Multi View Geometry (Spring '08)

Auto-Calibration

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Auto-Calibration

- Auto-Calibration:
  - ✓ How to determine **K** directly from multiple uncalibrated images
  - ✓ And then compute a metric reconstruction

- Goal:
  - ✓ Given projective reconstruction $\{\mathbf{P}^i, \mathbf{X}_j\}$
  - ✓ Determine rectifying homography **H** such that $\{\mathbf{P}^i\mathbf{H}, \mathbf{H}^{-1}\mathbf{X}_j\}$ is a metric reconstruction

$$\mathbf{x}^i = \mathbf{P}_M^i \mathbf{X}_M$$
$$\mathbf{P}_M^i = \mathbf{K}^i\left[\mathbf{R}^i \mid \mathbf{t}^i\right] = \mathbf{P}^i\mathbf{H}, \ i = 1, \cdots, m$$

Algebraic framework

- Rectifying homography H:

$$\mathbf{P}_M^1 = \mathbf{P}^1\mathbf{H}$$

$$\mathbf{P}^1 = [\mathbf{I}\,|\,0] \quad \mathbf{P}_M^1 = \mathbf{K}^1[\mathbf{I}\,|\,0]$$

$$\mathbf{K}^1[\mathbf{I}\,|\,0] = [\mathbf{I}\,|\,0]\mathbf{H}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{K}^1 & 0 \\ \mathbf{v}^T & 1 \end{bmatrix} \qquad \mathbf{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

- Plane at infinity $\boldsymbol{\pi}_\infty$

$$\boldsymbol{\pi}_\infty = \mathbf{H}^{-T}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{bmatrix} (\mathbf{K}^1)^{-T} & -(\mathbf{K}^1)^{-T}\mathbf{v} \\ 0 & 1 \end{bmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{pmatrix} -(\mathbf{K}^1)^{-T}\mathbf{v} \\ 1 \end{pmatrix}$$

$$\boldsymbol{\pi}_\infty = (\mathbf{p}^T, 1)^T \longrightarrow \mathbf{p} = -(\mathbf{K}^1)^{-T}\mathbf{v} \quad \Rightarrow \quad \mathbf{H} = \begin{bmatrix} \mathbf{K}^1 & 0 \\ -\mathbf{p}^T\mathbf{K}^1 & 1 \end{bmatrix}$$

8 DOF

Basic equation

- Basic equation for auto-calibration

$$\mathbf{H} = \begin{bmatrix} \mathbf{K}^1 & 0 \\ -\mathbf{p}^T\mathbf{K}^1 & 1 \end{bmatrix}$$

  - ✓ For $\mathbf{P}^i = [\mathbf{A}^i\,|\,\mathbf{a}^i]$

$$\mathbf{P}_M^i = \mathbf{P}^i\mathbf{H} = [\mathbf{A}^i\,|\,\mathbf{a}^i]\mathbf{H} \quad \text{for } i = 2,\ldots,m$$

$$\mathbf{K}^i\mathbf{R}^i = (\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)\mathbf{K}^1 \quad \text{*Consider 3x3 term}$$

$$\mathbf{R}^i = (\mathbf{K}^i)^{-1}(\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)\mathbf{K}^1$$

$$\downarrow \ \mathbf{R}\mathbf{R}^T = \mathbf{I}$$

$$\mathbf{K}^i\mathbf{K}^{iT} = (\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)\mathbf{K}^1\mathbf{K}^{1T}(\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)^T$$

  - ✓ In terms of AC (or DIAC) $\mathbf{K}^i\mathbf{K}^{iT} = \boldsymbol{\omega}^{*i}$

$$\boldsymbol{\omega}^{*i} = (\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)\boldsymbol{\omega}^{*1}(\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)^T$$

$$\boldsymbol{\omega}^i = (\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)^{-T}\boldsymbol{\omega}^1(\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)^{-1}$$

Basic equation

- How to find ω^i (ω^{*i}) and $\mathbf{p}$?
- Methods:
  - Calibration using the absolute dual quadric
  - The Kruppa equations
  - A stratified solution

Calibration using the DIAQ

- Absolute dual quadric $\mathbf{Q}_\infty^*$:
  - 4x4 homogeneous matrix of rank 3
  - PSD (positive semi-definite)
  - $\mathbf{Q}_\infty^* \boldsymbol{\pi}_\infty = \mathbf{0}$
  - $\mathbf{KK}^T = \boldsymbol{\omega}^* = \mathbf{PQ}_\infty^* \mathbf{P}^\mathrm{T}$
- Idea of calibration: $\text{constraints on } \mathbf{K} \longrightarrow \mathrm{Q}_\infty^* \xrightarrow{\text{decomposition}} \mathbf{H}$
- Simple property of absolute dual quadric
  - Canonical form in Euclidean frame

$$\tilde{\mathbf{I}} = \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0} \\ \mathbf{0}^\mathrm{T} & 0 \end{bmatrix}$$

$$\mathbf{X}_M = \mathbf{H}^{-1}\mathbf{X}$$

  - In a projective coordinate frame $\mathbf{Q}_\infty^* = \mathbf{H}\tilde{\mathbf{I}}\mathbf{H}^\mathrm{T}$

