Chapter 4. Plasma Heating

Reading assignment : Stacey Ch.5, Gross 6.5

1. Ohmic heating

= Intrinsic primary heating in tokamaks (RFP, stellarators ...) due to Joulian dissipation generated by currents through resistive plasma [Thermalization of kinetic energies of energetic electrons (accelerated by applied *E*) via Coulomb collision with plasma ions]

A. Heating power density

$$P_\Omega = \boldsymbol{j}\cdot\boldsymbol{E} = \eta j^2 \quad [W/m^2] \qquad (1)$$

where

$$\eta = \frac{m_e \nu_{ei}}{ne^2} \approx 2\times10^{-9} Z_{eff} \ln\Lambda\, T_e^{-3/2} \quad [\Omega - m] \qquad (2)$$

$$j = \frac{I}{\pi a^2} = \frac{2B_\Theta(a)}{a\mu_o} \quad [A/m^2] \qquad (3)$$

Safety factor $q(a) = aB^o_\Phi / R_o B_\Theta(a)$ (4)

(2)-(4) in (1) :

$$P_\Omega = 7\times10^{-2}\left(\frac{Z_{eff}}{T_e^{3/2}(keV)}\right)\left(\frac{1}{q(a)}\right)^2\left(\frac{B^o_\Phi}{R_o}\right)^2 \quad [MW/m^3] \qquad (5)$$

Notes)

P_Ω ↑ as

- Z_{eff}↑ but limited by radiation losses
- T_e ↓ intrinsically limited by η and P_{br}
- $q(a)$ ↓ limited by instabilities
- $(\frac{B^o_\Phi}{R_o})$ ↑ size and field

⇒ compact high-field tokamak

B. Maximum temperature

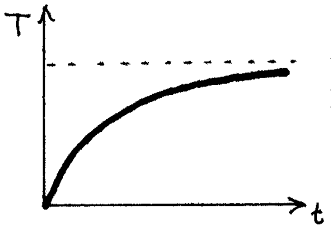
Assume $P_{br} \ll P_{in} \equiv \frac{3}{2} nkT/\tau_E$

$P_{tr} \equiv 3nkT/2\tau_E$: power loss due to heat conduction and convection

then $\frac{dP_{in}}{dt} = P_\Omega - P_{tr}$

$P_\Omega \downarrow$, $P_{tr} \uparrow$ as $T\uparrow$: intrinsic heating limitation

In steady state : $P_\Omega = P_{tr} \rightarrow T = T_{\max}$



When $\frac{dP_{in}}{dt} = 0$

$$P_\Omega = P_{tr} \quad \Rightarrow \quad \eta j^2 = \frac{3nkT}{2\tau_E} \tag{6}$$

Alcator scaling law :

$$\tau_E = 5\times 10^{-21} na^2 \tag{7}$$

(5), (7) in (6) :

$$T_{e\max}^{5/2}(keV) = 1.46\frac{Z_{eff}}{q(a)^2}\left(\frac{B_\Phi^o}{R_o/a}\right)^2 \tag{8}$$

Notes)

i) For conventional tokamak

$B_\Phi^o \approx 5T,\ \frac{R_o}{a} \approx 3-5,\ q(a) = 2-4$

$\Rightarrow \quad T_{emax} \approx \sim 100\ eV\ \sim 1\ keV$

ii) For heating up to $T_{ig} \approx 6\sim 8\ keV$

$q(a) = 2,\ Z_{eff} = 2,\ R_o/a \geq 2$

$\Rightarrow \quad B_\Phi^o \approx 20-30\ T$

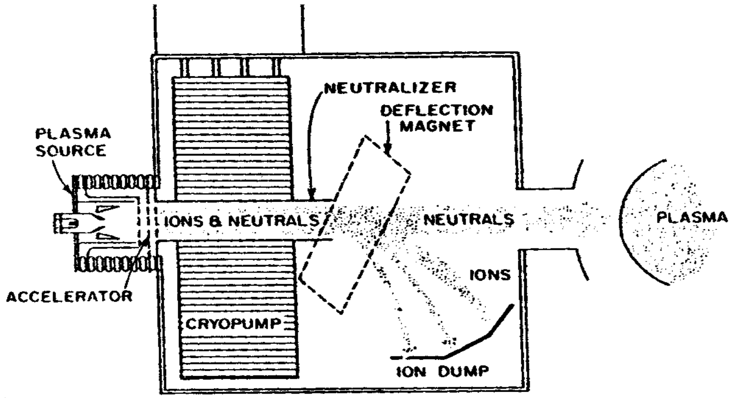
$\Rightarrow$ enormorous magnetic stress and currents

