Chapter 3.

Carrier Action

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Subject

1. 반도체의 전계가 가해졌을 때 전자는 어떻게 움직이고 전자의 이동도와 저항과는 어떤 관계에 있는가?

2. excess 캐리어의 확산은 전류에 어떻게 기여하는가?

3. 캐리어의 주입은 어떻게 할 수 있는가? 또, 주입된 캐리어들은 어떻게 되는가?
    1. 직접형 및 간접형 반도체
    2. 재결합 수명
    3. quasi-Fermi level

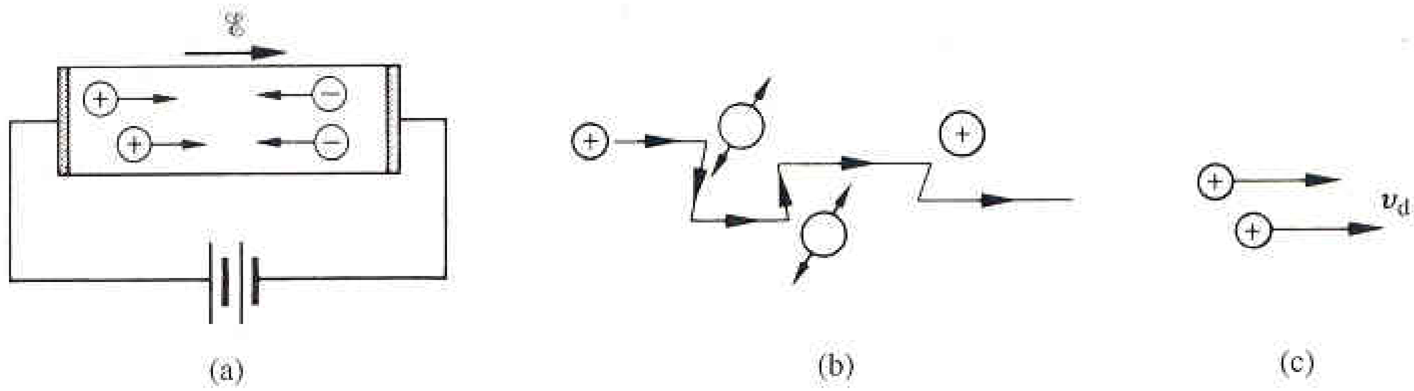
4. 확산과 재결합을 동시에 고려했을 때의 전류는?

Contents

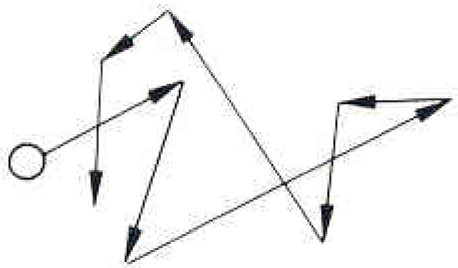
- Drift
- Diffusion
- Generation-Recombination
- Equations of State

❑ Drift

- Definition-Visualization
  - ✓ *Charged-particle motion in response to electric field*
  - ✓ An electric field tends to accelerate the +*q* charged holes in the direction of the electric field and the –*q* charged electrons in the opposite direction
  - ✓Collisions with impurity atoms and thermally agitated lattice atoms
  - ✓ Repeated periods of acceleration and subsequent decelerating collisions
  - ✓ Measurable quantities are macroscopic observables that reflect the average or overall motion of the carriers
  - ✓The drifting motion is actually superimposed upon the always-present thermal motion.
  - ✓ The thermal motion of the carriers is completely random and therefore averages out to zero, does not contribute to current

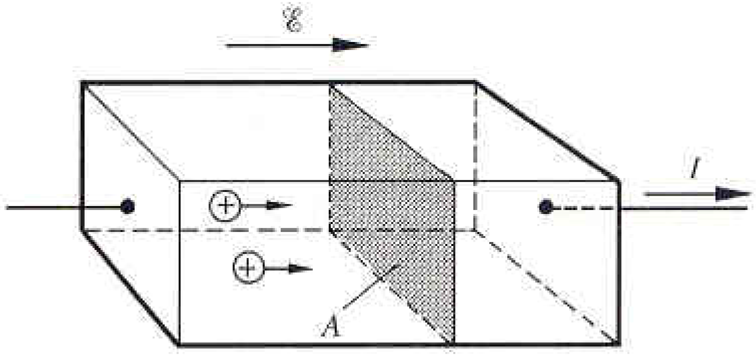


(a) Motion of carriers within a biased semiconductor bar; (b) drifting hole on a microscopic or atomic scale; (c) carrier drift on a macroscopic scale



Thermal motion of carrier

- Drift Current
  - ✓ *I* (current) = the charge per unit time crossing a plane oriented normal to the direction of flow



Expanded view of a biased *p*-type semiconductor bar of cross-sectional area *A*

$v_d t$ .... All holes this distance back from the v_d – normal plane will cross the plane in a time t

$v_d tA$ .... All holes in this volume will cross the plane in a time t

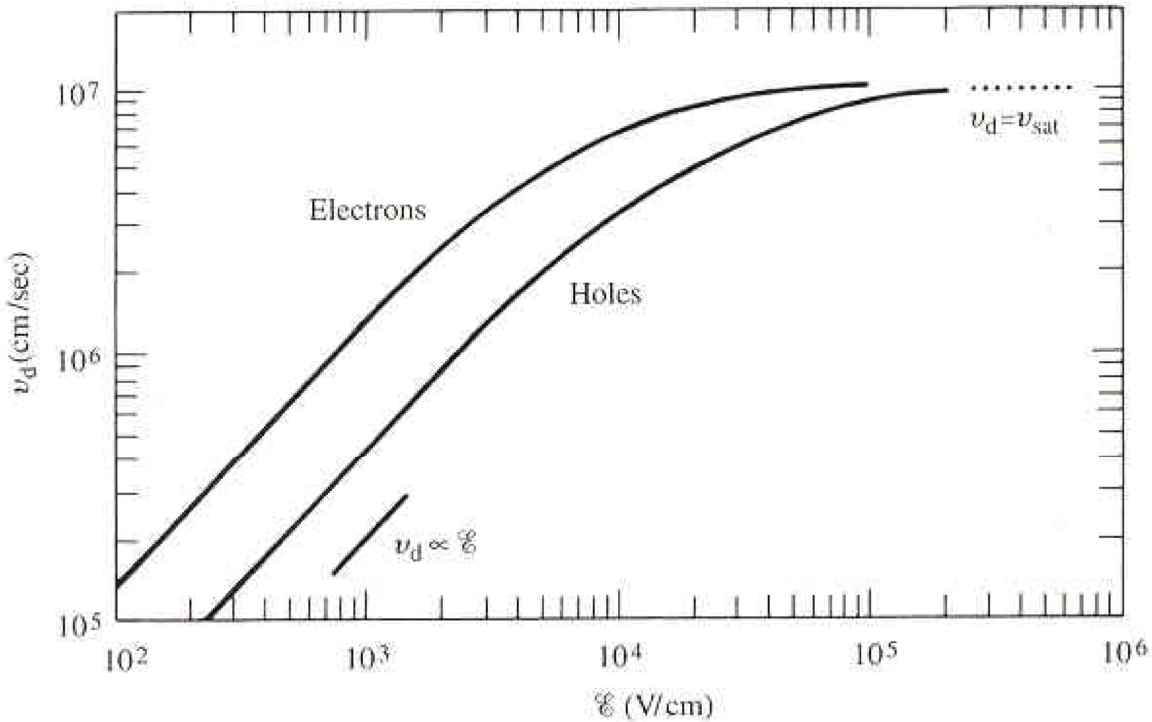
$pv_d tA$ .... Holes crossing the plane in a time t

$qpv_d tA$ ..... Charge crossing the plane in a time t

$qpv_d A$ ...... Charge crossing the plane per unit time

$$I_{\text{P|drift}} = qpv_{\text{d}}A \quad \text{hole drift current}$$

$$\mathbf{J}_{\text{P|drift}} = qp\vec{v}_{\text{d}}$$



The v_{d} is proportional to E at low electric fields, while at high electric fields v_{d} saturates and becomes independent of E

$$v_d = \frac{\mu_0 \mathscr{E}}{\left[1+\left(\frac{\mu_0 \mathscr{E}}{v_{sat}}\right)^{\beta}\right]^{1/\beta}} = \begin{cases} \mu_0 E & \ldots E \to 0 \\ v_{sat} & \ldots E \to \infty \end{cases}$$

where $\beta \cong 1$ for holes and $\beta \cong 2$ for electrons, μ_0 is the constant of proportionality between v_d and E at low to moderate electric fields, and v_{sat} is the limiting or saturation velocity

✓ In the low field limit $v_d = \mu_0 E$

$$\mathbf{J}_{P|drift} = q\mu_p p \mathscr{E}$$

$$\mathbf{J}_{N|drift} = q\mu_n n \mathscr{E}$$

$-q, \quad v_d = -\mu_n E, \quad \mathbf{J}_{N|drift} = -qnv_d$

- Mobility
  - ✓ Mobility is very important parameter in characterizing transport due to drift
  - ✓ Unit: cm^2/Vs
  - ✓ Varies inversely with the amount of scattering

