457.309 Hydraulics and Laboratory
.06 Hydraulic Scale Models

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Hydraulic Scale Models

- Model vs. Real Natural Flow
  - Natural flow is much more complex than modeled flow.
  - Natural flow is much larger than modeled flow.
- How we can mimic real flow with model?
  - Following dimensional numbers…

Tidal control gate

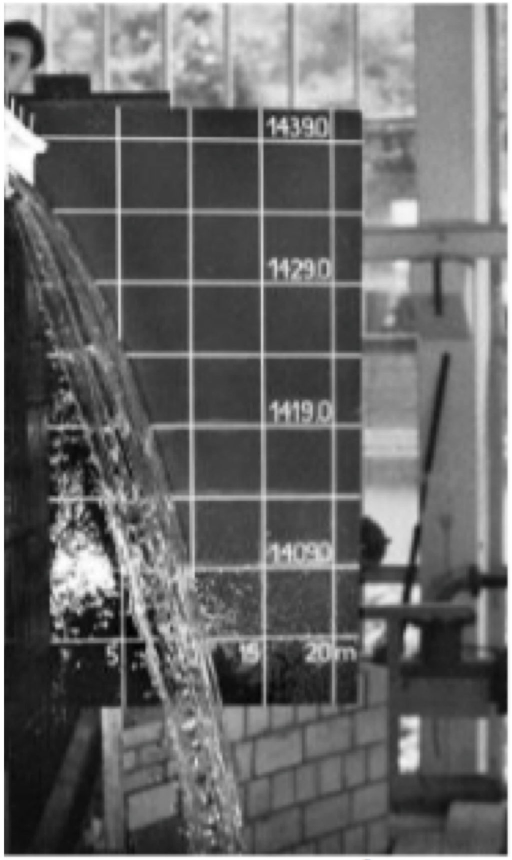
- Maeslant Barrier

Why do quantities between model and prototype disagree?

- Measurement effects:
  - due to non-identical measurement techniques used for data sampling in the model and prototype (intruding versus non-intruding measurement system etc.).
- Model effects:
  - Due to the incorrect reproduction of prototype features such as geometry (2D modelling, reflection from boundaries), flow or wave generation techniques or fluid properties
- Scale effects:
  - Due to the inability to keep each relevant force ratio constant between the he scale model and its real-world prototype.

Example of scale effects

Miniature universe

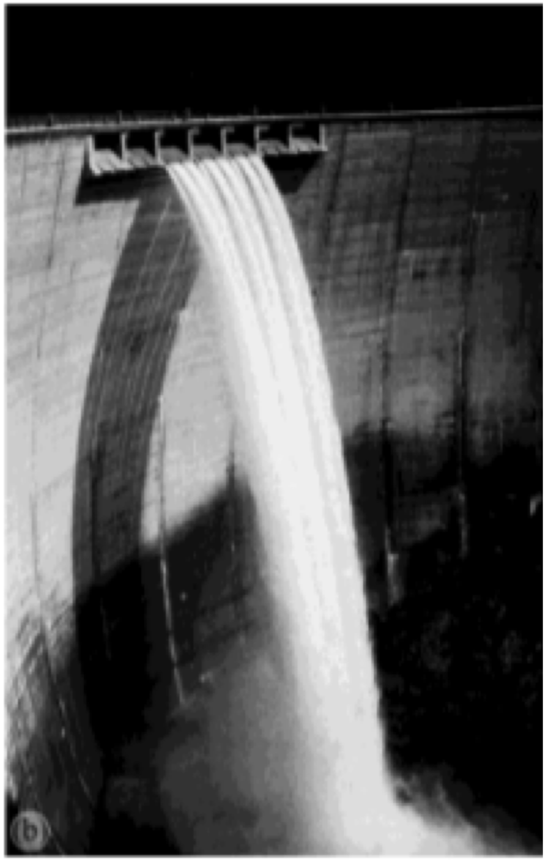


$1:\lambda = 1:30$

Jet trajectory ✓

Air concentration ✗

Real-world prototype



Types of Similarity

- If model fulfill the similarity condition, then we can use model to study natural flows.

