Machine Foundations

- Introduction :

  - the soil behavior should be elastic (∵ Repetitive dynamic load applied)

  the most important parameters are ;

  → the amplitude of motion of a machine at its operating frequency

  → the natural frequency of a machine 'foundation' soil system

- Machines (Types)

- Reciprocating machines : periodic unbalanced force, operating speed < 600 rpm
- Impact machines : impact load, operating speed : 60 ~ 150 blows/min
- Rotary machines : 3,000 rpm ~ 10,000 rpm

- Foundations (Types)

massive foundation { - block type foundation : large mass, smaller natural frequency
- box or caisson type : lighter than the block type

- complex or frame foundation : footing + wall & column + beam & slabs

relatively flexible

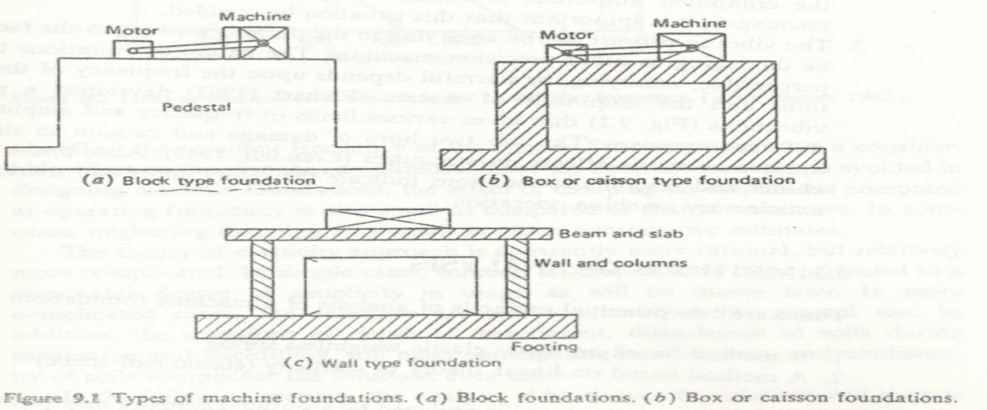


Figure 9.1 Types of machine foundations. (a) Block foundations. (b) Box or caisson foundations. (c) Complex foundations.

Satisfactory Machine Foundations(Requirements)

Pseudo-static
- Bearing capacity
- Settlement

\+

Dynamic
- no resonance
  - operating frequency should be smaller than the natural frequency of the foundation-soil system & the ratio < 0.5, If higher > 2 (upper w_n)
  - (unimportant case > 1.5 & < 0.6)
- The amplitude of motion < limiting value (refer to Fig 9.2)
- not be annoying the persons