Calibration using the DIAQ

- Equivalence to auto-calibration equations

$$\mathbf{Q}_\infty^* = \mathbf{H}\tilde{\mathbf{I}}\mathbf{H}^T = \begin{bmatrix} \mathbf{K}^1 & 0 \\ -\mathbf{p}^T\mathbf{K}^1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{K}^1 & 0 \\ -\mathbf{p}^T\mathbf{K}^1 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} \mathbf{K}^1\mathbf{K}^{1T} & -\mathbf{K}^1\mathbf{K}^{1T}\mathbf{p} \\ -\mathbf{p}^T\mathbf{K}^1\mathbf{K}^{1T} & \mathbf{p}^T\mathbf{K}^1\mathbf{K}^{1T}\mathbf{p} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega}^{*1} & -\boldsymbol{\omega}^{*1}\mathbf{p} \\ -\mathbf{p}^T\boldsymbol{\omega}^{*1} & \mathbf{p}^T\boldsymbol{\omega}^{*1}\mathbf{p} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{\omega}^{*i} = \mathbf{P}^i\mathbf{Q}_\infty^*\mathbf{P}^{iT} = (\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)\mathbf{K}^1\mathbf{K}^{1T}(\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T)^T$$

- How to estimate $\mathbf{Q}_\infty^*$:
  - ✓ Utilize constraints on the internal parameters of **K** (equivalently on ω^i or ω^{*i})
  - ✓ Linear solution
  - ✓ Non-linear solution
  - ✓ Iterative methods

Calibration using the DIAQ

- **Linear solution** for $\mathbf{Q}_\infty^*$ from a set of images

The forms of $\boldsymbol{\omega} = (\mathtt{KK}^\top)^{-1}$ and $\boldsymbol{\omega}^* = \boldsymbol{\omega}^{-1} = \mathtt{KK}^\top$ for a camera with calibration matrix K as in (5.10–p143) are

$$\boldsymbol{\omega}^* = \begin{bmatrix} \alpha_x^2 + s^2 + x_0^2 & s\alpha_y + x_0y_0 & x_0 \\ s\alpha_y + x_0y_0 & \alpha_y^2 + y_0^2 & y_0 \\ x_0 & y_0 & 1 \end{bmatrix} \quad (18.9)$$

and

$$\boldsymbol{\omega} = \frac{1}{\alpha_x^2\alpha_y^2} \begin{bmatrix} \alpha_y^2 & -s\alpha_y & -x_0\alpha_y^2 + y_0s\alpha_y \\ -s\alpha_y & \alpha_x^2 + s^2 & \alpha_y s x_0 - \alpha_x^2 y_0 - s^2 y_0 \\ -x_0\alpha_y^2 + y_0 s\alpha_y & \alpha_y s x_0 - \alpha_x^2 y_0 - s^2 y_0 & \alpha_x^2\alpha_y^2 + \alpha_x^2 y_0^2 + (\alpha_y x_0 - s y_0)^2 \end{bmatrix} \quad (18.10)$$

If the skew is zero, i.e. $s = 0$, then the expressions simplify to

$$\boldsymbol{\omega}^* = \begin{bmatrix} \alpha_x^2 + x_0^2 & x_0y_0 & x_0 \\ x_0y_0 & \alpha_y^2 + y_0^2 & y_0 \\ x_0 & y_0 & 1 \end{bmatrix} \quad (18.11)$$

and

$$\boldsymbol{\omega} = \frac{1}{\alpha_x^2\alpha_y^2} \begin{bmatrix} \alpha_y^2 & 0 & -\alpha_y^2 x_0 \\ 0 & \alpha_x^2 & -\alpha_x^2 y_0 \\ -\alpha_y^2 x_0 & -\alpha_x^2 y_0 & \alpha_x^2\alpha_y^2 + \alpha_y^2 x_0^2 + \alpha_x^2 y_0^2 \end{bmatrix} \quad (18.12)$$

Calibration using the DIAQ

- Specifying linear constraints on $\mathbf{Q}_\infty^*$
  - ✓ Assume that principle point is known and transformed to the origin.