: T_{ig} is not achievable only by resistive heating

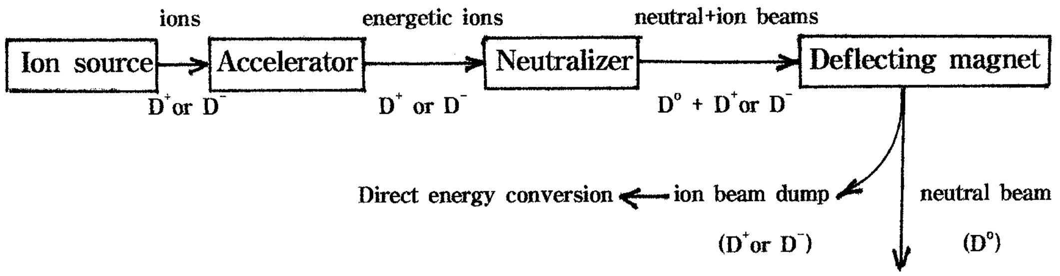
iii) Ohmic heating = Primary heating due to lower cost than other auxiliary heatings

2. NBI heating

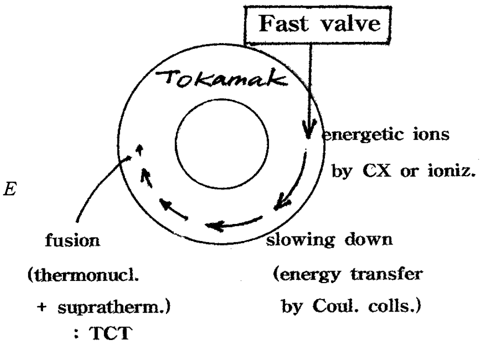
= Supplemental heating by energy transfer of neutral beam to the plasma through collisions



Gross$^{3)}$ Fig. 6.5 Schematic illustration of a neutral-beam injection system



① Enough energy for deep penetration
② Enough power for desired heating
③ Enough repetition rate and pulse length $> \tau_E$
④ Allowable impurity contamination



A. Ion source

1) Requirements

① Large-area uniform quiescent flux
of **high-current** [$\geq O(100\,A)$, $200-500\ mA/cm^2$] ions

② Large atomic ion fraction (D^+, D^-) > 75 %
⇒ adequate penetration

③ Low ion temperature (<< 1 eV) to minimize irreducible divergence of extracted ion beams due to random thermal motion of ions

2) Ion generation

① Positive ion generation by elec. discharge

$$D_2 + e \rightarrow \boldsymbol{D^+} + D + e + e$$

$$D_2 + e \rightarrow \boldsymbol{D_2^+} + e + e$$

$$D_2 + D_2^+ \rightarrow \boldsymbol{D_3^+} + D$$

② Negative ion generation

$D + e \rightarrow \boldsymbol{D^-} + h\upsilon$: Radiative attachment in high density gas

(Binding energy = 0.75 eV)

$D_2^* + e \rightarrow \boldsymbol{D^-} + D$: Dissociative electron attach. by elec. disch.

$\uparrow$ $\hookrightarrow \sim eV$ **(Volume Production)**

highly excited vibrational state

$D^+ + cathode\ surface(+Cs) \rightarrow \boldsymbol{D^-}$ **Surface Production** by elec. disch.

$D^o + cathode\ surface(+Cs) \rightarrow \boldsymbol{D^-}$

~ 100 eV range

$D^+ + M^o \rightarrow D^o + M^+$ (elect. attach.) **Double electron capture**

$D^o + M^o \rightarrow \boldsymbol{D^-} + M^+$ (elect. attach.)

D^+ from low energy ion source

alkali or alkali-earth metal vapor (Cs, Rb, Na, Sr, Mg)

3) Type of ion sources

① DuoPIGatron (ORNL) = Duoplasmatron + Penning Ionization Gauge

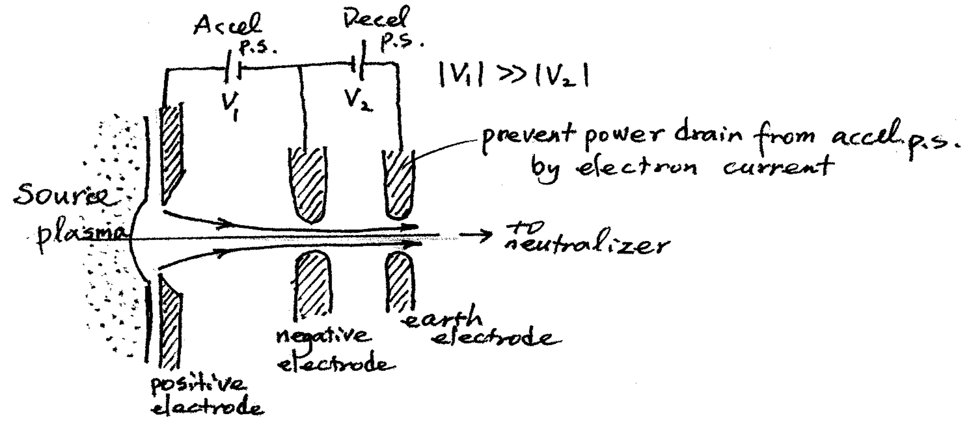
② Field-free sources (LBL type): arc disch. w/o $\boldsymbol{B}$

③ Magnetic bucket source (Culham type)

B. Beam forming system

= Ion extraction + acceleration + min. beam divergence (≤ 1°)