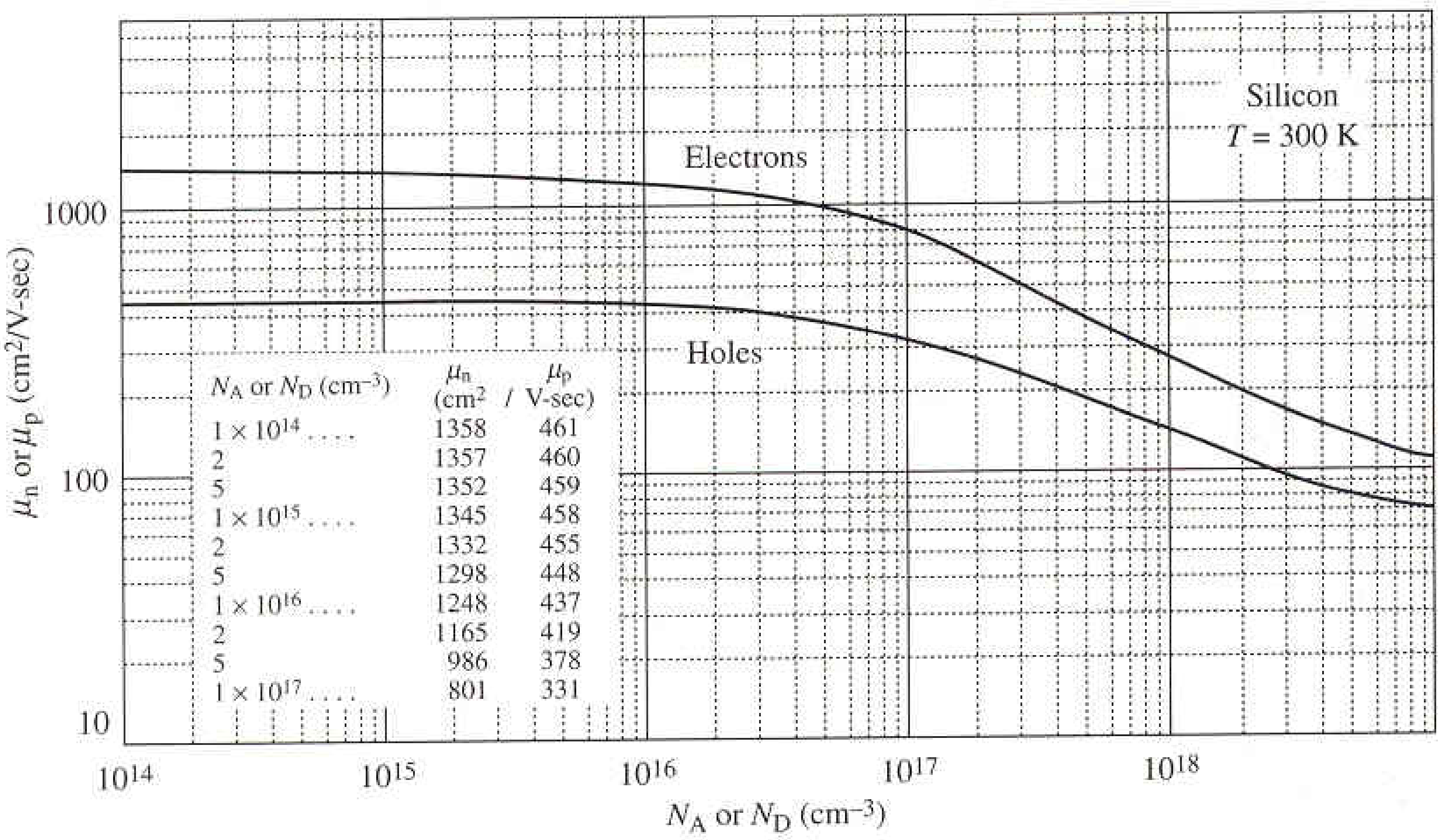
(i) Lattice scattering
(ii) Ionized impurity scattering

✓Displacement of atoms leads to lattice scattering

The internal field associated with the stationary array of atoms is already taken into account in m^*

✓ $\mu = q < \tau > / m^*$, where $< \tau >$ is the mean free time and m^* is the conductivity effective mass

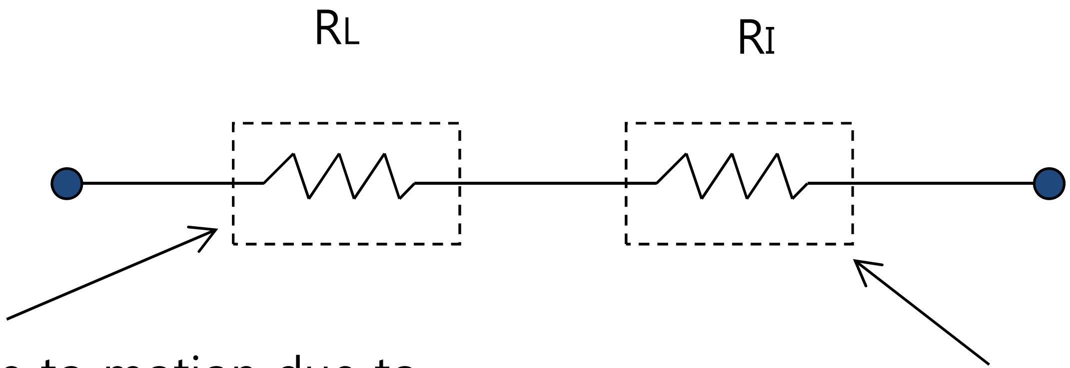
✓ The number of collisions decreases $< \tau >$ → μ varies inversely with the amount of scattering



N_A or N_D (cm^{-3})	μ_n (cm^2	μ_p / V-sec)
1×10^{14}	1358	461
2	1357	460
5	1352	459
1×10^{15}	1345	458
2	1332	455
5	1298	448
1×10^{16}	1248	437
2	1165	419
5	986	378
1×10^{17}	801	331

(a)

Room temperature carrier mobilities as a function of the dopant concentration in Si



Impedance to motion due to lattice scattering :

-No doping dependence.

-Decreases with decreasing T.

Impedance to motion due to ionized impurity scattering :

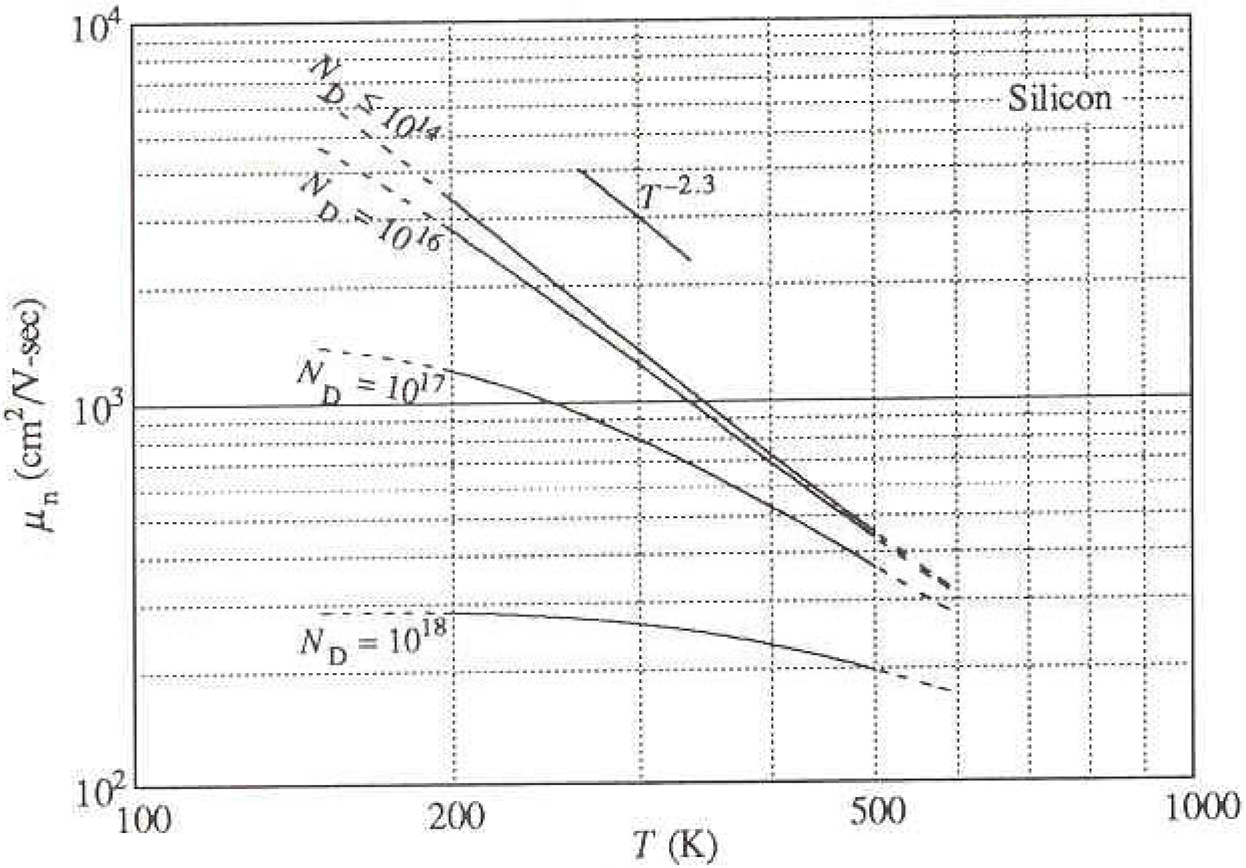
-Increases with N_A or N_D

-Increases with decreasing T.

