Multi View Geometry (Spring '08)

3D Reconstruction of Cameras and Structure

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Outline of reconstruction method

- **3D reconstruction of cameras and structure**
  - ✓ Reconstruction problem:

Given $\mathbf{x}_i$ and $\mathbf{x}_i$', compute **P**, **P**' and $\mathbf{X}_i$ such that

$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i \qquad \mathbf{x}'_i = \mathbf{P}'\mathbf{X}_i \qquad \text{for all } i$$

  - ✓ without additional information, we can reconstruct up to *projective* transformation

Outline of reconstruction method

- **Steps for reconstruction**
  - ✓ Compute **F** from correspondences features
  - ✓ Compute camera matrices **P**, **P'** from **F**
  - ✓ Compute 3D point $\mathbf{X}_i$ for each pair of corresponding points

- **Computation of F**
  - ✓ use $\mathbf{x}_i'\mathbf{F}\mathbf{x}_i = 0$, linear in coeff. **F**
  - ✓ 8 points (linear), 7 points (non-linear), points>8 (least-squares)
- **Computation of camera matrices**
  - ✓ Use $\mathbf{P} = [\mathbf{I} \mid \mathbf{0}]$ $\mathbf{P}' = [[\mathbf{e}']_\times \mathbf{F} \mid \mathbf{e}']$ or

    $\mathbf{P} = [\mathbf{I} \mid \mathbf{0}]$ $\mathbf{P}' = [[\mathbf{e}']_\times \mathbf{F} + \mathbf{e}'\mathbf{v}^T \mid \lambda\mathbf{e}']$

    Or factor $\mathbf{F} = [\mathbf{t}]_\times \mathbf{M}$,
    Then $\mathbf{P} = [\mathbf{I} \mid \mathbf{0}]$ and $\mathbf{P}' = [\mathbf{M} \mid \mathbf{t}]$
- **Triangulation**
  - ✓ Intersection of two backprojected rays

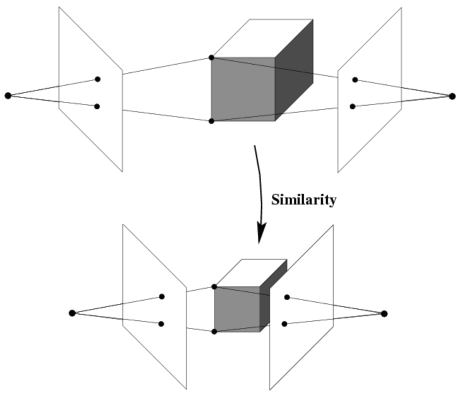
Reconstruction ambiguity

- If cameras are calibrated (**K** is known), reconstruction is up to **Similarity** transformation

$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i = \left(\mathbf{P}\mathbf{H}_S^{-1}\right)\left(\mathbf{H}_S\mathbf{X}_i\right)$$

$$\mathbf{P}\mathbf{H}_S^{-1} = \mathbf{K}[\mathbf{R} \mid \mathbf{t}]\begin{bmatrix} \mathbf{R}'^T & -\mathbf{R}'^T\mathbf{t}' \\ \mathbf{0} & \lambda \end{bmatrix}$$

$$= \mathbf{K}[\mathbf{R}\mathbf{R}'^T \mid -\mathbf{R}\mathbf{R}'^T\mathbf{t}' + \lambda\mathbf{t}]$$



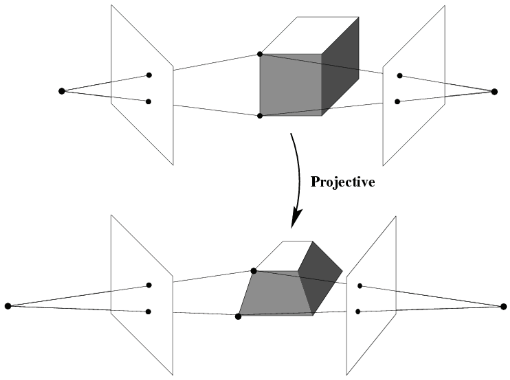
- similarity reconstruction

  = **Metric** and **Euclidean** reconstruction

Reconstruction ambiguity

- If cameras are uncalibrated, reconstruction is up to **Projective** transformation

