Multi View Geometry (Spring '08)

Three View Geometry

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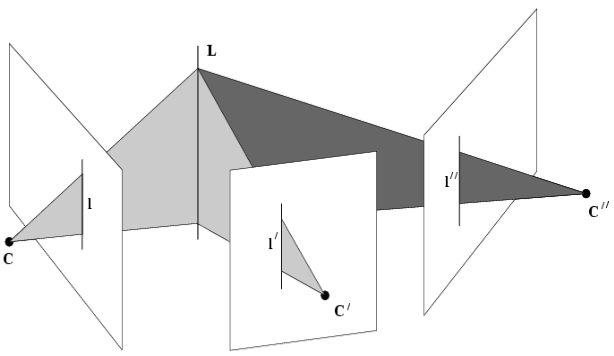
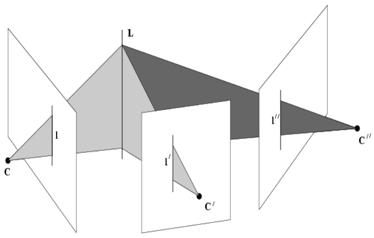
Three View Geometry

- The Trifocal Tensor
  - ✓ Defined for three views
  - ✓ Plays a similar role to the Fundamental matrix
  - ✓ Unlike **F**, trifocal tensor treats lines in three views
  - ✓ Mixed combination of lines and points are also treated



Geometry of three views

- Incidence relations for lines



$$\mathbf{l}_i \leftrightarrow \mathbf{l}'_i \leftrightarrow \mathbf{l}''_i$$

$$\mathbf{P} = [\mathbf{I} \,|\, \mathbf{0}] \quad \mathbf{P}' = [\mathbf{A} \,|\, \mathbf{a}_4] \quad \mathbf{P}''[\mathbf{B} \,|\, \mathbf{b}_4]$$

$$\boldsymbol{\pi} = \mathbf{P}^T\mathbf{l} = \begin{pmatrix} \mathbf{l} \\ 0 \end{pmatrix} \quad \boldsymbol{\pi}' = \mathbf{P}'^T\mathbf{l}' = \begin{pmatrix} \mathbf{A}^T\mathbf{l}' \\ \mathbf{a}_4{}^T\mathbf{l}' \end{pmatrix} \quad \boldsymbol{\pi}'' = \mathbf{P}''^T\mathbf{l}'' = \begin{pmatrix} \mathbf{B}^T\mathbf{l}'' \\ \mathbf{b}_4{}^T\mathbf{l}'' \end{pmatrix}$$

- $\mathbf{M} = [\boldsymbol{\pi}, \boldsymbol{\pi}', \boldsymbol{\pi}'']$ is rank 2, thus linearly dependent

$$\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3] = \begin{bmatrix} \mathbf{l} & \mathbf{A}^T\mathbf{l}' & \mathbf{B}^T\mathbf{l}'' \\ 0 & \mathbf{a}_4{}^T\mathbf{l}' & \mathbf{b}_4{}^T\mathbf{l}'' \end{bmatrix}$$

$$\mathbf{m}_1 = \alpha\mathbf{m}_2 + \beta\mathbf{m}_3 \Rightarrow 0 = \underbrace{\alpha\mathbf{a}_4{}^T\mathbf{l}'}_{k\mathbf{b}_4{}^T\mathbf{l}''} + \underbrace{\beta\mathbf{b}_4{}^T\mathbf{l}''}_{-k\mathbf{a}_4{}^T\mathbf{l}'} \Rightarrow \mathbf{l} = \left(\mathbf{b}_4{}^T\mathbf{l}''\right)\mathbf{A}^T\mathbf{l}' - \left(\mathbf{a}_4{}^T\mathbf{l}'\right)\mathbf{B}^T\mathbf{l}''$$