✓ Temperature dependence

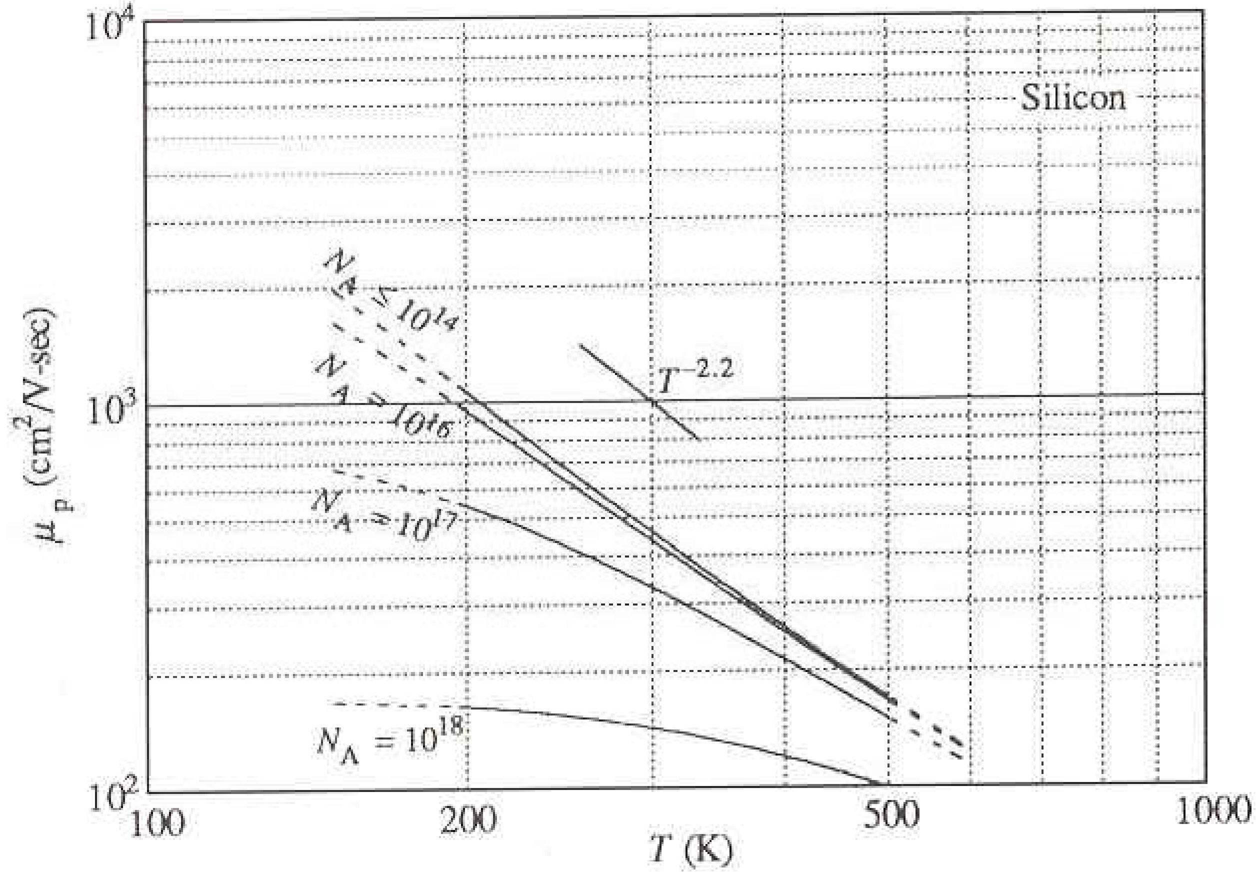
✓ Low doping limit: Decreasing temperature causes an ever-decreasing thermal agitation of the atoms, which decreases the lattice scattering

✓ Higher doping: Ionized impurities become more effective in deflecting the charged carriers as the temperature and hence the speed of the carriers decreases



(a)

Temperature dependence of electron mobility in Si for dopings ranging from < 10^{14} cm^{-3} to 10^{18} cm^{-3}.



(b)

Temperature dependence of hole mobility in Si for dopings ranging from < 10^{14} cm^{-3} to 10^{18} cm^{-3}.

- Resistivity
  - ✓ Resistivity (ρ) is defined as the proportionality constant between the electric field and the total current per unit area

$$\mathscr{E} = \rho \mathbf{J}$$

$$\mathbf{J} = \sigma \mathscr{E} = \frac{1}{\rho} \mathscr{E}$$

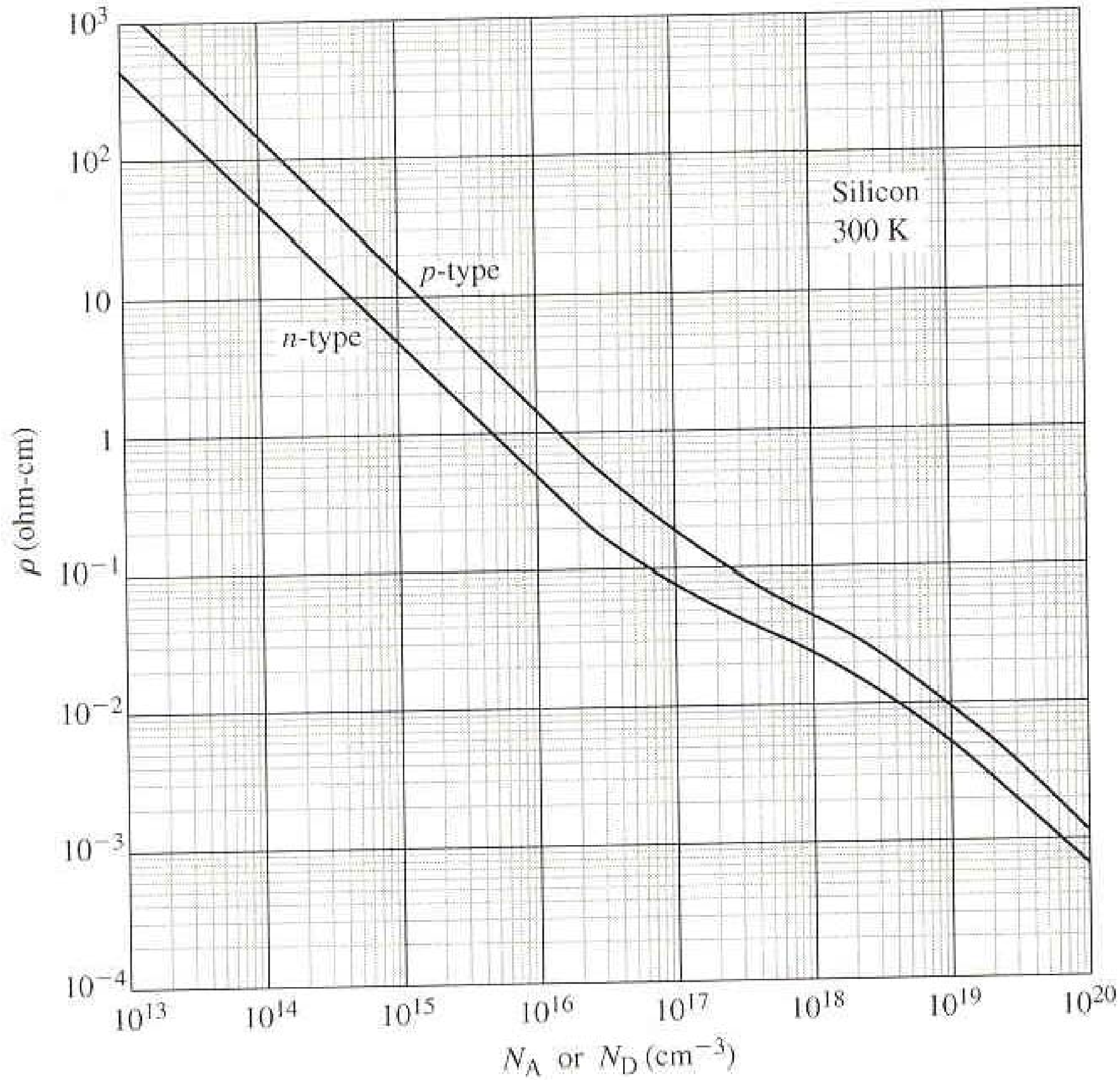
σ=1/ρ: conductivity

$$\mathbf{J}_{\text{drift}} = \mathbf{J}_{\text{N|drift}} + \mathbf{J}_{\text{P|drift}} = q(\mu_n n + \mu_p p)\mathscr{E}$$

