4. Shear & Diagonal Tension in Beams

DIAGONAL TENSION

RC BEAM without SHEAR REINFORCEMENT

RC BEAM with WEB REINFORCEMENT

KCI CODE PROVISION

DEEP BEAMS

ALTERNATIVE MODELS for SHEAR ANALYSIS & DESIGN

SHEAR FRICTION DESIGN METHOD

447.327
Theory of Reinforced Concrete and Lab. I
Spring 2008

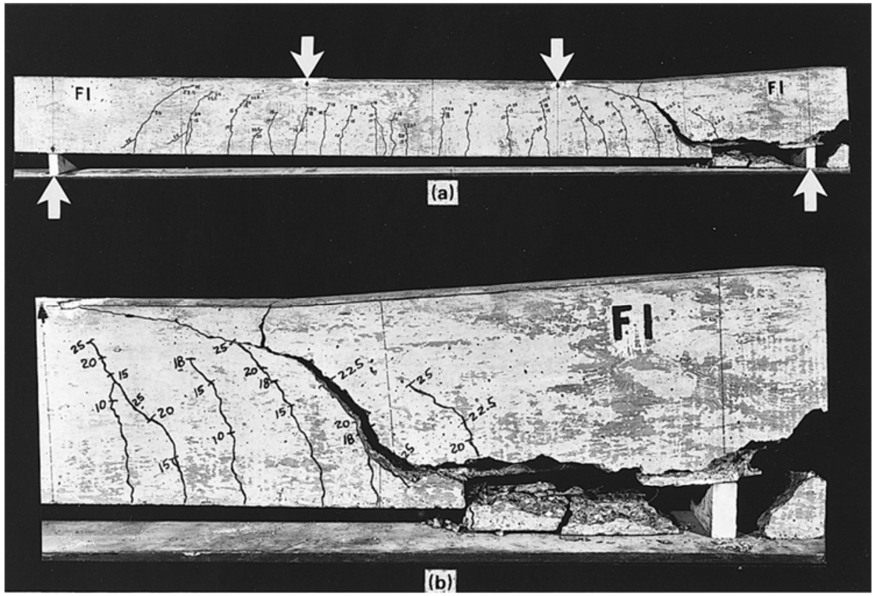
INTRODUCTION

Shear failure

- is often called diagonal tension failure.
- is very difficult to predict accurately, so not fully understood yet.
- shows sudden collapse without advance warning. (brittle)
- special shear reinforcement are provided to ensure flexural failure.

4. Shear & Diagonal Tension in Beams

INTRODUCTION



Typical shear failure

occurs immediately after the formation of the critical crack in the high-shear region near the support.

4. Shear & Diagonal Tension in Beams

INTRODUCTION

- Shear analysis and design are NOT really concerned with shear as such.

- The real concern is with diagonal tension stress ; resulting from the combination of shear stress and longitudinal flexural stress.

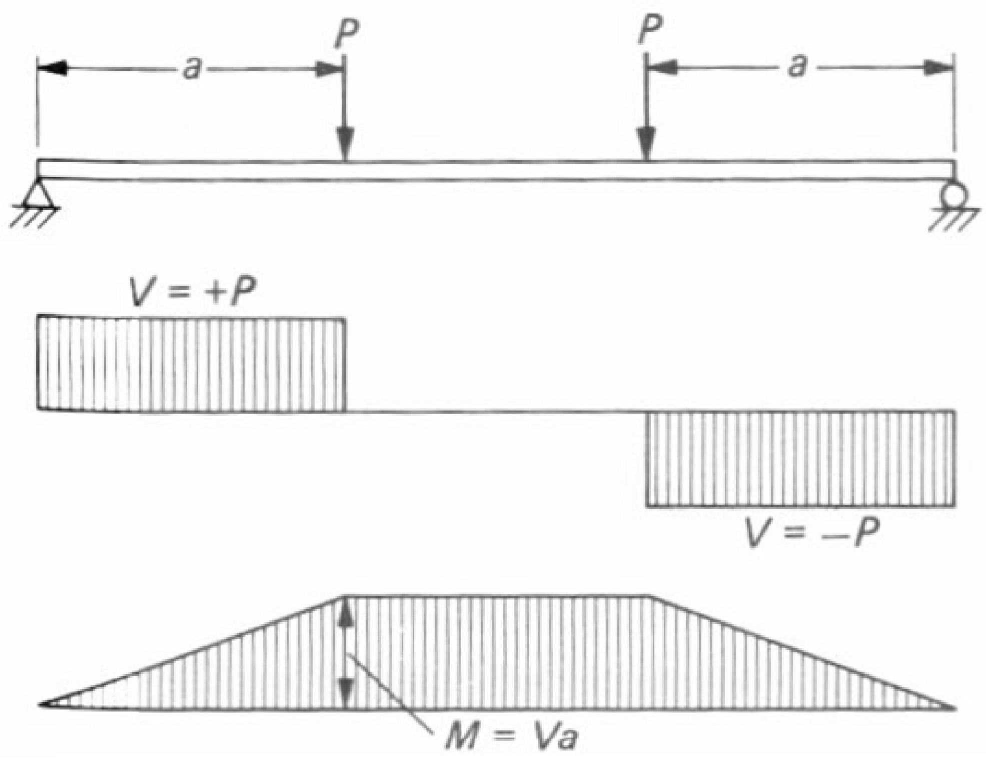
- We are going to deal with,
  1) KCI code provision
  2) variable angle truss model
  3) compression field theory
  4) shear-friction theory

4. Shear & Diagonal Tension in Beams

INTRODUCTION

S.F.D

B.M.D



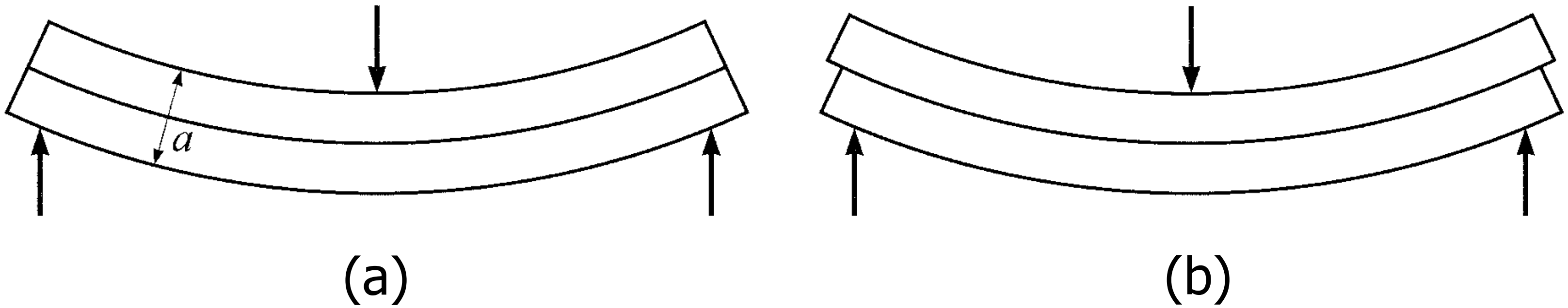
Stresses acting on section

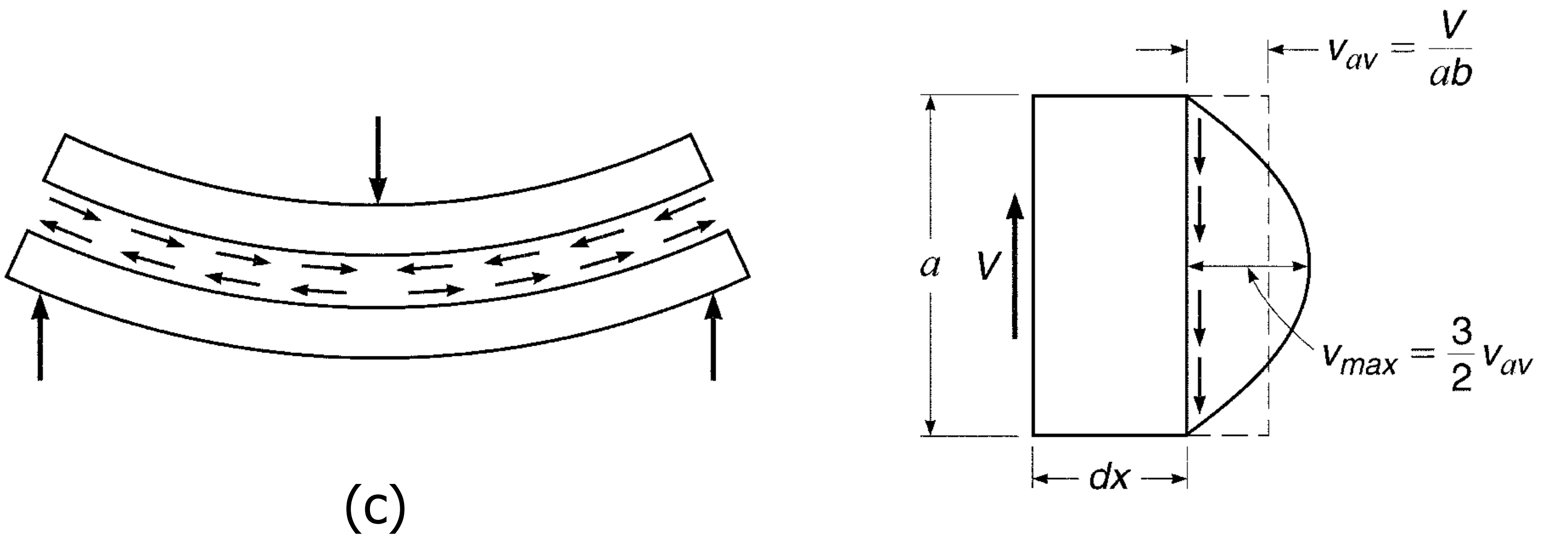
$$v = \frac{VQ}{Ib}$$

$$f = \frac{My}{I}$$

4. Shear & Diagonal Tension in Beams

DIAGONAL TENSION





4. Shear & Diagonal Tension in Beams

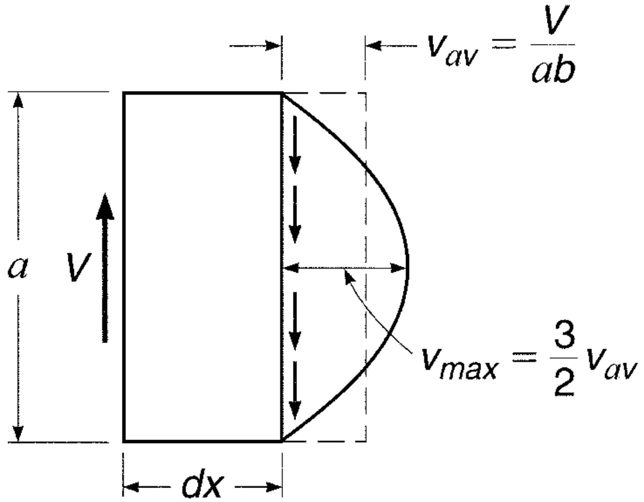
DIAGONAL TENSION

$$v = \frac{VQ}{Ib}$$

$$I = \frac{ba^3}{12}$$

$$Q_{max} = \left(\frac{ba}{2}\right) \times \left(\frac{a}{4}\right) = \frac{ba^2}{8}$$

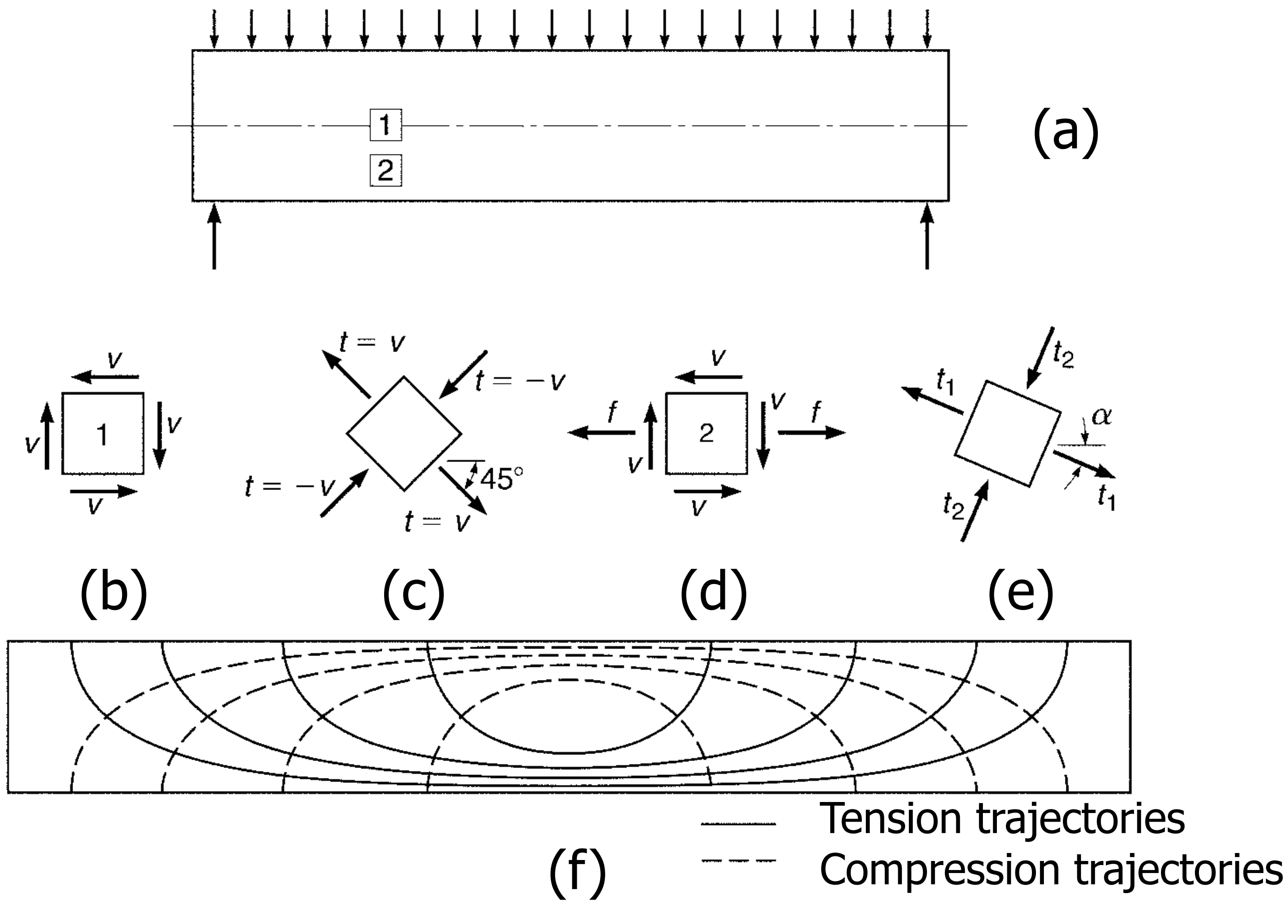
Note: The maximum 1st moment occurs at the neutral axis (NA).



$$v_{max} = \frac{3}{2}\left(\frac{V}{ba}\right) = 1.5\ v_{ave}$$

4. Shear & Diagonal Tension in Beams

DIAGONAL TENSION



RC BEAMS WITHOUT SHEAR REINFORCEMENT

- Plain concrete beam is controlled by tension failure according to elastic beam theory.

  ⇦ Shear has little influence on the beam strength.

- Tension reinforced beam shows quite different behavior.

  ⇨ Diagonal cracks occur ; which are distinguished from vertical flexural cracks.

4. Shear & Diagonal Tension in Beams

RC BEAMS WITHOUT SHEAR REINFORCEMENT

Criteria for Formation of Diagonal Cracks

- Diagonal tension stresses, *t* represent the combined effect of the shear and bending stress.

