

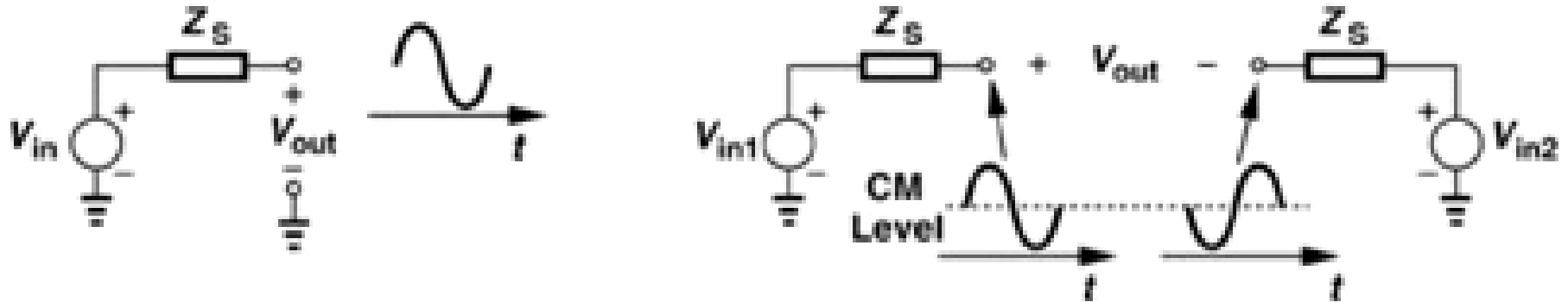
# Chapter 4

## Differential Amplifiers

# CMOS Differential Amplifiers

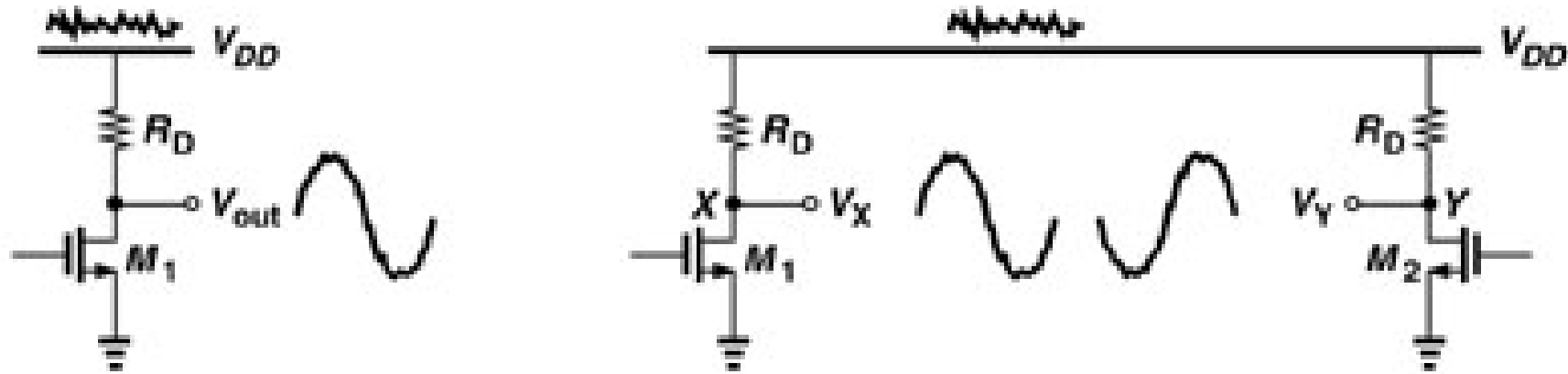
Basic Concepts-L15

# Single-ended vs. Differential Signals



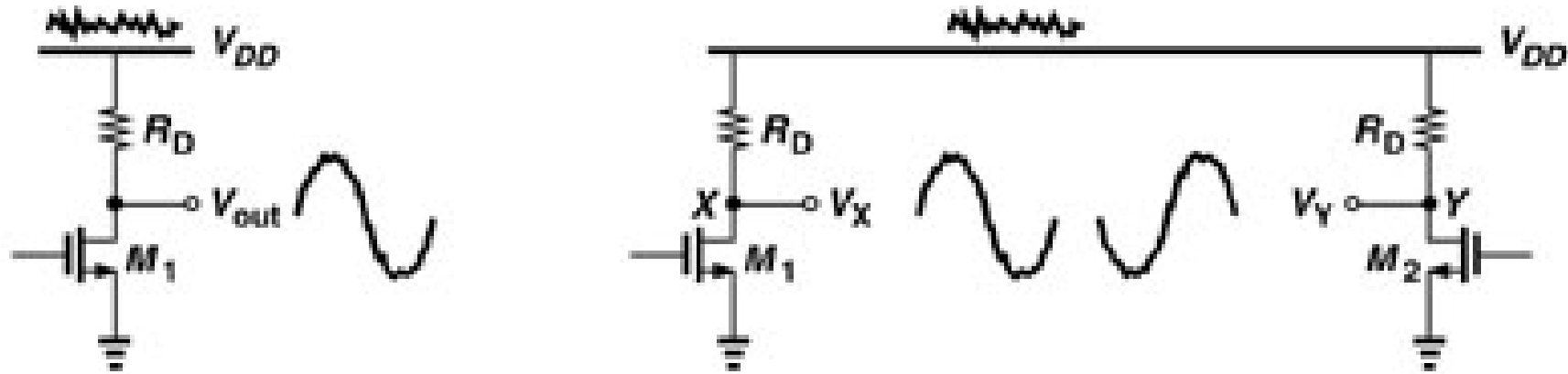
- Single-ended signal: Measured with respect to a fixed potential, typically ground.
- Differential signal: Measured between two symmetric nodes (nodes have equal and opposite signal excursions around a fixed potential, called a “common-mode” level)

# Why Differential?



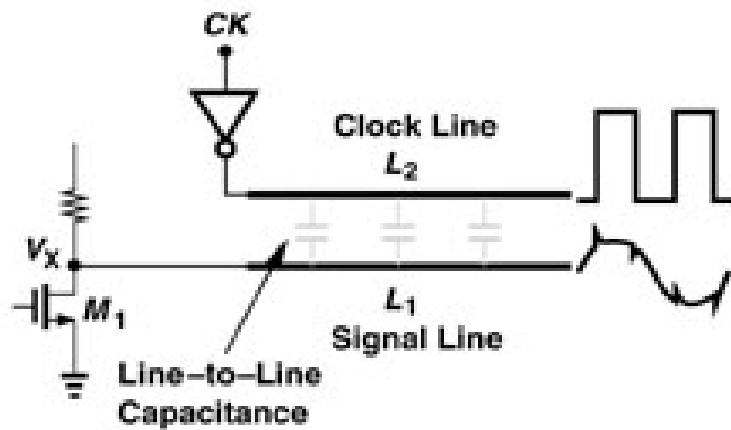
- In a single amplifier  $V_{DD}$  fluctuations appear directly on the amplified signal.
- In a differential pair, if we measure  $V_X - V_Y$  the fluctuations cancel out.

