Multi View Geometry (Spring '08)

More Single View Geometry

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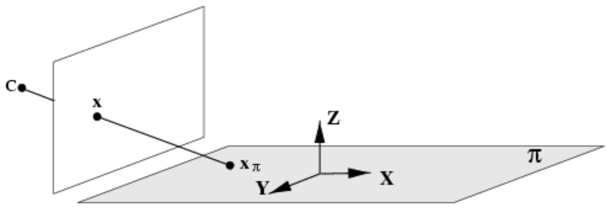
Actions of a Projective Camera on Planes



- Planar homography
  - ✓ For a point on π

$$\mathbf{x} = \mathrm{P}\mathbf{X} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4 \end{bmatrix} \begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_4 \end{bmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} = \mathbf{H}\mathbf{x}_\pi$$

planar homography

- The most general transformation that can occur between a scene plane and an image plane under perspective imaging is a plane projective transformation

  (affine camera-affine transformation)

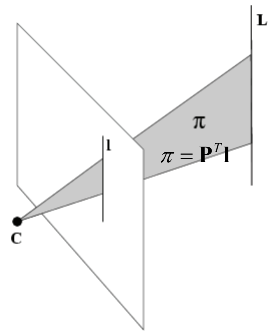
Actions of a Projective Camera on Lines

- Forward projection: Line to line

$$\begin{aligned} x(\mu) &= P(A + \mu B) = PA + \mu PB \\ &= a + \mu b \end{aligned}$$

- Back-projection: line to plane

$$\mathbf{l} \rightarrow \mathbf{P}^T\mathbf{l}$$

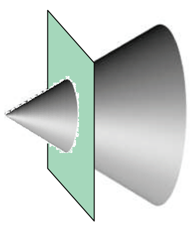


$\mathbf{x} = \mathbf{PX}$ and $\mathbf{x}^T\mathbf{l} = 0$
$\Rightarrow \mathbf{X}^T(\mathbf{P}^T\mathbf{l}) = 0$
$\Rightarrow \mathbf{P}^T\mathbf{l}$ is the back-projected plane of the line $\mathbf{l}$

Actions of a Projective Camera on Conics

- Back projection: conic to cone

$$\mathbf{C} \rightarrow \mathbf{Q}_{CO} = \mathbf{P}^T\mathbf{CP}$$



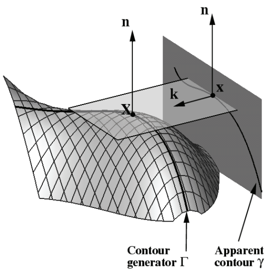
$\mathbf{x} = \mathbf{PX}$ and $\mathbf{x}^T\mathbf{Cx} = 0$
$\Rightarrow \mathbf{X}^T(\mathbf{P}^T\mathbf{CP})\mathbf{X} = 0$
$\Rightarrow \mathbf{P}^T\mathbf{CP}$ is the back-projected cone of the conic $\mathbf{C}$

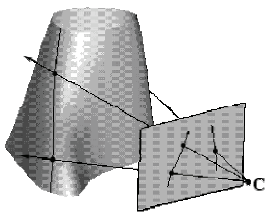
$\mathbf{Q}_{CO}$ has rank 3 and $\mathbf{C}$ is the null vector of $\mathbf{Q}_{CO}$,
$\mathbf{Q}_{CO}\mathbf{C} = \mathbf{P}^T\mathbf{C}(\mathbf{PC}) = \mathbf{0}$

- Ex: $\mathbf{P} = \mathbf{K}[\mathbf{I} \mid \mathbf{0}]$, $\mathbf{Q}_{co} = \begin{bmatrix} \mathbf{K}^T \\ \mathbf{0} \end{bmatrix} \mathbf{C}[\mathbf{K} \mid \mathbf{0}] = \begin{bmatrix} \mathbf{K}^T\mathbf{CK} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix}$

Images of Smooth Surfaces

- The contour generator Γ is the set of points **X** on **S** at which rays are tangent to the surface. The corresponding apparent contour γ is the set of points **x** which are the image of **X**, i.e. γ is the image of Γ
- The contour generator Γ depends only on position of projection center, γ depends also on rest of **P**



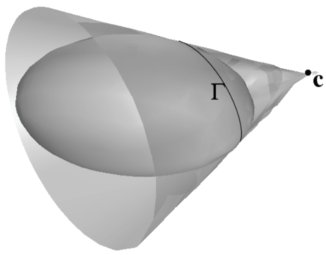


Actions of a Projective Camera on Quadrics

- Forward Projection: Quadric to conic

$$C^* = PQ^*P^T.$$

$$\begin{aligned}\pi^T Q^* \pi &= l^T P Q^* P^T l \\ &= l^T C^* l = 0\end{aligned}$$



pole – polar relationship

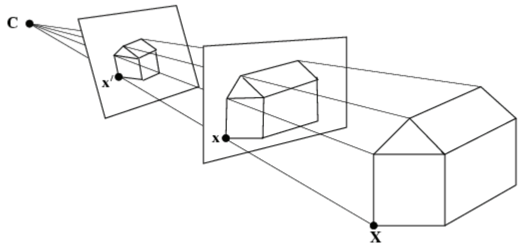
$$\boldsymbol{\pi}_\Gamma = \mathbf{QC}$$

- The cone with vertex **V** and tangent to the quadric **Q** (degenerate quadric) is

$$\mathbf{Q}_{CO} = (\mathbf{V}^T\mathbf{QV})\mathbf{Q} - (\mathbf{QV})(\mathbf{QV})^T \qquad \mathbf{Q}_{CO}\mathbf{V} = \mathbf{0}$$