$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i = \left(\mathbf{P}\mathbf{H}_{\mathrm{P}}^{-1}\right)\left(\mathbf{H}_{\mathrm{P}}\mathbf{X}_i\right)$$



The projective reconstruction theorem

- If a set of point correspondences in two views determine the funda mental matrix uniquely, then the scene and cameras may be reco nstructed from these correspondences alone, and any two such re constructions from these correspondences are projectively equival ent

- Consider two reconstructions

$$\left(\mathbf{P}_1, \mathbf{P}_1', \{\mathbf{X}_{1i}\}\right) \quad \left(\mathbf{P}_2, \mathbf{P}_2', \{\mathbf{X}_{2i}\}\right) \qquad \text{for} \quad \mathbf{x}_i \leftrightarrow \mathbf{x}_i'$$

$$\mathbf{P}_2 = \mathbf{P}_1\mathbf{H}^{-1} \quad \mathbf{P}_2' = \mathbf{P}_1'\mathbf{H}^{-1} \quad \mathbf{X}_2 = \mathbf{H}\mathbf{X}_1 \qquad \left(\text{except}: \mathbf{F}\mathbf{x}_i = \mathbf{x}_i'\mathbf{F} = 0\right)$$

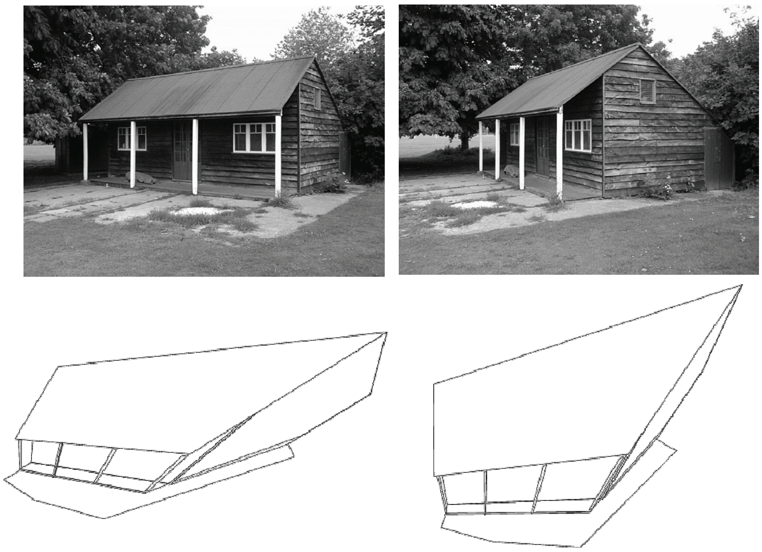
$$\Rightarrow \mathbf{P}_2\mathbf{X}_{2i} = \mathbf{P}_2\left(\mathbf{H}\mathbf{X}_{1i}\right) = \mathbf{P}_1\mathbf{H}^{-1}\mathbf{H}\mathbf{X}_{1i} = \mathbf{P}_1\mathbf{X}_{1i} = \mathbf{x}_i$$

- $\Rightarrow$ $\mathbf{X}_{2i}$ and $\mathbf{HX}_{1i}$ are along the same ray of $\mathbf{P}_2$, and that of $\mathbf{P'}_2$

- two possibilities: $\mathbf{X}_{2i}$=$\mathbf{HX}_{1i}$, or points along baseline