# Why Differential?

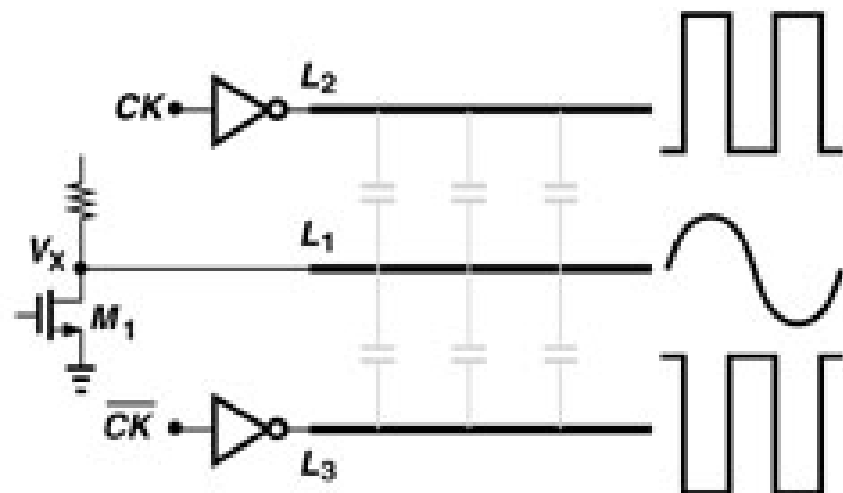
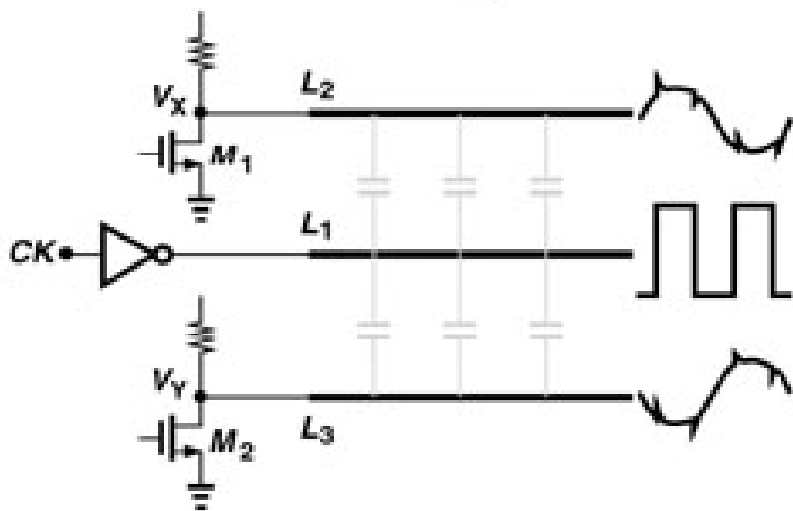


- In a single CS amplifier, the maximum swing is  $V_{DD} - (V_{GS} - V_{TH})$
- In a differential pair it can be shown that the swing of  $V_X - V_Y$  can reach  $2[V_{DD} - (V_{GS} - V_{TH})]$ .

# Clock Noise Reduction



(a)

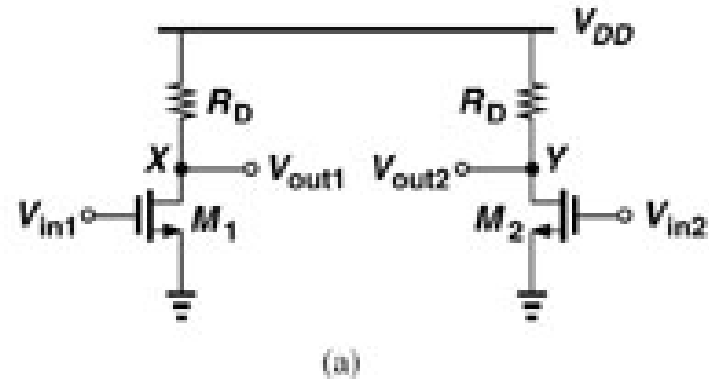


# Other advantages of differential amplifying

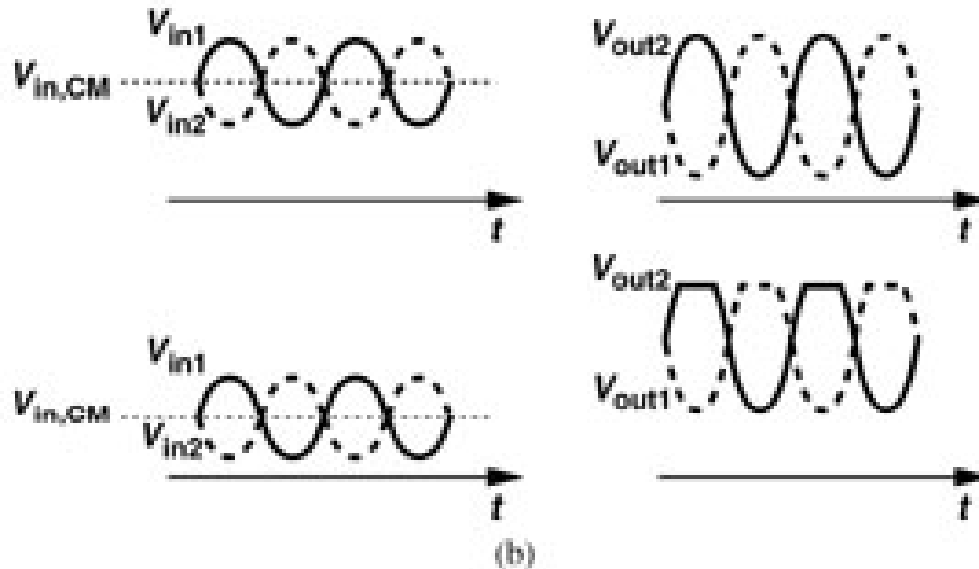
- Simpler biasing
- Higher linearity

# Simple Symmetric Differential Pair won't do!

Good features:  
Rejection of  
VDD  
fluctuations,  
Larger swing.