Fixed Camera Center

- Projective imaging devices



$$\mathbf{P} = \mathbf{KR}[\mathbf{I} \mid -\tilde{\mathbf{C}}], \mathbf{P}' = \mathbf{K}'\mathbf{R}'[\mathbf{I} \mid -\tilde{\mathbf{C}}]$$

$$\Rightarrow \mathbf{P}' = \mathbf{K}'\mathbf{R}'(\mathbf{KR})^{-1}\mathbf{P}$$

$$\mathbf{x}' = \mathtt{P}'\mathbf{X} = (\mathtt{K}'\mathtt{R}')(\mathtt{KR})^{-1}\mathtt{P}\mathbf{X} = (\mathtt{K}'\mathtt{R}')(\mathtt{KR})^{-1}\mathbf{x}.$$

$$\Rightarrow \mathbf{x}' = \mathbf{Hx} : \ \mathbf{H} = (\mathbf{K}'\mathbf{R}')(\mathbf{KR})^{-1}, \text{ planar homography}$$

Moving the image plane (zooming)

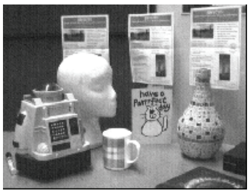
- Increase in focal length:
  - ✓ move image plane along the principal axis
  - ✓ magnification

$$\begin{aligned} \mathbf{x} &= \mathtt{K}[\mathtt{I} \mid \mathbf{0}]\mathbf{X} \\ \mathbf{x}' &= \mathtt{K}'[\mathtt{I} \mid \mathbf{0}]\mathbf{X} = \mathtt{K}'\mathtt{K}^{-1}\left(\mathtt{K}[\mathtt{I} \mid \mathbf{0}]\mathbf{X}\right) = \mathtt{K}'\mathtt{K}^{-1}\mathbf{x} \end{aligned}$$

$$\mathtt{K}'\mathtt{K}^{-1} = \begin{bmatrix} k\mathtt{I} & (1-k)\tilde{\mathbf{x}}_0 \\ \mathbf{0}^\top & 1 \end{bmatrix}. \quad k = \frac{f'}{f}$$

$$\begin{aligned} \mathtt{K}' &= \begin{bmatrix} k\mathtt{I} & (1-k)\tilde{\mathbf{x}}_0 \\ \mathbf{0}^\top & 1 \end{bmatrix}\mathtt{K} = \begin{bmatrix} k\mathtt{I} & (1-k)\tilde{\mathbf{x}}_0 \\ \mathbf{0}^\top & 1 \end{bmatrix}\begin{bmatrix} \mathtt{A} & \tilde{\mathbf{x}}_0 \\ \mathbf{0}^\top & 1 \end{bmatrix} \\ &= \begin{bmatrix} k\mathtt{A} & \tilde{\mathbf{x}}_0 \\ \mathbf{0}^\top & 1 \end{bmatrix} = \mathtt{K}\begin{bmatrix} k\mathtt{I} & \\ & 1 \end{bmatrix}. \end{aligned}$$

Camera rotation



$$\mathbf{x} = \mathbf{K}[\mathbf{I} \mid \mathbf{0}]\mathbf{X}$$

$$\mathbf{x}' = \mathbf{K}[\mathbf{R} \mid \mathbf{0}]\mathbf{X} = \mathbf{K}\mathbf{R}\mathbf{K}^{-1}\mathbf{K}[\mathbf{I} \mid \mathbf{0}]\mathbf{X} = \boxed{\mathbf{K}\mathbf{R}\mathbf{K}^{-1}}\mathbf{x}$$

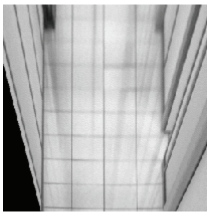
$\mathbf{H} = \mathbf{K}\mathbf{R}\mathbf{K}^{-1}$: conjugate rotation

three – values = $\{\mu, \mu e^{i\theta}, \mu e^{-i\theta}\}$

e_1 represent the direction of the rotation axis (vanishing point),

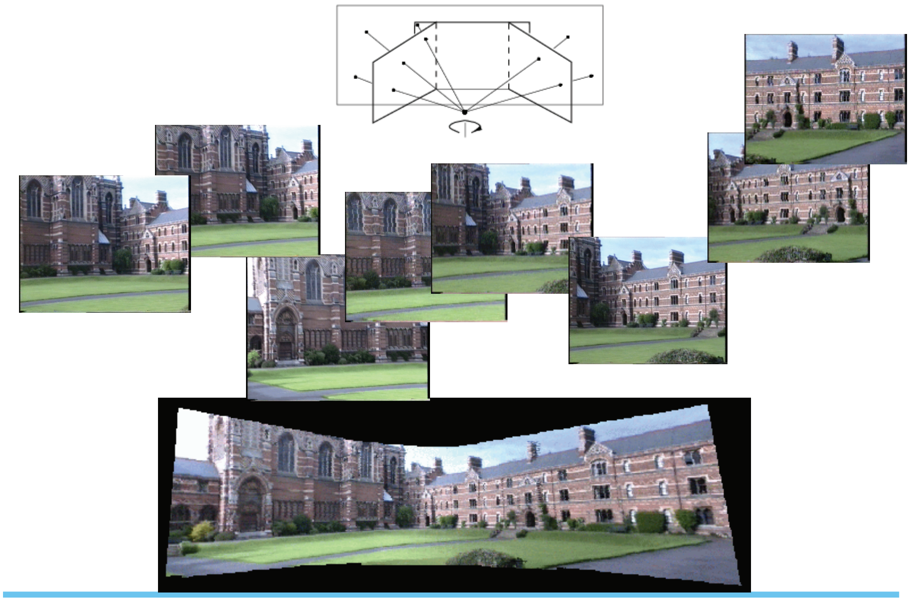
θ is the rotation angle

Applications – Synthetic Views


- Compute the homography that warps the image quadrilateral to a rectangle with the correct aspect ratio
- warp the image