The Trifocal Tensor

- Definition of trifocal tensor

$$\mathbf{l} = \left(\mathbf{b}_4{}^T\mathbf{l}''\right)\mathbf{A}^T\mathbf{l}' - \left(\mathbf{a}_4{}^T\mathbf{l}'\right)\mathbf{B}^T\mathbf{l}''$$
$$= \left(\mathbf{l}''^T\mathbf{b}_4\right)\mathbf{A}^T\mathbf{l}' - \left(\mathbf{l}'^T\mathbf{a}_4\right)\mathbf{B}^T\mathbf{l}''$$

$$\Rightarrow l_i = \mathbf{l}''^T\mathbf{b}_4\mathbf{a}_i{}^T\mathbf{l}' - \mathbf{l}'^T\mathbf{a}_4\mathbf{b}_i{}^T\mathbf{l}''$$
$$= \mathbf{l}'^T(\mathbf{a}_i\mathbf{b}_4^T - \mathbf{a}_4\mathbf{b}_i{}^T)\mathbf{l}''$$
$$= \mathbf{l}'^T\mathbf{T}_i\mathbf{l}''$$

- Trifocal tensor: $\boxed{\mathbf{T}_i = \mathbf{a}_i\mathbf{b}_4{}^T - \mathbf{a}_4\mathbf{b}_i{}^T}$

The Trifocal Tensor

- Trifocal Tensor = $\{\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3\}$

  $$\mathbf{l}^T = \mathbf{l}'^T[\mathbf{T}_i]\mathbf{l}'' = \mathbf{l}'^T[\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3]\mathbf{l}'' = (\mathbf{l}'^T\mathbf{T}_1\mathbf{l}'', \mathbf{l}'^T\mathbf{T}_2\mathbf{l}'', \mathbf{l}'^T\mathbf{T}_3\mathbf{l}'')$$

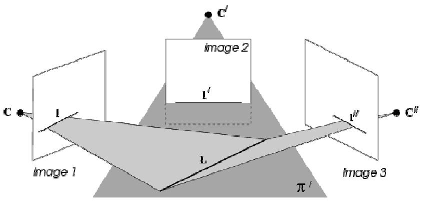
- Depends on image coordinates only, not involving 3D coordinates
- Similar relations exists, $\mathbf{l}'^T = \mathbf{l}^T[\mathbf{T}'_i]\mathbf{l}''$ and $\mathbf{l}''^T = \mathbf{l}^T[\mathbf{T}''_i]\mathbf{l}'$
- General expression is not as simple as $\mathbf{T}_i = \mathbf{a}_i\mathbf{b}_4^T - \mathbf{a}_4\mathbf{b}_i^T$
- DOF of **T**: 3x3x3=27 elements, 26 up to scale

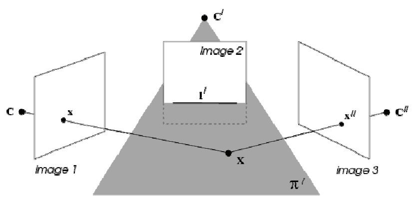
  3-view relations: 11x3-15=18 dof
- 8(=26-18) independent algebraic constraints on **T**

  (compare to 1 for **F**, i.e. rank-2)

The Trifocal Tensor

- Homographies induced by a plane





$$\mathbf{l} = \mathtt{H}^\top \mathbf{l}''$$

$$l_i = \mathbf{l}'^\top \mathtt{T}_i \mathbf{l}''$$

$$\mathtt{H} = [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3] \text{ with } \boxed{\mathbf{h}_i = \mathtt{T}_i{}^\top \mathbf{l}'}$$

$$\mathbf{x}'' = \mathtt{H}\mathbf{x}$$

$$\mathbf{x}'' = \mathtt{H}_{13}(\mathbf{l}')\,\mathbf{x}$$

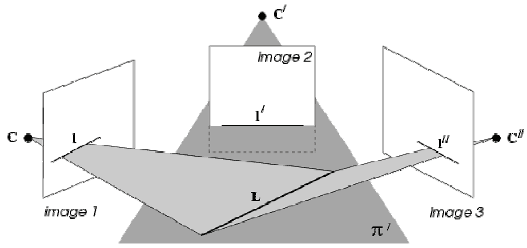
$$\boxed{\mathtt{H}_{13}(\mathbf{l}') = [\mathtt{T}_1{}^\top, \mathtt{T}_2{}^\top, \mathtt{T}_3{}^\top]\mathbf{l}'.}$$

$$\mathbf{x}' = \mathtt{H}_{12}(\mathbf{l}'')\,\mathbf{x}$$

$$\boxed{\mathtt{H}_{12}(\mathbf{l}'') = [\mathtt{T}_1, \mathtt{T}_2, \mathtt{T}_3]\mathbf{l}''}$$