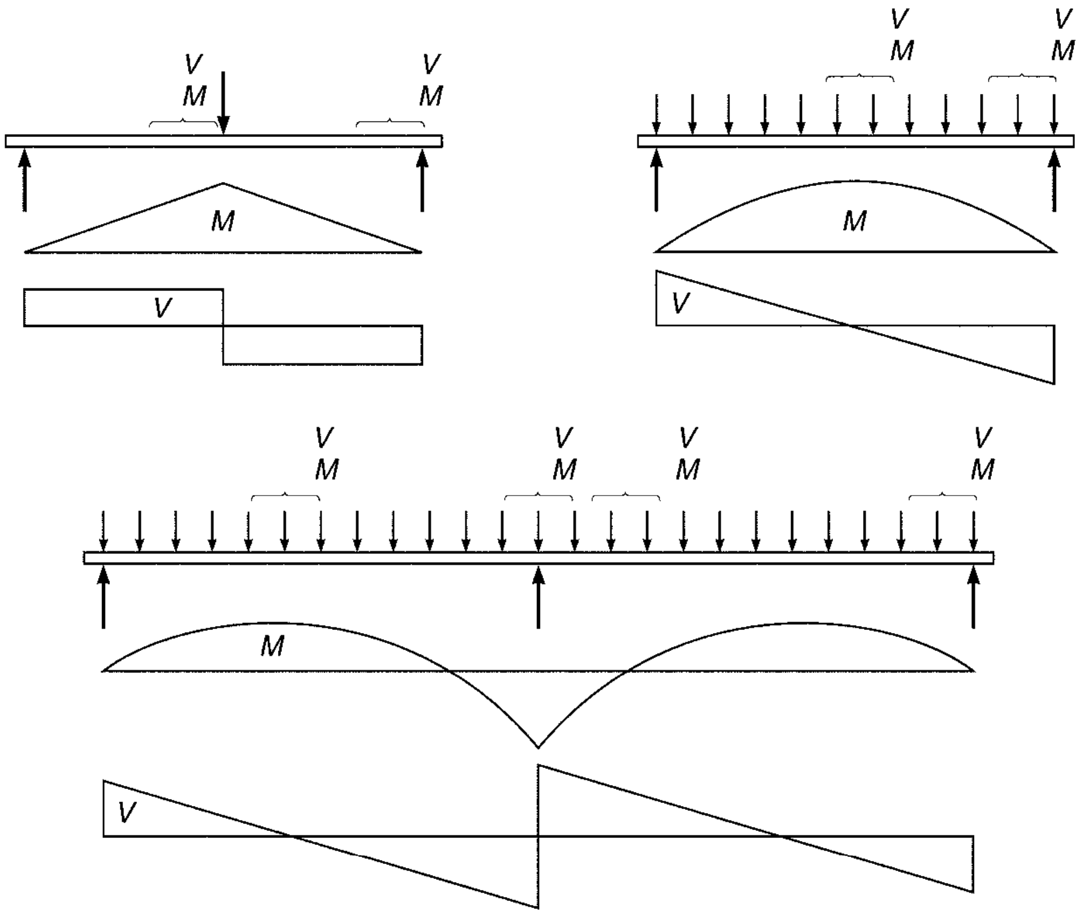
$$t = \frac{f}{2} + \sqrt{\frac{f^2}{4} + v^2}$$

- Therefore, the magnitude of *M* and *V* affect the value and the direction of diagonal tension stress.

See the Next page!

4. Shear & Diagonal Tension in Beams

Criteria for Formation of Diagonal Cracks



4. Shear & Diagonal Tension in Beams

Criteria for Formation of Diagonal Cracks

I) Large V and small M region

- average shear stress prior to crack formation is,

$$v = \frac{V}{bd} \tag{1}$$

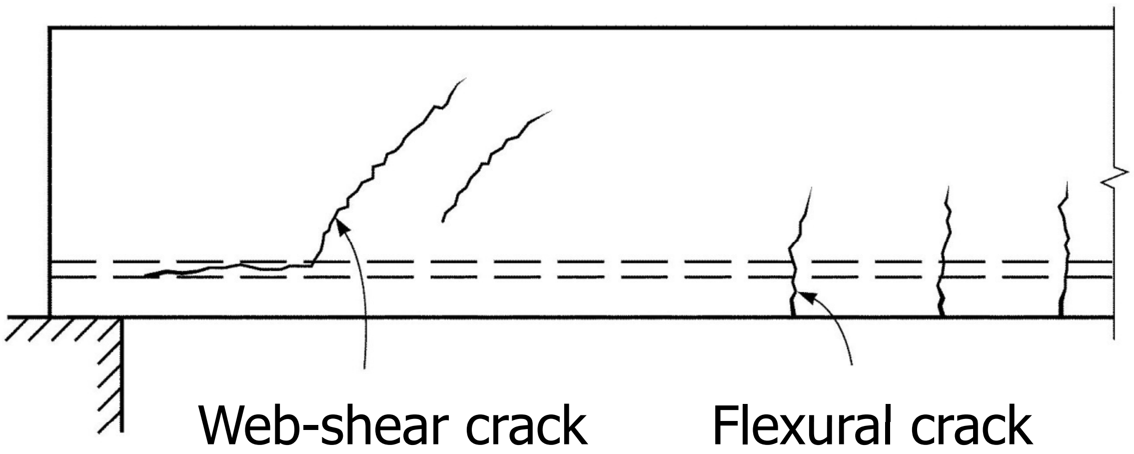
- The exact distribution of these shear stress over the depth is NOT known.

- It cannot be obtained from $v = VQ/Ib$, because this eqn. does not account for influence of tension reinforcement and because concrete is not an elastic homogeneous material.

4. Shear & Diagonal Tension in Beams

Criteria for Formation of Diagonal Cracks

- Eq (1) must be regarded merely as a measure of the average intensity of shear stresses in the section.
- The max. shear stress occurs at neutral axis with an angle of 45°



4. Shear & Diagonal Tension in Beams

Criteria for Formation of Diagonal Cracks

- Web-shear crack occurs when the diagonal tension stress near the neutral axis becomes equal to the tensile strength of the concrete.

$$v_{cr} = \frac{V_{cr}}{bd} = \underbrace{0.29\sqrt{f_{ck}}}_{\textit{empirical}} \qquad (2)$$

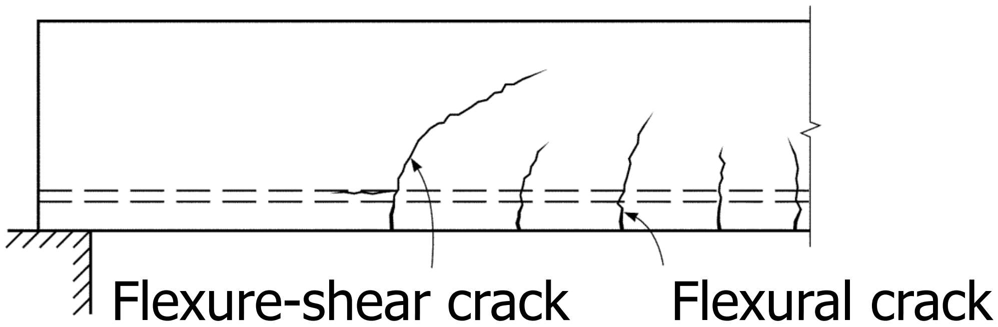
- Web shear cracking is relatively rare and occurs chiefly near supports of deep, thin-webbed beam or at inflections point of continuous beams.

4. Shear & Diagonal Tension in Beams

Criteria for Formation of Diagonal Cracks

II) Large *V* and large *M* region

- when the diagonal tension stress at the upper end of several flexural cracks exceeds the tensile strength of concrete, flexural-shear cracks occur.



- Flexural-shear cracks occurs when nominal shear stress reach,

$$v_{cr} = \frac{V_{cr}}{bd} = 0.16\sqrt{f_{ck}} \quad (3)$$

4. Shear & Diagonal Tension in Beams

Criteria for Formation of Diagonal Cracks

- Comparison of Eq. (2) and (3) indicates that diagonal crack development depends on the *V-M* ratio.

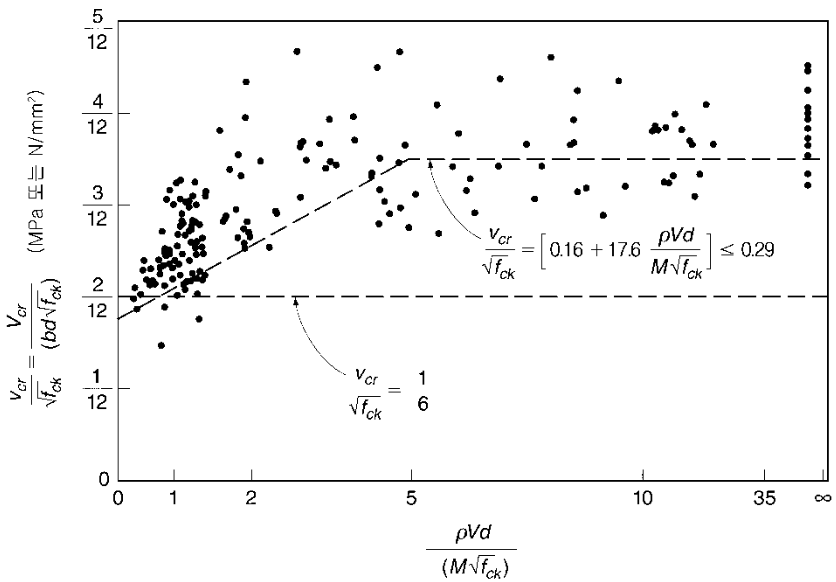
  ⇨ Large bending moments can reduce the shear force at which diagonal cracks form. (about to one-half)

$$v_{cr} = \frac{V_{cr}}{bd} = 0.16\sqrt{f_{ck}} + 17.6\frac{\rho V d}{M} \le 0.29\sqrt{f_{ck}} \qquad (4)$$

⇦ $$\frac{v}{f} = \frac{K_1(V/bd)}{K_2(M/bd^2)} = \frac{K_1}{K_2}\frac{Vd}{M}$$

4. Shear & Diagonal Tension in Beams

Criteria for Formation of Diagonal Cracks



The larger reinforcement ratio ρ shows the higher shear at which diagonal cracks form.

⇦ smaller and narrower flexural tension crack leaves a larger area of uncracked concrete

4. Shear & Diagonal Tension in Beams

RC BEAMS WITHOUT SHEAR REINFORCEMENT

Behavior of Diagonally Cracked Beams

Type 1

- Diagonal crack, once formed, immediately propagate from tensile reinforcement to top surface
- This process occurs chiefly in *shallower* beams.
  ; span-depth ratio of 8 or more (very common)
  ⇨ minimum shear reinforcements are required,
  even if calculation does not require it. (for ductility)

Exception where an unusually large safety factor is provided.
; some slabs or most footings does not need shear reinf.

4. Shear & Diagonal Tension in Beams

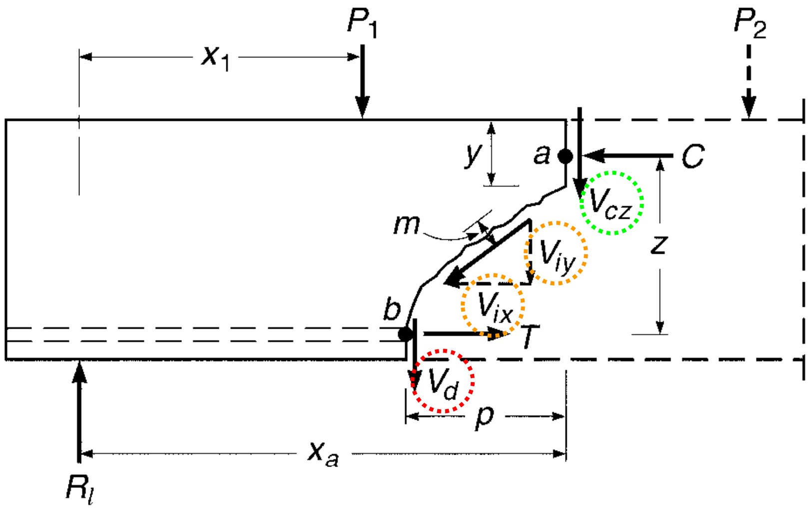
Behavior of Diagonally Cracked Beams

Type 2

- Diagonal crack, once formed, propagate *partially* into compression zone but stops shortly.
- In this case, no sudden collapse occurs.
  ; can resist higher load than that at which the diagonal crack first formed
- This process occurs chiefly in *deeper* beams.
  ; smaller span-depth ratio
  even if calculation does not require it. (for ductility)

4. Shear & Diagonal Tension in Beams

Behavior of Diagonally Cracked Beams

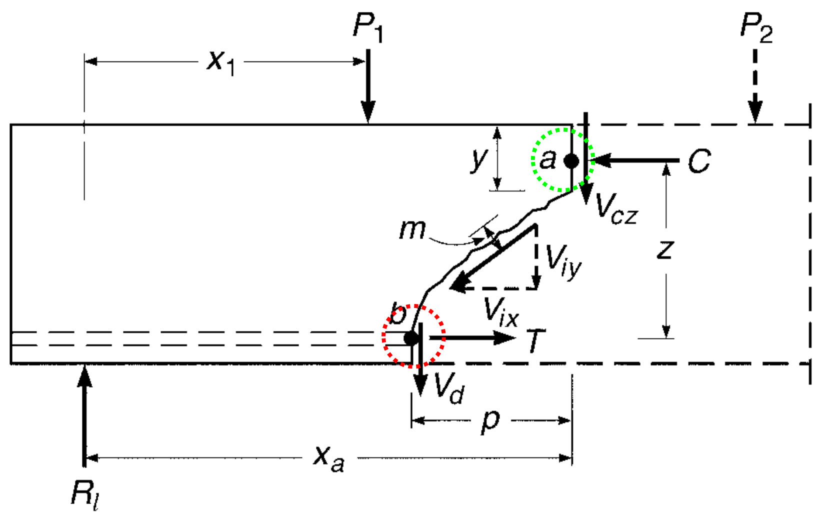


Total shear resistance $$V_{int} = V_{cz} + V_d + V_{iy} \qquad (6)$$

, where V_{cz} = shear resisted by uncracked compression zone
V_d = shear resisted by dowel action of reinforcement
V_{iy} = vertical component of aggregate interlocking

4. Shear & Diagonal Tension in Beams

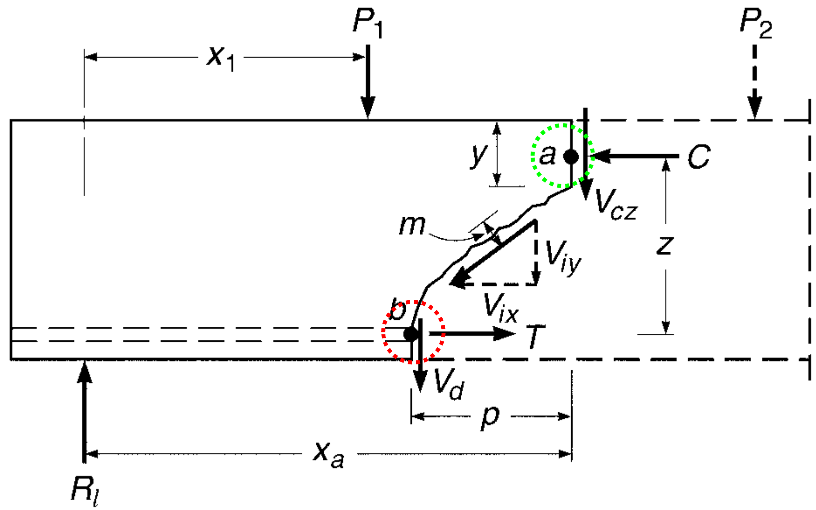
Behavior of Diagonally Cracked Beams



1. Before diagonal crack formation,
   average shear stress at the vertical section through pt. a is V_{ext}/bd.

4. Shear & Diagonal Tension in Beams

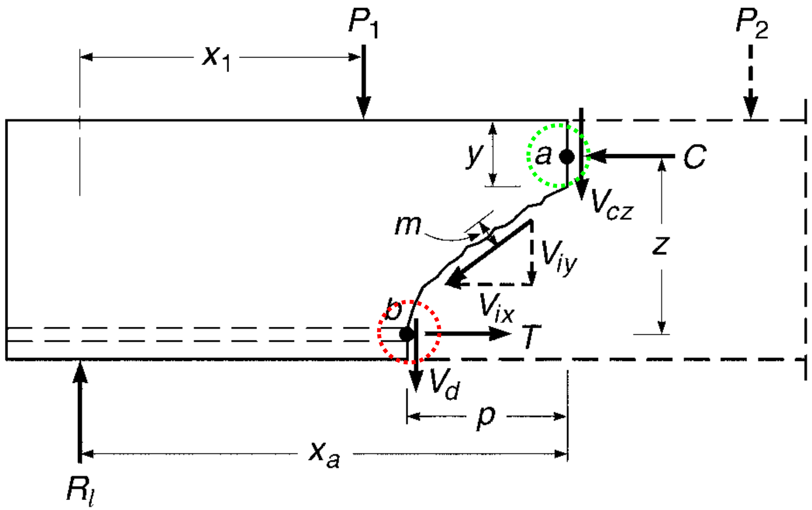
Behavior of Diagonally Cracked Beams



2. After diagonal crack formation, dowel shear + interface shear + uncracked concrete *yb* resist the external shear force.