⇒ multiple-aperture beam forming structures with accel-decel electrodes



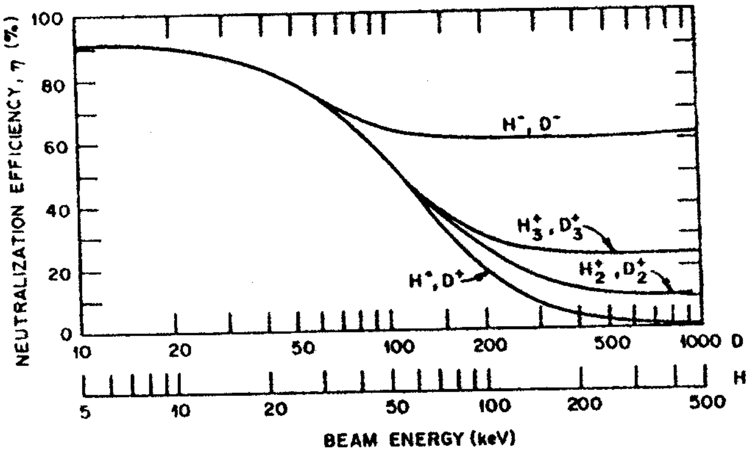
C. Neutralizer (D_2 gas cell $\sim 10^{-3}$ torr)

$[D^-,\ D_j^+\ (\ j=1,2,3)\ ] + D_2 \quad \Rightarrow \quad$ (Dissociation)

$\Rightarrow \quad D^0$: neutralization by cx

Neutralization efficiency:

$\eta_j \equiv$ (outgoing neutral beam power) / (entering ion beam power)

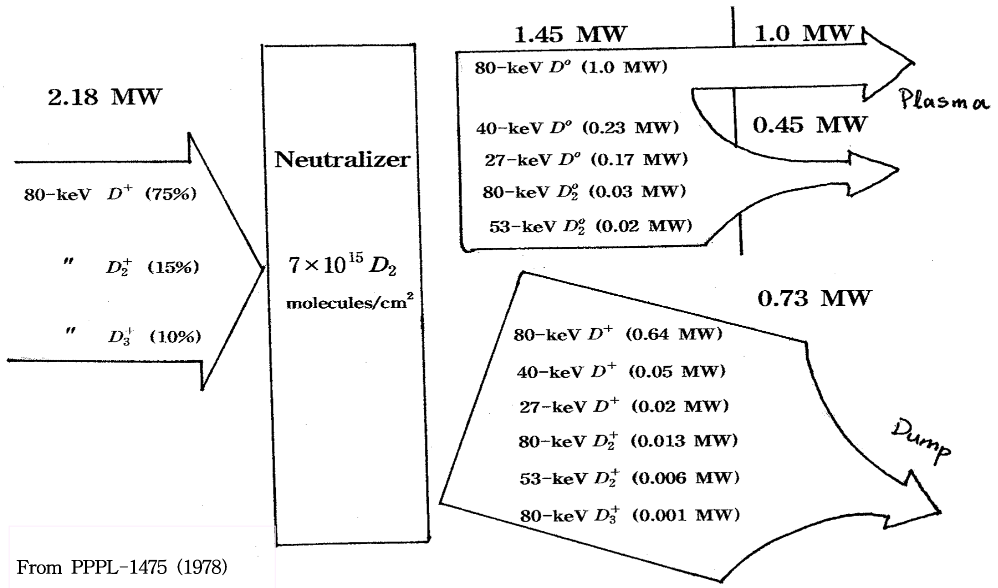


Gross[3] Fig. 6.6 Neutralization efficiency for different beam energies; efficiency equals power in neutral atoms out divided by power in ion beam entering neutralizer cell.

(Notes) $\eta_j \downarrow$ as $(\frac{eV_+}{D^+})\uparrow$ and $j\downarrow$

Highest η_j for D^-

Typical power flow for a 1-MW, 40-keV atomic deuterium injection system :



From PPPL-1475 (1978)

D. Ion beam dump and Vacuum pumps

1) Beam dump

Deflect by analyzing magnet

Minimize reionization losses

Prevent local power dump at undesirable place ($\sim kW/m^2$)

Possible application to direct evergy conversion

2) Pumping

Minimize reioninzaton losses

Prevent cold neutral particles from flowing into reactor plasma

Liquid He cryopumps ($\sim 10^6\ l/s$ for $\sim MW$ system)