The Trifocal Tensor

- Line–line–line Relation



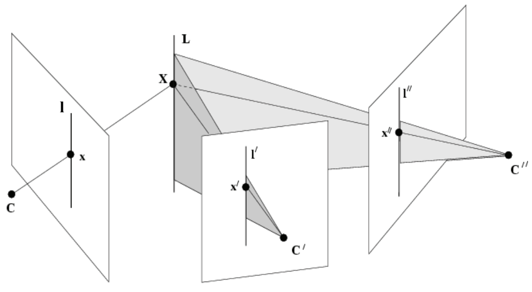
$$\mathbf{l}^\top = \mathbf{l}'^\top [\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3] \mathbf{l}'' \quad \text{(up to scale)}$$

- Eliminate scale factor : $(\mathbf{l}'^\top [\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3] \mathbf{l}'')[\mathbf{l}]_\times = \mathbf{0}^\top$

$$(\mathbf{l}'^\top [\mathrm{T}_i] \mathbf{l}'')[\mathbf{l}]_\times = \mathbf{0}^\top$$
$$(\mathbf{l}''^\top [\mathrm{T}_i{}^\top] \mathbf{l}')[\mathbf{l}]_\times = \mathbf{0}^\top$$

The Trifocal Tensor

- Point–line–line Relation



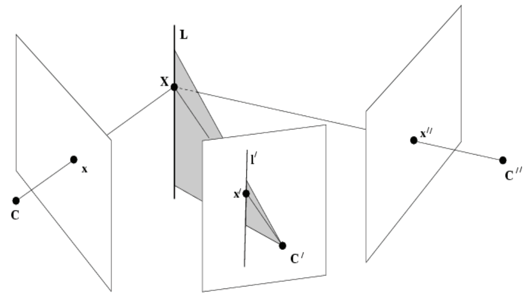
$$\mathbf{x}^\top \mathbf{l} = \textstyle\sum_i x^i l_i = 0 \qquad l_i = \mathbf{l}'^\top \mathrm{T}_i \mathbf{l}''^\top$$

$$\Longrightarrow \quad \mathbf{l}'^\top \Big(\sum_i x^i \mathrm{T}_i\Big) \mathbf{l}''^\top = 0$$

$\left(\sum_i x^i \mathrm{T}_i\right)$ 3 × 3 matrix

The Trifocal Tensor

- Point–line–point Relation

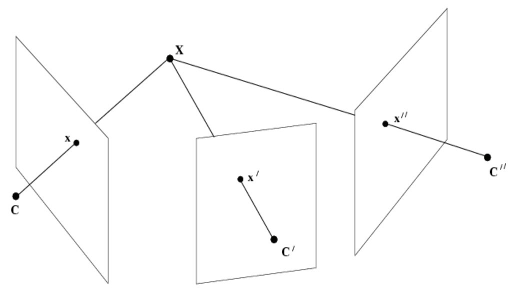


$$\mathbf{x}'' = \mathtt{H}_{13}(\mathbf{l}')\,\mathbf{x} = [\mathtt{T}_1{}^{\top}\mathbf{l}', \mathtt{T}_2{}^{\top}\mathbf{l}', \mathtt{T}_3{}^{\top}\mathbf{l}']\,\mathbf{x} = (\sum_i x^i \mathtt{T}_i{}^{\top})\mathbf{l}'$$

$$\Longrightarrow \quad \mathbf{x}''^{\top}[\mathbf{x}'']_\times = \boxed{\mathbf{l}'^{\top}(\sum_i x^i \mathtt{T}_i)[\mathbf{x}'']_\times = \mathbf{0}^{\top}},$$

The Trifocal Tensor

- Point–point–point Relation



$$\mathbf{l}'^{\top}(\sum_i x^i \mathtt{T}_i)[\mathbf{x}'']_\times = \mathbf{0}^{\top}$$

$$\mathbf{l}'^{\top} = \mathbf{y}'^{\top}[\mathbf{x}']_\times, \text{ any } \mathbf{y}' \textit{ is on } \mathbf{l}'$$

$$\Longrightarrow \quad \boxed{[\mathbf{x}']_\times(\sum_i x^i \mathtt{T}_i)[\mathbf{x}'']_\times = 0_{3\times 3}}$$