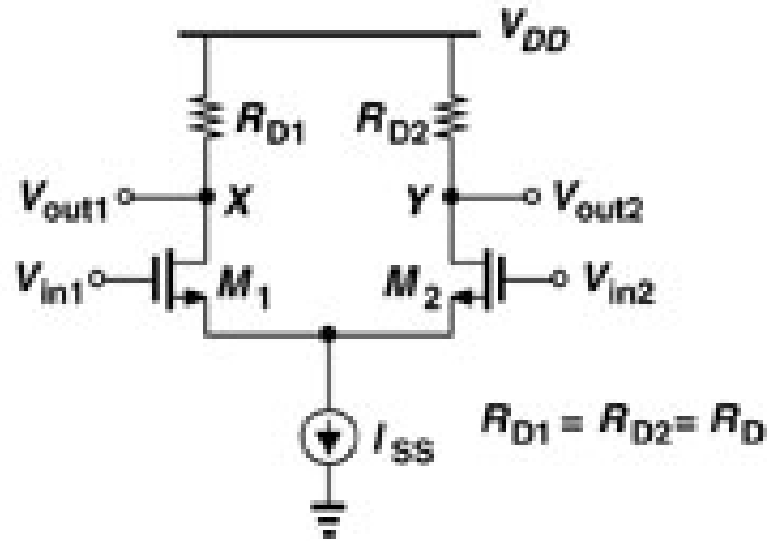


Key problem: Input  
signal common-  
mode affects bias  
conditions, and  
differential  
amplification.





Solution: Source-coupled (“long-tailed”) Pair,  
biasing with a current source



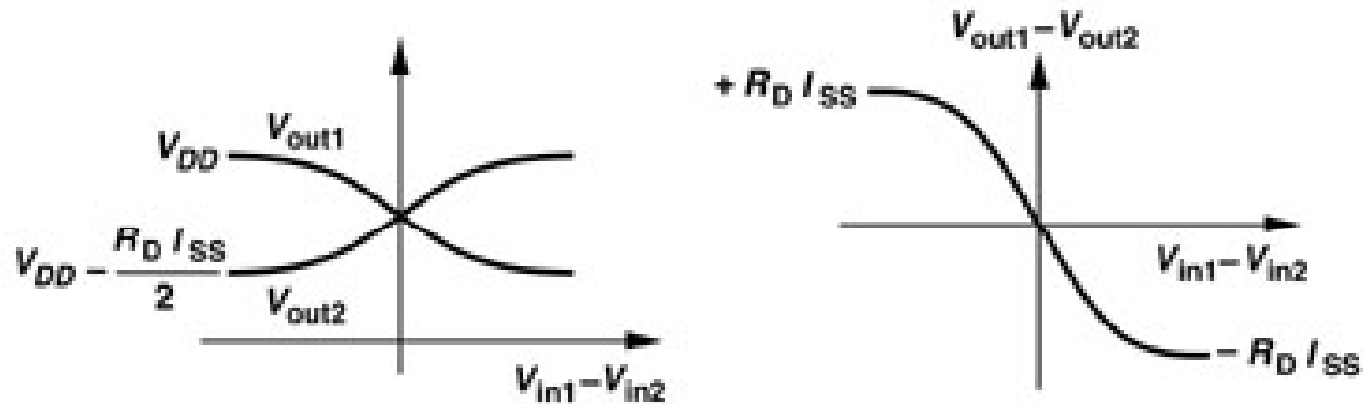
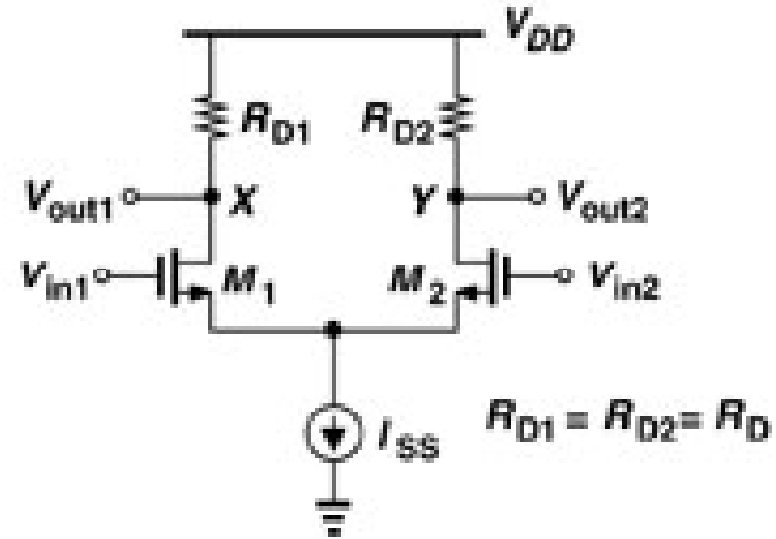
If  $V_{in1} = V_{in2}$  then  $I_{D1} = I_{D2} = I_{SS}/2$

Then  $V_{out1} = V_{out2} = V_{DD} - R_D I_{SS}/2$

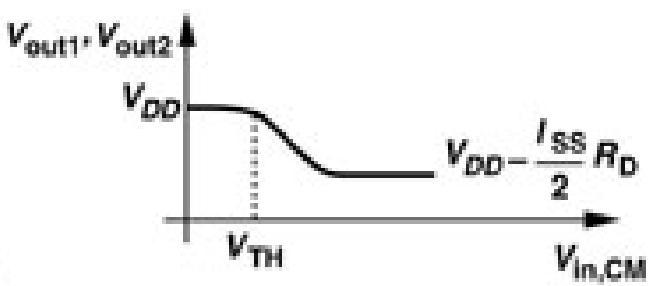
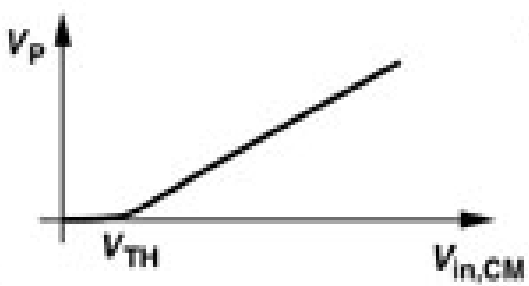
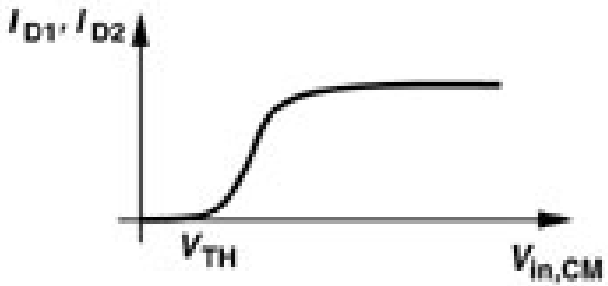
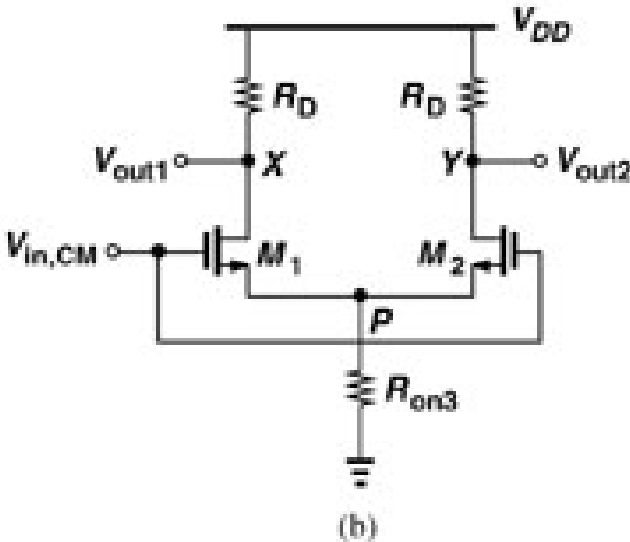
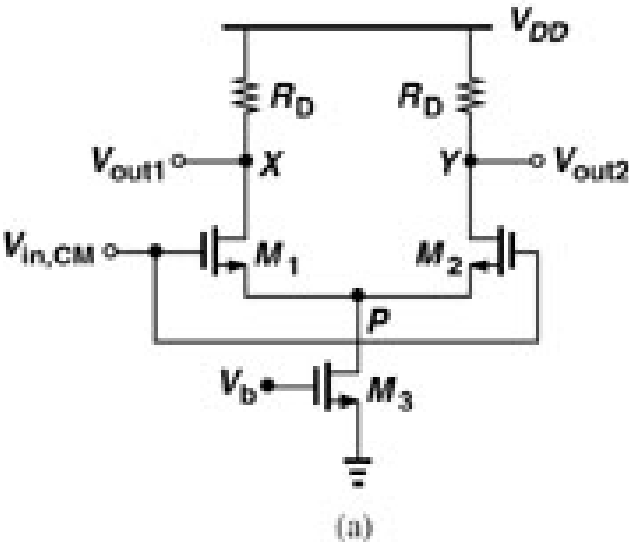
# “Current stealing “ phenomenon