$$\boldsymbol{\omega}^* = \begin{bmatrix} \alpha_x^2 + s^2 & s\alpha_y & 0 \\ s\alpha_y & \alpha_y^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\omega}_{13}^{*i} = (\mathbf{P}^i \mathbf{Q}_\infty^* \mathbf{P}^{iT})_{13} = \boldsymbol{\omega}_{23}^{*i} = (\mathbf{P}^i \mathbf{Q}_\infty^* \mathbf{P}^{iT})_{23} = 0$$

$$\boldsymbol{\omega}_{13}^{*i} = \mathbf{P}_1{}^i \mathbf{Q}_\infty^* \mathbf{P}_3{}^{iT} = \boldsymbol{\omega}_{23}^{*i} = \mathbf{P}_2{}^i \mathbf{Q}_\infty^* \mathbf{P}_3{}^{iT} = 0 \longrightarrow \text{2 equations}$$

  - ✓ DAQ may be parameterized linearly by 10 homogeneous parameters.

$$\mathbf{Ax} = 0$$

  - ✓ From five images a total of 10 equation are obtained.
    ※ 8DOF = 10 – 1(scale factor) – 1 {det(DAQ) = 0}

Calibration using the DIAQ

- Auto-calibration constraints derived from the DIAC

Condition	constraint	type	# constraints
zero skew	$\omega_{12}^*\omega_{33}^* = \omega_{13}^*\omega_{23}^*$	quadratic	m
principal point (p.p.) at origin	$\omega_{13}^* = \omega_{23}^* = 0$	linear	$2m$
zero skew (p.p. at origin)	$\omega_{12}^* = 0$	linear	m
fixed (unknown) aspect ratio (zero skew and p.p. at origin)	$\frac{\omega^{*i}_{11}}{\omega^{*i}_{22}} = \frac{\omega^{*j}_{11}}{\omega^{*j}_{22}}$	quadratic	$m-1$
known aspect ratio $r = \alpha_y/\alpha_x$ (zero skew and p.p. at origin)	$r^2\omega_{11}^* = \omega_{22}^*$	linear	m

- Example: variable focal length
  - ✓ Zero skew, principal point at origin, unit aspect ratio
  - ✓ 4 constraints per view $\longrightarrow$ if $m \geq 3$, unique solution exists

Calibration using the DIAQ

- **Nonlinear solution for $\mathbf{Q}_\infty^*$:**

  i) Constant internal parameter case (same cameras)

  $$\boldsymbol{\omega}^{*i} = \boldsymbol{\omega}^{*j} \longleftrightarrow \mathbf{P}^i \mathbf{Q}_\infty^* \mathbf{P}^{i\mathrm{T}} = \mathbf{P}^j \mathbf{Q}_\infty^* \mathbf{P}^{j\mathrm{T}} \text{ (up tp scale)}$$

  $$\omega_{11}^{*i} / \omega_{11}^{*j} = \omega_{12}^{*i} / \omega_{12}^{*j} = \omega_{13}^{*i} / \omega_{13}^{*j} = \omega_{22}^{*i} / \omega_{22}^{*j} = \omega_{23}^{*i} / \omega_{23}^{*j} = \omega_{33}^{*i} / \omega_{33}^{*j}$$

  ✓ 5 equation for 2 view

  ✓ 10 equations for 3 view ⟶ sufficient

  ii) Calibration assuming zero skew

  $$\omega_{12}^{*i}\omega_{33}^{*i} = \omega_{13}^{*i}\omega_{23}^{*i}$$

  ✓ Single quadric constraint per view

  ✓ m equations with m views + $\det \mathbf{Q}_\infty^* = 0$

Calibration using the DIAQ

- **Iterative methods**

  ✓ The known scale factor problem can be eliminated by using a matrix norm.

  ✓ Cost function

  $$\sum_i \left\| \mathbf{K}^i \mathbf{K}^{iT} - \mathbf{P}^i \mathbf{Q}_\infty^* \mathbf{P}^{iT} \right\|_F^2$$

  ✓ In case of the focal length is the only unknown per view, cost function would be minimized over $m+3$ parameters.

  (focal length of each view (total m) , and 3 parameters of **p**)

  ✓ Method

  - Get Initial value from linear solution
  - Use LM method with cost function to unknown parameter ($m+3$) .