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$$\rho = \frac{1}{q\mu_n N_D} \quad \ldots \; n-\text{type semiconductor}$$

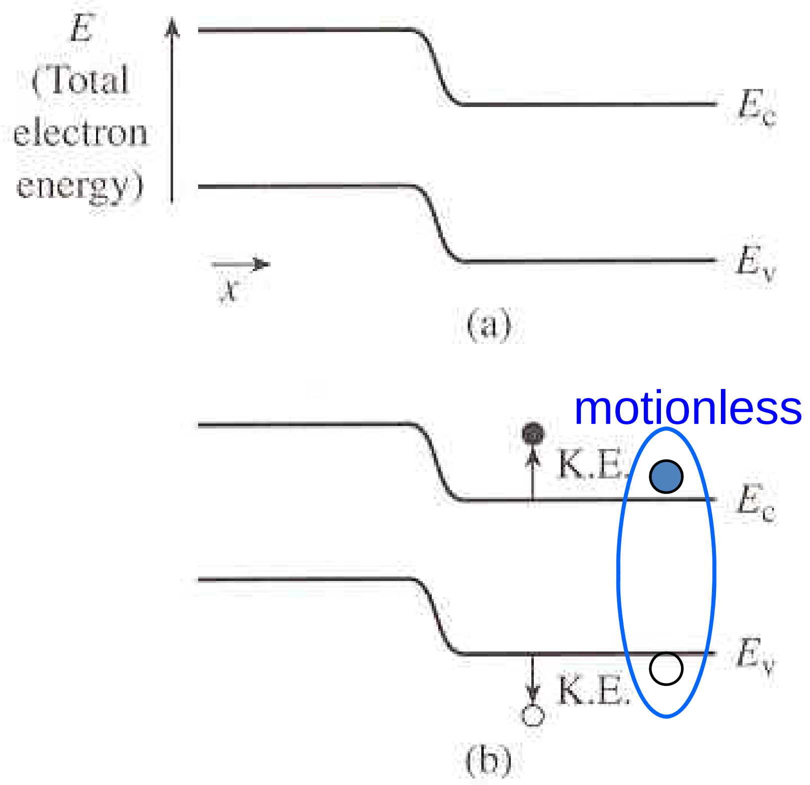
$$\rho = \frac{1}{q\mu_p N_A} \quad \ldots \; p-\text{type semiconductor}$$



(a)

Resistivity versus impurity concentration at 300 K in Si

- Band Bending
  - ✓When $\mathcal{E}$ exists the band energies become a function of position
  - ✓ If an energy of precisely E_G is added to break an atom-atom bond, the created electron and hole energies would be E_c and E_v, respectively, and the created carriers would be effectively motionless

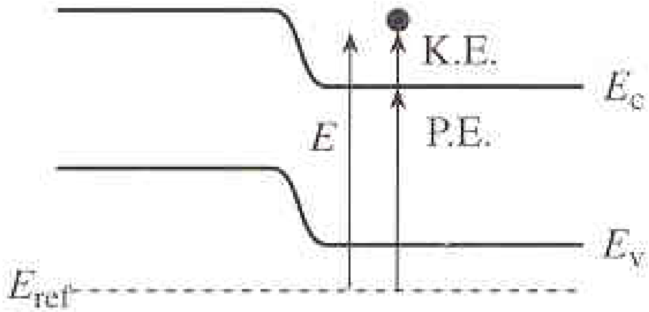


- ✓ $E-E_c$= K.E. of the electrons
- ✓ E_v-E= K.E. of the holes

$$\text{P.E.} = E_c - E_{ref}$$

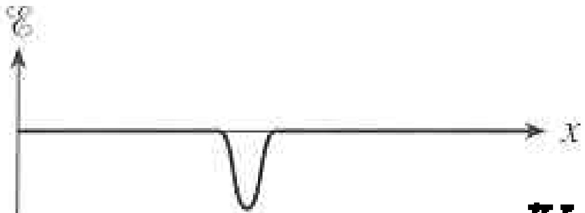
✓The potential energy of a –*q* charged particle is

$$\text{P.E.} = -qV \quad \rightarrow V = -\frac{1}{q}(E_c - E_{ref})$$



(c) $\text{P.E.} = E_c - E_{ref}$

(d)



(e) $\mathscr{E} = -\frac{dV}{dx}$

✓ By definition,

$$\mathscr{E} = -\nabla V$$

✓ In one dimension,

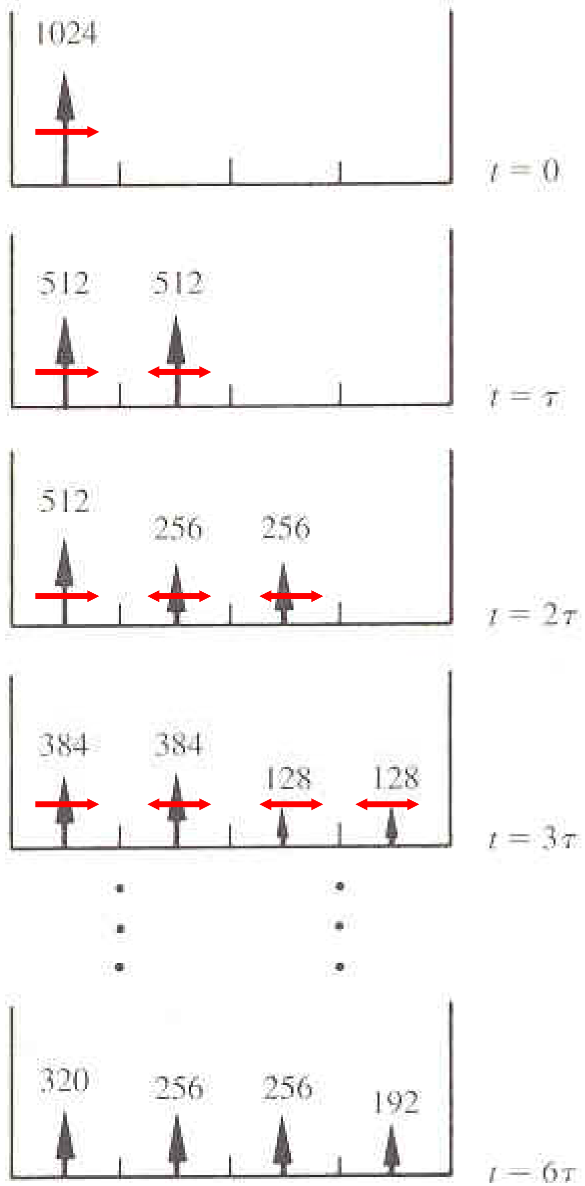
$$\mathscr{E} = -\frac{dV}{dx}$$

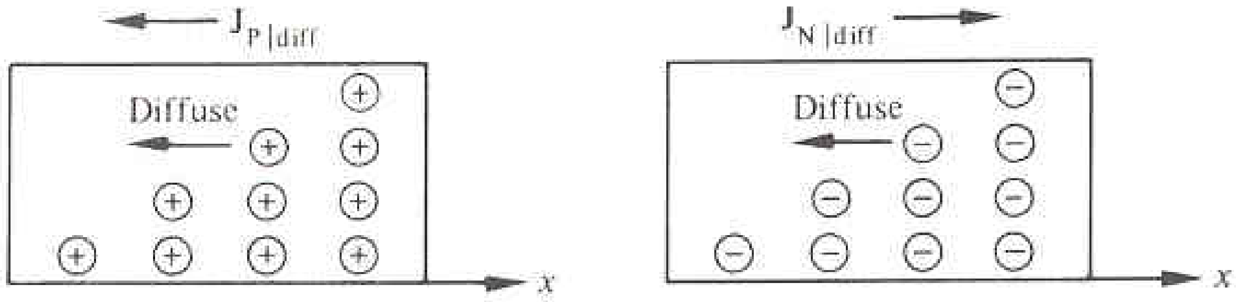
$$\mathscr{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

❑ Diffusion

• Definition-Visualization

✓ Diffusion is a process whereby particles tend to spread out or redistribute as a result of their random thermal motion, migrating on a macroscopic scale from regions of high concentration into region of low concentration





- Diffusion and Total Currents
  - ✓ Diffusion Currents: The greater the concentration gradient, the larger the flux
  - ✓ Using Fick's law,

$$F = -D\nabla\eta \; [\#/cm^2 \cdot s]$$

Diffusion constant

$$\mathbf{J}_{\mathrm{P|diff}} = -qD_{\mathrm{P}}\nabla p$$
$$\mathbf{J}_{\mathrm{N|diff}} = qD_{\mathrm{N}}\nabla n$$

$$[D] = cm^2 / s$$

✓ Total currents

$$\mathbf{J}_{\mathrm{P}} = \mathbf{J}_{\mathrm{P|drift}} + \mathbf{J}_{\mathrm{P|diff}} = q\mu_p p\mathscr{E} - qD_P\nabla p$$