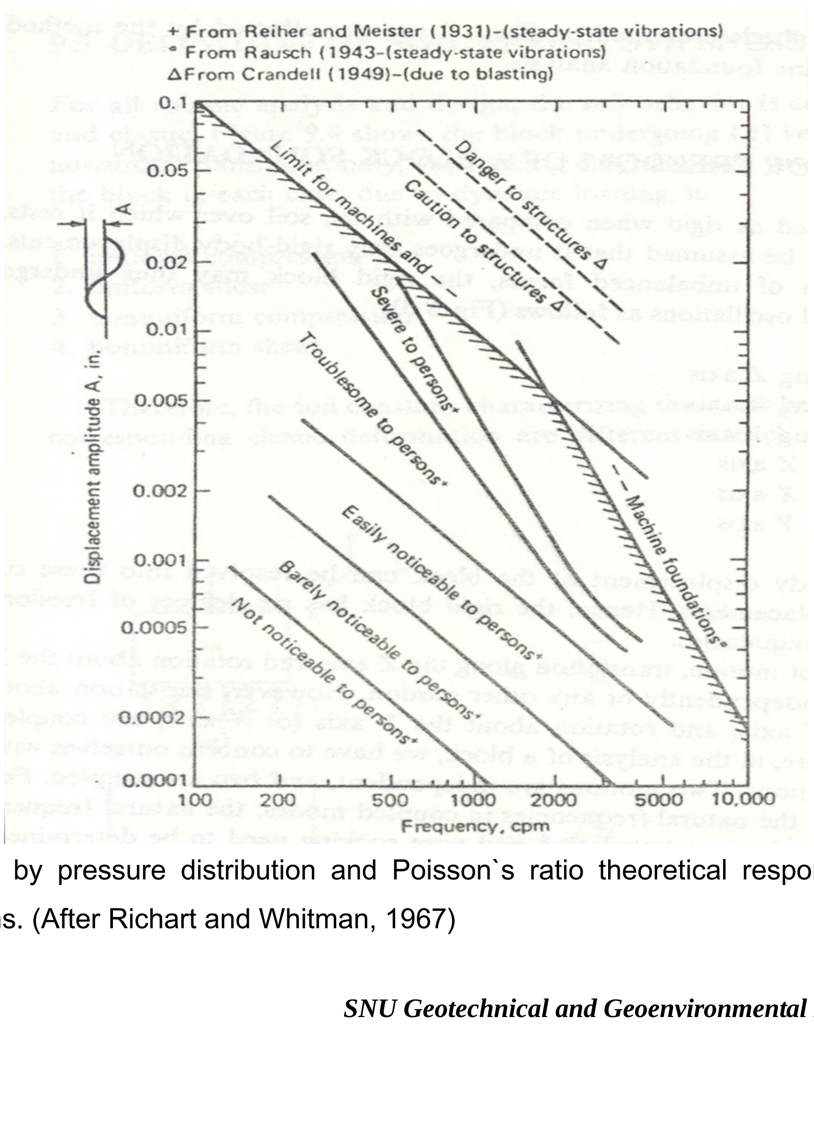


Fig 9.2a Effect by pressure distribution and Poisson`s ratio theoretical response for vertical footing vibrations. (After Richart and Whitman, 1967)

Methods of analysis

- linear elastic weightless spring (spring analysis)

- linear theory of elasticity on half space (theory of elasticity)

① soil → replaced by a spring
(spring constants obtained from dynamic subgrade reaction modulus)

damping may be introduced
↓
affect more on the resonant amplitude than on its frequency
↓
resonant frequency avoided
↓
damping may be neglected to result in rather conservative estimates

② more rational but relatively complicated

depth of embedment, disturbance of soils around the foundation, soil mass participating in vibrations, nonlinearity of soil properties.

Theory of elasticity approach

Preliminaries

'dynamic Boussinesq loading' : the oscillating vertical force at the surface

'dynamic reciprocity' (Lamb, 1904) : extension of Maxwell`s law of reciprocal deflections

Maxwell`s law : the deflection at point 1 in an elastic body due to a unit value of load at point 2 in that body is equal precisely to the deflection at point 2 due to a unit value of load applied at point 1.

Lamb`s demonstration : 1) the horizontal displacement produced at a point on the surface by an oscillating unit vertical force at that pt. has the same value as the vertical displacement at the same pt. by an oscillating horizontal unit force acting at the point, 2) Fig. 7-1

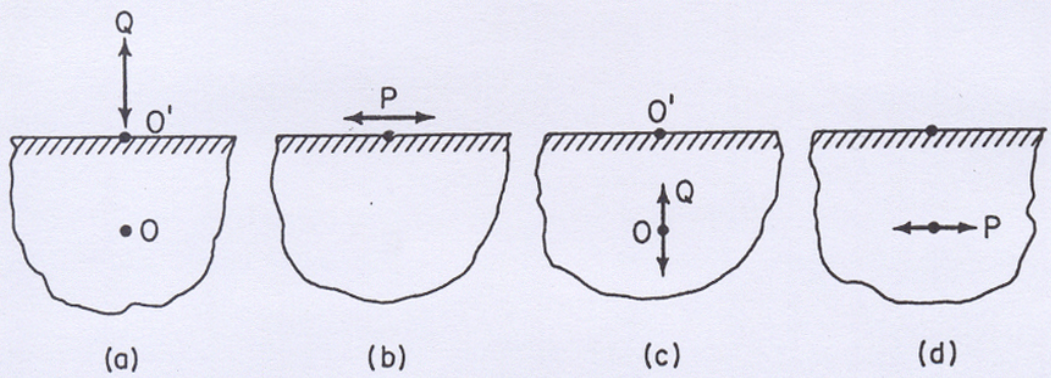


Figure 7-1. Lamb's problems for steady-state oscillating force or pulse loading acting at a point (three-dimensional) or along a line (two-dimensional). (a) For vertical loading at the surface. (b) For horizontal loading at the surface. (c) For vertical loading within the body. (d) For horizontal loading within the body.

