3. Flexural Analysis/Design of Beam

BENDING OF HOMOGENEOUS BEAMS

REINFORCED CONCRETE BEAM BEHAVIOR

DESIGN OF TENSION REINFORCED REC. BEAMS

DESIGN AIDS

PRACTICAL CONSIDERATIONS IN DESIGN

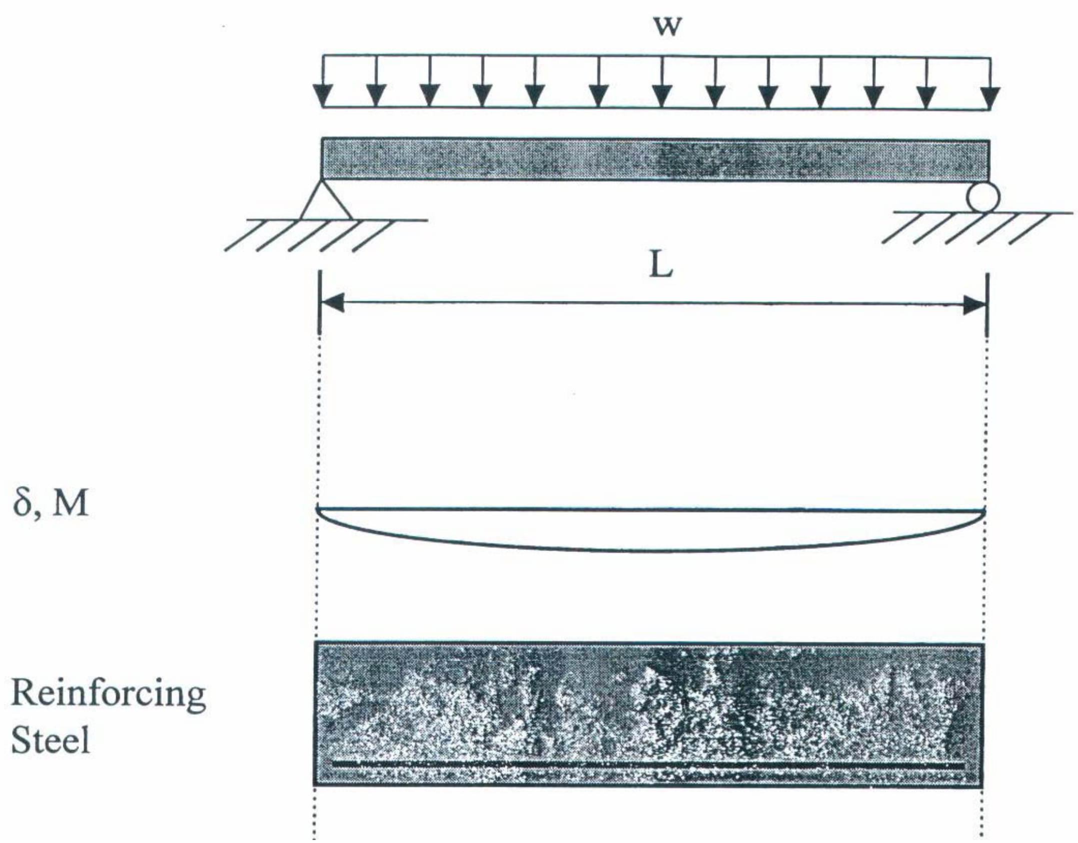
REC. BEAMS WITH TEN. AND COMP. REBAR

T BEAMS

447.327
Theory of Reinforced Concrete and Lab. I
Spring 2008

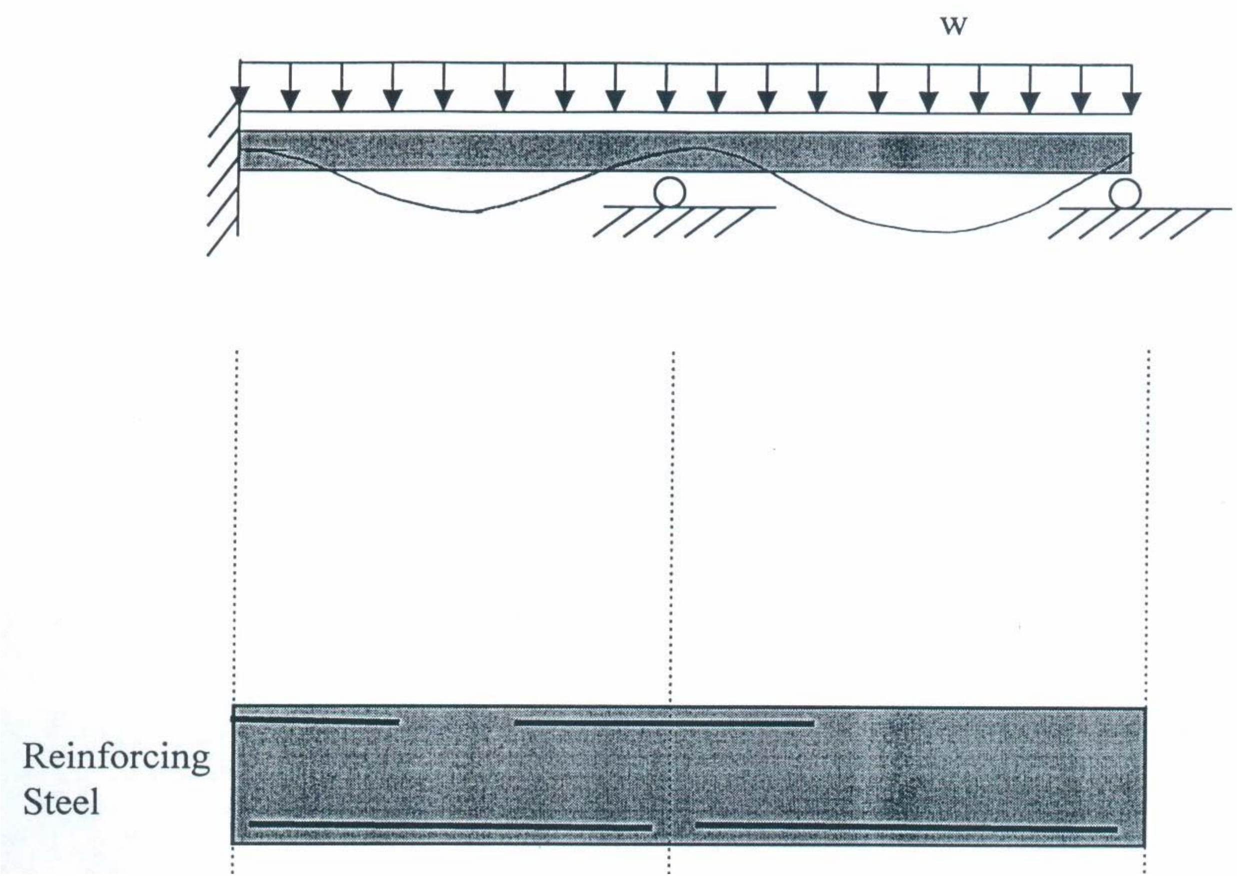
3. Flexural Analysis / Design of Beam

Typical Structures



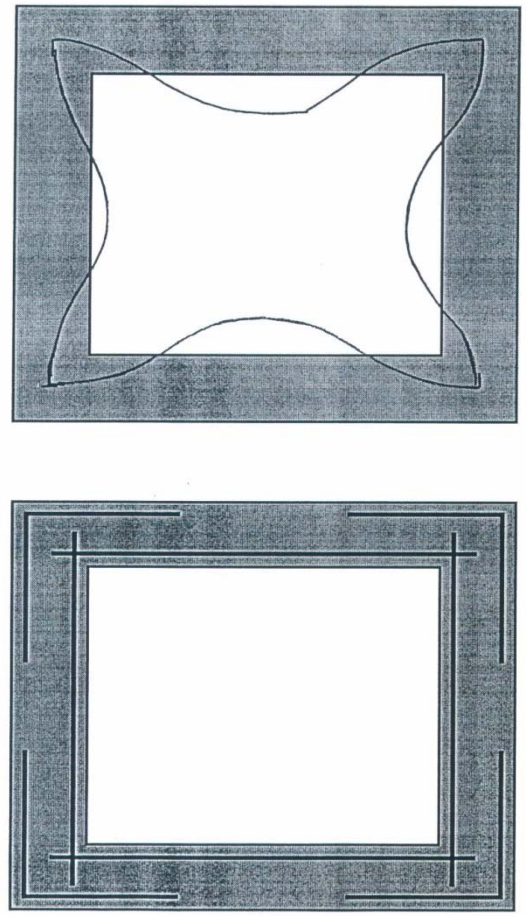
3. Flexural Analysis/Design of Beam

Typical Structures



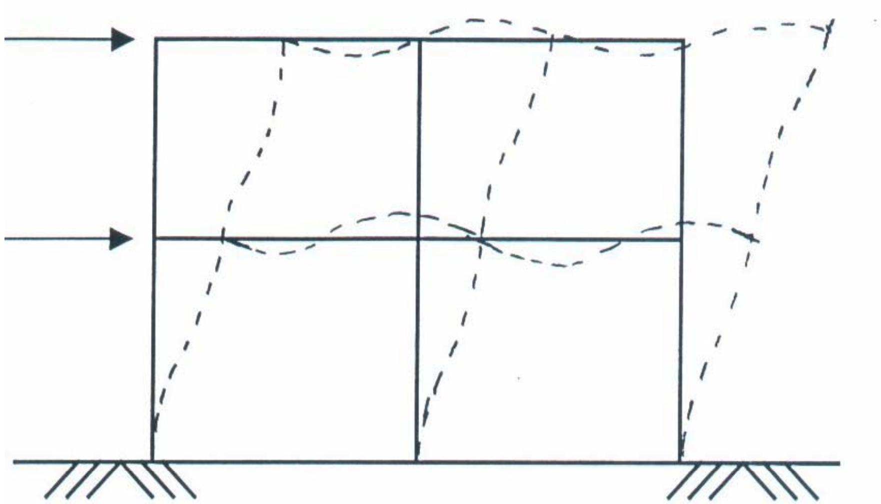
3. Flexural Analysis / Design of Beam

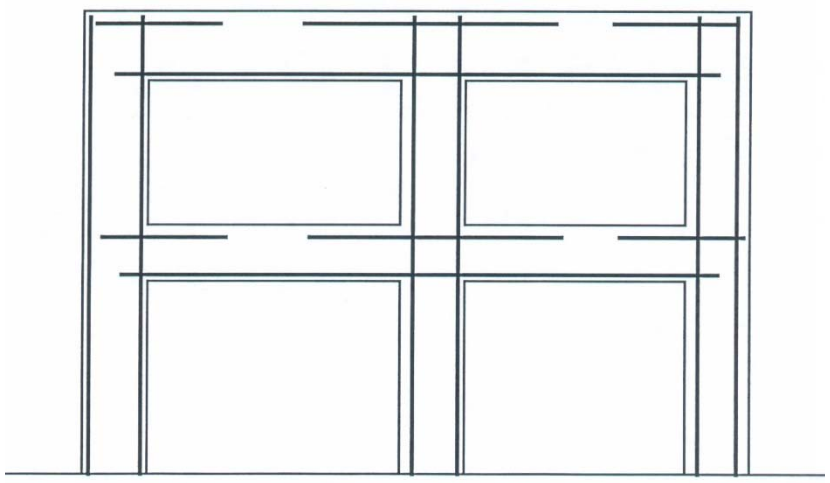
Typical Structures



3. Flexural Analysis / Design of Beam

Typical Structures





BENDING OF HOMOGENEOUS BEAMS

Concrete is homogeneous?
Reinforced Concrete is homogeneous?

The fundamental principles in the design and analysis of reinforced concrete are the same as those of homogeneous structural material.

Two components

Internal forces	normal to the section	- flexure
at any cross section	tangential to the section	- shear

3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Basic Assumptions in Flexural Design

1. A cross section that was plane before loading remains plane under load

⇨ unit strain in a beam above and below the neutral axis are proportional to the distance from that axis; strain distribution is linear ⇦ Bernoulli's hypothesis (not true for deep beams)

REINFORCED CONCRETE BEAM BEHAVIOR

Basic Assumptions in Flexural Design

2. Concrete is assumed to fail in compression, when $\varepsilon_c = \varepsilon_{cu}$ (limit state) = 0.003

3. Stress-strain relationship of reinforcement is assumed to be *elastoplastic* (elastic-perfectly plastic).
   ⇦ Strain hardening effect is neglected

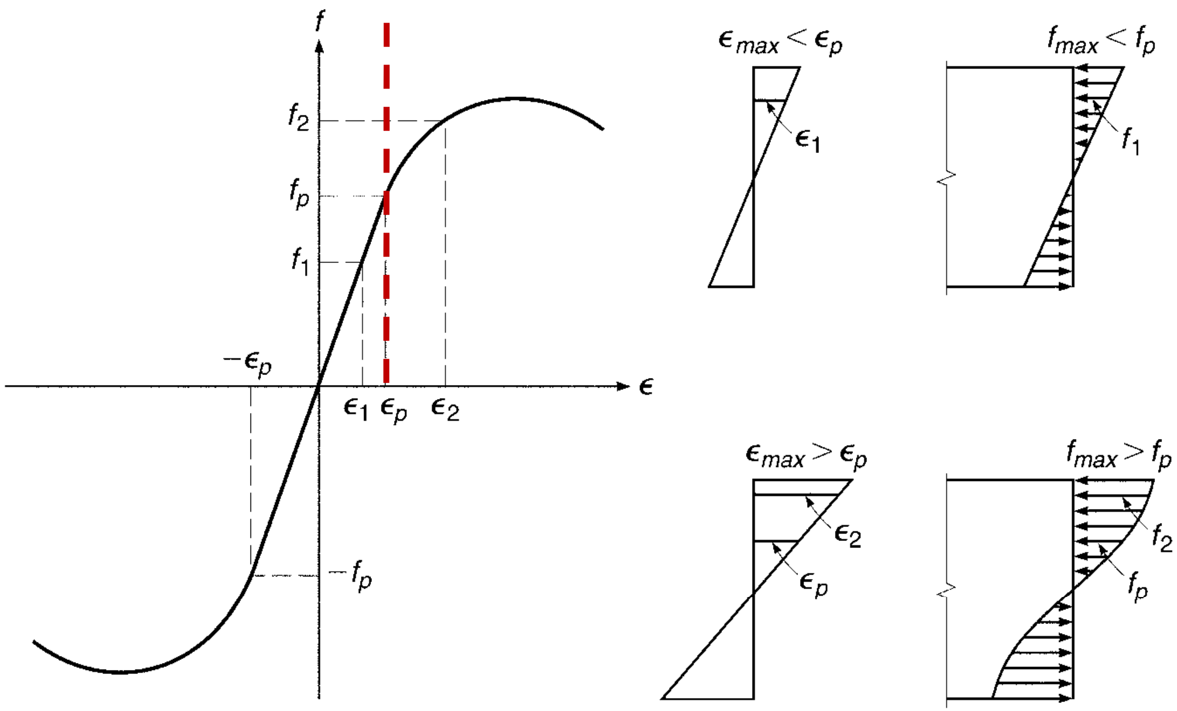
4. Tensile strength of concrete is neglected for calculation of flexural strength.

3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Basic Assumptions in Flexural Design

cf.) Typical s-s curve of homogeneous material



proportional

not proportional

REINFORCED CONCRETE BEAM BEHAVIOR

Basic Assumptions in Flexural Design

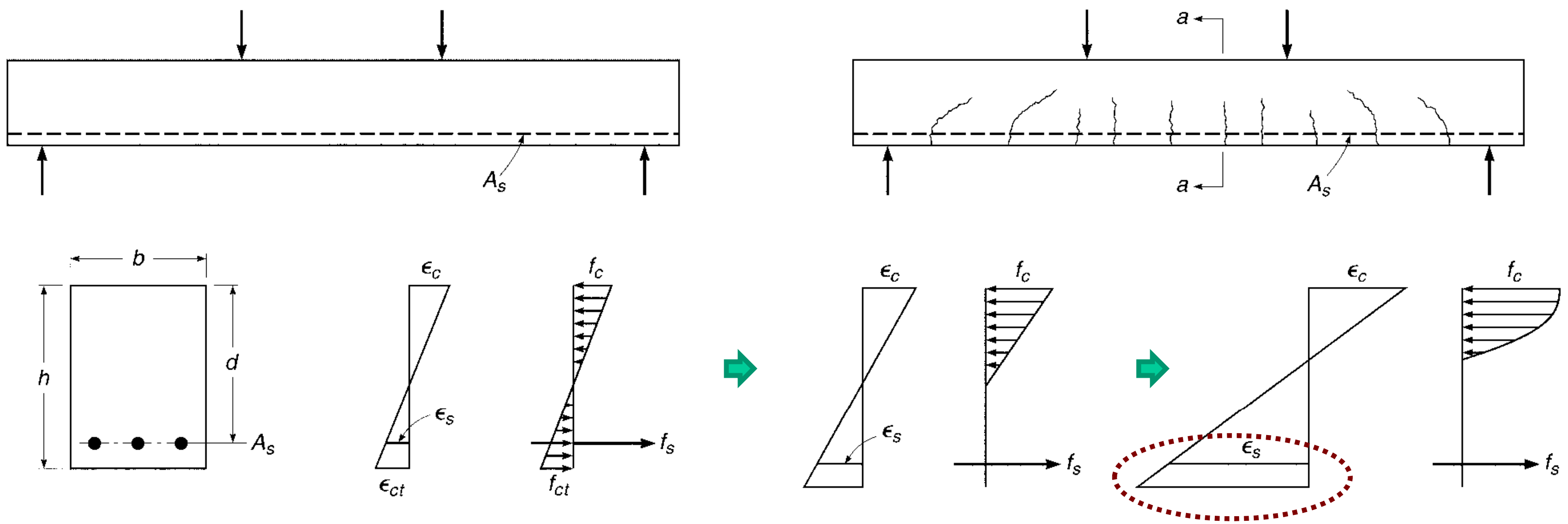
5. Compressive stress-strain relationship for concrete may be assumed to be any shape (rectangular, trapezoidal, parabolic, etc) that results in an acceptable prediction of strength.