4. Shear & Diagonal Tension in Beams

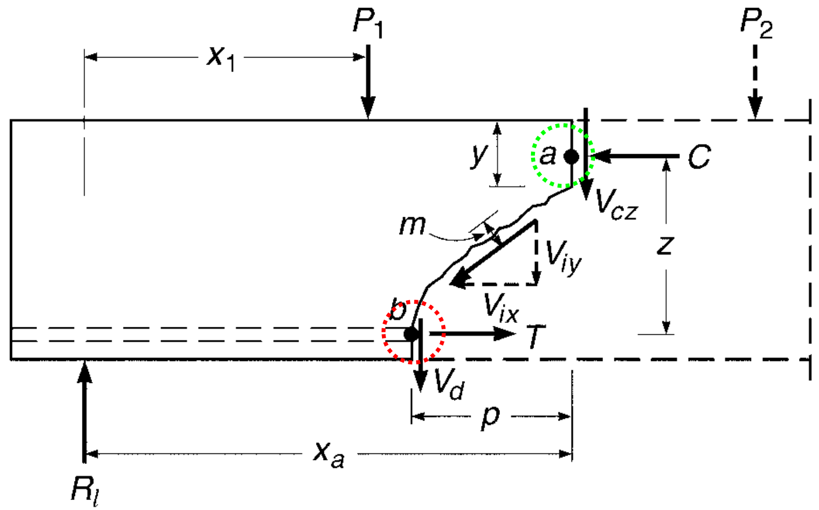
Behavior of Diagonally Cracked Beams



3. As tension splitting develops along the tension rebar, dowel shear and interface shear *decreases*
   ; the resulting shear stress on the remaining uncracked concrete *increases*

4. Shear & Diagonal Tension in Beams

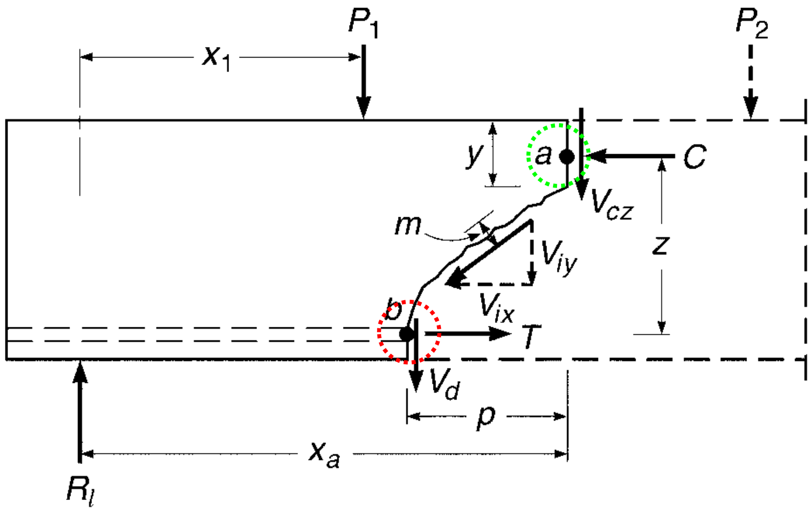
Behavior of Diagonally Cracked Beams



4. Compression force *C* also acts on *yb* smaller than that on which it acted before the crack was formed.
; diagonal crack formation *increases* the compression stress in the remaining uncracked concrete.

4. Shear & Diagonal Tension in Beams

Behavior of Diagonally Cracked Beams



5. *Before diagonal cracking*, tension at b is caused by the bending moment in vertical section through the pt. b. *After diagonal cracking*, tension at b is caused by the bending moment in vertical section through the pt. a.

4. Shear & Diagonal Tension in Beams

Behavior of Diagonally Cracked Beams

External moment at pt. a

$$M_{ext,a} = R_l x_a - P_1 (x_a - x_1)$$

Internal moment at pt. a

$$M_{int,a} = T_b z + V_d p + V_i m$$

Longitudinal tension in the steel at pt. b

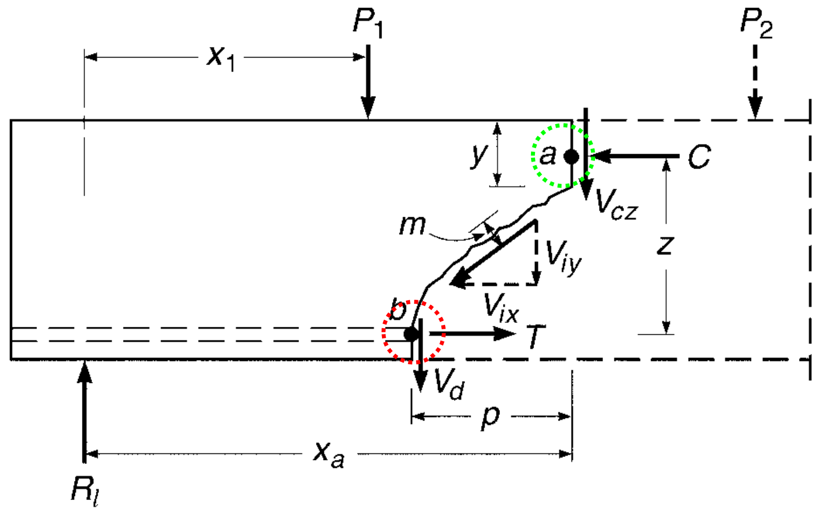
$$T_b = \frac{M_{ext,a} - V_d p + V_i m}{z}$$

$$\approx \frac{M_{ext,a}}{z}$$

V_d, V_i decrease with increasing crack opening, so *negligible*.

4. Shear & Diagonal Tension in Beams

Behavior of Diagonally Cracked Beams



6. Ultimate failure modes can be divided into three modes.
   - yielding of tension reinforcement
   - compression failure of concrete
   - splitting along the longitudinal bar (generally accompany crushing of remaining concrete)

4. Shear & Diagonal Tension in Beams

RC BEAMS with WEB REINFORCEMENT

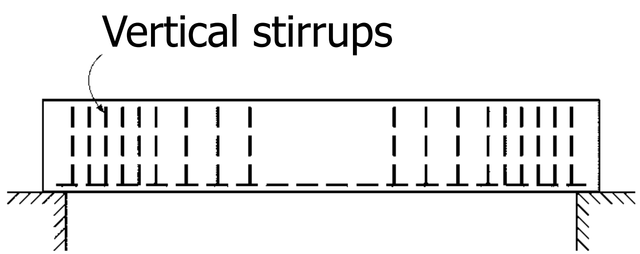
Types of web Reinforcement

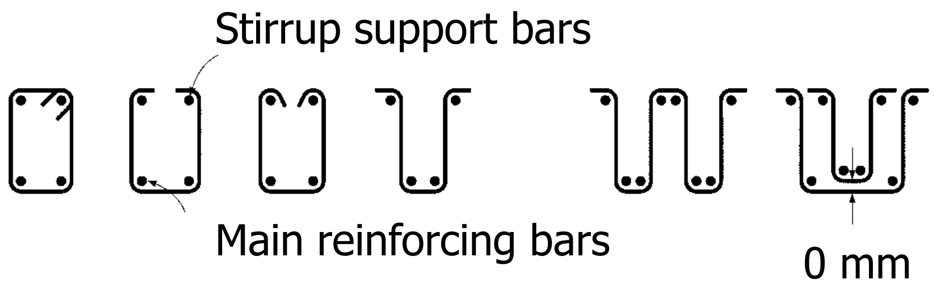
a) Vertical stirrup
: spaced at varying intervals
: conventionally D10~D16

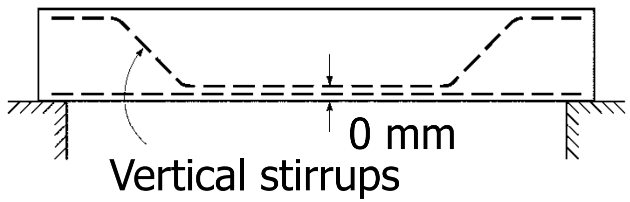
b) U-shaped bar
: most common

c) multi-leg stirrup

d) bent-up bar
: continuous beam etc.







4. Shear & Diagonal Tension in Beams

RC BEAMS with WEB REINFORCEMENT

Behavior of web-Reinforced Concrete Beams

Web reinforcement practically start working after development of diagonal crack.

Shear resistance mechanism of web reinforcement

1. Part of the shear force is resisted by the bars that traverse a particular crack.
2. Restricts the crack growth and reduce the crack propagation into compression zone.

4. Shear & Diagonal Tension in Beams

Shear resistance mechanism of web reinforcement

; this uncracked concrete resists the combined action of shear and compression

3. Web reinforcement counteract the widening of cracks

   ; very close crack surface yield a significant and reliable interlocking effect (V_i)

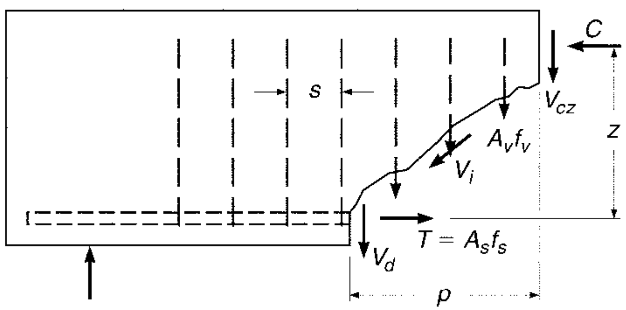
4. Tie of web reinforcement and longitudinal bar provides restraint against the splitting of concrete along the longitudinal rebar

   ; increases dowel action effect (V_d)

4. Shear & Diagonal Tension in Beams

Beams with vertical stirrups

Shear cracking stress is the same as that in a beam without web reinforcement



$$V_{ext} = V_{cz} + V_d + V_{iy} + \underline{V_s} \qquad (7)$$

,where $V_s = nA_v f_v$

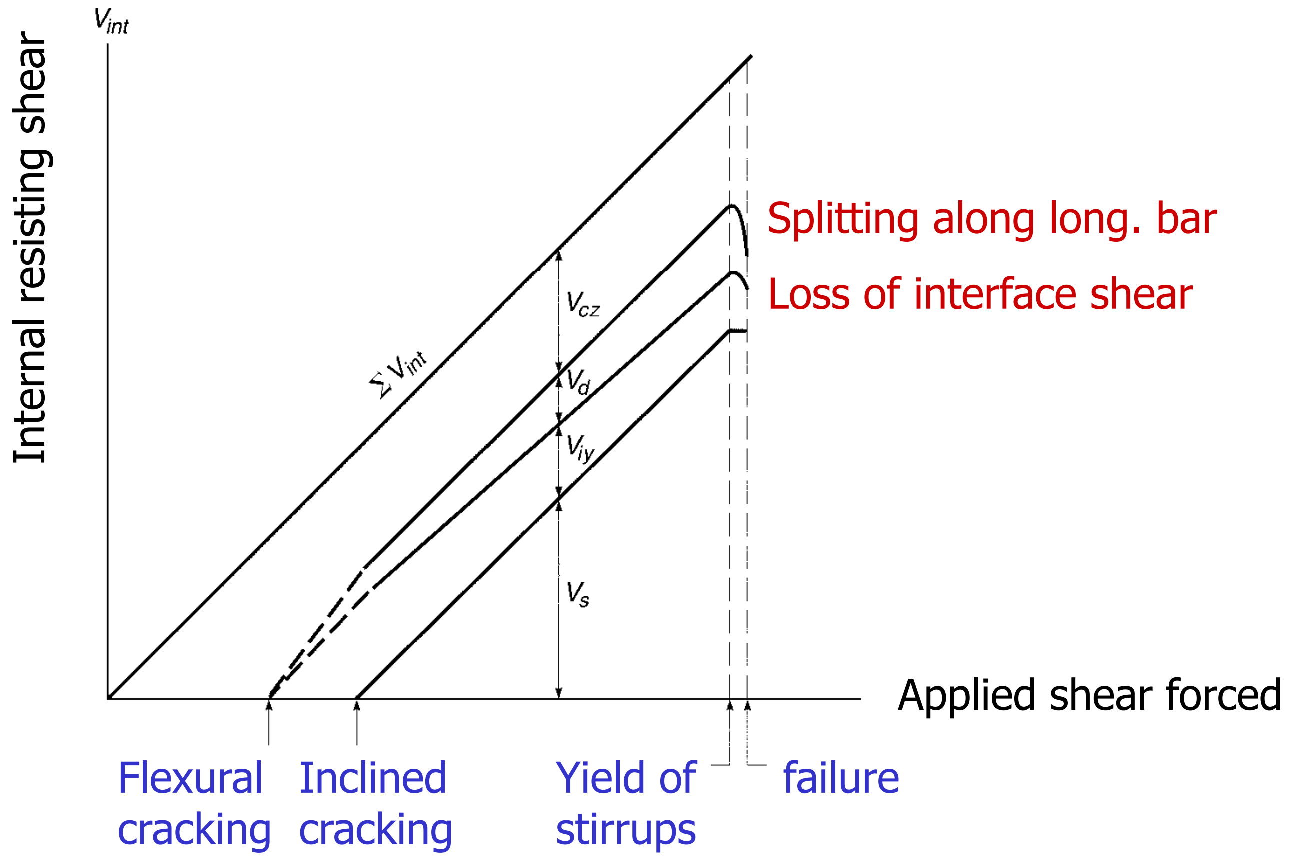
n : the number of stirrups traversing the crack=p/s

A_v : cross sectional area

(in case of U-shape stirrups, it is twice the area of one bar.)

4. Shear & Diagonal Tension in Beams

Beams with vertical stirrups



4. Shear & Diagonal Tension in Beams

Beams with vertical stirrups

- At yielding, total shear force carried by the stirrups is known but magnitudes of three other components are NOT.
- Cracking shear $V_{cr}=V_c$ (contribution of the concrete)

$$= (0.16\sqrt{f_{ck}} + 17.6\frac{\rho V d}{M})bd \leq 0.29\sqrt{f_{ck}}bd \qquad (8)$$

- Assuming the diagonal crack inclines by 45°

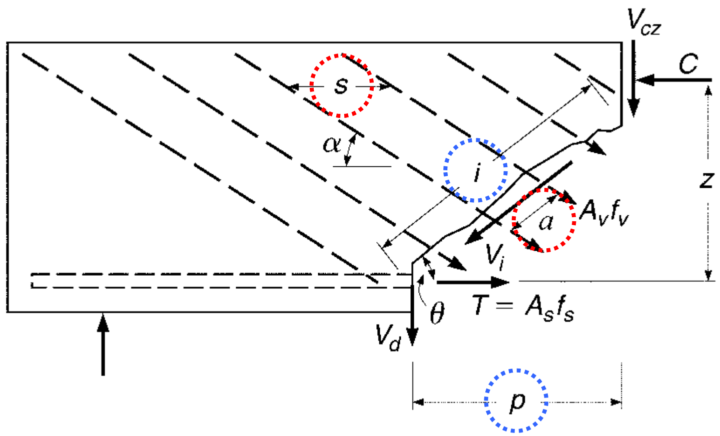
$$V_s = \frac{A_v f_y d}{s} \qquad (9)$$

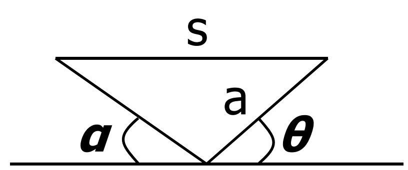
- Nominal shear strength of web reinforced beam

$$V_n = V_c + V_s \qquad (10)$$

4. Shear & Diagonal Tension in Beams

Beams with inclined bars





- The distance between bar measured parallel to the direction of the crack

$$a = \frac{s}{\sin\theta(\cot\theta + \cot\alpha)} \qquad (11)$$

- The number of bar crossing the crack

$$u = \frac{i}{a} = \frac{p}{s}(1 + \cot\alpha\tan\theta) \qquad (12)$$

4. Shear & Diagonal Tension in Beams

Beams with inclined bars

- Vertical component of the forces in all bars that cross crack

$$V_s = nA_v f_v \sin\alpha = A_v f_v \frac{p}{s}(\sin\alpha + \cos\alpha \tan\theta) \quad (13)$$

- Assuming diagonal crack to be 45°

$$V_s = A_v f_v \frac{d}{s}(\sin\alpha + \cos\alpha) \quad (14)$$

- Nominal shear strength of inclined web reinforcement

$$M_n = V_c + A_v f_v \frac{d}{s}(\sin\alpha + \cos\alpha) \quad (15)$$

4. Shear & Diagonal Tension in Beams

Beams with inclined bars

Note

- Eq. (13) and Eq. (14) are valid ONLY WHEN diagonal crack is traversed by at least one vertical or inclined bar.
- Therefore, upper limit on the spacing is required to ensure that the web reinforcement is actually effective.