Vertical oscillation

Lamb(1904) - response of elastic half space excited by oscillating vertical forces (line load & point load)

Reissner(1936) [Soil property parameters : G, υ, $\rho(=\gamma/g)$]

- developed an analytical solution for the periodic vertical displacement z_0 at the center of the circular loaded area (obtained this solution by integration of Lamb`s solution)

$$z_0 = \frac{P_0 \exp(iwt)}{G r_0}(f_1 + i f_2) \quad \ldots ①$$

where, P_0 : amplitude of the total force applied to the circular contact area

w : circular frequency of force application

G : shear modulus of the half-space

r_0 : radius of the circular contact area

f_1, f_2 : Reissner`s ‘displacement functions’

- f_1, f_2 ; Complicated functions of Poisson`s ratio and a dimensionless frequency term a_0

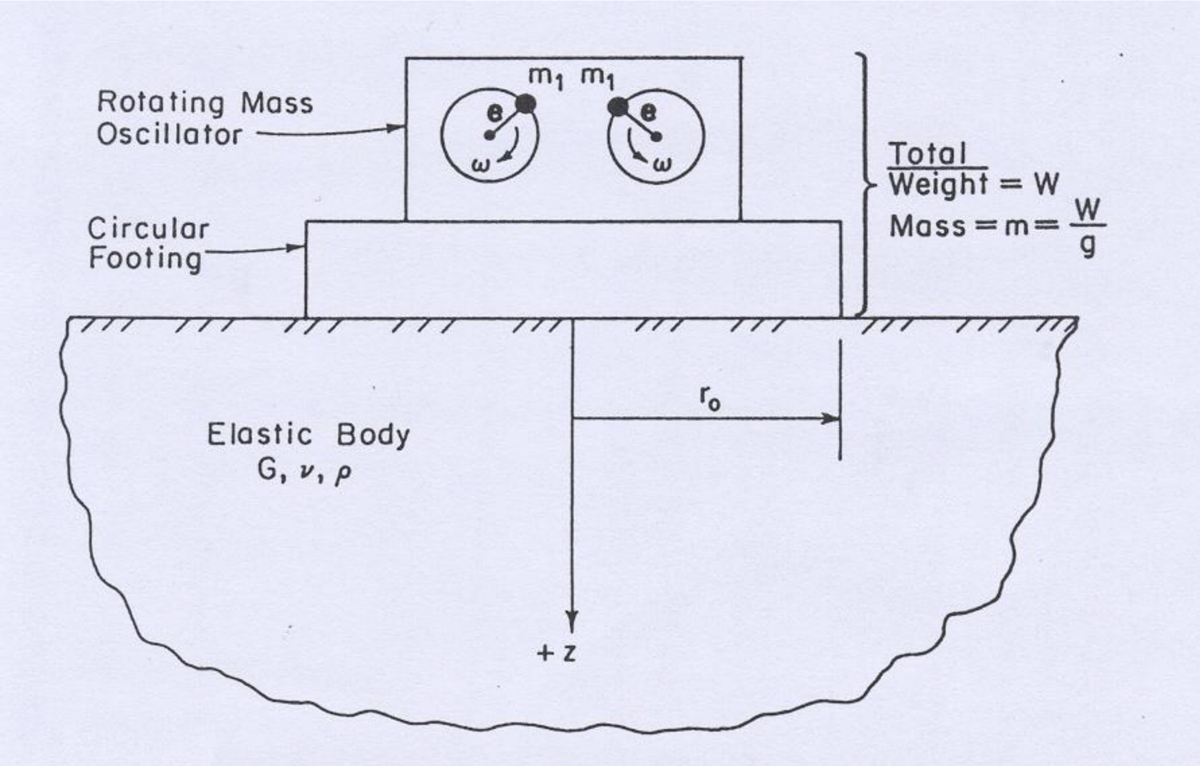
$$a_0 = w r_0 \sqrt{\frac{\rho}{G}} = \frac{w r_0}{V_s} \quad \ldots ②$$

where, V_s : shear wave velocity

- a and mass ratio, b : a second dimensionless term by Reissner

$$b = \frac{m}{\rho r_0^3} \quad \ldots ③$$

where, m : the total mass of the vibrating footing and exciting mechanism



- Eq. ③ describes a relation between the mass of rigid body and a particular mass of the elastic body (i.e. foundation soils)