The Trifocal Tensor

- Summary of incidence relations

(i) Line–line–line correspondence

$$\mathbf{l}'^{\top}[\mathtt{T}_1, \mathtt{T}_2, \mathtt{T}_3]\mathbf{l}'' = \mathbf{l}^{\top} \quad \text{or} \quad (\mathbf{l}'^{\top}[\mathtt{T}_1, \mathtt{T}_2, \mathtt{T}_3]\mathbf{l}'')\,[\mathbf{l}]_{\times} = \mathbf{0}^{\top}$$

(ii) Point–line–line correspondence

$$\mathbf{l}'^{\top}(\sum_i x^i \mathtt{T}_i)\mathbf{l}'' = 0 \quad \text{for a correspondence } \mathbf{x} \leftrightarrow \mathbf{l}' \leftrightarrow \mathbf{l}''$$

(iii) Point–line–point correspondence

$$\mathbf{l}'^{\top}(\sum_i x^i \mathtt{T}_i)[\mathbf{x}'']_{\times} = \mathbf{0}^{\top} \quad \text{for a correspondence } \mathbf{x} \leftrightarrow \mathbf{l}' \leftrightarrow \mathbf{x}''$$

(iv) Point–point–line correspondence

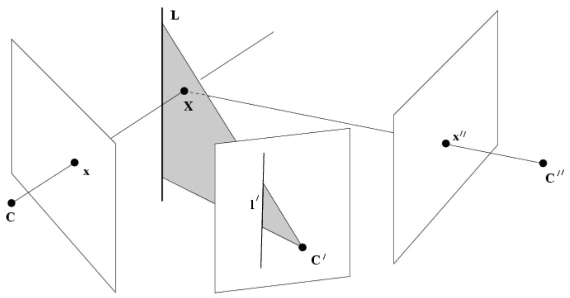
$$[\mathbf{x}']_{\times}(\sum_i x^i \mathtt{T}_i)\mathbf{l}'' = \mathbf{0} \quad \text{for a correspondence } \mathbf{x} \leftrightarrow \mathbf{x}' \leftrightarrow \mathbf{l}''$$

(v) Point–point–point correspondence

$$[\mathbf{x}']_{\times}(\sum_i x^i \mathtt{T}_i)[\mathbf{x}'']_{\times} = 0_{3\times 3}$$

The Trifocal Tensor

- Non-incidence configuration
  - ✓ Incidence in image does not guarantee incidence in space

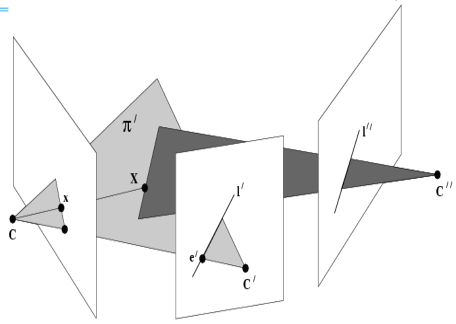


The Trifocal Tensor

- Epipolar lines

Point–line–line correspondence

$$\mathbf{l}'^{\top}(\sum_i x^i \mathtt{T}_i)\mathbf{l}'' = 0$$



✓ If **l'** is epipolar line, then the PLL relation holds for any **l''**,

$$\Longrightarrow \mathbf{l}'^{\top}(\sum_i x^i \mathtt{T}_i) = \mathbf{0}^{\top}$$

✓ Similarly if **l''** is epipolar line,

$$\Longrightarrow (\sum_i x^i \mathtt{T}_i)\mathbf{l}'' = \mathbf{0}$$

✓ with points $(1,0,0)^{\top}$, $(0,1,0)^{\top}$ and $(0,0,1)^{\top}$
$(\sum_i x^i \mathtt{T}_i)$ becomes $\mathtt{T}_1$, $\mathtt{T}_2$ and $\mathtt{T}_3$ respectively