4. Shear & Diagonal Tension in Beams

KCI CODE PROVISIONS FOR SHEAR DESIGN

KCI 7.2.1 design of beams for shear

$$V_u \le \phi V_n \tag{16}$$

for vertical stirrup

$$V_u \le \phi V_c + \frac{\phi A_v f_y d}{s} \tag{17}$$

for inclined stirrup

$$V_u \le \phi V_c + \frac{\phi A_v f_y d(\sin\alpha + \cos\alpha)}{s} \tag{18}$$

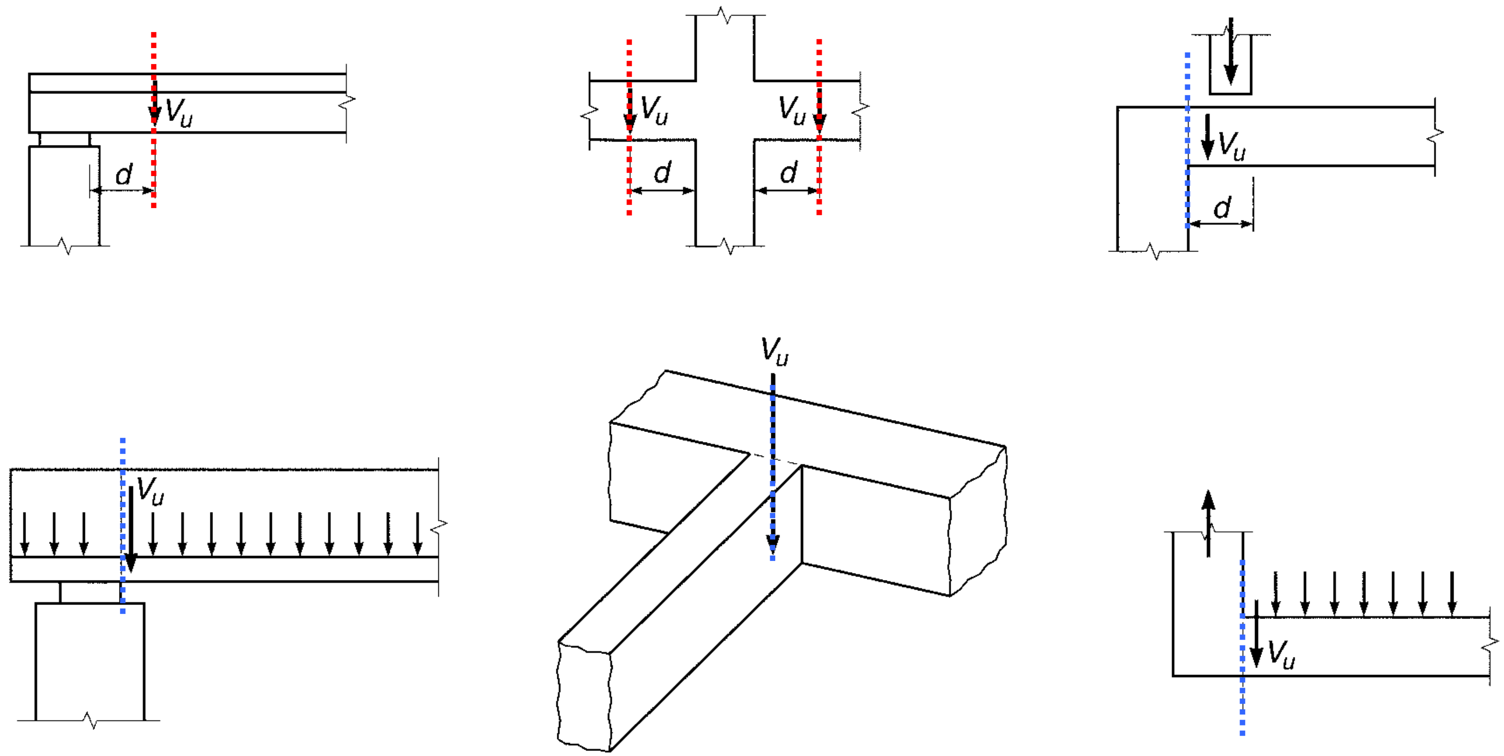
<u>Note</u>

- The strength reduction factor ϕ is 0.75
- More conservative than ϕ =0.85 for flexure

4. Shear & Diagonal Tension in Beams

KCI CODE PROVISIONS FOR SHEAR DESIGN

Location of Critical Section for Shear Design



4. Shear & Diagonal Tension in Beams

KCI CODE PROVISIONS FOR SHEAR DESIGN

Shear Strength Provided by the Concrete

- Generalized expression considering T-shape beam

$$V_c = \left(0.16\sqrt{f_{ck}} + 17.6 \frac{\rho_w V_u d}{M_u} \right) b_w d \quad \leq \quad 0.29\sqrt{f_{ck}} b_w d \qquad (19)$$

Note

· $V_u d / M_u$ should be taken less than 1.o

- Simple expression permitted by KCI 7.3.1

$$V_c = \frac{1}{6}\sqrt{f_{ck}} b_w d \qquad (20)$$

see slide 17

KCI CODE PROVISIONS FOR SHEAR DESIGN

Shear Strength Provided by the Concrete

Note

- Eq. (19) and Eq. (20) are based on beams with concrete compressive strength in the range of 21 to 35 MPa.
- Eq. (19) and Eq. (20) are only applicable to normal-weight concrete.
- For light-weight concrete, split-cylinder strength f_{sp} is used as a reliable measure.

4. Shear & Diagonal Tension in Beams

KCI CODE PROVISIONS FOR SHEAR DESIGN

Shear Strength Provided by the Concrete

Note

- For normal concrete f_{sp} is taken equal to $0.57\sqrt{f_{ck}}$

  ; KCI specifies that $f_{sp}/0.57$ shall be substitute for $\sqrt{f_{ck}}$ in all equation for V_c.

  ; if f_{sp} is not available, V_c calculated using $\sqrt{f_{ck}}$ must be multiplied by 0.75 for all-lightweight concrete
  0.85 for sand-lightweight concrete

4. Shear & Diagonal Tension in Beams

Minimum Web Reinforcement

Even though $V_u \leq \phi V_c$, KCI Code requires provision of at least a minimum web reinforcement

$$A_{v,\min} = 0.0625\sqrt{f_{ck}}\frac{b_w s}{f_y} \quad \geq \quad 0.35\frac{b_w s}{f_y} \qquad (13)$$

where $V_u > \frac{1}{2}\phi V_c$

Exceptions

1. Slabs, footings, and concrete joist construction
2. Total depth h of beams ≤ largest of 250mm, $2.5h_f$, or $0.5b_w$

⇦ Their capacity to redistribute internal forces before diagonal tension failure is confirmed by test & experience.

4. Shear & Diagonal Tension in Beams

Design of web Reinforcement

Spacing of web reinforcement

For vertical
$$s = \frac{\phi A_v f_y d}{V_u - \phi V_c} \tag{14}$$

For bent bar
$$s = \frac{\phi A_v f_y d(\sin\alpha + \cos\alpha)}{V_u - \phi V_c} \tag{15}$$

(α≥30° and only the center ¾ of the inclined part is effective)

Minimum spacing of web reinforcement

- It is undesirable to space vertical stirrups closer than 100mm

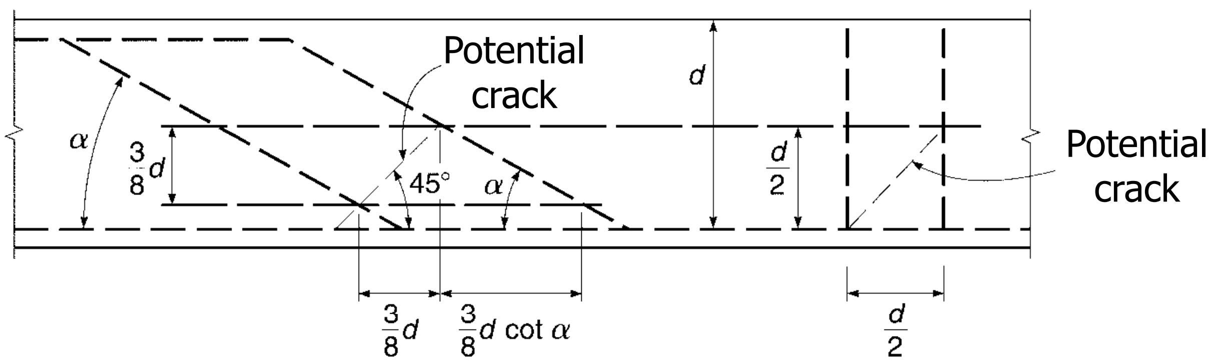
4. Shear & Diagonal Tension in Beams

Design of web Reinforcement

Maximum spacing of web reinforcement

- Basic assumptions :

1) web crack occurs with an angle of 45°
2) crack extends from the mid-depth *d*/2 to the tension rebar
3) crack is crossed by at least one line of web reinforcement



4. Shear & Diagonal Tension in Beams

Maximum spacing of web reinforcement

for $V_s \le 0.33\sqrt{f_{ck}}b_w d$, the smallest of

$$s_{\max} = \frac{A_v f_y}{0.0625\sqrt{f_{ck}}b_w} \le \frac{A_v f_y}{0.35b_w} \quad \text{or} \qquad (16)$$

$$= \frac{d}{2} \quad \text{or} \qquad (17)$$

$$= 600mm \qquad (18)$$

for $V_s > 0.33\sqrt{f_{ck}}b_w d$,
the above maximum spacing should be *halved*.

Note For longitudinal bars bent at 45°, Eq.(17) is replaced by
$s_{max}=3d/4$

4. Shear & Diagonal Tension in Beams

Design of web Reinforcement

additional KCI Code provisions

- The yield strength of the web reinforcement is limited to 400 MPa or less.
- In no cases, V_s cannot exceed $0.67\sqrt{f_{ck}}b_w d$ regardless of web steel used.

4. Shear & Diagonal Tension in Beams

Example 4.1

A rectangular beam is designed to carry a shear force V_u of 120kN. No web reinforcement is to be used, and f_{ck} is 27MPa. What is the minimum cross section if controlled by shear?

Solution

KCI requirement $\quad \underline{V_u < \frac{1}{2}\phi V_c} = \frac{1}{2}\phi\left(\frac{1}{6}\sqrt{f_{ck}}b_w d\right)$

$$\Rightarrow b_w d > \frac{12V_u}{\phi\sqrt{f_{ck}}} = \frac{(12)(120)(10^3)}{(0.75)(\sqrt{27})} = 349,500mm^2$$

$\therefore b_w = 475mm, d = 800mm$ is required

Region in which web Reinforcement is Required

Example 4.2

A simply supported rectangular beam.
b_w=400, d=600, clear span 8m,
factored load=110kN/m, f_{ck}=27MPa, A_s=4,910mm^2

What part of the beam is web reinforcement required?

Solution

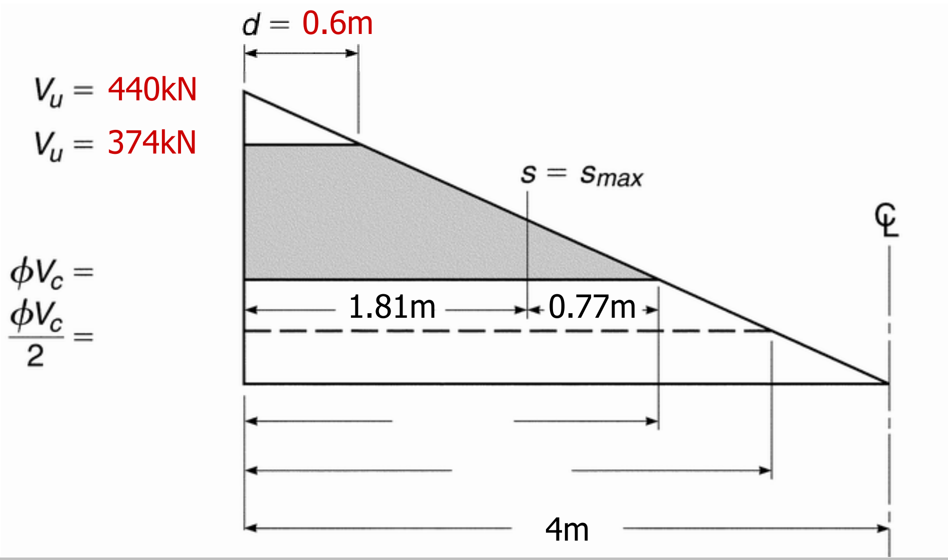
1) The maximum external shear force

$$V_u = (110)\left(\frac{8}{2}\right) = 440kN \qquad \text{at the ends of span}$$

2) At the shear critical section, a distance from the support

$$V_u = (110)\left(\frac{8}{2} - 0.6\right) = 374kN$$

Because, shear force varies linearly to zero at mid span



4. Shear & Diagonal Tension in Beams

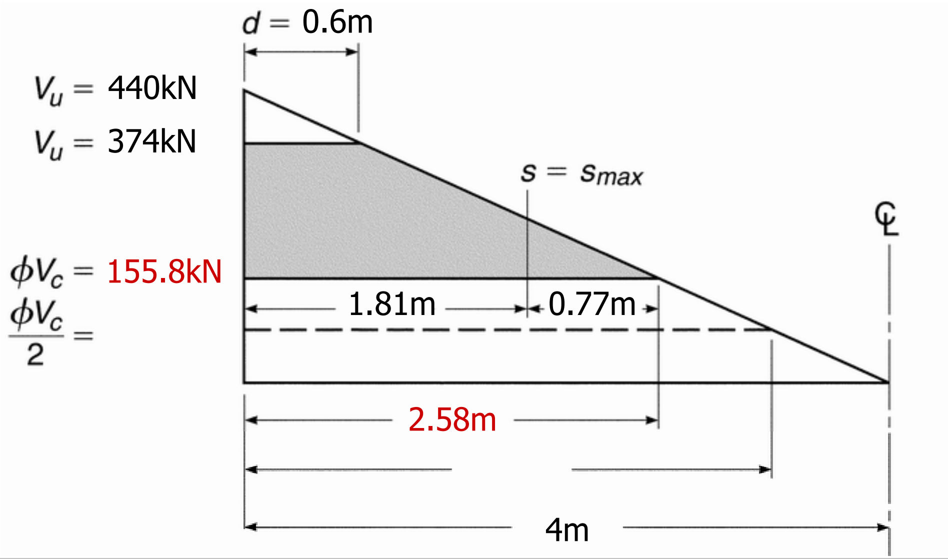
3) Concrete portion

$$V_c = \frac{1}{6}\sqrt{f_{ck}}b_w d = \frac{1}{6}(\sqrt{27})(400)(600) = 207.8kN$$

$$\Rightarrow \quad \phi V_c = (0.75)(207.8) = 155.8kN$$

; the point at which web reinforcement theoretically is no longer required is

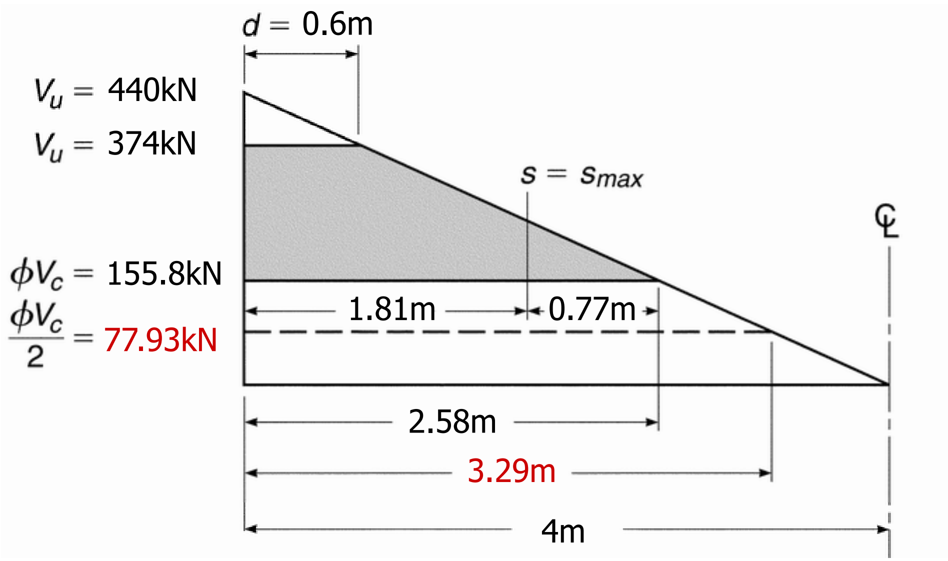
$$(4.0)\left(\frac{440-155.8}{440}\right) = 2.58m$$



4. Shear & Diagonal Tension in Beams

4) According to KCI Code, at least a minimum amount of web reinforcement is required wherever the shear force exceeds $\phi V_c/2$ (=77.93kN)

