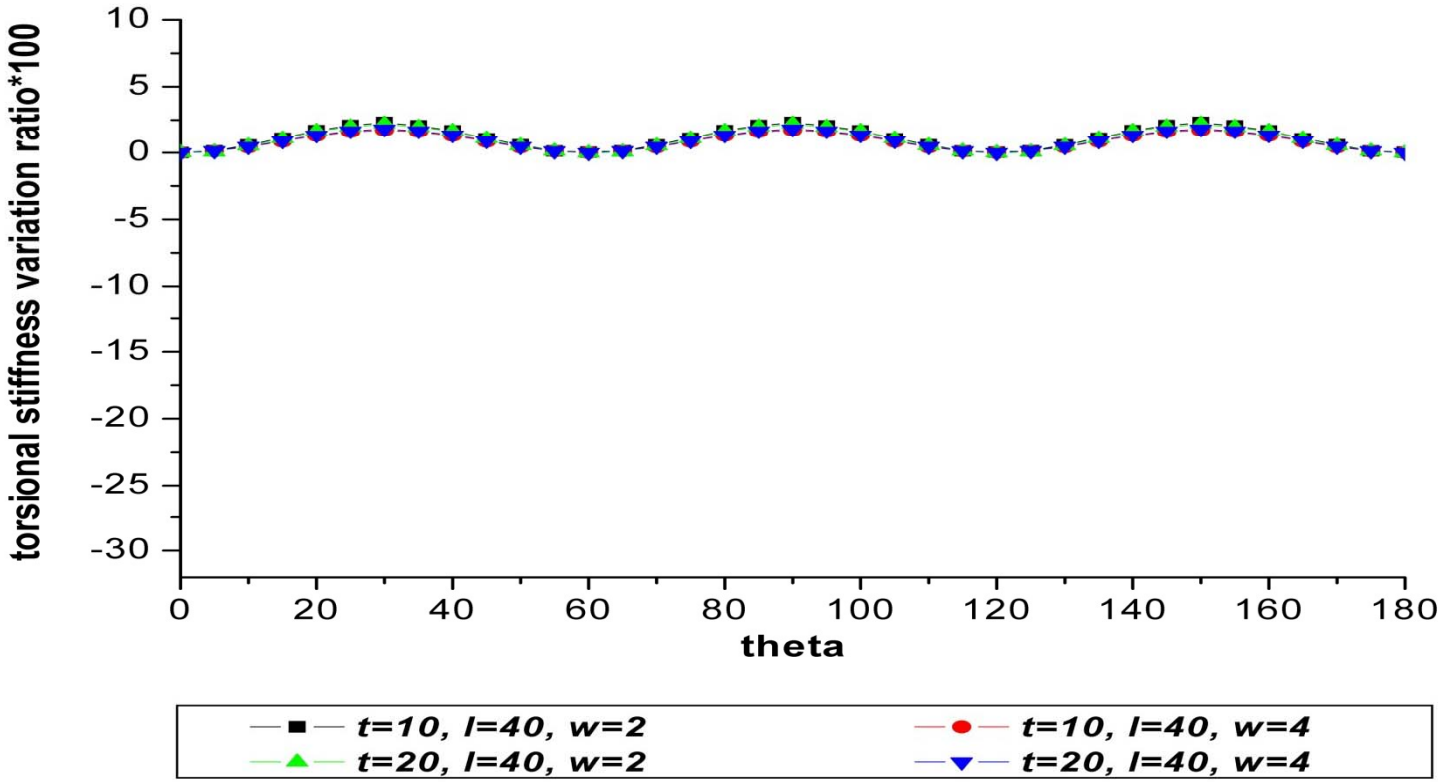
Mechanical Property for Single Crystal Silicon (cont'd)

- ***Torsional stiffness (4)***
  - Silicon (111)



The torsional stiffness on silicon (111) varies from 1.7 % to 2.3 %.

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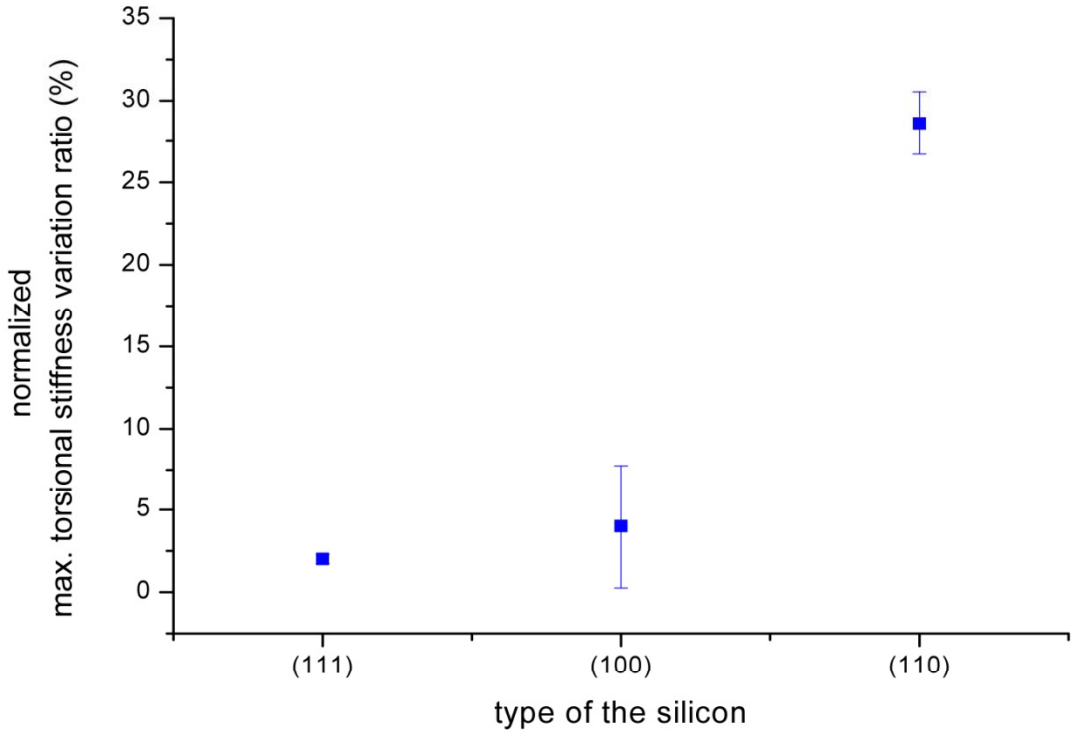
Mechanical Property for Single Crystal Silicon (cont'd)

