Ship Motion and Wave Load
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이규열

Advanced
Ship
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Laboratory

학습 목표 (왜 배우는가? 어디에 쓰는가?)

- 선박의 6자유도 운동(가속도, 속도, 변위)을 구함으로써 외부에서 주파수 w인 파가 올 때, 선박의 거동 확인
- 해양파에 의한 동적인 힘과 모멘트가 구조 설계에 반영됨

배울 내용

- 선박의 6자유도 운동 방정식 유도
- 6자유도 운동 방정식에 필요한 외력을 Laplace Equation[1)]과 Bernoulli Equation[2)]으로 부터 구함
- Hydrodynamic Force[3)]를 구하는 방법
  - Step1 : 2-D 단면의 velocity potential을 계산하고, 이로부터 hydrodynamic Force를 계산하는 방법 (Singularity distribution method)
  - Step2 : 2-D 단면에서 계산된 hydrodynamic Force를 3차원으로 확장하는 방법 (Strip method)

1) Laplace Equation : $\nabla^2 \Phi = 0$

2) Bernoulli Equation : $\rho \dfrac{\partial \Phi}{\partial t} + P + \dfrac{1}{2}\rho \left|\nabla \Phi\right|^2 + \rho g\, z = C$

3) Potential Virtual Inertial Force (“Added mass”),
Potential Wave Damping Force
Wave Exciting Force

참고 자료

- Text book

1) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997
2) Bhattacharyya, R. , Dynamics of Marine Vehicles, John Wiley & Sons, 1978
3) Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998
4) 이승건, 선박운동 조종론, 부산대학교 출판부, 2004
5) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001 (http://www.shipmotions.nl/index.html)
6) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program “ Seaway for Windows” , Delft University of Technology, 2003 (http://www.shipmotions.nl/index.html)
7) Tommy Pedersen, Wave Load Prediction – a Design Tool, PhD thesis, Department of naval architecture and offshore engineering, 2000
8) Cengel & Cimbala, Fluid Mechanics Mc Graw Hill,2005
9) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005

DOF : Degree Of Freedom

6DOF Equations of Motion

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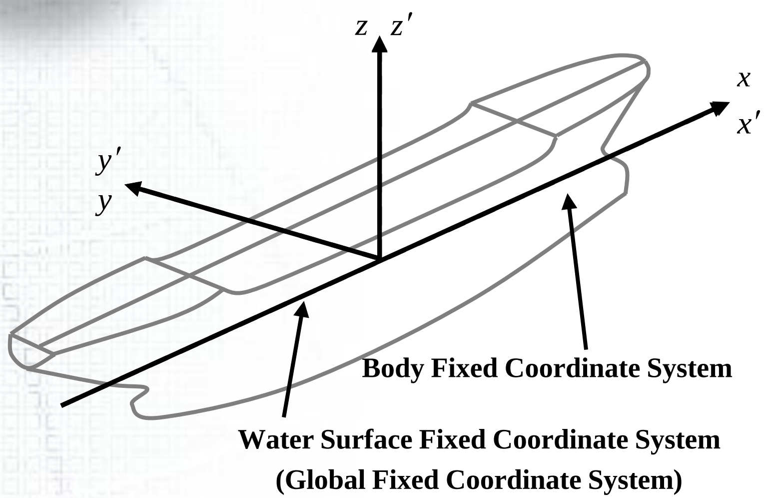
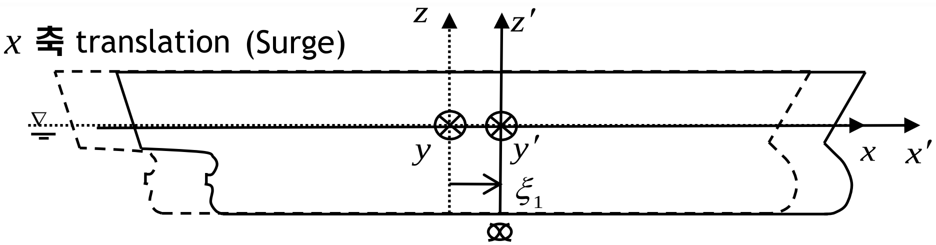
Coordinate System

x' 축 - 원점: Midship, (+): 선수
y' 축 - 원점: Centerline, (+): 좌현
z' 축 - 원점: 수선면, (+): 선박의 위

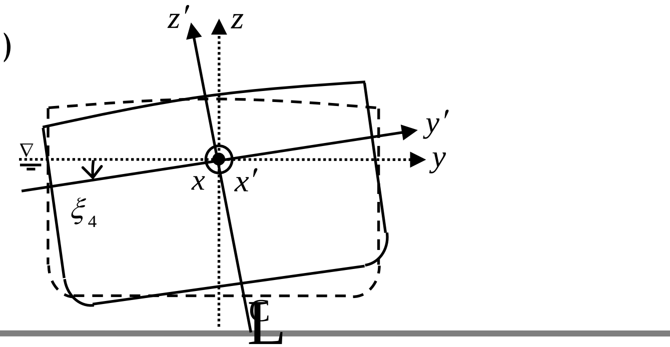
x 축 - 원점: Midship, (+): x'축을 포함하고 수선면과 직교인 평면과 수선면 사이의 교선
y 축 - 원점: Centerline, (+): z 축과 x 축의 외적 방향
z 축 - 원점: 수면, (+): 수선면에 수직한 위 방향



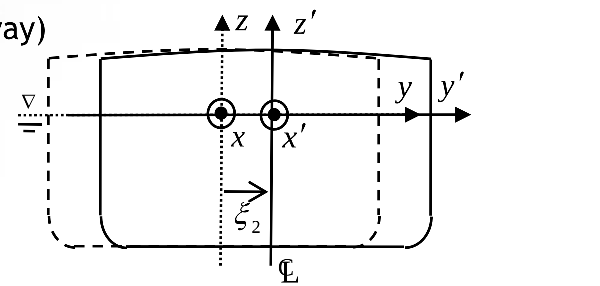
x 축 translation (Surge)



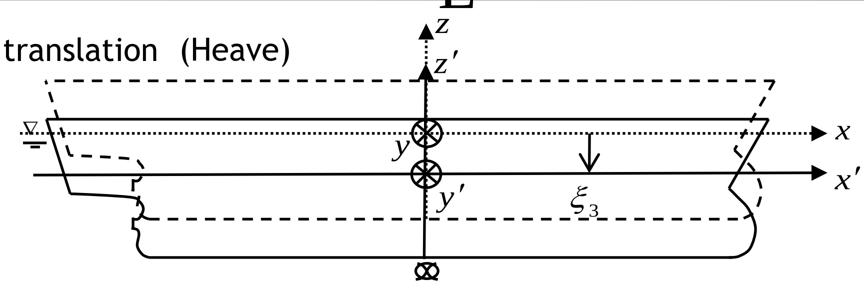
x 축 rotation (Roll)



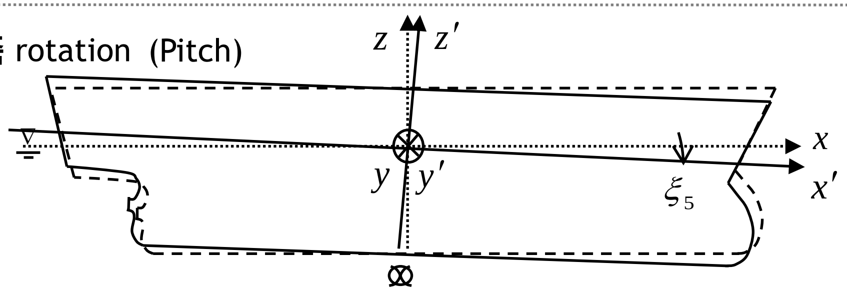
y 축 translation (Sway)



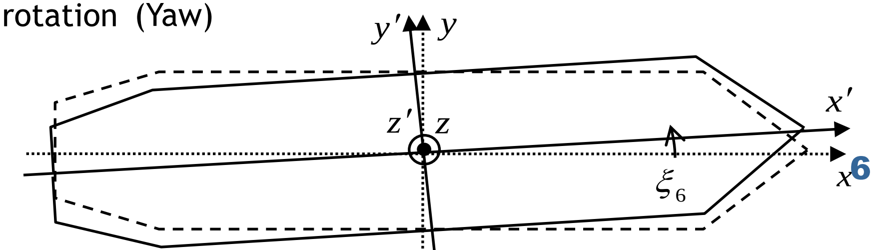
z 축 translation (Heave)



y 축 rotation (Pitch)



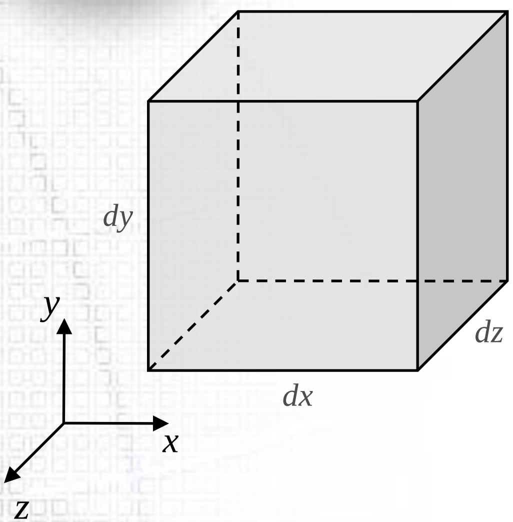
z 축 rotation (Yaw)



1) Cengel & Cimbala, Fluid Mechanics Mc Graw Hill,2005, p396~401

Cauchy Equation[1] 유도

미소 유체 요소



✓ **미소 유체 요소가 받는 힘** (Newton's 2nd Law)

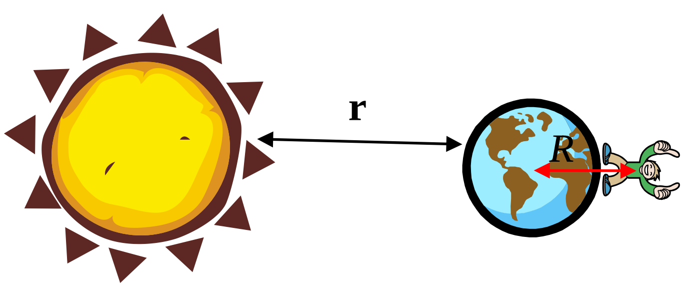
질량 X 가속도

$$m\frac{d\mathbf{V}}{dt} = \sum \mathbf{F} = \mathbf{F}_{Body} + \mathbf{F}_{Surface}$$

(체적력 + 표면력)

만유인력에 의한 힘

(질량이 있는 물체간에 서로를 끌어당기는 힘)



유체 중을 움직일 때 표면에 작용하는 힘

유체의 종류에 따라 다르지만, 저항하는 힘을 느낌

(Q) **공기중에서도 표면력이 있는가?**

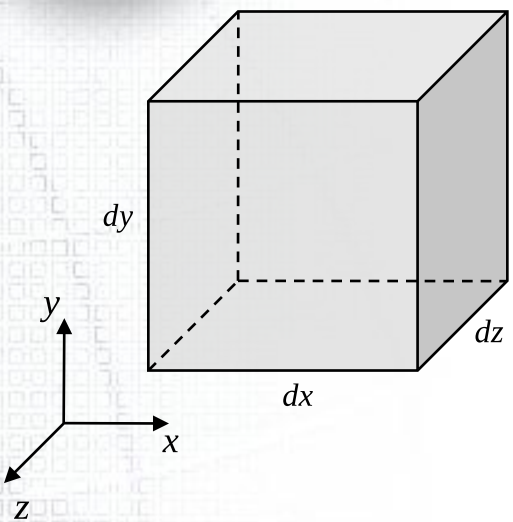
(A) **있다.**

(**밀도가 작아서 작용하는 힘을 못 느낄 뿐**)

1) Cengel & Cimbala, Fluid Mechanics Mc Graw Hill,2005, p396~401

Cauchy Equation[1] 유도

미소 유체 요소



✓ **미소 유체 요소가 받는 힘** (Newton's 2nd Law)

$$m\frac{d\mathbf{V}}{dt} = \rho\left(\frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}\right)dxdydz \;,\left(m = \rho dxdydz\right)$$

$$m\frac{d\mathbf{V}}{dt} = \sum \mathbf{F} = \mathbf{F}_{Body} + \mathbf{F}_{Surface}$$ **(체적력 + 표면력)**

$$\mathbf{F}_{Body} = \rho\mathbf{g}\,dxdydz$$

$$\mathbf{F}_{Surface} = \left[\nabla \bullet \sigma_{ij}\right]dxdydz$$

대입

$dxdydz$ **로 나누면**

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}\right)\cancel{dxdydz} = \rho\,\mathbf{g}\,\cancel{dxdydz} + \left[\nabla \bullet \sigma_{ij}\right]\cancel{dxdydz}$$

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}\right) = \rho\,\mathbf{g} + \nabla \bullet \sigma_{ij}$$ => Cauchy Equation

Summary (I)

※ 현재까지의 가정 정리

① 뉴턴 유체 (Newtonian fluid)

② 비압축성 유동 (Incompressible flow)

③ 비점성 유동 (Invicid flow)

④ 비회전 유동 (Irrotational flow)

Cauchy Equation : $$\rho \frac{d\mathbf{V}}{dt} = \rho\,\mathbf{g} + \nabla \bullet \sigma_{ij}$$

↓

Navier-Stokes Equation : $$\rho \frac{d\mathbf{V}}{dt} = \rho\,\mathbf{g} - \nabla P + \mu \nabla^2 \mathbf{V}$$

↓

Euler Equation : $$\rho \frac{d\mathbf{V}}{dt} = \rho\mathbf{g} - \nabla P$$

↓

Bernoulli Equation :

$$\rho \frac{\partial \Phi}{\partial t} + \frac{1}{2}\rho \left|\nabla \Phi\right|^2 + P + \rho\, gz = f(t)$$

Summary (II)

Φ : Velocity potential
μ : 점성 계수
P : 압력
$\mathbf{V}$: 유체의 속도

1) Cengel & Cimbala, Fluid Mechanics Mc Graw Hill,2005, p396~401
2) Cengel & Cimbala, Fluid Mechanics Mc Graw Hill,2005, p401~406
3) Cengel & Cimbala, Fluid Mechanics, Mc Graw Hill,2005, p450~452
4) Cengel & Cimbala, Fluid Mechanics, Mc Graw Hill,2005, p134~135
5) Cengel & Cimbala, Fluid Mechanics, Mc Graw Hill,2005, p179~182
6) Cengel & Cimbala, Fluid Mechanics, Mc Graw Hill,2005, p167~172
7) Erwin Kreyszig,Advanced Engineering Mathematics, Wiley,Ch12.PDE

Newton's 2nd Law

$$m\mathbf{a} = \sum \mathbf{F} = \text{(Body Force) + (Surface Force)}$$

⇓

Cauchy Equation[1]

$$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} + \nabla \bullet \sigma_{ij}$$

⇓ **뉴턴 유체, 비압축성**(incompressible)**이라면,**
Surface force**를 속도성분으로 표현 가능**

$$\left(\nabla \bullet \sigma_{ij} = -\nabla P + \mu \nabla^2 \mathbf{V}\right)$$

적용

Navier-Stokes Equation[2]

$$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} - \nabla P + \mu \nabla^2 \mathbf{V}$$

⇓ $\mu = 0$ (invicid)

Euler Equation[3]

$$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} - \nabla P$$

⇓ $\nabla \times \mathbf{V} = 0$ (irrotational[4])

$$\left(\mathbf{V} = \nabla \Phi\right)$$

Bernoulli Equation[5]

$$\rho \frac{\partial \Phi}{\partial t} + P + \frac{1}{2}\rho \left|\nabla \Phi\right|^2 + \rho g z = C$$

Continuity Equation[6] (**질량보존**)

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \rho \mathbf{V} = 0$$

⇓ (incompressible)

$$\nabla \bullet \mathbf{V} = 0$$

⇓ $\nabla \times \mathbf{V} = 0$ (irrotational[4])

$$\left(\mathbf{V} = \nabla \Phi\right)$$

$$\nabla^2 \Phi = 0$$

$$\left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0\right)$$

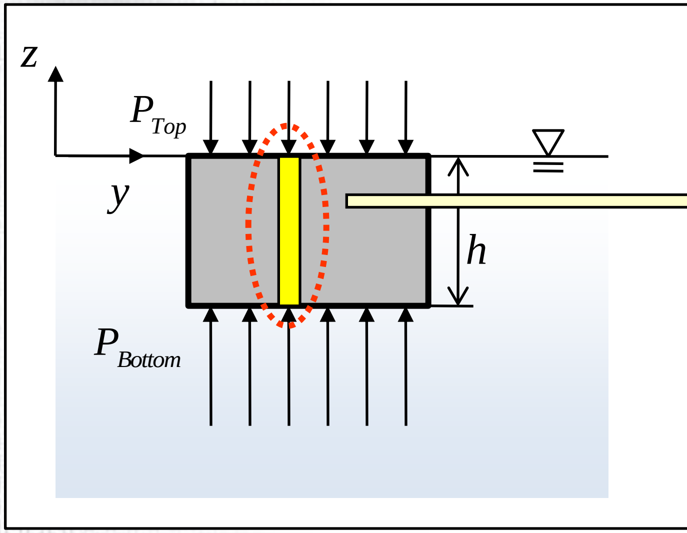
Laplace Equation[7]

유도 과정 중에 continuity Equation**이 사용되었으므로,** Bernoulli Equation**은 반드시** Laplace Equation**을 만족해야 한다.**

압력

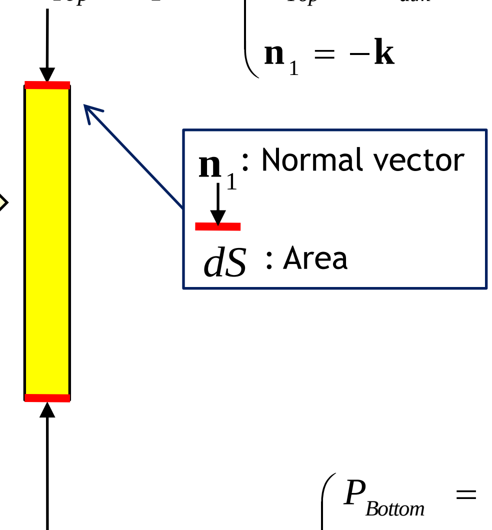
※ 압력(Pressure) : 단위 면적에 수직으로 작용하는 힘
즉, 힘을 구하기 위해서는 압력에 면적과
그 작용면의 법선 벡터(Normal Vector)를 곱해야 함

✓ 아래 물체에 작용하는 수직방향의 정적인 힘은?



: 물체 윗면의 미소 면적에 작용하는 힘

$$dF_{Top} = P_{Top} \cdot \mathbf{n}_1 dS \quad \begin{pmatrix} P_{Top} = P_{atm} - \rho g \cdot 0 \\ \mathbf{n}_1 = -\mathbf{k} \end{pmatrix}$$



$$dF_{Bottom} = P_{Bottom} \cdot \mathbf{n}_2 dS \quad \begin{pmatrix} P_{Bottom} = P_{atm} - \rho g h \\ \mathbf{n}_2 = \mathbf{k} \end{pmatrix}$$

: 물체 아랫면의 미소 면적에 작용하는 힘

$$\begin{aligned} dF &= dF_{Top} + dF_{Bottom} \\ &= P_{Top} \cdot \mathbf{n}_1 dS + P_{Bottom} \cdot \mathbf{n}_2 dS \\ &= \cancel{P_{atm}}(-\mathbf{k})dS + (\cancel{P_{atm}} - \rho g h)\mathbf{k} dS \\ &= -\rho g h \, \mathbf{k} dS = \mathbf{k}(-\rho g h \cdot dS) \end{aligned}$$

: 대기압에 의한 힘이 서로 상쇄됨

✓ Bernoulli Equation : $\left(where \;\; P_{Static} = P_{atm} + P_{Fluid} \right)$

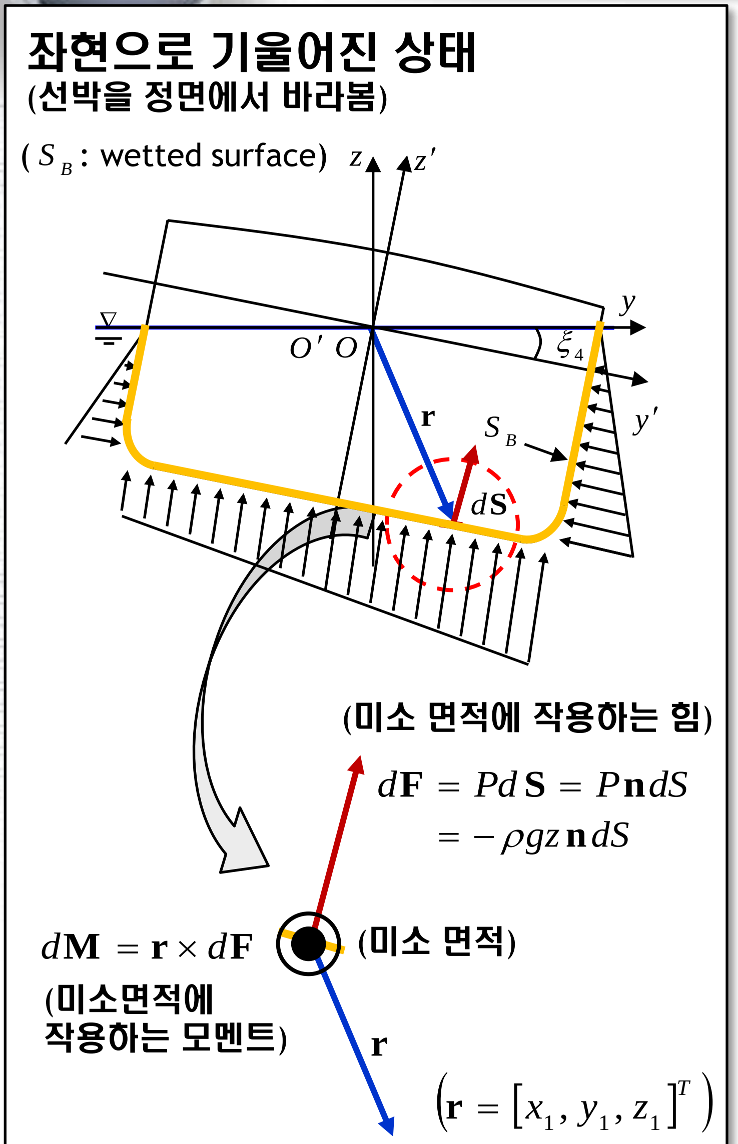
$$\rho \frac{\partial \Phi}{\partial t} + P + \frac{1}{2}\rho \left| \nabla \Phi \right|^2 + \rho g z = P_{atm}$$

$$\rho \frac{\partial \Phi}{\partial t} + \left(P_{atm} + P_{Fluid}\right) + \frac{1}{2}\rho \left| \nabla \Phi \right|^2 + \rho g z = P_{atm}$$

$$\boxed{\rho \frac{\partial \Phi}{\partial t} + P_{Fluid} + \frac{1}{2}\rho \left| \nabla \Phi \right|^2 + \rho g z = 0}$$

Force & moment acting on the surface

(S_B : wetted surface)



- Force : 표면에 작용하는 모든 힘을 적분하여 구함
  - ✓ 미소 면적에 작용하는 단위 길이당 힘 :

$$d\mathbf{F} = P \cdot d\mathbf{S} = P \cdot \mathbf{n}dS = -\rho gz \cdot \mathbf{n}dS$$

  - ✓ Total force

$$\mathbf{F} = \iint_{S_B} P\mathbf{n}dS$$

- Moment : (모멘트)=(거리) X (힘)
  - ✓ 미소 면적에 작용하는 단위 길이당 모멘트 :

$$d\mathbf{M} = \mathbf{r} \times d\mathbf{F} = \mathbf{r} \times P\mathbf{n}dS = (\mathbf{r} \times \mathbf{n})PdS$$

  - ✓ Total moment

$$\mathbf{M} = \iint_{S_B} P(\mathbf{r} \times \mathbf{n})dS$$

왜 r이 먼저 오는가? (좌표축에서 양의 방향을 고려함)

Notation

$$\left(\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ n_1 & n_2 & n_3 \end{vmatrix} = \mathbf{i}(y_1 n_3 - z_1 n_2) + \mathbf{j}(z_1 n_1 - x_1 n_3) + \mathbf{k}(x_1 n_2 - y_1 n_1) \right)$$

✓ Total force

$$\mathbf{F} = \iint_{S_B} P\mathbf{n}\,dS$$

성분별로 나눠쓰면,

$$\left(\begin{aligned} F_1 &= \iint_{S_B} P n_1 dS \\ F_2 &= \iint_{S_B} P n_2 dS \\ F_3 &= \iint_{S_B} P n_3 dS \end{aligned} \right)$$

$$(j = 1, \cdots, 6)$$

$$\boxed{F_j = \iint_{S_B} P n_j dS}$$

✓ Total moment

$$\mathbf{M} = \iint_{S_B} P(\mathbf{r} \times \mathbf{n}) dS$$

$$\left(\mathbf{r} = [x_1, y_1, z_1]^T\right)$$

성분별로 나눠쓰면,

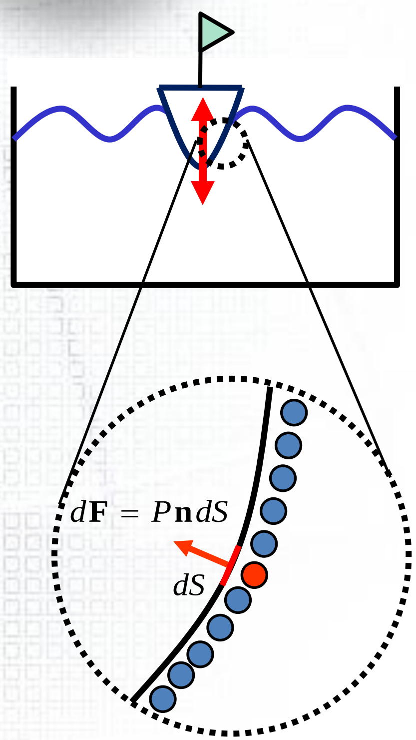
$$M_1 = \iint_{S_B} P(y_1 n_3 - z_1 n_2) dS \quad \left(F_4 = \iint_{S_B} P n_4 dS \right)$$

$$M_2 = \iint_{S_B} P(z_1 n_1 - x_1 n_3) dS \quad \left(F_5 = \iint_{S_B} P n_5 dS \right)$$

$$M_3 = \iint_{S_B} P(x_1 n_2 - y_1 n_1) dS \quad \left(F_6 = \iint_{S_B} P n_6 dS \right)$$

운동 방정식 유도 – 선박에 작용하는 힘

(변위 : $\mathbf{x} = [\xi_1, \cdots, \xi_6]^T$)
($\mathbf{M}, \mathbf{A}, \mathbf{B}, \mathbf{C}$: 6×6 *Matrix*)



$d\mathbf{F}$: 하나의 유체 입자가 선박 표면에 가하는 힘

dS : 미소 면적

$\mathbf{n}$: 미소 면적의 Normal 벡터

✓ **유체 압력** (From Bernoulli Eq.)

$$\rho \frac{\partial \Phi}{\partial t} + P + \cancel{\frac{1}{2}\rho |\nabla \Phi|^2} + \rho g z = 0$$

✓ Laplace Equation

$$\nabla^2 \Phi = 0$$

Solve

$$\Phi_T = \Phi_I + \Phi_D + \Phi_R$$

선형화

$$P = -\rho g z - \rho \frac{\partial \Phi_T}{\partial t} = -\rho g z - \rho \left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \Phi_D}{\partial t} + \frac{\partial \Phi_R}{\partial t} \right)$$

$$= P_{static} + P_{F.K} + P_D + P_R$$

유체 입자 하나가 표면에 주는 압력

선박의 침수 표면 전체에 대하여 적분
(유체에 의해 선박이 받는 힘과 모멘트)

$$\iint_{S_B} P\mathbf{n}dS$$

2-D → 3-D (Strip method)

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{Gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

Linearization

$\mathbf{F}_{Restoring}$ $(= -\mathbf{C}\mathbf{x})$ $\quad$ $\mathbf{F}_{exciting}$ $\quad$ $\mathbf{F}_R = -\mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$

added mass $\quad$ Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = -\mathbf{C}\mathbf{x} + \mathbf{F}_{exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

Motion RAO (Response Amplitude Operator)

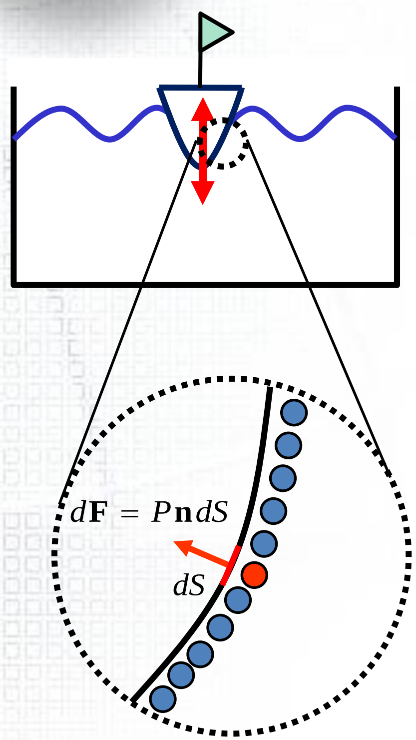
임의의 길이 x까지만 적분
(선박의 내부에 작용하는 S.F / B.M. 구함)