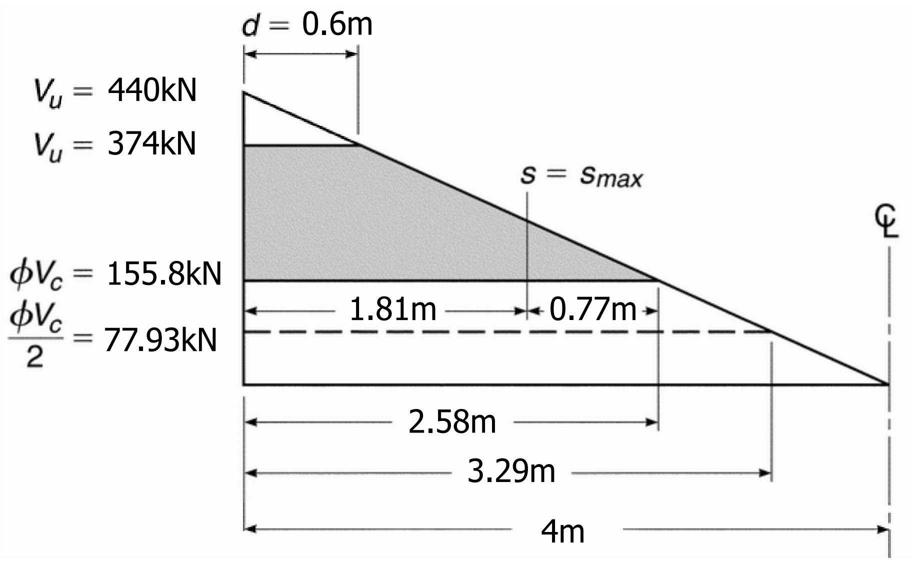
$$(4.0)\left(\frac{440-77.93}{440}\right)=3.29m$$



4. Shear & Diagonal Tension in Beams

Summary

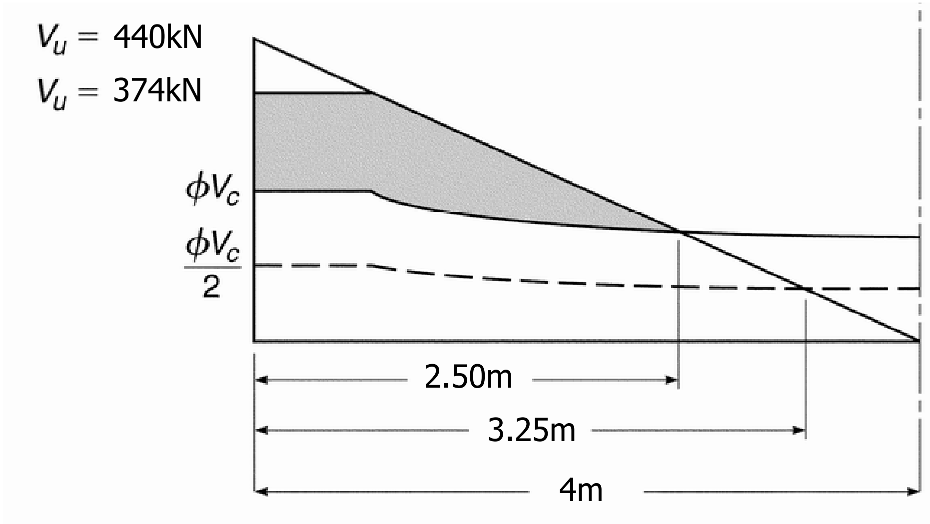
- At least the minimum web steel must be provided within a distance 3.29m from support.
- and within 2.58m the web steel must be provided for the shear force corresponding to the shaded area.



4. Shear & Diagonal Tension in Beams

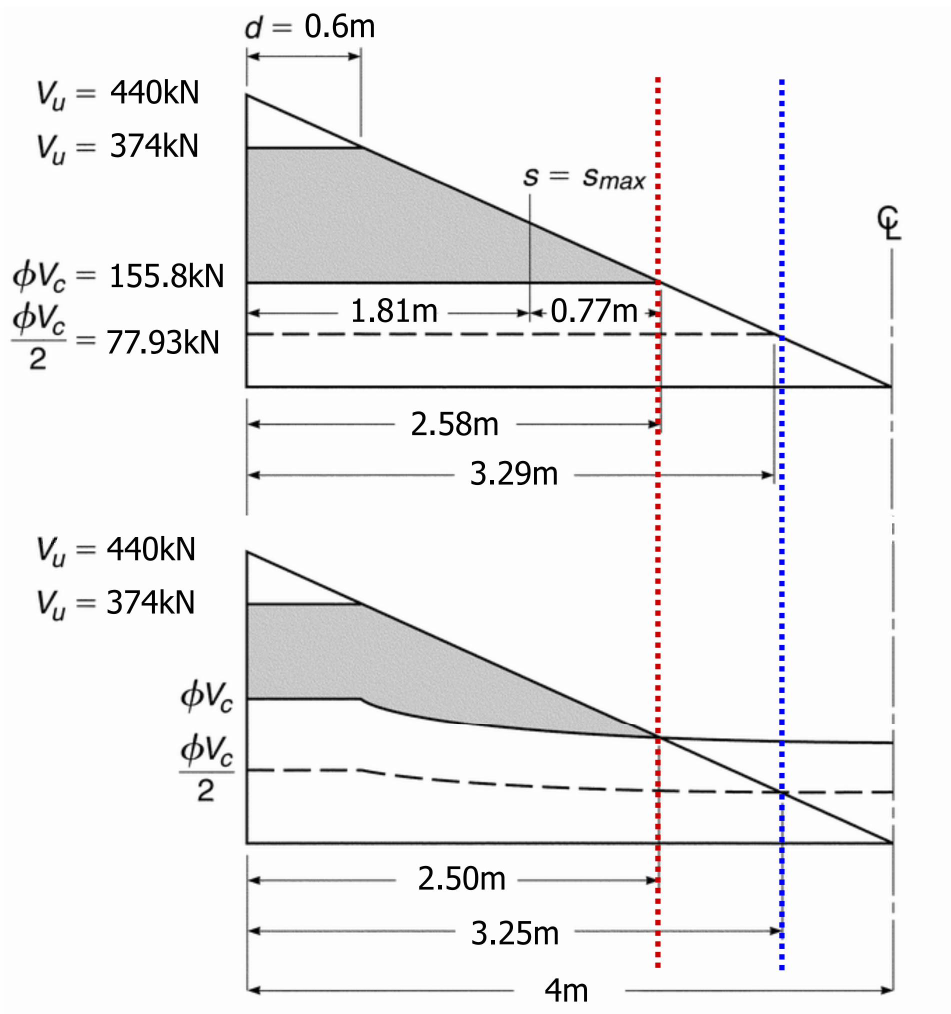
– If we use the alternative Eq. (19), V_c can be calculated with the varying ρ_w, V_u, and M_u along the span.

Distance from support (m)	M_u (kN·m)	V_u (kN)	V_c (kN)	$ØV_c$ (kN)
0	0	374	279	223.2
0.6	244	374	279	223.2
1.0	385	330	244	195.2
1.5	536	275	226	180.9
2.0	660	220	217	173.5
2.5	756	165	211	168.7
3.0	825	110	206	165.2
3.5	866	55	203	162.3
4.0	880	0	200	159.6



Comparison two methods

- The length over which web reinforcement is needed is nearly the same.
- The smaller shaded area of (b) means that the more accurate Eq. (19) can reduce the amount of web reinforcement.



4. Shear & Diagonal Tension in Beams

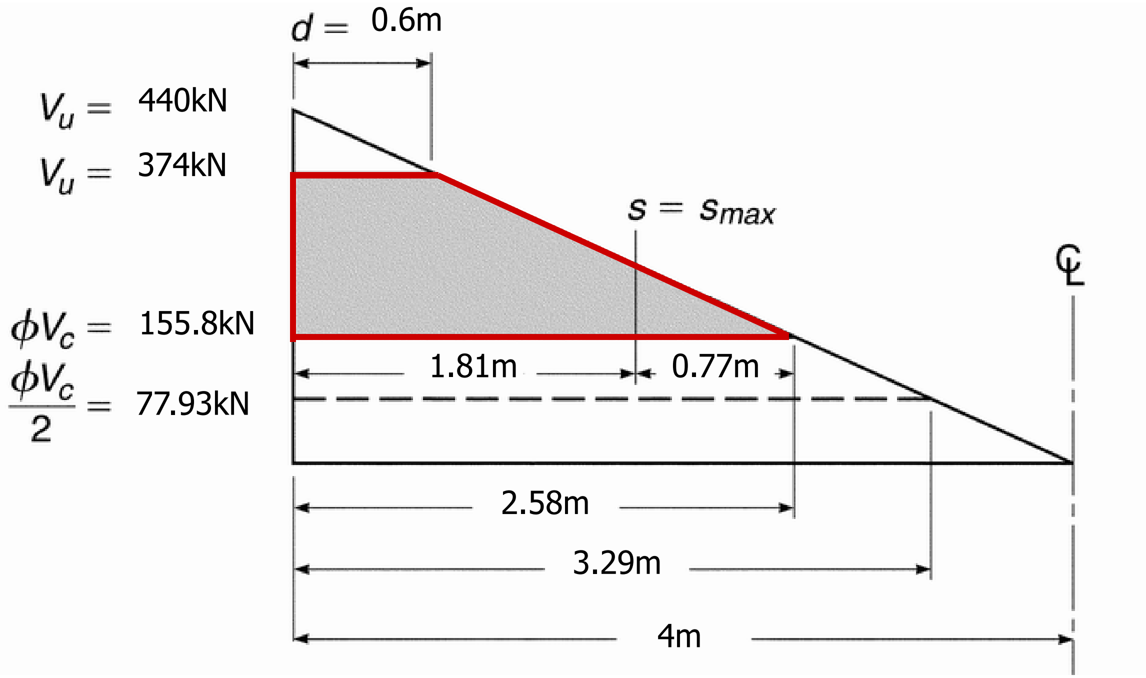
Design of web Reinforcement

Example 4.3

A simply supported rectangular beam.
b_w=400, d=600, clear span 8m,
factored load=110kN/m, f_{ck}=27MPa, A_s=4,910mm^2

using vertical U-shape stirrups with f_y=400MPa,
Design the web reinforcement.

Solution



Mission – make a design to resist the shear force corresponding to shaded area.

4. Shear & Diagonal Tension in Beams

$$\phi V_s = V_u - \phi V_c = 374 - 155.8 = 218.2kN$$

1) Maximum spacing

With D10 stirrups used for trial,

$$\phi V_s < 0.33\phi\sqrt{f_{ck}}b_w d = 309kN$$

i) $$\frac{A_v f_y}{0.35b_w} = \frac{(142)(400)}{(0.35)(400)} = 406mm$$

ii) $$\frac{d}{2} = \frac{600}{2} = \underline{300mm}$$ *controls*

iii) $$600mm$$

4. Shear & Diagonal Tension in Beams

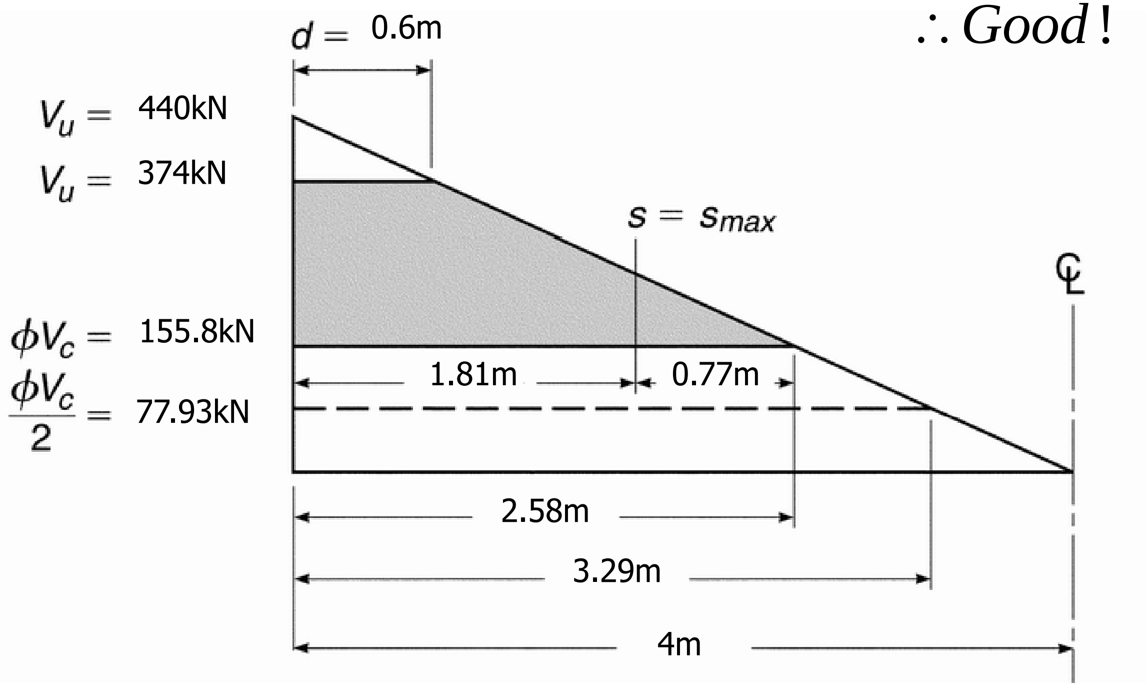
2) For the excessive shear V_u-ΦV_c at a distance d from the support.

$$s = \frac{\phi A_v f_y d}{V_u - \phi V_c} = \frac{(0.75)(142)(400)(600)}{(218.2)(10^3)} = \underline{117mm}$$

$$> 100mm$$

$$< 300mm$$

$$\therefore Good!$$



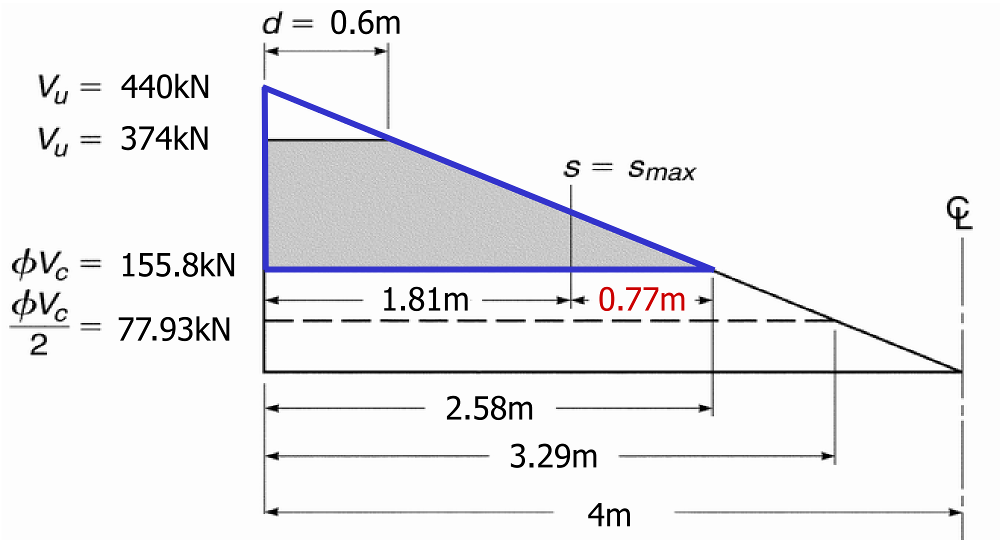
4. Shear & Diagonal Tension in Beams

3) For maximum spacing s_{max}=300mm.

$$V_u - \phi V_c = \frac{\phi A_v f_y d}{s} = \frac{(0.75)(142)(400)(600)}{(300)(10^3)} = 85.2kN$$

this value is attained at a distance 0.77m from the point of zero excess shear

⇦ 0.77=(2.58)(85.2)/(440-155.8)