Counting argument

- Consider *m* views
- Assume *k* parameters are known, *f* parameters are fixed but unknown *(k+f <5)*
- The number of total constraints: *mk + f(m-1)*
- The requirement for calibration

$$mk + (m-1)f \geq 8$$

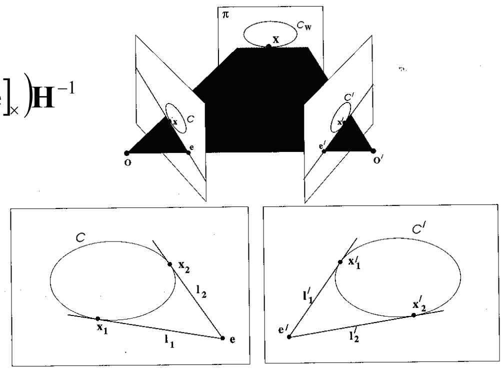
- **Limitations of absolute quadratic approach**
- Limitations of least-square algebraic solution
  - ✓ Solution does not satisfy the constraints precisely
  - ✓ Enforcing the PSD condition is difficult, leading spurious calibration

The Kruppa equations

- First auto-calibration method
- Two-view constraints on $\boldsymbol{\omega}^*$ that require only **F** to be known

$$[\mathbf{e}]_\times \mathbf{C}^* [\mathbf{e}]_\times \quad \text{(degenerate line conic)}$$

$$[\mathbf{e}']_\times \mathbf{C}^{*\prime} [\mathbf{e}']_\times = \mathbf{H}^{-T} \left([\mathbf{e}]_\times \mathbf{C}^* [\mathbf{e}]_\times \right) \mathbf{H}^{-1} = \mathbf{F}\mathbf{C}^*\mathbf{F}^T$$



The Kruppa equations

- Let $\mathbf{C}^* = \boldsymbol{\omega}^*, \mathbf{C}^{*'} = \boldsymbol{\omega}^{*'} (and\ \ \mathbf{H} = \mathbf{H}_\infty)$ then

$$[\mathbf{e}']_\times \boldsymbol{\omega}^{*'} [\mathbf{e}']_\times = \mathbf{F}\boldsymbol{\omega}^* \mathbf{F}^T$$

- This can be rewritten as

$$\begin{pmatrix} \mathbf{u}_2^T \boldsymbol{\omega}^{*'} \mathbf{u}_2 \\ -\mathbf{u}_1^T \boldsymbol{\omega}^{*'} \mathbf{u}_2 \\ \mathbf{u}_1^T \boldsymbol{\omega}^{*'} \mathbf{u}_2 \end{pmatrix} \times \begin{pmatrix} \sigma_1^2 \mathbf{v}_1^T \boldsymbol{\omega}^* \mathbf{v}_1 \\ \sigma_1 \sigma_2 \mathbf{v}_1^T \boldsymbol{\omega}^* \mathbf{v}_2 \\ \sigma_2^2 \mathbf{v}_2^T \boldsymbol{\omega}^* \mathbf{v}_2 \end{pmatrix} = 0$$

where

$$\mathbf{F} = \mathbf{U}\mathbf{D}\mathbf{V}^T = \mathbf{U}\begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & 0 \end{bmatrix}\mathbf{V}^T$$

  - ✓ 3 quadric equations for $\boldsymbol{\omega}_{ij}^*$ of $\boldsymbol{\omega}^*$

- For identical **K**s,

$$\boldsymbol{\omega}^* = \boldsymbol{\omega}^{*'} \Rightarrow \quad [\mathbf{e}']_\times \boldsymbol{\omega}^* [\mathbf{e}']_\times = \mathbf{F}\boldsymbol{\omega}^* \mathbf{F}^T$$

The Kruppa equations

- Example
  - ✓ Assume all the parameters are known except the focal length.
  - ✓ Then

$$\boldsymbol{\omega}^* = diag(\alpha^2, \alpha^2, 1), \quad \boldsymbol{\omega}^{*'} = diag(\alpha'^2, \alpha'^2, 1)$$

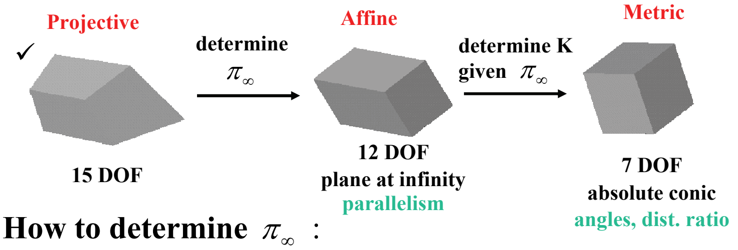
  - ✓ And the Kruppa equations become

$$\frac{\mathbf{u}_2^T \boldsymbol{\omega}^{*'} \mathbf{u}_2}{\sigma_1^2 \mathbf{v}_1^T \boldsymbol{\omega}^* \mathbf{v}_1} = \frac{\mathbf{u}_1^T \boldsymbol{\omega}^{*'} \mathbf{u}_2}{\sigma_1 \sigma_2 \mathbf{v}_1^T \boldsymbol{\omega}^* \mathbf{v}_2} = \frac{\mathbf{u}_1^T \boldsymbol{\omega}^{*'} \mathbf{u}_2}{\sigma_2^2 \mathbf{v}_2^T \boldsymbol{\omega}^* \mathbf{v}_2}$$