   ⇦ Equivalent rectangular stress distribution

3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

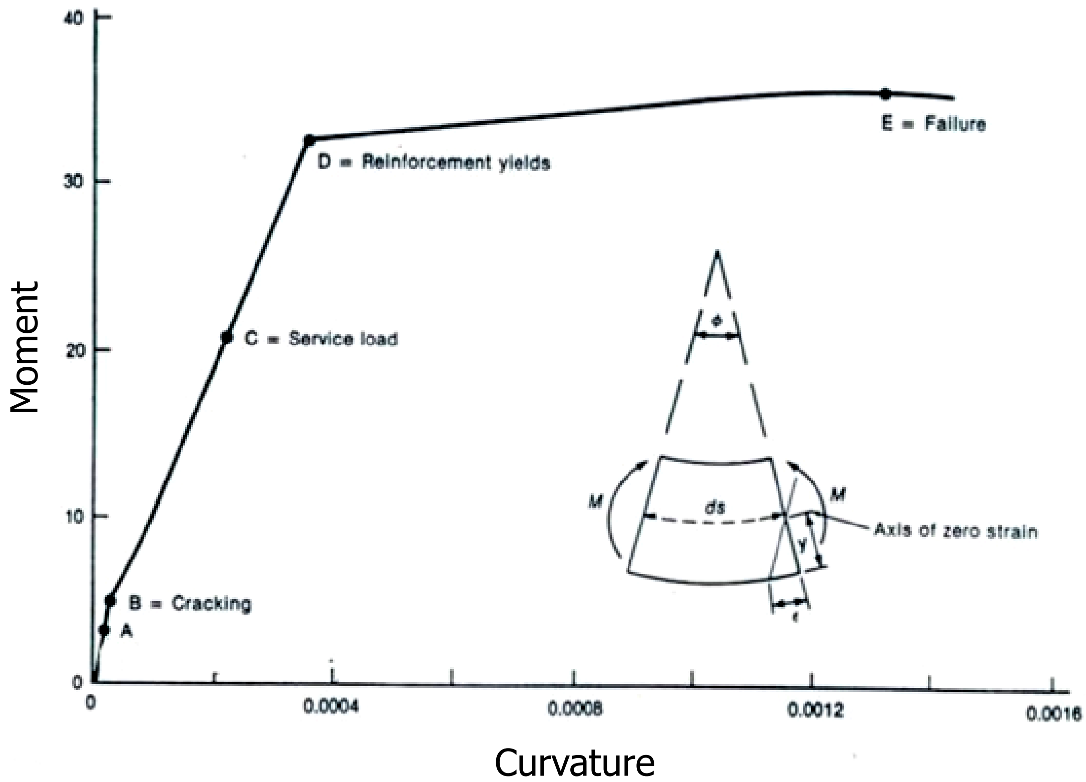
Behavior of RC beam under increasing load



3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Behavior of RC beam under increasing load

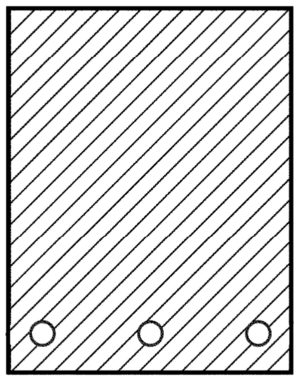


$$\phi = \frac{\varepsilon}{y} = \frac{M}{EI}$$

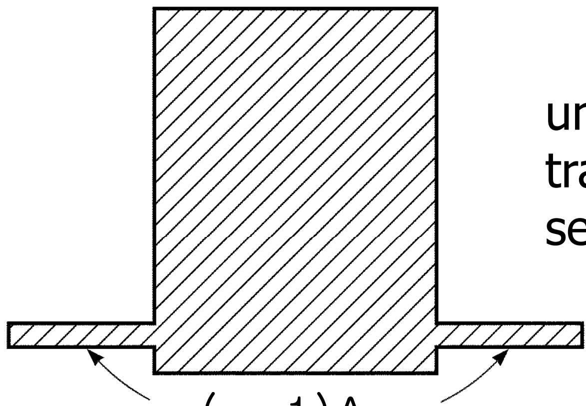
REINFORCED CONCRETE BEAM BEHAVIOR

Elastic uncracked Section

not homogeneous



homogeneous



uncracked transformed section

In elastic range

$$\varepsilon_c = \frac{f_c}{E_c} = \varepsilon_s = \frac{f_s}{E_s} \Rightarrow f_s = \frac{E_s}{E_c} f_c = n f_c$$

3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Example 3.1 (SI unit)

A rectangular beam

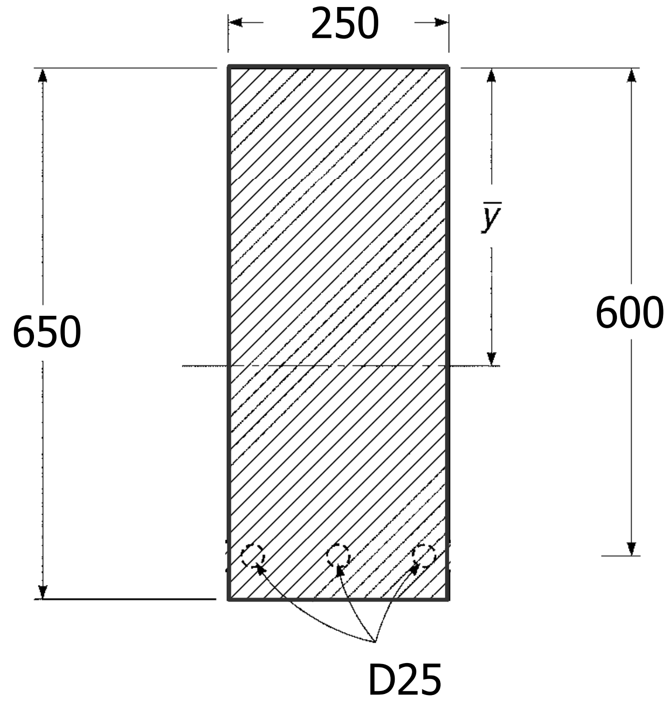
A_s = 1,520 mm^2

f_{cu} = 27 MPa (cylinder strength)

f_r = 3.5 MPa (modulus of rupture)

f_y = 400 MPa

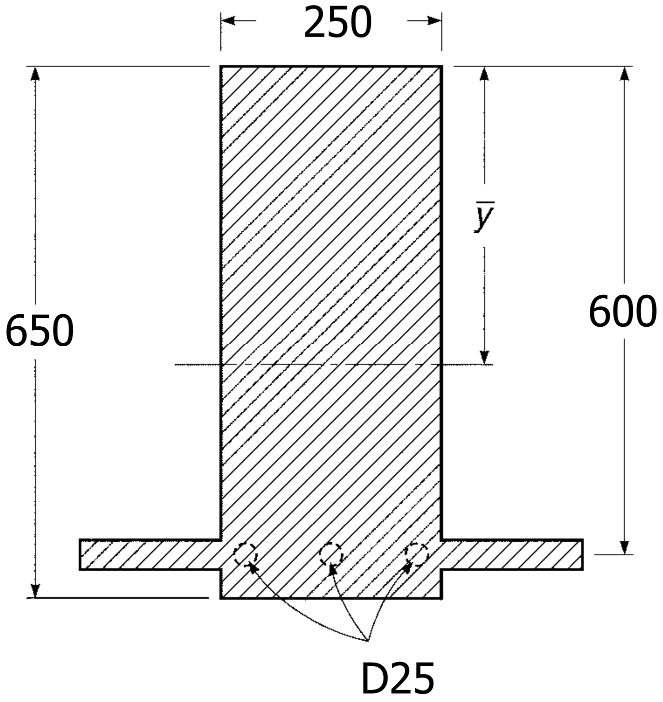
Calculate the stresses caused by a bending moment M = 60 kN·m



(unit: mm)

REINFORCED CONCRETE BEAM BEHAVIOR

Solution



transformed section

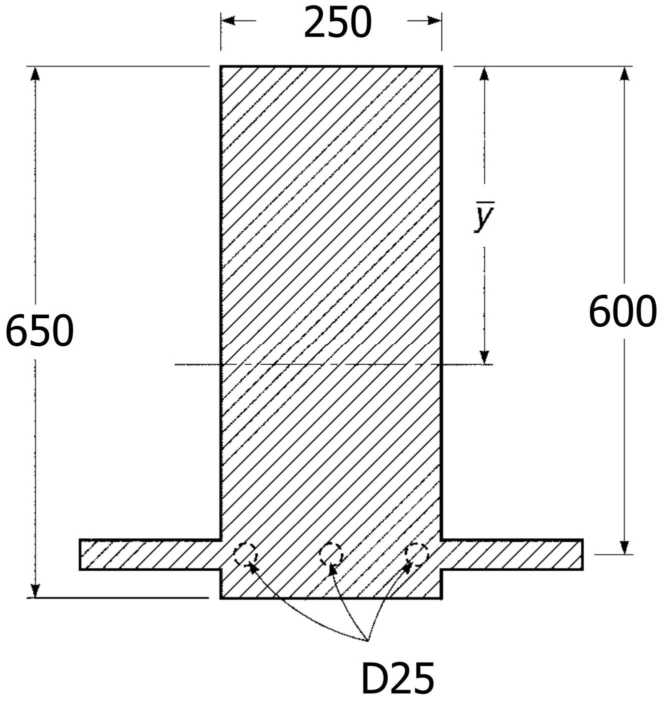
$$n = \frac{E_s}{E_c} = \frac{2.0\times10^5}{8,500\sqrt[3]{f_{cu}}} = 7.84 \approx 8$$

$$\underline{(n-1)A_s} = 7\times1,520 = 10,640\ mm^2$$

transformed area of rebars

REINFORCED CONCRETE BEAM BEHAVIOR

Solution



transformed section

Assuming the uncracked section,
neutral axis

$$\bar{y} = \frac{Q_x}{A} = \frac{\int y dA}{\int dA} = \frac{\frac{bh^2}{2} + (n-1)A_s d}{bh + (n-1)A_s}$$

$$= \underline{342\ mm}$$

$$I_x = \int y^2 dA = \frac{bh^3}{12} + \left(\frac{h}{2} - \bar{y}\right) bh$$

$$+ (d - \bar{y})^2 (n-1) A_s$$

$$= \underline{6,477 \times 10^6\ mm^4}$$

REINFORCED CONCRETE BEAM BEHAVIOR

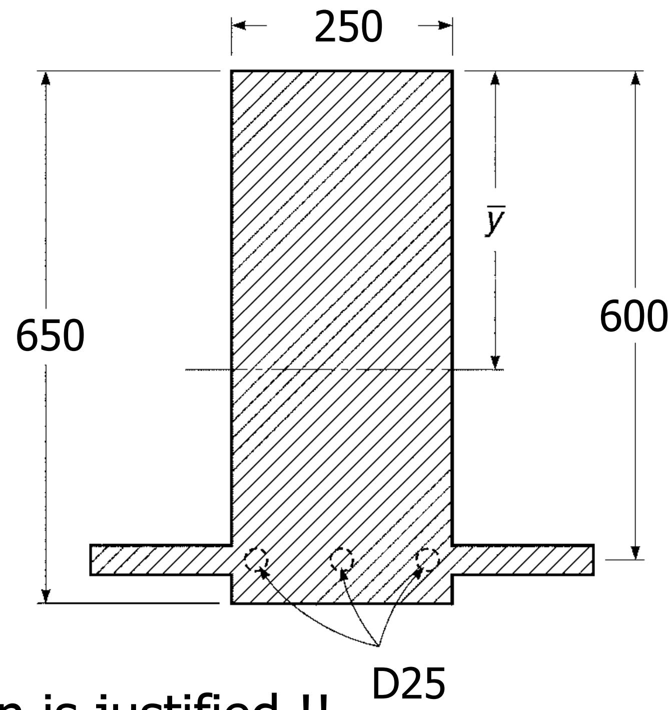
Solution

Compressive stress of concrete at the top fiber

$$f_c = \frac{M}{I_x}\bar{y} = 3.17\ MPa$$

Tension stress of concrete at the bottom fiber

$$f_{ct} = \frac{M}{I_x}(h - \bar{y}) = 2.85\ MPa \quad < \quad f_r$$



Assumption of uncracked, transformed section is justified !!

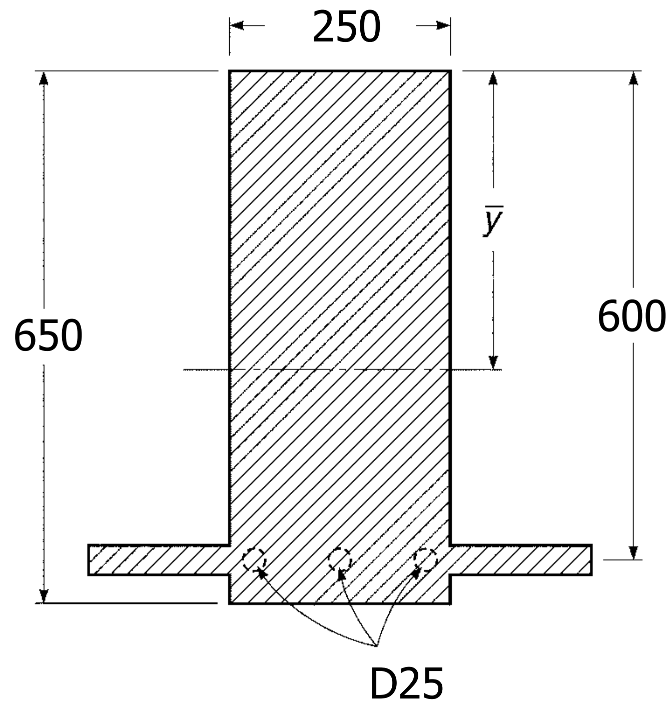
3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Solution

Stress in the tensile steel

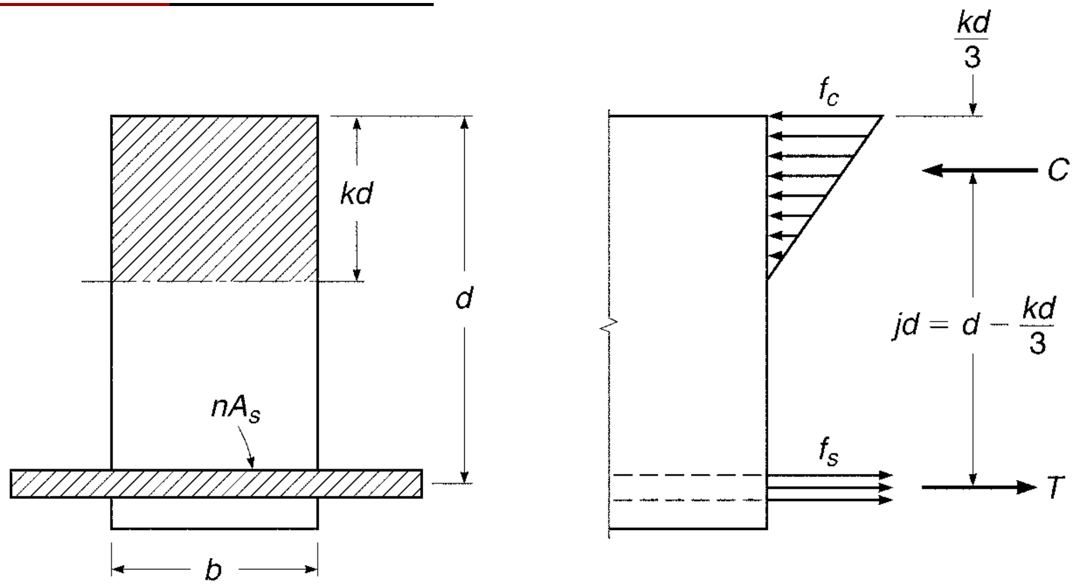
$$f_s = n\frac{M}{I_x}(d-\bar{y}) = 19.12\ MPa$$



Compare f_c and f_s with f_{cu} and f_y respectively!!!

REINFORCED CONCRETE BEAM BEHAVIOR

Elastic Cracked Section



This situation is under *service load state*

- $f_{ct} > f_r$
- $f_c < \frac{1}{2} f_{ck}$
- $f_s < f_y$

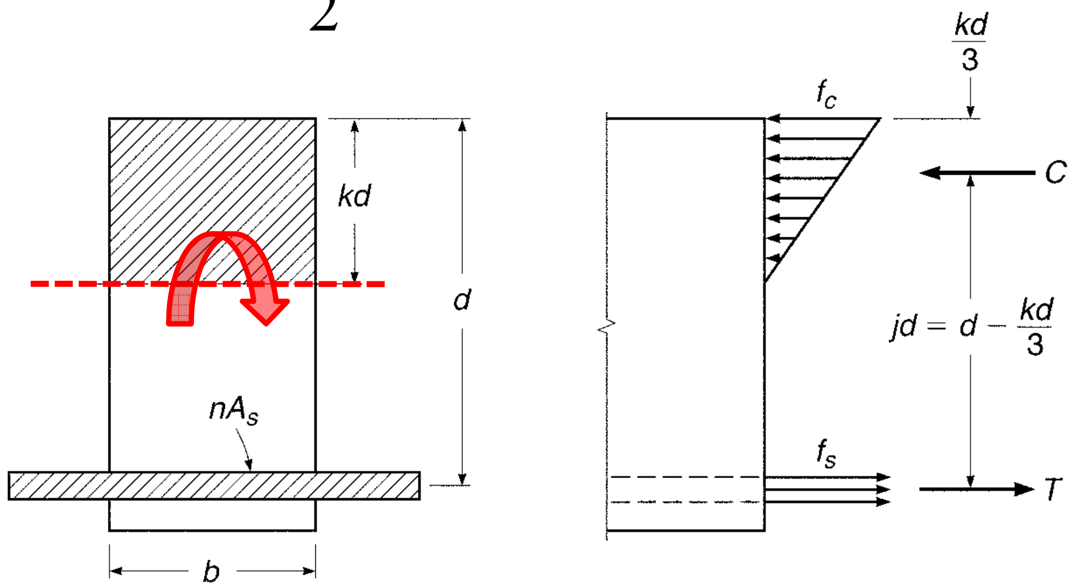
3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Elastic Cracked Section

To determine neutral axis

$$b(kd)\frac{kd}{2} - nA_s(d - kd) = 0 \qquad (1)$$

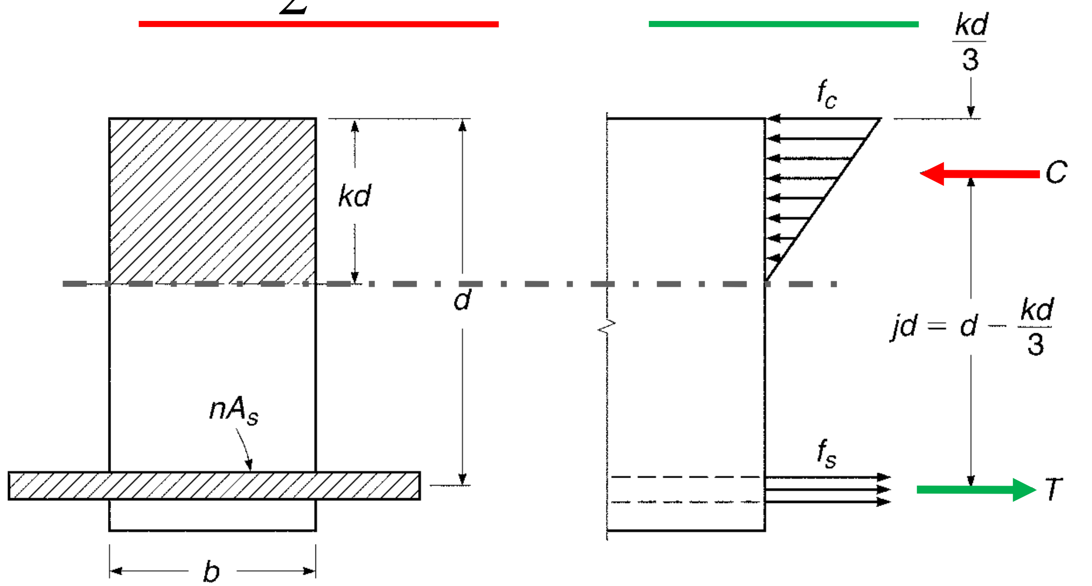


REINFORCED CONCRETE BEAM BEHAVIOR

Elastic Cracked Section

Tension & Comp. force

$$C = \frac{f_c}{2} b(kd) \qquad T = A_s f_s \qquad (2)$$



REINFORCED CONCRETE BEAM BEHAVIOR

Elastic Cracked Section

Bending moment about C

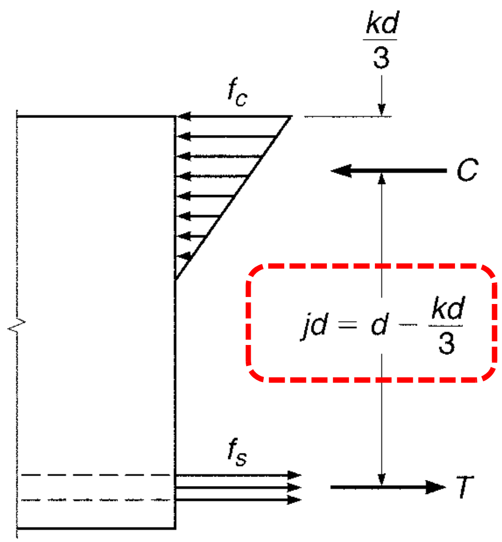
$$M = T(jd) = A_s f_s (jd) \quad (3)$$

$$\Rightarrow \quad f_s = \frac{M}{A_s (jd)} \quad (4)$$

Bending moment about T

$$M = C(jd) = \frac{f_c}{2} b(kd)(jd) \quad (5)$$

$$\Rightarrow \quad f_c = \frac{2M}{kjbd^2} \quad (6)$$



How to get *k* and *j* ?