4. Shear & Diagonal Tension in Beams

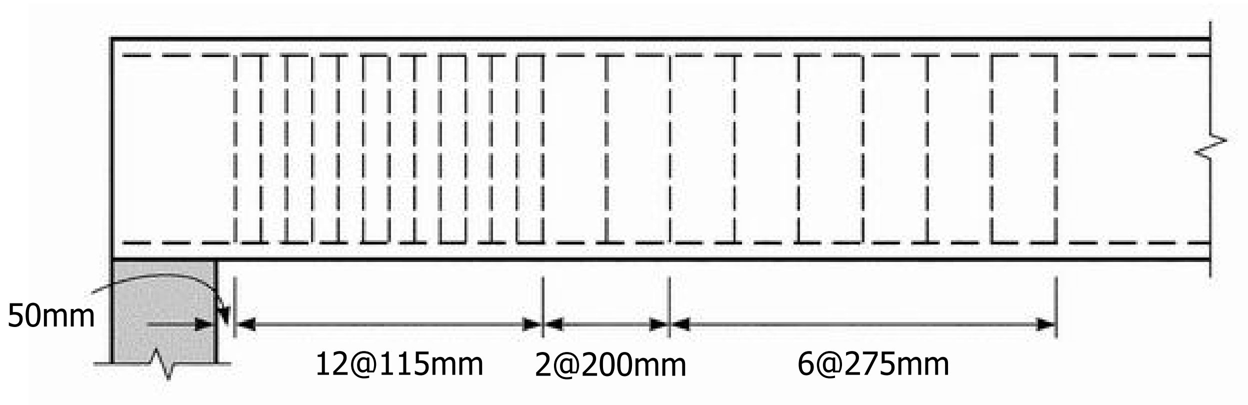
4) Location of first stirrup

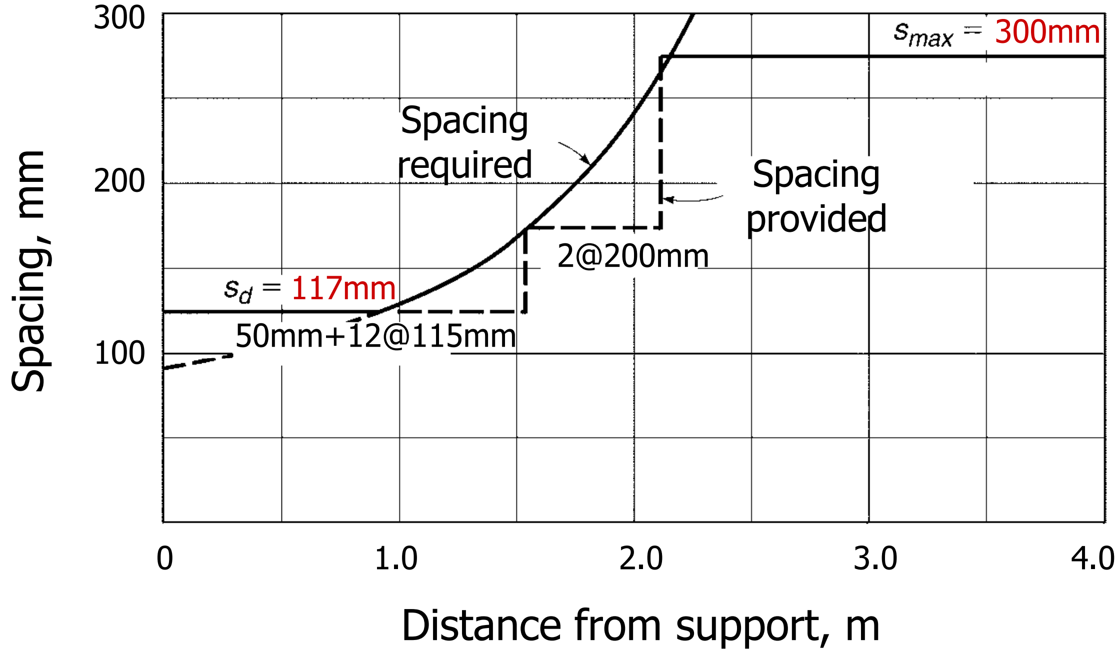
- No specific requirement
- usually placed at a distance s/2 from the support.

5) Now, we can array stirrups. For example,

1 space at 50mm	= 50mm
12 spaces at 115mm	= 1,380mm
2 spaces at 200mm	= 400mm
6 spaces at 275mm	= 1,650mm
Total	= 3,480mm

4. Shear & Diagonal Tension in Beams





DEEP BEAMS

- Structural elements loaded as beams but having a large depth/thickness ratio and a shear span/depth ratio not exceeding 2.5

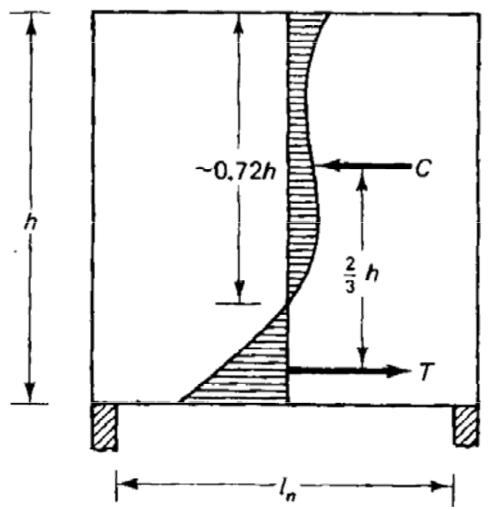
e.g) - floor slab under horizontal loads
- wall slab under vertical loads
- short span beam carrying heavy loads
- shear walls

DEEP BEAMS

- Because of the geometry of deep beams, they are subjected to two-dimensional state of stress.

⇨ Bernoulli's hypothesis is not valid any longer.

⇨ Stress distribution is quite different from that of normal beams.

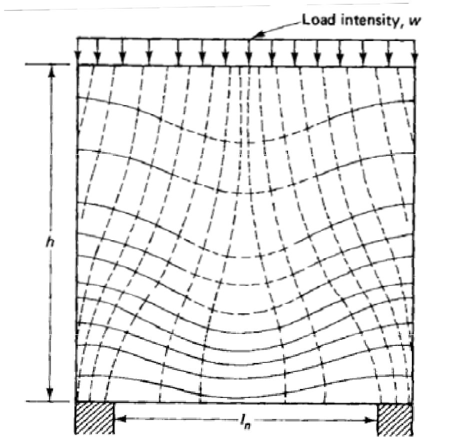


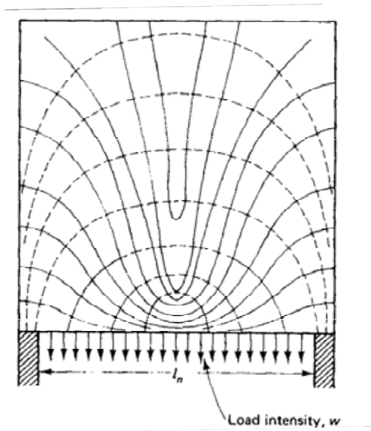
4. Shear & Diagonal Tension in Beams

DEEP BEAMS

- Tensile stress trajectories are steep and concentrated at midspan and compressive stress trajectories are concentrated at supports.
- Due to a higher compressive ARCH ACTION, V_c for deep beams will considerately exceed V_c for normal beams.

⇨ SHEAR in deep beam is a major consideration.





DEEP BEAMS

- Because the shear span is small,

  the compressive stress in the support region affect the magnitude and direction of the principal tensile stresses such that they becomes LESS inclined and lower in value.

4. Shear & Diagonal Tension in Beams

DEEP BEAMS

KCI Code provisions for deep beam (KCI 7.8)

- This criteria is only valid for shear design of deep beams ($a/d < 2.5$ and $l_n/d < 4.0$) loaded at the TOP

  where, a : shear span for concentrated load

  l_n : clear span for uniformly distributed load

- Location of critical section is

  for uniform load $x = 0.15 l_n < d$

  for concentrated load $x = 0.50 a < d$ (19)

4. Shear & Diagonal Tension in Beams

- The factored shear force V_u

$$V_u \le \phi(\frac{2}{3}\sqrt{f_{ck}}b_w d) \quad \text{for} \quad l_n / d < 2 \tag{20}$$

$$V_u \le \phi\left[\frac{1}{18}(10+\frac{l_n}{d})\right]\sqrt{f_{ck}}b_w d \quad \text{for} \quad 2 \le l_n / d \le 5 \tag{21}$$

if not, section to be enlarged.

- The nominal shear resisting force V_c of plain concrete

$$V_c = (3.5-2.5\frac{M_u}{V_u d})(0.16\sqrt{f_{ck}} + 17.6\rho_w \frac{V_u d}{M_u})b_w d \le 0.5\sqrt{f_{ck}}b_w d \tag{22}$$

, where the first terms

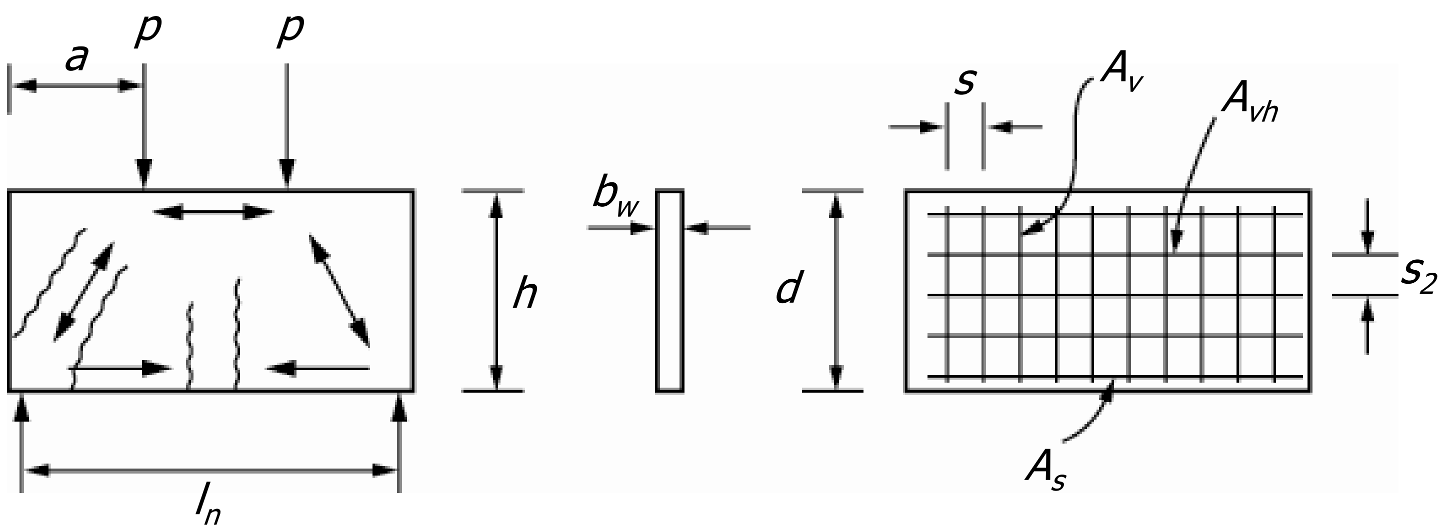
$$3.5-2.5\frac{M_u}{V_u d} \le 2.5 \tag{23}$$

4. Shear & Diagonal Tension in Beams

- When the factored shear V_u exceeds φV_c

$$V_s = \left[\frac{A_v}{s}\left(\frac{1+l_n/d}{12}\right) + \frac{A_{vh}}{s_2}\left(\frac{11-l_n/d}{12}\right) \right] f_y d \qquad (24)$$

, where A_v = total area of vertical reinforcement spaced at s
A_{vh} = total area of horizontal reinforcement spaced at s_h



4. Shear & Diagonal Tension in Beams

- maximum $s \leq d/5$ or 300mm
  maximum $s_h \leq d/5$ or 300mm

  whichever is smaller

and

minimum $A_v = 0.0025 b_w s$

minimum $A_{vh} = 0.0015 b_w s_h$

ALTERNATIVE MODELS FOR SHEAR ANALYSIS & DESIGN

KCI Code Provisions Deficiencies

- Design methods for shear and diagonal tension in beams are empirical
- "$V_s + V_c$" approach lacks a physical model for beams subjected to shear combined with bending.
- Each contribution of three components of V_c is not identified.

ALTERNATIVE MODELS FOR SHEAR ANALYSIS & DESIGN

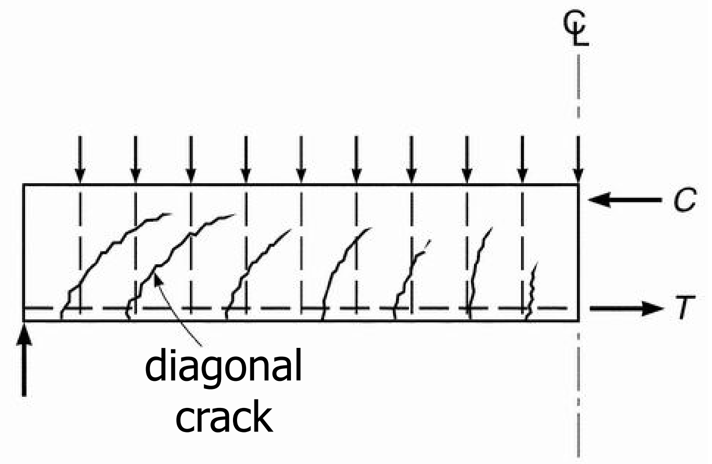
KCI Code Provisions Deficiencies

- Diagonal cracking loads (Eq.19)
  - overestimate beams with low reinforcement ratio(ρ<0.001),
  - overestimate the gain in shear strength by using high strength concrete,
  - underestimate the influence of $V_u d/M_u$ and
  - ignore the size effect that shear strength decreases as member size increase *s*.

4. Shear & Diagonal Tension in Beams

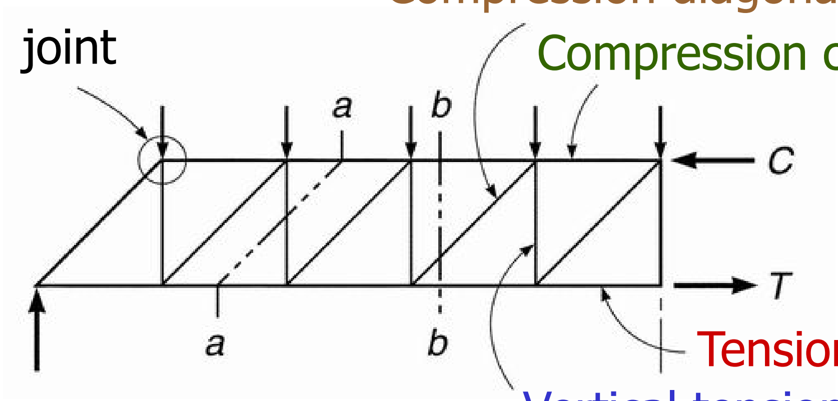
45° Truss Model

- Originally introduced by Ritter(1899) and Morsch(1902)
- This simple model has long provided the basis for the ACI/KCI Code design of shear steel.
- Consider a reinforced concrete beam subject to uniformly distributed load.



4. Shear & Diagonal Tension in Beams

45° Truss Model

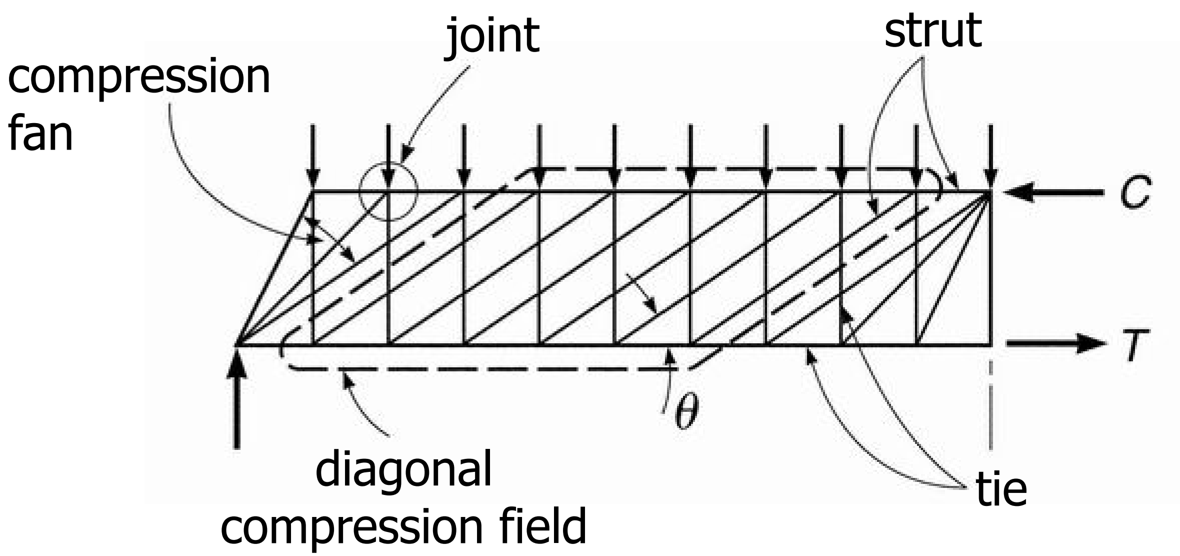


- longitudinal tension steel – tension chord
  concrete top flange – compression chord
  vertical stirrups – vertical tension web member
  the concrete between cracks - 45° compression diagonals
- This model is quite conservative for beams with small amount of web reinforcements.