- Anyway, 3D reconstruction is possible from pair of unc alibrated images

The projective reconstruction theorem



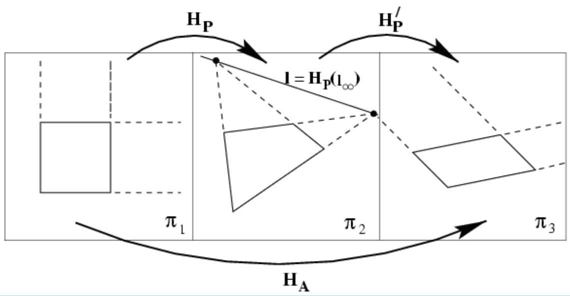
Stratified reconstruction

- **Stratified reconstruction steps:**
  (i) Projective reconstruction
  (ii) Affine reconstruction
  (iii) Metric reconstruction

$$\mathbf{H} = \begin{bmatrix} s\mathbf{RK} & \mathbf{t} \\ \mathbf{v}^{\mathsf{T}} & 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} s\mathbf{R} & \mathbf{t} \\ 0^{\mathsf{T}} & 1 \end{bmatrix}}_{\text{metric}} \underbrace{\begin{bmatrix} \mathbf{K} & 0 \\ 0^{\mathsf{T}} & 1 \end{bmatrix}}_{\text{affine}} \underbrace{\begin{bmatrix} \mathbf{I} & 0 \\ \mathbf{v}^{\mathsf{T}} & 1 \end{bmatrix}}_{\text{projective}}$$

- **Projective to affine**
  - ✓ Remember 2D case



Projective to affine

- Use of π_∞

$$(\mathbf{P}, \mathbf{P}', \{\mathbf{X}_i\})$$

$$\boldsymbol{\pi}_\infty = (A, B, C, D)^\mathsf{T} \mapsto (0,0,0,1)^\mathsf{T}$$

$$\mathbf{H}^{-\mathsf{T}}\boldsymbol{\pi}_\infty = (0,0,0,1)^\mathsf{T}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \mid \mathbf{0} \\ \boldsymbol{\pi}_\infty \end{bmatrix} \quad \text{(if } D \neq 0\text{)}$$

$$\mathbf{P}_\mathrm{A} = \mathbf{P}\mathbf{H}^{-1} \quad \mathbf{P}'_\mathrm{A} = \mathbf{P}'\mathbf{H}^{-1} \quad \mathbf{X}_\mathrm{A} = \mathbf{H}\mathbf{X}$$

- Projective with fixed π_∞ is affine transformation
- Sufficient information depending on application, e.g. mid-point, centroid, parallellism

Projective to affine – how to find π_∞

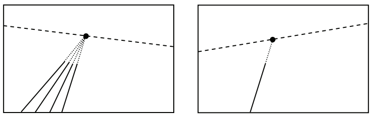
- **Translation motion**
- points at infinity are fixed for a pure translation
  $\Rightarrow$ $\mathbf{X}_i$ satisfying $\mathbf{x}_i \leftrightarrow \mathbf{x}_i$ is on π_∞
  $\Rightarrow$ Three such points determines π_∞

translational motion case:

$$\mathbf{F} = [\mathbf{e}]_\times = [\mathbf{e}']_\times \quad \Rightarrow \quad \mathbf{P} = [\mathbf{I} \mid \mathbf{0}] \quad \mathbf{P}' = [\mathbf{I} \mid \mathbf{e}']$$