Shear force, Bending moment

운동 방정식 유도 – 선박에 작용하는 힘

(변위 : $\mathbf{x} = [\xi_1, \cdots, \xi_6]^T$)

($\mathbf{M}, \mathbf{A}, \mathbf{B}, \mathbf{C}$: 6×6 *Matrix*)



$d\mathbf{F}$: 하나의 유체 입자가 선박 표면에 가하는 힘

dS : 미소 면적

$\mathbf{n}$: 미소 면적의 Normal 벡터

✓ **유체 압력** (From Bernoulli Eq.)

$$\rho \frac{\partial \Phi}{\partial t} + P + \cancel{\frac{1}{2}\rho |\nabla \Phi|^2} + \rho g z = 0$$

선형화

$$P = -\rho g z - \rho \frac{\partial \Phi_T}{\partial t} = -\rho g z - \rho \left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \Phi_D}{\partial t} + \frac{\partial \Phi_R}{\partial t} \right)$$

$$= P_{static} + P_{F.K} + P_D + P_R$$

유체 입자 하나가 표면에 주는 압력

✓ Laplace Equation **Step1**

$$\nabla^2 \Phi = 0$$

Solve

$$\Phi_T = \Phi_I + \Phi_D + \Phi_R$$

선박의 침수 표면 전체에 대하여 적분 (유체에 의해 선박이 받는 힘과 모멘트) **Step2**

$$\iint_{S_B} P\mathbf{n}dS$$

2-D → 3-D (Strip method)

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{Gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

Linearization

$\mathbf{F}_{Restoring}$ $(= -\mathbf{C}\mathbf{x})$ $\qquad$ $\mathbf{F}_{exciting}$ $\qquad$ $\mathbf{F}_R = -\mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$

added mass $\qquad$ Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = -\mathbf{C}\mathbf{x} + \mathbf{F}_{exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$ **Step3**

Motion RAO (Response Amplitude Operator)

임의의 길이 x까지만 적분 (선박의 내부에 작용하는 S.F / B.M. 구함) **Step4**

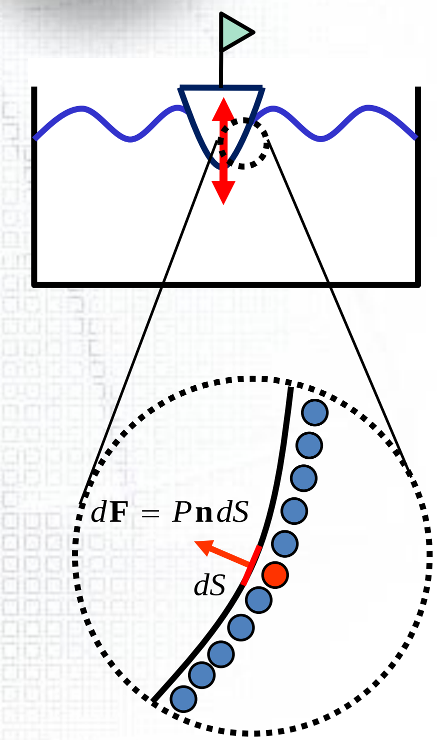
Shear force, Bending moment

Step1.
Velocity potential

운동 방정식 유도 – 선박에 작용하는 힘

(변위 : $\mathbf{x} = [\xi_1, \cdots, \xi_6]^T$)

($\mathbf{M}, \mathbf{A}, \mathbf{B}, \mathbf{C}$: 6×6 *Matrix*)



$d\mathbf{F}$: 하나의 유체 입자가 선박 표면에 가하는 힘

dS : 미소 면적

$\mathbf{n}$: 미소 면적의 Normal 벡터

✓ 유체 압력 (From Bernoulli Eq.)

$$\rho\frac{\partial\Phi}{\partial t} + P + \cancel{\frac{1}{2}\rho|\nabla\Phi|^2} + \rho gz = 0$$

선형화

$$P = -\rho gz - \rho\frac{\partial\Phi_T}{\partial t} = -\rho gz - \rho\left(\frac{\partial\Phi_I}{\partial t} + \frac{\partial\Phi_D}{\partial t} + \frac{\partial\Phi_R}{\partial t}\right)$$

$$= P_{static} + P_{F.K} + P_D + P_R$$

유체 입자 하나가 표면에 주는 압력

✓ Laplace Equation **Step1**

$$\nabla^2\Phi = 0$$

Solve

$$\Phi_T = \Phi_I + \Phi_D + \Phi_R$$

선박의 침수 표면 전체에 대하여 적분
(유체에 의해 선박이 받는 힘과 모멘트)

$$\iint_{S_B} P\mathbf{n}dS$$

Step2

2-D → 3-D (Strip method)

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{Gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

Linearization

$\mathbf{F}_{Restoring}$ $(= -\mathbf{C}\mathbf{x})$ $\mathbf{F}_{exciting}$ $\mathbf{F}_R = -\mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$

added mass

Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = -\mathbf{C}\mathbf{x} + \mathbf{F}_{exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

Step3

Motion RAO (Response Amplitude Operator)

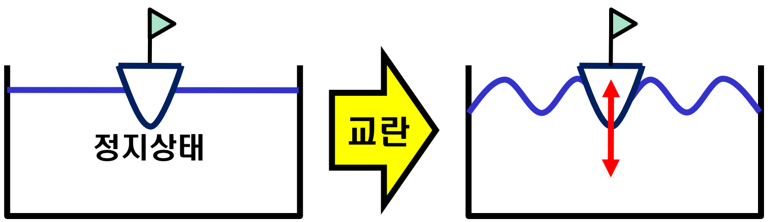
임의의 길이 x까지만 적분
(선박의 내부에 작용하는 S.F / B.M. 구함)

Step4

Shear force, Bending moment

1) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch 12.1 (pp 535~538)

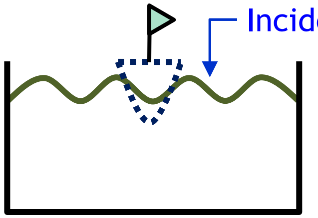
파랑 중 선박이 받는 힘



✓ **파랑 중 선박이 받는 힘**

: 유체장의 운동으로 인해 유체 입자의 속도,가속도,압력이 변하게 되고, 선박 표면의 유체 입자가 선박에 가하는 압력도 변하게 된다.

선형화[1)]된 힘으로 분해

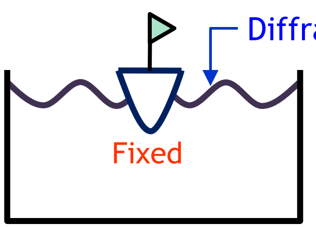


Incident wave velocity potential (Φ_I)

✓ **입사파가 선박에 의해 교란되지 않는다고 가정함**

➔ **입사파에 의한 힘** (Froude-Krylov Force)

+

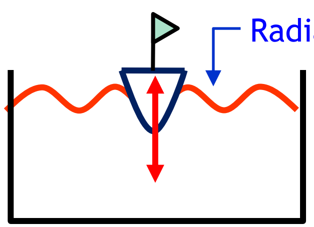


Diffraction wave velocity potential (Φ_D)

✓ **선박의 존재로 인하여 교란된 파에 의한 힘. 물체 고정**

➔ **산란파에 의한 힘** (Diffraction Force)

+



Radiation wave velocity potential (Φ_R)

✓ **정수 중에서 선박의 강제 진동으로 인해 작용하는 힘**

➔ **기진력에 의한 힘**(Radiation Force)

✓ Total Velocity Potential

$$\Phi_T = \Phi_I + \Phi_D + \Phi_R$$

Superposition Theorem

Laplace equation은 **선형 방정식이므로, 각의 해를 더한 것** (superposition)**도 해가 된다.**

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$

Assumption

Partial Differential Equation

$$\left(\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \right)$$

✓ **가정 정리 (유체)**

① **뉴턴 유체** (Newtonian fluid)

: **전단응력이 전단 변형률에 선형적으로 비례하는 유체**

② **비압축성 유동** (Incompressible flow)

③ **비점성 유동** (Invicid flow)

④ **비회전 유동** (Irrotational flow)

Laplace Equation

$\boxed{\nabla^2 \Phi = 0}$: Governing Equation

Bernoulli Equation

$$\rho \frac{\partial \Phi}{\partial t} + P + \frac{1}{2}\rho \left| \nabla \Phi \right|^2 + \rho g z = C$$

✓ **가정 정리 (Wave)**

⑤ Small amplitude water wave
(파장에 비해 파고가 작음)

⑥ Wave is periodic in space and time.

⑦ two dimensional water wave

✓ **가정 정리 (Ship)**

⑧ Resulting motion will be small

⑨ The hull is slender

⑩ Zero forward speed

⑩ The hull sections are wall-sided at the waterline

1) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 285~290
Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch 12.1 (pp 535~538)

Superposition of Velocity potential[1)]

Φ_I : Incident Wave V.P.
Φ_D : Diffraction V.P.
Φ_R : Radiation V.P.

✓ Decomposition of Velocity potential

$$\Phi_T(x,y,z,t) = \Phi_I(x,y,z,t) + \Phi_D(x,y,z,t) + \Phi_R(x,y,z,t)$$

$$= \underbrace{\{\phi_I(x,y,z) + \phi_D(x,y,z) + \phi_R(x,y,z)\}}_{\text{Time Independent Term (Complex)}} e^{i\omega t}$$

시간이 많이 지나 Steady 상태에서
Harmonic Motion
(Transient motion 고려안함)

Time Independent Term (Complex)

$$\Phi(x,y,z,t) = \mathrm{Re}\{\Phi_T(x,y,z,t)\}$$ (or Take an Imaginary term)

---

ex) If $\phi_I(x,y,z)$ is not a complex (real)

Let $\phi_I(x) = a$

$$\Phi_I = \phi_I(x)e^{i\omega t}$$ (Euler 공식)

$$= a(\cos\omega t + i\sin\omega t)$$

$$= a\cos\omega t + ia\sin\omega t$$

$$\mathrm{Re}\{\Phi_I\} = a\cos\omega t$$

If $\phi_I(x,y,z)$ is a complex

Let $\phi_I(x) = a + ib$

$$\Phi_I = \phi_I(x)e^{i\omega t}$$ (Euler 공식)

$$= (a+ib)(\cos\omega t + i\sin\omega t)$$

$$= (a\cos\omega t - b\sin\omega t) + i(b\cos\omega t + a\sin\omega t)$$

$$\mathrm{Re}\{\Phi_I\} = a\cos\omega t - b\sin\omega t = c\cos(\omega t - \varepsilon)$$

Phase가 나타남

1) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 285~290
Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch 12.1 (pp 535~538)

Superposition of Velocity potential[1)]

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✓ Decomposition of Velocity potential

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시간이 많이 지나 Steady 상태에서 Harmonic Motion (Transient motion 고려안함)

Time Independent Term (Complex)

파고(η_0)에 비례하는 Velocity potential

$$= \eta_0 \phi_I'(x,y,z) + \eta_0 \phi_D'(x,y,z) + \sum_{j=1}^{6} \xi_j^A \phi_j(x,y,z)$$

파고 운동변위의 크기

(η_0와 ξ_3^A는 주어지는 값)

※ $\phi_R(x,y,z) = \xi_1^A\phi_1 + \xi_2^A\phi_2 + \xi_3^A\phi_3 + \xi_4^A\phi_4 + \xi_5^A\phi_5 + \xi_6^A\phi_6 = \sum_{j=1}^{6} \xi_j^A \phi_j$

ϕ_j : 선박의 j방향 운동변위가 1일 때 Velocity Potential

ξ_j^A : 선박의 j방향 운동변위의 크기 (j = 4,5,6에서는 rotational angle in Radian)

ex) Heave 변위 0.5m, roll 변위 0.1rad 일 때,

$$\phi_R = 0.5\phi_3 + 0.1\phi_4$$

선박의 운동변위(Given)

$$\xi_j(t) = \xi_j^A e^{i\omega t}$$

크기(Amplitude)

$$\therefore \phi_T(x,y,z) = \phi_I(x,y,z) + \phi_D(x,y,z) + \sum_{j=1}^{6} \xi_j^A \phi_j(x,y,z)$$

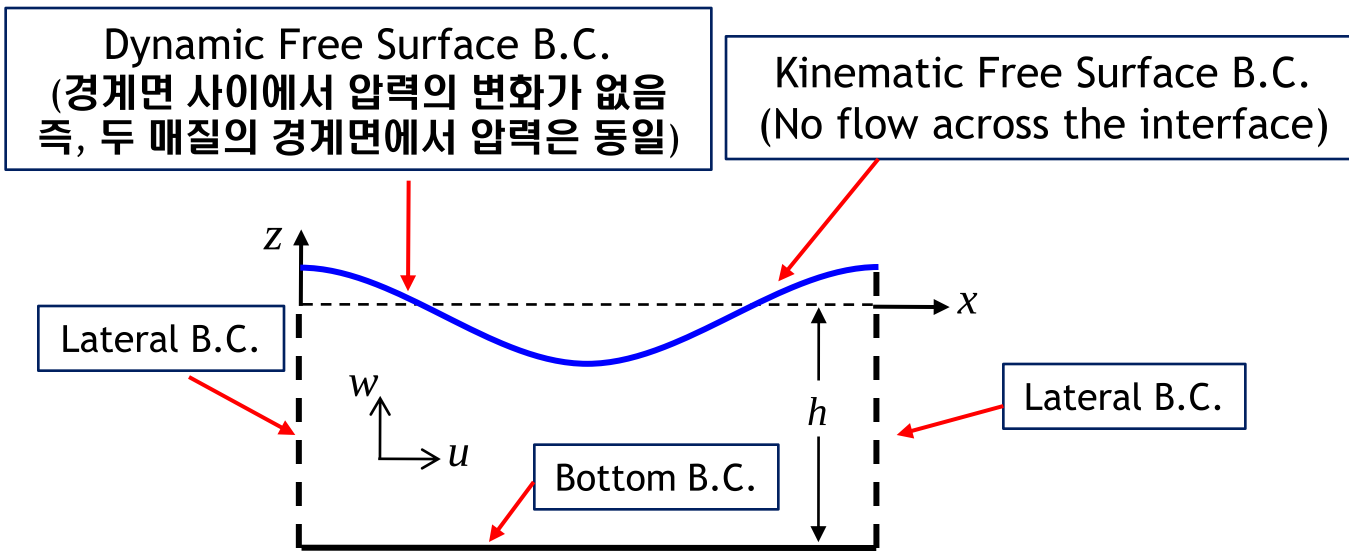
Incident Wave Velocity Potential (1)
: Boundary condition

Wave Equation

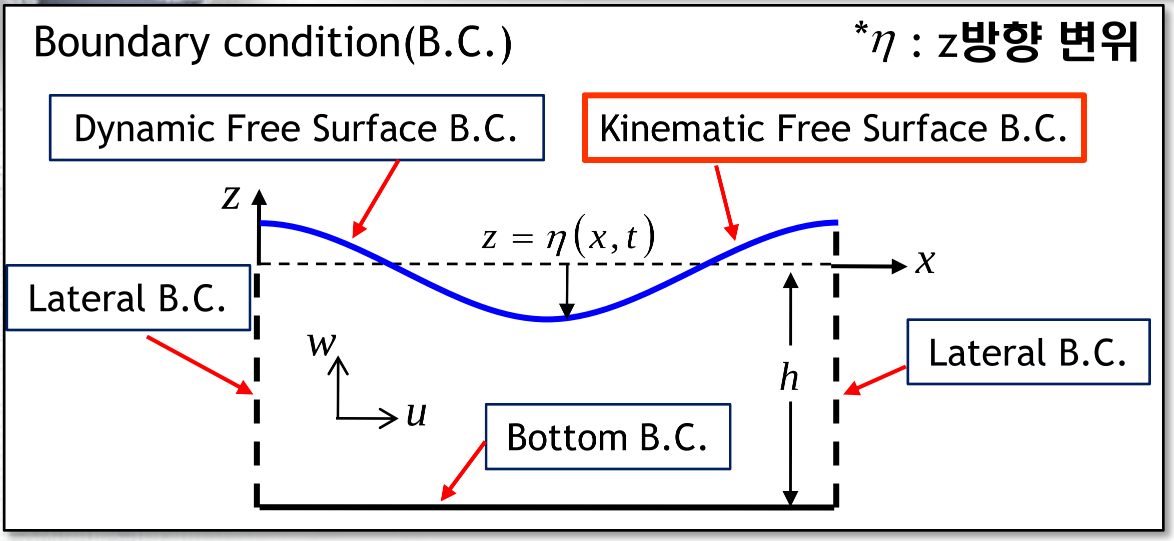
① Governing Equation :

$$\nabla^2 \Phi = 0$$

② Boundary condition(B.C.) :



Incident Wave Velocity Potential (2)
: Boundary condition



① Kinematic Free Surface B.C.(KFSBC)

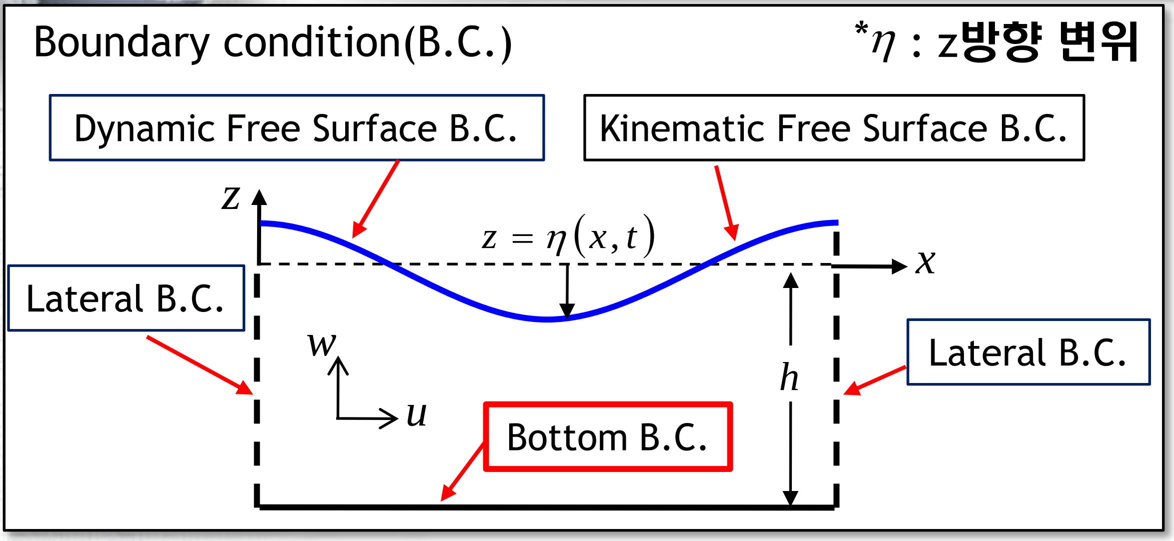
: **경계면 사이에 유동**(flow)**이 없기 위해서는 경계면에서 입자의 속도가 동일해야 한다.**

유체입자의 z 방향 속도 : $w = \dfrac{\partial \Phi}{\partial z}$

자유표면의 z 방향 속도 : $\dfrac{d\eta(x,t)}{dt} = \dfrac{\partial \eta}{\partial t} + \dfrac{\partial \eta}{\partial x}\dfrac{dx}{dt} = \dfrac{\partial \eta}{\partial t} + u\dfrac{\partial \eta}{\partial x} = \dfrac{\partial \eta}{\partial t} + \dfrac{\partial \Phi}{\partial x}\dfrac{\partial \eta}{\partial x}$

$$\therefore\ w = \frac{d\eta}{dt} -> \boxed{\frac{\partial \Phi}{\partial z} = \frac{\partial \eta}{\partial t} + \frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x}}$$

Incident Wave Velocity Potential (3)
: Boundary condition



② Bottom B.C. (BBC)

: **바닥면에서 유체가 스며들거나 바닥으로 침투하지 않는다면**(Impermeable) **다음 조건이 성립**

(바닥면의 속도) = (바닥면의 유체의 속도)

$$\therefore \left.\frac{\partial \Phi}{\partial n}\right|_{z=-h} = 0$$

→ **(좌변) : 바닥면은 고정되어 있으므로,**
(바닥면의 속도) = 0

→ **(우변) :**

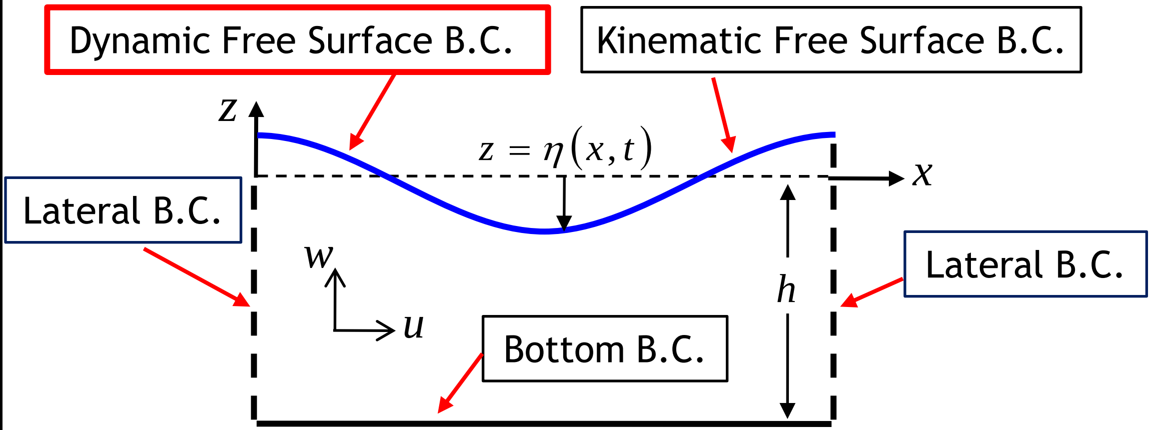
(바닥면 유체의 속도)$= \mathbf{V} \bullet \mathbf{n} = \left.\dfrac{\partial \Phi}{\partial n}\right|_{z=-h}$

만약, 바닥이 수심 z=-h**에서 평평하다고 가정하면**
(Horizontal bottom)

$$\boxed{\left.\frac{\partial \Phi}{\partial z}\right|_{z=-h} = 0}$$

Incident Wave Velocity Potential (4)
: Boundary condition

Boundary condition(B.C.) *η : z방향 변위



Bernoulli Equation

$$\rho\frac{\partial\Phi}{\partial t}+P+\frac{1}{2}\rho\left|\nabla\Phi\right|^2+\rho g\,z=C$$

③ Dynamic Free Surface B.C. (DFSBC) : **경계면에서 유체의 압력은 대기압과 같아야 함**

Wave**가 생성되기 전 상태를 고려하면,** $\mathbf{V}=\nabla\Phi=0,\ P=P_{atm}\quad(\text{on}\ \ z=0)$ **이므로,** $P_{atm}=C$

Wave**가 생성되었을 때**, Bernoulli Equation**에 의해 표면에서 유체의 압력은,**

$$\rho\frac{\partial\Phi}{\partial t}+P_{Surface}+\frac{1}{2}\rho\left|\nabla\Phi\right|^2+\rho g\,\eta=P_{atm}\quad(\text{on}\ \ z=\eta)$$

한편, 경계면에서 표면의 압력은 대기압과 같으므로,

$$\left(P_{Surface}=P_{atm}\quad(\text{on}\ \ z=\eta)\right)$$

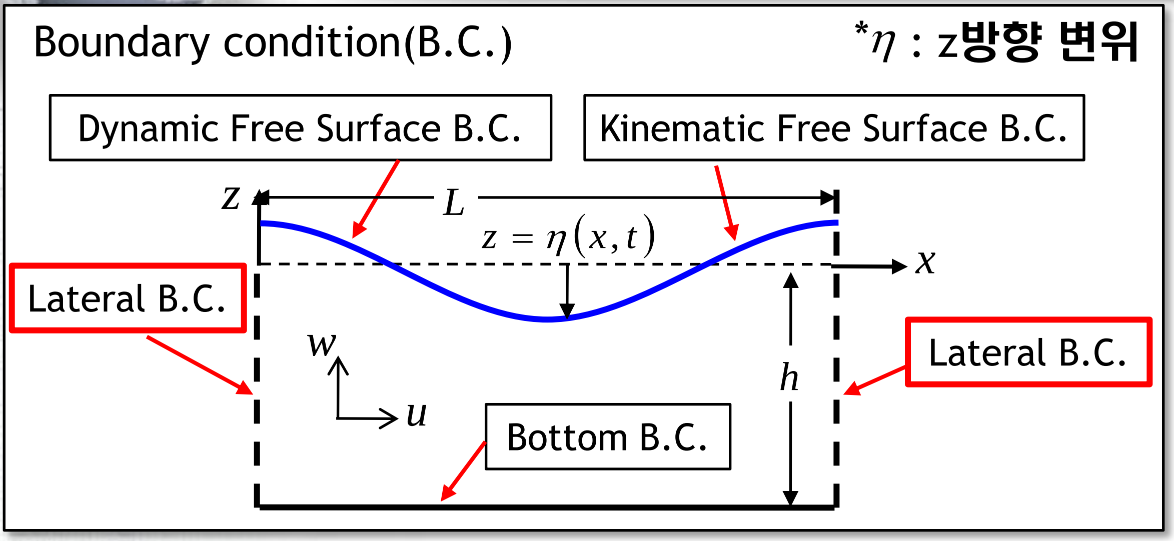
$$\rho\frac{\partial\Phi}{\partial t}+\cancel{P_{atm}}+\frac{1}{2}\rho\left|\nabla\Phi\right|^2+\rho g\,\eta=\cancel{P_{atm}}$$

양변을 ρ **로 나누면,**

$$\boxed{\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left|\nabla\Phi\right|^2+g\,\eta=0}$$

$$(\text{on}\ \ z=\eta)$$

Incident Wave Velocity Potential (5)
: Boundary condition



④ Lateral B.C. (DFSBC)

: **주기가 일정하다는 조건에 의해** Periodic lateral B.C**를 적용한다.**

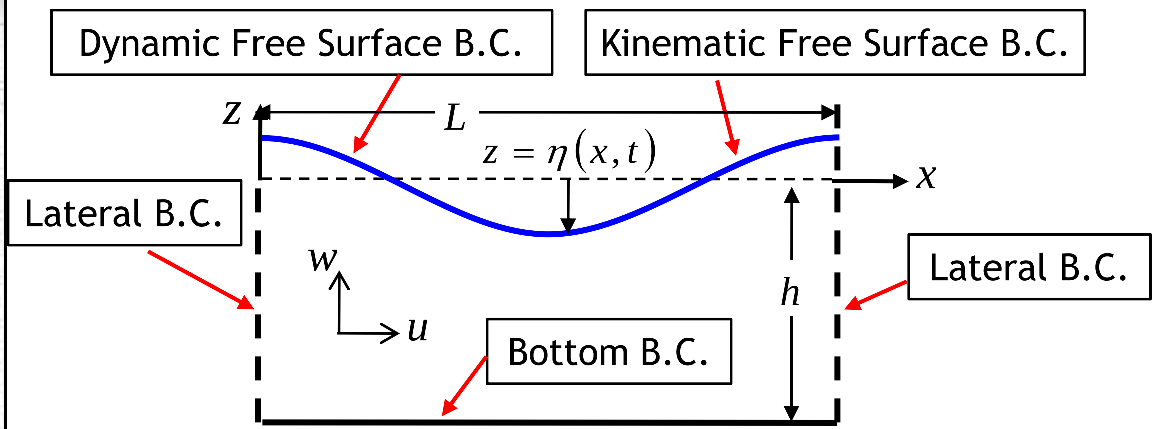
파의 주기(wave period)**를** T, **파장**(wave length)**을** L **이라고 하면, 다음이 성립한다.**

$$\Phi(x,z,t) = \Phi(x,z,t+T)$$

$$\Phi(x,z,t) = \Phi(x+L,z,t)$$

Incident Wave Velocity Potential (6)
: Boundary condition

Boundary condition(B.C.)　　　　*η : z방향 변위



<Summary of the 2-D periodic water wave boundary condition>

① Kinematic Free Surface B.C.(KFSBC)

$$\frac{\partial \Phi}{\partial z} - \frac{\partial \eta}{\partial t} - \frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x} = 0 \quad (\text{on} \quad z = \eta)$$

② Bottom B.C. (BBC)

$$\left.\frac{\partial \Phi}{\partial z}\right|_{z=-h} = 0$$

③ Dynamic Free Surface B.C. (DFSBC)

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left|\nabla \Phi\right|^2 + g\eta = 0 \quad (\text{on} \quad z = \eta)$$

④ Lateral B.C.

$$\Phi(x,z,t) = \Phi(x,z,t+T)$$

$$\Phi(x,z,t) = \Phi(x+L,z,t)$$

Incident Wave Velocity Potential (7)
: Linearization(선형화)

① Kinematic Free Surface B.C.(KFSBC)

$$\frac{\partial \Phi}{\partial z} - \frac{\partial \eta}{\partial t} - \frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x} = 0 \quad (\text{on} \;\; z = \eta)$$