E. Energy deposition in a plasma by NBI

1) Atomic and molecular processes of NB in a plasma

$\Rightarrow$ birth of energetic ions

① charge exchange

$$D^o + D^+ \quad -(\sigma_x)\rightarrow \quad D^+ + D$$

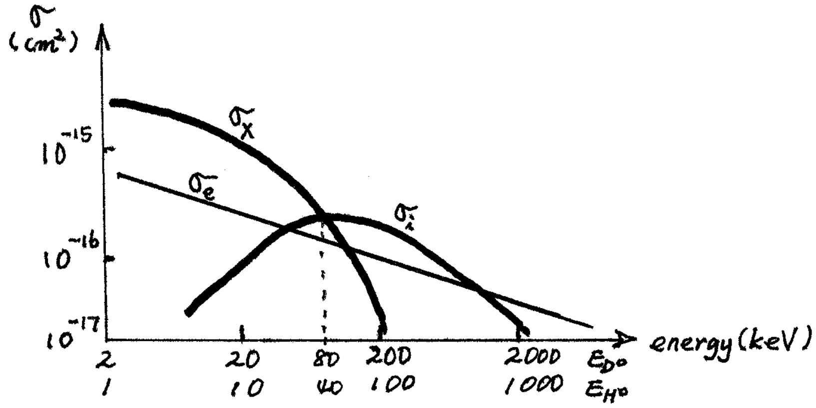
$$D_2^o + D^+ \quad -(\sigma_x)\rightarrow \quad D_2^+ + D$$

② electron ionization

$$D^o + e \quad -(\sigma_e)\rightarrow \quad D^+ + e + e$$

③ deuterium ionization

$$D^o + D^+ \quad -(\sigma_i)\rightarrow \quad D^+ + D^+ + e$$

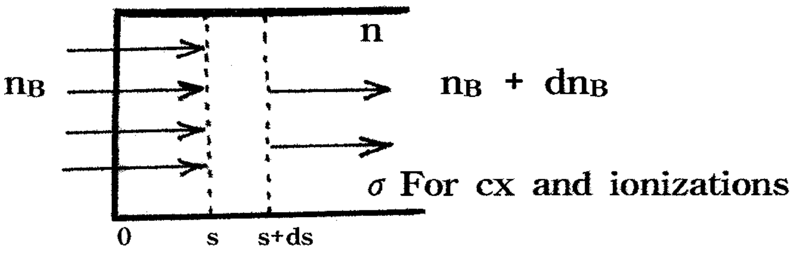


Notes) . $E_{D^o} \leq 80\ keV$: CX is domiant

. $E_{D^o} \geq 80\ keV$: Deut. ioniz. is dominant

. $\sigma_{DT} \approx 10^{-24} cm^2$ at $T_e \approx 40\ keV$

2) Attenuation of neutral beam



$dn_B = -\sigma n n_B ds$ → energetic ion birth distribution

$$\Rightarrow n_B(s) = n_B(0)\ \exp\left\{ -\int \sigma(s)\, n(s) ds \right\}$$

For a uniform plasma

$$n_B = n_B(0)\, e^{-\sigma n s} = n_B(0)\, e^{-s/\lambda} \tag{9}$$

where $\lambda \equiv \frac{1}{\sigma n}$ penetration(attenuation) length

3) Minium NB energy for effective plasma heating

General criterion for adequate penetration

$$\lambda \geq a/4 \tag{10}$$

where $\lambda \equiv \frac{1}{\sigma n_o z_{eff}^{\gamma}}$ (n_o: central density) (11)

σ from fitting of cross section curves in (11):

$$\lambda(m) = \frac{5.5\times 10^{17} E_B(keV)}{A(amu)\, n_o(m^{-3})\, Z_{eff}^{\gamma}} \tag{12}$$

(12) in (10):

$$\boldsymbol{E_B \geq 4.5\times 10^{-19} A\, n_o\, a\, Z_{eff}^{\gamma}} \quad \textit{(keV)} \tag{13}$$

Note)

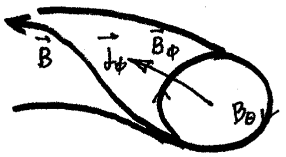
$n\tau_E = 5\times 10^{-21} n^2 a^2$: alcator scaling in (13)

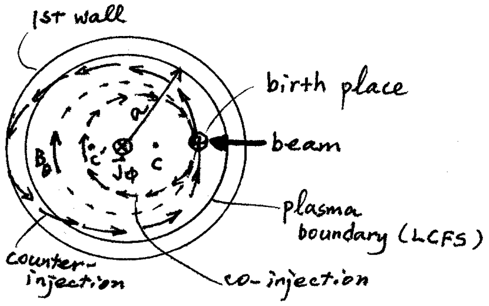
$n\tau_E = 10^{20}$: Lawson criterion

$$(E_B)_{\min} \approx 63.6 A Z_{eff}^{\gamma} \quad (\ keV)$$

$$\approx 100\ keV$$

4) Orbits of energetic particles





. Coinjection ($v \nearrow\nearrow j_\Phi$)

$ev \times B_\Theta = -\hat{r}$ direction force

$\Rightarrow$ outward shift of orbit center

. Counterinjection ($v \nearrow\swarrow j_\Phi$)

$ev \times B_\Theta = +\hat{r}$ direction force

$\Rightarrow$ inward shift

$\Rightarrow$ bad drift orbits $\Rightarrow$ energetic ion loss to wall/limiters

Best injection angle for maximum penetration and minimum orbital excursion = $10 \sim 20^\circ$ off perpendicular in coinjection direction