2 quadratic eqs. in α^2 and α'^2

A stratified solution

- **Calibration steps**



- **How to determine π_∞ :**

i) The modulus constraint

✓ Assumption : The internal parameters are constant.

$$\mathbf{A} - \mathbf{ap}^T = \mu \mathbf{KRK}^{-1} \longleftarrow \left(\mathbf{K}^i \mathbf{R}^i = (\mathbf{A}^i - \mathbf{a}^i \mathbf{p}^T) \mathbf{K}^1 \right)$$

Scale factor

$\mathbf{KRK}^{-1}$ is conjugate to a rotation $\rightarrow$ eigenvalues : $\{1, e^{i\theta}, e^{-i\theta}\}$

$$\boxed{\mathbf{A} - \mathbf{ap}^T \rightarrow \text{eigenvalues} : \{\mu, \mu e^{i\theta}, \mu e^{-i\theta}\}}$$

A stratified solution

$$\det\left(\lambda \mathbf{I} - \mathbf{A} + \mathbf{ap}^T\right) = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3)$$
$$= \lambda^3 - f_1 \lambda^2 + f_2 \lambda - f_3$$

where λ_i are the 3 eigenvalues.

$$f_1 = \lambda_1 + \lambda_2 + \lambda_3 = \mu(1 + 2\cos\theta)$$
$$f_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = \mu^2(1 + 2\cos\theta)$$
$$f_3 = \lambda_1\lambda_2\lambda_3 = \mu^3$$

Eliminating the scalar μ and angle θ we obtain

$$\boxed{f_3 f_1^3 = f_2^3}$$

$\det\left(\lambda \mathbf{I} - \mathbf{A} + \mathbf{ap}^T\right) \longrightarrow$ Quartic polynomial in the 3 elements p_i of $\mathbf{p}$

A stratified solution

ii) Other methods of finding π_∞

- ✓ Straightforward method (Using 3 vanishing points) [Chap.9]
- ✓ Pure translation : No rotation, constant internal parameter
  - π_∞ is determined uniquely.
- ✓ Using ADC $\mathbf{Q}^*_\infty \boldsymbol{\pi}_\infty = \mathbf{0}$
- ✓ Quasi-affine reconstruction. [Chap.20]
- ✓ Bounds imposed by cheirality.

A stratified solution

- **Affine to metric conversion**

$$\boldsymbol{\pi}_\infty(\mathbf{p}) \longrightarrow \mathbf{H}_\infty \longrightarrow \boldsymbol{\omega}(\boldsymbol{\omega}^*) \longrightarrow \mathbf{K}$$

✓ How to find the infinite homography $\mathbf{H}_\infty$

$\boldsymbol{\pi}_\infty = (\mathbf{p}^T, 1)^T$, and $\mathbf{P}^i = \left[\mathbf{A}^i \mid \mathbf{a}^i\right]$ are known

then

$$\mathbf{H}^i_\infty = \mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T \qquad ;\text{if } \mathbf{P}^1 = [\mathbf{I} \mid \mathbf{0}]$$

$$\mathbf{H}^i_\infty = \left(\mathbf{A}^i - \mathbf{a}^i\mathbf{p}^T\right)\left(\mathbf{A}^1 - \mathbf{a}^1\mathbf{p}^T\right)^{-1} \qquad ;\text{if } \mathbf{P}^1 = \left[\mathbf{A}^1 \mid \mathbf{a}^1\right]$$

A stratified solution

- ✓ Transformation of IAC (DIAC) through $\mathbf{H}_\infty$

$$\omega^{*i} = \mathbf{H}^i_\infty \omega^* \mathbf{H}^{i\,T}_\infty \quad \text{and} \quad \omega^i = \left(\mathbf{H}^i_\infty\right)^{-T} \omega \left(\mathbf{H}^i_\infty\right)^{-1}$$

$\mathbf{H}^i_\infty$ is normalized as $\det \mathbf{H}^i_\infty = 1$ for the scale factors.

- ✓ If fixed camera ($\omega^{*i} = \omega^*$)

$$\Rightarrow \mathbf{Ac} = \mathbf{0}$$

$\mathbf{A}$: 6x6 matrix

$\mathbf{c}$: conic ω^* written as a 6-vector

- ✓ If $m \geq 2$ views, $\mathbf{A}$ is a $6\,m$ X 6 matrix.

$$\mathbf{c} \rightarrow \omega \rightarrow \mathbf{K}$$

(↑ Cholesky decomposition)

$$\omega^* = \begin{bmatrix} f_x^2 + c_x^2 & c_x c_y & c_x \\ c_x c_y & f_y^2 + c_y^2 & c_y \\ c_x & c_y & 1 \end{bmatrix}$$

$$\omega = \frac{1}{f_x^2 f_y^2} \begin{bmatrix} f_y^2 & 0 & -f_y^2 c_x \\ 0 & f_x^2 & -f_x^2 c_y \\ -f_y^2 c_x & -f_x^2 c_y & f_x^2 f_y^2 + f_y^2 c_x^2 + f_x^2 c_y^2 \end{bmatrix}$$