Applications – Planar panoramic mosaicing



Applications – Planar panoramic mosaicing

- close-up: interlacing problem

Applications – Planar panoramic mosaicing

Projective (reduced) notation

- If canonical coordinates are chosen

$$X_1 = (1,0,0,0)^T, X_2 = (0,1,0,0)^T, X_3 = (0,0,1,0)^T, X_4 = (0,0,0,1)^T$$

$$x_1 = (1,0,0)^T, x_2 = (0,1,0)^T, x_3 = (0,0,1)^T, x_4 = (1,1,1)^T$$

- Then the camera matrix becomes

$$P = \begin{bmatrix} a & 0 & 0 & -d \\ 0 & b & 0 & -d \\ 0 & 0 & c & -d \end{bmatrix}$$

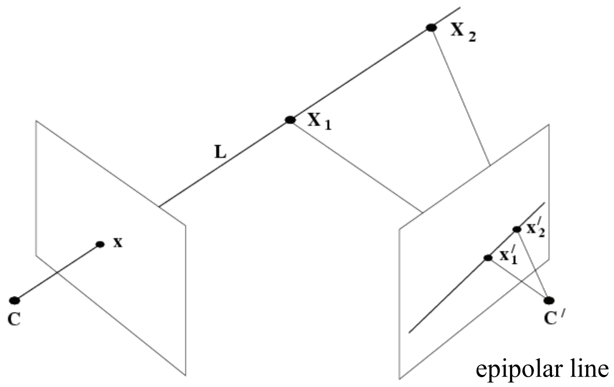
and the camera center is

$$C = (a^{-1}, b^{-1}, c^{-1}, d^{-1})^T$$

- This implies that all images with the same camera center are projectively equivalent

Moving the Camera Center

- When the camera center moves – *motion parallax* occurs



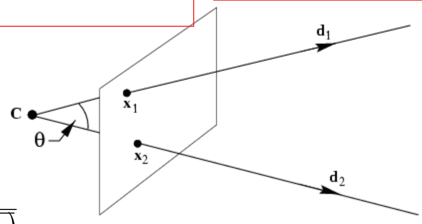
What does calibration give?

- Let $\tilde{\mathbf{X}} = \lambda\mathbf{d}$ be the back projected points of $\mathbf{x}$, then
  $\mathbf{x} = \mathbf{K}[\mathbf{I} \mid \mathbf{0}](\lambda\mathbf{d}^T, 1)^T = \mathbf{Kd}$
  $\Rightarrow \mathbf{d} = \mathbf{K}^{-1}\mathbf{x}$

in the camera Euclidean coordinates

- The angle between two rays

$$\cos\theta = \frac{\mathbf{d}_1^T\mathbf{d}_2}{\sqrt{(\mathbf{d}_1^T\mathbf{d}_1)(\mathbf{d}_2^T\mathbf{d}_2)}} = \frac{\mathbf{x}_1^T(\mathbf{K}^{-T}\mathbf{K}^{-1})\mathbf{x}_2}{\sqrt{(\mathbf{x}_1^T(\mathbf{K}^{-T}\mathbf{K}^{-1})\mathbf{x}_1)(\mathbf{x}_2^T(\mathbf{K}^{-T}\mathbf{K}^{-1})\mathbf{x}_2)}}$$



- A calibrated camera (**K** is known) is a direction sensor which can measure the direction of rays
- An image line **l** defines a plane through the camera center with normal $\mathbf{n}=\mathbf{K}^T\mathbf{l}$ measured in the camera's Euclidean frame

$$(\mathbf{d}^T\mathbf{n} = 0 \Rightarrow (\mathbf{K}^{-1}\mathbf{x})^T\mathbf{n} = 0 \Rightarrow \mathbf{x}^T(\mathbf{K}^{-T}\mathbf{n}) = 0 \Rightarrow \mathbf{l} = \mathbf{K}^{-T}\mathbf{n})$$

The image of the Absolute Conic

- mapping between π_∞ to an image is given by the planar homogaphy **x=Hd**, with **H=KR** Independent on **C**

$$\mathbf{x} = \mathtt{P}\mathbf{X}_\infty = \mathtt{KR}[\mathtt{I} \mid -\tilde{\mathbf{C}}] \begin{pmatrix} \mathbf{d} \\ 0 \end{pmatrix} = \mathtt{KR}\mathbf{d}.$$

- image of the absolute conic (IAC) is

$$\omega = \left(\mathbf{K}\mathbf{K}^{\mathrm{T}}\right)^{-1} = \mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1}$$

$\mathbf{C}' = \mathbf{H}^{-T}\mathbf{C}\mathbf{H}^{-1}$

For an absolute conic, $\mathbf{C} = \mathbf{\Omega}_\infty = \mathbf{I}$ on $\boldsymbol{\pi}_\infty$

$\boldsymbol{\omega} = (\mathbf{KR})^{-T}\mathbf{I}(\mathbf{KR})^{-1} = (\mathbf{KK}^{T})^{-1}$

The image of the Absolute Conic

- Remarks:
  - ✓ $\boldsymbol{\omega}$ depends only on the internal parameters K
  - ✓ The angle between two rays