REINFORCED CONCRETE BEAM BEHAVIOR

Elastic Cracked Section

Defining $\rho = \dfrac{A_s}{bd}$ *Then,* $A_s = \rho bd$ (7)

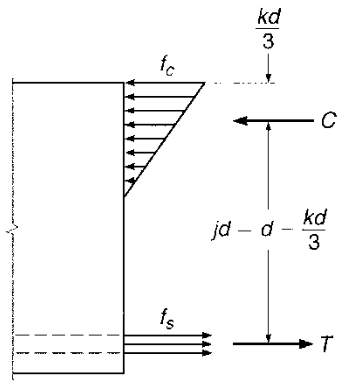
Substitute (7) into (1) and solve for k $b(kd)\dfrac{kd}{2} - nA_s(d - kd) = 0$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n \quad (8)$$

cf.) $jd = d - kd/3$

⇨ $j = 1 - \dfrac{k}{3}$

See Handout #3-3
Table A.6



REINFORCED CONCRETE BEAM BEHAVIOR

Example 3.2 (Quiz)

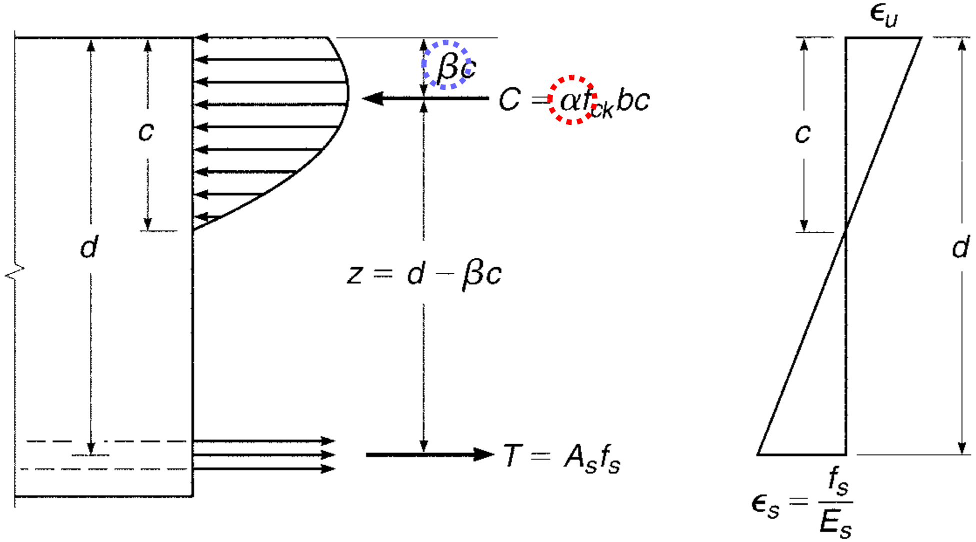
The beam of Example 3.1 is subjected to a bending moment M=120 kN·m (rather than 60 kN·m as previously).

Calculate the relevant properties and stress right away!!

3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

<u>Flexural Strength</u>



$$\alpha = \frac{f_{av}}{f_{ck}} \qquad (9)$$

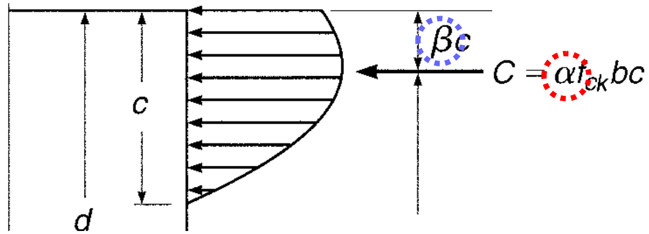
f_{av} = ave. compressive stress on the area bc

$$C = \alpha f_{ck} bc \qquad (10)$$

3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Flexural Strength



α

0.72	$f_{ck} \le 28$ MPa
decrease by 0.04 for every 7 MPa	28 MPa $\le f_{ck} \le 56$ MPa
0.56	$f_{ck} > 56$ MPa

β

0.425	$f_{ck} \le 28$ MPa
decrease by 0.025 for every 7 MPa	28 MPa $\le f_{ck} \le 56$ MPa
0.325	$f_{ck} > 56$ MPa

3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Flexural Strength

This values apply to compression zone with other cross sectional shapes (circular, triangular, etc)

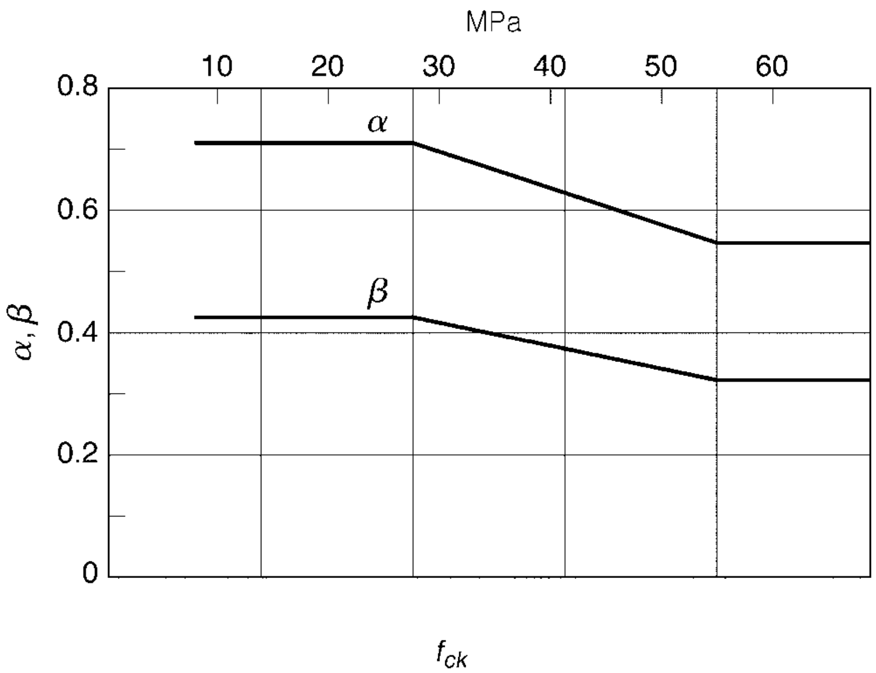
However, the analysis of those shapes becomes complex.

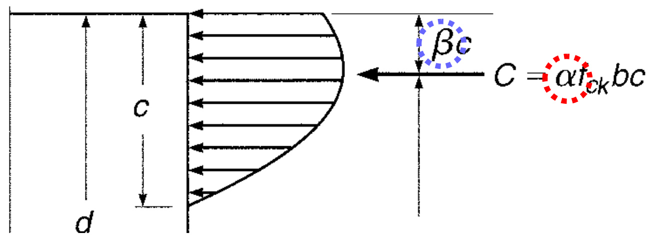
Note that to compute the flexural strength of the section, it is *not* necessary to know exact shape of the compression stress block. Only need to know C and its location.

These two quantities are expressed in α and β .

REINFORCED CONCRETE BEAM BEHAVIOR

Flexural Strength





⇦ The higher compressive strength, the more brittle.

3. Flexural Analysis/Design of Beam

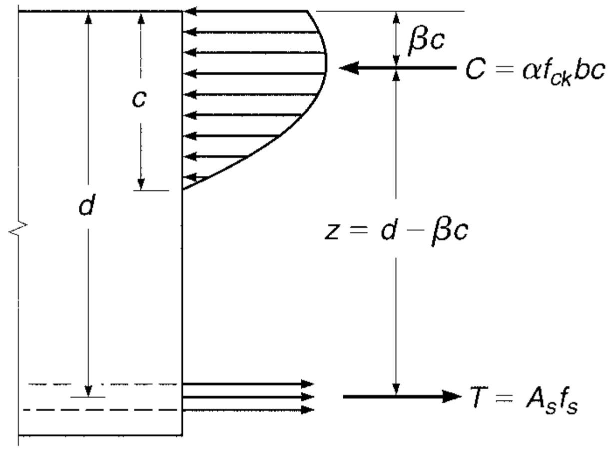
REINFORCED CONCRETE BEAM BEHAVIOR

Flexural Strength

Tension failure (ε_u<0.003, $f_s=f_y$)

Equilibrium

$$C = T \qquad \alpha f_{ck} bc = A_s f_s \qquad (11)$$



Bending moment

$$M = Tz = A_s f_s (d - \beta c) \qquad (12)$$

or

$$M = Cz = \alpha f_{ck} bc(d - \beta c) \qquad (13)$$

3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Tension failure (ε_u<0.003, $f_s=f_y$)

Neutral axis at steel yielding, $f_s = f_y$

From Eq.(11) $$c = \frac{A_s f_y}{\alpha f_{ck} b} = \frac{\rho f_y d}{\alpha f_{ck}} \qquad (14)$$

Nominal bending moment

$$M_n = A_s f_y \left(d - \beta c\right) = \rho b d f_y \left(d - \beta \frac{\rho f_y d}{\alpha f_{ck}}\right)$$

$$= \rho f_y b d^2 \left(1 - \beta \frac{\rho f_y}{\alpha f_{ck}}\right) = \underline{\rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_{ck}}\right)} \qquad (15)$$

3. Flexural Analysis / Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Flexural Strength

Compression failure (ε_u=0.003, $f_s < f_y$)

Hook's law

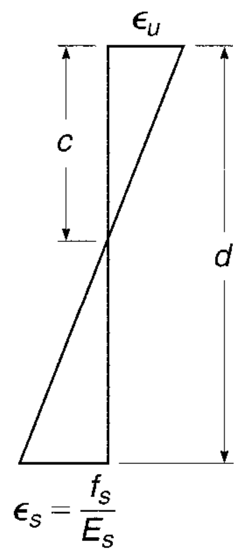
$$f_s = \varepsilon_s E_s \tag{16}$$

from strain diagram

$$f_s = \varepsilon_u E_s \frac{d-c}{c} \tag{17}$$

Equilibrium

$$\alpha f_{ck} bc = \underline{A_s f_s} = A_s \varepsilon_u E_s \frac{d-c}{c} \tag{18}$$



REINFORCED CONCRETE BEAM BEHAVIOR

Compression failure (ε_u=0.003, $f_s < f_y$)

Solving the quadratic for c

$$\alpha f_{ck} bc^2 + A_s \varepsilon_u E_s c - A_s \varepsilon_u E_s d = 0 \tag{19}$$

$$c = \tag{20}$$

$$f_s = \tag{21}$$

Nominal bending moment

$$M_n = \tag{22}$$

3. Flexural Analysis/Design of Beam

REINFORCED CONCRETE BEAM BEHAVIOR

Flexural Strength

Balanced reinforcement ratio ρ_b

The amount of reinforcement necessary for beam fail to by crushing of concrete at the same load causing the steel to yield; (ε_u=0.003, $f_s = f_y$)

- $\rho < \rho_b$ lightly reinforced, tension failure, ductile
- $\rho = \rho_b$ balanced, tension/comp. failure
- $\rho > \rho_b$ heavily reinforced, compression failure, brittle

REINFORCED CONCRETE BEAM BEHAVIOR

Balanced reinforcement ratio ρ_b

Balanced condition

$$f_s = f_y \qquad \varepsilon_y = \frac{f_y}{E_s} \tag{23}$$

Substitute Eq. (23) into Eq.(17) $\quad f_s = \varepsilon_u E_s \frac{d-c}{c}$

$$c = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} d \tag{24}$$

Substitute Eq. (24) into Eq.(11) $\quad \alpha f_{ck} bc = A_s f_s$

$$\rho_b = \frac{\alpha f_{ck}}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \tag{25}$$

REINFORCED CONCRETE BEAM BEHAVIOR

Example 3.3 (SI unit)

A rectangular beam

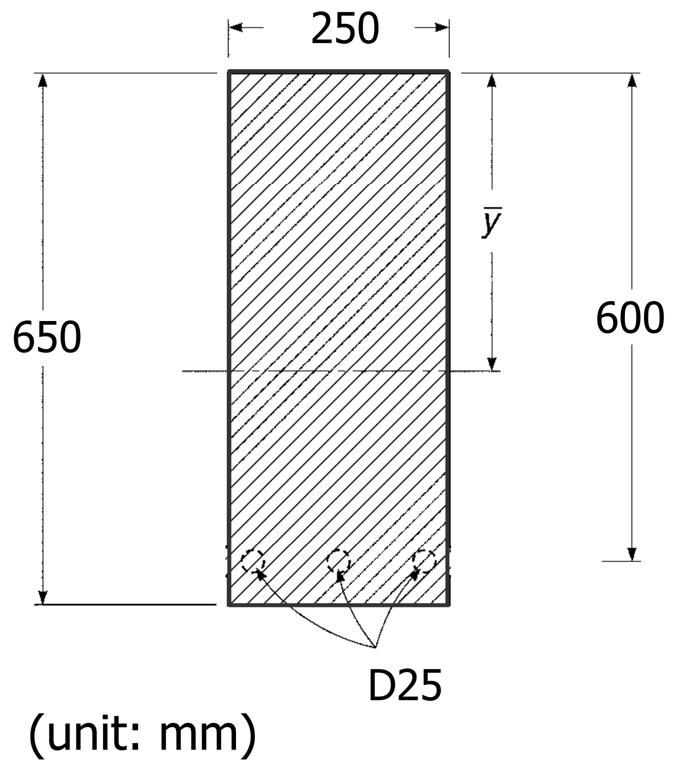
A_s = 1,520 mm^2

f_{cu} = 27 MPa (cylinder strength)

f_r = 3.5 MPa (modulus of rupture)

f_y = 400 MPa

Calculate the nominal moment M_n at which the beam will fail.



REINFORCED CONCRETE BEAM BEHAVIOR

Solution

Check whether this beam fail in tension or compression

$$\rho = \frac{A_s}{bd} = \frac{1,520}{(250)(600)} = 0.0101$$

$$\rho_b = \frac{\alpha f_{ck}}{f_y}\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = \frac{(0.72)(27)}{400}\frac{0.003}{0.003+0.002} = 0.0292 > \rho$$

⇨ The beam will fail in tension by yielding of the steel

REINFORCED CONCRETE BEAM BEHAVIOR

Solution

Using Eq. (15) for tension failure

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_{ck}}\right)$$

$$= (0.0101)(400)(250)(600)^2 \left(1 - 0.59 \frac{(0.0101)(400)}{27}\right)$$

$$= 332\ kN \cdot m$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Korea's design method is Ultimate Strength Design.
called as Limit States Design in the US and Europe

1. Proportioning for adequate strength

2. Checking the serviceability
   :deflections/crack width compared against limiting values

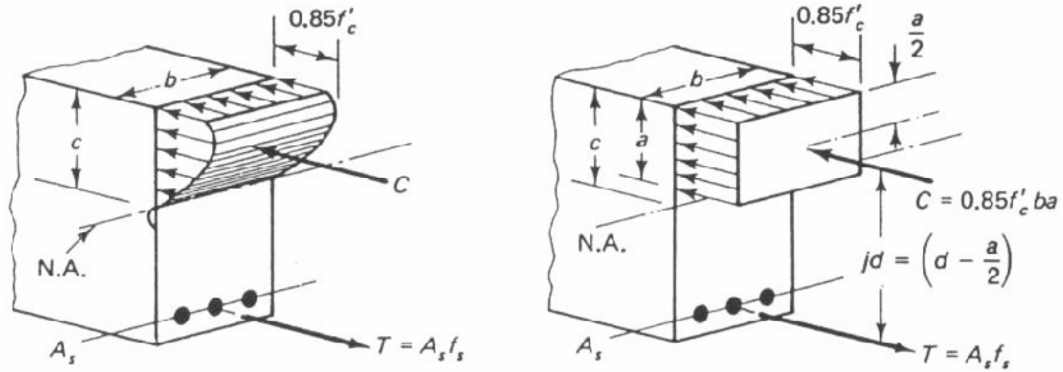
DESIGN OF TENSION REINFORCED REC. BEAMS

Equivalent Rectangular Stress Distribution

is called as Whitney's Block (Handout #3-1)

What if the actual stress block is replaced by an equivalent rectangular stress block for compression zone.