- ✓ Linear solution using IAC is more practical than using DIAC

A stratified solution

- Auto-calibration constraints derived from the IAC

Condition	constraint	type	# constraints
zero skew	$\omega_{12} = 0$	linear	m
principal point at origin	$\omega_{13} = \omega_{23} = 0$	linear	$2m$
known aspect ratio $r = \alpha_y/\alpha_x$ (assuming zero skew)	$\omega_{11} = r^2\omega_{22}$	linear	m
fixed (unknown) aspect ratio (assuming zero skew)	$\omega^i_{11}/\omega^i_{22} = \omega^j_{11}/\omega^j_{22}$	quadratic	$m-1$

- In order to make the solution satisfy the constraints precisely, one may impose the known info. as hard constraints

A stratified solution

<u>Objective</u>

Given a projective reconstruction $\{\mathtt{P}^i, \mathbf{X}_j\}$, where $\mathtt{P}^i = [\mathtt{A}^i \mid \mathbf{a}^i]$, determine a metric reconstruction via an intermediate affine reconstruction.

<u>Algorithm</u>

(i) **Affine rectification:** Determine the vector $\mathbf{p}$ that defines $\boldsymbol{\pi}_\infty$, using one of the methods described in section 18.5.1. At this point an affine reconstruction may be obtained as $\{\mathtt{P}^i\mathtt{H}_\mathtt{P}, \mathtt{H}_\mathtt{P}^{-1}\mathbf{X}_j\}$ with

$$\mathtt{H}_\mathtt{P} = \begin{bmatrix} \mathtt{I} & \mathbf{0} \\ -\mathbf{p}^\top & 1 \end{bmatrix}.$$

(ii) **Infinite homography:** Compute the infinite homography between the reference view and the others as

$$\mathtt{H}^i_\infty = \left(\mathtt{A}^i - \mathbf{a}^i\mathbf{p}^\top\right).$$

Normalize the matrix so that $\det \mathtt{H}^i_\infty = 1$.

(iii) **Compute ω:**

- In the case of constant calibration: rewrite the equations $\omega = (\mathtt{H}^i_\infty)^{-\top}\omega(\mathtt{H}^i_\infty)^{-1}$, $i = 1, \ldots, m$ as $\mathtt{A}\mathbf{c} = \mathbf{0}$ with $\mathtt{A}$ a $6m \times 6$ matrix, and $\mathbf{c}$ the elements of the conic ω arranged as a 6-vector, **or**
- For variable calibration parameters, use the equation $\omega^i = (\mathtt{H}^i_\infty)^{-\top}\omega(\mathtt{H}^i_\infty)^{-1}$ to express linear constraints on entries of ω^i (e.g. zero skew) as linear equations in the entries of ω.

(iv) Obtain a least-squares solution to $\mathtt{A}\mathbf{c} = \mathbf{0}$ via SVD.

(v) **Metric rectification:** Determine the camera matrix $\mathtt{K}$ from the Cholesky decomposition $\omega = (\mathtt{K}\mathtt{K}^\top)^{-1}$. Then a metric reconstruction is obtained as $\{\mathtt{P}^i\mathtt{H}_\mathtt{P}\mathtt{H}_\mathtt{A}, (\mathtt{H}_\mathtt{P}\mathtt{H}_\mathtt{A})^{-1}\mathbf{X}_j\}$ with

$$\mathtt{H}_\mathtt{A} = \begin{bmatrix} \mathtt{K} & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix}.$$

(vi) Use iterative least-squares minimization to improve the solution (see section 18.3.3).

Stratified auto-calibration algorithm using IAC constraints.

Calibration from rotating cameras

- Examples:
  - ✓ Pan-tilt zoom surveillance camera
  - ✓ broadcasting camera
  - ✓ handheld camcorder at a single position
- Determining $\mathbf{H}_\infty$ from rotating camera views

$$\mathbf{x}^i = \mathbf{H}^i\mathbf{x}$$

$$\mathbf{H}^i = \mathbf{H}^i_\infty = \mathbf{K}^i\mathbf{R}^i(\mathbf{K})^{-1}$$

- ✓ Example : Rotation about center with varying internal parameters
  - Zero skew $\left(\left(\mathbf{H}^i_\infty\right)^{-T}\boldsymbol{\omega}\left(\mathbf{H}^i_\infty\right)^{-1}\right)_{12} = 0$
  - Unit aspect ratio $\left(\left(\mathbf{H}^i_\infty\right)^{-T}\boldsymbol{\omega}\left(\mathbf{H}^i_\infty\right)^{-1}\right)_{11} = \left(\left(\mathbf{H}^i_\infty\right)^{-T}\boldsymbol{\omega}\left(\mathbf{H}^i_\infty\right)^{-1}\right)_{22}$

Calibration from rotating cameras

Objective

Given $m \geq 2$ views acquired by a camera rotating about its centre with fixed or varying internal parameters, compute the parameters of each camera. It is assumed that the rotations are not all about the same axis.