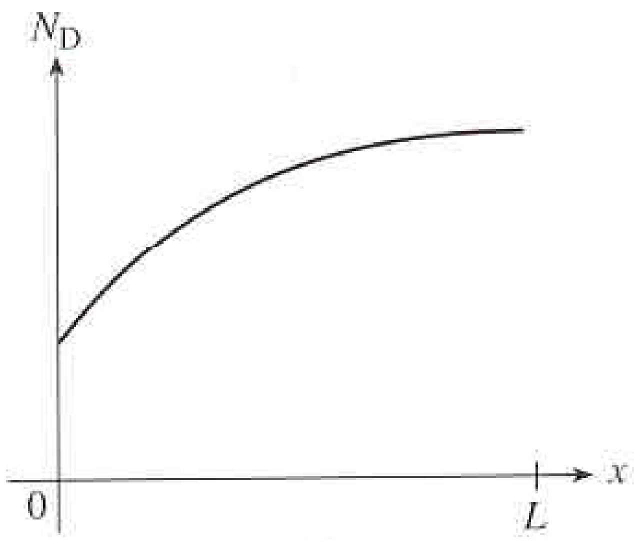
↕ drift ↕ diffusion

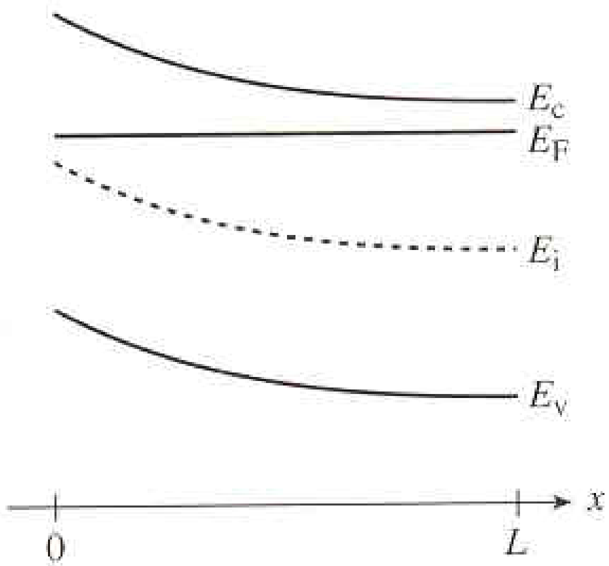
$$\mathbf{J}_{\mathrm{N}} = \mathbf{J}_{\mathrm{N|drift}} + \mathbf{J}_{\mathrm{N|diff}} = q\mu_n n\mathscr{E} + qD_N\nabla n$$

✓ Total particle currents

$$\mathbf{J} = \mathbf{J}_{\mathrm{N}} + \mathbf{J}_{\mathrm{P}}$$

- Relating Diffusion coefficients/Mobilities
  - ✓ Einstein relationship
  - ✓Consider a nonuniformly doped semiconductor under *equilibrium*
  - ✓ Constancy of the Fermi Level: nonuniformly doped *n*-type semiconductor as an example $dE_F / dx = 0$





✓ Under equilibrium conditions

$$\mathbf{J} = \mathbf{J}_N + \mathbf{J}_P = 0 \;\&\; \mathbf{J}_N = \mathbf{J}_P = 0$$

$$\begin{cases} q\mu_p p \mathscr{E} = qD_P \nabla p \\ q\mu_n n \mathscr{E} = -qD_N \nabla n \end{cases} \quad \text{for nonuniform doping}$$

Electron diffusion current flowing in the +x direction "Built-in" electric field in the –x direction → drift current in the –x direction

✓ Einstein relationship:

✓ Nondegenerate, nonuniformly doped semiconductor,Under equilibrium conditions, and focusing on the electrons, $\mathscr{E}$

$$J_{N|drift} + J_{N|diff} = q\mu_n n\mathscr{E} + qD_N \frac{dn}{dx} = 0$$

$$\because \mathscr{E} = \frac{1}{q}\frac{dE_i}{dx} \qquad \because n = n_i e^{(E_F - E_i)/kT}$$

✓ With $dE_F/dx=0$,

$$\frac{dn}{dx} = -\frac{n_i}{kT} e^{(E_F-E_i)/kT} \frac{dE_i}{dx} = -\frac{q}{kT} n\mathscr{E}$$

✓ Substituting

$$(qn\mathscr{E})\mu_n - (qn\mathscr{E})\frac{q}{kT} D_N = 0$$

$$\frac{D_N}{\mu_n} = \frac{kT}{q}$$

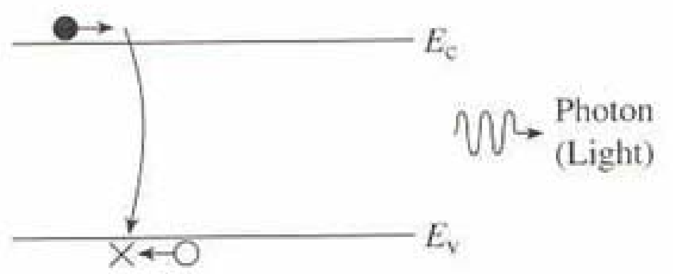
$$\frac{D_P}{\mu_p} = \frac{kT}{q}$$

✓ Einstein relationship is valid even under nonequilibrium

✓ Slightly modified forms result for degenerate materials

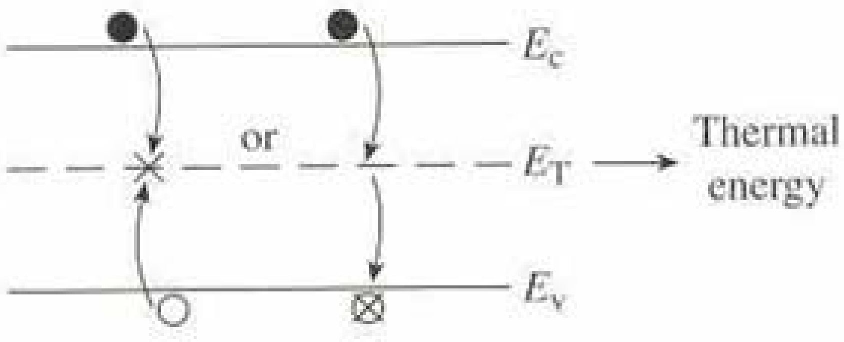
❑ Recombination-Generation

- Definition-Visualization
  - ✓ When a semiconductor is perturbed → an excess or deficit in the carrier concentrations → Recombination-generation
  - ✓ *Recombination*: a process whereby electrons and holes (carriers) are annihilated or destroyed
  - ✓ *Generation*: a process whereby electrons and holes are created
- Band-to-Band Recombination
  - ✓ The *direct* annihilation of an electron and a hole → the production of a photon (light)

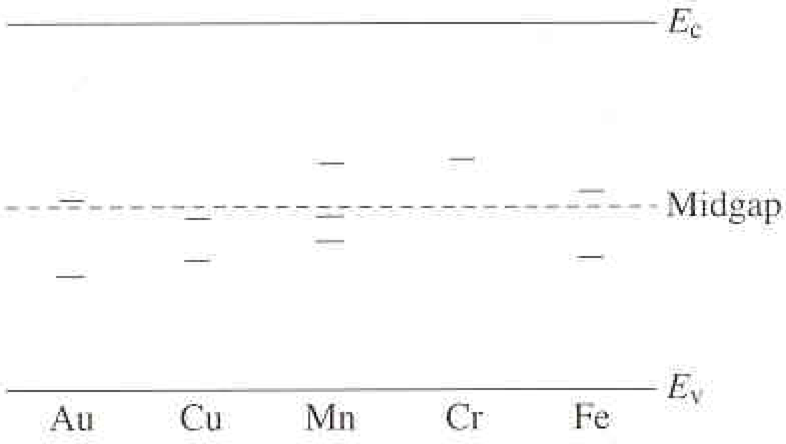


(a) Band-to-band recombination

- R-G Center Recombination
  - ✓ R-G centers are lattice defects or impurity atoms (Au)
  - ✓ The most important property of the R-G centers is the introduction of allowed electronic levels near the center of the band gap (E_T)
  - ✓Two-step process
  - ✓ R-G center recombination (or indirect recombination) typically releases thermal energy (heat) or, equivalently, produces lattice vibration

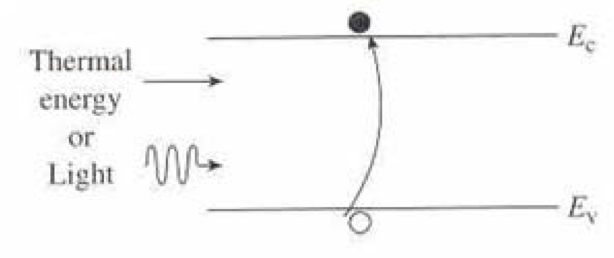


(b) R–G center recombination



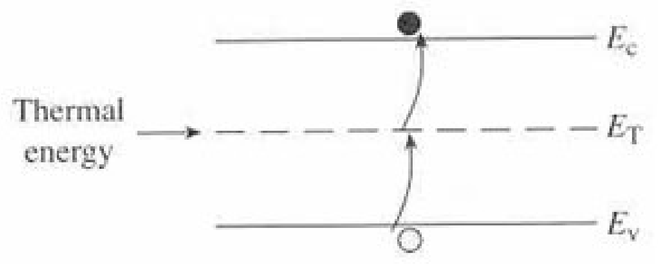
Near-midgap energy levels introduced by some common impurities in Si