Actual



Equivalent

3. Flexural Analysis / Design of Beam

DESIGN OF TENSION REINFORCED REC. BEAMS

Equivalent Rectangular Stress Distribution

Go to P.25

$$C_{actual} = \alpha f_{ck} bc = \gamma f_{ck} ab = C_{equi.}$$

$$\Rightarrow \quad \gamma = \frac{\alpha c}{a}$$

$$\Leftarrow \quad a = \beta_1 c$$

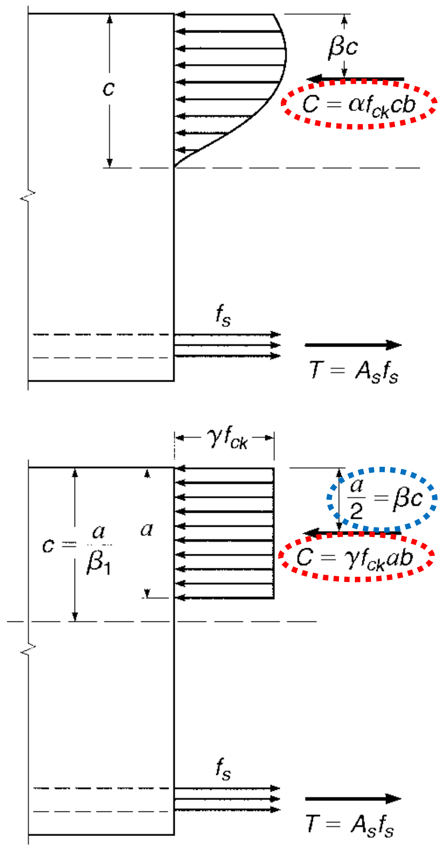
$$\gamma = \frac{\alpha}{\beta_1}$$

$$\frac{a}{2} = \beta c$$

$$\Leftarrow \quad a = \beta_1 c$$

$$\beta_1 = 2\beta$$

γ depends on a, β



3. Flexural Analysis/Design of Beam

DESIGN OF TENSION REINFORCED REC. BEAMS

Equivalent Rectangular Stress Distribution

	f_{ck}, MPa				
	< 28	35	42	49	56 ≤
α	0.72	0.68	0.64	0.60	0.56
β	0.425	0.400	0.375	0.350	0.325
$\beta_1 = 2\beta$	0.85	0.801	0.752	0.703	0.654
$\gamma = \alpha / \beta_1$	0.857	0.849	0.851	0.853	0.856

- γ is essentially independent of f_{ck}.
- $\beta_1 = 0.85 - 0.007 \times (f_{ck} - 28)$ and $0.65 \le \beta_1 \le 0.85$

DESIGN OF TENSION REINFORCED REC. BEAMS

Equivalent Rectangular Stress Distribution

- KCI 6.2.1(6) allows other shapes for the concrete stress block to be used in the calculations as long as they result in good agreement with test results.
- KCI6.2.1(5) makes a further simplification. Tensile stresses in the concrete may be neglected in the cals. ⇦ Contribution of tensile stresses of the concrete below N.A. is very small.

"Concrete Stress Distribution in Ultimate Strength Design"

Handout #3-2 *by Hognestad*

DESIGN OF TENSION REINFORCED REC. BEAMS

Balanced Strain Condition

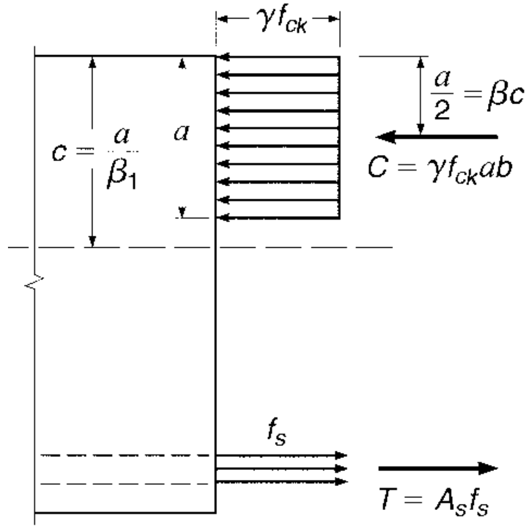
steel strain is exactly equal to ε_y and
concrete simultaneously reaches ε_u =0.003

Eq. (24)

$$c = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} d$$

Equilibrium C=T

$$A_s f_y = \rho_b bd f_y = 0.85 f_{ck} ab = 0.85 f_{ck} \beta_1 cb$$



DESIGN OF TENSION REINFORCED REC. BEAMS

Balanced Strain Condition

Balanced reinforcement ratio

$$\rho_b = 0.85\beta_1 \frac{f_{ck}c}{f_y d} = 0.85\beta_1 \frac{f_{ck}}{f_y}\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \qquad (26)$$

Apply ε_u =0.003 and E_s=200,000 MPa

$$\rho_b = 0.85\beta_1 \frac{f_{ck}}{f_y}\frac{0.003}{0.003 + \frac{f_y}{E_s}} = 0.85\beta_1 \frac{f_{ck}}{f_y}\frac{600}{600 + f_y} \qquad (27)$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Underreinforced Beams

In actual practice, ρ should be below ρ_b for the reasons,

1. Exactly $\rho = \rho_b$, then concrete reaches the comp. strain limit and steel reaches its yield stress.
2. Material properties are never known precisely.
3. Strain hardening can cause compressive failure, although ρ may be somewhat less than ρ_b.
4. The actual steel area provided, will always be equal to or larger than required based on ρ.
5. The extra ductility provided by low ρ provides warning prior to failure.

DESIGN OF TENSION REINFORCED REC. BEAMS

KCI Code Provisions for Underreinforced Beam

By the way, how to guarantee underreinforced beams?

⇨ KCI Code provides,

(1) The minimum tensile reinforcement strain allowed at nominal strength in the design of beam.

(2) Strength reduction factors that may depend on the tensile strain at nominal strength.

Note Both limitations are based on the net tensile strain ε_t of the rebar farthest from the compression face at the depth d_t. ($d \leq d_t$)

DESIGN OF TENSION REINFORCED REC. BEAMS

KCI Code Provisions for Underreinforced Beam

(1) For nonprestressed flexural members and members with factored axial compressive load less than $0.1f_{ck}A_g$, ε_t shall not be less than 0.004 (KCI 6.2.2(5))

Substitute d_t for d and ε_t for ε_y

$$c = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} d \quad \Rightarrow \quad \varepsilon_t = \varepsilon_u \frac{d_t - c}{c}$$

$$\rho_b = 0.85\beta_1 \frac{f_{ck}}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \quad \Rightarrow \quad \rho = 0.85\beta_1 \frac{f_{ck}}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$

The reinforcement ratio to produce a selected ε_t

3. Flexural Analysis/Design of Beam

DESIGN OF TENSION REINFORCED REC. BEAMS

KCI Code Provisions for Underreinforced Beam

Maximum reinforcement ratio (KCI 2007)

$$\rho_{\max} = 0.85\beta_1 \frac{f_{ck}}{f_y}\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} = 0.85\beta_1 \frac{f_{ck}}{f_y}\frac{\varepsilon_u}{\varepsilon_u + 0.004} \quad (29)$$

Cf.) (prior to KCI 2007 & ACI 2002)

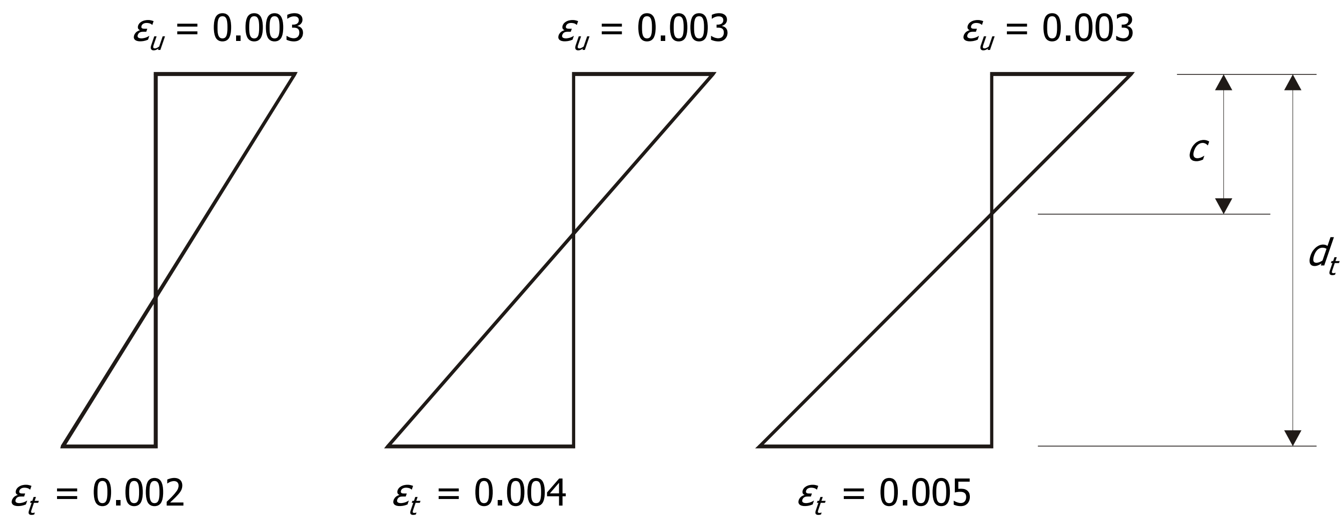
$$\rho_{\max} = 0.75\rho_b \quad (28)$$

⇨ ε_t = 0.00376 at ρ= 0.75ρ_b for f_y=400 MPa.

⇨ 0.00376 < 0.004 ,i.e., KCI 2007 is slightly conservative.

DESIGN OF TENSION REINFORCED REC. BEAMS

KCI Code Provisions for Underreinforced Beam



$$\frac{c}{d_t} = \frac{0.003}{0.003 + 0.002} = 0.600$$

$$\frac{c}{d_t} = \frac{0.003}{0.003 + 0.004} = 0.429$$

$$\frac{c}{d_t} = \frac{0.003}{0.003 + 0.005} = 0.375$$

Compression controlled member

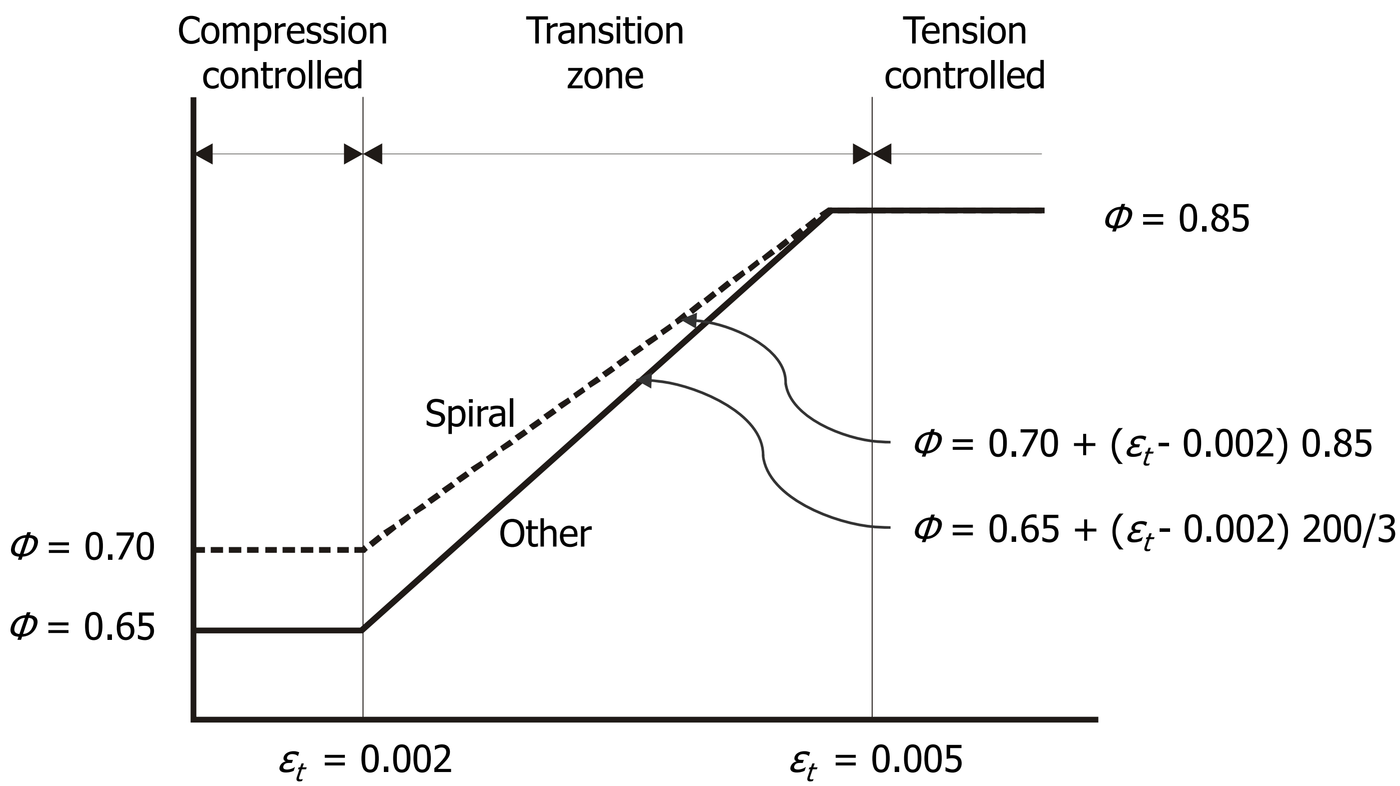
Minimum net strain for flexural member

Tension controlled member

DESIGN OF TENSION REINFORCED REC. BEAMS

KCI Code Provisions for Underreinforced Beam

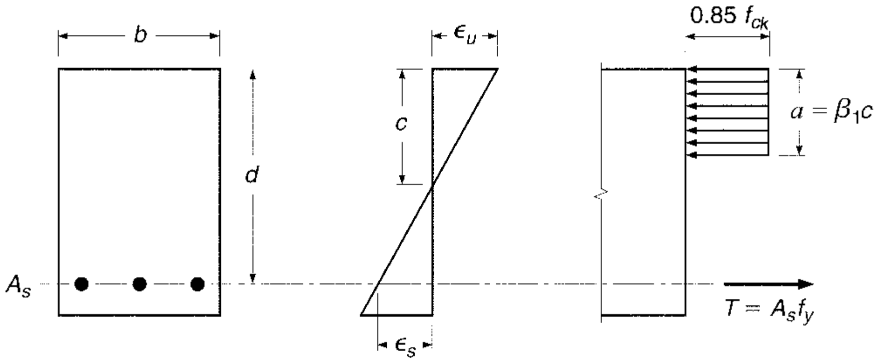
(2)



DESIGN OF TENSION REINFORCED REC. BEAMS

KCI Code Provisions for Underreinforced Beam

Nominal flexural strength



$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \quad (30)$$

$$a = \frac{A_s f_y}{0.85 f_{ck} b} \quad (31)$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Example 3.4 (the same as Ex.3.3)

A rectangular beam

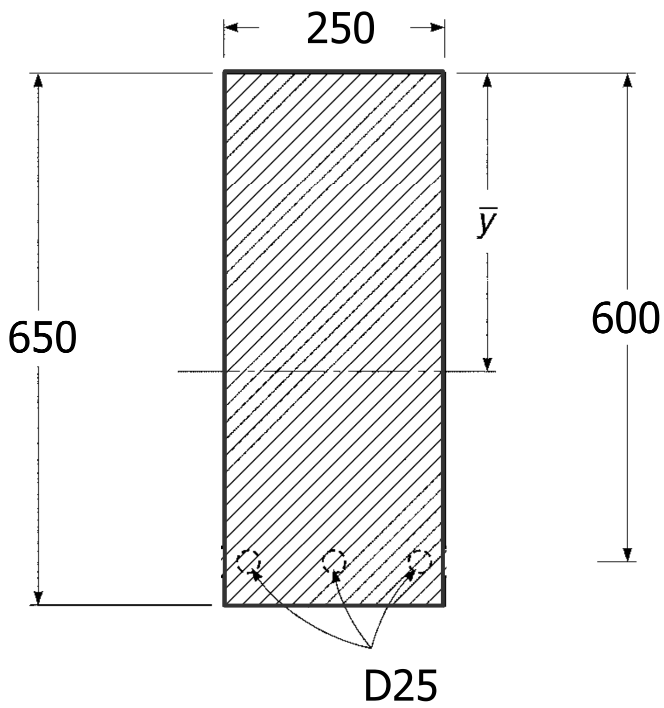
A_s = 1,520 mm^2

f_{cu} = 27 MPa (cylinder strength)

f_r = 3.5 MPa (modulus of rupture)

f_y = 400 MPa

Calculate the nominal strength M_n *using the equivalent stress block*.



(unit: mm)

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

Maximum reinforcement ratio

$$\rho_{\max} = 0.85\beta_1 \frac{f_{ck}}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004}$$

$$= (0.85)\underline{(0.85)} \frac{(27)}{(400)} \frac{0.003}{0.003 + 0.004} = 0.0209 > 0.0101 = \frac{A_s}{bd}$$

⇨ This beam is underreinforced (tension controlled) and will fail yielding of the steel

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

Depth of stress block

$$a = \frac{A_s f_y}{0.85 f_{ck} b} = \frac{(1520)(400)}{(0.85)(27)(250)} = 106\ mm$$

Nominal Strength

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) = (1520)(400)\left(600 - \frac{106}{2} \right) = 333\ kN \cdot m$$

Compare this with Example 3.3 !!!