If  $V_{in1} \gg V_{in2}$ , then  $M_2$  turns off and  $M_1$  steals all the current.

If  $V_{in1} \ll V_{in2}$  then  $M_2$  takes all the current.

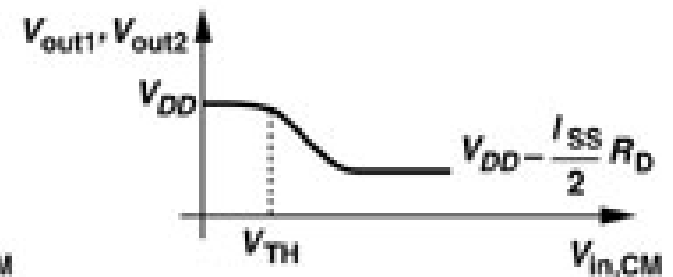
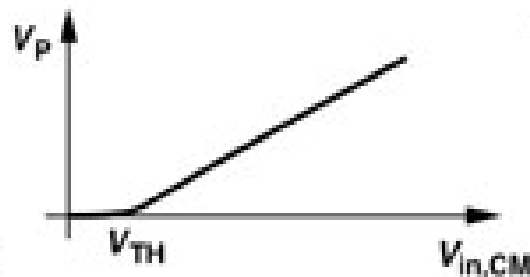
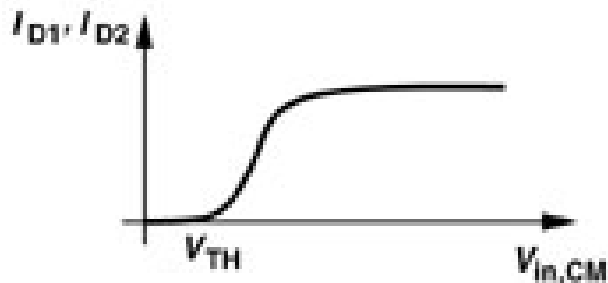
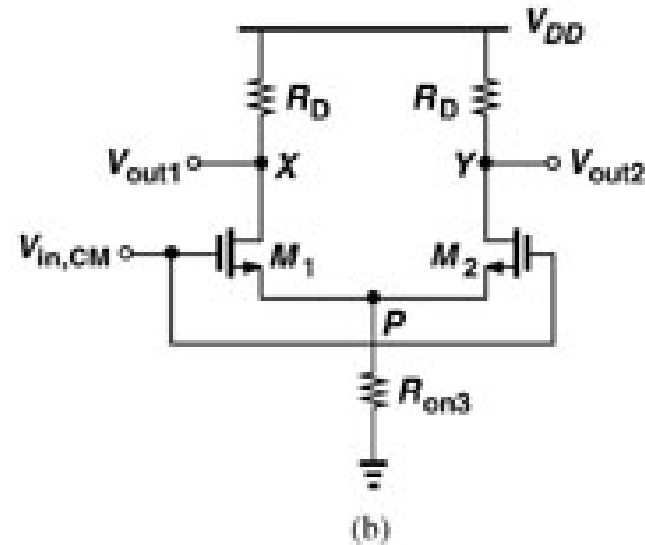
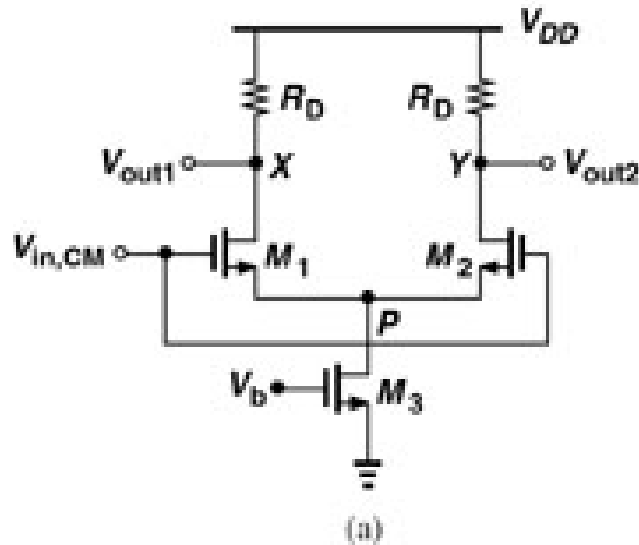