F. Feathres of NBI

① Applicable to both open and closed systems

② Heating, current drive, and refueling

③ In two-component torus (TCT)

. Suprathermal fusion

. Thermonuclear fusion $\Rightarrow$ Lower T_{ig} and Lawson criterion

G. NBI experiments

(cf) Stacey; Table 5.2.1, Table 5.2.2

Ignition in power tokamak

$\Rightarrow$ 50 *MJ*, $\geq$ 75 *MW*, $5 \sim 10$ *sec*, 200 *keV*

Recent NBI experiments in JT-60U :

350 *keV*,	13.5 *A*,	5.4 *MW*	H^-
400	18.4	6.4	D^-
500		10	Design values

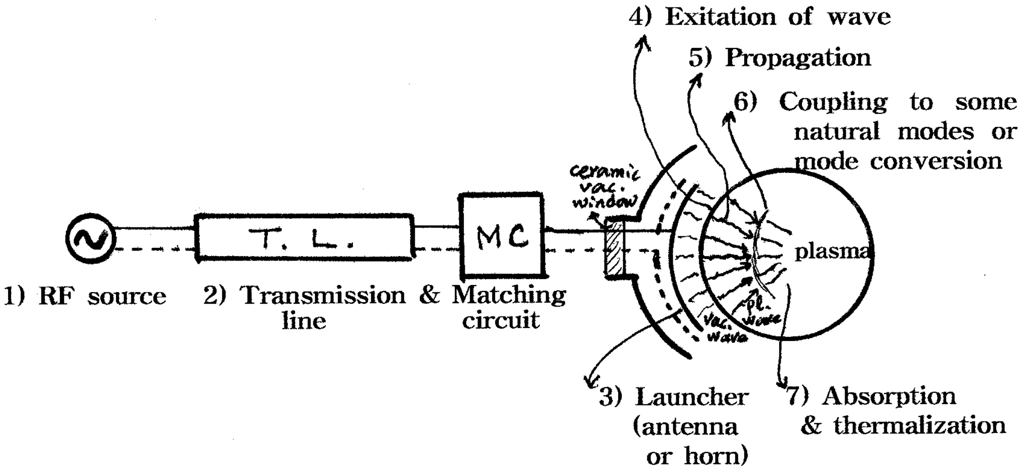
Planned in ITER :

0.5 *MeV* (heating), 1-2 *MeV* (CD)

Negative-ion based D^o : 1 *MeV*, 40 *A* → 50 *MW* with 3 units

3. RF (Radio Frequency) heating

A. RF (or MW) heating setup



1) Wave generation
   Ocsillator, Tube, Klystron, Gyrotron (elec. efficiency 40 ~90 %)
2) Transmission system
   Coaxial line (100 *kHz* - 100 *MHz*)
   Wave guide (100 *MHz* - 200 *GHz*) (~90 %)
3) Coupling system (Launcher)
   Antenna (100 *kHz* - 100 *MHz*)
   Horn (100 *MHz* - 200 *GHz*)
4) Launching of wave into plasma
   Pumping the wave into plasma through a coupling system
5) Propagation of externally-launched wave into plasma
   Accessibility of resonant region
   Penetration of wave into inhomogeneous plasma $\omega_{cutoff} < \omega < \omega_{res}$
6) Coupling of external wave to some natural plasma modes
   by resonance or mode conversion
7) Collisional of collisionless absorption of wave energy
   in plasma ions and electrons ⇒ heating
   Collisional damping: Coulomb collisions
   (e.g., Joulian heating)
   Collsionless damping: Landau(n=0) and cyclotron dampings(n=1,2, ...)
   $(\omega - k_{\parallel} v_{\parallel} \pm n\omega_c = 0)$
   Transit-time damping
   $(\boldsymbol{F}_{\parallel} = -\mu \nabla_{\parallel} B, \quad t_{\omega} \approx t_t)$

B. Natural mode frequencies (H$^+$, 10^{20} m^{-3}, 5 T)

$$f_{pe} = \omega_{pe}/2\pi = \sqrt{\frac{ne^2}{\varepsilon_o m}}/2\pi \approx 9\sqrt{n} \approx 90\ GHz$$

$$f_{pi} = \omega_{pi}/2\pi \approx 0.2\sqrt{\frac{n}{A}} \approx 2\ GHz$$

$$f_{ce} = \Omega_e/2\pi = \frac{eB}{m}/2\pi \approx 28B \approx 140\ GHz$$

$$f_{ci} = \Omega_i/2\pi \approx 15\frac{B}{A} \approx 75\ MHz$$

$$f_{LH} = \omega_{LH}/2\pi = \frac{\omega_{pi}\Omega_e}{\sqrt{\omega_{pe}^2+\Omega_e^2}}/2\pi \approx \omega_{pi}/2\pi \approx 1.6\ GHz$$