Algorithm

(i) **Inter-image homographies:** Compute the homography $\mathtt{H}^i$ between each view i and a reference view such that $\mathbf{x}^i = \mathtt{H}^i\mathbf{x}$ using, for example, algorithm 3.6-(p108). Normalize the matrices such that $\det \mathtt{H}^i = 1$.

(ii) **Compute ω:**

- In the case of constant calibration: rewrite the equations $\omega = (\mathtt{H}^i)^{-\top}\omega(\mathtt{H}^i)^{-1}$, $i = 1, \ldots, m$ as $\mathtt{A}\mathbf{c} = \mathbf{0}$ where $\mathtt{A}$ is a $6m \times 6$ matrix, and $\mathbf{c}$ the elements of the conic ω arranged as a 6-vector, **or**
- For variable calibration parameters, use the equation $\omega^i = (\mathtt{H}^i)^{-\top}\omega(\mathtt{H}^i)^{-1}$ to express linear constraints on entries of ω^i in table 18.4 (e.g. unit aspect ratio) as linear equations in the entries of ω.

(iii) **Compute $\mathtt{K}$:** Determine the Cholesky decomposition of ω as $\omega = \mathtt{U}\mathtt{U}^\top$, and thence $\mathtt{K} = \mathtt{U}^{-\top}$.

(iv) **Iterative improvement:** (Optional) Refine the linear estimate of $\mathtt{K}$ by minimizing

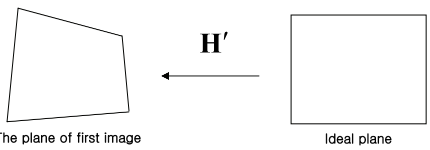
$$\sum_{i=2,m;\ j=1,n} d(\mathbf{x}_j^i, \mathtt{K}\mathtt{R}^i\mathbf{x}_j)^2$$

over $\mathtt{K}$ and $\mathtt{R}^i$, where $\mathbf{x}_j, \mathbf{x}_j^i$ are the position of the j-th point measured in the first and i-th images respectively. Initial estimates for the minimization are obtained from $\mathtt{K}$ and $\mathtt{R}^i = \mathtt{K}^{-1}\mathtt{H}^i\mathtt{K}$.

Auto-calibration from planes

- Potential applications
  - ✓ Ground plane, aerial images
- Auto-Calibration from scene planes

- Only requires homographies between planes
- No need of **F** or **T**



✓ Let the imaged circular points in the first view to be

$$\mathbf{c}_j = \mathbf{H}'(1, \pm i, 0)^T \qquad j = 1,2$$

✓ Then given $\mathbf{H}^i$, in the i-th view

$$\left(\mathbf{H}^i\mathbf{c}_j\right)^T \omega^i \left(\mathbf{H}^i\mathbf{c}_j\right) = 0,\ i = 1,\ldots,m \quad j = 1,2 \qquad \mathbf{H}^1 = \mathbf{I}$$

✓ Solution is possible if $2m \geq v + 4$ — # of parameters in $\mathbf{c}_j$

# of unknown parameters in K

Practical consideration

- Avoid ambiguous motion sequences
- Use as much info. you have
- Refine using bundle adjustment

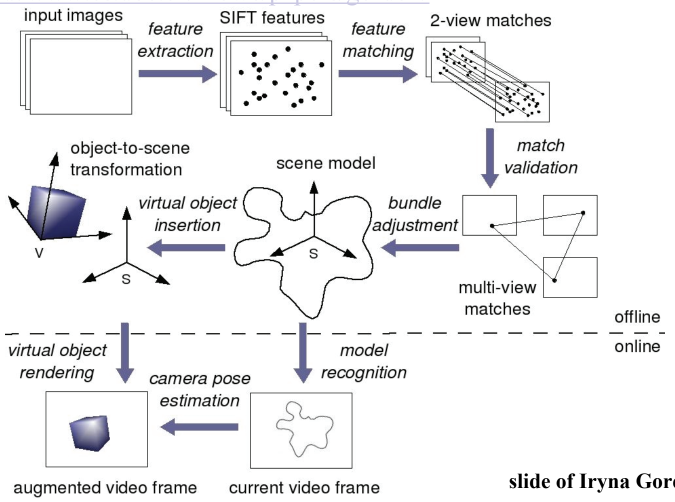
$$\arg\min_{\mathbf{P}^i, \mathbf{X}_j} \sum_{i=1}^{m} \sum_{j=1}^{n} D\left(\mathbf{x}_j^i, \mathbf{P}^i\left(\mathbf{X}_j\right)\right)^2$$