✓ Epipoles are the intersections of the left and right null-space of $\mathtt{T}_1$, $\mathtt{T}_2$ and $\mathtt{T}_3$

Properties of Trifocal Tensor

- Algibric properties of $\mathbf{T}_i$ matrices

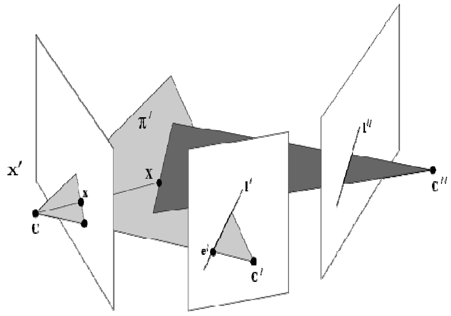
- Each matrix $\mathtt{T}_i$ has rank 2. This is evident from (14.1) since $\mathtt{T}_i = \mathbf{a}_i\mathbf{e}''^{\top} - \mathbf{e}'\mathbf{b}_i^{\top}$ is the sum of two outer products.
- The right null-vector of $\mathtt{T}_i$ is $\mathbf{l}''_i = \mathbf{e}''\times\mathbf{b}_i^{\top}$, and is the epipolar line in the third view for the point $\mathbf{x} = (1,0,0)^{\top}$, $(0,1,0)^{\top}$ or $(0,0,1)^{\top}$, as $i = 1,2$ or 3 respectively.
- The epipole $\mathbf{e}''$ is the common intersection of the epipolar lines $\mathbf{l}''_i$ for $i = 1,2,3$.
- The left null-vector of $\mathtt{T}_i$ is $\mathbf{l}'_i = \mathbf{e}'\times\mathbf{a}_i^{\top}$, and is the epipolar line in the second view for the point $\mathbf{x} = (1,0,0)^{\top}$, $(0,1,0)^{\top}$ or $(0,0,1)^{\top}$, as $i = 1,2$ or 3 respectively.
- The epipole $\mathbf{e}'$ is the common intersection of the epipolar lines $\mathbf{l}'_i$ for $i = 1,2,3$.
- The *sum* of the matrices $\mathtt{M}(\mathbf{x}) = (\sum_i x^i \mathtt{T}_i)$ also has rank 2. The right null-vector of $\mathtt{M}(\mathbf{x})$ is the epipolar line $\mathbf{l}''$ of $\mathbf{x}$ in the third view, and its left null-vector is the epipolar line $\mathbf{l}'$ of $\mathbf{x}$ in the second view.

Extracting F

- Extracting **F**

$$\mathbf{x}' = ([\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3]\mathbf{l}'')\,\mathbf{x}$$

$$\underset{\text{Epipolar line}}{\mathbf{l}'} = \underbrace{[\mathbf{e}']_\times ([\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3]\mathbf{l}'')}_{\mathbf{F}_{21}}\,\mathbf{x}$$



- A good choice for **l**” is **e**” (since **e**” is perpendicular to the right null space of $\mathbf{T}_i$)

$$\boxed{\mathrm{F}_{21} = [\mathbf{e}']_\times [\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3]\mathbf{e}''} \quad \boxed{\mathrm{F}_{31} = [\mathbf{e}'']_\times [\mathrm{T}_1^\top, \mathrm{T}_2^\top, \mathrm{T}_3^\top]\mathbf{e}'}$$

Retrieving camera matrices

- Computing camera matrices, $\mathbf{P}, \mathbf{P}', \mathbf{P}''$

$$\boxed{\mathbf{P} = [\mathbf{I} \,|\, \mathbf{0}]}$$

$$\mathbf{F}_{21} \Rightarrow \boxed{\mathbf{P}' = [[\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3]\mathbf{e}'' \,|\, \mathbf{e}']}$$

$$\mathbf{F}_{31} \Rightarrow \mathbf{P}'' = [[\mathbf{T}_1^T, \mathbf{T}_2^T, \mathbf{T}_3^T]\mathbf{e}' \,|\, \mathbf{e}''] \longrightarrow \text{incorrect}$$

$$\{\mathrm{P}, \mathrm{P}'\} \text{ and } \{\mathrm{P}, \mathrm{P}''\} \quad \text{ok, but not} \quad \{\mathrm{P}, \mathrm{P}', \mathrm{P}''\}$$