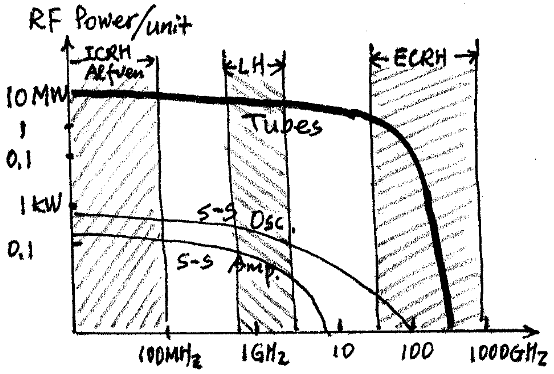
$$f_{UH} = \omega_{UH}/2\pi = \sqrt{\omega_{pe}^2+\Omega_e^2}/2\pi \approx 170\ GHz$$

$$f_{Alf} = \omega_{Alf}/2\pi = k_{\parallel}v_A/2\pi = \frac{1}{\lambda_{\parallel}}\frac{B}{\sqrt{\mu_o\rho}} \approx 1\ MHz$$

C. Possible wave heating regimes

Type	Freq.	Source	Transmission	Launcher	Absorption
TTMP	0.1-0.5 *MHz*	Oscillator (10*MW*, 90%)	coaxial line	coil anten.	T-T damping, Landau damping(L.D.)
Shear Alfven	~1 *MHz*	Tubes (10*MW*, 90%)	coaxial line	coil anten.	mode conv., eT-T, e-L.D.
Fast Alfven	1-10 *MHz*	Tubes (10*MW*, 90%)	coaxial line	coil anten.	cavity resonance, eT-T, e-L.D.
ICRF	25-100 *MHz*	Tubes (~5*MW*, 70%)	coaxial or ridged W.G.	coil anten. or cavity backed aperture anten.	ion-cycl. damp e-L.D., mode conv. (ion Berns.)
LHRH	1-5 *GHz*	Klystron (2*MW*, 60%)	W.G.	Phased W.G. array (Grill)	mode conv., i-, e-L.D., ion cycl. damp.
ECRH	50-200 *GHz*	Gyrotron (1*MW*, 40%)	W.G.	W.G. array Horn	ele. cycl. damp. mode conv. (e Berns), e-L.D.

D. RF power sources



Notes)

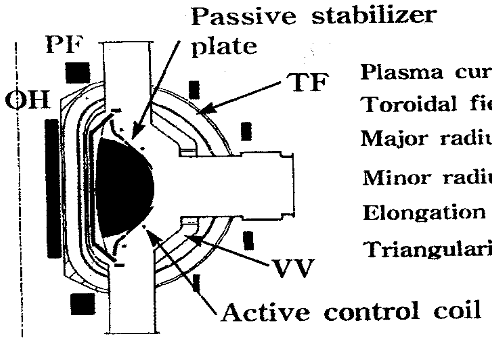
i) High power with high effic.
at low frequencies

ii) Simple coupling
at high frequencies

E. RF heating experiments: *(cf) Stacey* Table 5.3.2

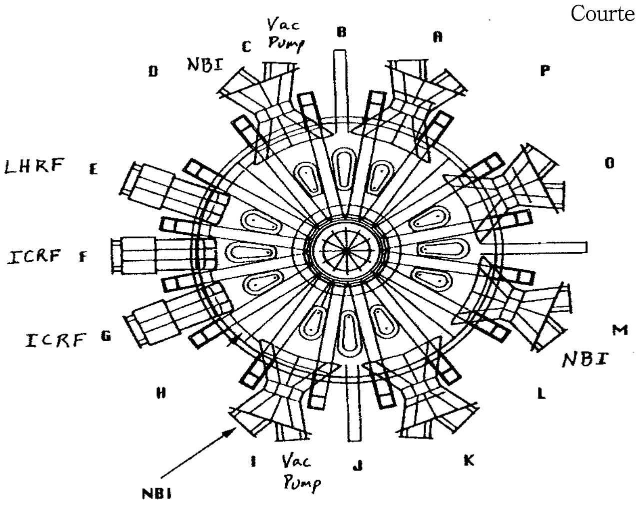
KSTAR Auxiliary Heating & Current Drive Systems

	Baseline	Upgrade	Remarks
Neutral Beam	8 MW 1 Co	24 MW 2 Co, 1 Ctr	120 keV ~ 300 sec
ICRF/FWCD	6 MW 1 Launcher	12 MW 2 Launchers	30-60 MHz ~300 sec
LHCD	1.5 MW	4.5 MW	3.7 GHz ~ 300 sec
ECH	0.5 MW		60 GHz ECH Start-up (0.5 sec)



Plasma current I_p = 2.0 MA
Toroidal field B_T = 3.5 T
Major radius R_0 = 1.8 m
Minor radius a_0 = 0.5 m
Elongation κ = 2.0
Triangularity δ = 0.8

Courtesy of NFRC[16)]



Summary of Auxiliary Heating Mechanism

- External Power Production

  Transformation of electric power into a form usable for plasma heating
- Conveyance

  Transfer and coupling of heating power to the edge of the plasma
- Propagation

  Movement of heating power into the plasma
- Absorption

  Absorption of the power into a particular species of plasma
- Thermalization

  Transfer of absorbed energy from particular plasma species to bulk plasma electrons and ions
- Side Effects

  Current drive, plasma rotation, fueling, etc.