- Remarks

- Reissner`s theory formed the basis for nearly all further studies on this subject

- However, his results did not completely agree with field test results because

1) permanent settlements developed during many tests(violating his assumptions)

2) the amplitudes of motion by the model field vibrators were so large that the vibrators jumped clear of the ground and acted as a hammer (the order of 2g~3g)

3) the assumption of a uniformly distributed pressure was not realistic

4) there was an error in the calculation of f_2

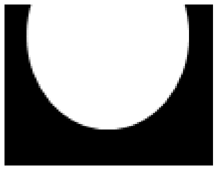
Quinlan(1953), Sung(1953)

The effect of changes in pressure distribution over the contact area is considered

- Pressure distributions

a) rigid base(approximation ← ∞ at the edge)

$$\sigma_z = \frac{P_0 \exp(iwt)}{2\pi r_0 \sqrt{r_0^{\,2} - r^2}} \quad \rightarrow$$

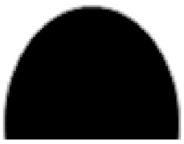


b) uniform

$$\sigma_z = \frac{P_0 \exp(iwt)}{\pi r_0^{\,2}} \quad \rightarrow$$

c) parabolic

$$\sigma_z = \frac{2P_0 (r_0^{\,2} - r^2) \exp(iwt)}{\pi r_0^{\,4}} \quad \rightarrow$$



- using the center displacements for the three cases, the amplitude – frequency curves obtained (Fig 9.12a) and the followings known :