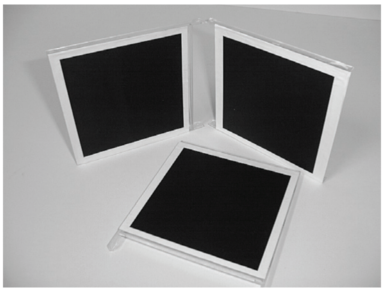
$$\cos\theta = \frac{\mathbf{x}_1^{\mathsf{T}}(\mathtt{K}^{-\mathsf{T}}\mathtt{K}^{-1})\mathbf{x}_2}{\sqrt{\mathbf{x}_1^{\mathsf{T}}(\mathtt{K}^{-\mathsf{T}}\mathtt{K}^{-1})\mathbf{x}_1}\sqrt{\mathbf{x}_2^{\mathsf{T}}(\mathtt{K}^{-\mathsf{T}}\mathtt{K}^{-1})\mathbf{x}_2}}$$

$$= \frac{\mathbf{x}_1^{\mathsf{T}}\omega\mathbf{x}_2}{\sqrt{\mathbf{x}_1^{\mathsf{T}}\omega\mathbf{x}_1}\sqrt{\mathbf{x}_2^{\mathsf{T}}\omega\mathbf{x}_2}}.$$

unchanged under any projective transformation of the image

  - ✓ Dual (line) conic (DIAC): $\omega^* = \omega^{-1} = \mathtt{KK}^{\mathsf{T}}$. Image of $\mathbf{Q}_\infty^*$
  - ✓ Once $\boldsymbol{\omega}$ or $\boldsymbol{\omega}^*$ is identified **K** can be determined uniquely by Cholesky factorization
  - ✓ A plane $\boldsymbol{\pi}$ intersects $\boldsymbol{\pi}_\infty$ in a line, and this line intersects $\boldsymbol{\Omega}_\infty$ in two points (circular points of $\boldsymbol{\pi}$). The imaged circular points lie on $\boldsymbol{\omega}$ at the points at which the vanishing line of the plane $\boldsymbol{\pi}$ intersects $\boldsymbol{\omega}$.

A simple calibration device



- Calibration steps:

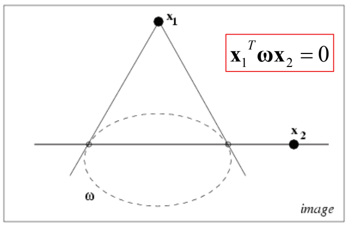
(i) For each square compute the homography H that maps its corner points, $(0,0)^\top, (1,0)^\top, (0,1)^\top, (1,1)^\top$, to their imaged points.

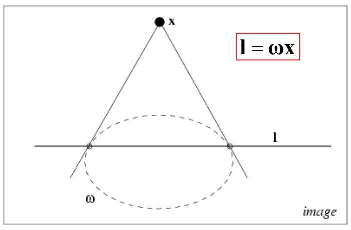
(ii) Compute the imaged circular points for the plane of that square as $\mathtt{H}(1, \pm i, 0)^\top$.

(iii) Fit a conic ω to the six imaged circular points using the design matrix of (1.4–p9).

(iv) Compute the calibration K from $\omega = (\mathtt{KK}^\top)^{-1}$ using the Cholesky factorization.

Orthogonality in the image





- *Suppose a line* $\mathbf{l}$ *back-projects to a plane* $\boldsymbol{\pi}$ *with normal direction* $\mathbf{n}$. *The normal is imaged at the point* $\mathtt{K}\mathbf{n}$, *and the line is the polar of this point, so that* $\mathbf{l} = \omega\mathtt{K}\mathbf{n} = (\mathtt{KK}^\top)^{-1}\mathtt{K}\mathbf{n} = \mathtt{K}^{-\top}\mathbf{n}$. *In short, the normal to* $\boldsymbol{\pi}$ *is* $\mathbf{n} = \mathtt{K}^\top\mathbf{l}$.

$$\cos\theta = \frac{\mathbf{x}_1^T \boldsymbol{\omega} \mathbf{x}_2}{\sqrt{(\mathbf{x}_1^T \boldsymbol{\omega} \mathbf{x}_1)(\mathbf{x}_2^T \boldsymbol{\omega} \mathbf{x}_2)}} \Rightarrow \cos\theta = \frac{\mathbf{d}_1^\top \Omega_\infty \mathbf{d}_2}{\sqrt{\mathbf{d}_1^\top \Omega_\infty \mathbf{d}_1}\sqrt{\mathbf{d}_2^\top \Omega_\infty \mathbf{d}_2}} .$$

$$\mathbf{x}_i = \mathbf{H}\mathbf{d}_i \text{ and } \boldsymbol{\omega} = \mathbf{H}^{-T}\boldsymbol{\Omega}_\infty \mathbf{H}^{-1}$$

Vanishing Points

- Let the point on a line through **A** with direction $\mathbf{D} = (\mathbf{d}^T, 0)^T$ be

  $$\mathbf{X}(\lambda) = \mathbf{A} + \lambda\mathbf{D} \qquad \mathbf{P} = \mathbf{K}[\mathbf{I} \mid \mathbf{0}]$$

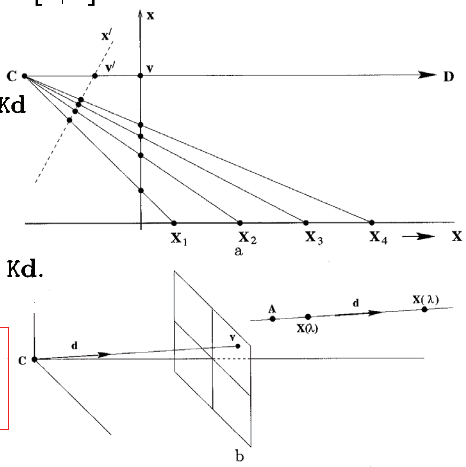
- Then

  $$\mathbf{x}(\lambda) = \mathtt{P}\mathbf{X}(\lambda) = \mathtt{P}\mathbf{A} + \lambda\mathtt{P}\mathbf{D} = \mathbf{a} + \lambda\mathtt{K}\mathbf{d}$$