# Common-Mode Response



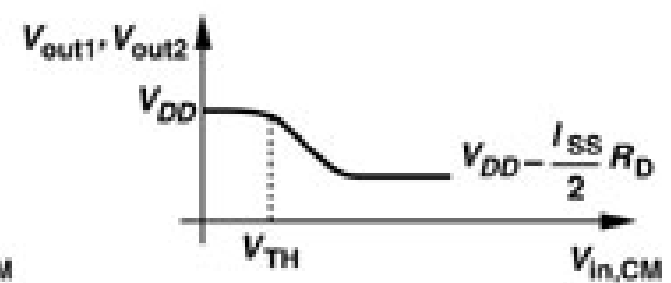
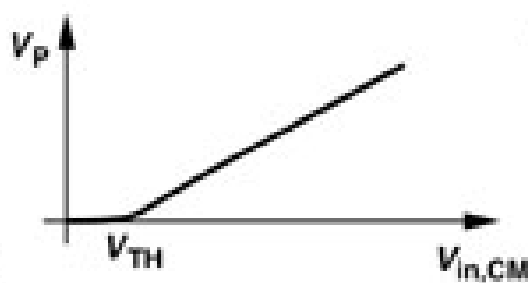
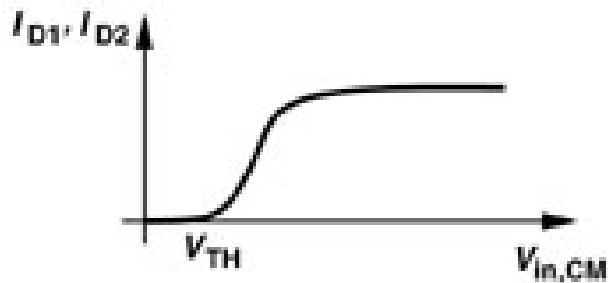
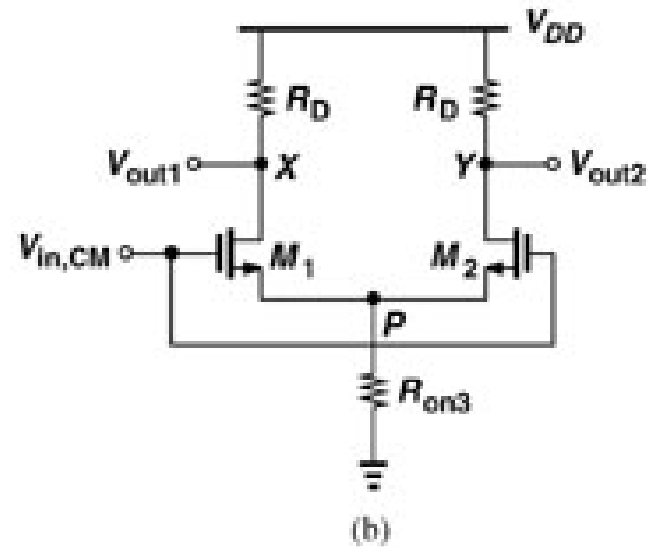
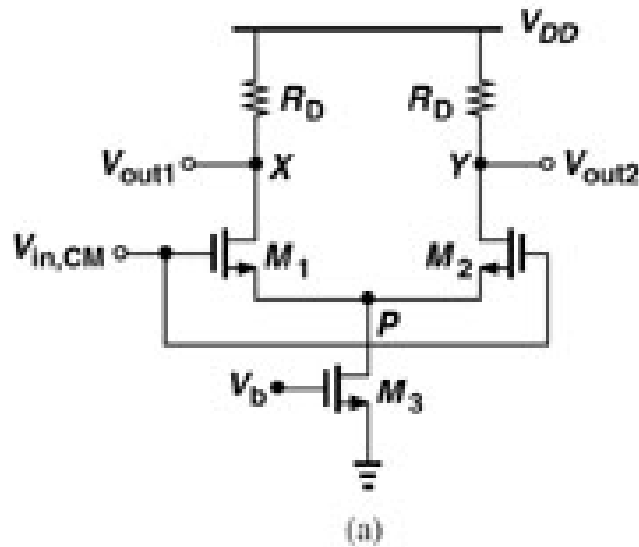
Can  $V_{in,CM}$  be arbitrarily large or small?

$V_{in,CM}$  must be large enough for  $M_3$  to be in saturation



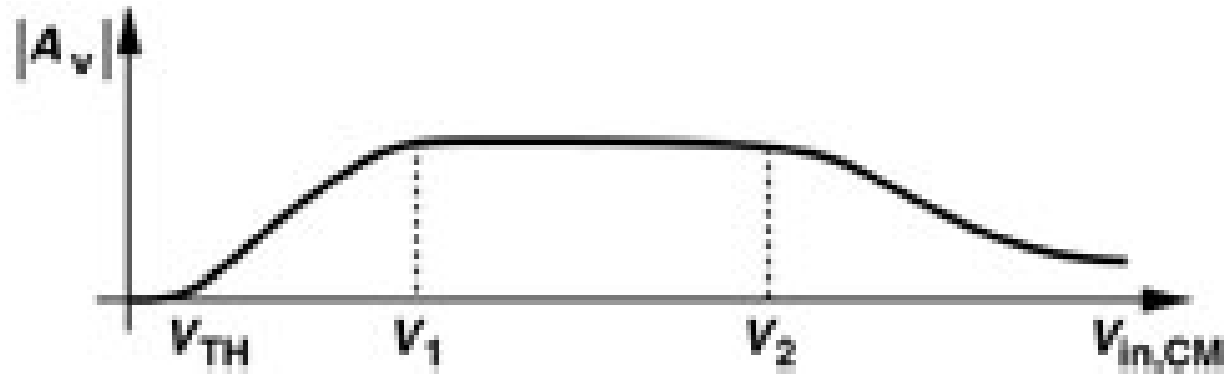
$M_1, M_2$  operate like Source Followers:  $V_P$  increases as  $V_{in,CM}$  increases – must be larger than  $V_b - V_{TH3}$

$V_{in,CM}$  must not be too large to keep  $M_1$  and  $M_2$  from entering Triode Mode



$$V_{GS1} + (V_{GS3} - V_{TH3}) \leq V_{in,CM} \leq \min[V_{DD} - R_D \frac{I_{SS}}{2} + V_{TH1}, V_{DD}]$$