Comparison of Heating Methods

Method	Power Production Technology	Coupl., Prop., Absorp. in Plasma
Ohmic	Simple (Large transformer)	Magnetic induction Joule heats electrons (hot center)
NBI (70's $P_{NBI} \geq 3P_{OH}$)	Difficult to generate beams Moderate efficiency (>50%)	Atomic phys. & Coul. colls. Easy to analyze
ICRH (80's $P_{ICRH} \geq 3P_{OH}$)	Commercially available High efficiency (>90%)	Complicated wave propagation Difficult coupling (30-100 MHz, $\lambda_{vac} < 10\ m$)
LHH (70's *demonstrated*)	Commercially available Moderate efficiency (>50%)	Complicated wave propagation Moderate coupling (> GHz, $\lambda_{vac} < 30\ cm$)
ECRH (80's *CD expts.*)	New technology (gyrotrons) Low efficiency (<25%) (300 kW, 90 GHz units)	Simple prop. for $\omega > \omega_{pe} > \Omega_e$ (geometrical optics) Easy coupling (> 60 GHz, $\lambda_{vac} < 3\ mm$)

(cf) **Ohmic :** simplest, but limited at high T_e

NBI, ECRH : simple phys., but difficult production technology

ICRH, LHH : complecated phys., but established technology

NBI, ICRH : used in present experiments

↪ favored for fusion reactor

4. Adiabatic magnetic compression

: Heating by increasing magnetic pressure adiabatically

A. Heating by adiabatic law

Compression time: $\tau_{comp} \equiv B / \dfrac{\partial B}{\partial t} \approx \sim 10^{-4} - 10^{-3} \; (\upsilon_{comp} < \upsilon_{th})$

If $\Omega_i^{-1} < \tau_{comp} < (\tau_{90})_L$ **and** $\tau_{comp} < \tau_m \equiv \mu_o \sigma L^2$,

$PV^{\gamma} = const$ **adiabatic reversible** **(14)**

where $\gamma = \dfrac{(\delta + 2)}{\delta}$

Notes)

i) . $\delta =$ **1 or 2** **if** $\tau_{comp} < (\tau_{90})_L$

. $\delta =$ **3** **if** $\tau_{comp} \geq (\tau_{90})_L$

ii) . $\delta < 3$ **for ions**

. $\delta = 3$ **for electrons**

$$(\tau_{90}^{ee})_L \approx (\tau_{90}^{ei})_L \approx \sqrt{\frac{m_e}{m_i}} (\tau_{90}^{ii})_L \approx \frac{m_e}{m_i} (\tau_{90}^{ie})_L$$

$P = nkT = nW, \quad V \propto n^{-1}$ **in (14)**

$$\boldsymbol{W n^{1-\gamma} = T n^{1-\gamma} = const} \qquad (15)$$

For $\tau_{comp} \ll (\tau_{90})_L$ **: 1-D comp. with** $W_{\parallel}$ $(\delta = 1)$

$$\text{(15)} \Rightarrow \frac{W_{\parallel}^{(1)}}{W_{\parallel}^{(2)}} = \left(\frac{n^{(2)}}{n^{(1)}} \right)^{1-\gamma}, \qquad W_{\perp}^{(2)} = W_{\perp}^{(1)}$$

For an initial isotropic distribution

$$W_{\parallel}^{(1)} = \frac{\delta}{3} W^{(1)}, \qquad W^{(1)} = W_{\parallel}^{(1)} + W_{\perp}^{(1)}$$

$$\therefore \frac{W^{(2)}}{W^{(1)}} \equiv \frac{W_{\parallel}^{(2)} + W_{\perp}^{(2)}}{W_{\parallel}^{(1)} + W_{\perp}^{(1)}}$$

$$= \frac{\left(\dfrac{n^{(2)}}{n^{(1)}} \right)^{\gamma - 1} \dfrac{\delta}{3} W^{(1)} + \dfrac{3 - \delta}{3} W^{(1)}}{\dfrac{\delta}{3} W^{(1)} + \dfrac{3 - \delta}{3} W^{(1)}}$$

$$= \frac{\left(\dfrac{n^{(1)}}{n^{(2)}} \right)^{\gamma - 1} \delta + 3 - \delta}{3} = \frac{T^{(2)}}{T^{(1)}} \qquad (16)$$

$$\Rightarrow \quad T_{1-D}^{(2)} > T_{2-D}^{(2)} > T_{3-D}^{(2)}$$

B. Scaling laws for tokamaks ($\delta=3$, $\gamma=5/3$)

$Wn^{-2/3} = Tn^{-2/3} = const$ (15)*

Magenetic flux conservation

$a^2B_\phi = const$: toroidal flux (17)

$RaB_\Theta = const$: poloidal flux (18)