- And

  $$\mathbf{v} = \lim_{\lambda\to\infty} \mathbf{x}(\lambda) = \lim_{\lambda\to\infty} (\mathbf{a} + \lambda\mathtt{K}\mathbf{d}) = \mathtt{K}\mathbf{d}.$$

$$\mathbf{v} = \mathtt{P}\mathbf{X}_\infty = \mathtt{K}[\mathtt{I} \mid \mathbf{0}]\begin{pmatrix}\mathbf{d}\\0\end{pmatrix} = \mathtt{K}\mathbf{d}.$$



Vanishing Points

- Camera rotation from vanishing points
- Assume two cameras with same **K** and rotation by **R**
- Vanishing points are depend on camera rotation but not position

$$\mathbf{v}_i = \mathbf{K}\mathbf{d}_i \Rightarrow \hat{\mathbf{d}}_i = \frac{\mathbf{K}^{-1}\mathbf{v}_i}{\left\|\mathbf{K}^{-1}\mathbf{v}_i\right\|}$$

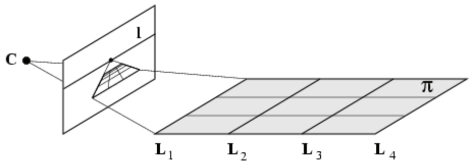
$$\mathbf{v}'_i = \mathbf{K}\mathbf{d}'_i \Rightarrow \hat{\mathbf{d}}'_i = \frac{\mathbf{K}^{-1}\mathbf{v}'_i}{\left\|\mathbf{K}^{-1}\mathbf{v}'_i\right\|}$$

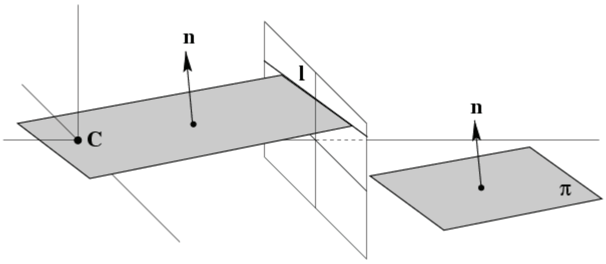
$$\hat{\mathbf{d}}'_i = \mathbf{R}\hat{\mathbf{d}}_i$$

- Thus, two corresponding directions are sufficient for determining **R**

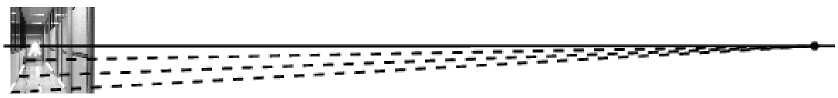
Vanishing Lines

- All planes with normal **n** have vanishing line $\mathbf{l} = \mathbf{K}^{-T}\mathbf{n}$
- Or $\mathbf{n} = \mathbf{K}^{T}\mathbf{l}$





Vanishing Lines



Orthogonality relations

- The angle between two scene lines using vanishing points:

$$\cos\theta = \frac{\mathbf{v}_1^{\mathrm{T}}\boldsymbol{\omega}\mathbf{v}_2}{\sqrt{\left(\mathbf{v}_1^{\mathrm{T}}\boldsymbol{\omega}\mathbf{v}_1\right)\left(\mathbf{v}_2^{\mathrm{T}}\boldsymbol{\omega}\mathbf{v}_2\right)}}$$

- The angle between two scene planes using vanishing lines:

$$\cos\theta = \frac{\mathbf{l}_1^{\mathrm{T}}\boldsymbol{\omega}^*\mathbf{l}_2}{\sqrt{\left(\mathbf{l}_1^{\mathrm{T}}\boldsymbol{\omega}^*\mathbf{l}_1\right)\left(\mathbf{l}_2^{\mathrm{T}}\boldsymbol{\omega}^*\mathbf{l}_2\right)}}$$

Orthogonality relations

- Orthogonality relations:
  - ✓ The vanishing points of two perpendicular lines

$$\mathbf{v}_1^T\boldsymbol{\omega}\mathbf{v}_2 = 0$$

  - ✓ The vanishing point of the normal direction to a plane and the plane vanishing line

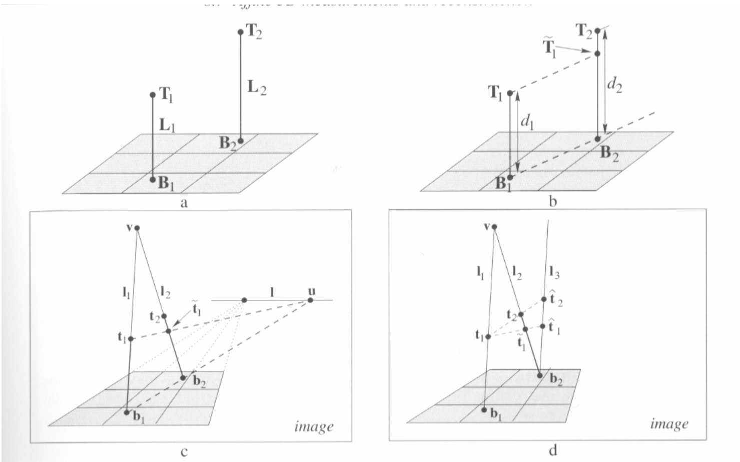
$$\mathbf{v} = \boldsymbol{\omega}^*\mathbf{l} \Rightarrow \mathbf{l} = \boldsymbol{\omega}\mathbf{v} \Rightarrow \mathbf{l}\times(\boldsymbol{\omega}\mathbf{v}) = \mathbf{0}$$