Tayler series로 전개하면,

(High Order Term)

$$\left(\frac{\partial \Phi}{\partial z} - \frac{\partial \eta}{\partial t} - \frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x}\right)_{z=\eta} = \left(\frac{\partial \Phi}{\partial z} - \frac{\partial \eta}{\partial t} - \cancel{\frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x}}\right)_{z=0} + \cancel{\eta \frac{\partial}{\partial z}\left(\frac{\partial \Phi}{\partial z} - \frac{\partial \eta}{\partial t} - \frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x}\right)_{z=0}} + \cancel{H.O.T} = 0$$

여기서 파장에 비해 파고가 작다고 가정했으므로, $\eta << 1$

$$u\Big|_{z=0} = \frac{\partial \Phi}{\partial x}\Big|_{z=0} << 1 \;\; , \;\; w\Big|_{z=0} = \frac{\partial \Phi}{\partial z}\Big|_{z=0} << 1$$

작은 텀이 두 개 이상 곱해진 경우를 무시하면,

$$\left(\frac{\partial \Phi}{\partial z} - \frac{\partial \eta}{\partial t}\right)_{z=0} = 0$$ => Linearized Kinematic Free Surface B.C.(KFSBC)

Incident Wave Velocity Potential (8)
: Linearization(선형화)

③ Dynamic Free Surface B.C. (DFSBC)

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left|\nabla \Phi\right|^2 + g\eta = 0 \quad (\text{on} \;\; z = \eta)$$

Tayler series로 **전개하면**,

(High Order Term)

$$\left(\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left|\nabla \Phi\right|^2 + g\eta\right)_{z=\eta} = \left(\frac{\partial \Phi}{\partial t} + \cancel{\frac{1}{2}\left|\nabla \Phi\right|^2} + g\eta\right)_{z=0} + \cancel{\eta \frac{\partial}{\partial z}\left(\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left|\nabla \Phi\right|^2 + g\eta\right)_{z=0}} + \cancel{H.O.T} = 0$$

여기서 파장에 비해 파고가 작다고 가정했으므로, $\eta << 1$

$$u\big|_{z=0} = \left.\frac{\partial \Phi}{\partial x}\right|_{z=0} << 1 \;\; , \;\; w\big|_{z=0} = \left.\frac{\partial \Phi}{\partial z}\right|_{z=0} << 1$$

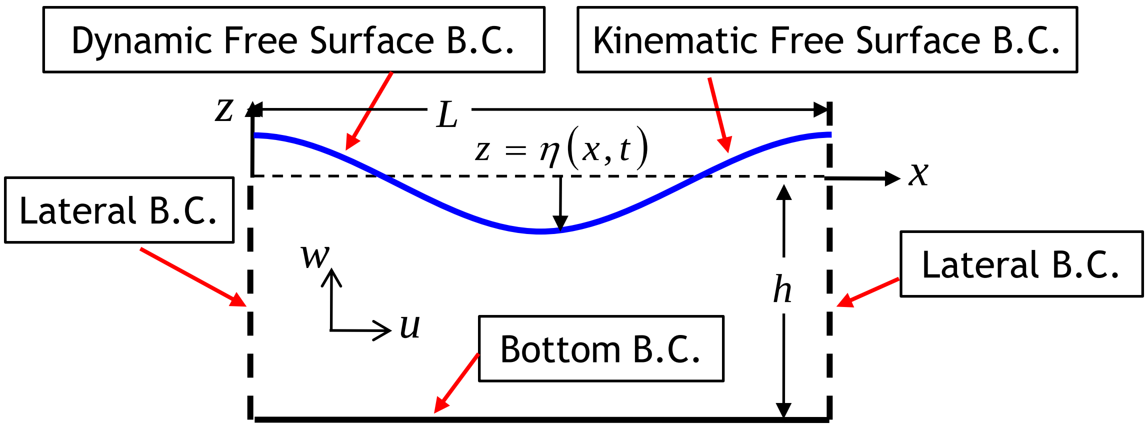
작은 텀이 두 개 이상 곱해진 경우를 무시하면,

$$\left(\frac{\partial \Phi}{\partial t} + g\eta\right)_{z=0} = 0 \;\; \Rightarrow \;\; \eta = -\frac{1}{g}\frac{\partial \Phi}{\partial t} \quad (\text{on} \;\; z = 0)$$

=> Linearized Dynamic Free Surface B.C.(DFSBC)

Incident Wave Velocity Potential (9)
: Boundary condition

Boundary condition(B.C.) *η : z방향 변위



<Summary of the 2-D periodic water wave boundary condition>

① Kinematic Free Surface B.C.(KFSBC)

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \eta}{\partial t} \quad (\text{on} \quad z = 0)$$

② Bottom B.C. (BBC)

$$\left.\frac{\partial \Phi}{\partial z}\right|_{z=-h} = 0$$

③ Dynamic Free Surface B.C. (DFSBC)

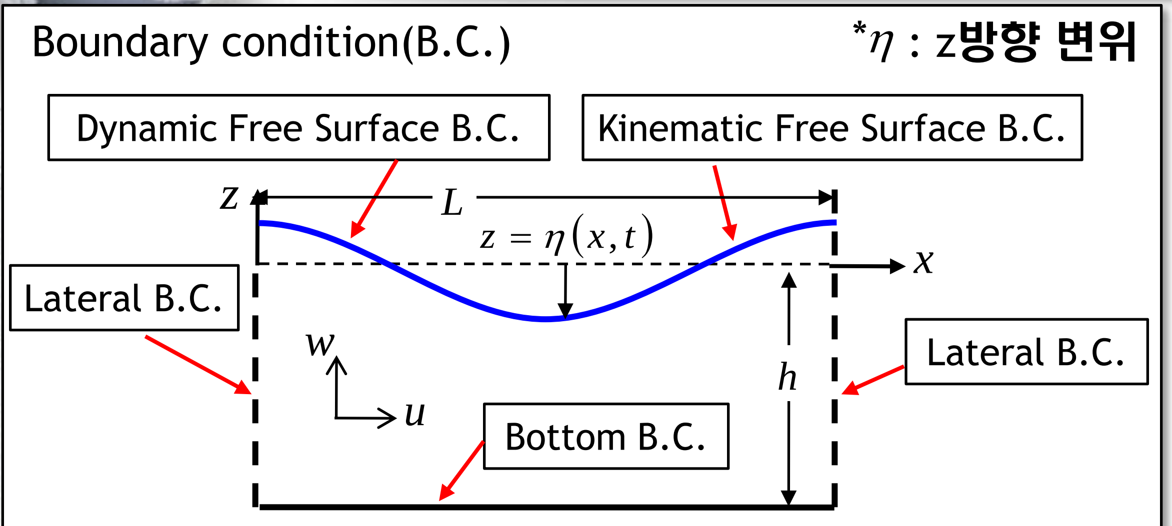
$$\eta = -\frac{1}{g}\frac{\partial \Phi}{\partial t} \quad (\text{on} \quad z = 0)$$

④ Lateral B.C.

$$\Phi(x,z,t) = \Phi(x,z,t+T)$$

$$\Phi(x,z,t) = \Phi(x+L,z,t)$$

Incident Wave Velocity Potential (10)
: Boundary condition



① Kinematic Free Surface B.C.(KFSBC)

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \eta}{\partial t} \quad (\text{on} \quad z = 0)$$

$$\frac{\partial \Phi}{\partial z} = -\frac{1}{g}\frac{\partial^2 \Phi}{\partial t^2}$$

③ Dynamic Free Surface B.C. (DFSBC)

$$\eta = -\frac{1}{g}\frac{\partial \Phi}{\partial t} \quad (\text{on} \quad z = 0) \xrightarrow{t\text{로 미분}} \frac{\partial \eta}{\partial t} = -\frac{1}{g}\frac{\partial^2 \Phi}{\partial t^2}$$

$$\boxed{\frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z} = 0 \quad (\text{on} \quad z = 0) \qquad (\Phi_{tt} + g\Phi_z = 0)}$$

=> Linearized Free Surface B.C.

Incident Wave Velocity Potential (11)
: Boundary condition

Boundary condition(B.C.)

$^{*}\eta$: z방향 변위

Linearized Free Surface B.C.

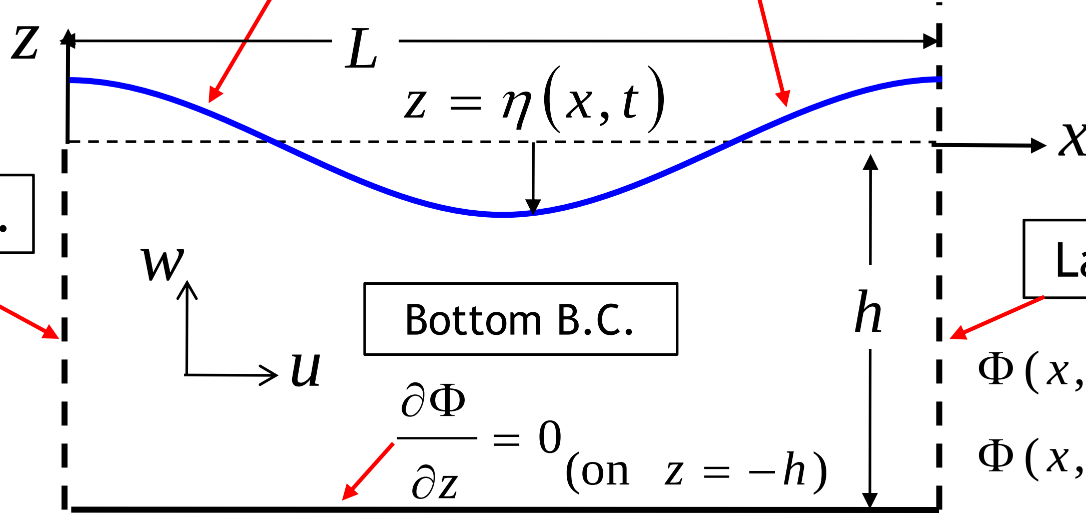
$$\Phi_{tt} + g\Phi_{z} = 0 \quad (\text{on} \quad z = 0)$$

Dynamic Free Surface B.C.

$$(\text{on} \quad z = 0) \qquad \eta = -\frac{1}{g}\frac{\partial \Phi}{\partial t}$$

Kinematic Free Surface B.C.

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \eta}{\partial t} \quad (\text{on} \quad z = 0)$$



Lateral B.C.

Bottom B.C.

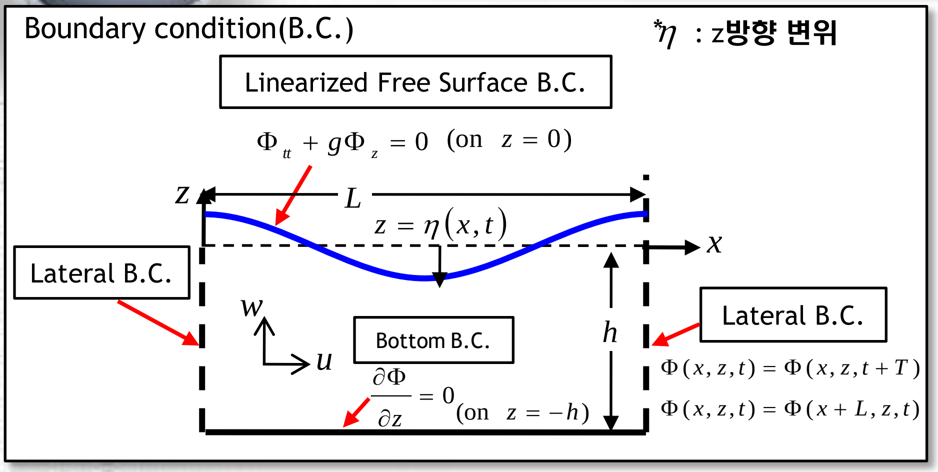
$$\frac{\partial \Phi}{\partial z} = 0 \quad (\text{on} \quad z = -h)$$

Lateral B.C.

$$\Phi(x,z,t) = \Phi(x,z,t+T)$$

$$\Phi(x,z,t) = \Phi(x+L,z,t)$$

Incident Wave Velocity Potential (12)
: Boundary condition



$\Phi = \Phi(x,z,t)$ 에서 시간에 대한 주기 함수로 가정하

시간 항을 분리하면,

$$\Phi = \mathrm{Re}\left\{\phi(x,z)\, e^{i\omega t}\right\}$$

($\phi(x,z)$: Complex amplitude of the velocity potential)

위 Velocity potential을 지배 방정식과
경계 조건에 대입하면,

① 지배 방정식

$$\nabla^2\Phi = \nabla^2\left(\phi(x,z)\, e^{i\omega t}\right) = e^{i\omega t}\left(\nabla^2\phi(x,z)\right) = 0$$

$$\boxed{\nabla^2\phi = 0\,, \quad \left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0\right)}$$

② Linearized Free Surface B.C.

$$\Phi_{tt} + g\Phi_z = \left(-\omega^2\phi\, e^{i\omega t} + g\,\phi_z\, e^{i\omega t}\right) = 0$$

$e^{i\omega t}$ 로 나누면,

$$\boxed{\therefore\; -\omega^2\phi + g\,\phi_z = 0} \quad (\text{on} \;\; z = 0)$$

③ Bottom B.C.

$$\frac{\partial \Phi}{\partial z} = e^{i\omega t}\frac{\partial \phi}{\partial z} = 0 \;\longrightarrow\; \boxed{\therefore\; \frac{\partial \phi}{\partial z} = 0} \quad (\text{on} \;\; z = -h)$$

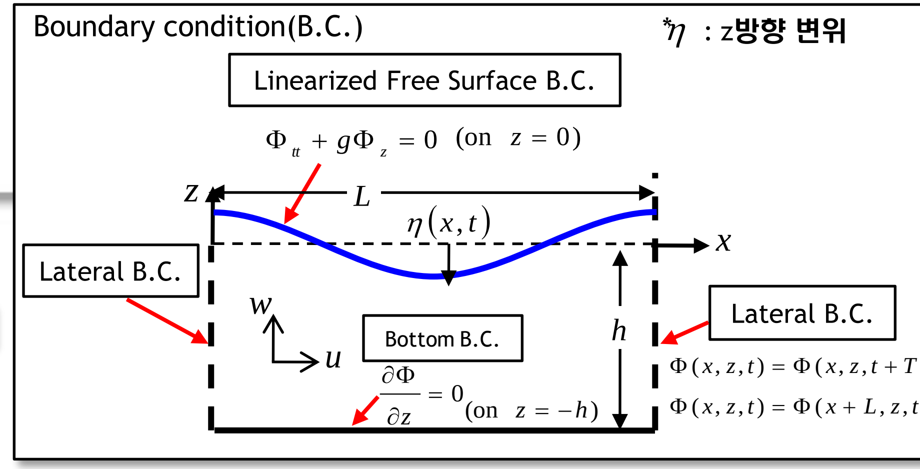
④ Lateral B.C.

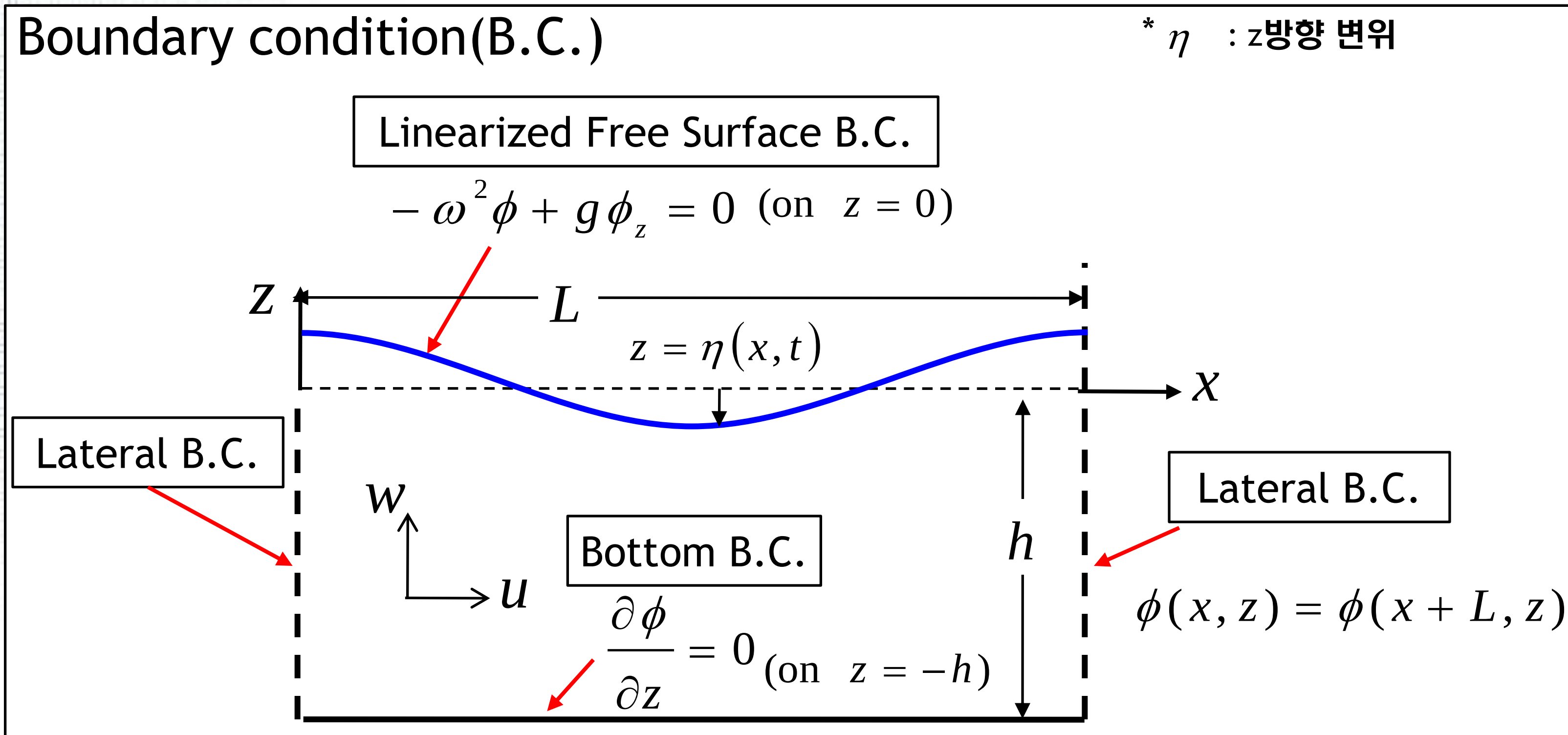
$$\Phi(x,z,t) = \Phi(x+L,z,t)$$

$$\downarrow$$

$$\boxed{\phi(x,z) = \phi(x+L,z)}$$

Incident Wave Velocity Potential (13)





1) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,2005,Ch12.5(p552~562)

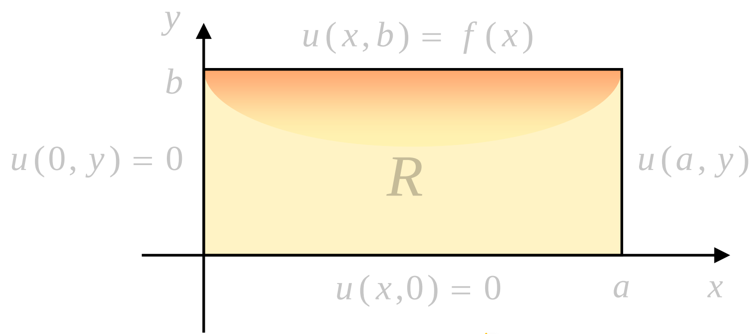
Incident Wave Velocity Potential (14)

공학 수학 Chapter 12.5[1)]
(2-D Heat equation of Steady state)

① Governing Equation :

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

② Boundary condition : (Dirichlet B.C.[1)])



동일한 방정식을 푸는데 각각 다른 경계 조건이 적용됨

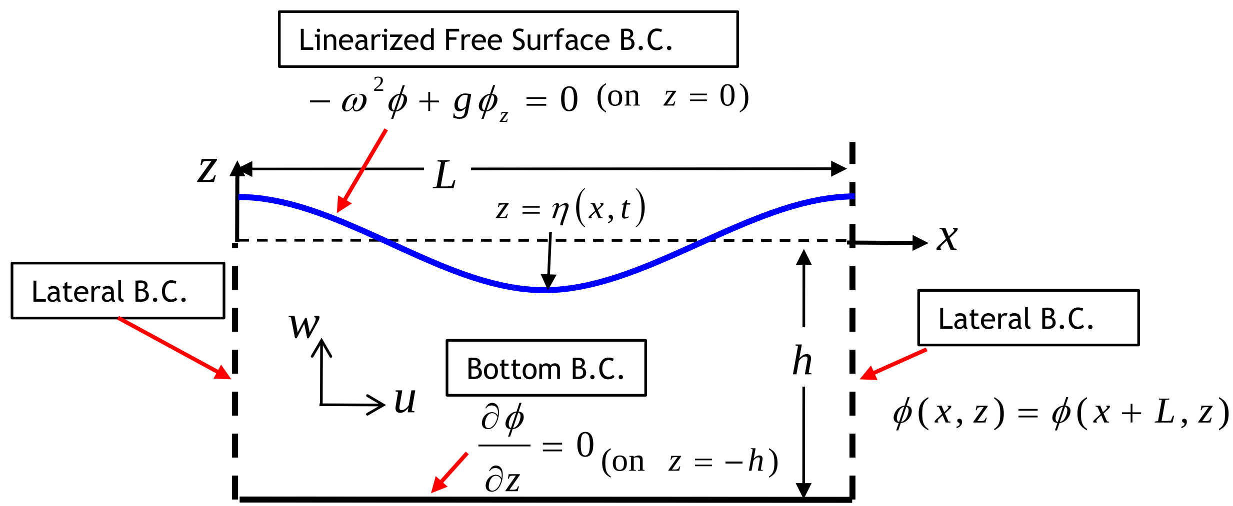
3학년 해양파 역학
(Wave Equation)

① Governing Equation :

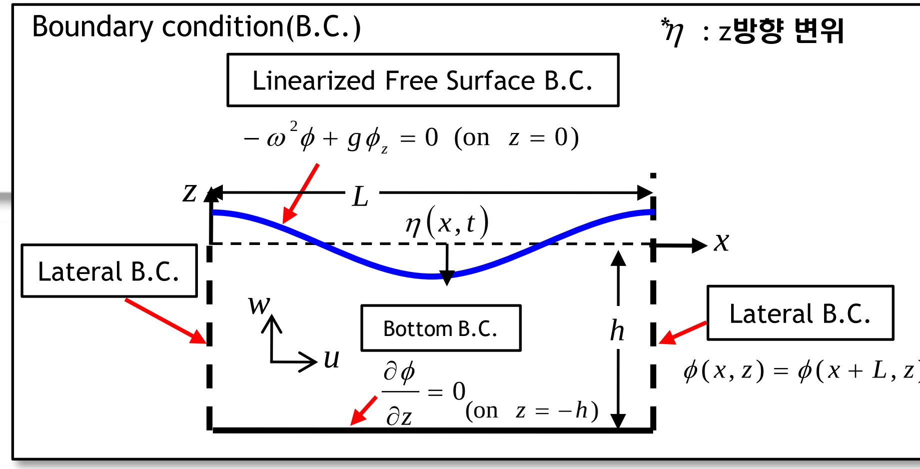
$$\nabla^2 \phi = 0$$

$$\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \right)$$

② Boundary condition(B.C.) : (Robin B.C.[1)])



Wave Equation 유도 (1)



Laplace Equation

$\nabla^2\phi = 0$: Governing Equation

① Velocity potential ϕ **는** x, z **의 함수 이므로,**
변수 분리법(separation of variables)**에 의해**
$\phi = F(x)\cdot G(z)$ **로 둘 수 있다.**

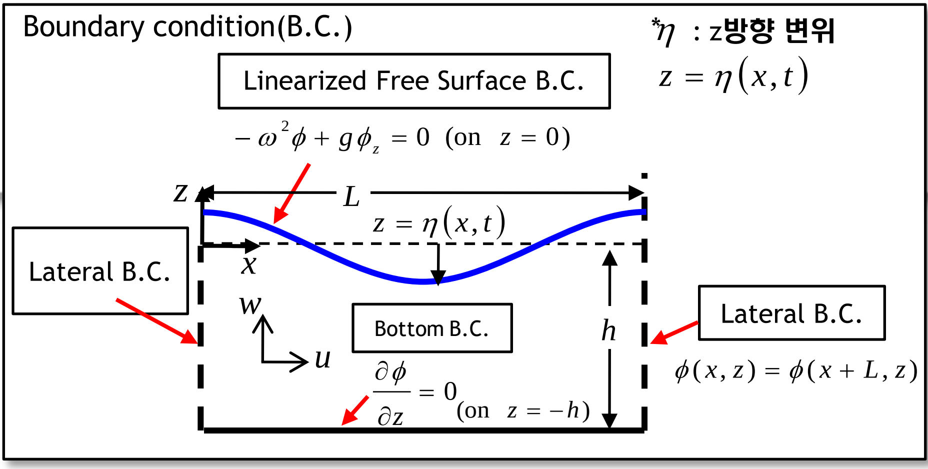
② Laplace Equation**에 대입하면,**

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{\partial^2 F}{\partial x^2}G + F\frac{\partial^2 G}{\partial z^2} = F_{xx}G + FG_{zz} = 0$$

$$\therefore F_{xx}G + FG_{zz} = 0 \xrightarrow{FG \text{ 로 나눔}} \frac{G_{zz}}{G} + \frac{F_{xx}}{F} = 0 \longrightarrow \frac{G_{zz}}{G} = -\frac{F_{xx}}{F} = p \longrightarrow \begin{cases} F_{xx} + pF = 0 \\ G_{zz} - pG = 0 \end{cases}$$

($\because$ x와 z만의 함수가 같은 것은 상수뿐)

Wave Equation 유도 (2)



③ p 의 부호에 따른 방정식의 해를 계산해 보면,

$$\begin{cases} F_{xx} + pF = 0 \\ G_{zz} - pG = 0 \end{cases}$$

(i) $p < 0$ 일 때, $p = -\nu^2$이라 하면,

$$F_{xx} - \nu^2 F = 0 \longrightarrow F = Ae^{\nu x} + Be^{-\nu x}$$

Lateral B.C.에 의해 x에 대한 주기함수여야 하는데 exponential 함수는 주기함수가 아님.

따라서 해가 될 수 없음.

(ii) $p = 0$ 일 때,

$$F_{xx} = 0 \longrightarrow F = Ax + B$$

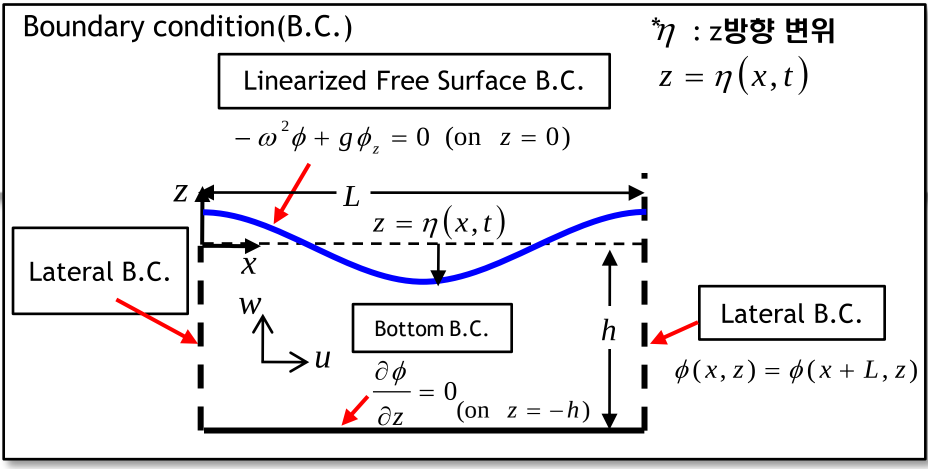
$$F(x) = Ax + B = F(x+L) = A(x+L) + B$$

$$\therefore F(x) = B$$

wave는 x에 따라서 주기적으로 '변'하는 데, 이를 만족하지 않음.