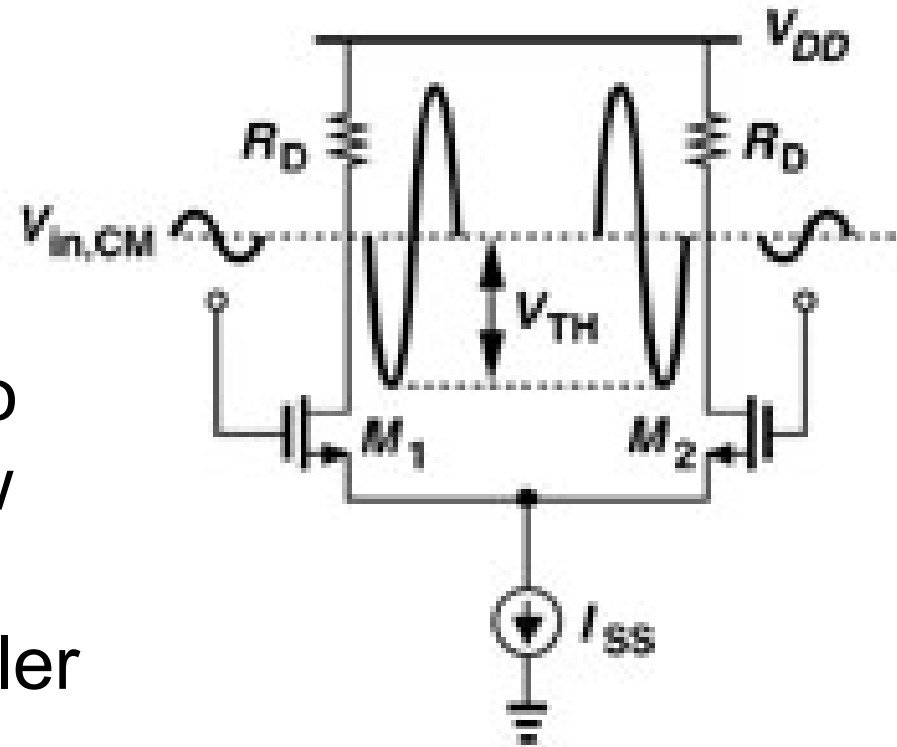
# Consequences of $V_{in,CM}$ going “off range”



- As long as common mode voltage is within the permitted range, differential gain is almost insensitive to it.
- Once too small or too large – gain falls off.

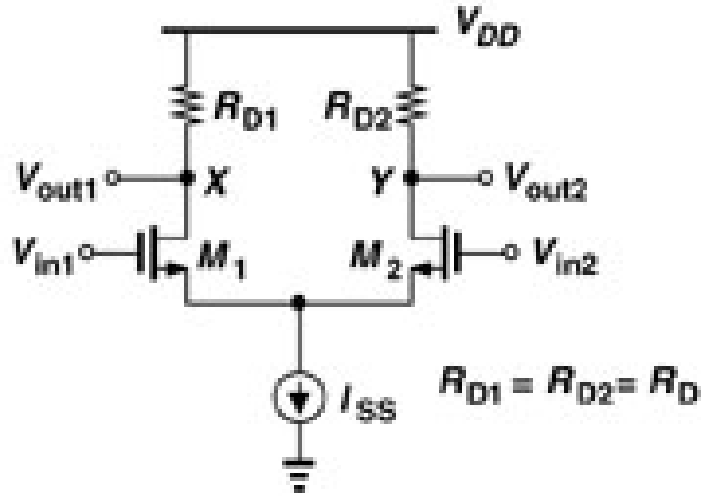
# Common-Mode Input vs. Output Swing Tradeoff

Each drain voltage can go as high as  $V_{DD}$  and as low as  $V_{in,CM} - V_{TH}$ .  
The larger  $V_{in,CM}$  the smaller the swing.



# Two types of Differential Gains

“Single-ended”:  $V_{out1}$  (or  $V_{out2}$ ) with respect to ground.

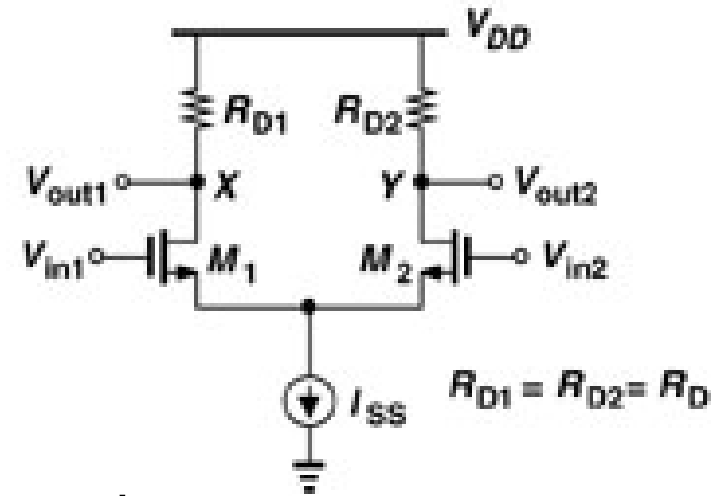


$$A_v(\text{diff}) = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = ?$$

$$A_v(\text{S.E.}) = \frac{V_{out1}}{V_{in1} - V_{in2}} = ?$$



# Current Division Mechanism



Calculate  $I_{D1}$  and  $I_{D2}$  in terms of  $V_{in1}$  and  $V_{in2}$ , assuming the circuit is symmetric,  $M_1$  and  $M_2$  are saturated, and  $\lambda=0$ .

Since the voltage at node P is equal to

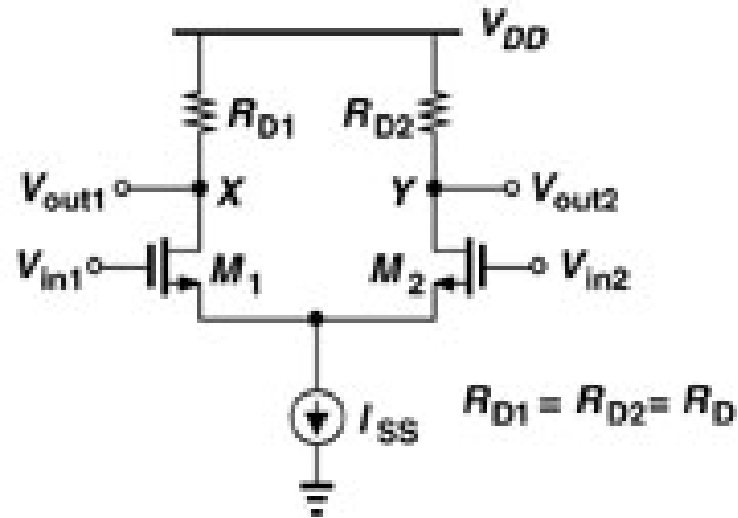
$$V_P = V_{in1} - V_{GS1} = V_{in2} - V_{GS2},$$

$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2}$$

$$V_{GS} = \sqrt{\frac{2I_D}{k_n' \frac{W}{L}}} + V_{TH}$$

# Current Division Mechanism

Given the inputs and  $I_{SS}$ : Solve two equations with two unknowns for the transistor currents:



$$V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{k_n' \frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{k_n' \frac{W}{L}}}$$