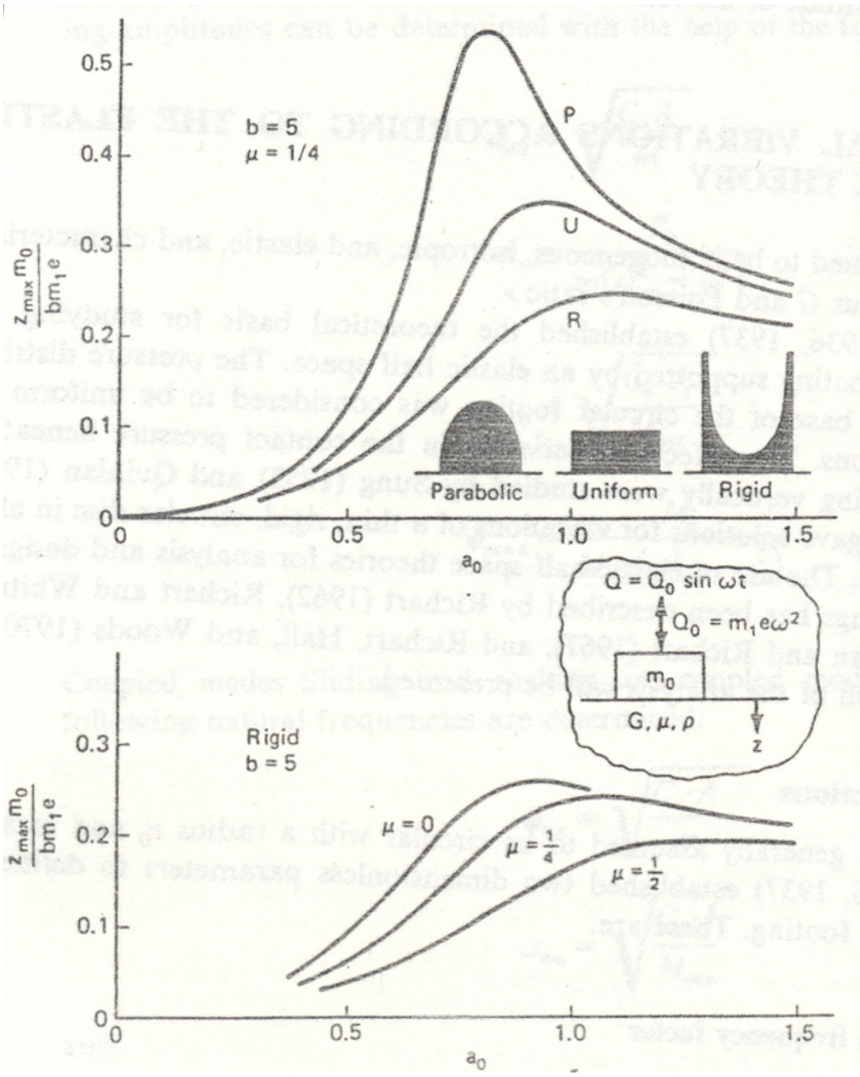
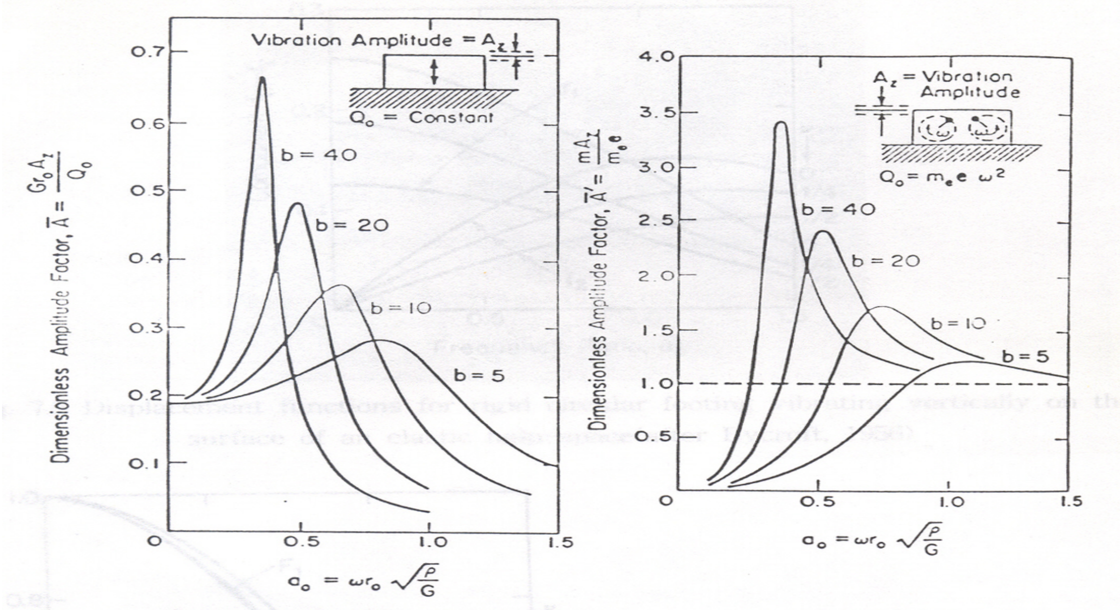


Fig 9.12a Effect by pressure distribution and Poisson`s ratio on theoretical for vertical footing vibrations. *(After Richart and Whitman, 1967)*

① (upper figure) as the load is concentrated nearer the center
→ the peak amplitude increases and the frequency at this peak is lowered
→ i.e. dynamic response of a foundation is influenced by the control of the flexibility of the foundation pad (Fistedis, 1957)

② (lower figure) the amplitude of motion is greater and the frequency at maximum amplitude is lower when $\upsilon = 0$
→ i.e. '$\upsilon = 0$' would represent the worst case of a greater motion at a lower frequency

- Influence of the mass ratio b



(a) For Constant Amplitude of Exciting Force.

(b) For Exciting Force Amplitude Dependent on Exciting Frequency.

Figure 7-4. Amplitude vs. frequency relations for vertical oscillation of a rigid circular footing on an elastic half-space ($\nu = \frac{1}{4}$). (*After Richart. 1962.*)

- Similar to Figs. 2-12 & 15 (*Prakash*)

→ illustrates a significant loss of energy by radiation of elastic waves (geometrical damping)

- Practically, b is smaller than 10 for the footings under vertical vibrations

→ vertical oscillations are usually highly damped & extreme amplitudes of motion do not occur

- Bycroft(1956)