- Three types of similarity
  - Geometric
  - Kinematic
  - Dynamic

Geometric similarity

- Different scale, generally smaller
  - Length

$$L_r = \frac{L_m}{L_p} = \frac{B_m}{B_p} = \frac{d_m}{d_p}$$

  - Area

$$A_r = \frac{A_m}{A_p} = \frac{B_m}{B_p}\frac{L_m}{L_p} = \frac{L_m^2}{L_p^2} = L_r^2$$

  - Volume

$$V_r = \frac{V_m}{V_p} = \frac{d_m}{d_p}\frac{B_m}{B_p}\frac{L_m}{L_p} = \frac{L_m^3}{L_p^3} = L_r^3$$

Kinematic similarity

- Similarity of motion
  - Stream line, boundary layer, etc
  - Kinematic similarity must ensure geometric similarity

$$V_r = \frac{V_m}{V_p} = const.$$

$$Also$$

$$Q_r = \frac{Q_m}{Q_p} = A_r V_r = L_r^2 V_r$$

$$T_r = \frac{T_m}{T_p} = L_r V_r^{-1}$$

$$a_r = \frac{a_m}{a_p} = V_r^{\ 2} T_r^{-1} = V_r^{\ 2} L_r^{-1}$$

  - Where a is the acceleration

Dynamic similarity

- Dynamic -> Force similarity !
- Three main forces
  - External force: Gravity force
  - Internal forces: Viscous and surface tension)
  - Resultant force : drag force, pressure force on hydraulic structures
- Those main forces ratio must be equivalent in both model and prototype (real)

Dynamic similarity

- Assuming that F_I, F_g, F_ν, F_σ, F_p, and F_{le}, are the inertia, gravity, viscous, surface tension, pressure, and elastic force respectively
  - Then

$$F_r = \frac{(F_I)_m}{(F_I)_p} = \frac{(F_g)_m}{(F_g)_p} = \frac{(F_\nu)_m}{(F_\nu)_p} = \frac{(F_\sigma)_m}{(F_\sigma)_p} = \frac{(F_e)_m}{(F_e)_p} = const.$$

  - Regardless of the specific type of the force, its dimensions are equivalent to the dimensions of mass (*M*) multiplies by acceleration (*a*).

$$F_r = M_r a_r = \left(\rho_r L_r^3\right)\left(V_r T_r^{-1}\right) = \rho_r L_r^2 V_r \left(L_r T_r^{-1}\right) = \rho_r L_r^2 V_r^2$$

Dynamic similarity

- Work $$W_r = F_r L_r = \rho_r L_r^3 V_r^2$$
- Power $$P_r = F_r V_r = \rho_r L_r^2 V_r^2 \cdot v_r = \rho_r L_r^2 V_r^3$$
- Pressure $$p_r = F_r L_r^{-2} = \rho_r L_r^2 V_r^2 \cdot L_r^{-2} = \rho_r V_r^2$$
- Specific gravity $$\gamma_r = F_r L_r^{-3} = \rho_r L_r^{-1} V_r^2$$

- It is clean that the ratios of all parameters in two systems, we can express with 3 main parameter ratios of geometric (length), kinematic(velocity), and dynamic(density).

$$\rho_r = \frac{\rho_m}{\rho_p}$$

- While length and density are given in the laboratory, the velocity is not determined well… We need to determine the dominating force.

Dominating forces

- In hydraulic problems, there are a number of forces exist. Howeve r, only one or two forces dominate.

- To reduce the cost and simplify the solution, we generally consider one or two forces.

Gravity force

- In open channels, natural rivers, hydraulic structures, wave actions, etc, gravity force is dominant.

$$\left(\frac{F_I}{F_g}\right)_m = \left(\frac{F_I}{F_g}\right)_p$$

- Or

$$\left(\frac{F_I}{F_g}\right)_r = 1$$

- Where $F_I = \rho L^2 V^{2;}$ and $F_g = \gamma L^3$

$$\left(\frac{\rho L^2 V^2}{\gamma L^3}\right)_r = 1$$

- Which can be written as

$$\left(\frac{V^2}{gL}\right)_r = 1 = \left(F_n^2\right)_r$$

Gravity force

$$\left(\frac{V^2}{gL}\right)_r = 1 = \left(F_n^2\right)_r$$

$$\left(F_n\right)_r = \left(\frac{V}{\sqrt{gL}}\right)_r = 1$$

- The above equation indicates that when gravity force dominates , the Froude number in the model must be exactly equal to that i n the prototype.