$$\mathbf{P}' = [[\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3]\mathbf{e}'' \,|\, \mathbf{e}'] \quad \Rightarrow \quad a_i = \mathrm{T}_i \mathrm{e}''$$

$$a_4 = \mathrm{e}', \quad b_4 = \mathrm{e}''$$

$$\mathrm{T}_i = \mathrm{T}_i \mathbf{e}'' \mathbf{e}''^\top - \mathbf{e}' \mathbf{b}_i^\top \quad (\mathbf{T}_i = \mathbf{a}_i \mathbf{b}_4^T - \mathbf{a}_4 \mathbf{b}_i^T)$$

$$\mathbf{b}_i = (\mathbf{e}'' \mathbf{e}''^\top - \mathrm{I})\mathrm{T}_i^\top \mathbf{e}'$$

$$\boxed{\mathrm{P}'' = [(\mathbf{e}'' \mathbf{e}''^\top - \mathrm{I})[\mathrm{T}_1^\top, \mathrm{T}_2^\top, \mathrm{T}_3^\top]\mathbf{e}' \,|\, \mathbf{e}'']}$$

Tensor notation

- Standard matrix-vector notation can not handle large number of indices

$$[T_1, T_2, T_3]$$

$$l'^{\top}(\sum_i x^i T_i) l''^{\top} = 0$$

- Thus, let us use Tensor notation instead

$$T_i^{jk}$$

$$x^i l'_j l''_k T_i^{jk} = 0$$

Tensor notation

- Collection of numbers, related to coordinate choice, indexed by one or more indices

$$A^{i_1, i_2, \cdots i_n}_{j_1, j_2, \cdots j_m}$$

- Valency = $(n + m)$

- Indices can be any value between 1 and the dimension of space ($d^{(n+m)}$ coefficients)

Tensor notation

- Conventions

$$A^{i_1, i_2, \cdots i_n}_{j_1, j_2, \cdots j_m}$$

contravariant (upper index)

covariant (lower index)

✓ ***Einstein's summation*** **(once above, once below) :**

$$A^i_j b_i = \sum_i A^i_j b_i$$

✓ ***Index rule:***

$$A^i_j b_i = c_j$$

- Transformation

$$x'^j = A^j_i x^i \quad (\textit{Covariant})$$

$$A^j_i l'_j = l_i \quad (\textit{Contravariant})$$

More on Tensors

- Kronecker delta

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \qquad \text{(valency 2 tensor)}$$

- Levi-Cevita epsilon

$$\varepsilon_{ijk} = \begin{cases} 1 & \text{when i,j,k is a even permutation of 1,2,3} \\ -1 & \text{when i,j,k is an odd permutation of 1,2,3} \\ 0 & \text{when at least two indices are equal} \end{cases}$$

(valency 3 tensor)

- Ex) dot product

  $\mathbf{u} \cdot \mathbf{v} = u_i v^i$

  cross product

  $(\mathbf{u} \times \mathbf{v})_k = l_k = u^i v^j \varepsilon_{ijk}$

  skew symmetric matrix

  $([\mathbf{x}]_\times)_{rs} = x^i \varepsilon_{irs}$

$$\det \begin{bmatrix} a^1 & a^2 & a^3 \\ b^1 & b^2 & b^3 \\ c^1 & c^2 & c^3 \end{bmatrix} = a^i b^j c^k \varepsilon_{ijk}$$

More on Tensors

- Let us denote
  col. vector row vector
  point $\mathbf{x} = (x^1, x^2, x^3)^T$, line $\mathbf{l} = (l_1, l_2, l_3)$
  $\mathbf{A} \rightarrow a^i_j$, i : contravariant (row), j : covariant (col.)
- Then $\mathbf{x}' = \mathbf{A}\mathbf{x} \rightarrow x'^i = \sum_j a^i_j x^j \rightarrow x'^i = a^i_j x^j$
  $$\mathbf{T}_i = \mathbf{a}_i \mathbf{b}_4^T - \mathbf{a}_4 \mathbf{b}_i^T \rightarrow T_i^{jk} = a_i^j b_4^k - a_4^j b_i^k$$
  $$l_i = \mathbf{l}'^T \mathbf{T}_i \mathbf{l}'' \rightarrow l_i = l'_j l''_k T_i^{jk}$$
- Note that in tensor multiplication, the order doesn't matter
  $$l'_j l''_k T_i^{jk} = l'_j T_i^{jk} l''_k = T_i^{jk} l'_j l''_k$$
- The index *i, j, k* of T_i^{jk} correspond to entities in the first, second and third views respectively $l''_j T_i^{jk}$ (X)