3. Flexural Analysis / Design of Beam

DESIGN OF TENSION REINFORCED REC. BEAMS

Nominal flexural strength (Alternative)

Eq(31) can be written w.r.t. ρ

$$a = \frac{\rho f_y d}{0.85 f_{ck}} \tag{32}$$

Nominal flexural strength

$$M_n = \underbrace{(\rho b d)}_{A_s} f_y \left(d - \underbrace{\frac{\rho f_y d}{1.7 f_{ck}}}_{a/2} \right) = \underline{\rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_{ck}} \right)} \tag{33}$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Nominal flexural strength (Alternative)

simplified expression of Eq.(33)

$$M_n = Rbd^2 \tag{34}$$

where,

$$R = \rho f_y (1 - 0.59 \frac{\rho f_y}{f_{ck}}) \tag{35}$$

flexural resistance factor R depends on

1. reinforcement ratio
2. material properties (Handout #3-3)

DESIGN OF TENSION REINFORCED REC. BEAMS

Design flexural strength

KCI Code Provisions

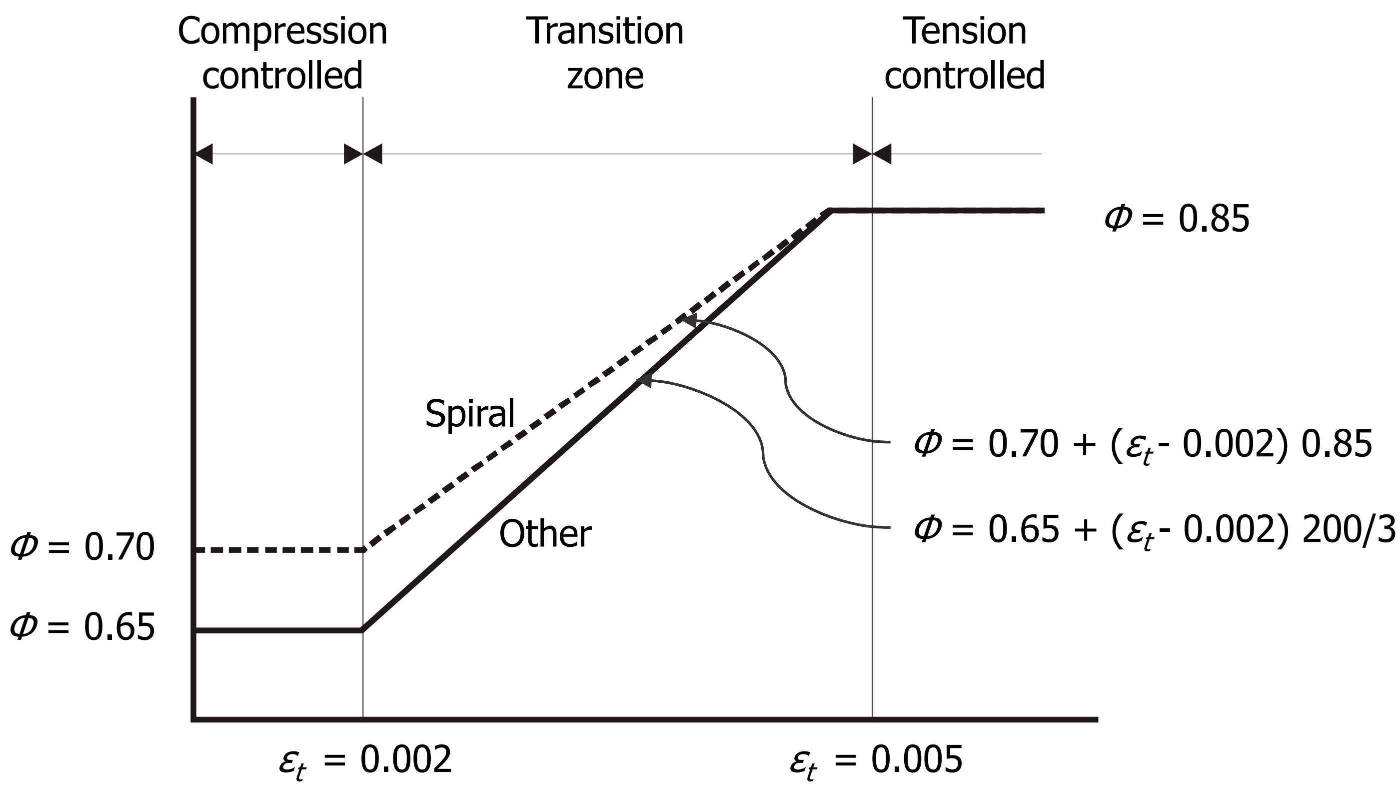
$$\underline{\phi M_n} = \phi A_s f_y (d - \frac{a}{2})$$

$$= \phi \rho f_y b d^2 (1 - 0.59 \frac{\rho f_y}{f_{ck}}) \quad (36)$$

$$= \phi R b d^2 \quad (37)$$

DESIGN OF TENSION REINFORCED REC. BEAMS

KCI Code Provisions for Underreinforced Beam



DESIGN OF TENSION REINFORCED REC. BEAMS

Example 3.4 (continued)

Calculate the design moment capacity for the beam analyzed in Example 3.4

Hint net tensile strain should be known.

3. Flexural Analysis/Design of Beam

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

$$c = \frac{a}{\beta_1} = \frac{106}{0.85} = 125\ mm$$

⇨ $$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} = 0.003 \frac{600 - 125}{125} = 0.0114 > 0.005$$

⇨ strength reduction factor is 0.85!!!

Design strength $\phi M_n = (0.85)(333) = 283\ kN \cdot m$

DESIGN OF TENSION REINFORCED REC. BEAMS

Minimum Reinforcement Ratio

; very lightly reinforced beams will also fail without warning. so, *lower limit* is required.

Rectangular cross section

$$A_{s.\min} = \frac{0.15\sqrt{f_{ck}}}{f_y} bd \quad (38)$$

Proof

Equating the cracking moment to the flexural strength, based under the assumptions, h=1.1d and internal lever arm = 0.95d

DESIGN OF TENSION REINFORCED REC. BEAMS

Proof

- *cracking moment M_{cr}*

$$M_{cr} = z \cdot f_r = A_s \cdot f_y \cdot \text{(internal lever arm)}$$

$$\Rightarrow \left(\frac{bh^2}{6}\right)\left(0.63\sqrt{f_{ck}}\right) = A_s f_y \left(0.95d\right)$$

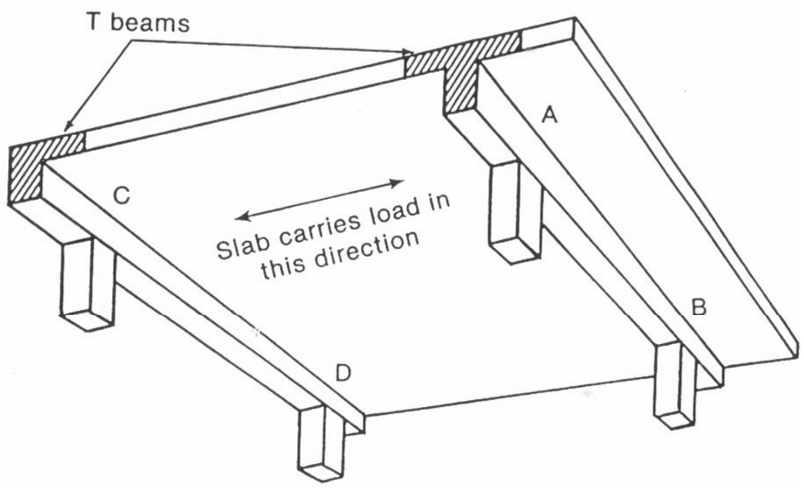
$$\Rightarrow A_{s.\min} = \frac{b(1.1d)^2(0.63\sqrt{f_{ck}})}{(0.95d)f_y(6)}$$

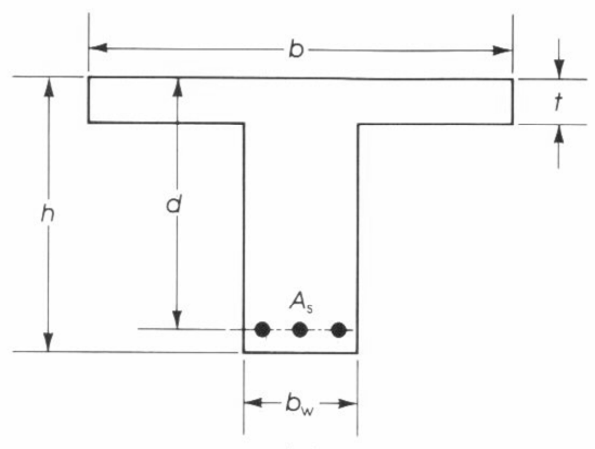
$$= \frac{0.134\sqrt{f_{ck}}bd}{f_y} \approx \underline{\frac{0.15\sqrt{f_{ck}}bd}{f_y}}$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Minimum Reinforcement Ratio

Similarly, T cross secton





(a)

DESIGN OF TENSION REINFORCED REC. BEAMS

Similarly, T cross secton

Flange in compression

$$A_{s,\min} = \frac{0.22\sqrt{f_{ck}}}{f_y} b_w d \qquad (39)$$

Flange in tension

$$A_{s,\min} = \frac{0.50\sqrt{f_{ck}}}{f_y} b_w d \qquad (40)$$

DESIGN OF TENSION REINFORCED REC. BEAMS

KCI Code provisions (6.3.2)

$$A_{s,\min} = \frac{0.25\sqrt{f_{ck}}}{f_y} b_w d \quad \geq \quad \frac{1.4}{f_y} b_w d \tag{41}$$

,where $b_w = b$, if rectangular cross section.

Exception (KCI 6.3.2 (2)) ~statically determinate T beam with a flange in tension, Eq.(41) replaced b_w by either b (effective flange width) or $2b_w$, whichever is smaller.

DESIGN OF TENSION REINFORCED REC. BEAMS

KCI Code provisions (6.3.2)

cf.) can be expressend w.r.t. reinforcement ratio

$$\rho_{\min} = \frac{0.25\sqrt{f_{ck}}}{f_y} \geq \frac{1.4}{f_y}$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Example 3.5 (analysis problem)

Rectangular beam b=300 mm, d=440 mm
Reinforced with four D29 in a row.
f_y=400 MPa, f_{ck}=27 MPa

What is the maximum moment that will be utilized in design, according to the KCI code?

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

From Table A.2 of Handout #3-3 $A_s = 2,570\ mm^2$

$$\rho = 2,570 /(300)(440) = 0.0195$$

$$\rho_{\max} = 0.85\beta_1 \frac{f_{ck}}{f_y}\frac{\varepsilon_u}{\varepsilon_u + 0.004}$$

$$= (0.85)(0.85)\frac{27}{400}\frac{0.003}{0.003+0.004} = 0.0209 > \rho$$

⇨ This beam will fail by tensile yielding

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

Nominal strength for this underreinforced beam

$$a = \frac{A_s f_y}{0.85 f_{ck} b} = \frac{(2,570)(440)}{(0.85)(27)(300)} = 149\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) = (2,570)(400)\left(440 - \frac{149}{2} \right) = 376\ kN \cdot m$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

Strength reduction factor φ?

$$c = \frac{a}{\beta_1} = \frac{149}{0.85} = 175\ mm$$

⇨ $$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} = 0.003 \frac{440 - 175}{175} = 0.00454 < 0.005$$

⇨ strength reduction factor is *not* 0.85!!!

⇨ $$\phi = 0.65 + (\varepsilon_t - 0.002)\frac{200}{3} = \underline{0.82}$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

Design strength

$$\phi M_n = (0.82)(376) = 308\ kN \cdot m$$

Check minimum reinforcement ratio

$$\rho_{\min} = \frac{0.25\sqrt{f_{ck}}}{f_y} \geq \frac{1.4}{f_y}$$

$$= \frac{0.25\sqrt{27}}{400} \geq \frac{1.4}{400} = 0.0035 < 0.0195 = \rho$$

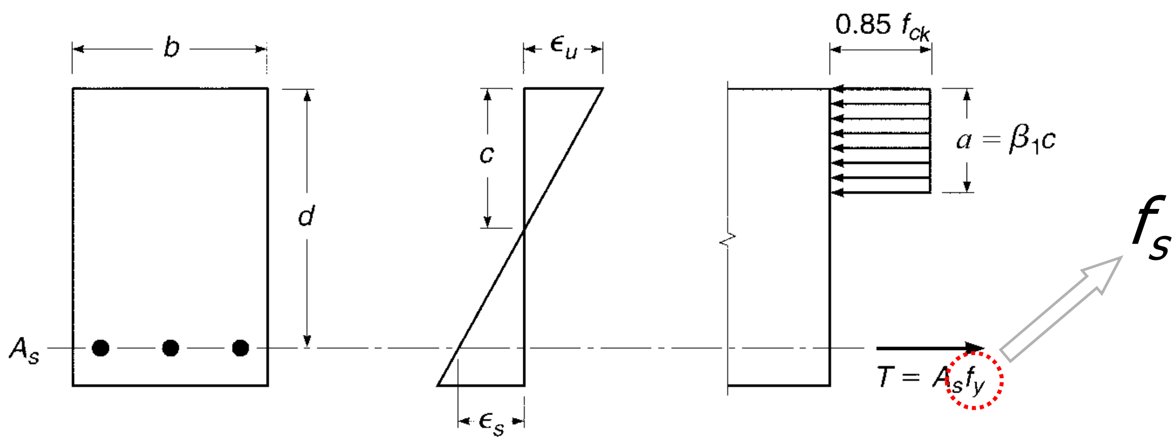
3. Flexural Analysis/Design of Beam

DESIGN OF TENSION REINFORCED REC. BEAMS

Overreinforced Beams

Occasionally it is necessary to calculate the flexural strength of an overreinforced ($f_s < f_y$) beams.

Such as, analysis of existing structures.



DESIGN OF TENSION REINFORCED REC. BEAMS

Overreinforced Beams

steel strain

$$\varepsilon_s = \varepsilon_u \frac{d-c}{c} \tag{42}$$

equilibrium

$$0.85\beta_1 f_{ck} bc = A_s \cdot f_s = \rho bd \cdot \varepsilon_s E_s$$

substitute Eq.(42) into above Eq. and define k_u=c/d

$$k_u^{\ 2} + m\rho k_u - m\rho = 0$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Overreinforced Beams

where, $\rho = A_s / bd$ and m is material parameter

$$m = \frac{E_s \varepsilon_u}{0.85 \beta_1 f_{ck}} \tag{43}$$

solving the quadratic equation

$$k_u = \sqrt{m\rho + \left(\frac{m\rho}{2}\right)^2} - \frac{m\rho}{2}$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Overreinforced Beams

neutral axis depth c

$$c = k_u d$$

stress-block depth a

$$a = \beta_1 c$$

nominal flexural strength

$$M_n = A_s f_s (d - \frac{a}{2}) = A_s (E_s \varepsilon_s)(d - \frac{a}{2}) \quad (44)$$

3. Flexural Analysis / Design of Beam

DESIGN AIDS

In practice, AIDS is very useful for both *analysis & design*

<u>1st approach for design</u>

1. Set the required strength equal to the design strength

$$M_u = \phi M_n = \phi R b d^2$$

2. Select an appropriate reinforcement ratio between ρ_{min} and ρ_{max} from Table A.4

   Often a ratio of about $0.5\ \rho_b$ will be an economical and practical choice.

3. Flexural Analysis/Design of Beam

DESIGN AIDS

1st approach for design

3. Find the flexural resistance factor from Table A.5, then

$$bd^2 = \frac{M_u}{\phi R}$$

4. Choose *b* and often an effective depth about 2~3 times is appropriate

5. Calculate the required steel area

$$A_s = \rho bd$$

6. Choose the size and number of bars from Table A.2

DESIGN AIDS

1st approach for design

7. Check that the selected beam will provide room for the bars chosen, with adequate concrete cover and spacing

2nd approach for design

1. Select b and d. Then calculate the required R

$$R = \frac{M_u}{\phi b d^2}$$

3. Flexural Analysis/Design of Beam

DESIGN AIDS

<u>2nd approach for design</u>

2. Find reinforcement ratio to meet the required *R* from Table A.5.
3. Calculate the required steel area.

$$A_s = \rho bd$$

4. Select the size and number of bars from Table A.2.
5. Check that the beam width is sufficient to contain the selected reinforcement.

3. Flexural Analysis/Design of Beam

PRACTICAL CONSIDERATIONS IN DESIGN

Concrete Protection for Reinforcement

cover thickness

; thickness of concrete cover outside of the outermost steel

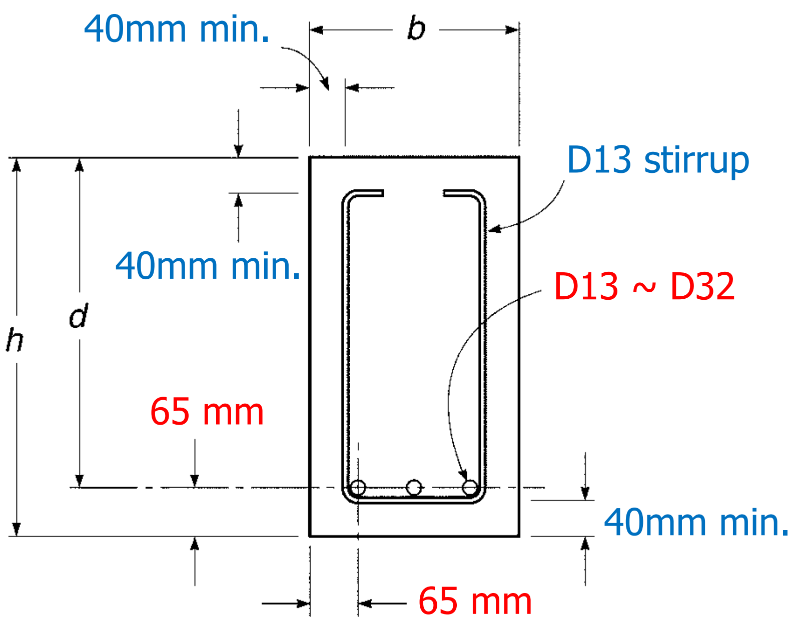
minimum concrete cover (KCI Code 5.4)

; To provide the steel with adequate concrete protection against fire and corrosion.

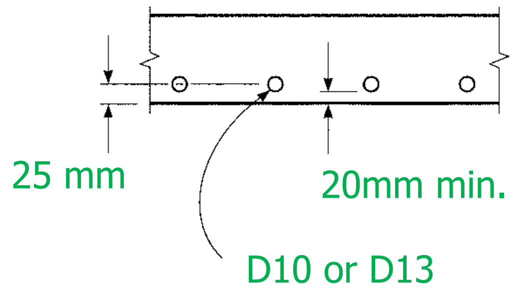
Make a hard copy of KCI Code 5.4 and attach it on the next page!!

PRACTICAL CONSIDERATIONS IN DESIGN

Concrete Protection for Reinforcement



Beam with stirrups



Slab

PRACTICAL CONSIDERATIONS IN DESIGN

Concrete Proportions

Maximum *material economy* ;

effective depth d = 2~3 times the width b

but, not always satisfy maximum structural economy

PRACTICAL CONSIDERATIONS IN DESIGN

Selection of Bars and Bar Spacing

- Often desirable to mix bar sizes to meet A_s more closely.
  ; limiting the variation in diameter of bars in a single layer
- Clear distance between adjacent bars shall not be less than the nominal bar diameter or 25mm (KCI Code 5.3.2)
- Max. number of bars in a beam of given width is limited.
  ; Table A.7 is limiting the maximum width of beam for a single layer of bars based on bar size and stirrup size.

PRACTICAL CONSIDERATIONS IN DESIGN

Selection of Bars and Bar Spacing

- The minimum number of bars in a single layer to control the flexural crack width. Table A.8
- In large girders and columns, it is sometimes advantageous to *bundle* rebars with two, three, or four bars. (KCI Code 5.3.2)

DESIGN OF TENSION REINFORCED REC. BEAMS

Example 3.6 (design problem)

Find the cross section of concrete and area of steel required for a simply supported rectangular beam.

- span = 4.5 m
- dead load = 19 kN/m
- live load = 31 kN/m
- f_{ck} = 27 MPa
- f_y = 400 MPa

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

load combination (KCI 3.3.2(1))

$$w_u = 1.2D + 1.6L = (1.2)(19) + (1.6)(31) = 72.4\,KN/m$$

$$M_u = \frac{w_u l^2}{8} = \frac{(72.4)(4.5)^2}{8} = 183.3\,KN \cdot m$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

determination of cross section

; depends on designer's choice of reinforcement ratio

; to minimize the concrete section, select the maximum permissible ρ

; to maintain φ=0.85, the maximum reinforcement ratio corresponding to a net tensile strain of 0.005 will be selected Why do not use ρ_{max} with ε_t=0.004 ?

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

1. reinforcement ratio for ε_t=0.005

$$\rho = 0.85\beta_1 \frac{f_{ck}}{f_y}\frac{\varepsilon_u}{\varepsilon_u + 0.005} = (0.85)^2 \frac{27}{400}\frac{0.003}{0.003+0.005} = 0.0183$$

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

2. Setting the *required flexural strength* equal to the *design flexural strength*

$$M_u = \phi M_n = \phi\rho f_y bd^2(1-0.59\frac{\rho f_y}{f_{ck}})$$

⇨ $$(183.3)(10^6) = (0.85)(0.0183)(400)bd^2\left(1-0.59\frac{(0.0183)(400)}{27}\right)$$

⇨ $$bd^2 = 35{,}070{,}000\,mm^3$$

be careful at UNIT!