따라서 해가 될 수 없음

Wave Equation 유도 (3)



$$\begin{cases} F_{xx} + pF = 0 \\ G_{zz} - pG = 0 \end{cases}$$

(iii) $p > 0$ 일 때, $p = k^2$이라 하면,

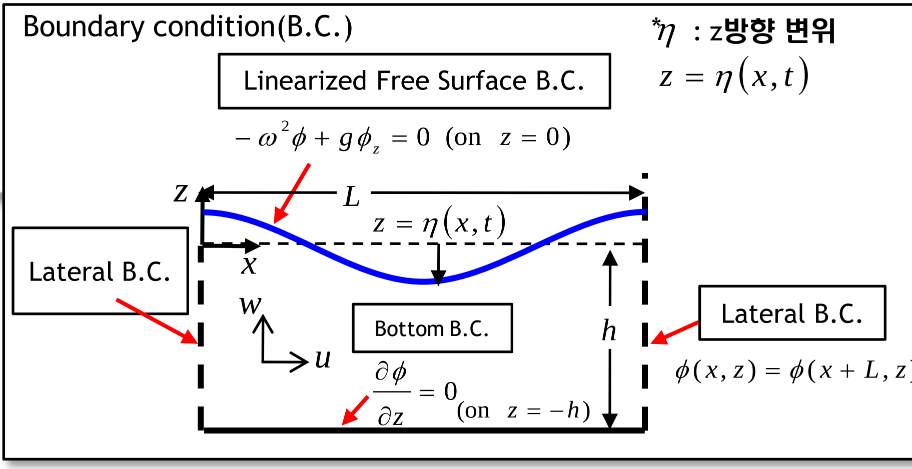
$$F_{xx} + k^2 F = 0 \longrightarrow \boxed{F = Ae^{ikx} + Be^{-ikx}}$$

(Euler 공식에 의해 $e^{ikx} = \cos kx + i \sin kx$ 이므로,
x에 대한 주기 함수의 성질을 가짐 -> Lateral B.C. 만족)

한편, $G_{zz} - pG = 0$ 에서

$$G_{zz} - k^2 G = 0 \longrightarrow \boxed{G(z) = Ce^{kz} + De^{-kz}}$$

Wave Equation 유도 (3)



$$F = Ae^{ikx} + Be^{-ikx}$$

$\downarrow$ **기저 변환**

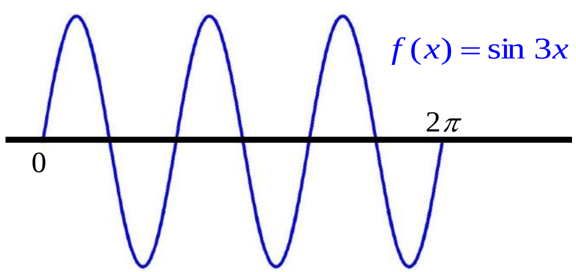
$$F(x) = A'\cos kx + B'\sin kx = F(x+L) = A'\cos(kx+kL) + B'\sin(kx+kL)$$

$$\therefore kL = 2\pi$$

(**하나의** wave**만 다루었으므로**, $2n\pi$ **라 하지 않고**, 2π **라 함**.)

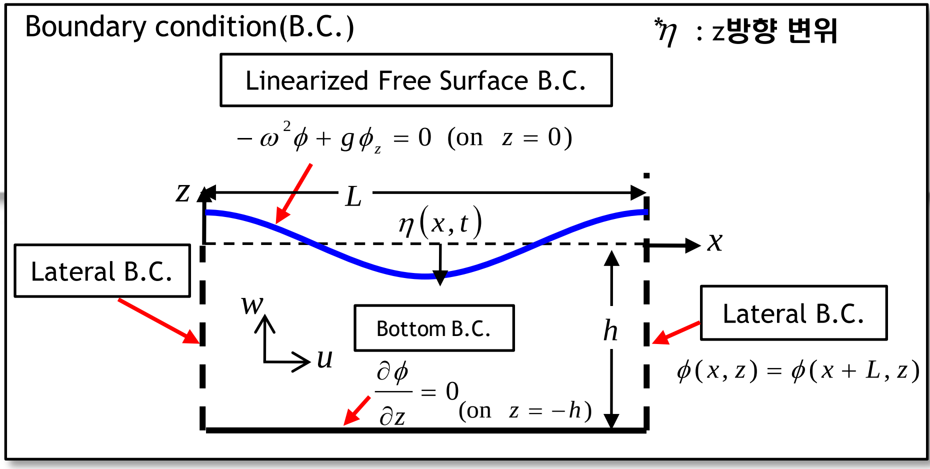
$$\therefore k = \frac{2\pi}{L}$$

wave number k : **한 주기**(2π) **내에 존재하는** wave**의 개수**



예를 들어 $f(x)=\sin 3x$ **인 경우**, wave number**는 3이며**, $x=0$**부터** 2π **사이에 3개의 파가 들어있다.**

Wave Equation 유도 (4)



④ Bottom B.C. 적용

$$\frac{\partial\phi}{\partial z} = F\frac{\partial G}{\partial z} = F\left(Cke^{kz} - Dke^{-kz}\right)$$

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=-h} = F\left(Cke^{-kh} - Dke^{kh}\right) = 0$$

F 나눔 $\Rightarrow Cke^{-kh} - Dke^{kh} = 0 \longrightarrow C = De^{2kh}$

$$\begin{cases} \phi(x,z) = F(x)\cdot G(z) \\ F = Ae^{ikx} + Be^{-ikx} \\ G = Ce^{kz} + De^{-kz} \end{cases}$$

$$G(z) = Ce^{kz} + De^{-kz} = De^{2kh}e^{kz} + De^{-kz} = D\left(e^{2kh}e^{kz} + e^{-kz}\right)$$

$$= De^{kh}\left(e^{kz+kh} + e^{-kz-kh}\right) = De^{kh}\left(e^{k(z+h)} + e^{-k(z+h)}\right)$$

z=0을 대입하면,

$$G(0) = De^{kh}\left(e^{kh} + e^{-kh}\right)$$

여기서 $G(0) = 1$ 을 만족하는 D를 정하면,

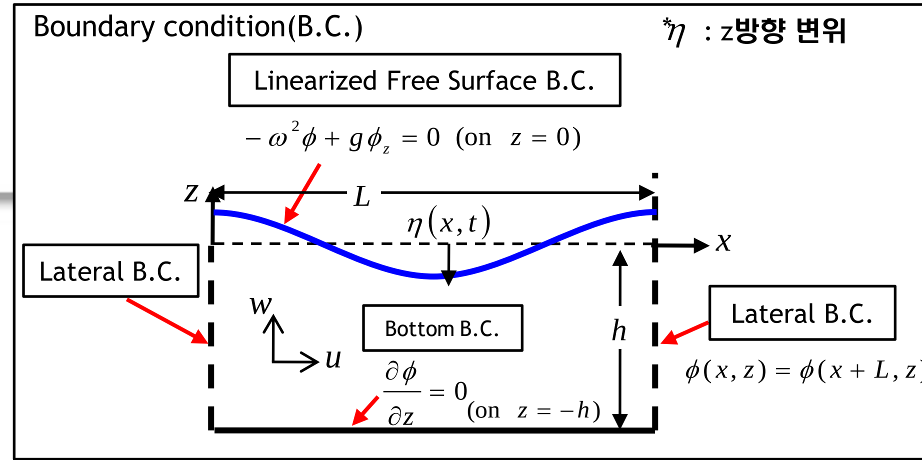
$$D = \frac{1}{e^{kh}\left(e^{kh} + e^{-kh}\right)}$$

$$G(z) = \frac{e^{kh}\left(e^{k(z+h)} + e^{-k(z+h)}\right)}{e^{kh}\left(e^{kh} + e^{-kh}\right)}$$

$$= \frac{e^{k(z+h)} + e^{-k(z+h)}}{e^{kh} + e^{-kh}} = \frac{\cosh\ k(z+h)}{\cosh\ kh}$$

$$\boxed{\therefore \phi(x,z) = \left(Ae^{ikx} + Be^{-ikx}\right)\frac{\cosh\ k(z+h)}{\cosh\ kh}}$$

Wave Equation 유도 (5)



⑤ Linearized Free Surface B.C. 적용

$$-\omega^2\phi + g\phi_z = 0$$

$\phi = F(x)G(z)$ 대입

$$-\omega^2 FG + gFG_z = 0$$

F 로 양변을 나눠줌

$$\phi(x,z) = F(x)\cdot G(z)$$

$$F = Ae^{ikx} + Be^{-ikx}$$

$$G = \frac{\cosh k(z+h)}{\cosh kh}$$

$$-\omega^2 G + gG_z = 0 \xrightarrow{G,\,G_z \text{ 대입}} -\omega^2\frac{\cosh k(z+h)}{\cosh kh} + gk\frac{\sinh k(z+h)}{\cosh kh} = 0$$

$$G(z) = \frac{\cosh k(z+h)}{\cosh kh}$$

$$G_z = \frac{dG}{dz} = k\frac{\sinh k(z+h)}{\cosh kh}$$

$z = 0$ 대입

$$\omega^2 = gk\frac{\sinh kh}{\cosh kh} = gk\tanh kh$$

$$\boxed{\therefore\ \omega^2 = gk\tanh kh}$$

=> dispersion relation

참고 – Dispersion Relation

h→∞

$$\lim_{h\to\infty}(\sinh kh) = \lim_{h\to\infty}\frac{e^{kh}-e^{-kh}}{2} = \frac{e^{kh}}{2}$$

$$\lim_{h\to\infty}(\cosh kh) = \lim_{h\to\infty}\frac{e^{kh}+e^{-kh}}{2} = \frac{e^{kh}}{2}$$

$$\lim_{h\to\infty}(\tanh kh) = \lim_{h\to\infty}\frac{\sinh kh}{\cosh kh} = \frac{e^{kh}/2}{e^{kh}/2} = 1$$

✓ Deep sea 일 때, (h→∞)

$$\omega^2 = gk \tanh kh \approx gk$$

$$\left(\omega = \frac{2\pi}{T}\right) \qquad \left(k = \frac{2\pi}{L}\right)$$

L :파장
T :주기

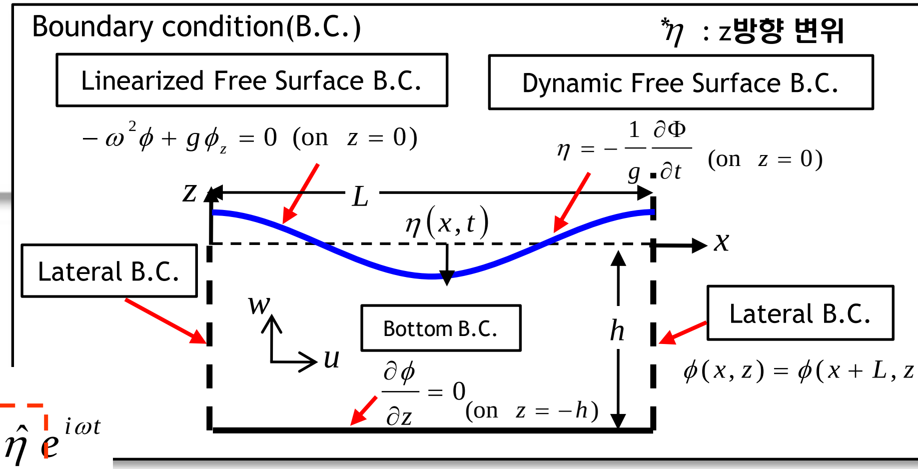
$$\left(\frac{2\pi}{T}\right)^2 = g\frac{2\pi}{L}$$

$$\frac{2\pi}{T^2} = \frac{g}{L}$$

=> 파장(길이)과 주기(시간)와의 관계식

즉, 장파일수록 주기가 길고, 단파일수록 주기가 짧아짐을 알 수 있다.

Wave Amplitude



✓ Dynamic Free Surface B.C. 적용

Dynamic Free Surface B.C. :

$$\eta = -\frac{1}{g}\frac{\partial\Phi}{\partial t} = -\frac{1}{g}\frac{\partial\left(\phi(x,z)e^{i\omega t}\right)}{\partial t} = \boxed{-\frac{i\omega}{g}}\phi e^{i\omega t} = \boxed{\hat{\eta}}\, e^{i\omega t}$$

Wave Amplitude

$$\phi(x,z) = \left(Ae^{ikx} + Be^{-ikx}\right)G(z)$$

$$G = \frac{\cosh\ k(z+h)}{\cosh\ kh}$$

$$\hat{\eta} = -\frac{i\omega}{g}\phi(x,z) = -\frac{i\omega}{g}\left(Ae^{ikx} + Be^{-ikx}\right)G(z)$$

$$\eta_0 = -\frac{\omega B}{g}, \Gamma = \frac{A}{B}$$ 라 두면

$$\hat{\eta} = -\frac{i\omega B}{g}\left(\frac{A}{B}e^{ikx} + Xe^{-ikx}\right)G(z)$$

$$= i\eta_0\left(e^{ikx} + \Gamma e^{-ikx}\right)G(z)$$

$z = 0$ 대입

$$\hat{\eta} = i\eta_0\left(\Gamma e^{ikx} + e^{-ikx}\right)$$

$$(\because G(0) = 1)$$

$\phi(x,z)$에 $A = \Gamma B, B = -\dfrac{g}{\omega}\eta_0$

$\hat{\eta} = i\eta_0\left(e^{ikx} + \Gamma e^{-ikx}\right)$ 대입

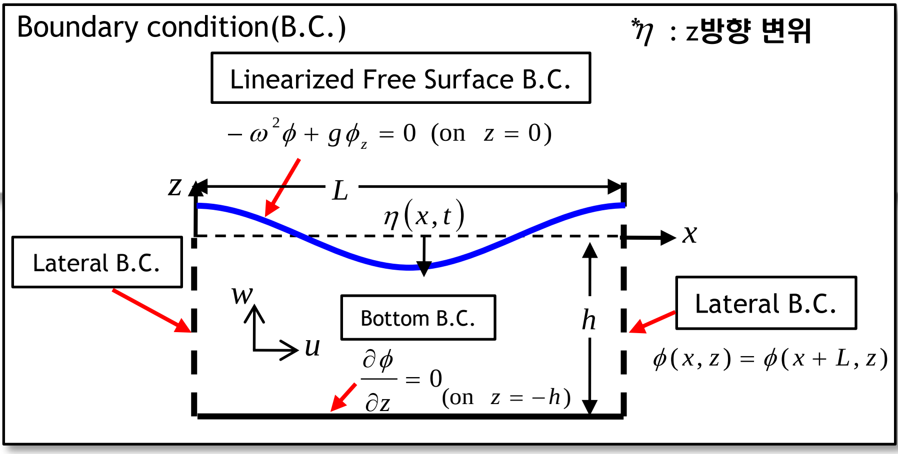
$$\phi(x,z) = \left(\Gamma Be^{ikx} + Be^{-ikx}\right)G(z) = B\left(\Gamma e^{ikx} + e^{-ikx}\right)G(z)$$

$$= -\frac{g}{\omega}\eta_0\left(\Gamma e^{ikx} + e^{-ikx}\right)G(z)$$

$$= -\frac{g}{i\omega}\hat{\eta}\,G(z)$$

(ϕ 는 파고(η)에 비례함)

Incident Wave Velocity Potential (15)



<Summary of the wave equation>

✓ $$\Phi_I(x,z,t) = \text{Re}\left\{\phi_I(x,z)e^{i\omega t}\right\}$$

✓ $$\phi_I(x,z) = -\frac{g}{\omega}\eta_0\left(\Gamma e^{ikx} + e^{-ikx}\right)G(z) \quad \left(G(z) = \frac{\cosh k(z+h)}{\cosh kh}\right)$$

If Deep water, $(h \to \infty)$

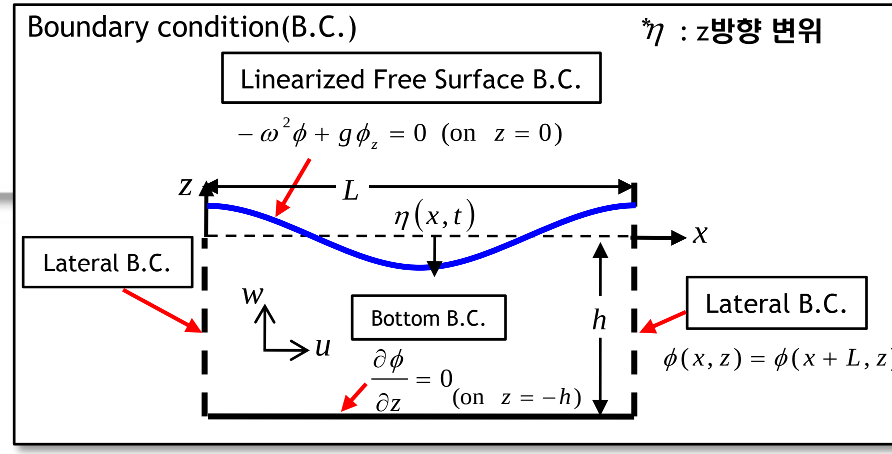
$$G(z) = e^{kz}$$

$$\left(\begin{aligned}\lim_{h\to\infty} G(z) &= \lim_{h\to\infty}\frac{\cosh k(z+h)}{\cosh kh} \\ &= \lim_{h\to\infty}\frac{e^{k(z+h)} - e^{-k(z+h)}}{e^{kh} - e^{-kh}} = \lim_{h\to\infty}\frac{e^{k(z+h)}}{e^{kh}} = e^{kz}\end{aligned}\right)$$

Plane progressive wave의 경우, (+)방향으로 진행파를 가정하면, $\Gamma = 0$

$$\phi_I(x,z) = -\frac{g}{\omega}\eta_0 e^{-ikx}\, e^{kz}$$

Incident Wave Velocity Potential (16)



Step1.
해상에서 파의 파고 및 주파수 계측

$\Rightarrow \eta_0, \omega$

Step2.
Dispersion Relation**으로 부터** Wave number **계산**

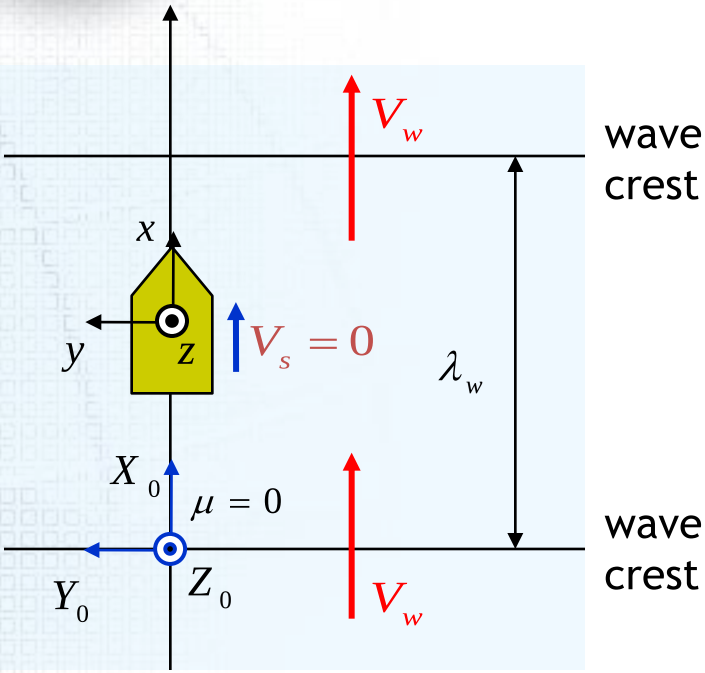
dispersion relation : $\omega^2 = \underline{gk \tanh kh}$

Step3.
Velocity Potential**식에 대입**

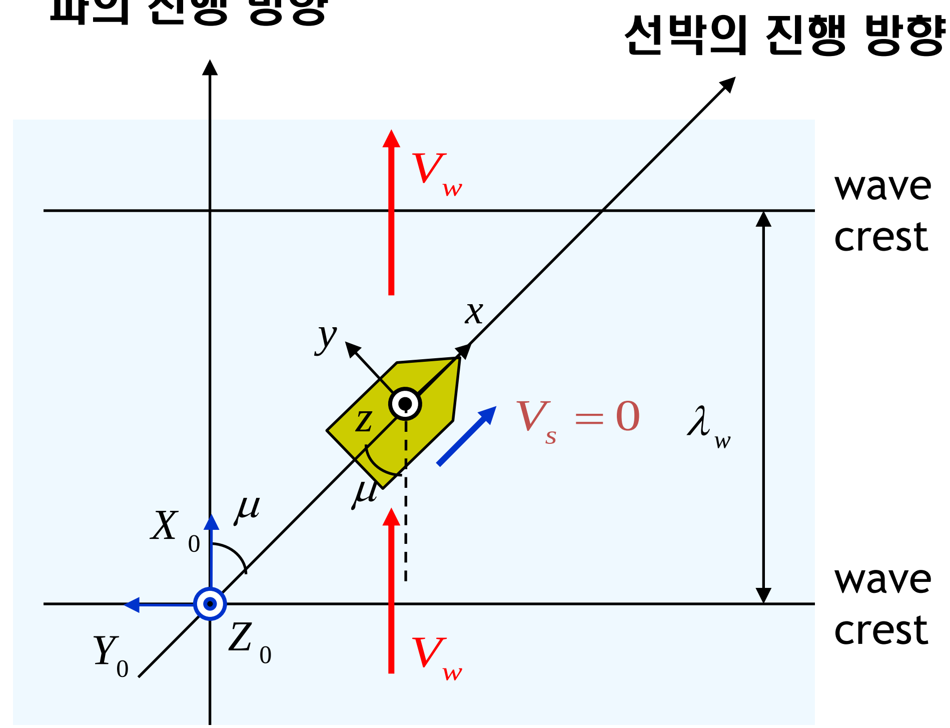
$$\phi_I(x,z) = -\frac{g}{\omega}\eta_0 e^{-ikx} e^{-kz}$$

Incident Wave Velocity Potential (18)

파의 진행 방향 =선박의 진행 방향



파의 진행 방향



$$\phi_I(x, y, z) = -\frac{g}{\omega}\eta_0 e^{-ikx} e^{kz}$$

파의 진행방향이
z축 기준으로
μ 만큼 회전

$$\phi_I(x, y, z) = -\frac{g}{\omega}\eta_0 e^{-ik(x\cos\mu - y\sin\mu)} e^{kz}$$

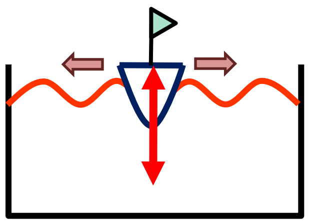
1) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 285~290

Radiation Wave Velocity Potential (1)

✓ Radiation wave velocity potential

$$\Phi_R(x,y,z,t)=\phi_R(x,y,z)e^{i\omega t}=\sum_{j=1}^{6}\xi_j\phi_j(x,y,z)e^{i\omega t}$$

정수중 선박의 강제 운동에 의해 발생한 힘



Radiation Force

✓ Boundary Condition[1)]

① Free surface condition

$$-\omega^2\phi_j+g\frac{\partial\phi_j}{\partial z}=0 \quad (\text{on} \quad z=0)$$

② Radiation Condition : **파가 무한이 발산하면 소멸됨**

$$\phi_j \propto e^{\mp iky}, \quad as \quad y\to\pm\infty \quad (j=1,\cdots,6)$$

③ Body boundary condition : **선박 표면에서 유체 입자와 표면의 속도가 동일함**

$$\frac{\partial\Phi_R}{\partial n}=V_n \;\Rightarrow\; \underline{\frac{\partial\phi_j}{\partial n}}=\underline{i\omega n_j}(\text{on} \quad S_B)$$

S_B : **침수표면**
V_n : **침수표면에 수직인 속도**
n_j : Generalized normal component

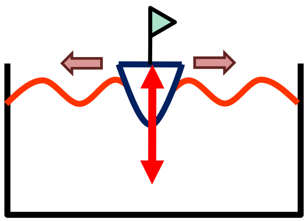
단위 진폭에 대한 유체 속도 성분 (선체 표면에 수직)

단위 진폭에 대한 선체 표면의 속도 성분 (선체 표면에 수직)

1) Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 50~55)

Radiation wave Velocity Potential (2)

정수중 선박의 강제 운동에 의해 발생한 힘



Radiation Force

Given

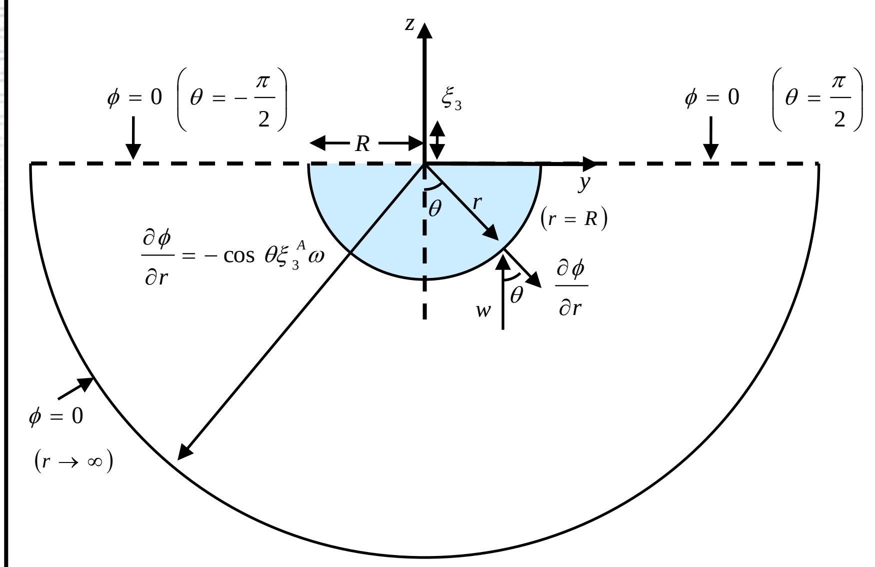
- Governing Equation : $\nabla^2 \phi_j = 0 \quad (j = 1, \cdots, 6)$
- Boundary Condition : $-\omega^2 \phi + g\phi_z = 0$; $\dfrac{\partial \phi_j}{\partial n} = i\omega n_j \quad (on \ \ S_B)$

  $\phi_j \propto e^{\mp iky}, \ \ as \ \ y \to \pm\infty$
- Motion of the ship : $\xi_j = \xi_j^A e^{i\omega t} \quad (j = 1, \cdots, 6)$

Find : $\phi_j \quad (j = 1, \cdots, 6)$

Difficult!!!