More on Tensors

Definition. The trifocal tensor $\mathcal{T}$ is a valency 3 tensor $\mathcal{T}_i^{jk}$ with two contravariant and one covariant indices. It is represented by a homogeneous $3 \times 3 \times 3$ array (i.e. 27 elements). It has 18 degrees of freedom.

Computation from camera matrices. If the canonical 3×4 camera matrices are

$$\mathtt{P} = [\mathtt{I} \mid \mathbf{0}], \quad \mathtt{P}' = [a^i_j], \quad \mathtt{P}'' = [b^i_j]$$

then

$$\mathcal{T}_i^{jk} = a_i^j b_4^k - a_4^j b_i^k.$$

See (16.12–*p*404) for computation from three general camera matrices.

Line transfer from corresponding lines in the second and third views to the first.

$$l_i = l'_j l''_k \mathcal{T}_i^{jk}$$

Transfer by a homography.

(i) **Point transfer from first to third view via a plane in the second**
The contraction $l'_j \mathcal{T}_i^{jk}$ is a homography mapping between the first and third views induced by a plane defined by the back-projection of the line $\mathbf{l}'$ in the second view.

$$x''^k = h_i^k x^i \quad \text{where} \quad h_i^k = l'_j \mathcal{T}_i^{jk}$$

(ii) **Point transfer from first to second view via a plane in the third**
The contraction $l''_k \mathcal{T}_i^{jk}$ is a homography mapping between the first and second views induced by a plane defined by the back-projection of the line $\mathbf{l}''$ in the third view.

$$x'^j = h_i^j x^i \quad \text{where} \quad h_k^j = l''_k \mathcal{T}_i^{jk}$$

Trilinearities

- Trilinearities

(i) Line–line–line correspondence

$$(l_r \epsilon^{ris}) l'_j l''_k \mathcal{T}_i^{jk} = 0^s$$

(ii) Point–line–line correspondence

$$x^i l'_j l''_k \mathcal{T}_i^{jk} = 0$$

(iii) Point–line–point correspondence

$$x^i l'_j (x''^k \epsilon_{kqs}) \mathcal{T}_i^{jq} = 0_s$$

(iv) Point–point–line correspondence

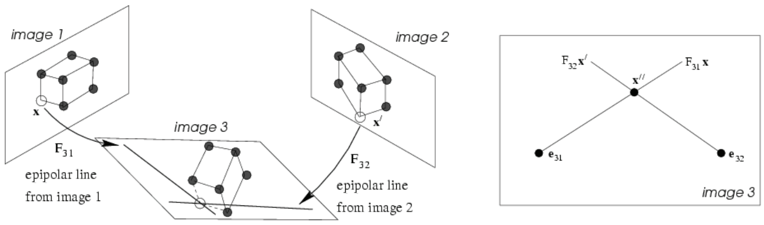
$$x^i (x'^j \epsilon_{jpr}) l''_k \mathcal{T}_i^{pk} = 0_r$$

(v) Point–point–point correspondence

$$x^i (x'^j \epsilon_{jpr})(x''^k \epsilon_{kqs}) \mathcal{T}_i^{pq} = 0_{rs}$$

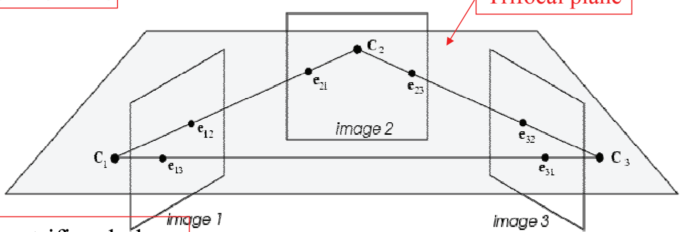
Transfer

- Transfer : epipolar transfer



$$\mathrm{x}'' = (\mathrm{F}_{31}\hat{\mathrm{x}}) \times (\mathrm{F}_{32}\hat{\mathrm{x}}')$$