  - ✓ The vanishing lines of two perpendicular planes

$$\mathbf{l}_1^T\boldsymbol{\omega}^*\mathbf{l}_2 = 0$$

Affine 3D measurements and Reconstruction

Result 8.24. *Given the vanishing line of the ground plane* $\mathbf{l}$ *and the vertical vanishing point* $\mathbf{v}$*, then the relative length of vertical line segments can be measured provided their end point lies on the ground plane.*



Affine 3D measurements and Reconstruction

Objective

Given the vanishing line of the ground plane $\mathbf{l}$ and the vertical vanishing point $\mathbf{v}$ and the top $(\mathbf{t}_1, \mathbf{t}_2)$ and base $(\mathbf{b}_1, \mathbf{b}_2)$ points of two line segments as in figure 8.20, compute the ratio of lengths of the line segments in the scene.

Algorithm

(i) Compute the vanishing point $\mathbf{u} = (\mathbf{b}_1 \times \mathbf{b}_2) \times \mathbf{l}$.

(ii) Compute the transferred point $\tilde{\mathbf{t}}_1 = (\mathbf{t}_1 \times \mathbf{u}) \times \mathbf{l}_2$ (where $\mathbf{l}_2 = \mathbf{v} \times \mathbf{b}_2$).

(iii) Represent the four points $\mathbf{b}_2, \tilde{\mathbf{t}}_1, \mathbf{t}_2$ and $\mathbf{v}$ on the image line $\mathbf{l}_1$ by their distance from $\mathbf{b}_2$, as $0, \tilde{t}_1, t_2$ and v respectively.

(iv) Compute a 1D projective transformation $\mathtt{H}_{2\times 2}$ mapping homogeneous coordinates $(0,1) \mapsto (0,1)$ and $(v,1) \mapsto (1,0)$ (which maps the vanishing point $\mathbf{v}$ to infinity). A suitable matrix is given by

$$\mathtt{H}_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 1 & -v \end{bmatrix}.$$

(v) The (scaled) distance of the scene points $\widetilde{\mathbf{T}}_1$ and $\mathbf{T}_2$ from $\mathbf{B}_2$ on $\mathbf{L}_2$ may then be obtained from the position of the points $\mathtt{H}_{2\times 2}(\tilde{t}_1, 1)^{\mathsf{T}}$ and $\mathtt{H}_{2\times 2}(t_2, 1)^{\mathsf{T}}$. Their distance ratio is then given by

$$\frac{d_1}{d_2} = \frac{\tilde{t}_1(v - t_2)}{t_2(v - \tilde{t}_1)}$$

Algorithm 8.1. *Computing scene length ratios from a single image.*

Affine 3D measurements and Reconstruction

- Measuring a person's height in a single image

Determining K from a single view

- Using five pairs of of vanishing points of perpendicular lines

$$\mathbf{v}_i^T \boldsymbol{\omega} \mathbf{v}_j = 0 \longrightarrow \boldsymbol{\omega} = \mathbf{K}^{-T}\mathbf{K}^{-1} \longrightarrow \mathbf{K}$$

- Similarly to pairs of vanishing point & lines, and vanishing lines & lines

- If $s = K_{12} = 0$ and $[\omega_{ij}] = \boldsymbol{\omega} = \mathbf{K}^{-T}\mathbf{K}^{-1}$ then $\omega_{12} = \omega_{21} = 0$.
  If in addition $\alpha_x = K_{11} = K_{22} = \alpha_y$, then $\omega_{12} = \omega_{21}$

- Thus, for a camera with zero skew and square pixels, three orthogonal vanishing points are sufficient for determining **K**

Determining K from a single view

- Three sources of constraints on ω
  - ✓ Metric plane imaged with known homography
  - ✓ Vanishing points and lines corresponding to perpendicular directions
  - ✓ "internal constraints" such as zero skew or square pixels

Condition	constraint	type	# constraints
vanishing points $\mathbf{v}_1, \mathbf{v}_2$ corresponding to orthogonal lines	$\mathbf{v}_1^\mathsf{T}\omega\mathbf{v}_2 = 0$	linear	1
vanishing point $\mathbf{v}$ and vanishing line $\mathbf{l}$ corresponding to orthogonal line and plane	$[\mathbf{l}]_\times \omega \mathbf{v} = \mathbf{0}$	linear	2
metric plane imaged with known homography $\mathtt{H} = [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3]$	$\mathbf{h}_1^\mathsf{T}\omega\mathbf{h}_2 = 0$ $\mathbf{h}_1^\mathsf{T}\omega\mathbf{h}_1 = \mathbf{h}_2^\mathsf{T}\omega\mathbf{h}_2$	linear	2
zero skew	$\omega_{12} = \omega_{21} = 0$	linear	1
square pixels	$\omega_{12} = \omega_{21} = 0$ $\omega_{11} = \omega_{22}$	linear	2

Table 8.1. **Scene and internal constraints on ω.**

Determining K from a single view

Objective

Compute $\mathtt{K}$ via ω by combining scene and internal constraints.