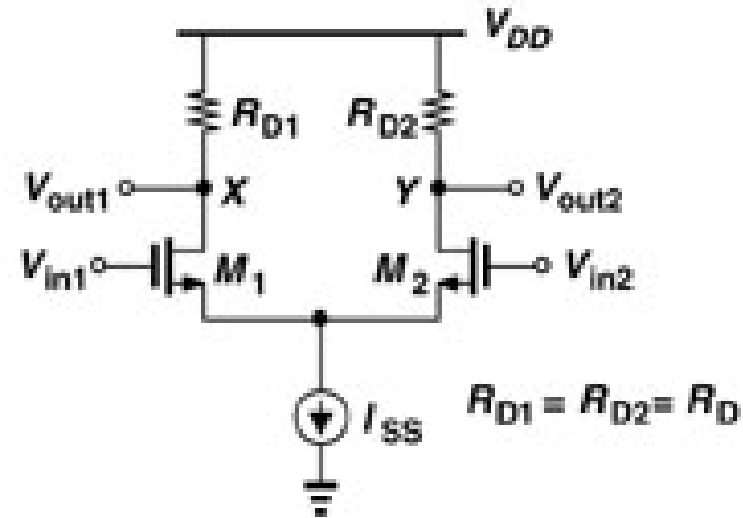
$$I_{SS} = I_{D1} + I_{D2}$$

# CMOS Differential Amplifiers

Small-Signal Differential Gain-L16

# Current Difference Properties

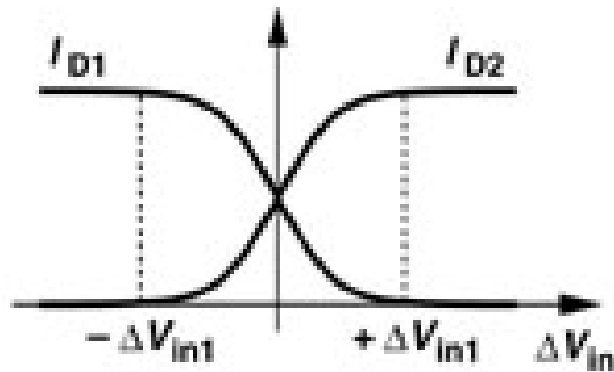
Even though each current is an even function of its respective gate-source voltage, the current difference is an odd function of the input voltage difference



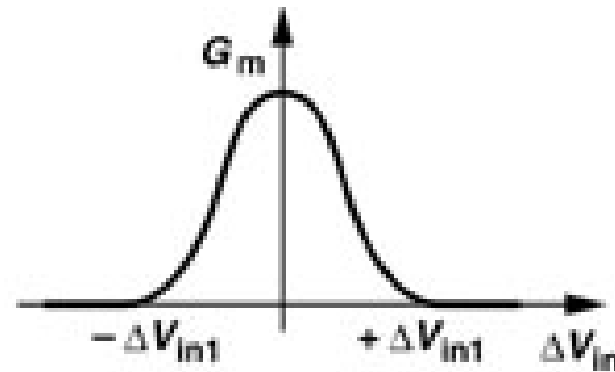
$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{OX} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{OX} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

# Differential Transconductance Gain vs. Input Voltage

$$\frac{\partial \Delta I_D}{\partial \Delta V_{in}} = G_m = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \frac{\frac{4I_{SS}}{\mu_n C_{ox} W / L} - 2(V_{in1} - V_{in2})^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} W / L} - (V_{in1} - V_{in2})^2}}$$



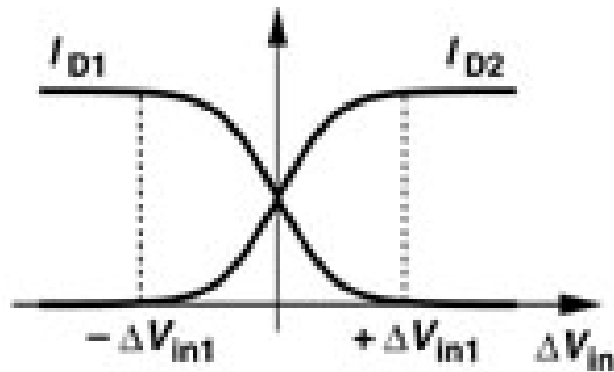
(a)



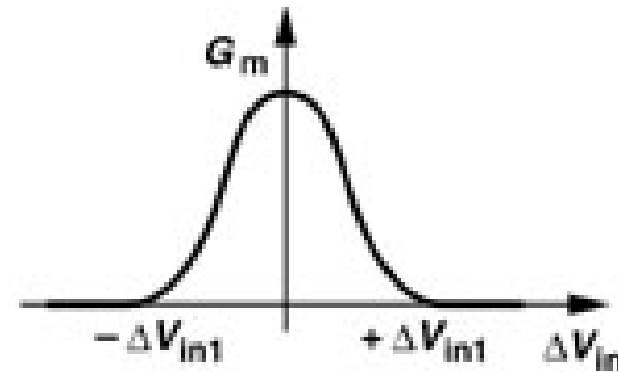
(b)

# Maximum Differential Transconductance Gain Occurs at $\Delta V_{in}=0$

$$G_{m,\max} = \sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right) I_{SS}}$$



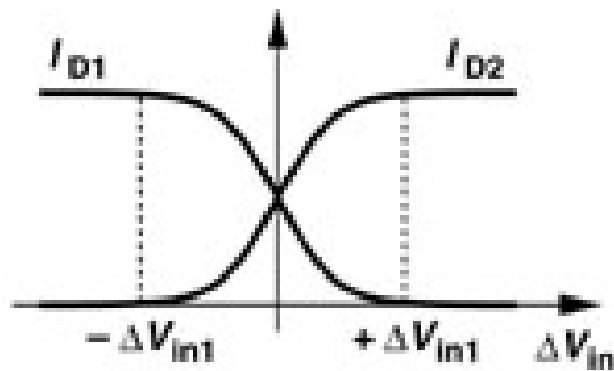
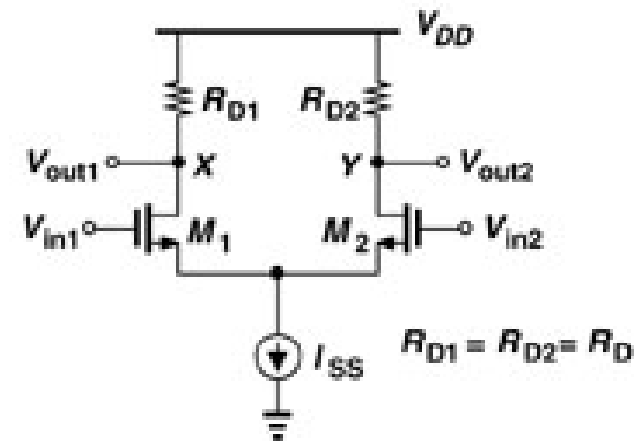
(a)