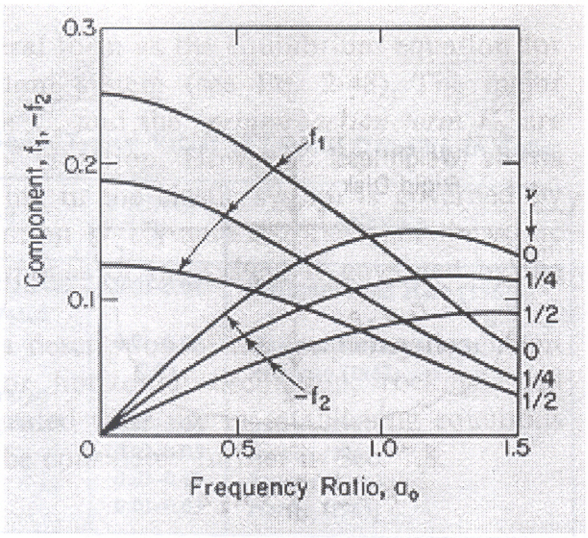


Fig. 7.5 Displacement functions for rigid circular footing vibrating vertically on the surface of an elastic halp-space(after Bycroft, 1956)

- at $a_0 = 0$ (static case), $f_2 = 0$

→ f_1 must produce the static displacement when substituted into Eq. ①

$$z_0 = \frac{P_0 \exp(iwt)}{Gr_0}(f_1 + if_2)$$

$$\rightarrow z_s = \frac{P_0}{Gr_0} f_1 = \frac{P_0(1-\upsilon)}{4Gr_0}$$ (for rigid circular)

$$\rightarrow f_1 = \frac{1-\upsilon}{4}$$ if $a_0 = 0$ (compare w/ Fig 7.5)

- the displacement function f_2 essentially describes 'damping' in the system

Hsieh`s Equations(1962)

- Established the expression for 'geometrical damping'
- weightless, rigid, circular disk of radius r_0 subjected to $P = P_0 \exp(iwt)$,

then, the vertical displacement is

$$z = \frac{P_0 \exp(iwt)}{Gr_0}(f_1 + if_2)$$

- By differentiating w.r.t. time,

$$\frac{dz}{dt} = \frac{P_0 w \exp(iwt)}{Gr_0}(if_1 - f_2)$$

thus,

$$f_1 wz - f_2 \frac{dz}{dt} = \frac{P_0 w}{Gr_0}(f_1^2 + f_2^2)\exp(iwt)$$
$$= \frac{Pw}{Gr_0}(f_1^2 + f_2^2)$$

or, $P = -\frac{Gr_0}{w}\frac{f_2}{f_1^2 + f_2^2}\frac{dz}{dt} + Gr_0 \frac{f_1}{f_1^2 + f_2^2} z$

let, $C_z = -\frac{Gr_0}{w}\frac{f_2}{f_1^2 + f_2^2}\left(= \frac{r_0^2}{a_0}\sqrt{G\rho}\left(\frac{-f_2}{f_1^2 + f_2^2}\right)\right)$

& $K_z = Gr_0 \frac{f_1}{f_1^2 + f_2^2}$

then, $P = C_z \frac{dz}{dt} + K_z Z \quad \ldots$ 1

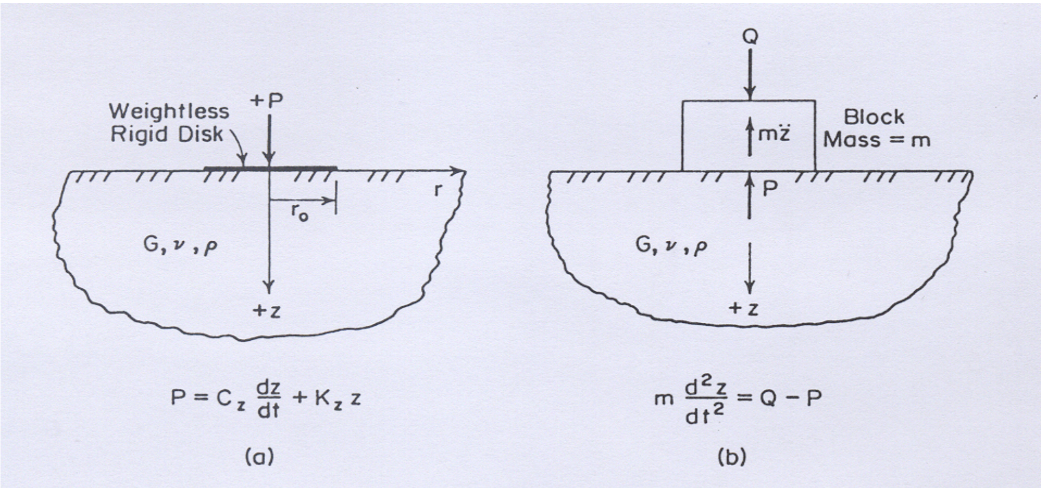


Figure 7-6. Notation for Hsieh's equations.