Algorithm

(i) Represent ω as a homogeneous 6-vector $\mathbf{w} = (w_1, w_2, w_3, w_4, w_5, w_6)^\mathsf{T}$ where:

$$\omega = \begin{bmatrix} w_1 & w_2 & w_4 \\ w_2 & w_3 & w_5 \\ w_4 & w_5 & w_6 \end{bmatrix}$$

(ii) Each available constraint from table 8.1 may be written as $\mathbf{a}^\mathsf{T}\mathbf{w} = 0$. For example, for the orthogonality constraint $\mathbf{u}^\mathsf{T}\omega\mathbf{v} = 0$, where $\mathbf{u} = (u_1, u_2, u_3)^\mathsf{T}$ and $\mathbf{v} = (v_1, v_2, v_3)^\mathsf{T}$, the 6-vector $\mathbf{a}$ is given by

$$\mathbf{a} = (v_1u_1, v_1u_2 + v_2u_1, v_2u_2, v_1u_3 + v_3u_1, v_2u_3 + v_3u_2, v_3u_3)^\mathsf{T}.$$

Similar constraints vectors are obtained from the other sources of scene and internal constraints. For example a metric plane generates two such constraints.

(iii) Stack the equations $\mathbf{a}^\mathsf{T}\mathbf{w} = 0$ from each constraint in the form $\mathtt{A}\mathbf{w} = \mathbf{0}$, where $\mathtt{A}$ is a $n \times 6$ matrix for n constraints.

(iv) Solve for $\mathbf{w}$ using the SVD as in algorithm 4.2(p109). This determines ω.

(v) Decompose ω into $\mathtt{K}$ using matrix inversion and Cholesky factorization (see section A4.2.1(p582)).

Algorithm 8.2. *Computing* $\mathtt{K}$ *from scene and internal constraints.*

Determining K using vanishing points and lines

Fig. 7.20. *For the case that image skew is zero and the aspect ratio unity* **the principal point is the orthocentre of an orthogonal triad of vanishing points**. *(a) Original image. (b) Three sets of parallel lines in the scene, with each set having direction orthogonal to the others. (c) The principal point is the orthocentre of the triangle with the vanishing points as vertices.*

$$\mathtt{K} = \begin{bmatrix} 1162.9 & 0 & 548.27 \\ 0 & 1162.9 & 404.25 \\ 0 & 0 & 1 \end{bmatrix}.$$

Determining K using vanishing points and lines

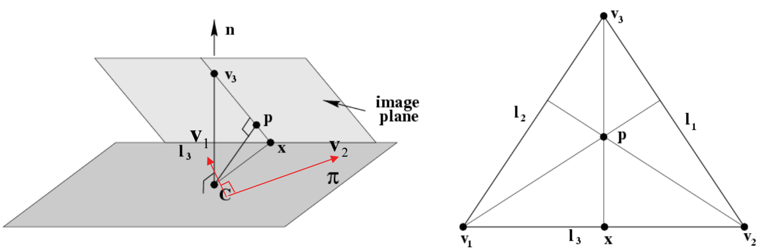


Fig. 8.23. **Geometric construction of the principal point.** *The vanishing line $\mathbf{l}_3$ back-projects to a plane π with normal $\mathbf{n}$. The vanishing point $\mathbf{v}_3$ back-projects to a line orthogonal to the plane π. (a) The normal $\mathbf{n}$ of the plane π through the camera centre $\mathbf{C}$ and the principal axis define a plane, which intersects the image in the line $\mathbf{l} = \langle \mathbf{v}_3, \mathbf{x} \rangle$. The line $\mathbf{l}_3$ is the intersection of π with the image plane, and is also its vanishing line. The point $\mathbf{v}_3$ is the intersection of the normal with the image plane, and is also its vanishing point. Clearly the principal point lies on $\mathbf{l}$, and $\mathbf{l}$ and $\mathbf{l}_3$ are perpendicular on the image plane. (b) The principal point may be determined from three such constraints as the orthocentre of the triangle.*

Determining K using vanishing points and lines

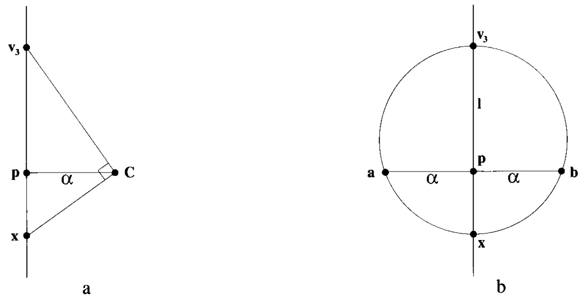


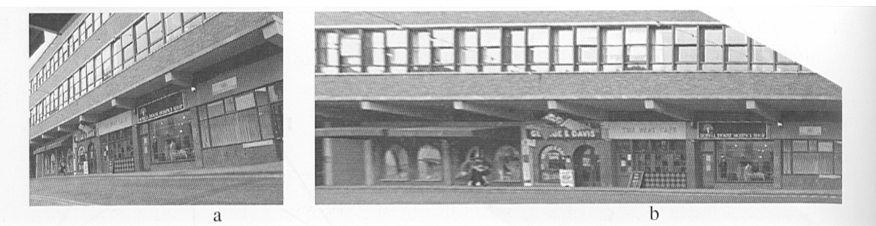
Fig. 8.24. **Geometric construction of the focal length.** *(a) Consider the plane defined by the camera centre* **C***, principal point and one of the vanishing points, e.g.* $\mathbf{v}_3$ *as shown in figure 8.23(a). The rays from* **C** *to* $\mathbf{v}_3$ *and* **x** *are perpendicular to each other. The focal length,* α*, is the distance from the camera centre to the image plane. By similar triangles,* $\alpha^2 = d(\mathbf{p}, \mathbf{v}_3)d(\mathbf{p}, \mathbf{x})$*, where* $d(\mathbf{u}, \mathbf{v})$ *is the distance between the points* **u** *and* **v***. (b) In the image a circle is drawn with diameter the line between* $\mathbf{v}_3$ *and* **x***. A line through* **p** *perpendicular to* $\langle \mathbf{v}_3, \mathbf{x} \rangle$ *meets the circle in two points* **a** *and* **b***. The focal length equals the distance* $d(\mathbf{p}, \mathbf{a})$*.*