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

3. Select an adequate width and height, as

b=220 mm and d=400 mm bd^2=35,200,000 mm^2

b=200 mm and d=420 mm bd^2=35,280,000 mm^2

.
.
.

See calculation.xls

⇨ b=200 mm and d=420 mm are selected.

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

determination of the reinforcement amount

1. $A_s = \rho bd = (0.0183)(200)(420) = 1,537\ mm^2$

2. 4@D22=1,548 mm^2
   3@D25=1,520 mm^2
   2@D32=1,522 mm^2 See Handout #3-3 Table A.2

3. assuming concrete cover = 70mm, then h=490mm

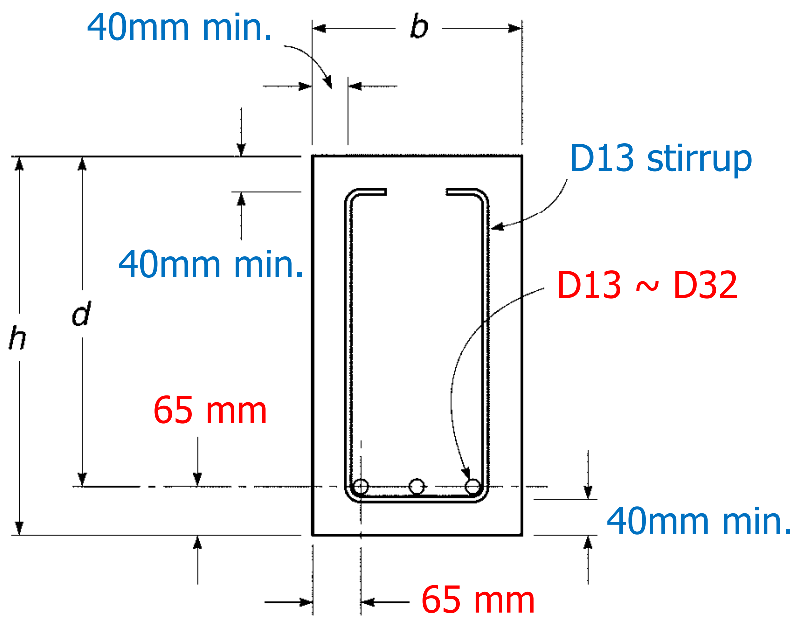
DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

Check cover and bar spacing

40+10+32+32+32+10+40

=196 mm < 200 mm O.K.



DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

<Iterative Method>

1. assuming a reasonable value of *a* is equal to 135 mm

$$M_u = \phi M_n = \phi A_s f_y (d - \frac{a}{2}) \quad (36)$$

⇨ $$A_s = \frac{(183.3)(10^6)}{(0.85)(400)(420 - 135/2)} = \underline{1,529 mm^2}$$

Compare with 1,537mm²

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

2. checking the assumed *a*

See calculation.xls

$$a = \frac{A_s f_y}{0.85 f_{ck} b} = \frac{(1529)(400)}{(0.85)(27)(200)} = 133\ mm \approx 135\ mm$$

Note

1. Assumed *a* is very close to the calculated
2. No further calculation require
3. This method converges very rapidly

3. Flexural Analysis/Design of Beam

DESIGN OF TENSION REINFORCED REC. BEAMS

Solution

Note

- Infinite number of solutions exist.
- $\rho_{\min} \le \rho \le \rho_{\max}$
- Larger cross section + less reinforcement can be economical and reduce deflection.
- Simplicity of construction should be considered in selection of reinforcement.
- Economical design typically have $0.50\rho_{\max} \le \rho \le 0.75\rho_{\max}$

DESIGN OF TENSION REINFORCED REC. BEAMS

Example 3.7 ~ 3.11

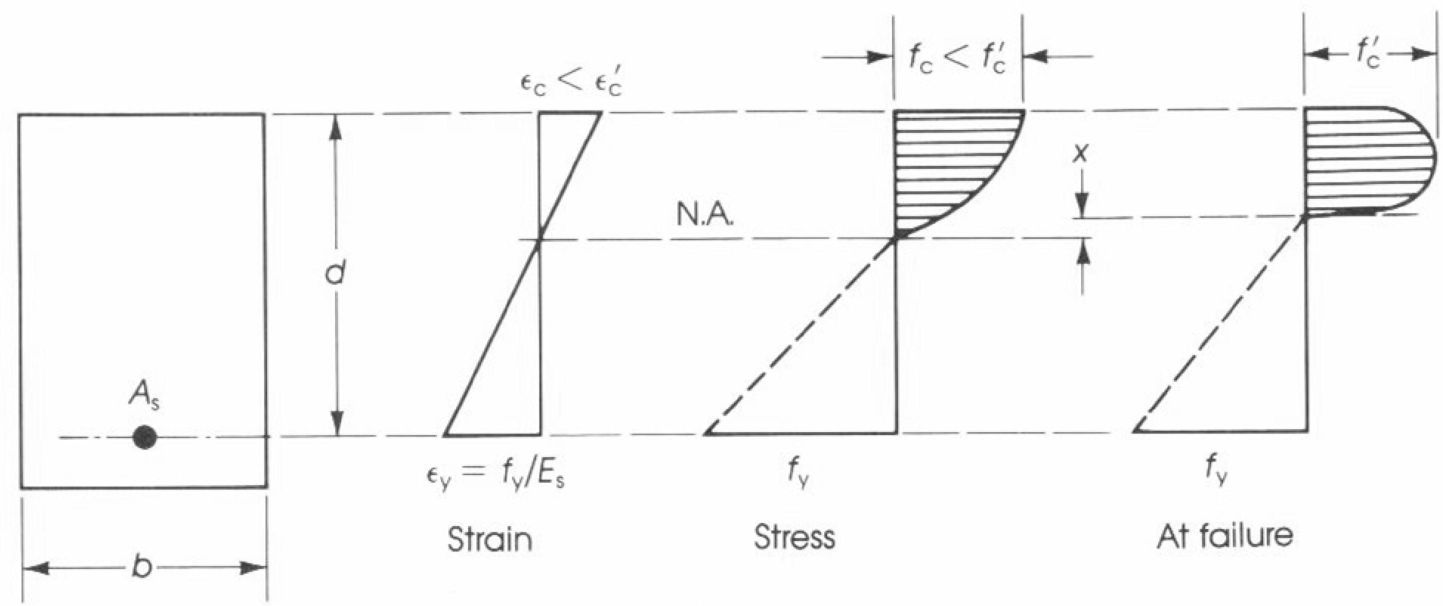
Example_Solution_1.pdf

SUMMARY

Under Reinforced Section

Steel may reach its yield strength before the concrete reaches its maximum.

$(f_c' = f_{ck},\ \varepsilon_c' = \varepsilon_u)$

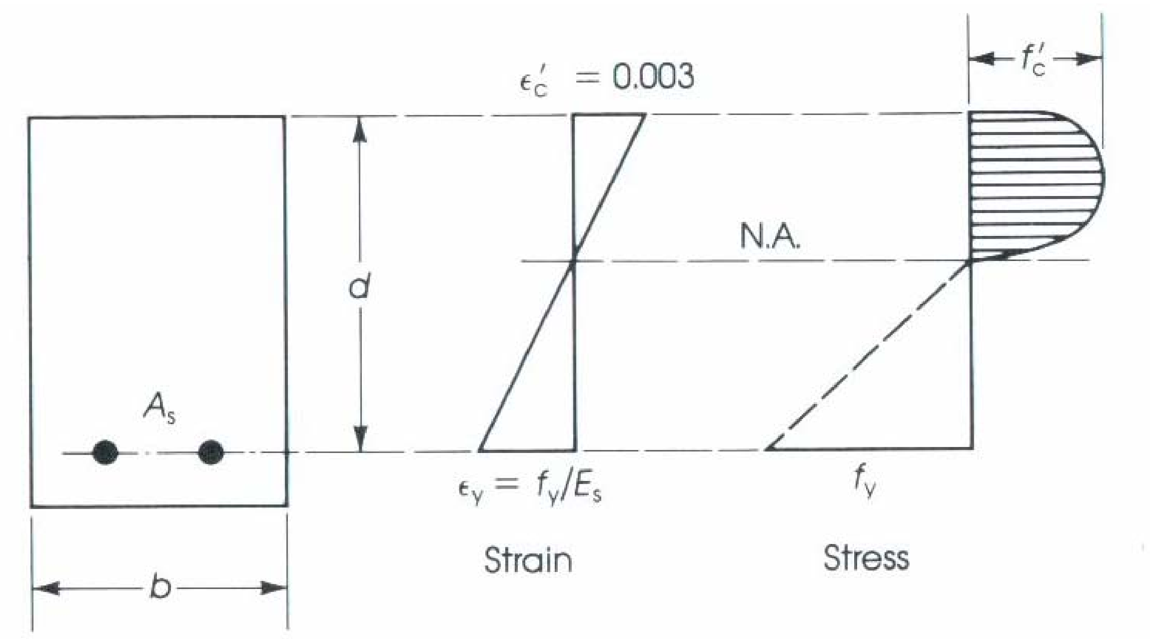


SUMMARY

Balanced Section

Steel reaches yield at same time as concrete reaches ultimate strength.

$(f_c' = f_{ck},\ \varepsilon_c' = \varepsilon_u)$

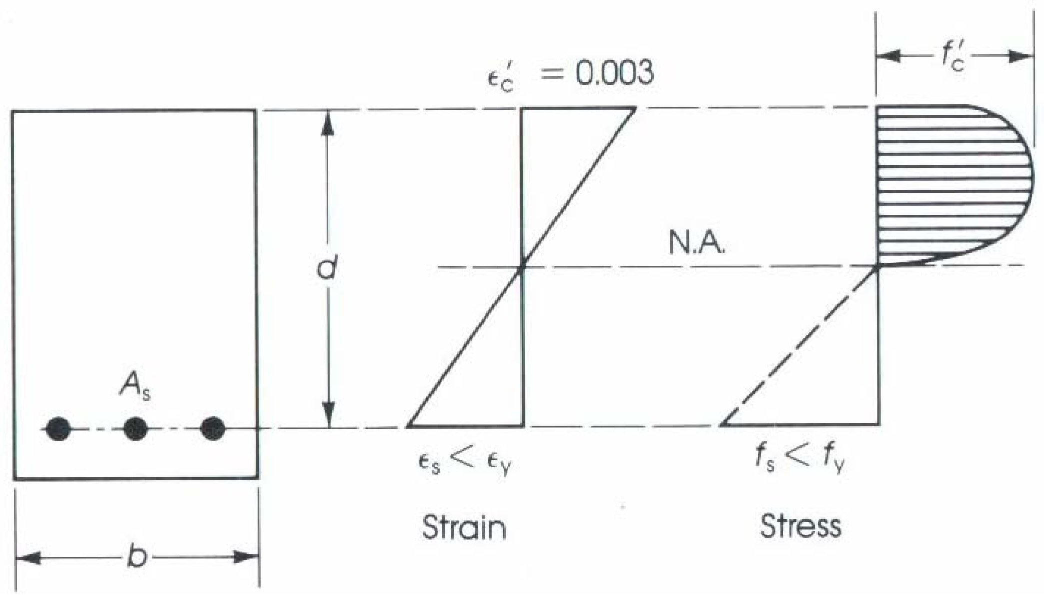


SUMMARY

Over Reinforced Section

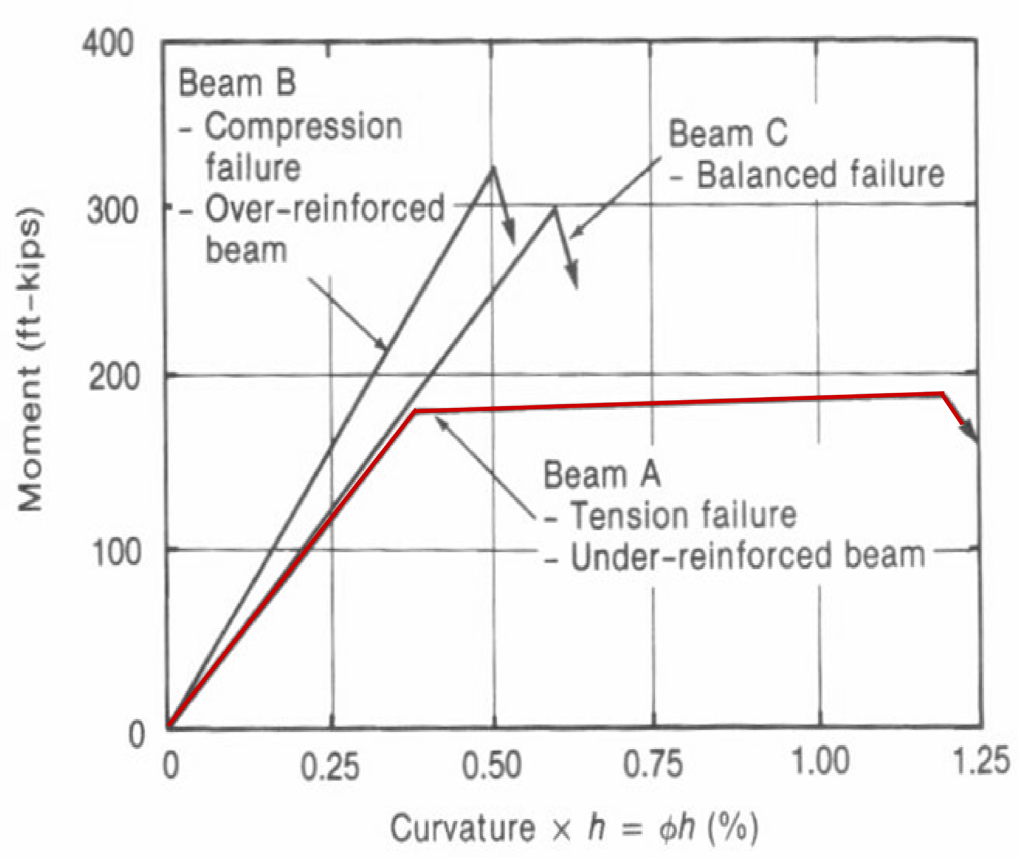
Concrete may fail before the the yield of steel due to the presence of a high percentage of steel in the section.

$(f_c' = f_{ck},\ \varepsilon_c' = \varepsilon_u)$



3. Flexural Analysis/Design of Beam

SUMMARY



most desirable

REC. BEAMS with TEN. & COMP. REBARS

Why Compression Steels are Needed?

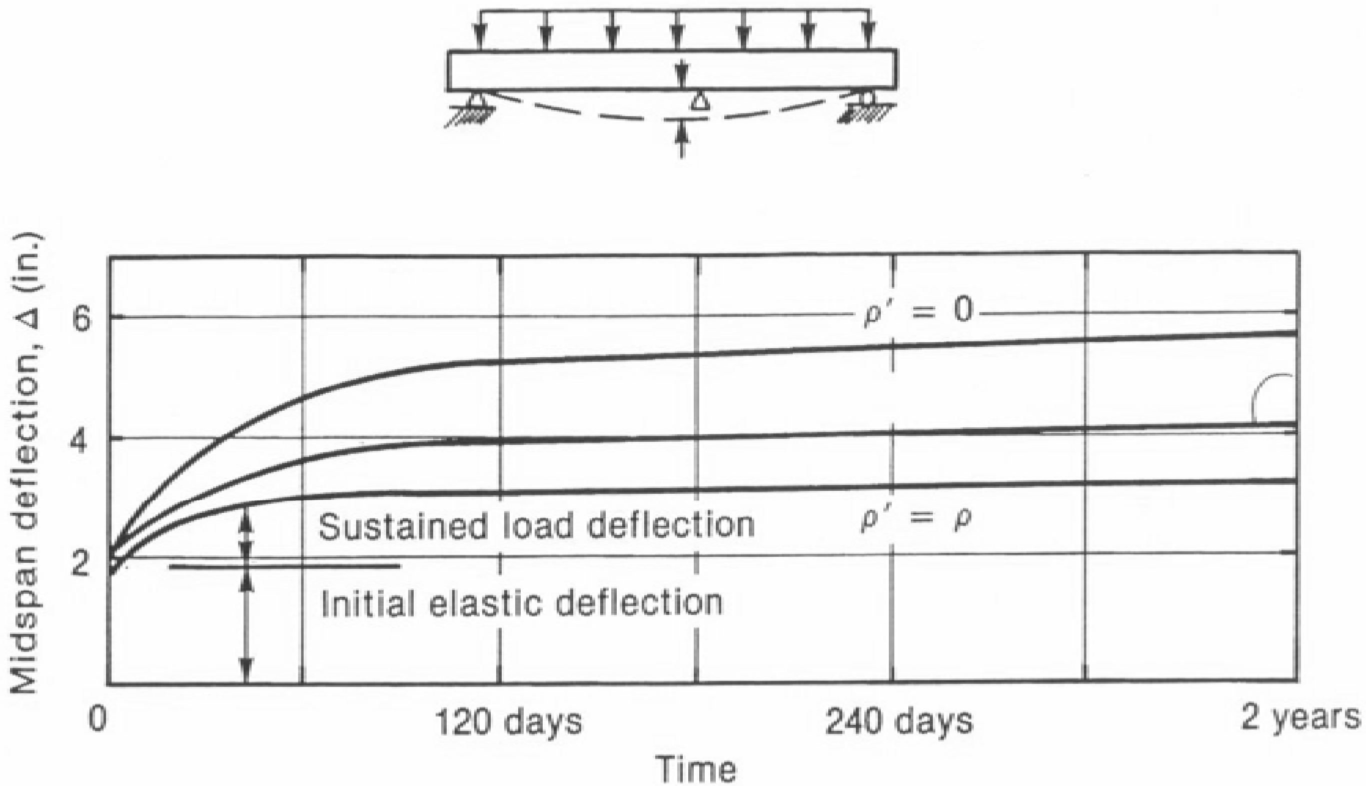
1. To increase the compression resistance of a beam of which dimension is limited by architectural or the other consideration.

2. To reduce long-term deflections of members.
   ; transferring load to compression steel induces the reduction of compressive stress in concrete and less creep.

3. Flexural Analysis / Design of Beam

REC. BEAMS with TEN. & COMP. REBARS

<u>Why Compression Steels are Needed?</u>



3. Flexural Analysis/Design of Beam

REC. BEAMS with TEN. & COMP. REBARS

Why Compression Steels are Needed?