✓ Enforce constraints or priors on intrinsics during minimization

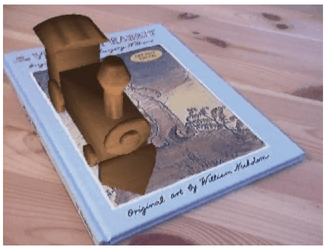
Some Recent Applications

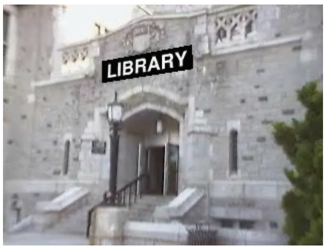
- **AR (Augmented Reality)**

http://www.cs.ubc.ca/~lowe/papers/gordon/

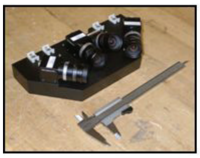


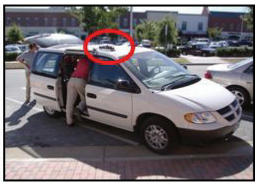
slide of Iryna Gordon

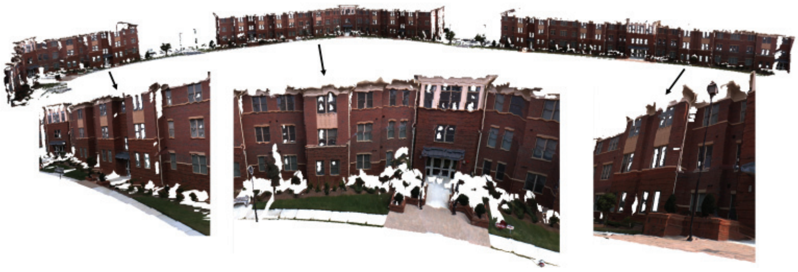




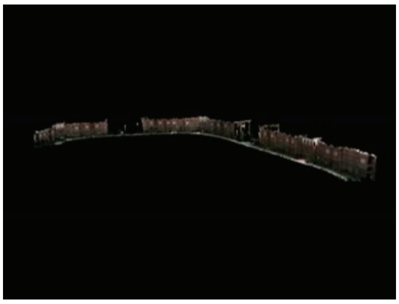
- **Urban 3D Modelling from Video**
  (http://www.cs.unc.edu/Research/urbanscape/)







Some Recent Applications

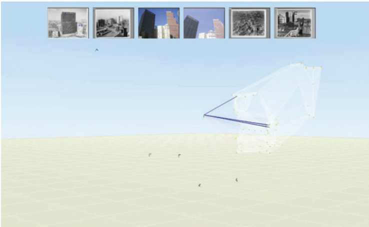


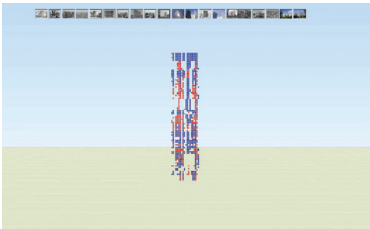
Some Recent Applications

Some Recent Applications

- **4D Cities:** Spatio-Temporal Reconstruction from Images

  http://www.cc.gatech.edu/4d-cities/dhtml/index.html



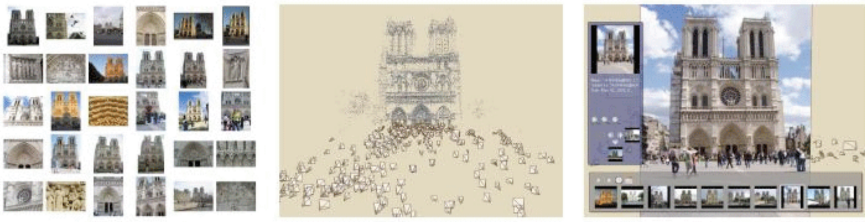


Some Recent Applications

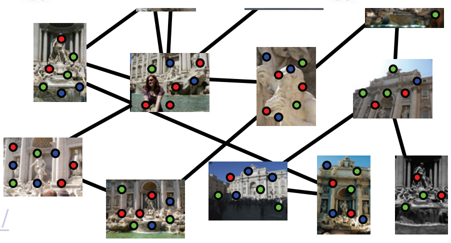
Photo Tourism

Exploring photo collections in 3D

Microsoft



(a) (b) (c)



http://phototour.cs.washington.edu/

Some Recent Applications