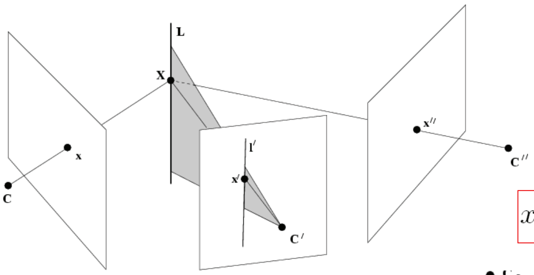
Trifocal plane



Fails for any **X** on trifocal plane

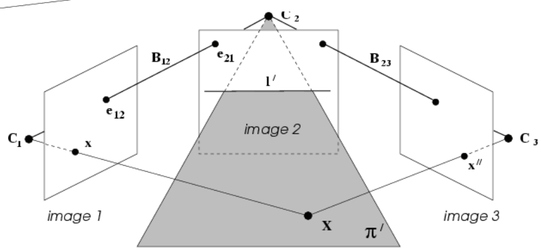
Transfer

- Transfer : trifocal tensor



$$x''^k = \hat{x}^i l'_j \mathcal{T}_i^{jk}$$

Avoid **l'**=epipolar line



Transfer

- Transfer : trifocal tensor
  - ✓ Point transfer

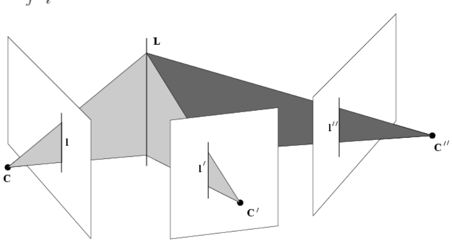
(i) Compute $\mathtt{F}_{21}$ from the trifocal tensor (by the method given in algorithm 14.1), and correct $\mathbf{x} \leftrightarrow \mathbf{x}'$ to the exact correspondence $\hat{\mathbf{x}} \leftrightarrow \hat{\mathbf{x}}'$ using algorithm 11.1-(p304).

(ii) Compute the line $\mathbf{l}'$ through $\hat{\mathbf{x}}'$ and perpendicular to $\mathbf{l}'_e = \mathtt{F}_{21}\hat{\mathbf{x}}$. If $\mathbf{l}'_e = (l_1, l_2, l_3)^\top$ and $\hat{\mathbf{x}}' = (\hat{x}_1, \hat{x}_2, 1)^\top$, then $\mathbf{l}' = (l_2, -l_1, -\hat{x}_1 l_2 + \hat{x}_2 l_1)^\top$.

(iii) The transferred point is $x''^k = \hat{x}^i l'_j \mathcal{T}_i^{jk}$.

  - ✓ Line transfer

$$l_i = l'_j l''_k \mathcal{T}_i^{jk}$$



- Degenerate when known lines are corresponding epipolar lines

Relation between F and T

- Fundamental matries $\underset{\bigcirc}{\overset{?}{\rightleftarrows}}$ Trifocal tensor
- $\mathbf{F}_{12} \longrightarrow$ **P, P'**
- **Algorithm for P"**

  (i) Select a set of matching points $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ in the first two images, satisfying $\mathbf{x}'^{\top}_i \mathtt{F}_{21}\mathbf{x}_i = 0$, and use triangulation to determine the corresponding 3D points $\mathbf{X}_i$.

  (ii) Use epipolar transfer to determine the corresponding points $\mathbf{x}''_i$ in the third image, using the fundamental matrices $\mathtt{F}_{31}$ and $\mathtt{F}_{32}$.

  (iii) Solve for the camera matrix $\mathtt{P}''$ from the set of 3D–2D correspondences $\mathbf{X}_i \leftrightarrow \mathbf{x}''_i$.

- Degeneracy
  - ✓ The second step in this algorithm will fail in the case where the point $\mathbf{X}_i$ lies in the trifocal plane.
- The trifocal tensor can be determined from three fundamental matrices alone *iff* the three cameras are not colinear