- ***Torsional stiffness (5)***
  - Summary for results



The average for the normalized maximum torsional stiffness variation ratio (%)

[Ref] D. Kwak et al., Proceedings of IMECE 2003, Washington D.C, USA, November 2003.

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Electrostatic Forces

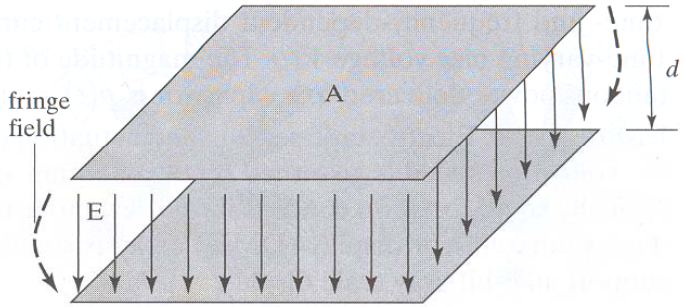
- ***Parallel Plate Capacitor:***
  - Capacitance:

$$C = Q/V = \varepsilon_0 \varepsilon_r A/d$$

ε_0; dielectric constant of free space

$(\approx 8.854 \times 10^{-12} F/m)$

ε_r; dielectric permittivity



  - Stored energy:

$$W = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} : \; C = \frac{\varepsilon A}{d}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

  - Electrostatic force between plates:

$$F_x = -\frac{\partial W}{\partial x} = -\frac{1}{2}\frac{\varepsilon A}{d^2}V^2 = \frac{1}{2}\frac{CV^2}{d}$$

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Electrostatic Actuation

- ***Positioning of capacitor plate:***

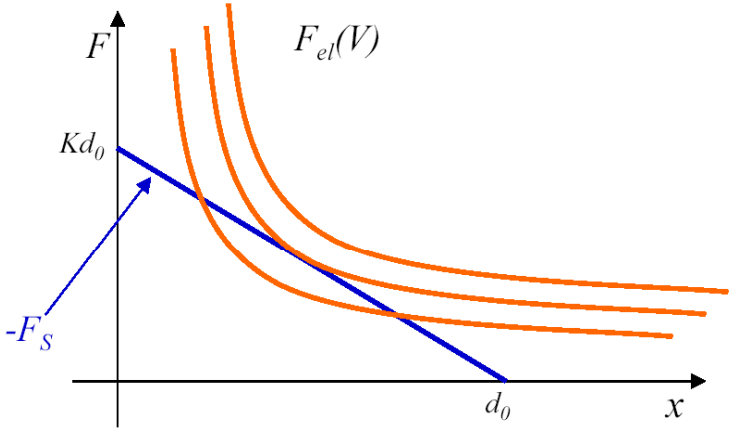
$$F_{el} = -\frac{1}{2}\varepsilon_0 \varepsilon_r A V^2 / x^2 < 0$$

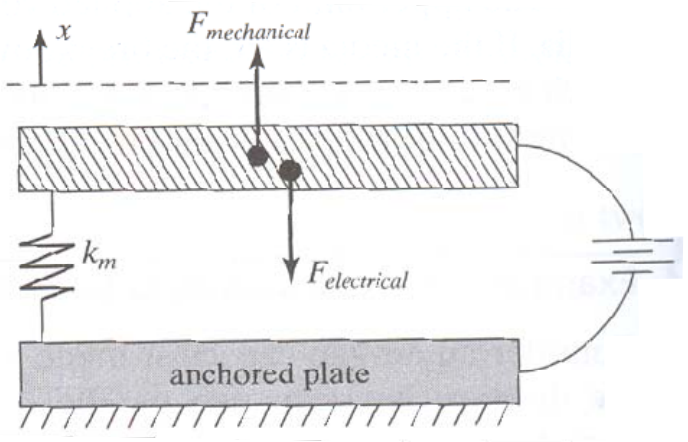
$$F_S = K(x - d_0) < 0$$

where d_0:distance at rest

(no applied voltage)

- ***Stable equilibrium when*** $F_{el} = F_S$





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Pull-In Point

The higher *V*, the closer the plate is pulled in. $F_{el} \rightarrow \infty$ when $d \rightarrow 0$. What is the closest stable distance x_{min}?