- Generation Processes
  - ✓thermal energy > E_G → *direct thermal generation*
  - ✓light with an energy > E_G → *photogeneration*



(d) Band-to-band generation

- ✓ The thermally assisted generation of carriers with R-G centers

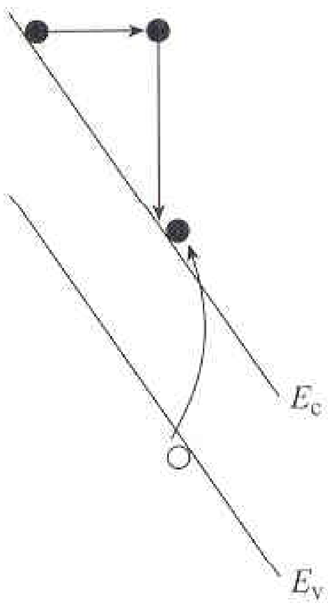


(e) R–G center generation

✓Impact ionization

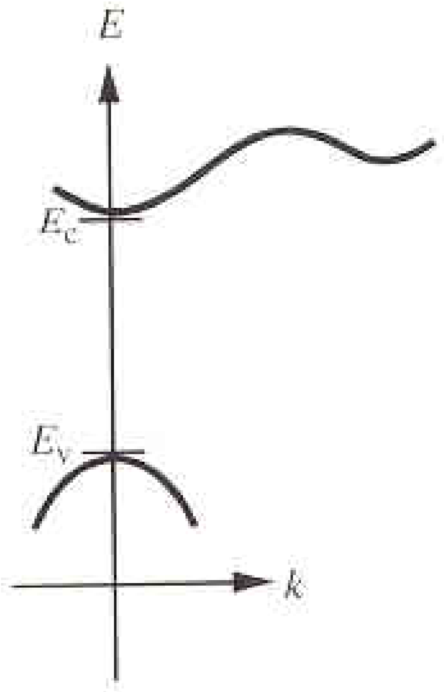
✓ e-h is produced as a results of energy released when a highly energetic carrier collide with the lattice

✓High ε -field regions



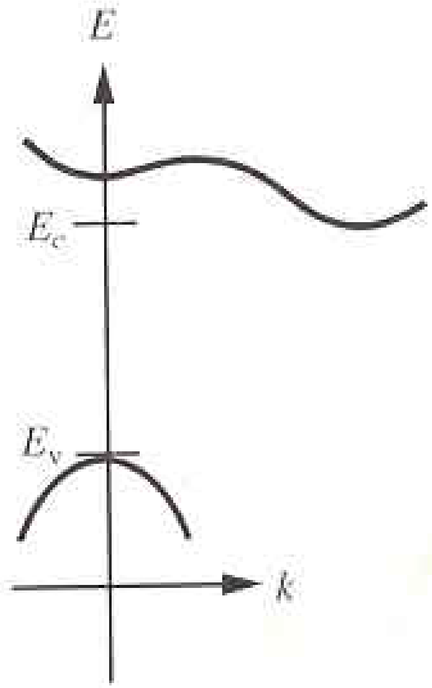
(f) Carrier generation via impact ionization

- Momentum Considerations
  - ✓One need be concerned only with the dorminat process
  - ✓Crystal momentum in addition to energy must be conserved.
  - ✓The momentum of an electron in an energy band can assume only certain quantized values.
  - ✓where **k** is a parameter proportional to the electron momentum



(a) Direct semiconductor

GaAs

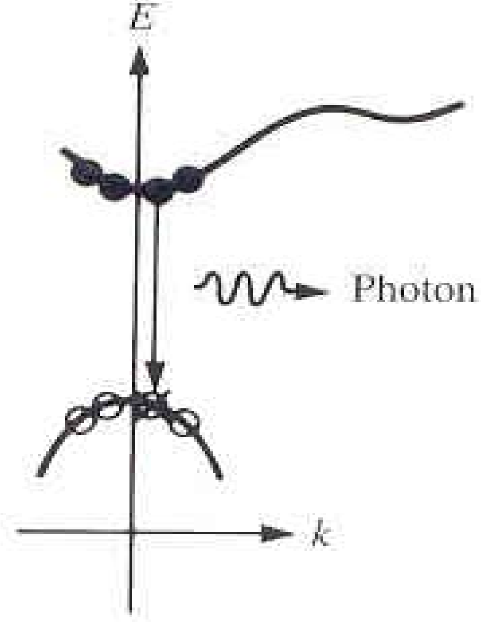


(b) Indirect semiconductor

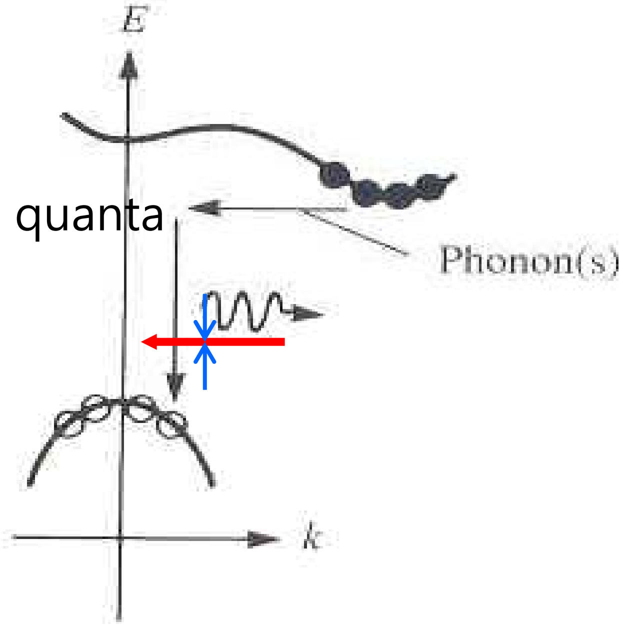
Si, Ge

Photons: light

Phonons: lattice vibration quanta



(a) Direct semiconductor



(b) Indirect semiconductor

Photons, being massless entities, carry very little momentum, and a photon-assisted transition is essentially vertical on the *E*-**k** plot

The thermal energy associated with lattice vibrations (phonons) is very small (in the 10-50 meV range), whereas the phonon momentum is comparatively large. A phonon-assisted transition is essentially horizontal on the *E*-**k** plot. The emission of a photon must be accompanied by the emission or absorption of a phonon.

- R-G Statistics
  - ✓ It is the time rate of change in the carrier concentrations ($\partial n/\partial t$, $\partial p/\partial t$) that must be specified
  - ✓ B-to-B recombination is totally negligible compared to R-G center recombination in Si
  - ✓ Even in direct materials, the R-G center mechanism is often the dominant process.

• Indirect Thermal Recombination-Generation

n_0, p_0 …. carrier concentrations when equilibrium conditions prevail

n, p ……. carrier concentrations under arbitrary conditions

$\Delta n = n - n_0$ … deviations in the carrier concentrations from their equilibrium values
$\Delta p = p - p_0$ …

N_T ………. number of R-G centers/cm^3

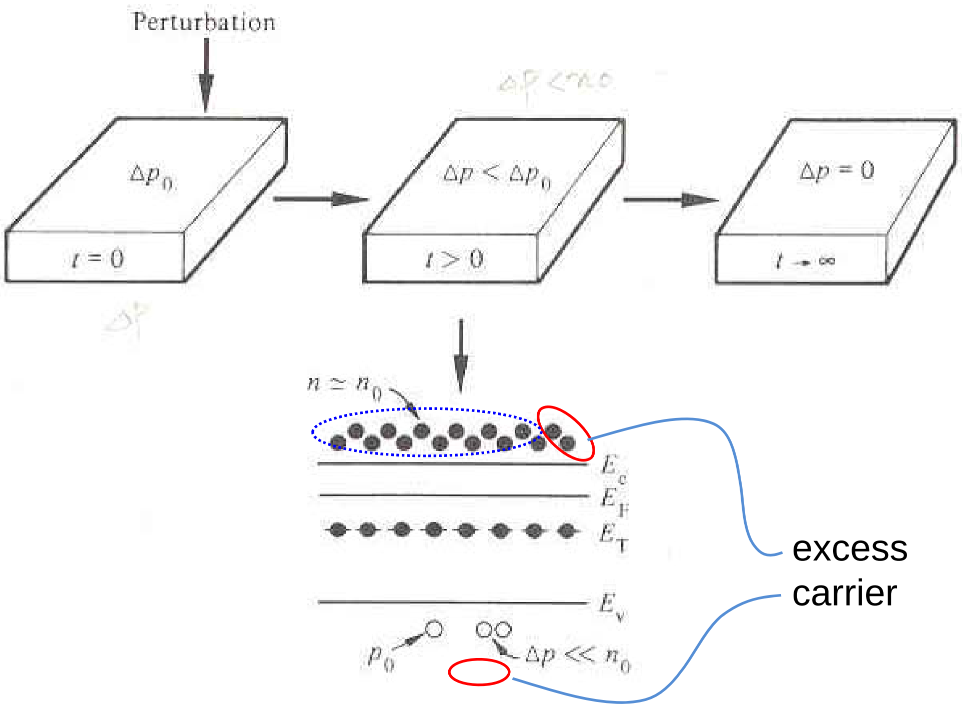
$$\text{low-level injection implies}$$
$$\Delta p \ll n_0, \quad n \cong n_0 \quad \text{in an } n\text{-type material}$$
$$\Delta n \ll p_0, \quad p \cong p_0 \quad \text{in a } p\text{-type material}$$

✓A specific example of N_D=10^{14} cm^{-3} Si subject to a perturbation where $\Delta n = \Delta p = 10^9$ cm^{-3}

$$n_0 \cong N_D = 10^{14} cm^{-3} \rightarrow \quad n = n_0 + \Delta n \cong n_0$$

$$p_0 \cong n_i^2 / N_D \cong 10^6 cm^{-3} \rightarrow \quad p = p_0 + \Delta p \cong \Delta p$$

✓Although the majority carrier concentration remains essentially unperturbed under low-level injection, the minority carrier concentration can increase by many orders of magnitude.