4. Shear & Diagonal Tension in Beams

Variable Angle Truss Model

- Recently the truss concept has been greatly extended by Schlaich, Thurlimann, Marti, Collins, MacGregor, etc.
- It was realized that the angle of inclination of the concrete strut may range between 25° and 65°



4. Shear & Diagonal Tension in Beams

Variable Angle Truss Model

Improved model components

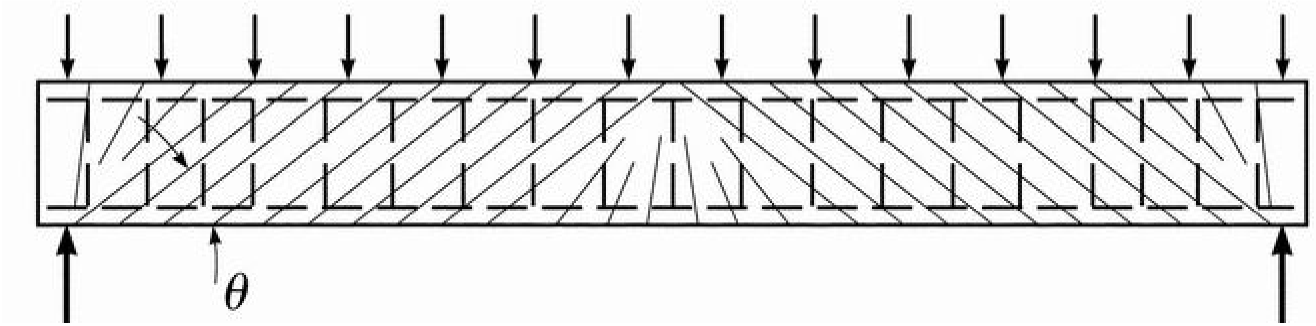
1) Strut or concrete compression member uniaxially loaded.
2) Ties or steel tension member
3) Pin connected joint at the member intersection
4) Compression fans, which forms at the supports or under concentrated loads, transmitting the forces into the beam.
5) Diagonal compression field, occurring where parallel compression struts transmit forces from one stirrup to another.

4. Shear & Diagonal Tension in Beams

Compression Field Theory

- is mandatory for shear design in AASHTO LRFD Bridge Design Specification of the U.S. *(Handout 4-1)*
- accounts for requirements of compatibility as well as equilibrium and incorporates stress-strain relationship of material.

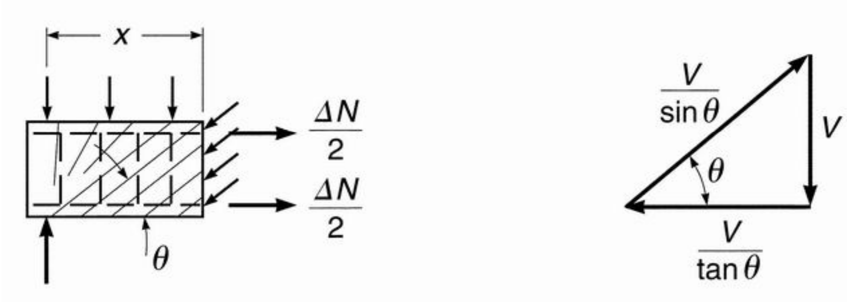
⇨ can predict not only the failure load but also the complete load-deformation response.



4. Shear & Diagonal Tension in Beams

Compression Field Theory

- the net shear V at a section a distance x from the support is resisted by the vertical component of the diagonal compression force in the concrete struts.



- The horizontal component of the compression in the struts must be equilibrated by the total tension force ΔN in the longitudinal steel.

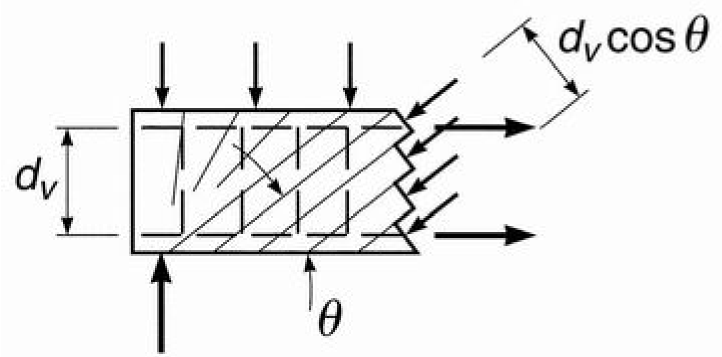
$$\Delta N = \frac{V}{\tan\theta} = V\cot\theta \qquad (25)$$

⇨ These forces superimpose on the longitudinal forces due to flexure.

4. Shear & Diagonal Tension in Beams

Compression Field Theory

- letting the effective depth for shear calculation d_v, the distance between longitudinal force resultants.



- The diagonal compressive stress in a web having b_v is,

$$f_d = \frac{V}{b_v d_v \sin\theta \cos\theta} \qquad (26)$$

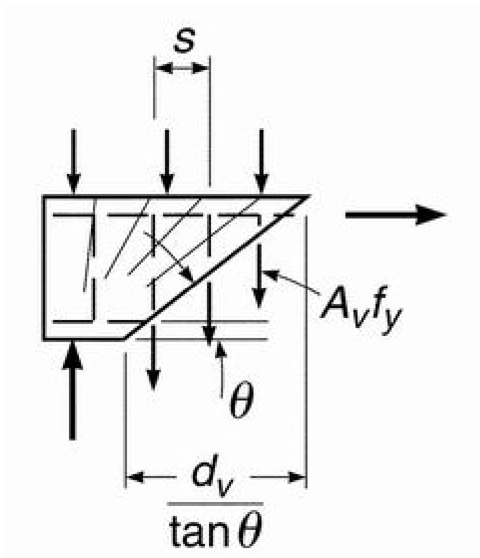
$$\Leftarrow \quad \frac{1}{\sin\theta} = d_v \cos\theta \cdot f_d \cdot b_v$$

4. Shear & Diagonal Tension in Beams

Compression Field Theory

- The tensile force in the vertical stirrups, each having area A_v and assumed to act at the yield stress f_y and uniformly spaced at s,

$$A_v f_y = \frac{V \cdot s \cdot \tan\theta}{d_v} \qquad (27)$$



4. Shear & Diagonal Tension in Beams

Compression Field Theory

Note

1. Vertical stirrups within the length $d_v/\tan\theta$ can be designed to resist the lowest shear that occurs within this length, i.e., the shear at the right end.
2. The angle θ range from 20° to 75°, but it is economical to use an angle θ somewhat less than 45°.
3. If a lower slope angle is selected, less vertical reinforcement but more longitudinal reinforcement will be required, and the compression in the concrete diagonals will be increased.

4. Shear & Diagonal Tension in Beams

Modified Compression Field Theory *(Handout 4-2)*

- The cracked concrete is treated as a new material with its own stress-strain relationships including the ability to carry tension following crack formation.
- As the diagonal tensile strain in the concrete INCREASES, the compressive strength and s-s curve of the concrete in the diagonal compression struts DECREASES.
- Equilibrium Compatibility, Constitutive relationship are formulated in terms of average stress and average strains.
- Variability of inclination angle and stress-strain softening effects are considered.

4. Shear & Diagonal Tension in Beams

AASHTO LRFD Bridge Design Specification

- is based on MCFT and nominal shear capacity V_n is

$$V_n = V_c + V_s$$

$$= 0.25 f_{ck} b_v d_v \quad (28)$$

, where b_v is web width(= b_w in KCI) and d_v is effective depth in shear (distance between the centroids of the tensile and compressive forces), but not less than *0.9d*

4. Shear & Diagonal Tension in Beams

AASHTO LRFD Bridge Design Specification

- Each contribution of concrete and steel

$$V_c = 0.083\beta\sqrt{f_{ck}}\,b_v d_v \tag{29}$$

$$V_s = \frac{A_v f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}{s} \tag{30}$$

, where β is the concrete tensile stress factor. (indicates the ability of diagonally cracked concrete to resist tension) Also, control the angle of diagonal tension crack.

4. Shear & Diagonal Tension in Beams

AASHTO LRFD Bridge Design Specification

- β and θ are determined by the average shear stress and longitudinal strain of concrete ε_x,

$$v_u = \frac{V_u}{b_v d_v} \qquad (31)$$

$$\varepsilon_x = \frac{M_u / d_v - 0.5N_u + V_u}{\alpha(E_s A_s)} \leq 0.002 \qquad (32)$$

, where α=2 for beams containing at least minimum transverse reinforcements

α=1 for beams with less than the minimum transverse reinforcements

, and N_u is positive for compression.

4. Shear & Diagonal Tension in Beams

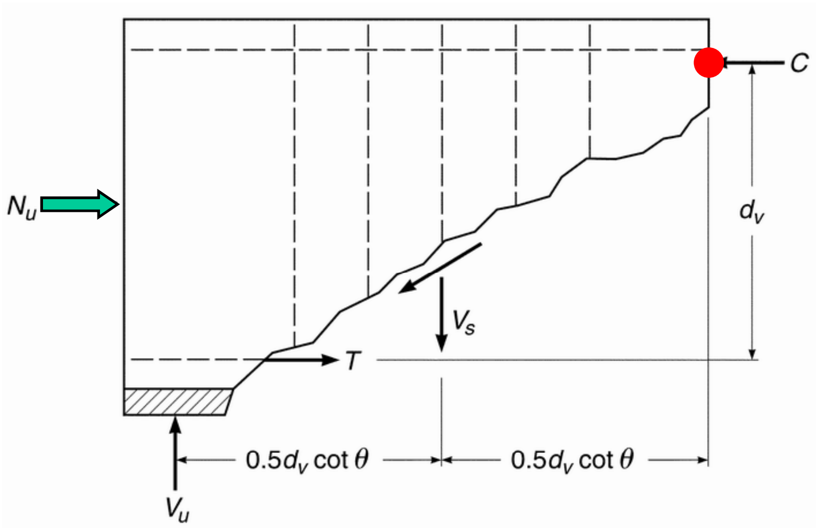
Table 4.1 For sections with at least minimum transverse reinforcement

v_u/f_{ck}		$\varepsilon_x \times 1{,}000$										
		≤ −0.20	≤ −0.10	≤ −0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00	≤1.50	≤2.00
≤0.075	θ	22.3	20.4	21.0	21.8	24.3	26.6	30.5	33.7	36.4	40.8	43.9
	β	6.32	4.75	4.10	3.75	3.24	2.94	2.59	2.38	2.23	1.95	1.67
≤0.1	θ	18.1	20.4	21.4	22.5	24.9	27.1	30.8	34.0	36.7	40.8	43.1
	β	3.79	3.38	3.24	3.14	2.91	2.75	2.50	2.32	2.18	1.93	1.69
≤0.125	θ	19.9	21.9	22.8	23.7	25.9	27.9	31.4	34.4	37.0	41.0	43.2
	β	3.18	2.99	2.94	2.87	2.74	2.62	2.42	2.26	2.13	1.9	1.67
≤0.150	θ	21.6	23.3	24.2	25.0	26.9	28.8	32.1	34.9	37.3	40.5	42.8
	β	2.88	2.79	2.78	2.72	2.60	2.52	2.36	2.21	2.08	1.82	1.61
≤0.175	θ	23.2	24.7	25.5	26.2	28.0	29.7	32.7	35.2	36.8	39.7	42.2
	β	2.73	2.66	2.65	2.60	2.52	2.44	2.28	2.14	1.96	1.71	1.54
≤0.200	θ	24.7	26.1	26.7	27.4	29.0	30.6	32.8	34.5	36.1	39.2	41.7
	β	2.63	2.59	2.52	2.51	2.43	2.37	2.14	1.94	1.79	1.61	1.47
≤0.225	θ	26.1	27.3	27.9	28.5	30.0	30.8	32.3	34.0	35.7	38.8	41.4
	β	2.53	2.45	2.42	2.40	2.34	2.14	1.86	1.73	1.64	1.51	1.39
≤0.250	θ	27.5	28.6	29.1	29.7	30.6	31.3	32.8	34.3	35.8	38.6	41.2
	β	2.39	2.39	2.33	2.33	2.12	1.93	1.70	1.58	1.50	1.38	1.29

4. Shear & Diagonal Tension in Beams

AASHTO LRFD Bridge Design Specification

- The strength of the longitudinal reinforcement must be adequate to carry the additional forced induced by shear.



Take moment at the point on which C acting,

$$\frac{M_u}{\phi} = (d_v)T + (0.5d_v)\frac{N_u}{\phi} - (d_v \cot\theta)\frac{V_u}{\phi} + (0.5d_v \cot\theta)V_s$$

$$A_s f_y \geq T = \frac{M_u}{\phi d_v} - \frac{0.5N_u}{\phi} + \left(\frac{V_u}{\phi} - 0.5V_s\right)\cot\theta \qquad (33)$$

4. Shear & Diagonal Tension in Beams

AASHTO LRFD Bridge Design Specification

- For members with less than the minimum transverse reinforcement, the following table give the optimum β and θ as a function of ε_x and a *crack spacing parameter* s_x.
  - s_x should be the lesser of either d_v or the maximum distance between layers of *longitudinal crack control rebar*.
  - $A_s \leq 0.003\ b_v s_x$ *(see 5.75 &5.76p of handout 4-1)*
- Following table is for 19mm coarse aggregate.
  - For other aggregate size a_g, an *equivalent* parameter should be used.

$$s_{xe} = s_x \frac{35}{a_g + 16} \qquad (34)$$

Table 4.2 For sections with less than minimum transverse reinforcement (coarse aggregate size=19mm)

s_{xe} (mm)		$\varepsilon_x \times 1{,}000$										
		≤ −0.20	≤ −0.10	≤ −0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00	≤1.50	≤2.00
≤130	θ	25.4	25.5	25.9	26.4	27.7	28.9	30.9	32.4	33.7	35.6	37.2
	β	6.36	6.06	5.56	5.15	4.41	3.91	3.26	2.86	2.58	2.21	1.96
≤250	θ	27.6	27.6	28.3	29.3	31.6	33.5	36.3	38.4	40.1	42.7	44.7
	β	5.78	5.78	5.38	4.89	4.05	3.52	2.88	2.50	2.23	1.88	1.65
≤380	θ	29.5	29.5	29.7	31.1	34.1	36.5	39.9	42.4	44.4	47.4	49.7
	β	5.34	5.34	5.27	4.73	3.82	3.28	2.64	2.26	2.01	1.68	1.46
≤500	θ	31.2	31.2	31.2	32.3	36.0	38.8	42.7	45.5	47.6	50.9	53.4
	β	4.99	4.99	4.99	4.61	3.65	3.09	2.46	2.09	1.85	1.52	1.31
≤750	θ	34.1	34.1	34.1	34.2	38.9	42.3	46.9	50.1	52.6	56.3	59.0
	β	4.46	4.46	4.46	4.43	3.39	2.82	2.19	1.84	1.60	1.30	1.10
≤1000	θ	36.6	36.6	36.6	36.6	41.2	45.0	50.2	53.7	56.3	60.2	63.0
	β	4.06	4.06	4.06	4.06	3.20	2.62	2.00	1.66	1.43	1.14	0.95
≤1500	θ	40.8	40.8	40.8	40.8	44.5	49.2	55.1	58.9	61.8	65.8	68.6
	β	3.50	3.50	3.50	3.50	2.92	2.32	1.72	1.40	1.18	0.92	0.75
≤2000	θ	44.3	44.3	44.3	44.33	47.1	52.3	58.7	62.8	65.7	69.7	72.4
	β	3.10	3.10	3.10	3.10	2.71	2.11	1.52	1.21	1.01	0.76	0.62

4. Shear & Diagonal Tension in Beams

AASHTO LRFD Bridge Design Specification

- Critical section is located at a distance equal to the larger of $0.5d_v cot\theta$ and d_v from the support.

  ⇦ load is not always applied to the upper surface.

- Minimum amount of transverse reinforcement

$$A_v = \frac{0.083\sqrt{f_{ck}} b_v s}{f_y} \quad \text{for} \quad V_u \geq 0.5\phi V_c \tag{35}$$

- Maximum spacing transverse reinforcement

$$s_{max} \leq 0.8d_v \leq 600mm \quad \text{for} \quad v_u \leq 0.125 f_{ck}$$

$$s_{max} \leq 0.4d_v \leq 300mm \quad \text{for} \quad v_u \geq 0.125 f_{ck}$$

4. Shear & Diagonal Tension in Beams

Shear Design by AASHTO Design Specification

Example 4.5

A simply supported rectangular beam.
b_w=400, d=600, clear span 8m,
factored load=110kN/m, f_{ck}=27MPa, A_s=4,910mm^2

using vertical U-shape stirrups with f_y=400MPa,
Design the web reinforcement.

Use KCI load factors and strength reduction factor φ=0.9 for shear as used in AASHTO Bridge Design Specification.

4. Shear & Diagonal Tension in Beams

Solution

1) For simplicity, set d_v=0.9d=540mm : minimum allowable value.
2) M_u and V_u according to distance from support are tabulated in Table 4-3.

Table 4-3 Shear Design Example

Distance from Support, m	M_u kN-m	V_u kN	V_c kN	φV_c kN
0	0	374	279	223.2
0.6	244	374	279	223.2
1.0	385	330	244	195.2
1.5	536	275	226	180.9
2.0	660	220	217	173.5
2.5	756	165	211	168.7
3.0	825	110	206	165.2
3.5	866	55	203	162.3
4.0	880	0	200	159.6

4. Shear & Diagonal Tension in Beams

3) Critical section is located at

   d_v=540mm or 0.5$d_v cot\theta$, whichever is larger.