- ***F_{el} and F_S must be tangential:***

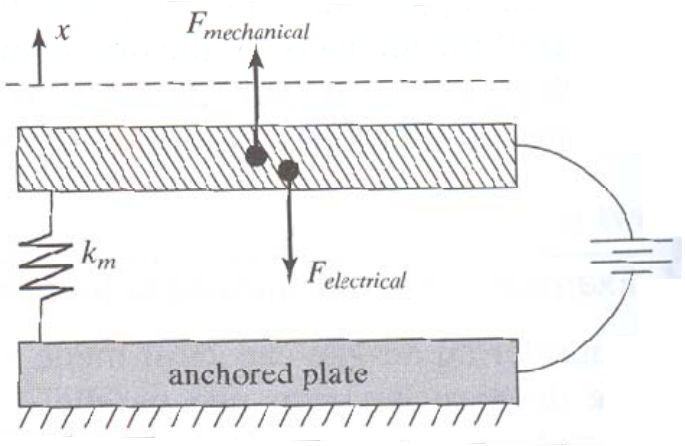
  $\varepsilon_0 \varepsilon_r A / x^3 V^2 = K$, so

  $V^2 = Kx^3 / \varepsilon_0 \varepsilon_r A$

- ***Substitute into F_{el}=F_S to get***

  $x_{\min} = 2/3 d_0$

  can control x only from $2/3 d_0$ to d_0



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Electrostatic Comb Drive

- ***Capacitance is approximately:***

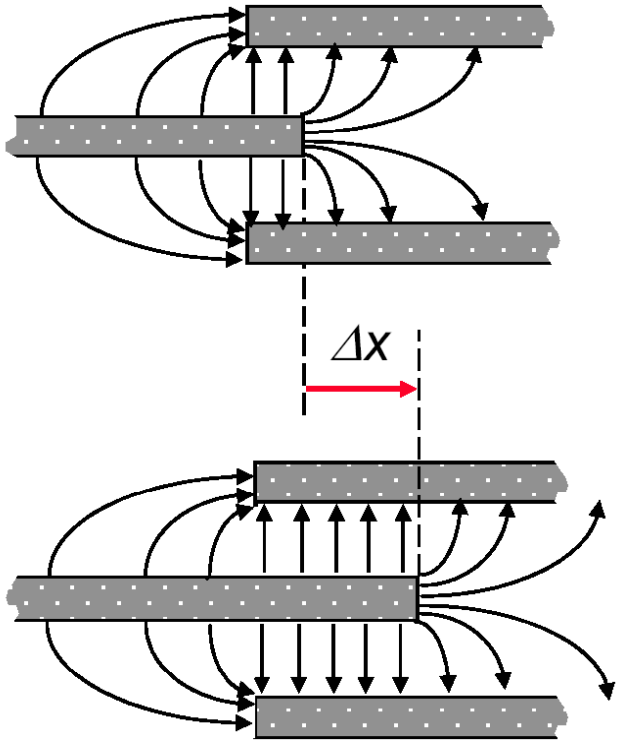
$$C = \varepsilon_0 \varepsilon_r A / d \qquad (L >> d)$$
$$= 2n\varepsilon_0 \varepsilon_r lh / d \qquad \text{(ignore fringing field effect)}$$

- ***Change in capacitance when moving by $\triangle x$:***

$$\Delta C = \varepsilon_0 \varepsilon_r \Delta A / d$$
$$= 2n\varepsilon_0 \varepsilon_r \Delta x h / d$$

- ***Electrostatic force :***

$$F_{el} = 1/2 V^2 dC / dx = n\varepsilon_0 \varepsilon_r h / dV^2$$



Electric field distribution in com-finger gaps

Note: F_{el} is independent of $\triangle x$ over wide range (fringing field), and quadratic in V.

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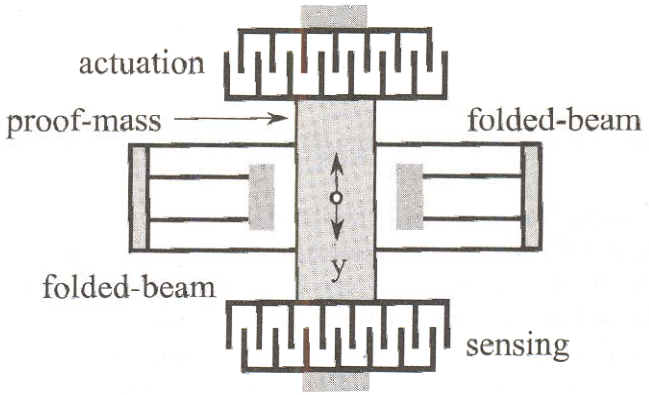
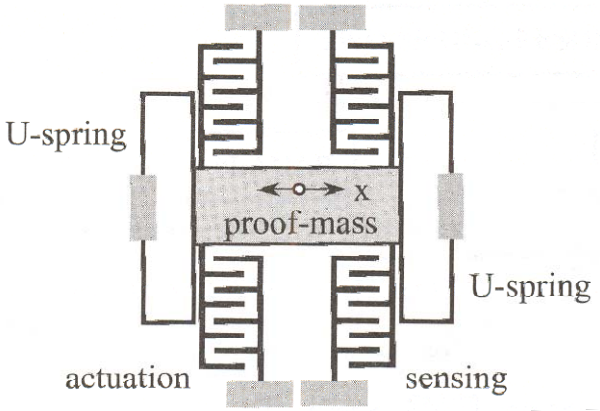
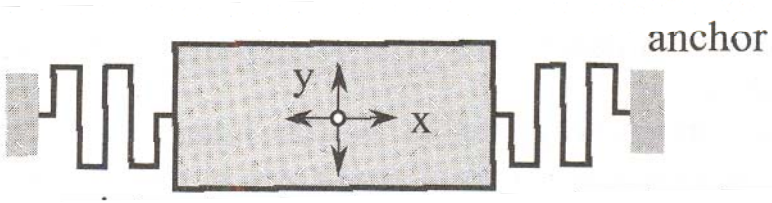
Comb Drive Design