3. For minimum-moment loading (Sec 12.2)
   ; according to the applied load, occasionally negative moment can occur.

4. To increase ductility.
   ; additional steel reduces stress block depth, along with the increase of steel strain.

   ⇨ larger curvature

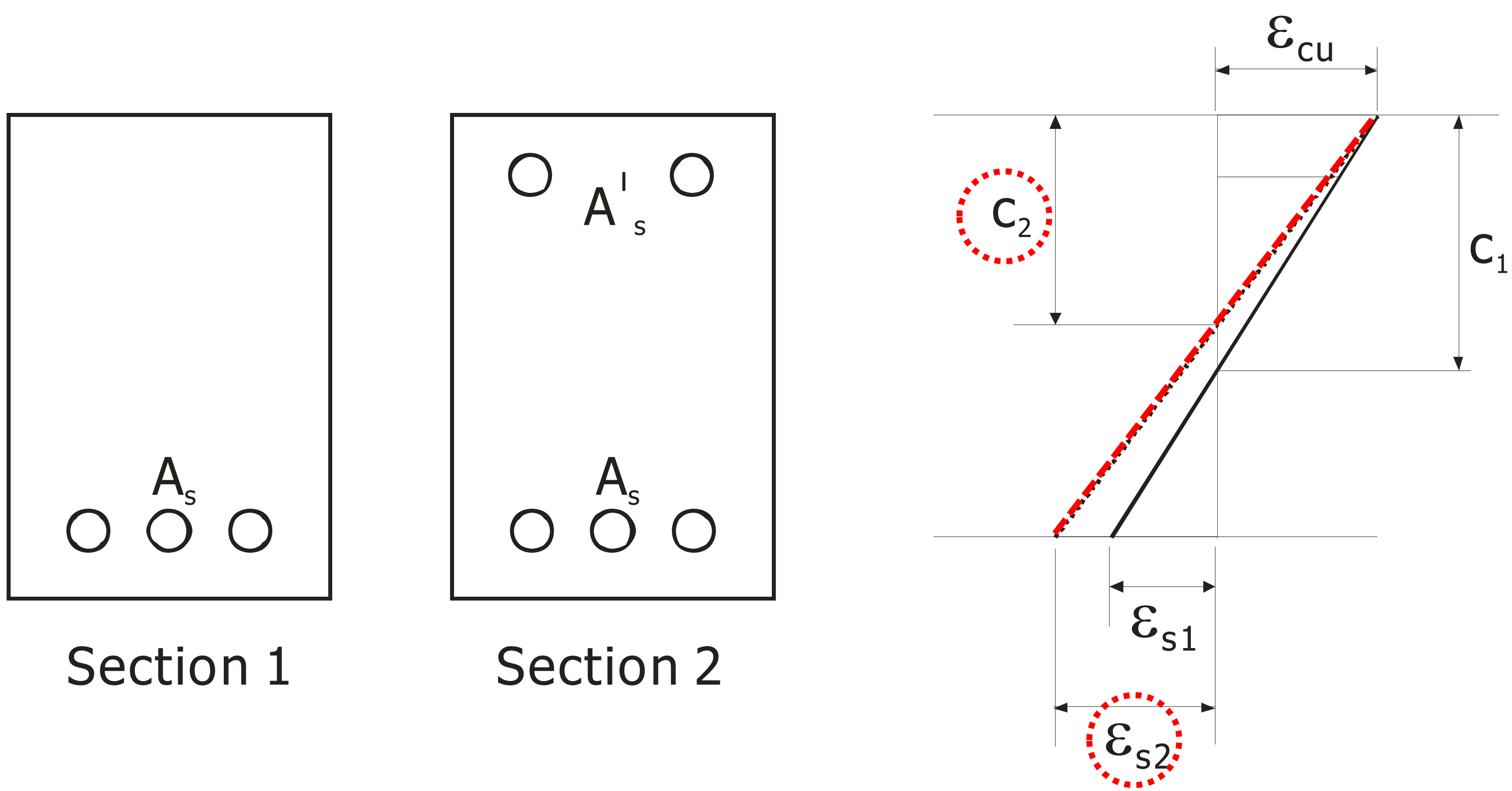
REC. BEAMS with TEN. & COMP. REBARS

Why Compression Steels are Needed?

5. For the ease of fabrication
   ; a role as stirrup-support bars continuous throughout the beam span.

REC. BEAMS with TEN. & COMP. REBARS

Comparison Singly RB and Doubly RB



REC. BEAMS with TEN. & COMP. REBARS

Comparison Singly RB and Doubly RB

Section 1	Section 2
$c_1 = \dfrac{A_s f_s}{0.85 f_{ck} b \beta_1}$	$c_2 = \dfrac{A_s f_s - A_s' f_s'}{0.85 f_{ck} b \beta_1}$

Additional A_s' strengthens compression zone so that less concrete is needed to resist a given force.

⇨ *N.A. goes up and steel strain increases.*

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Failure Types of Doubly Reinforced Beams

1. Both tension and compression steels yield.
2. Tension steel yields but compression steel does not.
3. *Tension steel does not yield but compression steel does.*
4. *Neither tension nor compression steel yield.*

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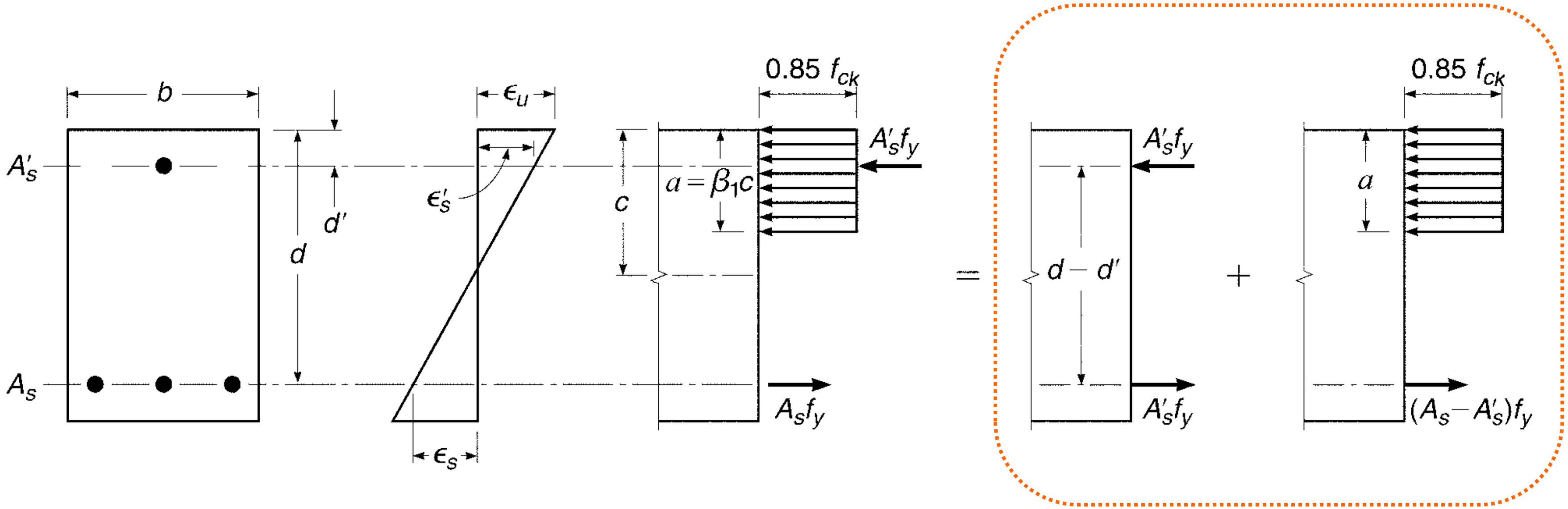
Tension and Comp. Steel Both at Yield Stress



3. Flexural Analysis/Design of Beam

REC. BEAMS with TEN. & COMP. REBARS

<u>Tension and Comp. Steel Both at Yield Stress</u>

If $\rho \leq \rho_b$ Effect of compression steel can be disregarded, since such a beam will be controlled by steel yielding.

- Internal lever arm of the resisting moment is little affected by the presence of the compression bar

If $\rho > \rho_b$ Beam should be considered as a doubly reinforced beam.

3. Flexural Analysis/Design of Beam

REC. BEAMS with TEN. & COMP. REBARS

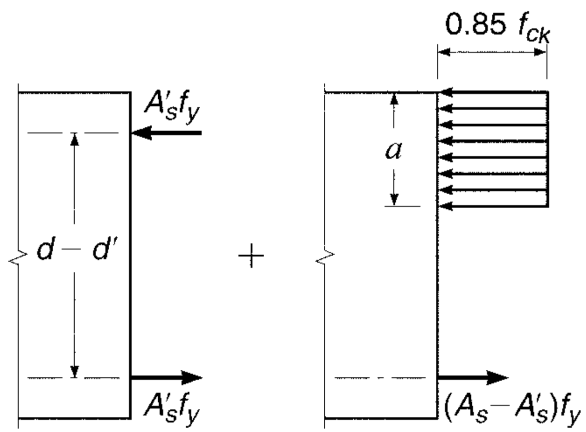
<u>Tension and Comp. Steel Both at Yield Stress</u>

The total resistance force (M_n)

$= M_{n1}$ *by* A_s' + M_{n2} *by* $(A_s - A_s')$

$$M_{n1} = A_s' f_y (d - d') \quad (45)$$

$$M_{n2} = (A_s - A_s') f_y (d - \frac{a}{2}) \quad (46)$$



<u>the depth of stress block</u>

$$a = \frac{(A_s - A_s') f_y}{0.85 f_{ck} b} \quad (47)$$

REC. BEAMS with TEN. & COMP. REBARS

Tension and Comp. Steel Both at Yield Stress

apply $A_s = \rho bd$ and $A_s' = \rho' bd$

$$a = \frac{(\rho - \rho')f_y d}{0.85 f_{ck}} \tag{47}$$

total nominal resisting moment

$$M_n = M_{n1} + M_{n2}$$

$$= A_s' f_y (d - d') + (A_s - A_s') f_y (d - \frac{a}{2}) \tag{48}$$

3. Flexural Analysis/Design of Beam

REC. BEAMS with TEN. & COMP. REBARS

Tension and Comp. Steel Both at Yield Stress

balanced reinforcement ratio

$$\bar{\rho}_b = \rho_b + \rho' \qquad (49)$$

,where ρ_b is balanced reinforcement ratio of singly reinforced beam. (Eq. (27))

$$\rho_b = 0.85\beta_1 \frac{f_{ck}}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = 0.85\beta_1 \frac{f_{ck}}{f_y} \frac{600}{600 + f_y}$$

Maximum reinforcement ratio

$$\bar{\rho}_{\max} = \rho_{\max} + \rho' \qquad (50)$$

$$\rho_{\max} = 0.85\beta_1 \frac{f_{ck}}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004}$$

REC. BEAMS with TEN. & COMP. REBARS

Compression Steel below Yield Stress

Eq. (48) is valid only if the compression steel yields at beam failure.

In many cases, however, compression steel is below the yield stress.

Whether the compression steel will yield or not can be determined as follows.

3. Flexural Analysis / Design of Beam

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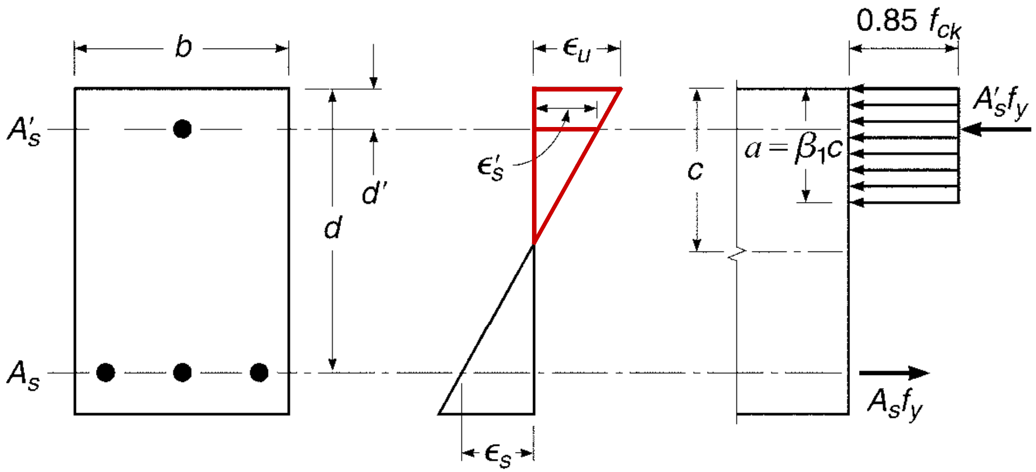
Compression Steel below Yield Stress



let, $\varepsilon_s' = \varepsilon_y$, then from geometry

$$\frac{c}{d'} = \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} \qquad (51)$$

REC. BEAMS with TEN. & COMP. REBARS

Compression Steel below Yield Stress

check minimum tensile reinforcement ratio $\bar{\rho}_{cy}$

; to ensure yielding of compression steel

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_{ck}}{f_y} \frac{d'}{d} \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho' \qquad (52)$$

Compare with Eq. (26) $\rho_b = 0.85\beta_1 \frac{f_{ck}}{f_y}\frac{c}{d} = 0.85\beta_1 \frac{f_{ck}}{f_y}\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

If $\rho < \bar{\rho}_{cy}$, neutral axis is sufficiently high that the compression steel stress at failure is less than the yield stress. ($f_s' < f_y$)

3. Flexural Analysis/Design of Beam

REC. BEAMS with TEN. & COMP. REBARS

Compression Steel below Yield Stress

balanced reinforcement ratio

$$\bar{\rho}_b = \rho_b + \rho' \frac{f_s'}{f_y} \qquad (53)$$

Compare with Eq. (49)

,where $$f_s' = E_s \varepsilon_s' = E_s \left(\varepsilon_u - \frac{d'}{d}\left(\varepsilon_u + \varepsilon_y\right) \right) \le f_y \qquad (54)$$

maximum reinforcement ratio

$$f_s' = E_s \left(\varepsilon_u - \frac{d'}{d}\left(\varepsilon_u + 0.004\right) \right) \le f_y$$

$$\bar{\rho}_{\max} = \rho_{\max} + \rho' \frac{f_s'}{f_y} \qquad (55)$$

REC. BEAMS with TEN. & COMP. REBARS

Compression Steel below Yield Stress

Note Equations for compression steel stress f_s' apply only for beams with exact strain values in the extreme tensile steel of ε_y or ε_t=0.004

If tensile reinforcement ratio $\rho \leq \bar{\rho}_b$ and $\rho \leq \bar{\rho}_{cy}$, then the tensile steel yields at failure, but the compression steel does not reach the yield.

Therefore, new equations for compression steel stress and flexural strength are needed.

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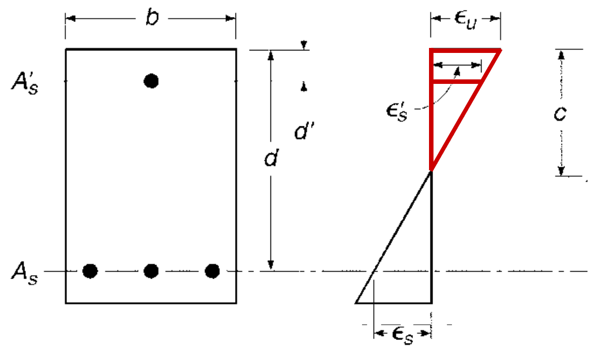
Compression Steel below Yield Stress

comp. steel stress



$$\varepsilon_s' : \varepsilon_u = c - d' : c$$

$$f_s' = \varepsilon_s' E_s = \varepsilon_u E_s \frac{c - d'}{c}$$

where c is still unknown

force equilibrium

$$T = C_c + C_s$$

$$\Rightarrow \quad A_s f_y = 0.85 f_{ck} \beta_1 c b + A_s' \varepsilon_u E_s \frac{c - d'}{c} \qquad (57)$$

REC. BEAMS with TEN. & COMP. REBARS

Compression Steel below Yield Stress

solve the quadratic Eq. (57) for c

then apply $a = \beta_1 c$

nominal flexural strength

$$M_n = 0.85 f_{ck} ab\left(d - \frac{a}{2}\right) + A_s' f_s' \left(d - d'\right) \qquad (58)$$

Note must ensure that compression steel does not buckle using lateral ties (KCI 5.5.1)

REC. BEAMS with TEN. & COMP. REBARS

Analysis Procedure of Doubly Reinforced Beam.

1. Check the tensile reinforcement ratio $\rho < \bar{\rho}_b$ (Eq.(53)) using Eq.(54)
2. Calculate $\bar{\rho}_{cy}$ from Eq.(52) determining whether compression steel yields or not.
3. Compare $\bar{\rho}_{cy}$ with actual tensile reinforcement ratio ρ.
   - i) $\rho \geq \bar{\rho}_{cy}$ then $f_s' = f_y$ ⇨ M_n=Eq.(48)
   - ii) $\rho < \bar{\rho}_{cy}$ then $f_s' < f_y$ ⇨ M_n=Eq.(58)

c should be calculated previously

REC. BEAMS with TEN. & COMP. REBARS

Design Procedure of Doubly Reinforced Beams

Note

- Direct solution is impossible because, steel areas to be provided depends on the steel stress, which are NOT KNOWN before the section is proportioned.
- Assume $f_s'=f_y$ but this must be Confirmed.

3. Flexural Analysis / Design of Beam

REC. BEAMS with TEN. & COMP. REBARS

<u>Design Procedure of Doubly Reinforced Beams</u>

1. Calculate maximum moment can be resisted by $\rho = \rho_{max}$ or ρ for $\varepsilon_t = 0.005$ to ensure $\varphi=0.85$

   Corresponding $A_s=\rho_{max}bd$

$$M_n = A_s f_y (d - \frac{a}{2})$$

with $$a = \frac{A_s f_y}{0.85 f_{ck} b}$$

REC. BEAMS with TEN. & COMP. REBARS

Design Procedure of Doubly Reinforced Beams

2. Find the excess moment, if any, that must be resisted and set $M_2=M_n$.

   In other words, if M_n (from step 1) < M_u/φ, then set $M_2=M_n$

$$\Rightarrow \quad M_1 = \frac{M_u}{\phi} - M_2$$

(cont.)

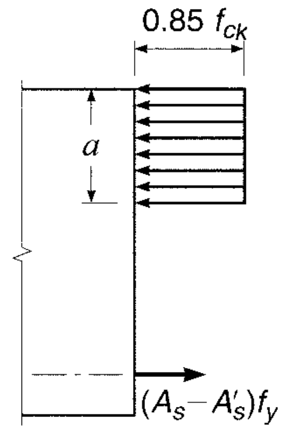
REC. BEAMS with TEN. & COMP. REBARS

Design Procedure of Doubly Reinforced Beams

2. Now, A_s from step1 is defined as A_{s2}.

   Here, A_{s2} is the part of the tension steel area that works with compression force in the concrete.

$$\Rightarrow \quad A_s - A_s' = A_{s2}$$



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Design Procedure of Doubly Reinforced Beams

3. Tentatively assume $f_s' = f_y$, then

$$A_s' = \frac{M_1}{f_y(d - d')}$$

4. Add an additional amount of tensile steel $A_{s1} = A_s'$.
   Thus, $A_s = A_{s2}$ (step 2) $+ A_{s1}$

5. Analyze the doubly reinforced beam to see if $f_s' = f_y$;
   that is, check $\rho > \bar{\rho}_{cy}$ to ensure yielding of the compression steel at failure.