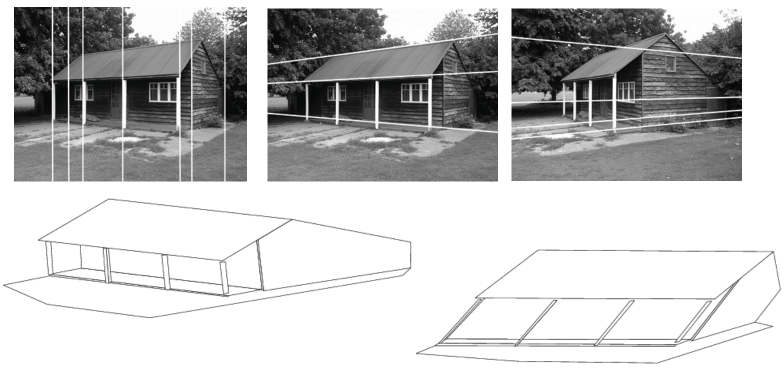
Projective to affine - how to find π_∞

- **Scene constraints**
  - ✓ **Parallel lines**
    - Parallel lines intesect at infinity
    - Reconstruction of corresponding vanishing point yields point on plane at infinity
    - 3 sets of parallel lines allow to uniquely determine π_∞
  - ✓ Remarks:
    - noise problem in finding intersections of parallel lines
    - obtaining vanishing point in one image is sufficient



Projective to affine – how to find π_∞

- ✓ **Distance ratios on a line**
  known distance ratio along a line allow to determine point at infinity (same as 2D case)

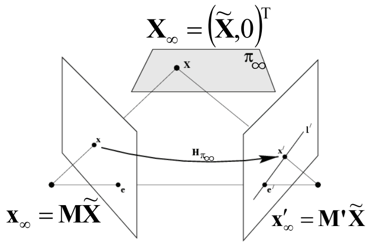


Projective to affine

- **The infinity homography**

$$\mathbf{P} = [\mathbf{M} \mid \mathbf{m}] \qquad \mathbf{P}' = [\mathbf{M}' \mid \mathbf{m}']$$

$$\boxed{\mathbf{H}_\infty = \mathbf{M}'\mathbf{M}^{-1}}$$



✓ unchanged under affine transformations

$$\mathbf{P} = [\mathbf{M} \mid \mathbf{m}] \begin{bmatrix} \mathbf{A} & \mathbf{a} \\ \mathbf{0} & 1 \end{bmatrix} = [\mathbf{MA} \mid \mathbf{Ma} + \mathbf{m}]$$

$$\mathbf{x}'_\infty \rightarrow \boldsymbol{\pi}_\infty \rightarrow \mathbf{x}_\infty$$
$$\mathbf{x}'_\infty = \mathbf{M}'\mathbf{M}^{-1}\mathbf{x}_\infty = \mathbf{H}_\infty \mathbf{x}_\infty$$

$$\mathbf{H}_\infty = \mathbf{M}'\mathbf{A}\mathbf{A}^{-1}\mathbf{M}^{-1} = \mathbf{M}'\mathbf{M}^{-1}$$

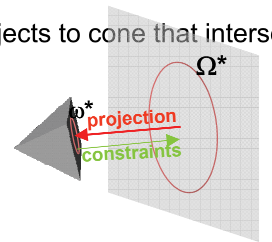
✓ affine reconstruction

$$\text{determine } \mathbf{H}_\infty \Rightarrow \boxed{\mathbf{P} = [\mathbf{I} \mid \mathbf{0}] \quad \mathbf{P}' = [\mathbf{H}_\infty \mid \mathbf{e}']}$$

Affine to metric

- **Use of Ω_∞ and IAC**
  - ✓ Find the transformation **H** which maps the absolute conic onto π_∞ $\qquad \Omega_\infty : X^2 + Y^2 + Z^2 = 0, \text{ on } \boldsymbol{\pi}_\infty$
  - ✓ Applying **H** to the affine reconstruction, then projective transformation relating original and reconstruction is a <u>similarity transformation</u>
  - ✓ Find IAC, then

    IAC ω back-projects to cone that intersects π_∞ in Ω_∞



  - ✓ note that IAC is independent of particular reconstruction

Affine to metric

- Given $\mathbf{P} = [\mathbf{M} \mid \mathbf{m}],\ \boldsymbol{\omega}$
  Possible transform from affine to metric is

$$\boxed{\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}} \qquad \mathbf{A}\mathbf{A}^T = \left(\mathbf{M}^T \boldsymbol{\omega} \mathbf{M}\right)^{-1}$$