4) V_u at critical section

$$V_u = (110)(\frac{8}{2} - 0.54) = 380.6kN$$

5) Maximum spacing of stirrups

$$v_u = \frac{V_u}{b_v d_v} = \frac{(380.6)(10^3)}{(400)(540)} < 0.125 f_{ck} = (0.125)(27)$$

$$\therefore s_{\max} \le 0.8 d_v = (0.8)(540) = 432mm < 600mm$$

4. Shear & Diagonal Tension in Beams

6) Check minimum amount of transverse reinforcement

$$A_v \geq \frac{0.083\sqrt{f_{ck}}\, b_v s}{f_y}$$

$$\Rightarrow \quad 143 \geq \frac{(0.083)(\sqrt{27})(400)s}{400}$$

$$\therefore s \leq 332mm$$

from 5) and 6), maximum spacing of D10 stirrups is,

$$s_{\max} = 332mm$$

4. Shear & Diagonal Tension in Beams

7) Using Eq. (32), ε_x can be calculated with α=2 for beams containing at least the minimum transverse reinforcement. Otherwise α=1

$$\varepsilon_x = \frac{M_u / 540 + V_u}{\alpha(2\times10^5)(4,910)} \qquad (N_u = 0)$$

8) ε_x and v_u/f_{ck} are tabulated along with M_u and V_u in Table 4.4

9) β and θ are selected from Table 4.1 and 4.2 for sections with/without minimum stirrups.

4. Shear & Diagonal Tension in Beams

10) If the section meets the minimum stirrup criterion, β is used to calculate V_c using

$$V_c = 0.083\beta\sqrt{f_{ck}}\,b_v d_v$$

11) which are then used, along with θ, to calculate V_s and stirrup spacing s (See Table 4-4)

12) For transverse reinforcement less than the minimum, the values of β are based on v_u/f_{ck} and s_x

13) Crack spacing parameter s_x is the lesser of either d_v or the maximum distance between layers of longitudinal bar. In this case,

$$s_x = d_v = 540mm$$ ⇦ No layers

Table 4-4 MCFT Design Example using φ=0.9 for Shear

x m	M_u kN-m	V_u kN	φV_c for at Least Minimum Stirrups								φV_c for Less Than Minimum Stirrups			
			$\varepsilon_x \times 1000$ ($a=2$)	v_u/f_{ck}	β	θ	V_c kN	φV_c kN	V_s kN	s mm	β	V_c kN	φV_c kN	$\varphi V_c/2$ kN
0.0	0.0	440	0.22	0.075	2.94	26.6	274	246	215	285	2.19	204	184	92
0.5	206.3	385	0.39	0.066	2.59	30.5	241	217	187	279	1.60	149	134	67
1.0	385.0	330	0.53	0.057	2.38	33.7	222	200	145	317	1.30	121	109	54
1.5	536.3	275	0.65	0.047	2.38	33.7	222	200	84	549	1.30	121	109	54
2.0	660.0	220	0.73	0.038	2.23	33.7	222	200	23	2,023	1.10	102	92	46
2.5	756.3	165	0.80	0.028	2.23	36.4	208	187	-	-	1.10	102	92	46
3.0	825.0	110	0.83	0.019	2.23	36.4	208	187	-	-	1.10	102	92	46
3.5	866.3	55	0.84	0.009	2.23	36.4	208	187	-	-	1.10	102	92	46
4.0	880.0	0	0.83	0.000	2.23	36.4	208	187	-	-	1.10	102	92	46

4. Shear & Diagonal Tension in Beams

14) Equivalent crack spacing parameter $s_{xe}=s_x$, because a_g=19mm

15) These values of β in 12) as used to determine the point $V_u \leq \varphi V_c/2$

16)

1 space at 140mm = 140mm

4 spaces at 250mm = 1,000mm

8 spaces at 300mm = 2,400mm

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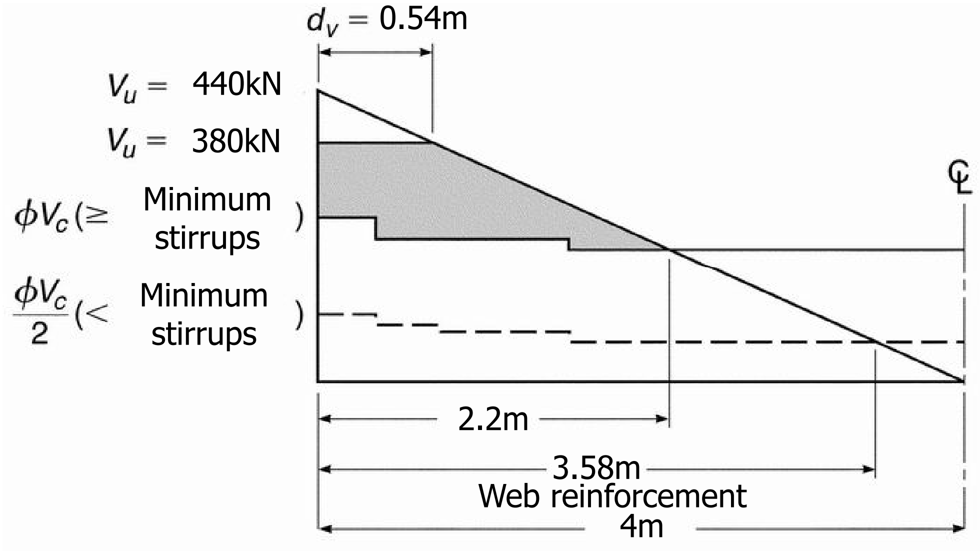
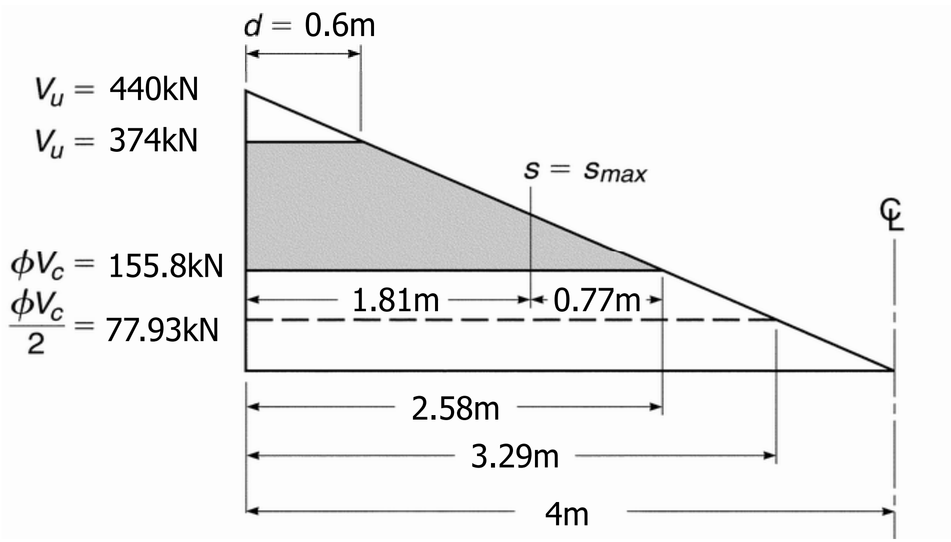
total = 3,540mm

4. Shear & Diagonal Tension in Beams

by AASHTO Code

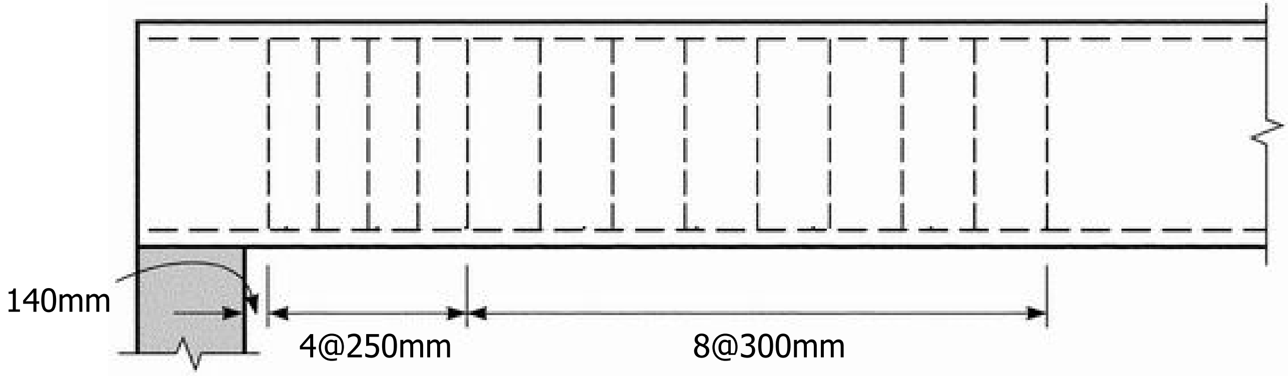


by KCI Code

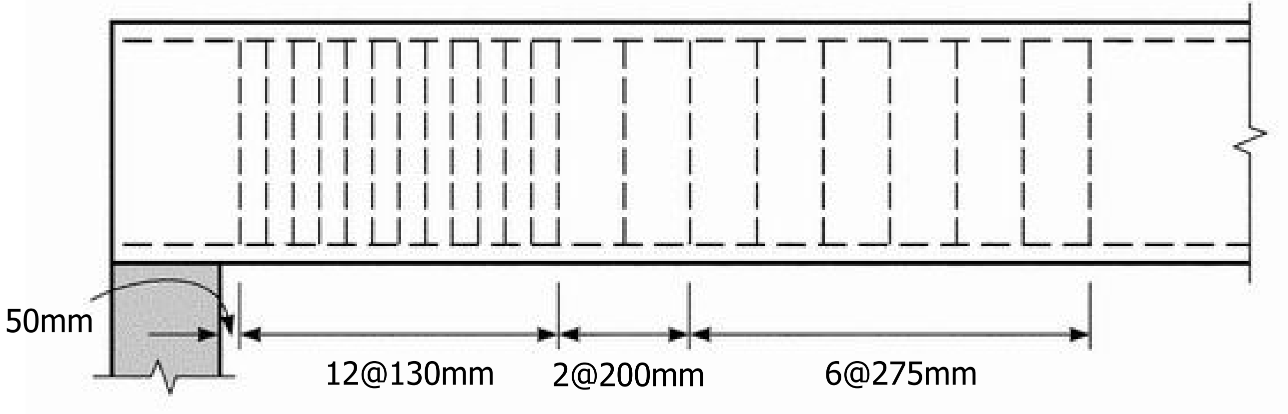


4. Shear & Diagonal Tension in Beams

by AASHTO Code



by KCI Code



4. Shear & Diagonal Tension in Beams

<u>Note</u>

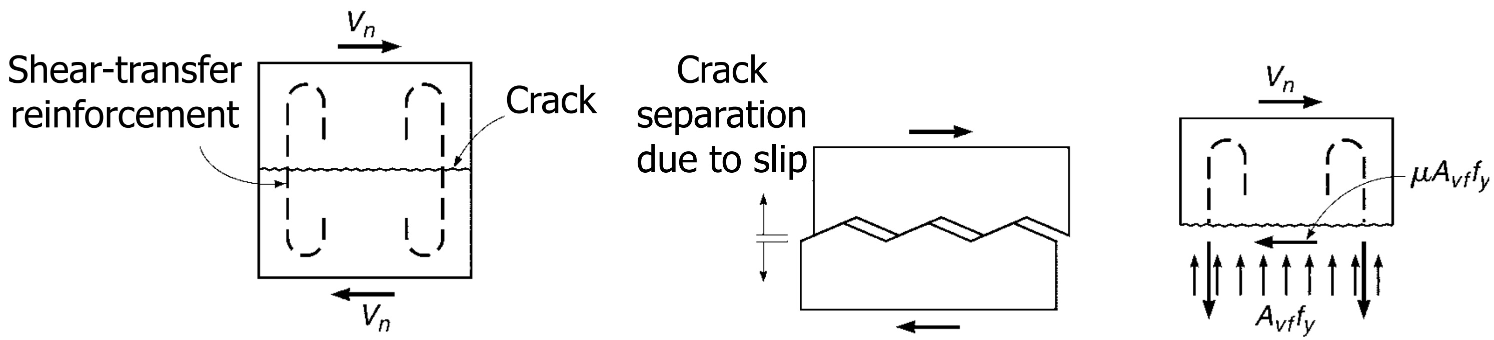
- For this example, V_s is selected based on V_u at each point, not the minimum V_u one a crack with angle θ
  - ⇨ This simplifies the design procedure and results in some what more conservative design
- MCFT base design is more economic one than that by KCI Code.
- Based on MCFT, shear increases the force in the flexural steel. But only effect on the location of steel termination, not on the maximum tensile force in the steel.

SHEAR FRICTION DESIGN METHOD

Direct shear may cause failure of reinforced concrete member

- In the vicinity of connection of precast structures
- In composite section of cast-in-place concrete and precast concrete or steel members

⇦ Necessary reinforcement can be obtained using shear-friction design method



4. Shear & Diagonal Tension in Beams

- The resistance to sliding is

$$V_n = \mu A_{vf} f_y \qquad (36)$$

, where μ is a coefficient of friction and
A_{vf} is the total area of steel crossing the crack

- Letting $\rho = A_{vf}/A_c$, where A_c is the area of the cracked surface

$$v_n = \mu \rho f_y \qquad (37)$$

- KCI Code 7.6 are based on Eq. (36) φ=0.8 and V_n should not exceed the smaller of $0.20 f_{ck} A_c$ or $5.6 A_c$ (Newton)

4. Shear & Diagonal Tension in Beams

- Recommendations for friction factor μ

- Concrete placed monolithically	1.4λ
- Concrete placed against hardened concrete with surface intentionally roughened	1.0λ
- Concrete placed against hardened concrete not intentionally roughened	0.6λ
- Concrete anchored to as-rolled structural steel by headed studs or reinforcing bars	0.7λ

,where λ=1.00 for normal-weight concrete

=0.85 for sand-lightweight concrete

=0.75 for all-lightweight concrete

4. Shear & Diagonal Tension in Beams

- f_y may not exceed 400 MPa

  compression/tension force across the shear plane must be considered.

  the surface roughness means a full amplitude of approx. 6mm

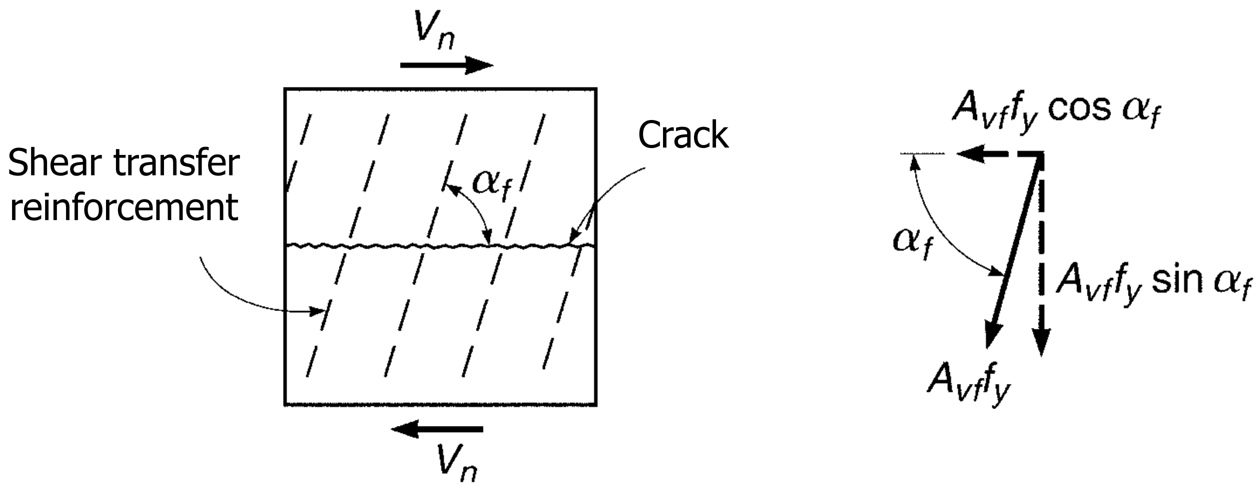
- The required steel area for the factored shear force V_n

$$A_{vf} = \frac{V_u}{\phi \mu f_y} \tag{38}$$

4. Shear & Diagonal Tension in Beams

- For inclined reinforcement

$$V_n = A_{vf} f_y (\mu \sin \alpha_f + \cos \alpha_f) \qquad (39)$$



, where α_f should not exceed 90°