- In the case(b)

$$\frac{W}{g}\frac{d^2z}{dt^2} = Q - P$$

Q : external periodic force
W : weight of footing

- Sub. Into Eq. ①, & $\frac{W}{g} = m$

$$m\frac{d^2z}{dt^2} + C_z\frac{dz}{dt} + K_z z = Q = Q_0 \exp(iwt)$$

- same as the governing equation for the damped-single-degree-of freedom system. Difference is that the damping term(C_z) & the spring-reaction term K_z are both functions of the frequency of vibration($f_1, f_2 \leftarrow a_0, \upsilon$)

Lysmer`s analog(1965)

① Introduced a new displacement function

$$F = \frac{4}{1-\upsilon} f = F_1 + iF_2 , \quad f = f_1 + if_2$$

- Independent of υ

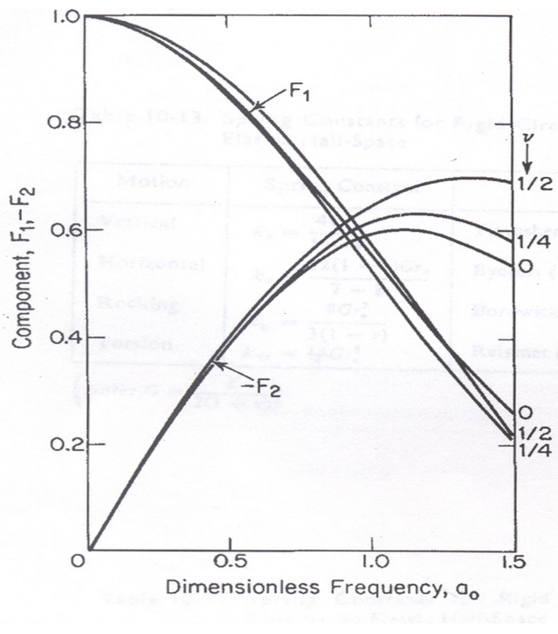


Fig 7.7 Variation of components of F with Poisson's ratio (after Lysmer and Richart, 1966)

- Extended the solution up to a_0 of 8 ($\leftarrow a_0$ of 1.5)

  Remember that $f_1 = \frac{1-\upsilon}{4}$, when $a_0 = 0$

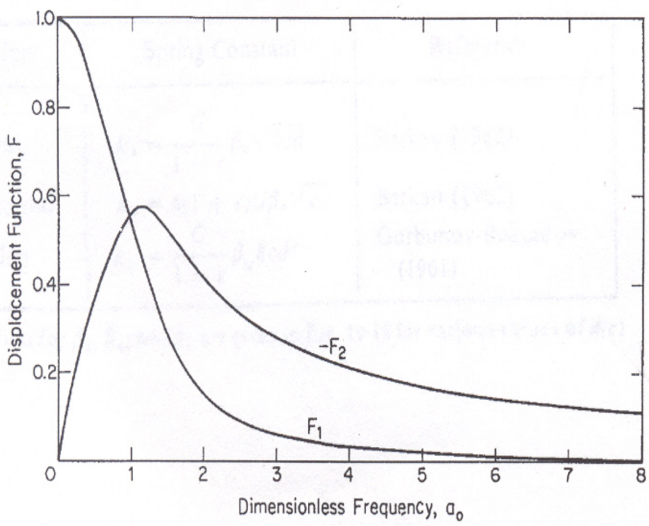


Fig 7.8 Displacement function F for vertical vibration of a weightless rigid circular disk(m=0) (after Lysmer and Richart, 1966)