⇩ Considering Simple Example[1)]



Given : $\nabla^2 \phi_3 = 0$

- Boundary Condition :

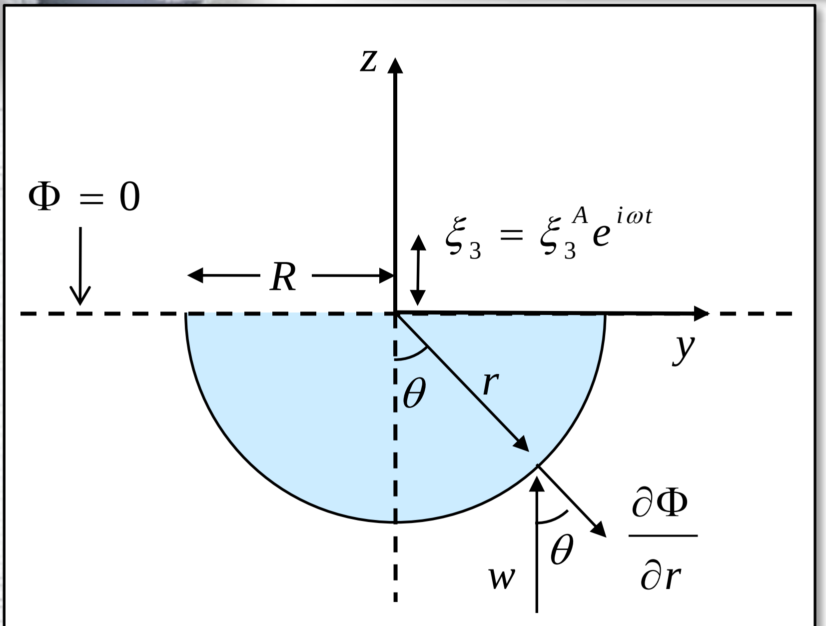
$$\frac{\partial \phi_3}{\partial r} = -\cos \theta i \omega \qquad (r = R)$$

$$\phi_3 = 0 \quad (r \to \infty), \qquad \phi_3 = 0 \quad \left(\theta = -\frac{\pi}{2}, \theta = \frac{\pi}{2}\right)$$

- Motion of the ship : $\xi_3(t) = \xi_3^A e^{i\omega t}$

Find : ϕ_3

Radiation Velocity Potential (3)



✓ 물체 표면 경계 조건

: 물체 표면의 속도는 유체의 속도와 동일함

(kinematic Boundary Condition)

※ 원통 좌표계 사용 (Polar Coordinate)

$$(y, z) = (r\cos\theta, r\sin\theta)\left(r = R, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right)$$

물체 표면에서 유체의 속도 $= \dfrac{\partial \Phi}{\partial r}$

$$\Phi(r,\theta,t) = \xi_3^A \phi_3(r,\theta) e^{i\omega t}$$

$$\frac{\partial \Phi}{\partial r} = \xi_3^A \frac{\partial \phi_3}{\partial r} e^{i\omega t}$$

강체의 z방향 속도 (어느 지점에서나 동일함)

$$w = \frac{d\xi_3}{dt} = \xi_3^A i\omega e^{i\omega t}$$

강체의 반지름 방향 속도 성분

$$w_r = w\cos\theta = \cos\theta\, \xi_3^A i\omega e^{i\omega t}$$

$$\frac{\partial \Phi}{\partial r} = w_r$$

(위쪽이 +이므로)

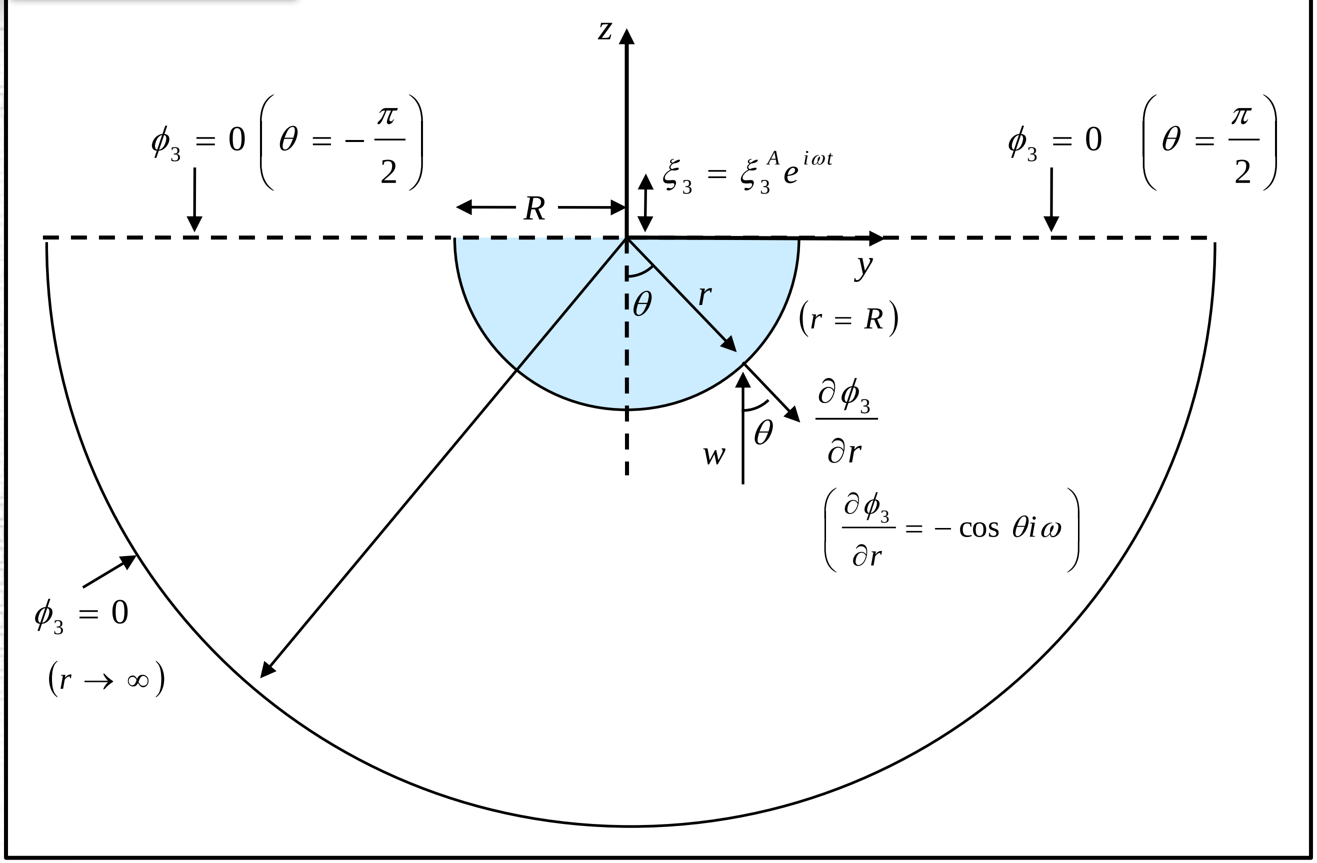
$$\xi_3^A \frac{\partial \phi_3}{\partial r} e^{i\omega t} = -\cos\theta\, \xi_3^A i\omega e^{i\omega t} \quad (r = R)$$

$$\therefore \frac{\partial \phi_3}{\partial r} = -\cos\theta\, i\omega$$

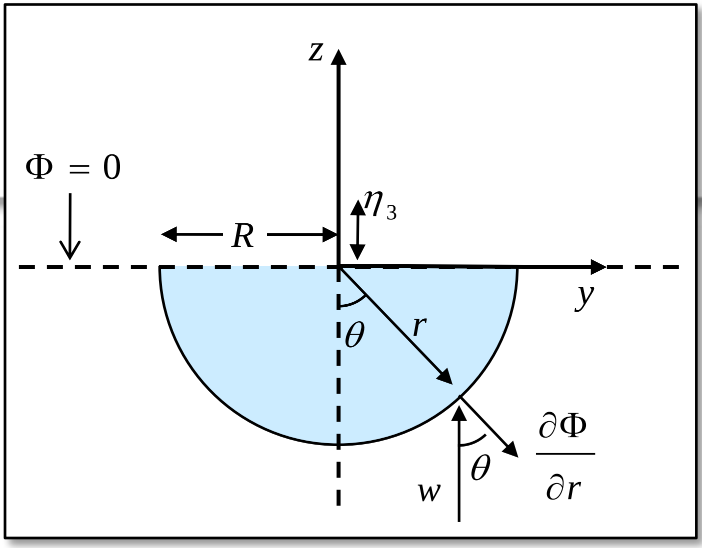
1) Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 50~55)

Radiation Wave Velocity Potential (4)

✓ 경계 조건



Radiation Wave Velocity Potential (5)



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_3}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi_3}{\partial \theta^2} = 0$$

$$\downarrow$$

$$\frac{1}{r}\frac{\partial \phi_3}{\partial r} + \frac{\partial^2 \phi_3}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \phi_3}{\partial \theta^2} = 0$$

$$\downarrow$$

$$r^2\frac{\partial^2 \phi_3}{\partial r^2} + \frac{\partial \phi_3}{\partial r} + \frac{\partial^2 \phi_3}{\partial \theta^2} = 0$$

$$\downarrow$$

$$r^2 \phi_{3rr} + r\phi_{3r} + \phi_{3\theta\theta} = 0$$

$$\downarrow$$

$\phi_3(r,\theta) = F(r)G(\theta)$라 하면,

$$r^2 F_{rr} G + rF_r G + FG_{\theta\theta} = 0$$

$$\downarrow$$

$$\frac{r^2 F_{rr} + rF_r}{F} = -\frac{G_{\theta\theta}}{G} = k^2$$

$$\downarrow$$

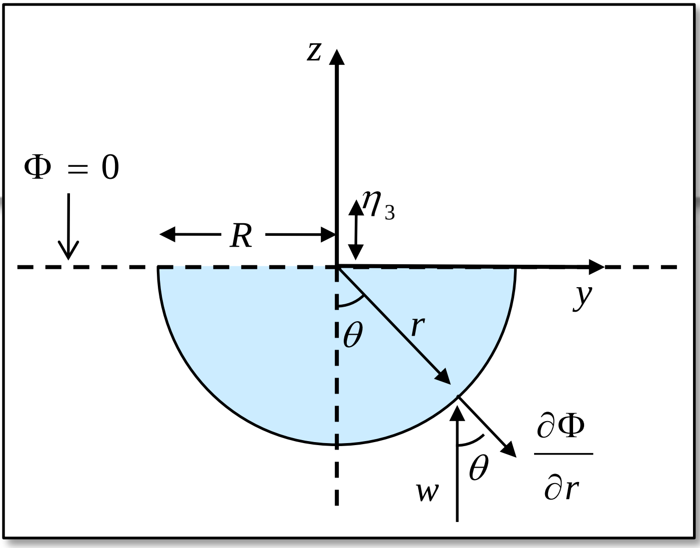
$$r^2 F_{rr} + rF_r - k^2 F = 0, \quad G_{\theta\theta} + k^2 G = 0$$

(Euler-Cauchy Equation)

$$F(r) = Ar^k + Br^{-k} \qquad G(\theta) = Ce^{ik\theta} + De^{-ik\theta}$$

$$\phi_3(r,\theta) = \left(Ar^k + Br^{-k}\right)\cdot\left(Ce^{ik\theta} + De^{-ik\theta}\right)$$

Radiation Wave Velocity Potential (6)



$$\phi(r,\theta) = \left(Ar^{k} + Br^{-k}\right)\cdot\left(Ce^{ik\theta} + De^{-ik\theta}\right)$$

$\downarrow$

경계 조건 (1)을 대입하면,

$$\frac{\partial \phi_3}{\partial r} = -\cos\theta i\omega \qquad (r = R)$$

$$\left.\frac{\partial \phi_3}{\partial r}\right|_{r=R} = \left(AkR^{k-1} - BkR^{-k-1}\right)\cdot\left(Ce^{ik\theta} + De^{-ik\theta}\right) = -\cos\theta i\omega$$

$\downarrow$

$$AkR^{k-1} - BkR^{-k-1} = -\omega \longrightarrow AR - BR^{-2} = -\omega$$

$$Ce^{ik\theta} + De^{-ik\theta} = i\cos\theta$$

$\downarrow$

$$C(\cos k\theta + i\sin k\theta) + D(\cos k\theta - i\sin k\theta) = i\cos\theta$$

$$(C + D)\cos k\theta + i(C - D)\sin k\theta = i\cos\theta$$

$\downarrow$

$$k = 1, C = D = i/2 \longrightarrow G(\theta) = i\cos\theta$$

한편, r이 무한대일 경우 수렴해야함.
k=1이므로, A=0이 되어야 한다.

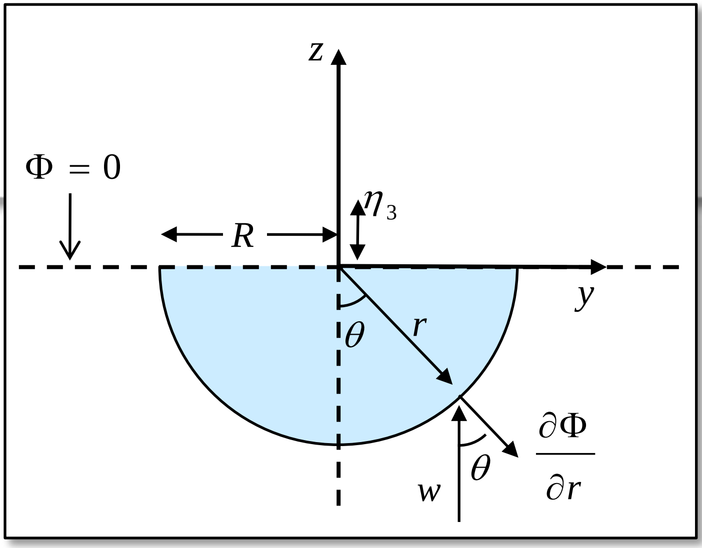
$$-BR^{-2} = -\omega$$

$\downarrow$

$$B = R^{2}\omega$$

$$F(r) = R^{2}\omega r^{-1} \longleftarrow B = R^{2}\omega$$

Radiation Wave Velocity Potential (7)



$$F(r) = R^2 \omega r^{-1} = \omega \frac{R^2}{r}$$

$$G(\theta) = i \cos \theta$$

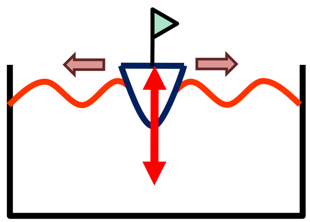
$$\phi_3(r,\theta) = F(r)G(\theta) = \frac{R^2}{r} \omega i \cos \theta$$

$$\therefore \Phi_3 = \xi_3^A \phi_3(r,\theta) e^{i\omega t} = \xi_3^A \frac{R^2}{r} \omega i \cos \theta \cdot e^{i\omega t}$$

Radiation Wave Velocity Potential (8)

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-22~23
Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch 17,Ch18
이승건, 선박운동 조종론, 부산대학교 출판부, 2004, pp93~101, pp91~93
2) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-30~36
Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 104~108)
이승건, 선박운동 조종론, 부산대학교 출판부, 2004, pp93~101

정수중 선박의 강제 운동에 의해 발생한 힘



Radiation Force

Given

- Governing Equation : $\nabla^2 \phi_j = 0 \quad (j = 1, \cdots, 6)$
- Boundary Condition : $-\omega^2 \phi + g\phi_z = 0$; $\dfrac{\partial \phi_j}{\partial n} = i\omega n_j \quad (on \ \ S_B)$

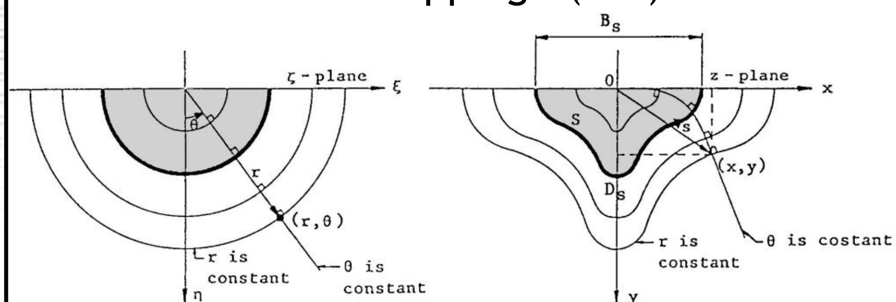
  $\phi_j \propto e^{\mp iky}, \ \ as \ \ y \to \pm\infty$
- Motion of the ship : $\xi_j = \xi_j^A e^{i\omega t} \quad (j = 1, \cdots, 6)$

Find : $\phi_j \quad (j = 1, \cdots, 6)$

Difficult!!!

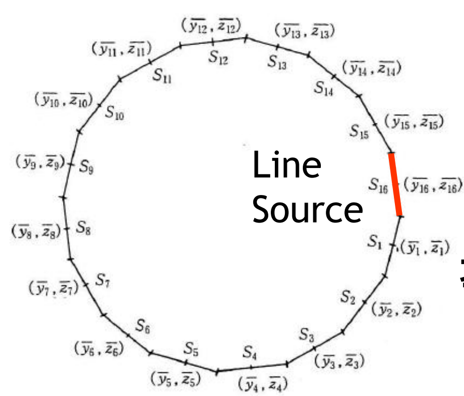
How to solve ?

Lewis Conformal Mapping[1] (2-D)



Mapping Function : $w = 1 + \dfrac{a_1}{\zeta} + \dfrac{a_3}{\zeta^3}$

Singularity Distribution Method[2] (2-D)



각 Line마다 Source 분포.
경계 조건을 만족하도록
Source Strength 구함.

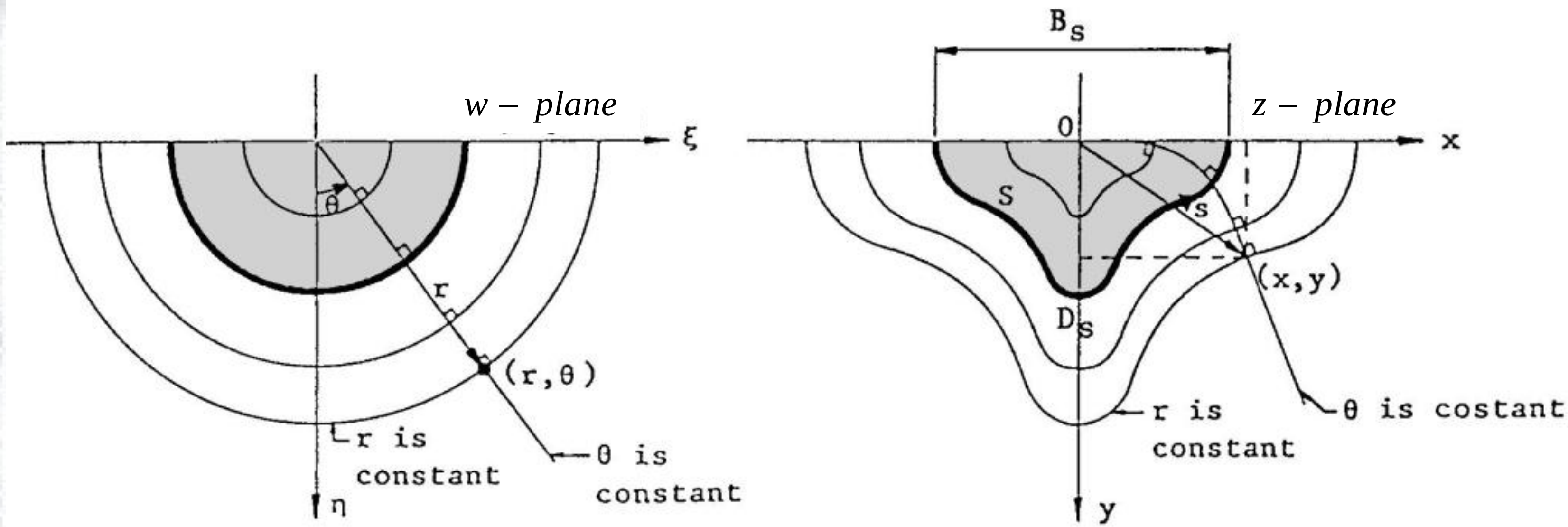
Panel Method (3-D)

· · · (기타 방법)

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-22~23
Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch 17,Ch18
이승건, 선박운동 조종론, 부산대학교 출판부, 2004, pp93~101, pp91~93

Radiation Velocity Potential (9)

✓ Lewis Conformal Mapping[1)] (2-D)



Mapping Function : $w = 1 + \frac{a_1}{\zeta} + \frac{a_3}{\zeta^3}$

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-30~36
Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 104~108)
이승건, 선박운동 조종론, 부산대학교 출판부, 2004, pp93~101

Radiation Velocity Potential (10)

✓ Singularity Distribution Method[2)] (2-D)

Laplace Equation을 만족함

물체 표면에 특이점 (source, doublet, vortex)**을 분포시켜 수학적으로 물체 경계면을 생성시키고, 이들 특이점들의 강도**(Strength)**를 구하여 전체 유장의** velocity potential**을 구하는 방법**

※ Laplace equation on polar coordinate

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right)}_{=0}+\underbrace{\frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}}_{=0}=0$$

Let 2-D source $\phi=\ln r$

$$\frac{\partial \phi}{\partial r}=\frac{1}{r},\quad r\frac{\partial \phi}{\partial r}=1,\quad \frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right)=0,\quad \frac{\partial^2 \phi}{\partial \theta^2}=0$$

Given : **특이점** (source, doublet, vortex)

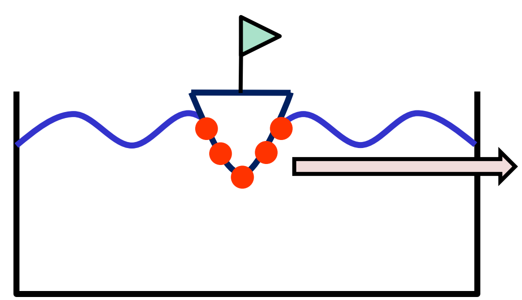
Find : **특이점의 강도**(Strength)

Find Given

$$\phi=\sum_{m=1}^{N} q_m \phi_m$$

N개의 미지수가 존재함

➔ **N개의 경계조건으로 부터 방정식 구함**



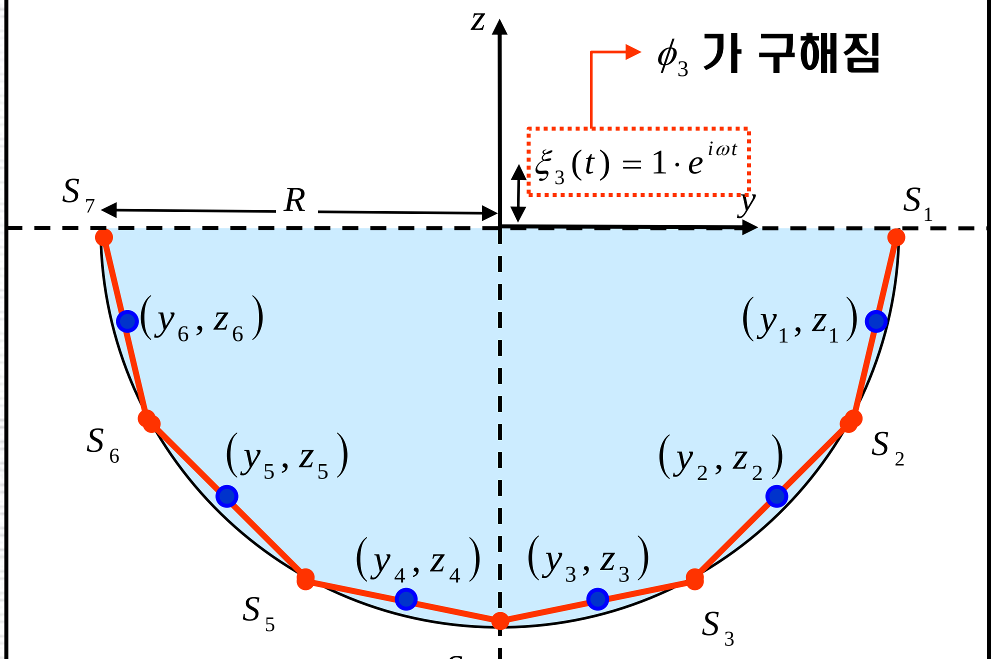
$$\frac{\partial \phi_m}{\partial n}=V_n$$

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-30~36
Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 104~108)
이승건, 선박운동 조종론, 부산대학교 출판부, 2004, pp93~101

※ 2차원 source : $q \ln r$

Radiation Velocity Potential (11)

ex) 반원이 $\xi_3(t) = 1 \cdot e^{i\omega t}$ 로 운동 중일 때, Velocity potential을 구하시오.



(y_m, z_m): $\overline{S_m S_{m+1}}$ 의 중점

Step 1. 반원을 6등분 한다.$(S_1, \cdots, S_6)$

Step 2. 등분된 점과 점 사이를 선으로 연결하고, 각 Line segment에 Line source를 분포시킨다. 한 Line에 분포된 source는 같은 강도(strength)를 가짐

- 점 $(\eta(s), \zeta(s))$ 에 위치한 크기 q_j 인 source

➔ $q_m \ln \sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}$

- Line($\overline{S_m S_{m+1}}$)에 분포된 Line source

➔ $\Delta\phi_m = q_m \int_{\overline{S_m S_{m+1}}} \ln \sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds$

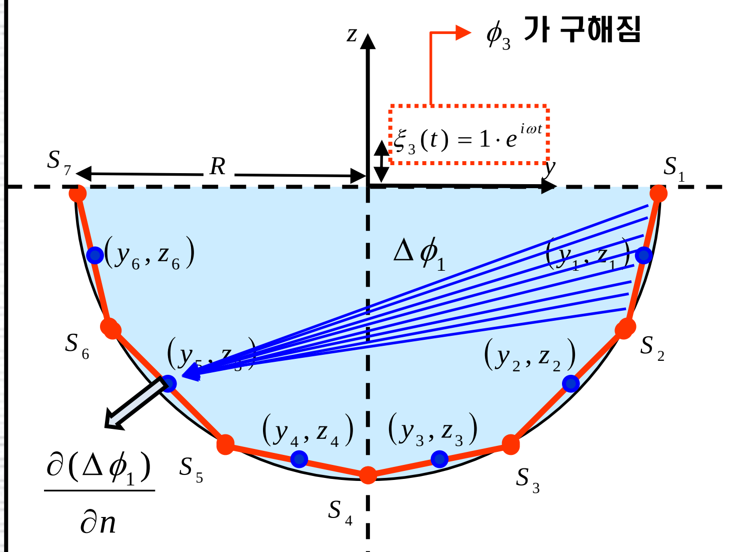
Step 3. Source를 다음과 같이 각 Line Source들의 합으로 가정함

$$\phi_3(y, z) = \sum_{m=1}^{6} \Delta\phi_m = \sum_{m=1}^{6} q_m \int_{\overline{S_m S_{m+1}}} \ln \sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds$$

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-30~36
Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 104~108)
이승건, 선박운동 조종론, 부산대학교 출판부, 2004, pp93~101

Radiation Velocity Potential (12)

ex) **반원이** $\xi_3(t) = 1 \cdot e^{i\omega t}$ **로 운동 중일 때,**
Velocity potential**을 구하시오.**



Step 4. **물체 경계 조건**(Body boundary condition)

$$\frac{\partial \phi_3}{\partial n} = i\omega n_3$$

➔ LHS: $\left.\frac{\partial \phi_3}{\partial n}\right|_{(y_m, z_m)} = q_1 \frac{\partial}{\partial n}\left[\int_{S_1S_2} \ln\sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_m, z_m)}$

$$+ q_2 \frac{\partial}{\partial n}\left[\int_{S_2S_3} \ln\sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_m, z_m)}$$

$$+ \cdots$$

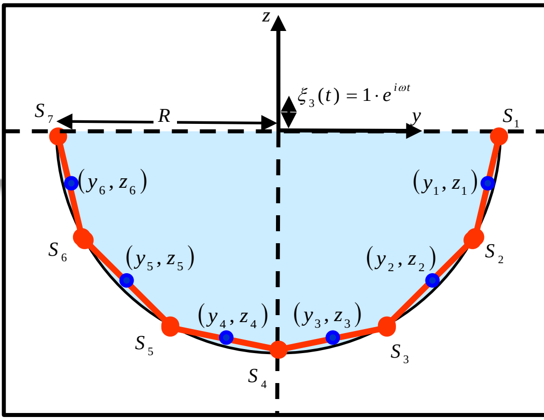
$$+ q_6 \frac{\partial}{\partial n}\left[\int_{S_6S_7} \ln\sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_m, z_m)}$$

➔ RHS : $i\omega n_3 = -i\omega \cos\theta \big|_{(y_m, z_m)}$

$$\phi_3(y, z) = \sum_{m=1}^{6} \Delta\phi_m = \sum_{m=1}^{6} q_m \int_{S_m S_{m+1}} \ln\sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds$$

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-30~36
Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 104~108)
이승건, 선박운동 조종론, 부산대학교 출판부, 2004, pp93~101

Radiation Velocity Potential (13)



물체 경계 조건(Body boundary condition)

$$q_1 \frac{\partial}{\partial n}\left[\int_{\overline{S_1 S_2}} \ln \sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_1,z_1)} + \cdots + q_6 \frac{\partial}{\partial n}\left[\int_{\overline{S_6 S_7}} \ln \sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_1,z_1)} = -i\omega \cos\theta_{(y_1,z_1)}$$

$$q_1 \frac{\partial}{\partial n}\left[\int_{\overline{S_1 S_2}} \ln \sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_2,z_2)} + \cdots + q_6 \frac{\partial}{\partial n}\left[\int_{\overline{S_6 S_7}} \ln \sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_2,z_2)} = -i\omega \cos\theta_{(y_2,z_2)}$$

$$\vdots$$

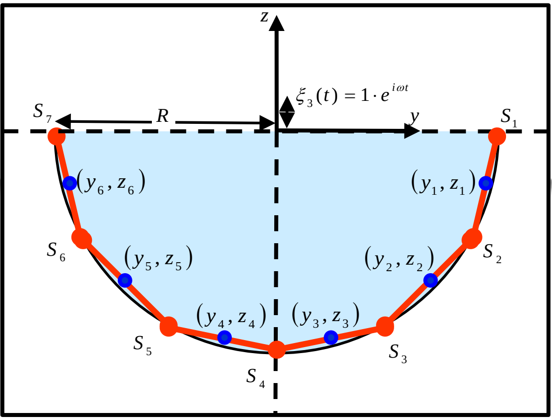
$$q_1 \frac{\partial}{\partial n}\left[\int_{\overline{S_1 S_2}} \ln \sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_6,z_6)} + \cdots + q_6 \frac{\partial}{\partial n}\left[\int_{\overline{S_6 S_7}} \ln \sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_6,z_6)} = -i\omega \cos\theta_{(y_6,z_6)}$$

방정식 : 6개

미지수 : 6개 $q_1, \cdots, q_6$ ⇨ Now we can find the solution !!!