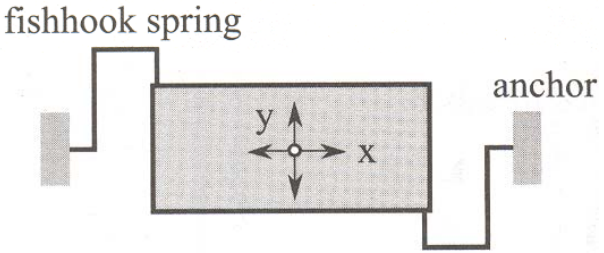
[U spring suspension type]



[Folded suspension type]



[Serpentine suspension type]



[Fishhook suspension type]

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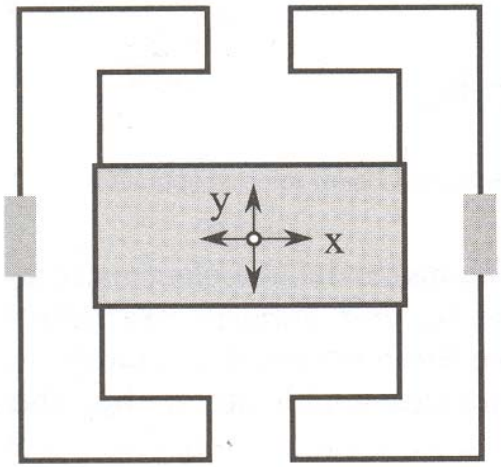
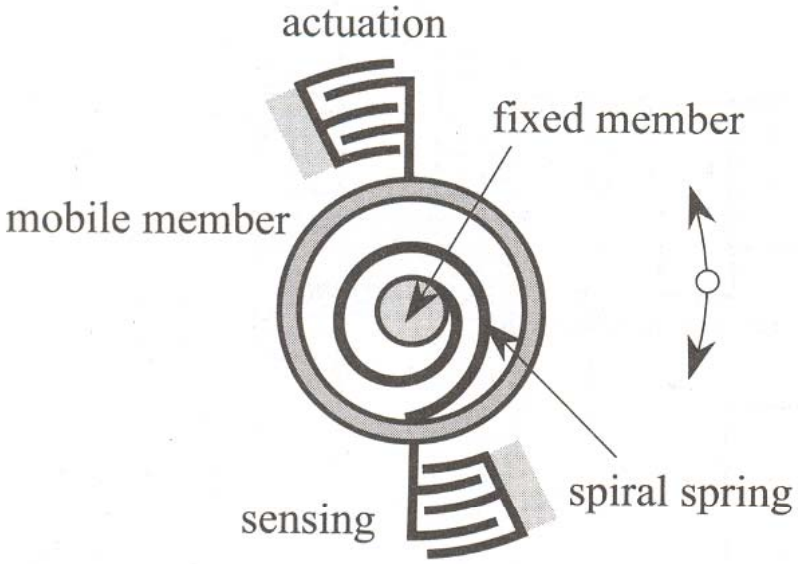
Comb Drive Design (cont'd)



[Bent beam serpentine suspension type]



[Spiral spring suspension type]

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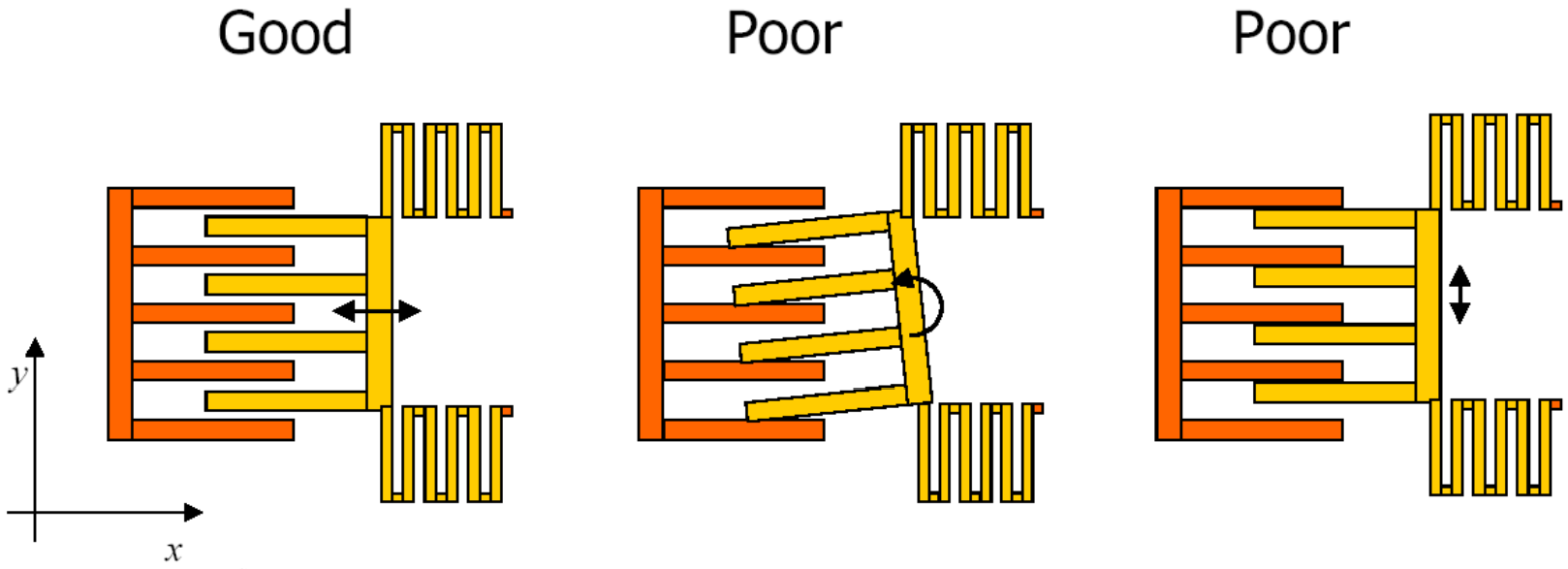
Comb Drive Failure Modes