- The velocity ratio for gravity problems

$$V_r = \sqrt{g_r L_r}$$

- Gravity does not change in model and real world, so

$$V_r = \sqrt{L_r}$$

Example: A 1:10 scale model is constructed for a hydraulic structure. Determine the ratios of force and pressure.

If the force and pressure on the model are equal to 1.90kN and 500N/m^2, respectively, what are the corresponding values in the prototype? Assume the same fluid is used in both the model and prototype.

- Solution:

$$L_r = \frac{1}{10}$$

Using force ration equation

$$F_r = \rho_r L_r^2 V_r^2$$

where

$$\rho_r = 1, \quad V_r = L_r^{1/2}$$

Example: A 1:10 scale model is constructed for a hydraulic structure. Determine the ratios of force and pressure.

$$F_r = L_r^3 = \frac{1}{1000}$$

$$F_r = \rho_r L_r^2 V_r^2$$

$$\rho_r = 1, \quad V_r = L_r^{1/2}$$

$$p_r = \frac{F_r}{A_r} = \rho_r L_r^2 V_r^3 L_r^{-2} = \rho_r V_r^3$$

$$p_r = \frac{1}{10}$$

$$\frac{F_m}{F_p} = \frac{1}{1000} = \frac{1.9}{F_p}$$

Then F_p=1.9X1000=1900kN.

$$p_r = \frac{p_m}{p_p} = \frac{1}{10} = \frac{500}{p_p}$$

$$p_p = 5kN/m^2$$

Viscous Force

- Viscous force mainly appear in the pipe flow, or partially submerged object.
  - Viscous force dominant means that flow is laminar
  - In the opposite way, if viscous force is ignored, what kind of flow can occur?

    Turbulent flow

    Inviscid flow
- Following Newton's law of viscosity

$$\tau = \mu \frac{dv}{dy}$$

τ is shear stress due to viscosity of μ which is dynamic viscosity

Viscous Force

- Following Newton's law of viscosity

$$\tau = \mu \frac{dv}{dy}$$

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- Rewrite in dimensional form, then

$$\frac{F_v}{L^2} = \mu \frac{v}{L}$$

$$\therefore F_v = \mu v L$$

For dynamic similarity, the ratio of two main forces of inertia and viscosity must be equivalent.

Viscous Force

- For dynamic similarity, the ratio of two main forces of inertia and viscosity must be equivalent.

$$\therefore \left(\frac{F_I}{F_v}\right)_m = \left(\frac{F_I}{F_v}\right)_p = const.$$

$$or \quad \left(\frac{F_I}{F_v}\right)_r = 1$$

- Let's think in the different way,

$$\frac{F_I}{F_v} = \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho L V}{\mu} = \frac{L V}{\mu} = R_n$$

It means that the Reynolds number is the force balance of inertia and viscous

Viscous Force

- Then we can conclude that for dynamically similar system with a viscous force dominating the flow, we have

$$(R_n)_r = \left(\frac{\rho L V}{\mu}\right)_r = 1$$

- Reynolds number ratio of model and prototype (real) must be 1.

$$V_r = \mu_r \rho_r^{-1} L_r^{-1} = \nu_r L_r^{-1}$$

- Let's say we use the same fluids in both systems, then viscosity must be equal.

$$V_r = \nu_r L_r^{-1} = 1 \cdot L_r^{-1}$$

Gravity and viscous forces

- Let's think drag forces on floating body like ships. Two component are important (for design or experiment)
  1. Wave resistance (gravitational force)
  2. Viscous resistance (viscosity)
  3. (the importance of inertia need not to be discussed)