② a modified mass ratio

$$B_z = \frac{1-\upsilon}{4} b = \frac{1-\upsilon}{4} \frac{m}{\rho r_0^3}$$

Now, the influence of Poisson`s ratio eliminated

③ With the new F & B_z, the amplitude of oscillator motion becomes

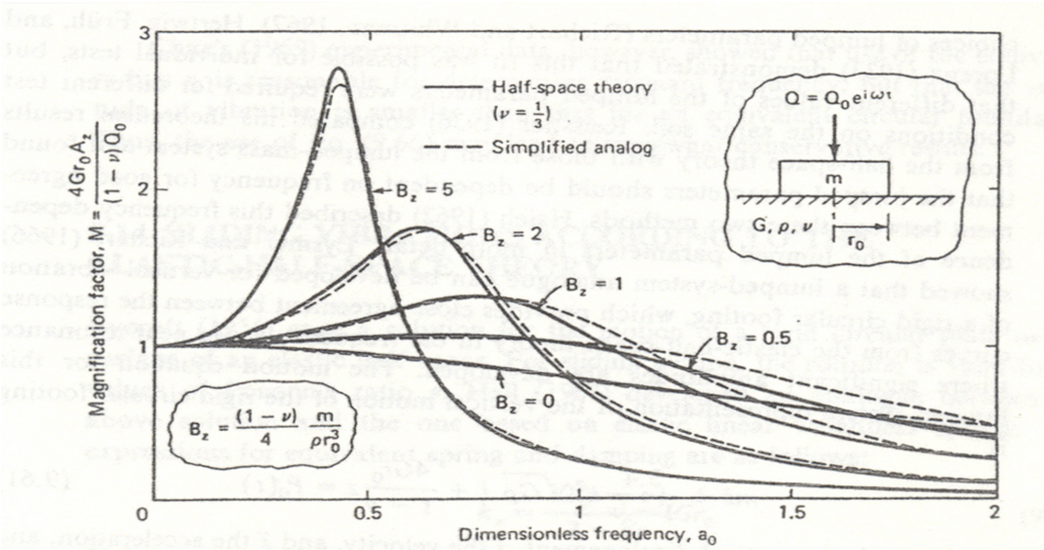
$$A_z = \frac{m_e e}{m} M a_0^2 B_z = \frac{m_e e}{m} Mr$$, Mr : the magnification factor

④ constant values of the effective damping and spring factors are chosen

(independent of a_0)

$$k_z = \frac{4Gr_0}{1-\upsilon}$$

$$C_z = \frac{3.4{r_0}^2}{(1-\upsilon)}\sqrt{\rho G}\ , \ \ (0 < a_0 < 1.0)$$



⑤ The Eq. of motion for Lysmer`s analog

$$m\ddot{z} + \frac{3.4{r_0}^2}{(1-\upsilon)}\sqrt{\rho G}\,\dot{z} + \frac{4Gr_0}{(1-\upsilon)}z = Q$$: Equation for Lumped-parameter system

For Vertical Oscillation

- the damping factor, $D = \dfrac{C}{C_c}$

$$C_c = 2\sqrt{k_z m} = 2\sqrt{\frac{4Gr_0 m}{(1-\upsilon)}}$$

$$D = \frac{C_z}{C_c} = \frac{0.425}{\sqrt{B_z}}$$

- the resonant frequency

constant force Q_0 : $f_m = \dfrac{1}{2m}\sqrt{\dfrac{k_z}{m}}\sqrt{1-2D^2} = \dfrac{1}{2m}\dfrac{V_s}{r_0}\dfrac{\sqrt{B_z - 0.36}}{B_z}$

rotating mass excitation : $f_{mr} = \dfrac{V_s}{2\pi r_0}\sqrt{\dfrac{0.9}{B_z - 0.45}}$

- (these approximations valid for $B_z \geq 1$)

- maximum amplitude of oscillation

constant force Q_0 : $A_{zm} = \dfrac{Q_0(1-\upsilon)}{4Gr_0}\dfrac{B_z}{0.85\sqrt{B_z - 0.18}}$

rotating mass excitation : $A_{zrm} = \dfrac{m_e e}{m}\dfrac{B_z}{0.85\sqrt{B_z - 0.18}}$

- the phase angle Φ

$$\tan\phi = \frac{0.85a_0}{B_z a_0^{\,2} - 1}$$

- equivalent radius : $r_0 = \sqrt{\dfrac{a \times b}{\pi}}$

- Lysmer established the bridge between elasticity theory & spring analysis