Determining K using vanishing points and lines

- Assume $s = 0$, $\alpha_x = \alpha_y$, and $x_0 = y_0 = 0$, then

  $$\boldsymbol{\omega} = diag\begin{pmatrix} \frac{1}{f^2} & \frac{1}{f^2} & 1 \end{pmatrix}$$

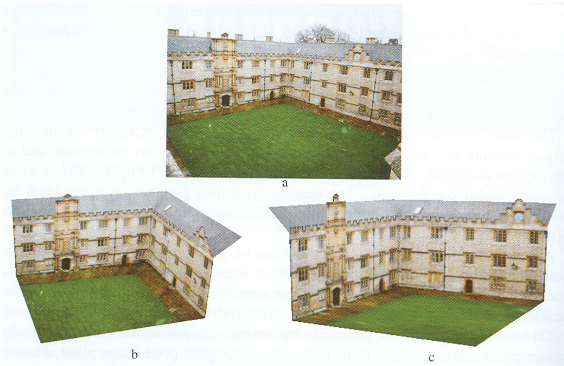
- Only one constraints is needed to determine f (and thus **K**)
- We can synthesize a fronto-parallel view using **K** and vanishing line **l**



a b

Single View Reconstruction

- Piecewise planar graphical model from a single image
  - ✓ Assume three orthogonal planes
  - ✓ Determine **K** using three vanishing points and square pixels
  - ✓ Proportioned width and height are determined by rectification
  - ✓ Find homography for texture mapping for each plane using **K** and **l**

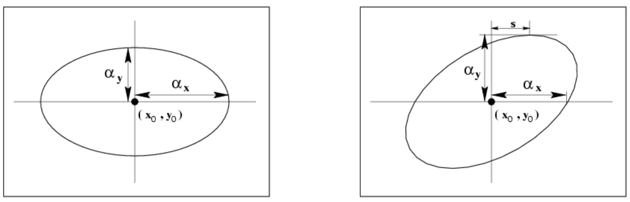


Calibrating Conic

- Image of a cone with apex angle 45° and axis coinciding with the principal axis
- Let $\mathbf{P} = \mathbf{K}[\mathbf{I} \,|\, \mathbf{0}]$ and the point on the cone be $X^2 + Y^2 = Z^2$, then the calibrating conic is an affine transformation of a unit circle

Unit circle

$$C = K^{-\top} \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix} K^{-1}$$



Calibrating Conic

- Orthogonality: $\mathbf{C} = \mathbf{K}^{-T}\mathbf{D}\mathbf{K}^{-1},\ \mathbf{D} = diag[1,1,-1]$

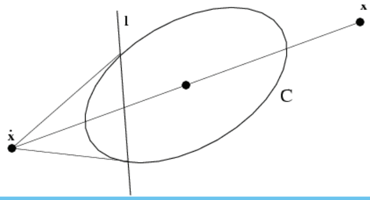
$$\longrightarrow \quad \mathtt{C} = (\mathtt{K}^{-\mathsf{T}}\mathtt{K}^{-1})(\mathtt{KDK}^{-1}) = \omega \mathtt{S}$$

Thus, **x** and **x'** are orthogonal if

$$0 = \mathbf{x}'^T \boldsymbol{\omega} \mathbf{x} = \mathbf{x}'^T \mathbf{C}\mathbf{S}^{-1}\mathbf{x}$$
$$= \mathbf{x}'^T \mathbf{C}\mathbf{S}\mathbf{x} = \mathbf{x}'^T \mathbf{C}\dot{\mathbf{x}}$$

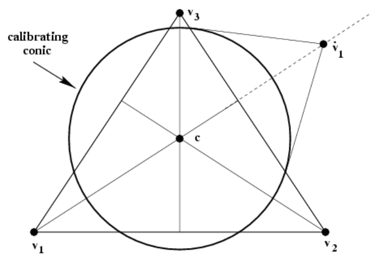
$\dot{\mathbf{x}} = \mathbf{S}\mathbf{x}$; reflection point of $\mathbf{x}$ w.r.t. the center of $\mathbf{C}$

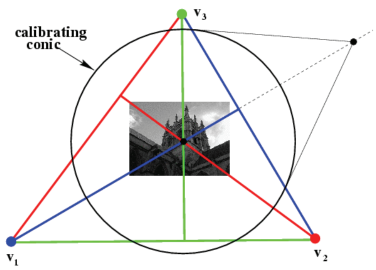
Result 7.19. *The line in an image corresponding to the plane perpendicular to a ray through image point* $\mathbf{x}$ *is the polar* $\mathtt{C}\dot{\mathbf{x}}$ *of the reflected point* $\dot{\mathbf{x}}$ *with respect to the calibrating conic.*



Determining K using vanishing points and lines

- Finding calibrating conic using orthogonal vanishing points





(i) First, construct the triangle with vertices the three vanishing points $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.
(ii) The centre of $\mathtt{C}$ is the orthocentre of the triangle.
(iii) Reflect one of the vanishing points (say $\mathbf{v}_1$) in the centre to get $\dot{\mathbf{v}}_1$.
(iv) The radius of $\mathtt{C}$ is determined by the condition that the polar of $\dot{\mathbf{v}}_1$ is the line passing though $\mathbf{v}_2$ and $\mathbf{v}_3$.