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-30~36
Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 104~108)
이승건, 선박운동 조종론, 부산대학교 출판부, 2004, pp93~101

Radiation Velocity Potential (14)



Step 5. **방정식을** Matrix **형태로 나타내면,**

$$\underbrace{q_1 \frac{\partial}{\partial n}\left[\int_{\overline{S_1S_2}} \ln\sqrt{(y-\eta(s))^2+(z-\zeta(s))^2}\,ds\right]_{(y_1,z_1)}}_{I_{11}} + \cdots + \underbrace{q_6 \frac{\partial}{\partial n}\left[\int_{\overline{S_6S_7}} \ln\sqrt{(y-\eta(s))^2+(z-\zeta(s))^2}\,ds\right]_{(y_1,z_1)}}_{I_{16}} = -i\omega\cos\theta_{(y_1,z_1)}$$

$$\Rightarrow \left.\begin{aligned} q_1 I_{11} + \cdots + q_6 I_{16} &= b_1 \\ q_1 I_{21} + \cdots + q_6 I_{26} &= b_2 \\ &\vdots \\ q_1 I_{61} + \cdots + q_6 I_{66} &= b_6 \end{aligned}\right\} \Rightarrow \mathbf{A}\mathbf{q} = \mathbf{b} \quad \left(\mathbf{A} = \begin{bmatrix} I_{11} & \cdots & I_{16} \\ \vdots & \ddots & \vdots \\ I_{61} & \cdots & I_{66} \end{bmatrix}, \mathbf{q} = \begin{bmatrix} q_1 \\ \vdots \\ q_6 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_6 \end{bmatrix} \right)$$

$$\Downarrow$$

$$\mathbf{q} = \mathbf{A}^{-1}\mathbf{b}$$

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-30~36
Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 104~108)
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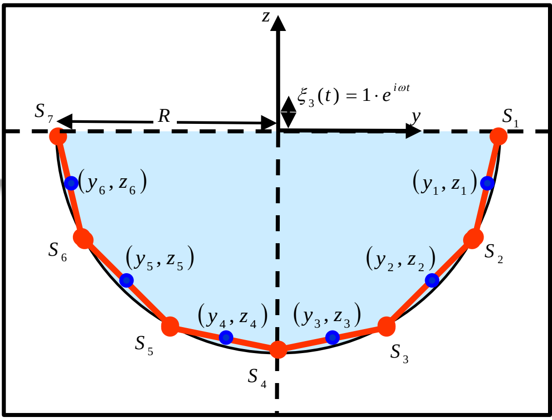
Radiation Velocity Potential (15)



(참고) I_{jk} 의 계산

$$f(y,z) = \ln\sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}$$ **라 하면,**

$$\frac{\partial f(y,z)}{\partial n} = \nabla f \bullet \mathbf{n} = \left(\frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \bullet (n_2, n_3) \qquad \left(\begin{aligned} \frac{\partial f}{\partial y} &= \frac{y-\eta(s)}{(y-\eta(s))^2 + (z-\zeta(s))^2} \\ \frac{\partial f}{\partial z} &= \frac{z-\zeta(s)}{(y-\eta(s))^2 + (z-\zeta(s))^2} \end{aligned}\right)$$

$$\Downarrow$$

$$I_{jk} = \frac{\partial}{\partial n}\left[\int_{\overline{S_k S_{k+1}}} \ln\sqrt{(y-\eta(s))^2 + (z-\zeta(s))^2}\, ds\right]_{(y_j, z_j)}$$

$$= \int_{\overline{S_k S_{k+1}}} \left\{ n_2 \frac{y_j - \eta(s)}{(y_j-\eta(s))^2 + (z_j-\zeta(s))^2} + n_3 \frac{z_j - \zeta(s)}{(y_j-\eta(s))^2 + (z_j-\zeta(s))^2} \right\} ds$$

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft Univ. of Technology, 2001, Ch 7-30~36
Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 104~108)
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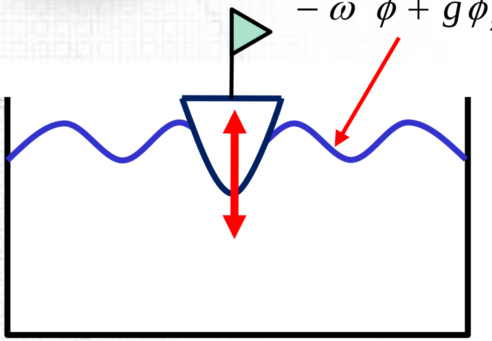
Radiation Wave Velocity Potential (16)

Linearized Free Surface B.C.

$$-\omega^2\phi + g\phi_z = 0 \quad (\text{on} \quad z = 0)$$



따라서, 단순한 형태의 2차원 source $(q \ln r)$ 대신
Free surface condition을 만족하는 Green function을 사용함

ex) Green function introduced by Wehausen and Laitone(1960)

$$G(z,\zeta,t) = \frac{1}{2\pi}\left\{ \ln(z-\zeta) - \ln(z-\bar{\zeta}) + 2\cdot PV\int_0^\infty \frac{e^{-ik(z-\bar{\zeta})}}{\nu - k}dk \right\}\cos\omega t$$

$$- e^{-i\nu(z-\bar{\zeta})}\sin\omega t$$

complex notation : $z = x + iy, \zeta = \xi + i\eta$

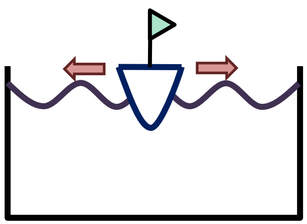
Wave number : $\nu (= \omega^2 / g)$

Wehausen,J.V.,Laitone,E.V.(1960), *Surface Waves*,
Handbuch der hysik,edited by S.Fluegge, Vol.9,Fluid Dynamics 3,Springer Verlag,Berlin,Germany,pp 446~778

1) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 285~290

Diffraction Wave Velocity Potential (1)

산란파에 의한 힘



Diffraction Force

✓ Diffraction wave velocity potential

$$\Phi_D(x, y, z, t) = \phi_D(x, y, z)e^{i\omega t}$$

✓ Boundary Condition[1)]

① Free surface condition

$$-\omega^2\phi_D + g\frac{\partial \phi_D}{\partial z} = 0 \quad (\text{on} \quad z = 0)$$

② Radiation Condition : **파가 무한이 발산하면 소멸됨**

$$\phi_D \propto e^{\mp iky}, \quad as \quad y \rightarrow \pm\infty$$

③ Body boundary condition : **선박 표면에서 유체 입자의 속도가** Zero

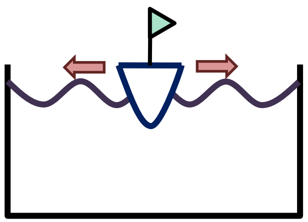
$$V_n = 0 \quad (on \quad S_B) \quad \Rightarrow \quad \frac{\partial(\phi_I + \phi_D)}{\partial n} = 0 \quad \Rightarrow \quad \frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n} \quad (on \quad S_B)$$

S_B : **침수표면**

V_n : **침수표면에 수직인 속도**

Diffraction Wave Velocity Potential (2)

산란파에 의한 힘



Diffraction Force

Given
- Governing Equation : $\nabla^2 \phi_D = 0$
- Boundary Condition : $-\omega^2 \phi_D + g \dfrac{\partial \phi_D}{\partial z} = 0$

  $\phi_D \propto e^{\mp iky}, \quad as \quad y \to \pm\infty$

  $\dfrac{\partial \phi_D}{\partial n} = -\dfrac{\partial \phi_I}{\partial n} \quad (on \quad S_B)$

Find : ϕ_D

ϕ_D 를 직접 구할 수도 있지만, Body Boundary Condition과

Green 2nd Theorem을 사용하여, ϕ_D 를 ϕ_I 와 ϕ_k 로 대체 가능

ϕ_I, ϕ_D are the solutions of Laplace equation.

Both satisfy $\nabla^2 \phi_I = 0$, $\nabla^2 \phi_D = 0$

$$\iint_S \phi_D \frac{\partial \phi_k}{\partial n} dA = \iint_S \phi_k \frac{\partial \phi_D}{\partial n} dA$$

1) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch10.7 (pp 458~462)

Proof) Green Theorem

Divergence Theorem of Gauss[1)]

$$\nabla \bullet \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

(Theorem 1) Divergence Theorem of Gauss
(Transformation Between Triple and Surface Integrals)

Let T be a closed bounded region in space whose boundary is a piecewise smooth orientable surface S.

$\mathbf{F}(x, y, z)$: a vector function that is continuous and has continuous first partial derivatives in T

$$(2) \quad \iiint_T \nabla \bullet \mathbf{F}\, dV = \iint_S \mathbf{F} \bullet \mathbf{n}\, dA$$

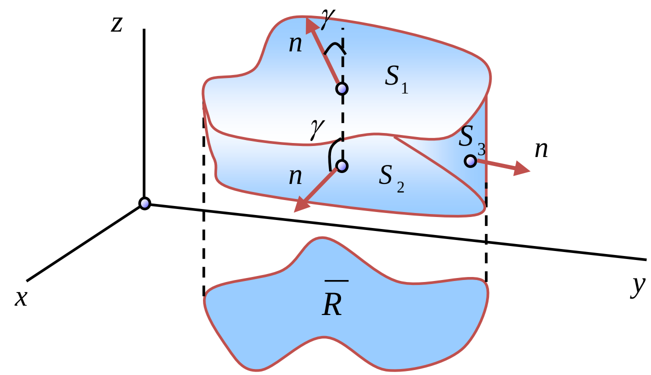


Fig. 250. Example of a special region

Using component, $\mathbf{F} = [F_1, F_2, F_3],\ n = [\cos\alpha, \cos\beta, \cos\gamma]$

$$(2') \quad \iiint_T \left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right) dxdydz = \iint_S (F_1\cos\alpha + F_2\cos\beta + F_3\cos\gamma)\, dA$$

$$= \iint_S (F_1\, dydz + F_2\, dzdx + F_3\, dxdy)$$

Proof) Green Theorem

1) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch10.8 (pp 466~467)

Divergence Theorem : $\iiint_T \nabla \bullet \mathbf{F}\, dV = \iint_S \mathbf{F} \bullet \mathbf{n}\, dA$

(Example 4) Let $\mathbf{F} = f \nabla g$

LHS : $\nabla \bullet \mathbf{F} = \nabla \bullet (f \nabla g) = \nabla \bullet \left(\left[f \frac{\partial g}{\partial x}, f \frac{\partial g}{\partial y}, f \frac{\partial g}{\partial z} \right] \right)$

$$= \left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial x} + f \frac{\partial^2 g}{\partial x^2} \right) + \left(\frac{\partial f}{\partial y}\frac{\partial g}{\partial y} + f \frac{\partial^2 g}{\partial y^2} \right) + \left(\frac{\partial f}{\partial z}\frac{\partial g}{\partial z} + f \frac{\partial^2 g}{\partial z^2} \right)$$

$$= f \nabla^2 g + \nabla f \bullet \nabla g$$

RHS : $\mathbf{F} \bullet \mathbf{n} = \mathbf{n} \bullet \mathbf{F} = \mathbf{n} \bullet (f \nabla g) = f (\mathbf{n} \bullet \nabla g) = f \frac{\partial g}{\partial n}$

(1) Green's first formula

$$\iiint_T \left(f \nabla^2 g + \nabla f \bullet \nabla g \right) dV = \iint_S f \frac{\partial g}{\partial n} dA$$

Proof) Green Theorem

1) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch10.8 (pp 466~467)

(1) Green's first formula

$$\iiint_T \left(f\nabla^2 g + \nabla f \bullet \nabla g\right)dV = \iint_S f\frac{\partial g}{\partial n}dA$$

(Example 4) Let $\mathbf{F} = f\nabla g$

$$\iiint_T \left(f\nabla^2 g + \nabla f \bullet \nabla g\right)dV = \iint_S f\frac{\partial g}{\partial n}dA \quad \text{-->} \text{ ①}$$

Let $\mathbf{F} = g\nabla f$

$$\iiint_T \left(g\nabla^2 f + \nabla g \bullet \nabla f\right)dV = \iint_S g\frac{\partial f}{\partial n}dA \quad \text{-->} \text{ ②}$$

① - ② :

(2) Green's second formula

$$\iiint_T \left(f\nabla^2 g - g\nabla^2 f\right)dV = \iint_S \left(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\right)dA$$

Proof) Green Theorem

1) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch10.8 (pp 466~467)

(2) Green's second formula

$$\iiint_T \left(f \nabla^2 g - g \nabla^2 f \right) dV = \iint_S \left(f \frac{\partial g}{\partial n} - g \frac{\partial f}{\partial n} \right) dA$$

If f, g are the solutions of Laplace equation

Both satisfy $\nabla^2 f = 0 \ , \ \nabla^2 g = 0$

From Green's 2nd formula, we can derive an equation (3)

(3) $$\iint_S \left(f \frac{\partial g}{\partial n} - g \frac{\partial f}{\partial n} \right) dA = 0$$

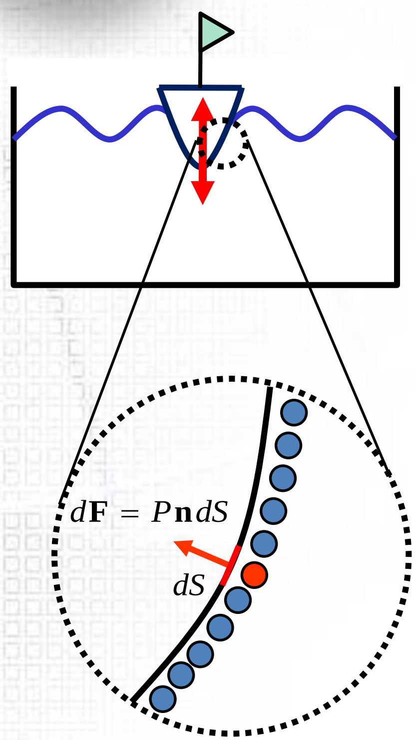
$\Downarrow$ 항을 분리하여, 두 번째 항을 우변으로 넘김

(3') $$\iint_S f \frac{\partial g}{\partial n} dA = \iint_S g \frac{\partial f}{\partial n} dA$$

Step2. Forces & Moments acting on the ship

운동 방정식 유도 – 선박에 작용하는 힘

(변위 : $\mathbf{x} = [\xi_1, \cdots, \xi_6]^T$)
($\mathbf{M}, \mathbf{A}, \mathbf{B}, \mathbf{C}$: 6×6 *Matrix*)



$d\mathbf{F}$: 하나의 유체 입자가 선박 표면에 가하는 힘

dS : 미소 면적

$\mathbf{n}$: 미소 면적의 Normal 벡터

✓ 유체 압력 (From Bernoulli Eq.)

$$\rho \frac{\partial \Phi}{\partial t} + P + \cancel{\frac{1}{2}\rho |\nabla \Phi|^2} + \rho g z = 0$$

선형화

$$P = -\rho g z - \rho \frac{\partial \Phi_T}{\partial t} = -\rho g z - \rho \left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \Phi_D}{\partial t} + \frac{\partial \Phi_R}{\partial t} \right)$$

$$= P_{static} + P_{F.K} + P_D + P_R$$

유체 입자 하나가 표면에 주는 압력

✓ Laplace Equation **Step1**

$$\nabla^2 \Phi = 0$$

Solve

$$\Phi_T = \Phi_I + \Phi_D + \Phi_R$$

Step2

선박의 침수 표면 전체에 대하여 적분
(유체에 의해 선박이 받는 힘과 모멘트)

$$\iint_{S_B} P\mathbf{n}\,dS$$

2-D → 3-D (Strip method)

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{Gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

Linearization

$\mathbf{F}_{Restoring}\ (= -\mathbf{C}\mathbf{x})$ $\quad \mathbf{F}_{exciting}$ $\quad \mathbf{F}_R = -\mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$

added mass — Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = -\mathbf{C}\mathbf{x} + \mathbf{F}_{exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

Step3

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

Motion RAO (Response Amplitude Operator)

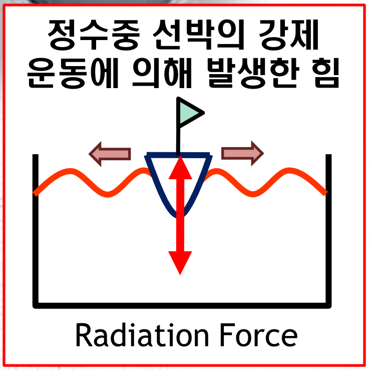
Step4

임의의 길이 x까지만 적분
(선박의 내부에 작용하는 S.F / B.M. 구함)

Shear force, Bending moment

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theroy program“ Seaway for Windows”, Delft University of Technology, 2003, pp30~33
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 295~300

Radiation Force (F_R) (1)



✓ Radiation Wave Velocity Potential

$$\Phi_R(x,y,z,t) = \phi_R(x,y,z)e^{i\omega t}$$

$$\phi_R(x,y,z) = \sum_{j=1}^{6} \underline{\xi_j^A \phi_j(x,y,z)}$$

- B.C.과 Laplace Eq. 으로부터 구한 것
- 변위 ξ_j^A 는 주어진 값 $\left(\xi_j(t) = \xi_j^A e^{i\omega t}\right)$

✓ Radiation Force

$$P_R = -\rho \frac{\partial \Phi_R}{\partial t} = -\rho i\omega \sum_{j=1}^{6} \xi_j^A \phi_j(x,y,z)e^{i\omega t}$$

$$\underline{\mathbf{F}_R = \iint_{S_B} P_R \mathbf{n}\, dS}$$

Consider
k^{th} component
(k=3이면, Heave Force)

$$F_{R,k} = \iint_{S_B} P_R n_k\, dS = \iint_{S_B} \left(-\rho i\omega \sum_{j=1}^{6} \xi_j^A \phi_j(x,y,z)e^{i\omega t} \right) n_k\, dS$$

$$= -\rho \iint_{S_B} \left(\sum_{j=1}^{6} \xi_j^A \phi_j \right) e^{i\omega t} i\omega n_k\, dS$$

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theroy program“ Seaway for Windows”, Delft University of Technology, 2003, pp30~33
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 295~300

Radiation Force (F_R) (2)

(Continue)

$$F_{R,k} = -\rho \iint_{S_B} \left(\sum_{j=1}^{6} \xi_j^A \phi_j \right) e^{i\omega t} i\omega n_k \, dS$$

$\dfrac{\partial \phi_k}{\partial n} = i\omega n_k$ 대입

$$= -\rho \iint_{S_B} \left(\sum_{j=1}^{6} \xi_j^A \phi_j \right) e^{i\omega t} \frac{\partial \phi_k}{\partial n} dS$$

$$= -\rho \left(\iint_{S_B} \xi_1^A \phi_1 e^{i\omega t} \frac{\partial \phi_k}{\partial n} dS + \iint_{S_B} \xi_2^A \phi_2 e^{i\omega t} \frac{\partial \phi_k}{\partial n} dS + \cdots + \iint_{S_B} \xi_6^A \phi_6 e^{i\omega t} \frac{\partial \phi_k}{\partial n} dS \right)$$

$$= -\rho \sum_{j=1}^{6} \left(\iint_{S_B} \xi_j^A \phi_j e^{i\omega t} \frac{\partial \phi_k}{\partial n} dS \right)$$

$$= \sum_{j=1}^{6} \xi_j^A \left(-\rho \iint_{S_B} \phi_j \frac{\partial \phi_k}{\partial n} dS \right) e^{i\omega t}$$

$F_{jk} = -\rho \iint_{S_B} \phi_j \dfrac{\partial \phi_k}{\partial n} dS$ 로 치환

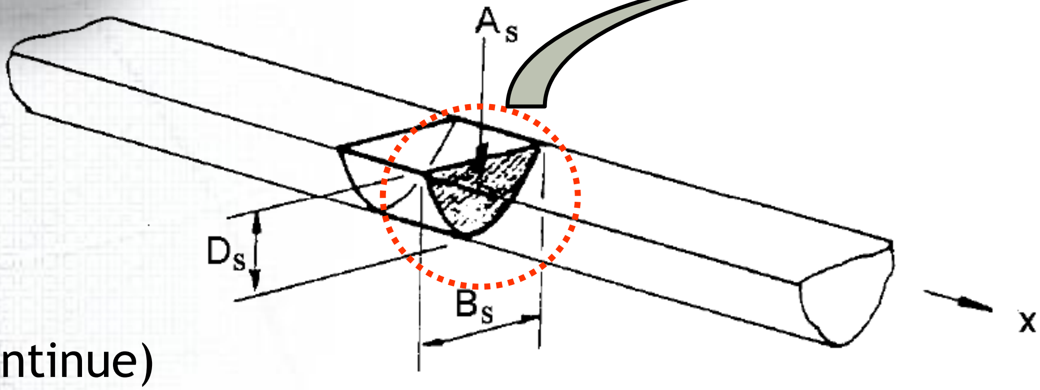
(F_{jk} : j방향 운동으로 인해 나타나는 k방향 힘)

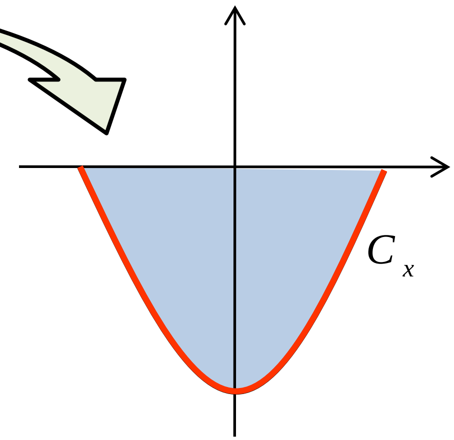
$$= \sum_{j=1}^{6} \xi_j^A F_{jk} e^{i\omega t}$$

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theroy program" Seaway for Windows", Delft University of Technology, 2003, pp30~33
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 295~300

Radiation Force (F_R) (3)

(Slender body)





(Continue)

$$F_{R,k} = \sum_{j=1}^{6} \xi_j^A F_{jk} e^{i\omega t}$$

$$F_{jk} = -\rho \iint_{S_B} \phi_j \frac{\partial \phi_k}{\partial n} dS = \int_0^L \left(-\rho \int_{C_x} \phi_j \frac{\partial \phi_k}{\partial n} dl \right) dx = \underline{\int_0^L f_{jk}\, dx}$$

(2-D 단면에서 구한 힘 또는 모멘트를 길이 방향으로 적분하여 선박 전체에 작용하는 힘 또는 모멘트를 계산 = "Strip Theory")

$$f_{jk} = -\rho \int_{C_x} \phi_j \frac{\partial \phi_k}{\partial n} dl$$ (2-D 단면에서 j방향 운동으로 인해 나타나는 k방향 힘)

$$= \omega^2 a_{jk} - i\omega b_{jk}$$

(2-D 단면의 Added mass)

(2-D 단면의 Damping Coefficient)

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program“ Seaway for Windows”, Delft University of Technology, 2003, pp30~33
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 295~300

Radiation Force (F_R) (4)

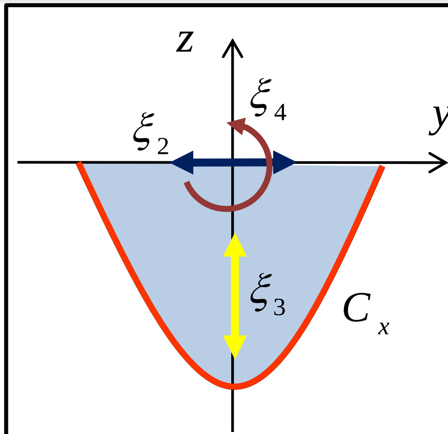
(Continue)

$$f_{jk} = -\rho \int_{c_x} \phi_j \frac{\partial \phi_k}{\partial n} dl$$

$$= \omega^2 a_{jk} - i\omega b_{jk}$$

(2-D 단면의 Added mass)

(2-D 단면의 Damping Coefficient)



변위 : $\xi_j = \xi_j^A e^{i\omega t}$

속도 : $\dot{\xi}_j = \xi_j^A i\omega e^{i\omega t}$

가속도 : $\ddot{\xi}_j = -\xi_j^A \omega^2 e^{i\omega t}$

$$F_{jk} = \int_0^L f_{jk} dx = \omega^2 \int_0^L a_{jk} dx - i\omega \int_0^L b_{jk} dx = \omega^2 A_{jk} - i\omega B_{jk}$$

(Added mass)

(Damping Coefficient)

$$F_{R,k} = \sum_{j=1}^{6} \xi_j^A F_{jk} e^{i\omega t} = \sum_{j=1}^{6} \xi_j^A e^{i\omega t} (\omega^2 A_{jk} - i\omega B_{jk})$$

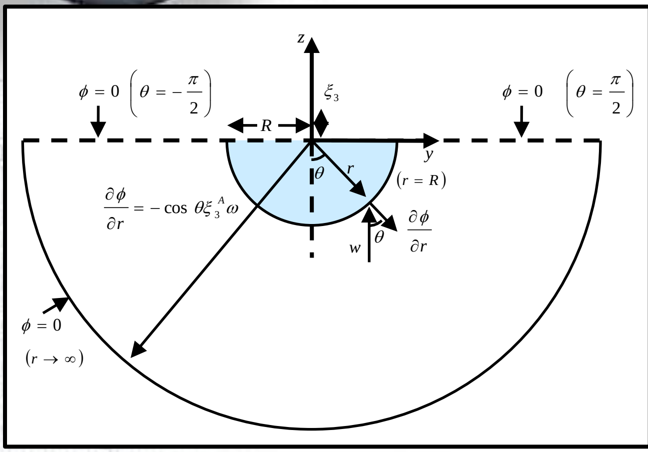
$$= \sum_{j=1}^{6} (\underbrace{\xi_j^A \omega^2 e^{i\omega t}}_{-\ddot{\xi}_j} A_{jk} - \underbrace{\xi_j^A i\omega e^{i\omega t}}_{-\dot{\xi}_j} B_{jk}) = \sum_{j=1}^{6} (-\ddot{\xi}_j A_{jk} - \dot{\xi}_j B_{jk})$$

(가속도에 비례)

(속도에 비례)

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program“ Seaway for Windows”, Delft University of Technology, 2003, pp30~33
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 295~30
3) Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 50~55)

Radiation Force (F_R) (5)



ex) 2차원 반원의 Velocity potential이 주어져 있다고 했을 때, Heave 방향 Added mass 및 Damping Coefficient를 구하시오

- 선박의 운동 변위 : $\xi_3(t) = \xi_3^A e^{i\omega t}$

- Radiation Wave Velocity Potential :

$$\Phi_3(r,\theta,t) = \phi_3(r,\theta)e^{i\omega t} = \xi_3^A \frac{R^2}{r}\omega i \cos\theta \cdot e^{i\omega t}$$

sol) $$f_{33} = -\rho\int_{c_x} \phi_3 \frac{\partial \phi_3}{\partial n} dl \ , (on \quad r = R) \quad \Rightarrow \quad f_{33} = -\rho\int_{c_x} R\omega^2 \cos^2\theta \cdot dl$$

$$\phi_3(r,\theta) = \frac{R^2}{r}\omega i \cos\theta$$

$$\frac{\partial \phi_3}{\partial n} = \frac{\partial \phi_3}{\partial r} = -\frac{R^2}{r^2}\omega i \cos\theta$$

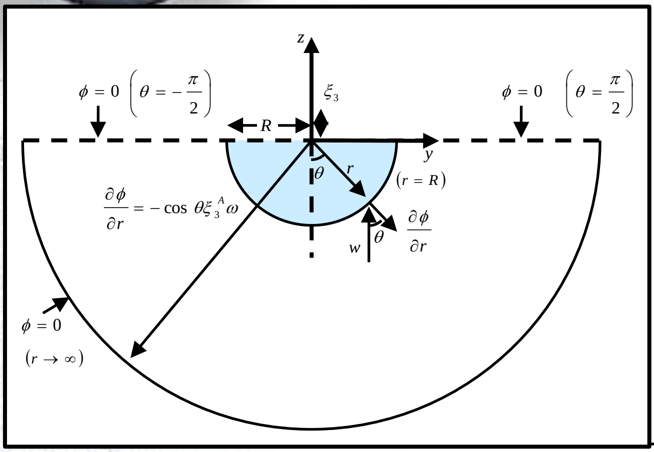
$$\Rightarrow \quad \phi_3 \frac{\partial \phi_3}{\partial n} = \left(\frac{R^2}{r}\omega i \cos\theta\right) \times \left(-\frac{R^2}{r^2}\omega i \cos\theta\right)$$

$$= -\frac{R^4}{r^3}\omega^2 i^2 \cos^2\theta = \frac{R^4}{r^3}\omega^2 \cos^2\theta$$

$$= R\omega^2 \cos^2\theta \quad (on \quad r = R)$$

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program" Seaway for Windows", Delft University of Technology, 2003, pp30~33
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 295~300
3) Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 50~55)

Radiation Force (F_R) (6)



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$$\Phi_3(r,\theta,t) = \phi_3(r,\theta)e^{i\omega t} = \xi_3^A \frac{R^2}{r}\omega i \cos\theta \cdot e^{i\omega t}$$

sol) $$f_{33} = -\rho\int_{c_x} \phi_3 \frac{\partial \phi_3}{\partial n} dl$$

$$= -\rho\int_{c_x} R\omega^2 \cos^2\theta dl$$

$$dl = Rd\theta$$

$$f_{33} = -\rho\int_{-\pi/2}^{\pi/2} R\omega^2 \cos^2\theta \cdot Rd\theta$$

$$= -\rho R^2\omega^2\int_{-\pi/2}^{\pi/2} \cos^2\theta d\theta$$

$$= -\rho R^2\omega^2\int_{-\pi/2}^{\pi/2} \frac{1+\cos 2\theta}{2} d\theta$$

$$= -\rho R^2\omega^2\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_{-\pi/2}^{\pi/2}$$

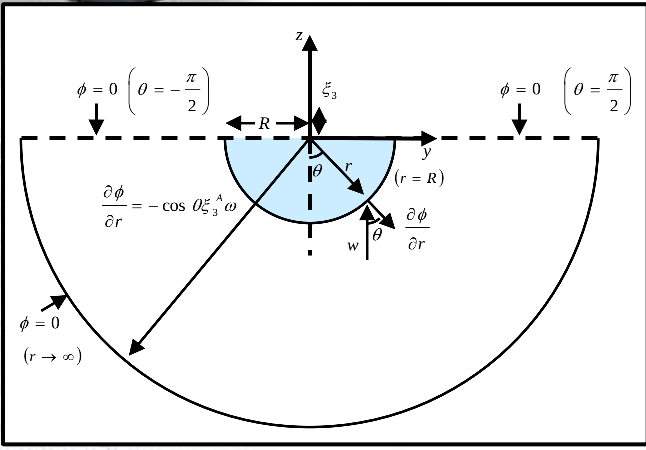
$$= -\rho R^2\omega^2 \frac{\pi}{2}$$

$$= -\omega^2 \underbrace{\left(\frac{\pi R^2}{2}\rho\right)}_{a_{33}}$$

(반원 단면의 질량과 동일함)

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program“ Seaway for Windows”, Delft University of Technology, 2003, pp30~33
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 295~300
3) Faltinsen, O.M. , Sea loads on ships and offshore structures, Cambridge Univ. Press, 1998, Ch3 (pp 50~55)

Radiation Force (F_R) (7)



ex) 2차원 반원의 Velocity potential이 주어져 있다고 했을 때, Heave 방향 Added mass 및 Damping Coefficient를 구하시오

- 선박의 운동 변위 : $\xi_3(t) = \xi_3^A e^{i\omega t}$

- Radiation Wave Velocity Potential :

$$\Phi_3(r,\theta,t) = \phi_3(r,\theta)e^{i\omega t} = \xi_3^A \frac{R^2}{r} \omega i \cos\theta \cdot e^{i\omega t}$$

sol)

$$f_{33} = -\omega^2 \left(\frac{\pi R^2}{2} \rho \right)$$

$$F_{33} = \xi_3^A e^{i\omega t} f_{33} = \underbrace{-\xi_3^A \omega^2 e^{i\omega t}}_{= \ddot{\xi}_3} \underbrace{\left(\frac{\pi R^2}{2} \rho \right)}_{= a_{33}} = \ddot{\xi}_3 a_{33}$$

변위 : $\xi_3(t) = \xi_3^A e^{i\omega t}$

속도 : $\dot{\xi}_3(t) = \xi_3^A i\omega e^{i\omega t}$

가속도 : $\ddot{\xi}_3(t) = -\xi_3^A \omega^2 e^{i\omega t}$

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001,8-5~10

Radiation Force (F_R) (8)

How to find added mass and damping coefficient ???