(cholesky factorisation)

✓ Proof :

$$\mathbf{P}_{\mathrm{M}} = \mathbf{P}\mathbf{H}^{-1} = [\mathbf{M}\mathbf{A} \mid \mathbf{m}] = [\mathbf{M}_M \mid \mathbf{m}]$$

Then the ideal AC on should obey

$$\boldsymbol{\omega}^* = \mathbf{H}_d \mathbf{I} \mathbf{H}_d^T$$

$$\left(\boldsymbol{\Omega}^* = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \right)$$

by the planar homography $\mathbf{H}_d = \mathbf{M}_M\ (= \mathbf{K}\mathbf{R})$

$$\Rightarrow \boldsymbol{\omega}^* = \mathbf{M}_{\mathrm{M}} \mathbf{M}_{\mathrm{M}}{}^T = \mathbf{M}\mathbf{A}\mathbf{A}^T\mathbf{M}^T$$

$$\Rightarrow \mathbf{M}^{-1} \boldsymbol{\omega}^{-1} \mathbf{M}^{-T} = \mathbf{A}\mathbf{A}^T$$

Affine to metric-how to determine IAC

- Orthogonality
  - ✓ vanishing points corresponding to orthogonal directions

$$\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2 = 0$$

  - ✓ vanishing line and vanishing point corresponding to plane and normal direction

$$\mathbf{l} = \boldsymbol{\omega} \mathbf{v}$$

- Knowing internal parameter

$$\boldsymbol{\omega} = \mathbf{K}^{-T} \mathbf{K}^{-1}$$

  - ✓ Rectangular pixel $s = 0,\quad \omega_{12} = \omega_{21} = 0$
  - ✓ Square pixel $\alpha_x = \alpha_y,\ \ \omega_{11} = \omega_{22}$

Affine to metric-how to determine IAC

- Same camera for all images
  - ✓ same intrinsics $\Rightarrow$ same image of the absolute conic $\omega' = \omega$
    e.g. moving cameras
  - ✓ given sufficient images there is in general only one conic that projects to the same image in all images, i.e. the absolute conic

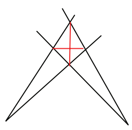
    This approach is called *self-calibration,* see later in CH. 19
  - ✓ transfer of IAC: $\omega' = \mathbf{H}_\infty^{-T} \omega \mathbf{H}_\infty^{-1}$

Affine to metric

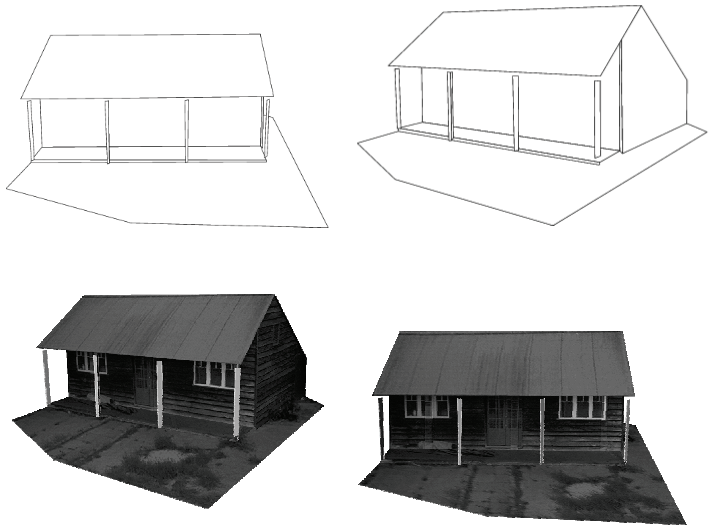
- **Direct metric reconstruction using ω**
  - ✓ **Approach 1 :**

    $$\omega = \mathbf{K}^{-T}\mathbf{K}^{-1} \Rightarrow \mathbf{K} \text{ for each camera}$$