- ***Comb drives require low stiffness in x direction but high stiffness in y, z direction as well as rotations.***

Note: comb fingers are in unstable equilibrium with respect to the y direction.



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Summary of Translation motion

- ***Acceleration, velocity, distance***

  $a = \dot{v} = \ddot{x}$

- ***Force, momentum***

  F

  $p = mv = Ft$

- ***Kinetic energy***

  $E = \frac{1}{2} mv^2$

- ***Dynamic (spring, damper, mass)***

  $F = Kx + b\dot{x} + m\ddot{x}$

- ***Oscillation (assume b=0)***

  $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad (Hz) \qquad \omega = \sqrt{\frac{K}{m}}$

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Summary of Rotation motion

- ***Angular acceleration, angular velocity, angle***

  $\alpha = \dot{\omega} = \ddot{\phi}$

- ***Torque, angular momentum***

  $\vec{T} = \vec{r} \times \vec{F}$

  $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

- ***Kinetic energy***

  $E = \frac{1}{2} I\omega^2$

- ***Dynamic (moment of inertia)***

  $T = K\phi + \beta\dot{\phi} + I\ddot{\phi}$

- ***Oscillation (assume b=0)***

  $f = \frac{1}{2\pi}\sqrt{\frac{K}{I}}$

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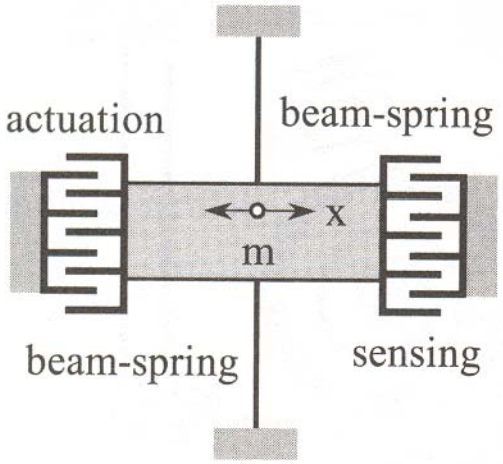
Lumped-parameter Model

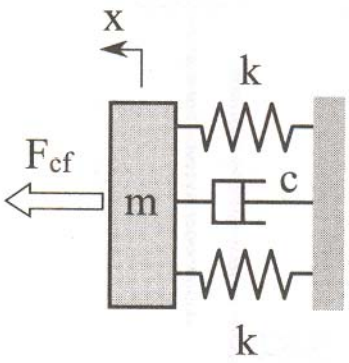
- The dynamic of motion is based on the lumped-parameter model
  - ***Input: external fore F, Output: displacement x***

$$m\ddot{x}(t) + b\dot{x}(t) + Kx(t) = F_0 \sin(wt)$$

where m : mass, b : damping, K : stiffness



Schematic microdevice



Lumped-parameter model

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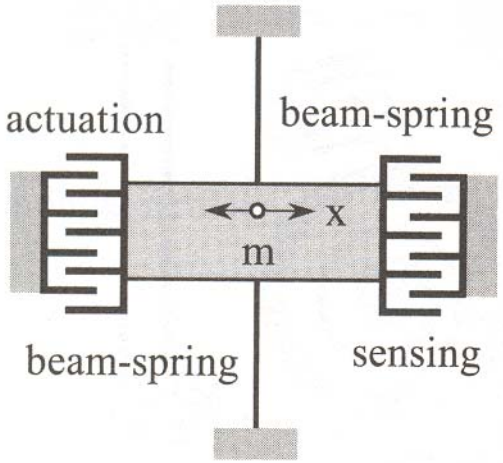
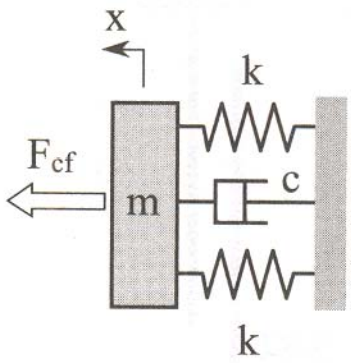
Lumped-parameter Model (cont'd)

- ***Transfer function:***

$$H(s) = \frac{x}{F} = \frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{K}{m}}$$



Schematic microdevice



Lumped-parameter model

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Resonators

- ***Analogy between mechanical and electrical system:***
  - Mass *m* – Inductivity *L*
  - Spring *K* – Capacitance *1/C*
  - Damping *b* – Resistance R

    (depending where R is placed in circuit)
- ***Solution to 2nd order differential equation:***

