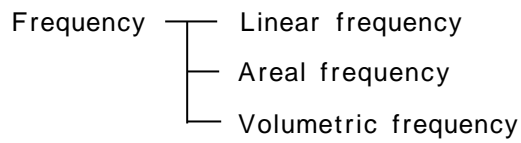


## 4. Discontinuity frequency



### Linear frequency

The normal linear frequency ( ) has a relationship with the linear frequency ( ) as follow.

—

where is an acute angle between a scanline and a joint normal.

a) Determination of the normal linear frequency of a joint set using s scanlines non-parallel to each other:

- weighted mean of the normal linear frequencies

b) Determination of the normal linear frequencies of D joint sets using s scanlines non-parallel to each other (Karzulovic & Goodman (1985)):

- total linear frequency from the j<sup>th</sup> scanline

Setting a theoretical total linear frequency and an observed one the error of frequency in the j<sup>th</sup> scanline ( ) and the sum of squared errors ( ) are as follows.

,

which can be solved using tensor notation as below.

Representing , ,

Because  $\sigma$  is a function of  $\rho$  (or  $\theta$ ), the minimum error can be obtained by partial differentiation of it with respect to  $\rho$  :

which can be expressed in matrix form as follow.

(refer to p.100)

#### Areal frequency

- Definition: the number of joint trace centers in unit area of plane (rock exposure)
- Setting an acute angle between a sampling window and a joint plane  $\theta$ , the relationship between normal areal frequency  $(N)$  and areal frequency  $(A)$  is as follow.

—————

- Sampling windows in practical survey have finite size and it is frequently hard to judge whether the trace centers are in the window or not. The end-point estimator (Mauldon, 1998) is very useful in these cases.

—————

This estimator defines the areal frequency as a sum of contained traces and a half of dissecting traces divided by sampling area. Lyman (2003) proved that the end-point estimator is an unbiased maximum likelihood estimator and at the same time it is an optimal estimator having the least variance.

#### Volumetric frequency

- Necessary for joint 3D modelling
- Direct measurement is almost impossible. Indirect estimation is needed.
- Difficult to find out joint centers

Total discontinuity area / rock mass volume is alternatively adopted.

- $\rho$ ,  $\theta$ ,  $\sigma$  (mean diameter) or size (diameter) distribution are required to estimate the volumetry frequency.

a) Calculation of  $\rho$  using

When  $\theta$  is an acute angle between a scanline and a joint normal,  $L$  is the length of scanline and  $r$  is a joint radius the number of joint intersections in a scanline can be calculated as below.

Combining  $N = \frac{L}{2r} \sin \theta$  and  $N = \frac{L}{2r} \cos \theta$  makes

$$N = \frac{L}{2r} \sqrt{\sin^2 \theta + \cos^2 \theta} = \frac{L}{2r}$$

To calculate the total number of joints having various radii, set

$N = \sum N_i$  and  $N_i = \frac{L}{2r_i}$ , which makes

$$N = \frac{L}{2} \sum \frac{1}{r_i}$$

b) Calculation of  $N$  using  $A$

When  $\theta$  is an acute angle between a sampling window and a joint plane,  $A$  is an area of the sampling window and  $d$  is joint diameter, the number of joints whose trace center is located in the sampling window is as follow

.

Rewritten with respect to  $\theta$  :

$$N = \frac{A}{d} \sum \frac{1}{\cos \theta}$$

For various  $\theta$ ,

makes  $N = \frac{A}{d} \sum \frac{1}{\cos \theta}$

### Discontinuity Frequency Extrema

- Meaning: Maxima + minima of the total linear frequency for all possible orientations of scanlines

- Assumption: Each joint set consists of parallel joints.

- Terminology:

$N$  : total linear frequency,  $N_i$  : normal linear frequency of the  $i^{\text{th}}$  set,

$\mathbf{n}_i$  : normal vector of the  $i^{\text{th}}$  set,  $\mathbf{v}$  : orientation vector,  $n$  : number of sets

- Mathematical expression

$$\left| \frac{\omega}{\omega_0} \right| = \frac{1}{\sqrt{1 - \frac{D}{2} \cos^2 \theta}}$$

where the rightmost equation is applied to the case  $\theta$  is adjusted to make an acute angle with  $\omega/\omega_0$ .

- Frequency locus when  $D=1$  and joints have strike/dip as NS/90:
  
- Frequency locus when  $D=1$  and joints have strike/dip as EW/90:
  
- Frequency locus when  $D=2$  and each joint set has strike/dip as NS/90 and EW/90, respectively:
  
- Frequency locus when  $D=3$  and all joint sets are vertical and their strikes are  $60^\circ$  apart each other: Home work
  
- General case of joint set orientation
  - 1) Single joint set

2) Two joint sets

3) Three joint sets

- Local(global) minimum / maximum

- Study of Priest & Samaniego (1983)

Joint traces belonging to three sets were generated in a virtual sampling plane. The normal linear frequency of each set is taken to be 0.8, 2.8, 1.6 ( $\text{m}^{-1}$ ) and the trace length is set to follow negative exponential distributions whose mean length

is 2m, 6m and 4m, respectively. The trace orientation is assumed to obey normal distributions whose standard deviation is  $0^\circ$  or  $15^\circ$ . Variation of total linear frequency according to scanline orientation is shown in Fig. 4.7.

#### Discontinuity occurrence

-Even though the linear frequency of each joint set does not obey Poisson distribution the total linear frequency of a few joint sets usually obey Poisson distribution. When the total linear frequency of all the joint sets obey Poisson distribution, the total areal frequency and total volumetric frequency can be also considered to obey Poisson distributions.