(17) $\Rightarrow$ $B_\phi \propto a^{-2}$

(18) $\Rightarrow$ $B_\Theta \propto a^{-1}R^{-1}$

$$I_\phi = \frac{2\pi a}{\mu_o}B_\Theta \propto R^{-1}$$

$$n = \frac{N}{V} = \frac{N}{2\pi R\pi a^2} \propto a^{-2}R^{-1}$$

(15)* $\Rightarrow T \propto n^{2/3} \propto a^{-4/3}R^{-2/3}$ (19)

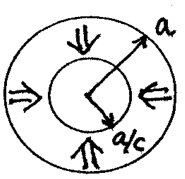
$$\beta_p = \frac{nT}{B_\Theta^2} \approx a^{-4/3}R^{1/3}$$

$$\frac{\beta}{\beta_p} = \frac{B_\Theta^2}{B_\phi^2 + B_\Theta^2} \propto \left(\frac{B_\Theta}{B_\phi}\right)^2 \propto a^2R^{-2}$$

$$\beta \propto a^2R^{-2}\beta_p \propto a^{2/3}R^{-5/3}$$

C. Compression schemes in tokamak

1) Type A = comp. of a at const. R by $B_\phi\uparrow$ ($I_{TF}\uparrow$)



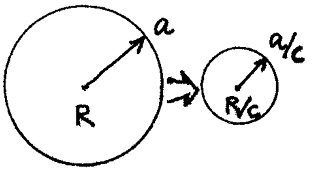
a $\rightarrow c^{-1}a$, $R \rightarrow R$

$B_\phi \rightarrow c^2B_\phi$

$T \rightarrow c^{4/3}T$

2) Type B = comp. of a and R at $\frac{a}{R}$=const. by $I_\phi\uparrow$ and $B_v\uparrow$

$\hookrightarrow B_\Theta\uparrow$ $\hookrightarrow B_\phi\uparrow$

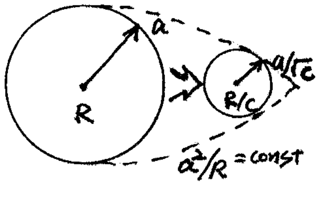


a $\rightarrow c^{-1}a$, $R \rightarrow c^{-1}R$

$I \rightarrow cI$

$B_\phi \rightarrow c^2B_\phi$, $T \rightarrow c^2T$

3) Type C = comp. of a and R at $\frac{a^2}{R}$=const. by $B_v\uparrow$, B_ϕ=const



$a \rightarrow c^{-1/2}a$, $R \rightarrow c^{-1}R$

$B_\phi \rightarrow B_\phi$ at fixed pt.

$B_\phi \rightarrow cB_\phi$ at plasma

$I_\phi \rightarrow cI_\phi$, $T \rightarrow c^{4/3}T$

Note) $B_\phi a^2 = const$

$B_\phi = B_\phi^o R_o/R$

$\Rightarrow a^2/R = const$

$\Rightarrow a\downarrow$ as $R\downarrow$

(automatically)

D. Features

1) Can heat to ignition
2) No additional sources (NBI or RF)
3) Larger tokamak to accomodate compressed plasma
4) Difficult to control plasma shape and size
5) Technical problem due to high-power pulse operation

5. Fusion α-particle heating

= Intrinsic self-heating by Coulomb collision of fusion α particles with plasma particles in D-T reactions

A. Charged particle source

$R_{DT}(r) = n_D(r) n_T(r) \langle \sigma v \rangle_f$: Birth distribution of α

Ideal igintion temperature for self-sustaining reactor

$R_{DT} E_\alpha = P_{br} \quad \Rightarrow \quad T_{ig} \approx 4\ keV$

B. α-*particle loss fraction* by radial excursion due to drifts

$\langle F_L \rangle = f(r, \triangle r)$

where r : born flux surface.

$\triangle r$: radial excursion

$$\triangle r(\theta) \propto \frac{r/R}{\Omega_\theta} \propto \frac{r/a}{(R/a) B_\theta(r)}$$

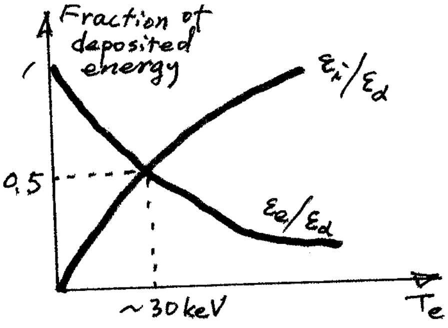
$$B_\theta(r) = \frac{\mu_o}{2\pi r} \int_0^r j_\phi(r') 2\pi r' dr'$$

$$j_\phi(r) = j_\phi^o \left(1 - \frac{r^2}{a^2}\right)^\nu$$

$\langle F_L \rangle \downarrow$ as $I_\phi \uparrow$, $\nu \uparrow$, $(\frac{r}{a}) \downarrow$, $(\frac{R}{a}) \uparrow$

$\Rightarrow$ more than 90 % of α can be confined by $I_\phi \geq 5\ MA$

C. Sharing of ε_α between plasma ions and electrons



Note) High T_e improve ion heating

Homework : Stacey 1 ~ 6