$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad \text{or} \quad \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

where $\omega_0 = 2\pi f_o$: natural frequency

$\omega_0 = \sqrt{\frac{K}{m}}$: mechanical system, $\omega_0 = \sqrt{\frac{1}{LC}}$: electrical system

Q: quality factor $Q = 1/2\xi$ (ξ : *damping ratio*)

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Mechanical Resonators

- ***Frequency and phase shift under damping:***

$$x(t) = Ae^{-t/2\tau}\cos(\omega_1 t + \varphi)$$

$$\text{where } \tau = \frac{m}{b} : \text{damping time}$$

$$\omega_1 = \omega_0\sqrt{1 - \frac{1}{4\omega_0{}^2\tau^2}} = \omega_0\sqrt{1 - \frac{b^2}{4Km}} = \omega_0\sqrt{1-\xi^2} = \omega_d$$

φ: phase shift

- ***Energy dissipation:*** $E(t) = E_0 e^{-t/\tau}$

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Quality Factor

- ***Definition: Quality factor (Q factor)***
  - Ratio of stored energy and lost energy:

$$Q = 2\pi \frac{E}{|\Delta E|} = 2\pi \frac{\tau}{T} = \omega_0 \tau$$

  - Mechanical system: $Q = \omega_0 \frac{m}{b} = \frac{\sqrt{Km}}{b}$

  - Similar for electric systems: (a) $Q = \omega_0 \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

(b) $Q = \omega_0 RC = R\sqrt{\frac{C}{L}}$

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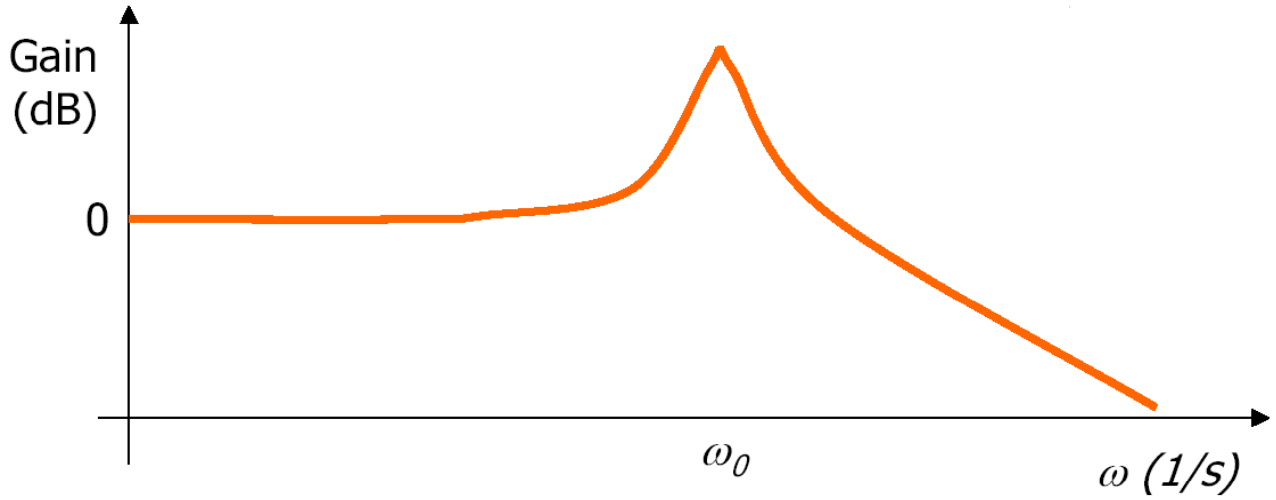
Quality Factor (cont'd)

- **How fast does energy dissipate?**

$$\tau = \frac{Q}{\omega_0} = \frac{m}{b} \text{ (mechanical)}$$

- **What is the maximum amplitude for a given frequency?**

  : At resonance, amplitude is Q times the DC response



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Summary: Mechanical/Electrical Resonator

- **Mechanical resonator :**

$$m\ddot{x}(t) + b\dot{x}(t) + Kx(t) = 0$$

m : mass, b: damping, K: stiffness

natural frequency $\omega_0 = \sqrt{K/m}$ (for small b)