✓ The greater the number of filled R-G centers, the greater the probability of a hole annihilating transition and the faster the rate of recombination

✓Under equilibrium, essentially all of the R-G centers are filled with electrons because $E_F >> E_T$

✓With $\Delta p << n_0$, electrons always vastly outnumber holes and rapidly fill R-G levels that become vacant → # of filled centers during the relaxation ≅ N_T

$$\left.\frac{\partial p}{\partial t}\right|_R \propto N_T$$

✓ The number of hole-annihilating transitions should increase almost linearly with the number of holes

✓ Introducing a proportionality constant, c_p,

$$\left.\frac{\partial p}{\partial t}\right|_{\mathrm{R}} = -c_{\mathrm{p}} N_{\mathrm{T}}\, p$$

✓$\partial p/\partial t|_G$ depends only on # of empty R-G centers

$$\left.\frac{\partial p}{\partial t}\right|_G = \left.\frac{\partial p}{\partial t}\right|_{G-equiliburium}$$

✓The recombination and generation rates must precisely balance under equilibrium conditions, or $\partial p/\partial t|_G = \partial p/\partial t|_{G\text{-equilibrium}} = -\ \partial p/\partial t|_{R\text{-equilibrium}}$

$$\left.\frac{\partial p}{\partial t}\right|_{\mathrm{G}} = c_{\mathrm{p}} N_{\mathrm{T}} p_0$$

✓ The net rate

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{\mathrm{i-thermal}\\ \mathrm{R-G}}} = \left.\frac{\partial p}{\partial t}\right|_{\mathrm{R}} + \left.\frac{\partial p}{\partial t}\right|_{\mathrm{G}} = -c_{\mathrm{p}} N_{\mathrm{T}} (p - p_0)$$

$$\text{or} \quad \left.\frac{\partial p}{\partial t}\right|_{\substack{\mathrm{i-thermal}\\ \mathrm{R-G}}} = -c_{\mathrm{p}} N_{\mathrm{T}} \Delta p \qquad \text{for holes in an } n-\text{type material}$$

✓ An analogous set of arguments yields

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\mathrm{i-thermal}\\ \mathrm{R-G}}} = -c_{\mathrm{n}} N_{\mathrm{T}} \Delta n \qquad \text{for electrons in a } p-\text{type material}$$

✓ c_n and c_p are referred to as the capture coefficients

✓ $c_p N_T$ and $c_n N_T$ must have units of 1/time

$$\tau_p = \frac{1}{c_p N_T}, \quad \tau_n = \frac{1}{c_n N_T}$$

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{i-thermal} \\ \text{R-G}}} = -\frac{\Delta p}{\tau_p} \quad \text{for holes in an } n\text{-type material}$$

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{i-thermal} \\ \text{R-G}}} = -\frac{\Delta n}{\tau_n} \quad \text{for electrons in a } p\text{-type material}$$

3.3.4 Minority Carrier Lifetime
General Information

- ***The average excess hole lifetime <t> can be computed***

  : $$<t>=\tau_n \quad (or \quad \tau_p)$$

- $\tau_n \quad (or \quad \tau_p)$***: minority carrier lifetimes***

❑ Equations of State

- Continuity Equations
  - ✓ Carrier action – whether it be drift, diffusion, indirect or direct thermal recombination, indirect or direct generation, or some other type of carrier action – gives rise to a change in the carrier concentrations with time
  - ✓ Let's combine all the mechanisms

$$\frac{\partial n}{\partial t} = \left.\frac{\partial n}{\partial t}\right|_{\text{drift}} + \left.\frac{\partial n}{\partial t}\right|_{\text{diff}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{other processes}\\ \text{(light, etc.)}}}$$

$$\frac{\partial p}{\partial t} = \left.\frac{\partial p}{\partial t}\right|_{\text{drift}} + \left.\frac{\partial p}{\partial t}\right|_{\text{diff}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{other processes}\\ \text{(light, etc.)}}}$$

$$\left.\frac{\partial n}{\partial t}\right|_{drift} + \left.\frac{\partial n}{\partial t}\right|_{diff} = \frac{1}{q}\left(\frac{\partial J_{Nx}}{\partial x} + \frac{\partial J_{Ny}}{\partial y} + \frac{\partial J_{Nz}}{\partial z}\right) = \frac{1}{q}\nabla \cdot J_N$$

$$\left.\frac{\partial p}{\partial t}\right|_{drift} + \left.\frac{\partial p}{\partial t}\right|_{diff} = -\frac{1}{q}\left(\frac{\partial J_{Px}}{\partial x} + \frac{\partial J_{Py}}{\partial y} + \frac{\partial J_{Pz}}{\partial z}\right) = -\frac{1}{q}\nabla \cdot J_P$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot J_N + \left.\frac{\partial n}{\partial t}\right|_{\substack{thermal \\ R-G}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{other \\ processes}}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot J_P + \left.\frac{\partial p}{\partial t}\right|_{\substack{thermal \\ R-G}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{other \\ processes}}$$

• Minority Carrier Diffusion Equations

✓ Simplifying assumptions:

(1) *One-dimensional*
(2) The analysis is limited or restricted to *minority carriers*
(3) $\mathscr{E} \cong 0$.
(4) The equilibrium minority carrier concentrations are not a function of position $n_0 \neq n_0(x),\ p_0 \neq p_0(x)$
(5) *Low level injection*
(6) *Indirect* thermal R-G is the dominant thermal R-G mechanism
(7) There are no "other processes", except possibly photogeneration

$$\frac{1}{q}\nabla \cdot \mathbf{J}_N \rightarrow \frac{1}{q}\frac{\partial J_N}{\partial x}$$

$$\mathbf{J_N} = q\mu_n n \mathscr{E} + qD_N \frac{\partial n}{\partial x} \cong qD_N \frac{\partial n}{\partial x}$$

✓ $n = n_0 + \Delta n$

$$\frac{\partial n}{\partial x} = \frac{\partial n_0}{\partial x} + \frac{\partial \Delta n}{\partial x} = \frac{\partial \Delta n}{\partial x}$$

$$\frac{1}{q}\nabla \cdot \mathbf{J}_\mathrm{N} \rightarrow D_\mathrm{N}\frac{\partial^2 \Delta n}{\partial x^2}$$

✓ With low level injection,

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -\frac{\Delta n}{\tau_\mathrm{n}}$$

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{other}\\ \text{processes}}} = G_\mathrm{L}$$

→ G_L=0 without illumination

✓ The equilibrium electron concentration is never a function of time

$$\frac{\partial n}{\partial t} = \frac{\partial n_0}{\partial t} + \frac{\partial \Delta n}{\partial t} = \frac{\partial \Delta n}{\partial t}$$

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

Minority carrier diffusion equations

- Simplifications and Solutions

✓ Steady state

$$\frac{\partial \Delta n_p}{\partial t} \rightarrow 0 \quad \left(\frac{\partial \Delta p_n}{\partial t} \rightarrow 0 \right)$$

✓ No concentration gradient or no diffusion current

$$D_N \frac{\partial^2 \Delta n_p}{\partial x^2} \rightarrow 0 \quad \left(D_P \frac{\partial^2 \Delta p_n}{\partial x^2} \rightarrow 0 \right)$$