- Using dimensional analysis, the drag force *F*, the fluid velocity *V*, a characteristic length of the body, *L*, density *r*, viscosity, *m*, the gravitational acceleration *g*,

$$F = \rho L^2 V^2 \varphi\left(F_n, R_n\right)$$

Gravity and viscous forces

- For the dynamic similarity, both F_n and R_n should be the same value in the two systems.

$$\left(F_n\right)_m = \left(F_n\right)_p \quad or \quad \left(F_n\right)_r = 1$$
$$\left(R_n\right)_m = \left(R_n\right)_p \quad or \quad \left(R_n\right)_r = 1$$

In viscous relationship, we drove

$$V_r = \mu_r \rho_r^{-1} L_r^{-1} = \nu_r L_r^{-1}$$

In gravity relationship,

$$V_r = \sqrt{g_r L_r}$$

Both system has same gravitational acceleration, then

$$L_r^{1/2} = \nu_r L_r^{-1}, \quad \nu_r = L_r^{3/2}$$

- So, if you need to reduce size of model then use different material for experiment from in the real world

Distorted models

- If we cannot maintain full geometric similarity for economic, practical or physical, we should use distorted model.

☞ Roughness height is impossible to change…

- For example, 10km river with 1km width and 4m depth.
- But, hydraulic laboratory has only dimension of 100m ×30m. Then we need

$$L_r = \frac{1}{100} \quad for \quad length$$

$$\therefore \quad L_r = \frac{B_m}{B_p} = \frac{1}{100} \quad (10m \quad for \quad B)$$

$$L_r = \frac{H_m}{H_p} = \frac{1}{100} \quad (4cm \quad for \quad H)$$

Distorted models

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$$L_r = \frac{H_m}{H_p} = \frac{1}{100} \qquad (4cm \quad for \quad H)$$

- When H=4cm, surface tension may be important. But, in real situation, surface tension is not crucial.

- So, in the practical perspectives, we consider two different scales for vertical and horizontal.

Distorted model

- "… are used extensively in rivers and harbors as flow depths are quite small as compared to other areal dimensions.
- When Z_r is taken as 1:100, while L_r is taken as 1:200 or even 1:500.

Distorted model

- In distorted models, we will only discuss gravity force as a dominating force.

- Most distorted models are built for rives, harbors, waves and other systems where flow mainly controlled by the gravity force.

$$V_r = g_r^{1/2} Z_r^{1/2}$$

- The above equation is similar to before, we use here *Z* instead of *L*. since fluid velocities are created by vertical heads.

$$V_r = \sqrt{g_r L_r}$$

Distorted model

- The gravity acceleration can be assumed as constant. Then simply

$$V_r = Z_r^{1/2}$$

- Be careful whether we consider ratios for horizontal or vertical. For example, the ratio for a horizontal area A_{rh} is

$$A_{r_h} = L_r^2$$

- The ratio for the vertical area A_{rv} is

$$A_{r_v} = L_r Z_r$$

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Distorted model

- *Discharge*

$$Q_{r_h} = A_{r_v} \cdot V_r = L_r Z_r \cdot Z_r^{1/2} = L_r Z_r^{3/2}$$

$$Q_{r_v} = A_{r_h} \cdot V_r = L_r{}^2 Z_r^{1/2}$$

- *Time*

$$T_{r_h} = \frac{L_r}{V_r} = L_r \; Z_r^{-1/2}$$

$$T_{r_v} = \frac{Z_r}{V_r} = Z_r^{1/2}$$

- *Force*

$$F_{r_h} = \rho_r Q_{r_h} V_r = \rho_r A_{r_v} V_r{}^2 = \rho_r \left(L_r Z_r\right) Z_r = \rho_r L_r Z_r^2$$

$$F_{r_v} = \rho_r Q_{r_v} V_r = \rho_r A_{r_h} V_r{}^2 = \rho_r L_r{}^2 Z_r$$

- *Work*

$$W_{r_h} = F_{r_h} \cdot L_r$$

$$W_{r_h} = \rho_r L_r \; Z_r^2 \cdot L_r = \rho_r L_r{}^2 Z_r^2$$