3. Flexural Analysis / Design of Beam

REC. BEAMS with TEN. & COMP. REBARS

Design Procedure of Doubly Reinforced Beams

6. If $\rho < \bar{\rho}_{cy}$, then $f_s' < f_y$ and the compression steel must be increased to provide the needed force as follows.

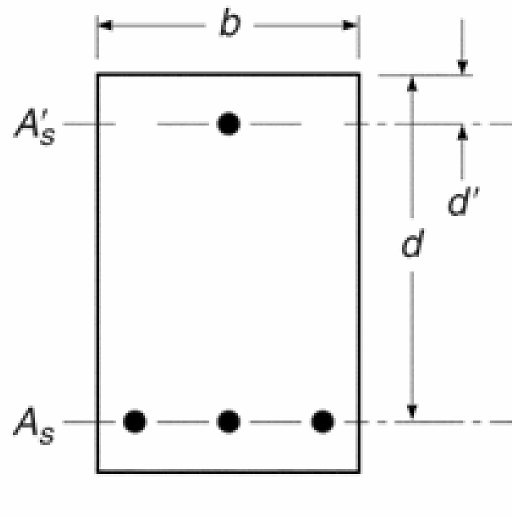
$$a = \frac{(A_s - A_s')f_y}{0.85 f_{ck} b} \quad \Leftarrow \quad c = \frac{a}{\beta_1}$$

$$f_s' = \varepsilon_u E_s \frac{c - d'}{c}$$

$$A'_{s,revised} f_s' = A'_{s,trial} f_y \qquad A'_{s,revised} = A'_{s,trial} \frac{f_y}{f_s'}$$

REC. BEAMS with TEN. & COMP. REBARS

Example 3.12 (Analysis)



b = 300 mm

d = 600 mm, d' = 65 mm

A_s = 4,765 mm^2 (6-No.32 in two rows)

A_s' = 1,013 mm^2 (2-No.25)

f_y = 400 MPa, f_{ck} = 35 MPa

Calculate the design moment capacity of the beam.

Example_Solution_2.pdf

REC. BEAMS with TEN. & COMP. REBARS

Example 3.13 (Design)

What steel area(s) must be provided?

service live load = 37 kN/m

calculated dead load = 16 kN/m

simple span length = 5.4 m

b = 250, h = 500 mm

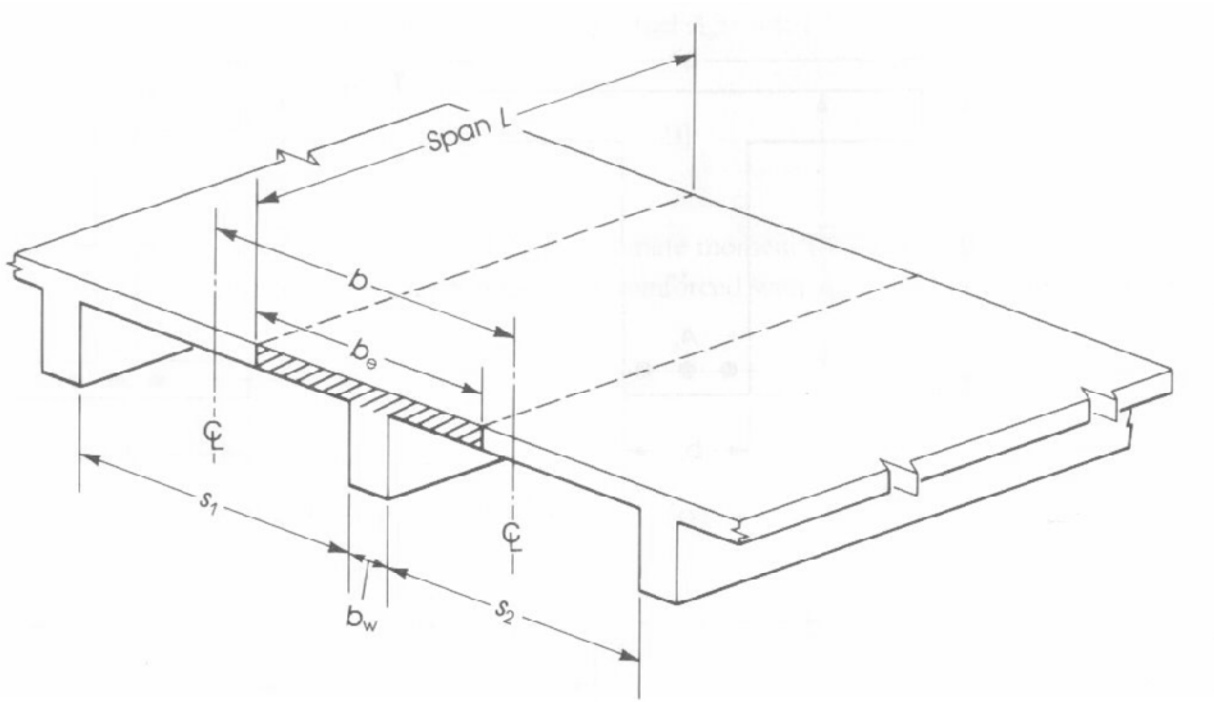
f_y = 400 MPa, f_{ck} = 27 MPa

Example_Solution_2.pdf

T BEAMS

- Reinforced concrete floors, roofs, decks are almost monolithic.
  - ⇦ cast in once
- Upper part of such structures resist longitudinal compression.
- The resulting beam cross section can be considered as T-shaped one.
- We call '-' as flange, while 'l' web.

T BEAMS



T BEAMS

Note

The upper part of T beam is stressed laterally due to slab action.

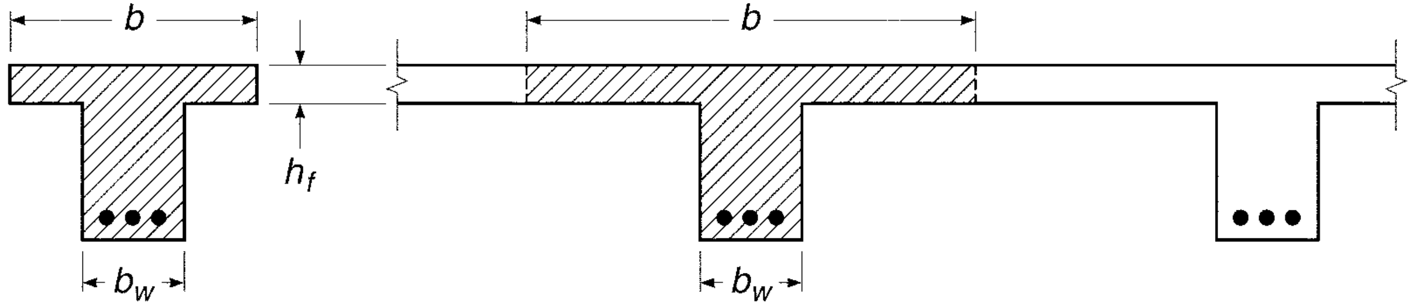
Transverse compression at the bottom surface of slab can increase longitudinal compressive strength by 25%.

Transverse tension at the top surface of slab can decrease longitudinal tensile strength.

Neither effect is considered in DESIGN.

T BEAMS

Effective Flange width



(a) (b)

In (b) of the above figure, the element of the flange between the webs are less stressed than the element directly over the web due to the effect of SHEAR deformation of the flange.

3. Flexural Analysis / Design of Beam

T BEAMS

<u>The criteria for effective width</u> (KCI 3.4.8)

; for the convenience of design assuming uniform stress at maximum value

- For symmetrical T beams, the effective width b should be selected to be the minimum value out of the follows.

  1) $16h_f + b_w$

  2) one-fourth the span length of the beam

  3) distance between the center of adjacent slab

3. Flexural Analysis / Design of Beam

T BEAMS

<u>The criteria for effective width</u> (KCI 3.4.8)

2. For L beams having a slab on one side only, the effective width shall not be exceed.

   1) $6h_f + b_w$

   2) one-twelfth the span length of the beam + b_w

   3) one-half the clear distance to the next beam + b_w

3. Flexural Analysis / Design of Beam

T BEAMS

The criteria for effective width (KCI 3.4.8)

3. For isolated T beams in which the flange is used only for the purpose of providing additional compression area,

   1) $h_f \geq b_w/2$

   2) total flange width $\leq 4b_w$

3. Flexural Analysis/Design of Beam

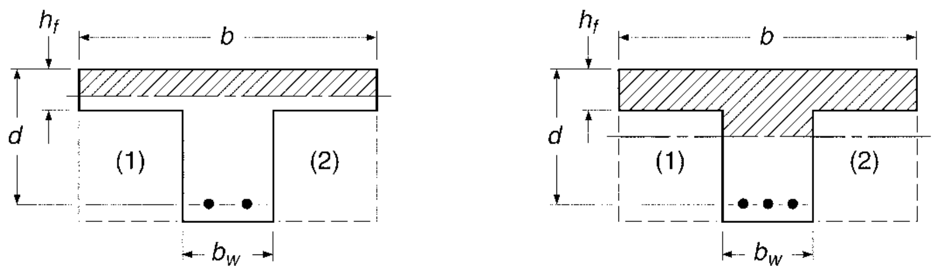
T BEAMS

Strength Analysis

- Neutral axis of T beam can either in the flange or in the web, that is, $c \leq h_f$ or $c \geq h_f$

- $c \leq h_f$, then T beam can be analyzed as a rectangular beam.

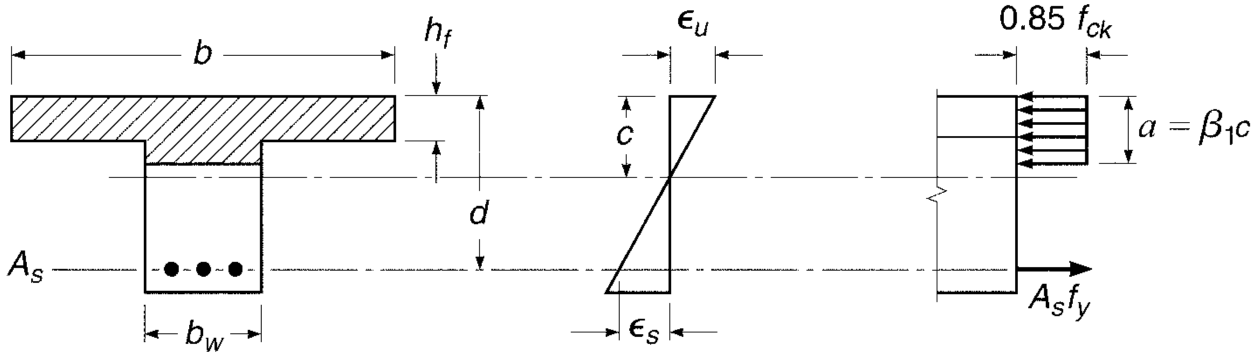
  $c > h_f$, then the actual T-shaped compression zone should be considered.



3. Flexural Analysis / Design of Beam

T BEAMS

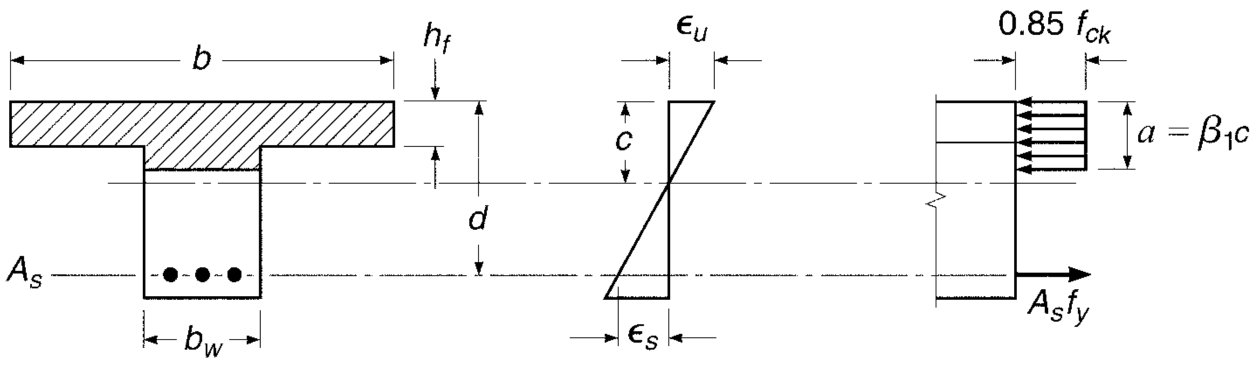
<u>Strength Analysis</u>



- In T beam analysis, equivalent stress block (Whitney's block) is still valid throughout the extensive researches.

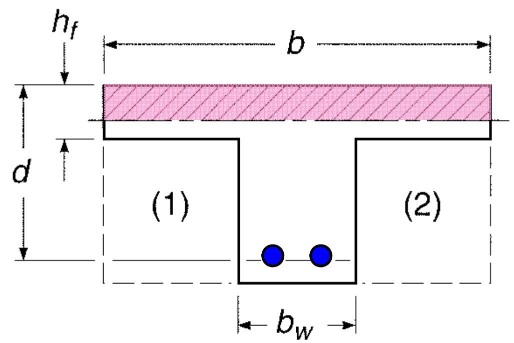
T BEAMS

Strength Analysis



$$a = \frac{A_s f_y}{0.85 f_{ck} b} = \frac{\rho f_y d}{0.85 f_{ck}}$$

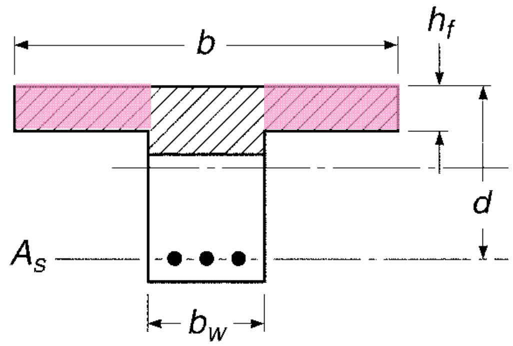
- $a \le h_f$, then T beam can be treated as



T BEAMS

Strength Analysis ($a > h_f$)

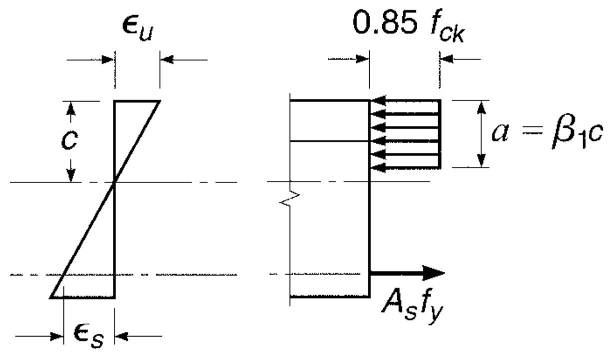
- For the convenience, A_s can be divided into TWO parts.



(1) for A_{sf} steel amount corresponding to overhanging portion

$$A_{sf} = \frac{0.85 f_{ck} (b - b_w) h_f}{f_y}$$

$$M_{n1} = A_{sf} f_y \left(d - \frac{h_f}{2} \right)$$



T BEAMS

Strength Analysis ($a > h_f$)

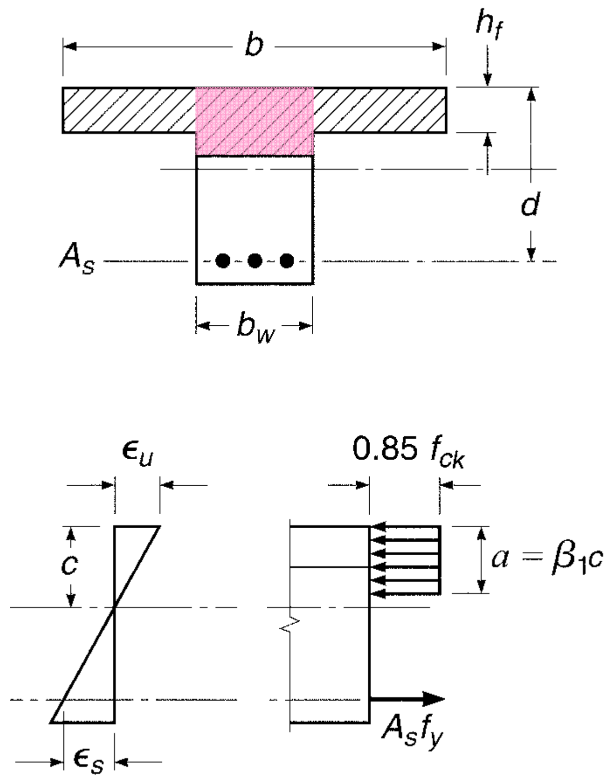
(2) for remaining A_s-A_{sf}

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_{ck} b_w}$$

$$M_{n2} = (A_s - A_{sf}) f_y \left(d - \frac{a}{2} \right)$$

total nominal moment

$$M_n = M_{n1} + M_{n2}$$



T BEAMS

Strength Analysis ($a > h_f$)

It should be noted that ρ_w and ρ_f are defined for rectangular cross section $b_w d$.

$$\rho_w = A_s / b_w d \qquad \rho_f = A_{sf} / b_w d$$

Maximum reinforcement ratio

$$\rho_{w,\max} = \rho_{\max} + \rho_f$$

where ρ_{max} is as previously defined for a rectangular cross section.

T BEAMS

Strength Analysis ($a > h_f$)

Minimum reinforcement ratio

As for rectangular beam, Eq.(41) can be applied

$$A_{s,\min} = \frac{0.25\sqrt{f_{ck}}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d \qquad (41)$$

T BEAMS

Design Procedure of T beams

(1) Determine flange thickness h_f based on flexural requirements of the slab

(2) Determine the effective flange width b according to KCI.

(3) Choose web dimensions b_w and d based on either of the following:

(a) negative bending requirements at the support, if a continuous T beam

(b) shear requirement (chapter 4)

3. Flexural Analysis / Design of Beam

T BEAMS

Design Procedure of T beams

(4) Calculate a trial value of A_s, assuming that a does not exceed h_f , with beam width equal to flange width b.

using ordinary rectangular beam design method.

(5) For the trial A_s, check $a \leq h_f$. If $a > h_f$, revise A_s using T beam equation.

(6) Check $\varepsilon_t \geq 0.004$

(7) Check $\rho_w \geq \rho_{w,min}$

REC. BEAMS with TEN. & COMP. REBARS

Example 3.14 (Analysis)

An Isolated T beam

b = 700 mm, h_f = 150 mm

b_w = 250 mm, h = 750 mm

A_s = 4,765 mm^2 (6-No.32 in two rows)

; The centroid of the bar group is 660 mm from the top of the beam

f_y = 400 MPa, f_{ck} = 21 MPa

Calculate the design moment capacity of the beam.

Example_Solution_2.pdf

REC. BEAMS with TEN. & COMP. REBARS

Example 3.15 (Design)

A floor system consists of a 80mm concrete slab supported by continuous T beams with a span 7.3 m span, 1.2 m on centers.

Web dimensions, as determined by negative moment requirements at the supports, are b_w = 280 mm and d = 500 mm.

What tensile steel area must be provided at midspan to resist a factored moment of 723 kN-m?

f_y = 400 MPa, f_{ck} = 21 MPa

Example_Solution_2.pdf