    Calibrated reconstruction using the Essential matrix **E**
  - ✓ **Approach 2 :**
    - Compute projective roeconstruction
    - Back-project ω from both images
    - Intersection defines Ω_∞ and its support plane π_∞

    (in general two solutions) 

Affine to metric



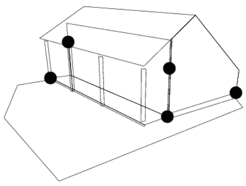
Direct reconstruction using ground truth

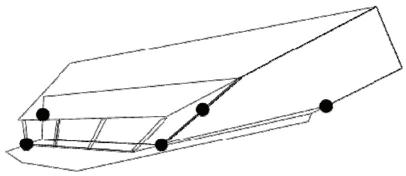
- Direct reconstruction
  - ✓ Use control points $\mathbf{X}_{Ei}$ with know coordinates to go from projective to metric

$$\mathbf{X}_{Ei} = \mathbf{H}\mathbf{X}_i$$

$$\mathbf{x}_i = \mathbf{P}\mathbf{H}^{-1}\mathbf{X}_{Ei}$$

(2 lin. eq. in $\mathbf{H}^{-1}$ per view,
3 for two views)
H: 15 DOF $\Rightarrow n \geq 5$





Summary

Objective

Given two uncalibrated images compute ($\mathbf{P}_M$,$\mathbf{P}'_M$,{$\mathbf{X}_{Mi}$})
(i.e. within similarity of original scene and cameras)

Algorithm

(i) Compute projective reconstruction ($\mathbf{P}$,$\mathbf{P}'$,{$\mathbf{X}_i$})
  (a) Compute F from $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$
  (b) Compute $\mathbf{P}$,$\mathbf{P}'$ from $\mathbf{F}$ → $\mathbf{P}=[\mathbf{I}\,|\,\mathbf{0}]\quad \mathbf{P}'=[[\mathbf{e}']_\times \mathbf{F}\,|\,\mathbf{e}']$ or $\mathbf{P}=[\mathbf{I}\,|\,\mathbf{0}]\quad \mathbf{P}'=[[\mathbf{e}']_\times \mathbf{F}+\mathbf{e}'\mathbf{v}^T\,|\,\lambda\mathbf{e}']$ $\quad \mathbf{e}'^T\mathbf{F}=0$
  (c) Triangulate $\mathbf{X}_i$ from $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$

(ii) Rectify reconstruction from projective to metric
  Direct method: compute H from control points

$$\mathbf{P}_M=\mathbf{P}\mathbf{H}^{-1},\quad \mathbf{P}'_M=\mathbf{P}'\mathbf{H}^{-1},\quad \mathbf{X}_{Mi}=\mathbf{H}\mathbf{X}_i$$

  Stratified method:
  (a) Affine reconstruction: compute π_∞

$$\mathbf{H}=\begin{bmatrix}\mathbf{I}\,|\,\mathbf{0}\\ \boldsymbol{\pi}_\infty\end{bmatrix}$$

  (b) Metric reconstruction: compute IAC ω

$$\mathbf{H}=\begin{bmatrix}\mathbf{A}^{-1} & \mathbf{0}\\ \mathbf{0} & 1\end{bmatrix}\qquad \mathbf{A}\mathbf{A}^T=\left(\mathbf{M}^T\boldsymbol{\omega}\mathbf{M}\right)^{-1}$$

Summary

Image information provided	View relations and projective objects	3-space objects	reconstruction ambiguity
point correspondences	**F**		projective
point correspondences including vanishing points	**F**,**H**$_\infty$	π_∞	affine
Points correspondences and internal camera calibration	**F**,**H**$_\infty$ ω,ω'	π_∞ Ω_∞	metric