단면의 정보로 부터 선박의 added mass**와** Damping Coefficient **구하기 위해서는**
각 단면의 $a_{jk}, b_{jk}\ (j,k=1,\cdots,6)$ **를 구한 뒤, 길이 방향으로 적분한다.** (Strip Theory)

- Radiation wave velocity potential (ϕ_j : **선박의** j**방향 운동변위가 1일 때** Velocity Potential)

$$\boxed{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6} \xrightarrow{\text{대입}} f_{jk} = -\rho\int_{c_x} \phi_j \frac{\partial \phi_k}{\partial n} dl = \omega^2 a_{jk} - i\omega b_{jk}$$

- Added mass component

$$A_{jk} = \int_0^L a_{jk}\,dx$$

- Damping coefficient component

$$B_{jk} = \int_0^L b_{jk}\,dx$$

- Added mass matrix

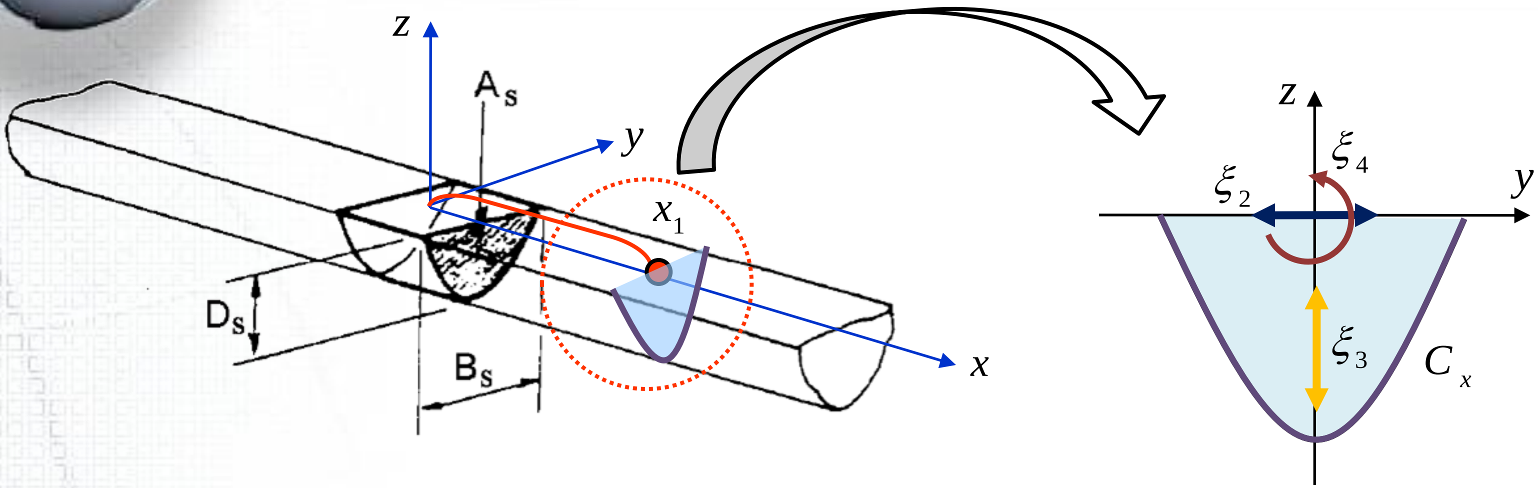
$$\mathbf{A}_{6\times6} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{16} \\ A_{21} & A_{22} & & \\ \vdots & & \ddots & \vdots \\ A_{61} & & & A_{66} \end{bmatrix}$$

- Damping coefficient matrix

$$\mathbf{B}_{6\times6} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{16} \\ B_{21} & B_{22} & & \\ \vdots & & \ddots & \vdots \\ B_{61} & & & B_{66} \end{bmatrix}$$

6개 Velocity potential을 모두 구해야 Matrix를 구할 수 있음

Strip Theory : Definition & Assumption



✓ Strip Theory

: 각 2차원 단면의 유체력 계수 (Added mass, Damping Coefficient) 및 Wave exciting force를 구한 후, 이를 길이 방향으로 적분하여 전체의 유체력을 구하는 근사적 방법

✓ Assumption

(1) Resulting motion will be small

(2) The hull is slender

(3) Forward speed of the ship should be relatively low

(4) The frequency of encounter should not be too low or too high

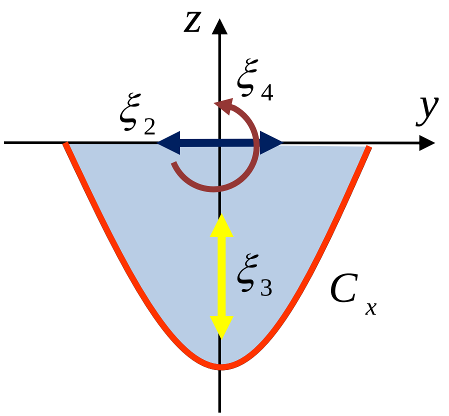
(5) The hull sections are wall-sided at the waterline

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001,8-5~10

Radiation Force (F_R) (9)

다음 중 2-D 단면에서 구할 수 있는 것은? (ϕ_j : 선박의 j방향 운동변위가 1일 때 Velocity Potential)

$\cancel{\phi_1}$ ⋮ ϕ_2 ⋮ ϕ_3 ⋮ ϕ_4 ⋮ $\cancel{\phi_5}$ ⋮ $\cancel{\phi_6}$



y축 병진 운동 : $\xi_2(t) = \xi_2^A e^{i\omega t}$

z축 병진 운동 : $\xi_3(t) = \xi_3^A e^{i\omega t}$

x축 회전 운동 : $\xi_4(t) = \xi_4^A e^{i\omega t}$

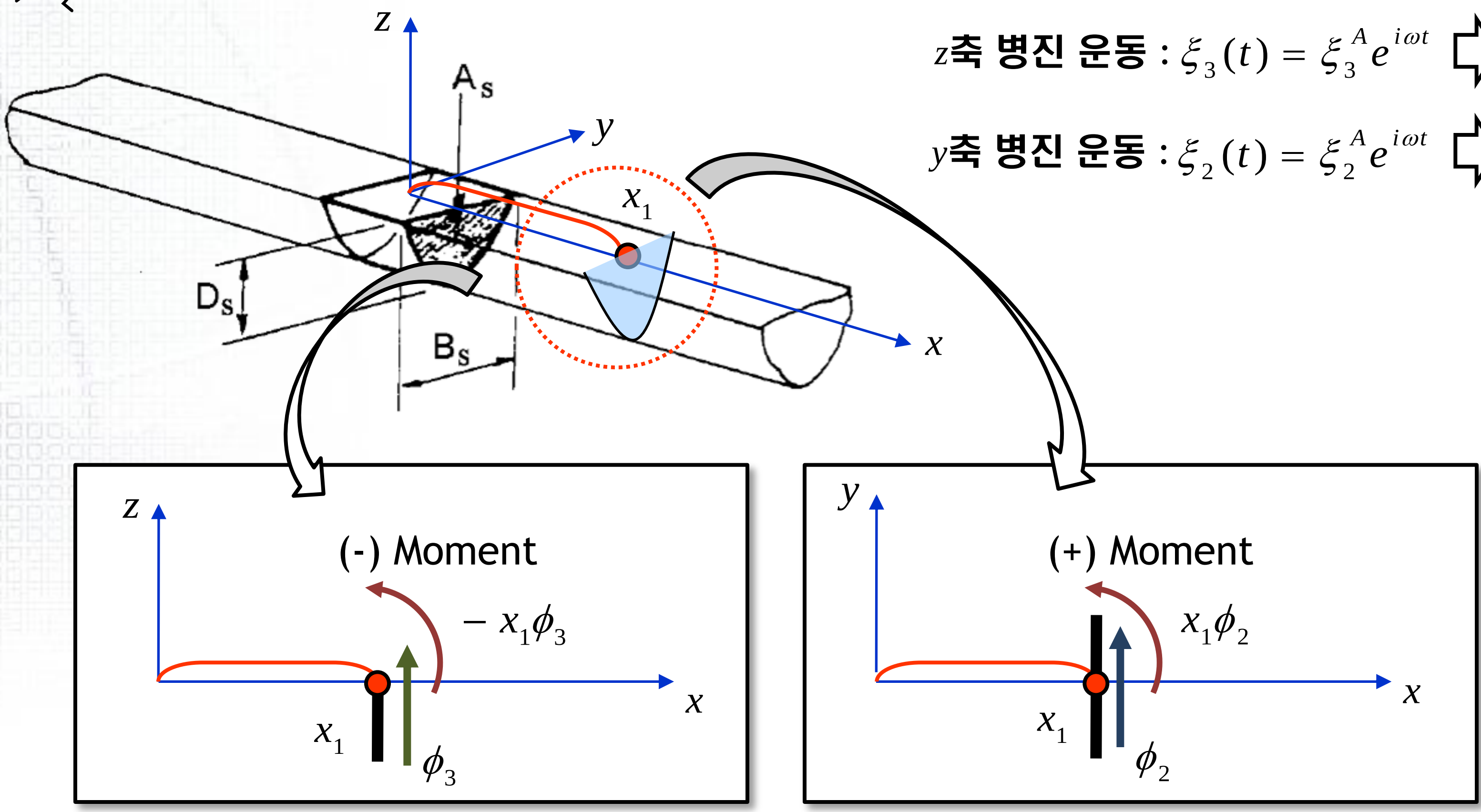
1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001,8-5~10

Radiation Force (F_R) (10)

ϕ_1, ϕ_5, ϕ_6 은 어떻게 구할 수 있을까? (ϕ_j : 선박의 j방향 운동변위가 1일 때 Velocity Potential)

z축 병진 운동 : $\xi_3(t) = \xi_3^A e^{i\omega t}$ ⇨ $\phi_3,\ \phi_5 = -x_1\phi_3$

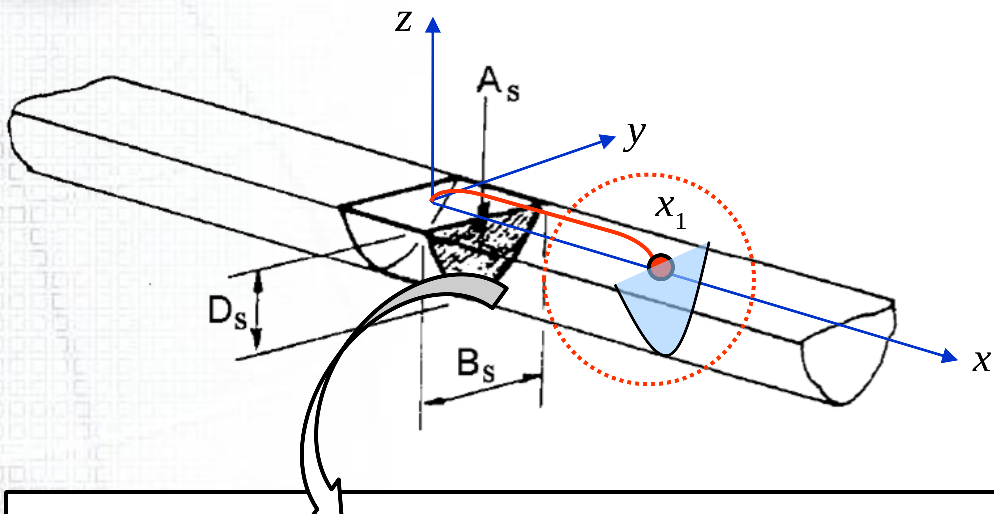
y축 병진 운동 : $\xi_2(t) = \xi_2^A e^{i\omega t}$ ⇨ $\phi_2,\ \phi_6 = x_1\phi_2$

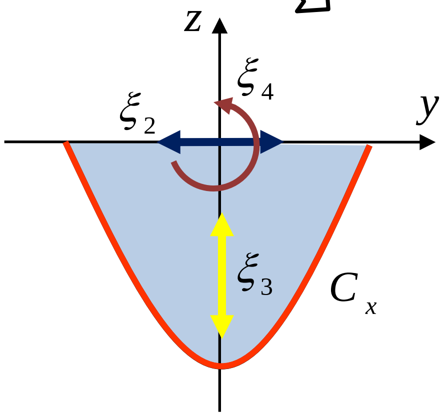


※ ϕ_1 은 일반적인 2-D strip theory로 구할 수 없다.
따라서, 경험식 또는 길이 방향 단면을 사용하여 계산함

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001,8-5~10

Radiation Force (F_R) (11)





y축 병진 운동 : $\xi_2(t) = \xi_2^A e^{i\omega t}$

z축 병진 운동 : $\xi_3(t) = \xi_3^A e^{i\omega t}$

x축 회전 운동 : $\xi_4(t) = \xi_4^A e^{i\omega t}$

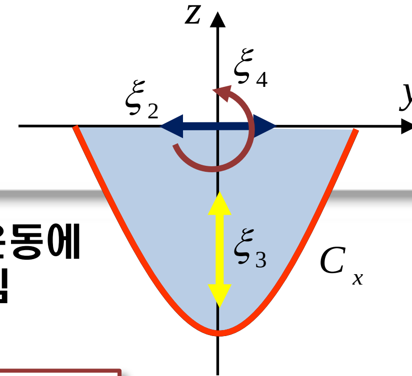
✓ Conclusion

2-D 단면의 세 velocity potential ϕ_2, ϕ_3, ϕ_4 을 구하고, $\phi_5 = -x\phi_3$, $\phi_6 = x\phi_2$ 의 관계식을 사용하여 다른 velocity potential을 구한다.

즉, 2-D 단면의 ϕ_2, ϕ_3, ϕ_4 만 구하면 된다.

1) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001,8-5~10

Radiation Force (F_R) (12)



Given : ϕ_2, ϕ_3, ϕ_4

$$\left(f_{jk} = -\rho \int_{c_x} \phi_j \frac{\partial \phi_k}{\partial n} dl \right)$$

※ f_{jk} : 2-D 단면에서 j 방향 운동에 의해 나타나는 k 방향 힘

$$f_{22} = -\rho \int_{c_x} \phi_2 \frac{\partial \phi_2}{\partial n} dl = \boxed{\omega^2 a_{22} - i\omega b_{22}}, \quad f_{33} = -\rho \int_{c_x} \phi_3 \frac{\partial \phi_3}{\partial n} dl = \boxed{\omega^2 a_{33} - i\omega b_{33}}$$

$$f_{44} = -\rho \int_{c_x} \phi_4 \frac{\partial \phi_4}{\partial n} dl = \boxed{\omega^2 a_{44} - i\omega b_{44}}, \quad f_{24} = -\rho \int_{c_x} \phi_2 \frac{\partial \phi_4}{\partial n} dl = \boxed{\omega^2 a_{24} - i\omega b_{24}}$$

$$f_{55} = -\rho \int_{c_x} \phi_5 \frac{\partial \phi_5}{\partial n} dl = -\rho \int_{c_x} (-x\phi_3) \frac{\partial (-x\phi_3)}{\partial n} dl = -\rho x^2 \int_{c_x} \phi_3 \frac{\partial \phi_3}{\partial n} dl = \boxed{x^2 \left(\omega^2 a_{33} - i\omega b_{33} \right)}$$

$$f_{66} = -\rho \int_{c_x} \phi_6 \frac{\partial \phi_6}{\partial n} dl = -\rho \int_{c_x} (x\phi_2) \frac{\partial (x\phi_2)}{\partial n} dl = -\rho x^2 \int_{c_x} \phi_2 \frac{\partial \phi_2}{\partial n} dl = \boxed{x^2 \left(\omega^2 a_{22} - i\omega b_{22} \right)}$$

$$f_{35} = -\rho \int_{c_x} \phi_3 \frac{\partial \phi_5}{\partial n} dl = -\rho \int_{c_x} \phi_3 \frac{\partial (-x\phi_3)}{\partial n} dl = -\rho(-x) \int_{c_x} \phi_3 \frac{\partial \phi_3}{\partial n} dl = \boxed{-x \left(\omega^2 a_{33} - i\omega b_{33} \right)}$$

$$f_{46} = -\rho \int_{c_x} \phi_4 \frac{\partial \phi_6}{\partial n} dl = -\rho \int_{c_x} \phi_4 \frac{\partial (x\phi_2)}{\partial n} dl = -\rho x \int_{c_x} \phi_4 \frac{\partial \phi_2}{\partial n} dl = \boxed{-x \left(\omega^2 a_{42} - i\omega b_{42} \right)}$$

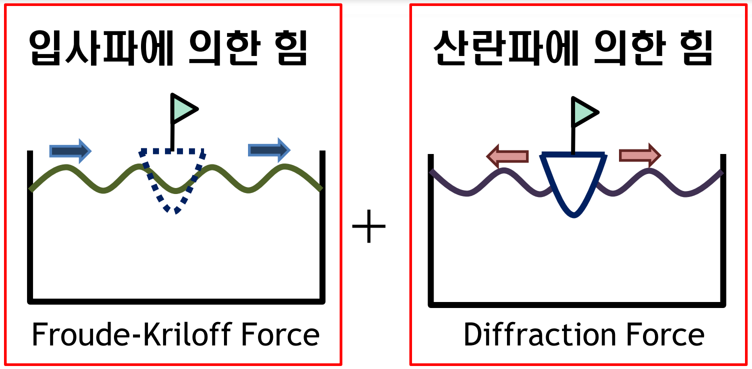
$(a_{22}, b_{22}), (a_{24}, b_{24}), (a_{33}, b_{33}), (a_{44}, b_{44})$ 만 알고 있으면,
added mass 및 damping coefficient를 구할 수 있다.

※ $\rho \int_{c_x} \phi_2 \frac{\partial \phi_4}{\partial n} dl = \rho \int_{c_x} \phi_4 \frac{\partial \phi_2}{\partial n} dl$

이므로, $a_{24} = a_{42}$, $b_{24} = b_{42}$

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program“ Seaway for Windows”, Delft University of Technology, 2003, pp36~38
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 300~307

Froude-Krylov Force & Diffraction Force (1)



✓ Incident Wave & Diffraction Velocity Potential

$$\Phi_I(x,y,z,t)=\phi_I(x,y,z)e^{i\omega t}$$

$$\left(\phi_I(x,y,z)=-\frac{g}{\omega}\eta_0 e^{-ik(x\cos\mu-y\sin\mu)}e^{kz}\right)$$

$$\Phi_D(x,y,z,t)=\phi_D(x,y,z)e^{i\omega t}\qquad\left(\text{Body B.C. : }\frac{\partial\phi_D}{\partial n}=-\frac{\partial\phi_I}{\partial n}\quad(on\quad S_B)\right)$$

✓ Froude Krylov Force & Diffraction Force

$$P_{FK}=-\rho\frac{\partial\Phi_I}{\partial t}=-\rho\phi_I(x,y,z)i\omega e^{i\omega t}\ ,\quad P_D=-\rho\frac{\partial\Phi_D}{\partial t}=-\rho\phi_D(x,y,z)i\omega e^{i\omega t}$$

$$\mathbf{F}_{FK}+\mathbf{F}_D=\iint_{S_B}(P_{FK}+P_D)\mathbf{n}\,dS$$

Consider
k^{th} component
(k=3이면, Heave Force)

$$F_{FK,k}+F_{D,k}=\iint_{S_B}(P_{FK}+P_D)n_k\,dS=-\rho\iint_{S_B}(\phi_I+\phi_D)e^{i\omega t}i\omega n_k\,dS$$

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program" Seaway for Windows", Delft University of Technology, 2003, pp36~38
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 300~307

Froude-Krylov Force & Diffraction Force (2)

(Continue)

Already found

k^{th} radiation velocity potential : ϕ_k

Incident wave velocity potential : $\phi_I = -\frac{g}{\omega}\eta_0 e^{-ik(x\cos\mu - y\sin\mu)} e^{kz}$

⇓ 대입

$$F_{FK,k} + F_{D,k} = -\rho e^{i\omega t}\iint_{S_B}\left(\phi_I \frac{\partial\phi_k}{\partial n} - \phi_k \frac{\partial\phi_I}{\partial n}\right)dS$$

$$= -\rho e^{i\omega t}\int_L \int_{C_x}\left(\phi_I \frac{\partial\phi_k}{\partial n} - \phi_k \frac{\partial\phi_I}{\partial n}\right)dl\,dx$$

$$= -\rho e^{i\omega t}\int_L (f_k + h_k)dx$$

$f_k = \int_{C_x}\phi_I \frac{\partial\phi_k}{\partial n}dl$: 2-D 단면에 작용하는 Froude-Krylov force

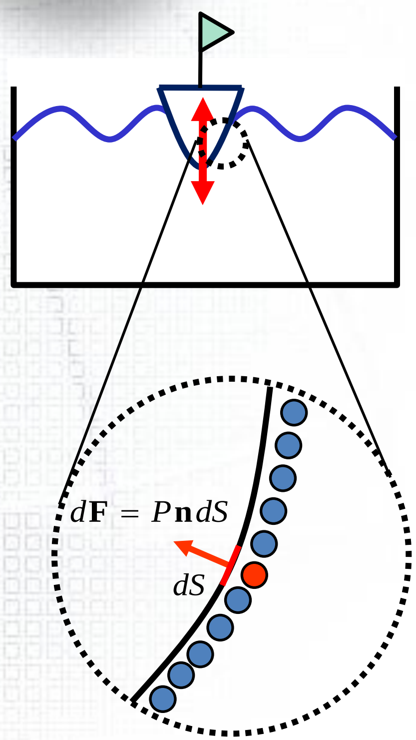
$h_k = -\int_{C_x}\phi_k \frac{\partial\phi_I}{\partial n}dl$: 2-D 단면에 작용하는 Diffraction force

Step3.
6DOF Equations of Ship Motion

운동 방정식 유도 – 선박에 작용하는 힘

(변위 : $\mathbf{x} = [\xi_1, \cdots, \xi_6]^T$)

($\mathbf{M}, \mathbf{A}, \mathbf{B}, \mathbf{C}$: 6×6 *Matrix*)



$d\mathbf{F}$: 하나의 유체 입자가 선박 표면에 가하는 힘

dS : 미소 면적

$\mathbf{n}$: 미소 면적의 Normal 벡터

✓ 유체 압력 (From Bernoulli Eq.)

$$\rho\frac{\partial\Phi}{\partial t} + P + \cancel{\frac{1}{2}\rho\left|\nabla\Phi\right|^2} + \rho g z = 0$$

선형화

✓ Laplace Equation **Step1**

$$\nabla^2\Phi = 0$$

Solve

$$\Phi_T = \Phi_I + \Phi_D + \Phi_R$$

$$P = -\rho g z - \rho\frac{\partial\Phi_T}{\partial t} = -\rho g z - \rho\left(\frac{\partial\Phi_I}{\partial t} + \frac{\partial\Phi_D}{\partial t} + \frac{\partial\Phi_R}{\partial t}\right)$$

$$= P_{static} + P_{F.K} + P_D + P_R$$

유체 입자 하나가 표면에 주는 압력

선박의 침수 표면 전체에 대하여 적분
(유체에 의해 선박이 받는 힘과 모멘트)

$$\iint_{S_B} P\mathbf{n}dS$$

Step2

2-D → 3-D (Strip method)

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{Gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

Linearization

$\mathbf{F}_{Restoring}$ $(= -\mathbf{C}\mathbf{x})$ $\mathbf{F}_{exciting}$ $\mathbf{F}_R = -\mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$

added mass

Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = -\mathbf{C}\mathbf{x} + \mathbf{F}_{exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

Step3

Motion RAO (Response Amplitude Operator)

임의의 길이 x까지만 적분
(선박의 내부에 작용하는 S.F / B.M. 구함)

Step4

Shear force, Bending moment

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program" Seaway for Windows", Delft University of Technology, 2003, pp38~42
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 307~311
3) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001,pp8-1~4

6DOF Equations of Ship Motion (1)

✓ 6DOF Equations of Ship Motion : 6 coupled equation

(변위 : $\mathbf{x} = [\xi_1, \cdots, \xi_6]^T$)

($\mathbf{M}, \mathbf{A}, \mathbf{B}, \mathbf{C}$: 6×6 *Matrix*)

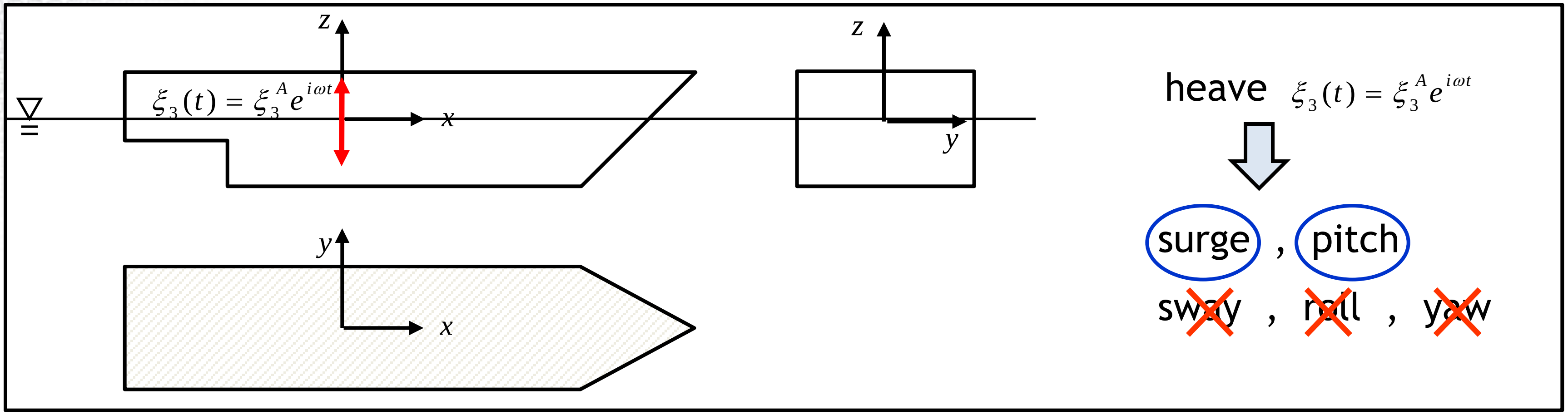
$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

✓ Assumption

1. Slender body (**선박의 길이에 비해 폭이 작음**)
   ➔ **물체의** x**축 병진 운동에 의한** Velocity potential ϕ_1 **이 작음** (Surge **운동은 독립적으로 취급**)
2. Lateral symmetry (symmetric about xz-plane) & small amplitude motion
   ➔ **물체 운동이** **종운동**(Longitudinal motion) **과** **횡운동**(Transverse motion)**으로 나뉨**

surge,heave,pitch ⟺ sway,roll,yaw

서로 영향을 주지 않음



1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program" Seaway for Windows", Delft University of Technology, 2003, pp38~42
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 307~311
3) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001,pp8-1~4

6DOF Equations of Ship Motion (2)

✓ 6DOF Equations of Ship Motion
: 3 kinds of coupled motions (surge-heave-pitch, sway-roll-yaw)

$$\mathbf{x} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & mz_c & 0 \\ 0 & m & 0 & -mz_c & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mz_c & 0 & I_{xx} & 0 & -I_{xz} \\ mz_c & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & -I_{zx} & 0 & I_{zz} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} B_{11} & 0 & B_{13} & 0 & B_{15} & 0 \\ 0 & B_{22} & 0 & B_{24} & 0 & B_{26} \\ B_{31} & 0 & B_{33} & 0 & B_{35} & 0 \\ 0 & B_{42} & 0 & B_{44} & 0 & B_{46} \\ B_{51} & 0 & B_{53} & 0 & B_{55} & 0 \\ 0 & B_{62} & 0 & B_{64} & 0 & B_{66} \end{bmatrix}$$

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

$$\mathbf{F}_{exciting} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & C_{53} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program" Seaway for Windows", Delft University of Technology, 2003, pp38~42
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 307~311
3) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001,pp8-1~4