- **Torsional resonator :**

$$I\ddot{\theta}(t) + b\dot{\theta}(t) + k\theta(t) = 0$$

I : moment of inertia, b: damping, k: stiffness

natural frequency $\omega_0 = \sqrt{k/I}$

- **Electrical resonator :**

$$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = 0$$

L : inductivity, R: resistance, C: capacitance

natural frequency $\omega_0 = \sqrt{1/LC}$

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Measurement

V_P : DC bias

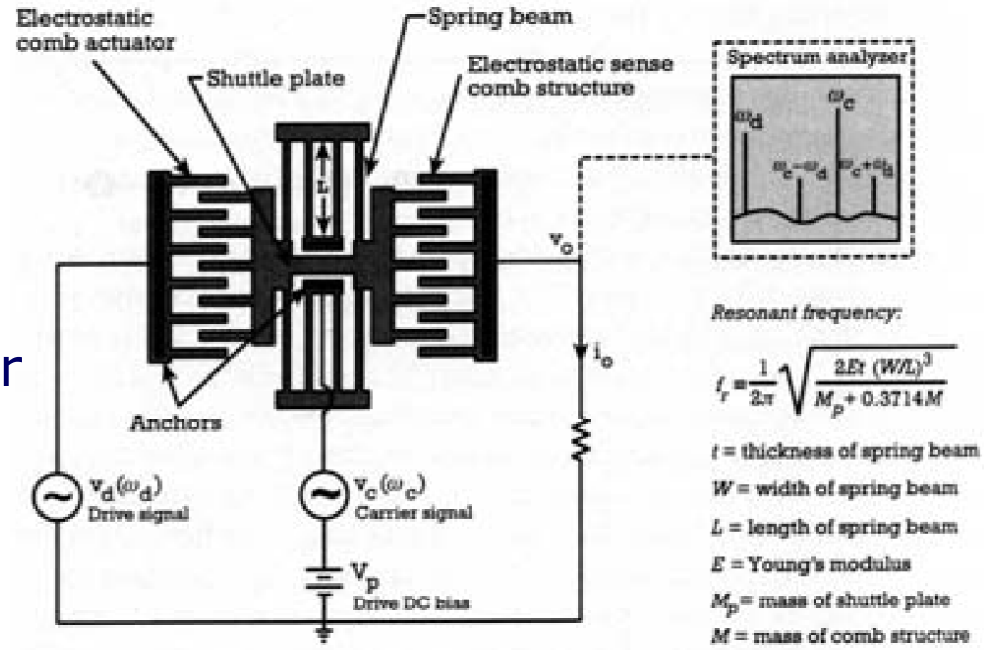
V_d : drive signal at ω_d

I_0 : output current $\propto V_P dC/dt$

Feedback

via transimpedance amplifier

Vc carrier signal at ω_c



Output signal:

frequency spectrum includes ω_d, ω_c, but also $\omega_c \pm \omega_d$

basis for frequency transfer

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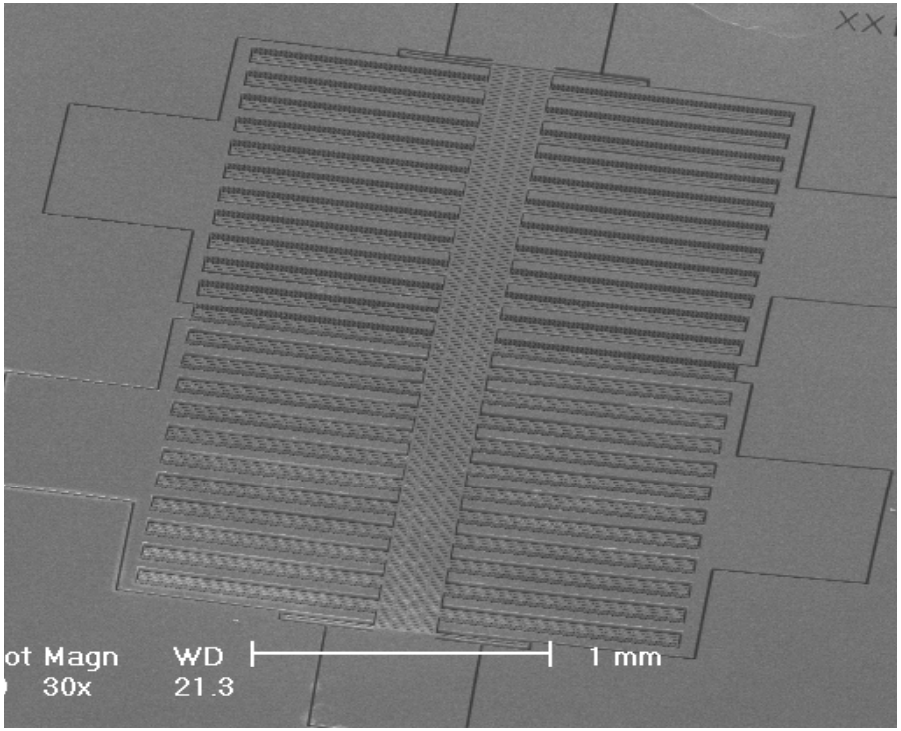
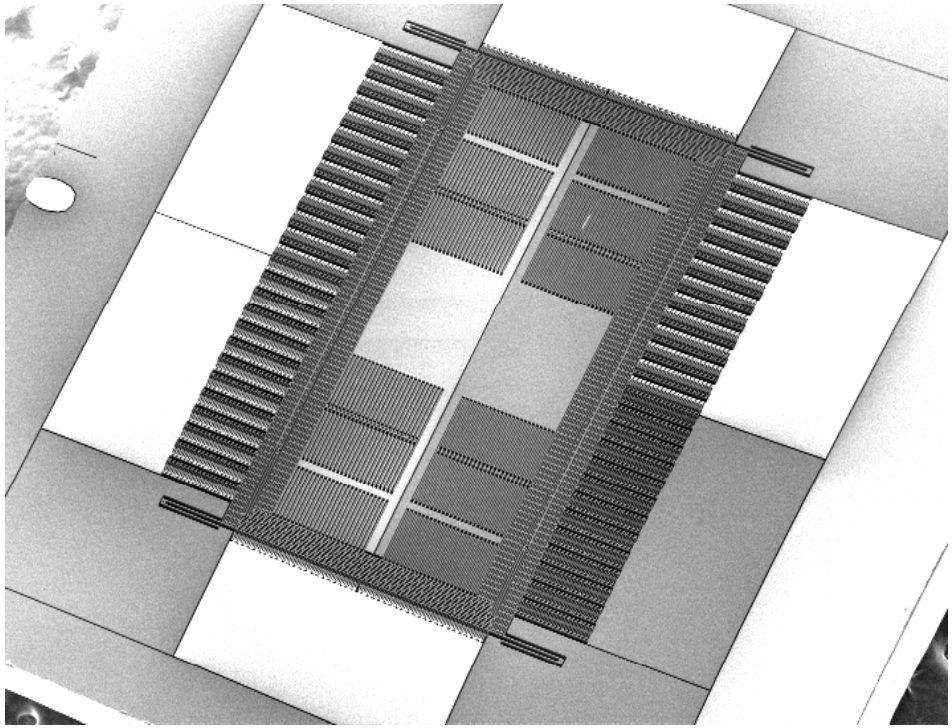
Application of Comb Drive