✓ No drift current or $\mathscr{E} = 0$ → no further simplification

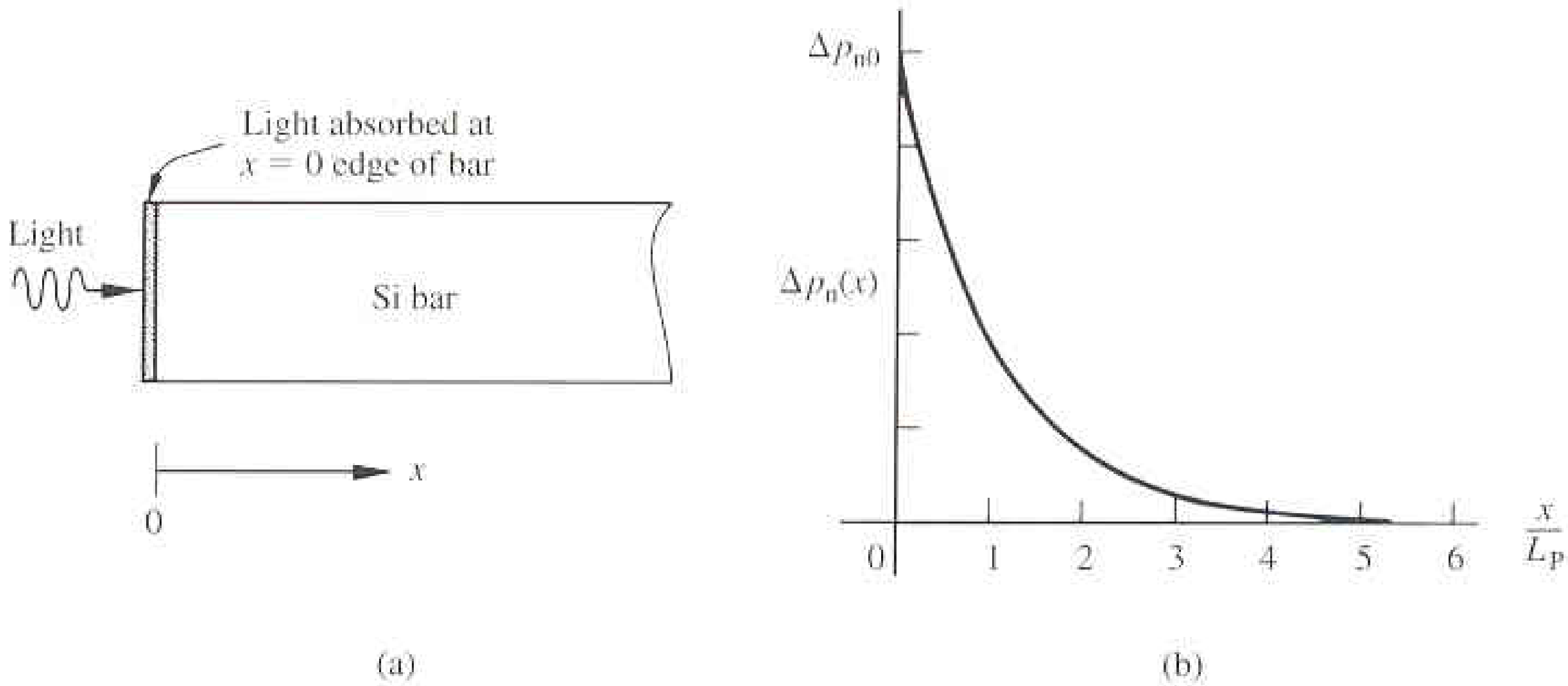
✓ No thermal R-G

$$\frac{\Delta n_p}{\tau_n} \rightarrow 0 \quad \left(\frac{\Delta p_n}{\tau_p} \rightarrow 0 \right)$$

✓ No light

$$G_L \rightarrow 0$$

✓ Sample problem 2: As shown below, the x=0 end of a uniformly doped semi-infinite bar of silicon with $N_D=10^{15}$ cm^{-3} is illuminated so as to create $\Delta p_{n0}=10^{10}$ cm^{-3} excess holes at x=0. The wavelength of the illumination is such that no light penetrates into the interior (x>0) of the bar. Determine Δp_n(x).



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✓ Solution: at x=0, $\Delta p_n(0)= \Delta p_{n0}= 10^{10}$ cm^{-3} , and $\Delta p_n \rightarrow 0$ as $x \rightarrow \infty$

✓ The light first creates excess carriers right at x=0

✓G_L=0 for x>0

✓Diffusion and recombination

✓ As the diffusing holes move into the bar their numbers are reduced by recombination

✓ Under steady state conditions it is reasonable to expect an excess distribution of holes near x=0, with $\Delta p_n(x)$ monotonically decreasing from Δp_{n0} at x=0 to Δp_{n0}= 0 as $x \rightarrow \infty$

✓ E ≅ 0 ? Yes

1>Excess hole pile-up is very small $(\Delta p_n\big|_{\max} \cong n_i)$

2>The majority carriers redistribute in such a way to partly cancel the minority carrier charge.

✓ Under steady state conditions with G_L=0 for x > 0

$$D_p \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} = 0 \quad \text{for} \quad x > 0$$

$$\Delta p_n\big|_{x=0^+} = \Delta p_n\big|_{x=0} = \Delta p_{n0}$$

$$\Delta p_n\big|_{x\rightarrow\infty} = 0$$

✓ The general solution

$$\Delta p_n(x) = Ae^{-x/L_P} + Be^{x/L_P} \quad \text{where } L_p \equiv \sqrt{D_p \tau_p}$$

exp(x/L_p) → ∞ as x → ∞ $\quad \Rightarrow B = 0$

✓ With x=0,

$$A = \Delta p_{n0}$$

$$\Delta p_n(x) = \Delta p_{n0} e^{-x/L_P} \quad \Leftarrow \text{solution}$$

❑ Supplemental Concepts

- Diffusion Lengths
  - ✓ minority carrier diffusion lengths

$$L_P \equiv \sqrt{D_P \tau_p}$$

$$L_{\mathrm{N}} \equiv \sqrt{D_{\mathrm{N}}\tau_{\mathrm{n}}}$$

✓L_{P} and L_{N} represent the average distance minority carriers can diffuse into a sea of majority carriers before being annihilated

✓ The average position of the excess minority carriers inside the semiconductor bar is

$$\langle x \rangle = \int_0^\infty x\Delta p_{\mathrm{n}}(x)dx \Big/ \int_0^\infty \Delta p_{\mathrm{n}}(x)dx = L_{\mathrm{P}}$$

• Quasi-Fermi Levels

✓ *Quasi-Fermi* levels are energy levels used to specify the carrier concentrations under nonequilibrium conditions

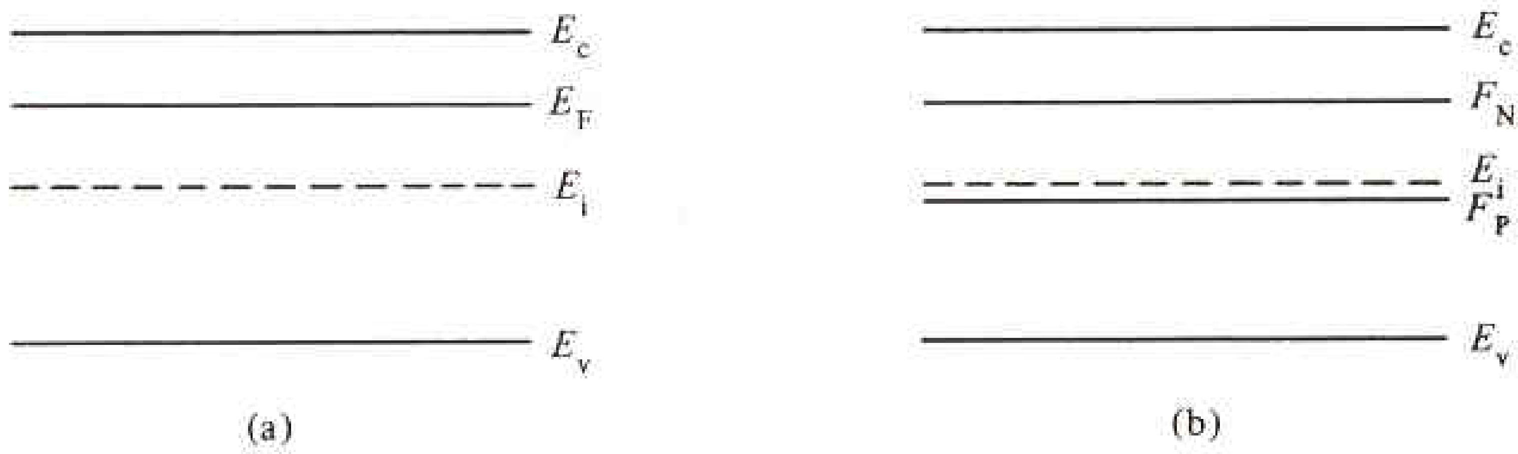
✓ Equilibrium conditions prevailed prior to t=0, with $n_0=N_D=10^{15}$ cm^{-3} and $p_0=10^5$ cm^{-3}

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$p_0 = n_i e^{(E_i - E_F)/kT}$$

✓ $\Delta p_n = G_L \tau_p = 10^{11}\ \mathrm{cm}^{-3},\ p = p_0 + \Delta p \cong 10^{11}\ \mathrm{cm}^{-3},\ n \cong n_0 = 10^{15}\,cm^{-3}$

✓ The convenience of being able to deduce the carrier concentrations by inspection from the energy band diagram is extended to nonequilibrium conditions through the use of quasi-Fermi levels by introducing F_N and F_P .



Equilibrium conditions   Nonequilibrium conditions

$$n \equiv n_i e^{(F_N - E_i)/kT} \quad \text{or} \quad F_N \equiv E_i + kT \ln\left(\frac{n}{n_i}\right)$$

$$p \equiv n_i e^{(E_i - F_P)/kT} \quad \text{or} \quad F_P \equiv E_i - kT \ln\left(\frac{p}{n_i}\right)$$