6DOF Equations of Ship Motion (3)

✓ 6DOF Equations of Ship Motion

: 3 kinds of coupled motions (surge-heave-pitch, sway-roll-yaw)

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & mz_c & 0 \\ 0 & m & 0 & -mz_c & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mz_c & 0 & I_{xx} & 0 & -I_{xz} \\ mz_c & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & -I_{zx} & 0 & I_{zz} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} B_{11} & 0 & B_{13} & 0 & B_{15} & 0 \\ 0 & B_{22} & 0 & B_{24} & 0 & B_{26} \\ B_{31} & 0 & B_{33} & 0 & B_{35} & 0 \\ 0 & B_{42} & 0 & B_{44} & 0 & B_{46} \\ B_{51} & 0 & B_{53} & 0 & B_{55} & 0 \\ 0 & B_{62} & 0 & B_{64} & 0 & B_{66} \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix}$$

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting} \qquad \mathbf{F}_{exciting} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & C_{53} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

➔ heave-pitch motion :

$$\boxed{\begin{bmatrix} m + A_{33} & A_{35} \\ A_{53} & A_{55} + I_{xx} \end{bmatrix}} \begin{bmatrix} \ddot{\xi}_3 \\ \ddot{\xi}_5 \end{bmatrix} + \boxed{\begin{bmatrix} B_{33} & B_{35} \\ B_{53} & B_{55} \end{bmatrix}} \begin{bmatrix} \dot{\xi}_3 \\ \dot{\xi}_5 \end{bmatrix} + \begin{bmatrix} C_{33} & C_{35} \\ C_{53} & C_{55} \end{bmatrix} \begin{bmatrix} \xi_3 \\ \xi_5 \end{bmatrix} = \boxed{\begin{bmatrix} F_3 \\ F_5 \end{bmatrix}}$$

Heave & Pitch

안의 값은 Library File에서 구할 수 있음

$$\left(A_{jk}^{0} = \int_L a_{jk}\, dx \,,\, B_{jk}^{0} = \int_L b_{jk}\, dx \right)$$

$$A_{33} = \int_L a_{33}\, dx - \frac{U}{\omega^2} b_{33}^{A}$$

$$B_{33} = \int_L b_{33}\, dx + U a_{33}^{A}$$

$$A_{35} = -\int_L x a_{33}\, dx - \frac{U}{\omega^2} B_{33}^{0} + \frac{U}{\omega^2} x_A b_{33}^{A} - \frac{U^2}{\omega^2} a_{33}^{A}$$

$$B_{35} = -\int_L x b_{33}\, dx + U A_{33}^{0} - U x_A a_{33}^{A} - \frac{U^2}{\omega^2} b_{33}^{A}$$

$$A_{53} = -\int_L x a_{33}\, dx + \frac{U}{\omega^2} B_{33}^{0} + \frac{U}{\omega^2} x_A b_{33}^{A}$$

$$B_{53} = -\int_L x b_{33}\, dx - U A_{33}^{0} - U x_A a_{33}^{A}$$

$$A_{55} = \int_L x^2 a_{33}\, dx + \frac{U^2}{\omega^2} A_{33}^{0} - \frac{U}{\omega^2} x_A b_{33}^{A} + \frac{U^2}{\omega^2} x_A a_{33}^{A}$$

$$B_{55} = \int_L x^2 b_{33}\, dx + \frac{U^2}{\omega^2} B_{33}^{0} + U x_A^2 a_{33}^{A} + \frac{U^2}{\omega^2} x_A b_{33}^{A}$$

$$F_3 = \rho\alpha \int_L \left(f_3 + h_3 \right) dx + \rho\alpha \frac{U}{i\omega} h_3^{A}$$

$$F_5 = \rho\alpha \int_L \left[x\left(f_3 + h_3 \right) + \rho\alpha \frac{U}{i\omega} h_3 \right] dx - \rho\alpha \frac{U}{i\omega} x_A h_3^{A}$$

U : 선박의 전진 속도	h_j : Sectional Diffraction force (j[th] mode)
ρ : 유체의 밀도	ω : Encounter wave frequency
α : Wave amplitude	$x_A, a_{jk}^{A}, b_{jk}^{A}$: Values at the aftermost section
f_j : Sectional Froude Krylov force (j[th] mode)	

1) Journee, J.M.J. , Adegeest, L.J.M. ,Theoretical Manual of Strip Theory program" Seaway for Windows", Delft University of Technology, 2003, pp38~42
2) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 307~311
3) Journee, J.M.J. , Massie, W.W. , Offshore Hydrodynamics, Delft University of Technology, 2001,pp8-1~4

6DOF Equations of Ship Motion (4)

✓ 6DOF Equations of Ship Motion
: 3 kinds of coupled motions (surge-heave-pitch, sway-roll-yaw)

$$(\mathbf{M}+\mathbf{A})\ddot{\mathbf{x}}+\mathbf{B}\dot{\mathbf{x}}+\mathbf{C}\mathbf{x}=\mathbf{F}_{exciting}$$

$$\mathbf{M}=\begin{bmatrix} m & 0 & 0 & 0 & mz_c & 0 \\ 0 & m & 0 & -mz_c & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mz_c & 0 & I_{xx} & 0 & -I_{xz} \\ mz_c & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & -I_{zx} & 0 & I_{zz} \end{bmatrix} \quad \mathbf{B}=\begin{bmatrix} B_{11} & 0 & B_{13} & 0 & B_{15} & 0 \\ 0 & B_{22} & 0 & B_{24} & 0 & B_{26} \\ B_{31} & 0 & B_{33} & 0 & B_{35} & 0 \\ 0 & B_{42} & 0 & B_{44} & 0 & B_{46} \\ B_{51} & 0 & B_{53} & 0 & B_{55} & 0 \\ 0 & B_{62} & 0 & B_{64} & 0 & B_{66} \end{bmatrix}$$

$$\mathbf{x}=\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix} \quad \mathbf{F}_{exciting}=\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}$$

$$\mathbf{A}=\begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix} \quad \mathbf{C}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & C_{53} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

➔ sway-roll-yaw :

$$\boxed{\begin{bmatrix} m+A_{22} & -mz_c+A_{24} & A_{26} \\ -mz_c+A_{42} & I_{yy}+A_{44} & -I_{xz}+A_{46} \\ A_{62} & -I_{zx}+A_{64} & I_{zz}+A_{66} \end{bmatrix}}\begin{bmatrix} \ddot{\xi}_2 \\ \ddot{\xi}_4 \\ \ddot{\xi}_6 \end{bmatrix}+\boxed{\begin{bmatrix} B_{22} & B_{24} & B_{26} \\ B_{42} & B_{44} & B_{46} \\ B_{62} & B_{64} & B_{66} \end{bmatrix}}\begin{bmatrix} \dot{\xi}_2 \\ \dot{\xi}_4 \\ \dot{\xi}_6 \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 \\ 0 & C_{44} & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \xi_2 \\ \xi_4 \\ \xi_6 \end{bmatrix}=\boxed{\begin{bmatrix} F_2 \\ F_4 \\ F_6 \end{bmatrix}}$$

Sway & Roll & Yaw

$$\left(A_{jk}^{0} = \int_L a_{jk}\,dx ,\ B_{jk}^{0} = \int_L b_{jk}\,dx \right)$$

$$A_{22} = \int_L a_{22}\,dx - \frac{U}{\omega^2} b_{22}^{A}$$

$$B_{22} = \int_L b_{22}\,dx + U a_{22}^{A}$$

$$A_{24} = A_{42} = \int_L a_{24}\,dx - \frac{U}{\omega^2} b_{24}^{A}$$

$$B_{24} = B_{42} = \int_L b_{24}\,dx + U a_{24}^{A}$$

$$A_{26} = \int_L x a_{22}\,dx + \frac{U}{\omega^2} B_{22}^{0} - \frac{U}{\omega^2} x_A b_{22}^{A} + \frac{U^2}{\omega^2} a_{22}^{A}$$

$$B_{26} = \int_L x b_{22}\,dx - U A_{22}^{0} + U x_A a_{22}^{A} + \frac{U^2}{\omega^2} b_{22}^{A}$$

$$A_{44} = \int_L a_{44}\,dx - \frac{U}{\omega^2} b_{44}^{A}$$

$$B_{44} = \int_L b_{44}\,dx + U a_{44}^{A} + B_{44}^{*}$$

$$A_{46} = \int_L x a_{24}\,dx + \frac{U}{\omega^2} B_{24}^{0} - \frac{U}{\omega^2} x_A b_{24}^{A} + \frac{U^2}{\omega^2} a_{24}^{A}$$

$$B_{46} = \int_L x b_{24}\,dx - U A_{24}^{0} + U x_A a_{24}^{A} + \frac{U^2}{\omega^2} b_{24}^{A}$$

$$A_{62} = \int_L x a_{22}\,dx - \frac{U}{\omega^2} B_{22}^{0} - \frac{U}{\omega^2} x_A b_{22}^{A}$$

$$B_{62} = \int_L x b_{22}\,dx + U A_{22}^{0} + U x_A a_{22}^{A}$$

$$A_{64} = \int_L x a_{24}\,dx - \frac{U}{\omega^2} B_{24}^{0} - \frac{U}{\omega^2} x_A b_{24}^{A}$$

$$B_{64} = \int_L x b_{24}\,dx + U A_{24}^{0} + U x_A a_{24}^{A}$$

$$A_{66} = \int_L x^2 a_{22}\,dx + \frac{U^2}{\omega^2} A_{22}^{0} - \frac{U}{\omega^2} x_A^2 b_{22}^{A} + \frac{U^2}{\omega^2} x_A a_{22}^{A}$$

$$B_{66} = \int_L x^2 b_{22}\,dx + \frac{U^2}{\omega^2} B_{22}^{0} + U x_A^2 a_{22}^{A} + \frac{U^2}{\omega^2} x_A b_{22}^{A}$$

U : **선박의 전진 속도**

ρ : **유체의 밀도**

α : Wave amplitude

f_j : Sectional Froude Krylov force (j[th] mode)

h_j : Sectional Diffraction force (j[th] mode)

ω : Encounter wave frequency

$x_A, a_{jk}^{A}, b_{jk}^{A}$: Values at the aftermost section

B_4^{*} : Roll Damping

Sway & Roll & Yaw

$$\left(A_{jk}^{0} = \int_L a_{jk}\, dx ,\ B_{jk}^{0} = \int_L b_{jk}\, dx \right)$$

$$F_2 = \rho\alpha \int_L (f_2 + h_2) dx + \rho\alpha \frac{U}{i\omega} h_2^A$$

$$F_4 = \rho\alpha \int_L (f_4 + h_4) dx + \rho\alpha \frac{U}{i\omega} h_4^A$$

$$F_6 = \rho\alpha \int_L \left[x(f_2 + h_2) + \rho\alpha \frac{U}{i\omega} h_2 \right] dx + \rho\alpha \frac{U}{i\omega} x_A h_2^A$$

U : 선박의 전진 속도

ρ : 유체의 밀도

α : Wave amplitude

f_j : Sectional Froude Krylov force (j^{th} mode)

h_j : Sectional Diffraction force (j^{th} mode)

ω : Encounter wave frequency

x_A, a_{jk}^A, b_{jk}^A : Values at the aftermost section

B_4^* : Roll Damping

Strip Theory :

Given

각 단면의 Added mass 및 Damping Coefficient, Wave exciting force

$(a_{22}, a_{24}, a_{33}, a_{44})$ $(b_{22}, b_{24}, b_{33}, b_{44})$ $(f_2, h_2, f_3, h_3, f_4, h_4)$

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & mz_c & 0 \\ 0 & m & 0 & -mz_c & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mz_c & 0 & I_{xx} & 0 & -I_{xz} \\ mz_c & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & -I_{zx} & 0 & I_{zz} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} B_{11} & 0 & B_{13} & 0 & B_{15} & 0 \\ 0 & B_{22} & 0 & B_{24} & 0 & B_{26} \\ B_{31} & 0 & B_{33} & 0 & B_{35} & 0 \\ 0 & B_{42} & 0 & B_{44} & 0 & B_{46} \\ B_{51} & 0 & B_{53} & 0 & B_{55} & 0 \\ 0 & B_{62} & 0 & B_{64} & 0 & B_{66} \end{bmatrix}$$

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

$$\mathbf{F}_{exciting} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & C_{53} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find

1) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1977, pp 307~310

Ship Motion in Regular waves
: RAO(Response Amplitude Operator)

✓ 선박의 6자유도 운동 방정식

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

ex) Heave

$$(M + A_{33})\ddot{\xi}_3 + B_{33}\dot{\xi}_3 + C_{33}\xi_3 = F_{exciting\ ,3}$$

$$\xi_3(t) = \xi_3^A e^{i\omega t}$$
$$\dot{\xi}_3(t) = i\omega\, \xi_3^A e^{i\omega t}$$
$$\ddot{\xi}_3(t) = -\omega^2 \xi_3^A e^{i\omega t}$$

$$F_{exciting\ ,3} = F_3^A e^{i\omega t} = \eta_0 f_3^A e^{i\omega t}$$

(η_0 : Wave Amplitude, Real)

(f_3^A : Wave exciting force Amplitude, Complex)

$$(M + A_{33})(-\omega^2 \xi_3^A e^{i\omega t}) + B_{33}(i\omega\, \xi_3^A e^{i\omega t}) + C_{33}(\xi_3^A e^{i\omega t}) = \eta_0 f_3^A e^{i\omega t}$$

$$\left\{-\omega^2 (M + A_{33}) + i\omega B_{33} + C_{33}\right\}\xi_3^A e^{i\omega t} = \eta_0 f_3^A e^{i\omega t}$$

$$\underbrace{\left\{-\omega^2 (M + A_{33}) + i\omega B_{33} + C_{33}\right\}}_{= \mathbf{D}\ (\rightarrow \text{Complex})}\xi_3^A = \eta_0 f_3^A \Rightarrow \xi_3^A = \eta_0 f_3^A D^{-1} \Rightarrow \frac{\xi_3^A}{\eta_0} = f_3^A \mathbf{D}^{-1}$$

✓ RAO(Response Amplitude Operator)
: 1m wave height를 가지는
주파수 ω인 wave에 대한
선박의 6자유도 운동 변위

1) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1977, pp 307~310

Ship Motion in Regular waves
: RAO(Response Amplitude Operator)

✓ **선박의 6자유도 운동 방정식**

$$(\mathbf{M}+\mathbf{A})\ddot{\mathbf{x}}+\mathbf{B}\dot{\mathbf{x}}+\mathbf{C}\mathbf{x}=\mathbf{F}_{exciting}$$

General Case

$$\mathbf{x}=\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\\ \xi_4\\ \xi_5\\ \xi_6\end{bmatrix}=\begin{bmatrix}\xi_1^A\\ \xi_2^A\\ \xi_3^A\\ \xi_4^A\\ \xi_5^A\\ \xi_6^A\end{bmatrix}e^{i\omega t}=\mathbf{x}^Ae^{i\omega t},\ \dot{\mathbf{x}}=i\omega\mathbf{x}^Ae^{i\omega t},\quad \ddot{\mathbf{x}}=-\omega^2\mathbf{x}^Ae^{i\omega t},\quad \mathbf{F}_{exciting}=\eta_0\begin{bmatrix}f_1^A\\ f_2^A\\ f_3^A\\ f_4^A\\ f_5^A\\ f_6^A\end{bmatrix}e^{i\omega t}=\eta_0\mathbf{f}^Ae^{i\omega t}$$

$$(\mathbf{M}+\mathbf{A})\left(-\omega^2\mathbf{x}^Ae^{i\omega t}\right)+\mathbf{B}\left(i\omega\mathbf{x}^Ae^{i\omega t}\right)+\mathbf{C}\left(\mathbf{x}^Ae^{i\omega t}\right)=\eta_0\mathbf{f}^Ae^{i\omega t}$$

$$\left\{-\omega^2(\mathbf{M}+\mathbf{A})+i\omega\mathbf{B}+\mathbf{C}\right\}\mathbf{x}^Ae^{i\omega t}=\eta_0\mathbf{f}^Ae^{i\omega t}$$

$$\underbrace{\left\{-\omega^2(\mathbf{M}+\mathbf{A})+i\omega\mathbf{B}+\mathbf{C}\right\}}_{=\mathbf{D}}\mathbf{x}^A=\eta_0\mathbf{f}^A \Rightarrow \mathbf{x}^A=\eta_0\mathbf{D}^{-1}\mathbf{f}^A \Rightarrow \frac{\mathbf{x}^A}{\eta_0}=\mathbf{D}^{-1}\mathbf{f}^A$$

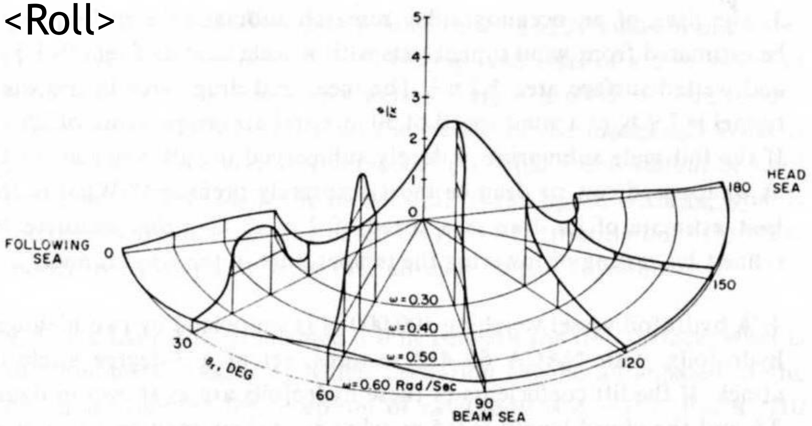
✓ RAO(Response Amplitude Operator)
: 1m wave height를 가지는
주파수 ω인 wave에 대한
선박의 6자유도 운동 변위

1) 그림 출처 : Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1977, pp 47

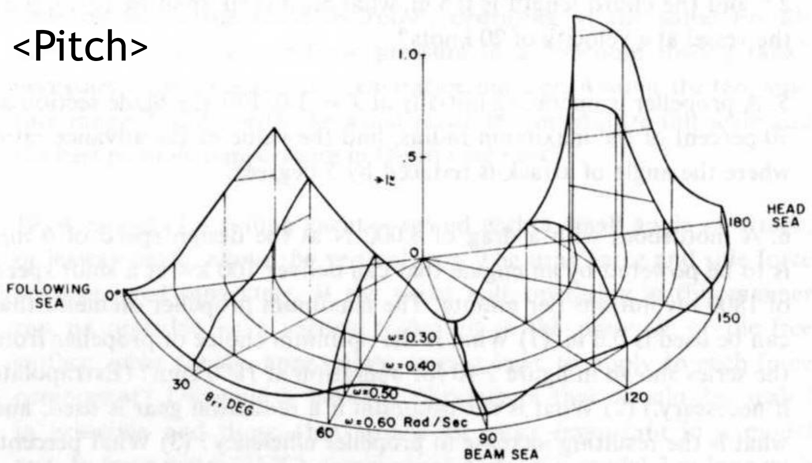
Ship Motion in Regular waves
: RAO(Response Amplitude Operator)

✓ Example of RAO

<Roll>



<Pitch>



2.17
Roll and pitch response of a 319 m ship at 25 knots. The motions are non-dimensionalized in terms of the maximum wave slope $2\pi A/\lambda$. (From Wachnik and Zarnick 1965; reproduced by permission of the Society of Naval Architects and Marine Engineers)

Step 4.
Shear force & Bending moment (SWBM[1]), VWBM[2]))

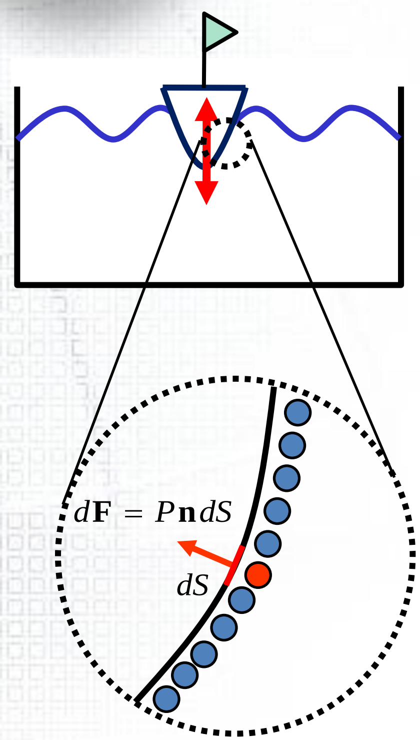
1) SWBM : Still Water Bending Moment

2) VWBM : Vertical Wave Bending Moment

운동 방정식 유도 – 선박에 작용하는 힘

(변위 : $\mathbf{x} = [\xi_1, \cdots, \xi_6]^T$)

($\mathbf{M}, \mathbf{A}, \mathbf{B}, \mathbf{C}$: 6×6 *Matrix*)



$d\mathbf{F}$: 하나의 유체 입자가 선박 표면에 가하는 힘

dS : 미소 면적

$\mathbf{n}$: 미소 면적의 Normal 벡터

✓ 유체 압력 (From Bernoulli Eq.)

$$\rho\frac{\partial\Phi}{\partial t} + P + \cancel{\frac{1}{2}\rho\left|\nabla\Phi\right|^2} + \rho gz = 0$$

선형화

$$P = -\rho gz - \rho\frac{\partial\Phi_T}{\partial t} = -\rho gz - \rho\left(\frac{\partial\Phi_I}{\partial t} + \frac{\partial\Phi_D}{\partial t} + \frac{\partial\Phi_R}{\partial t}\right)$$

$$= P_{static} + P_{F.K} + P_D + P_R$$

유체 입자 하나가 표면에 주는 압력

✓ Laplace Equation **Step1**

$$\nabla^2\Phi = 0$$

Solve

$$\Phi_T = \Phi_I + \Phi_D + \Phi_R$$

Step2

선박의 침수 표면 전체에 대하여 적분
(유체에 의해 선박이 받는 힘과 모멘트)

$$\iint_{S_B} P\mathbf{n}dS$$

2-D → 3-D (Strip method)

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{Gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

Linearization

$\mathbf{F}_{Restoring}$ $(= -\mathbf{C}\mathbf{x})$ $\mathbf{F}_{exciting}$ $\mathbf{F}_R = -\mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$

added mass Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = -\mathbf{C}\mathbf{x} + \mathbf{F}_{exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

Step3

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{exciting}$$

Motion RAO (Response Amplitude Operator)

Step4

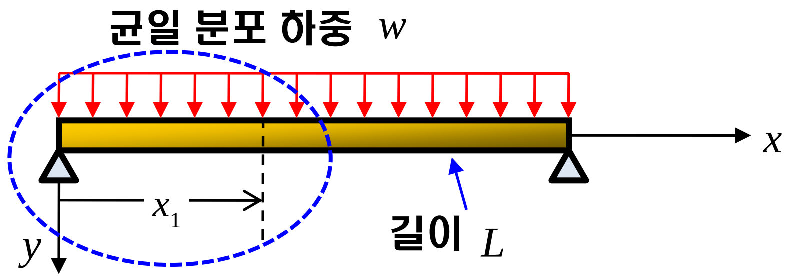
임의의 길이 x까지만 적분
(선박의 내부에 작용하는 S.F / B.M. 구함)

Shear force, Bending moment

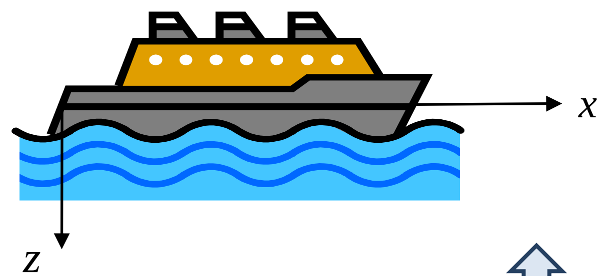
1) 임상전 편저, 재료역학, 문운당,2001, Chapter 8, pp 265~275

Review : 재료역학[1)]

보에 균일 하중 w가 작용하고 있다.
왼쪽 끝(x=0)에서 x_1만큼 떨어진 지점에서의
전단 응력과 굽힘 모멘트를 구하시오.

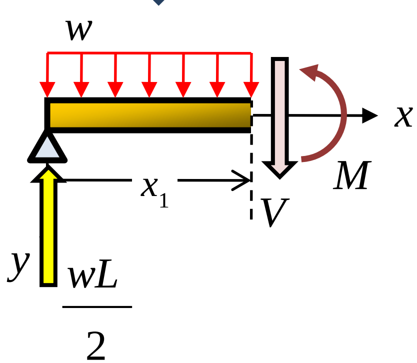


선미로부터 x1만큼 떨어진 지점의
전단 응력과 굽힘 모멘트는?



Application

자유 물체도



- y축 방향 힘의 평형 조건

$$V - \frac{wL}{2} + \int_0^{x_1} w dx = 0 \Rightarrow V(x_1) = \underline{\frac{wL}{2} - \int_0^{x_1} w dx}$$

➔ x_1이전까지 작용한 힘의 합

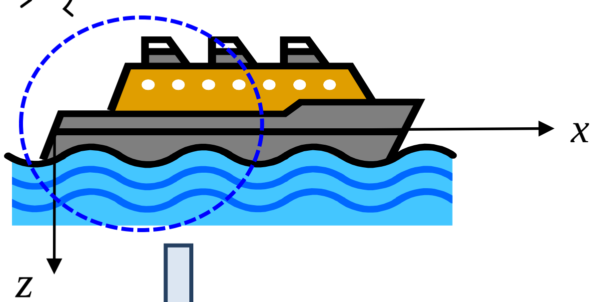
- 모멘트의 평형 조건(x=x_1기준)

$$M - \frac{wL}{2} x_1 + \int_0^{x_1} w(x_1 - x)\, dx = 0 \Rightarrow M(x_1) = \underline{\frac{wL}{2} x_1 - \int_0^{x_1} w(x_1 - x)\, dx}$$

➔ x_1이전까지 작용한 모멘트의 합

Shear force & Bending moment acting on the ship

선미로부터 x_1만큼 떨어진 지점의 전단 응력과 굽힘 모멘트는?

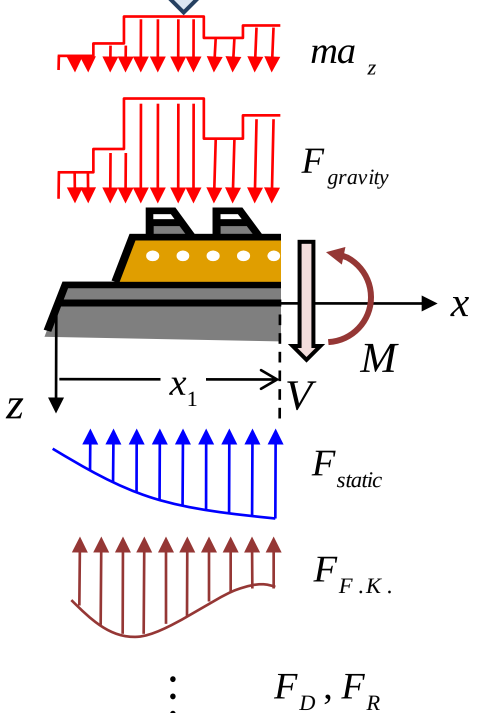


선박이 가속도 운동 중이므로, 동적 평형 상태일 때 작용하는 힘을 고려한다. (D'alembert Principle)

$$m\ddot{\mathbf{x}} = \sum \mathbf{F} = \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_{D} + \mathbf{F}_{R}$$: 운동 방정식

$$\Downarrow$$

$$\left(\mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{F.K} + \mathbf{F}_{D} + \mathbf{F}_{R}\right) - m\ddot{\mathbf{x}} = 0$$: 동적 평형 상태



z방향 힘 성분만 고려 (Heave & Pitch의 z방향 고려)

$$F(x) = F_{gravity}(x) + F_{static}(x) + F_{F.K}(x) + F_D(x) + F_R(x)$$

: x위치의 단면에 작용하는 수직 방향의 유체력

z방향 가속도

$$\boxed{\left(a_z = \ddot{z} - x\ddot{\theta}\right)}$$

운동 방정식 풀이로 부터 얻은 값

- z축 방향 힘의 평형 조건

$$V(x_1) = \int_{AP}^{x_1} \{F(x) - m(x)a_z\} dx$$

- 모멘트의 평형 조건(x=x_1기준)

$$M(x_1) = \int_{AP}^{x_1} V(x)dx = \int_{AP}^{x_1} (x_1 - x)\{m(x)(\ddot{z} - x\ddot{\theta}) - F(x)\} dx$$

Shear force & Bending moment acting on the ship

- 각 단면에 작용하는 힘 (Load)

x위치의 단면에 작용하는 수직 방향의 유체력

$$F(x) = F_{gravity}(x) + F_{static}(x) + F_{F.K}(x) + F_D(x) + F_R(x)$$

$$q(x) = F(x) - m(x)a_z$$

$$= \underbrace{F_{gravity}(x) + F_{static}(x)}_{q_{static}(x)} + \underbrace{F_{F.K}(x) + F_D(x) + F_R(x) - m(x)a_z}_{q_{dynamic}(x)}$$

$$= q_{static}(x) + q_{dynamic}(x)$$