Accelerometer (NML SNU)



Gyroscope (NML SNU)

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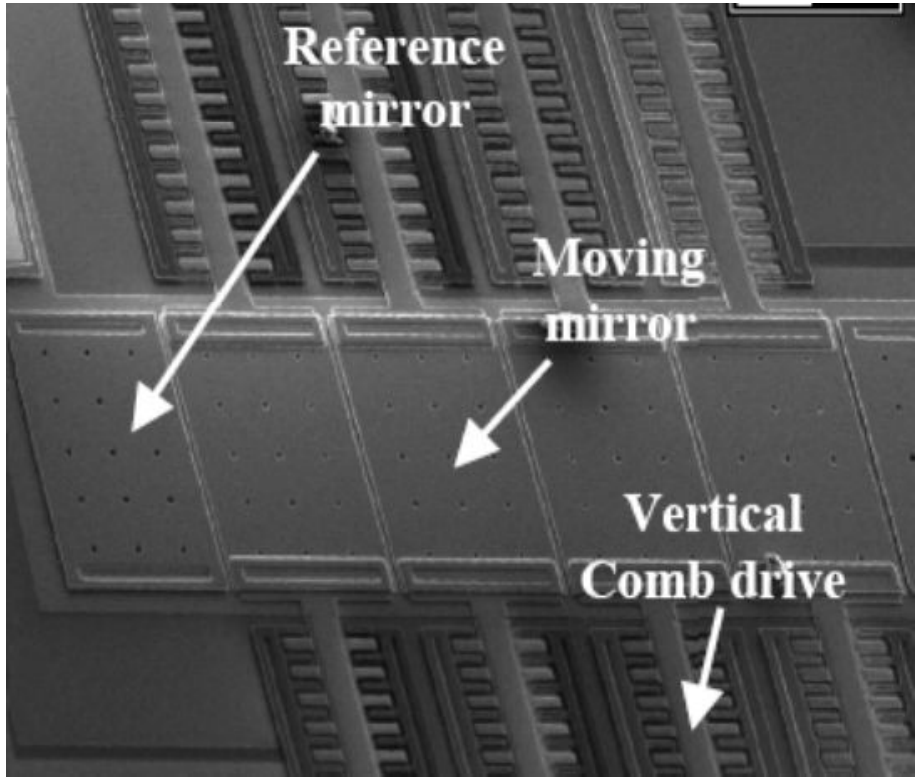
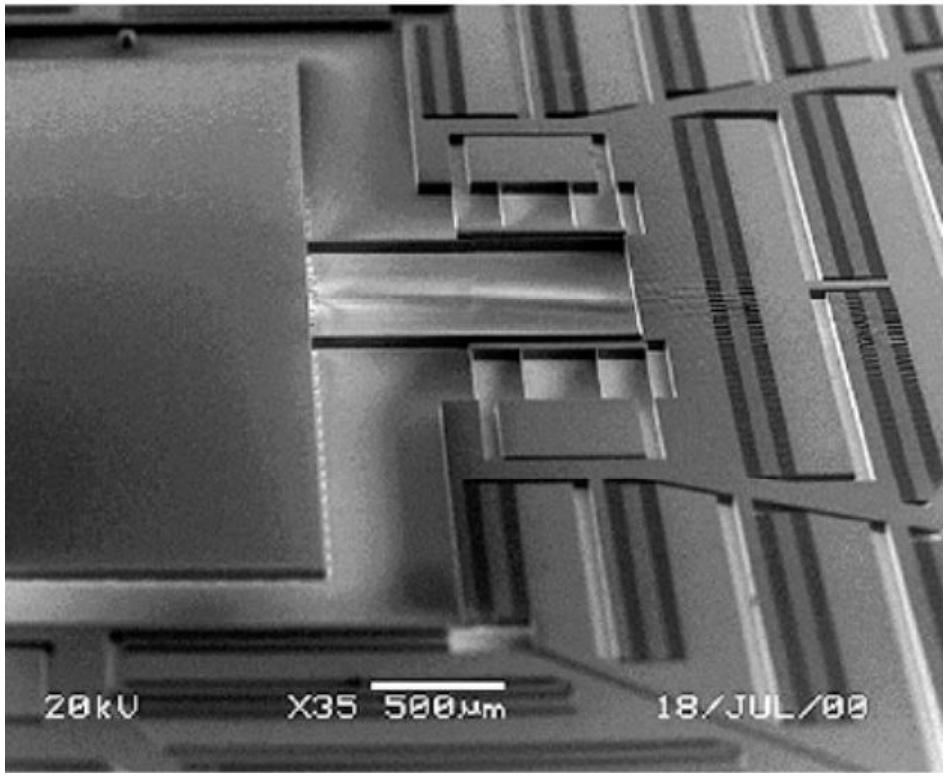
Application of Comb Drive



Micromirror (UC Davis & Stanford)



X-Y stage (MiSA SNU)

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