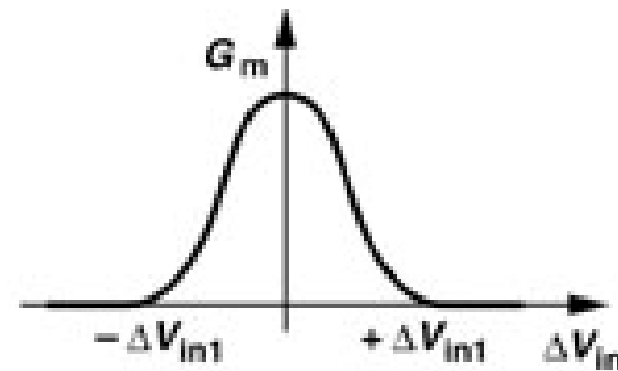
(b)

# Differential Voltage Gain

$$V_{out1} - V_{out2} = R_D \Delta I_D = R_D G_m \Delta V_{in}$$



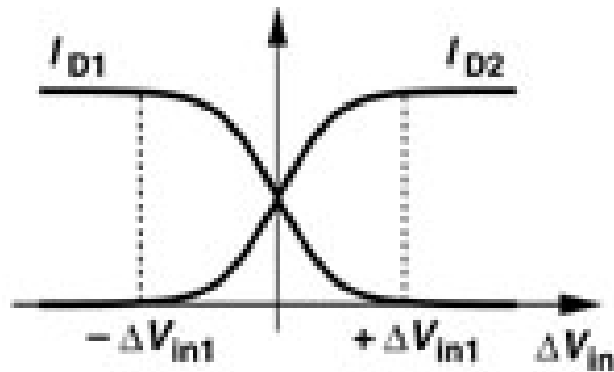
(a)



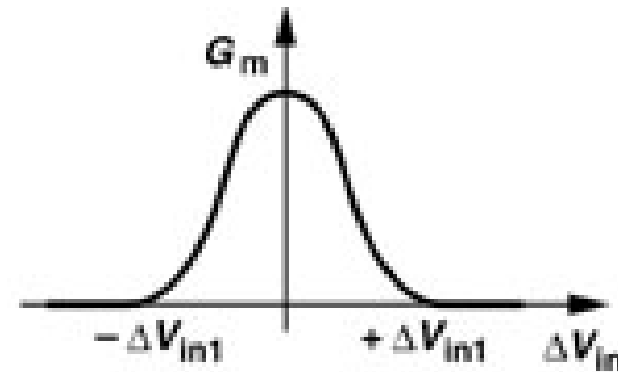
(b)

# Differential Voltage Gain near $\Delta V_{in} = 0$

$$|A_V| = \frac{\Delta V_{out}}{\Delta V_{in}} = G_{m,\max} R_D = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS} R_D}$$



(a)

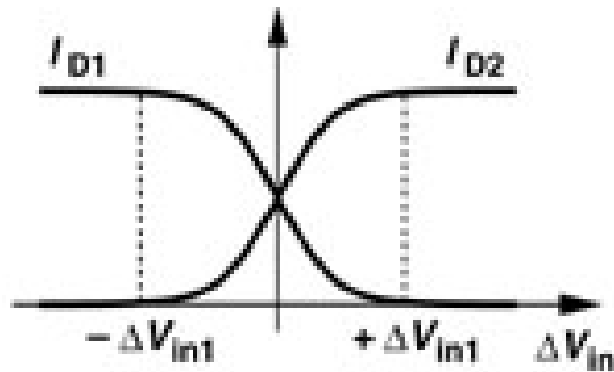


(b)

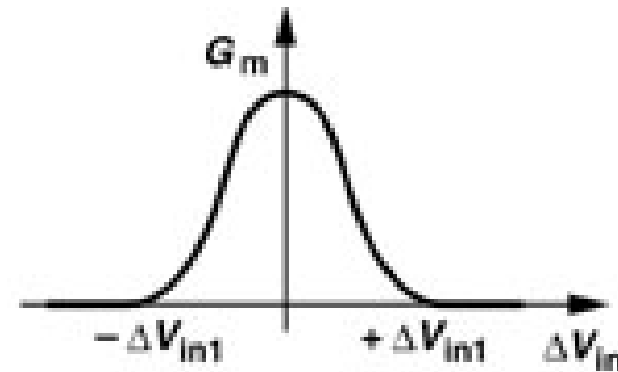


Differential Transconductance Gain Falls to  
Zero at  $\Delta V_{in} = \Delta V_{in1}$

$$\Delta V_{in1} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$



(a)



(b)

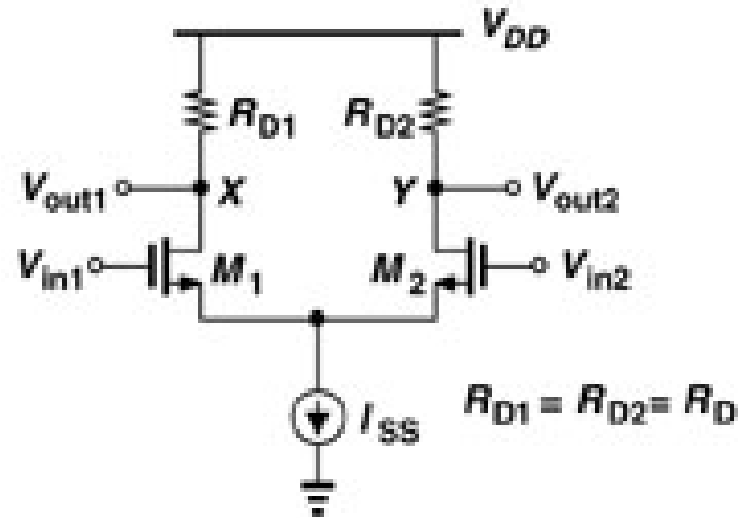
# Current Difference Properties

It appears as if  $\Delta I_D$  also becomes zero at

$$\Delta V_{in2} = (4I_{SS} / (\mu_n C_{OX} W/L))^{1/2}$$

But this is incorrect.

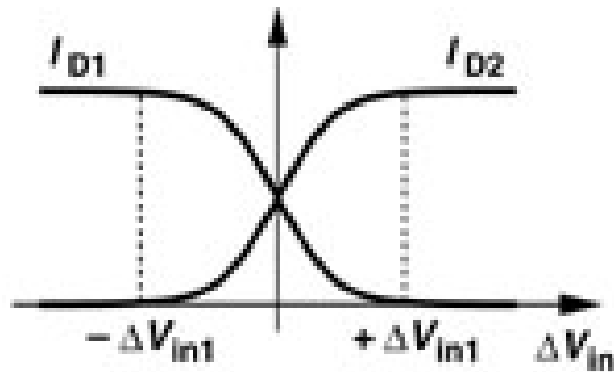
$\Delta V_{in2} > \Delta V_{in1}$  at which a total current stealing occurs.



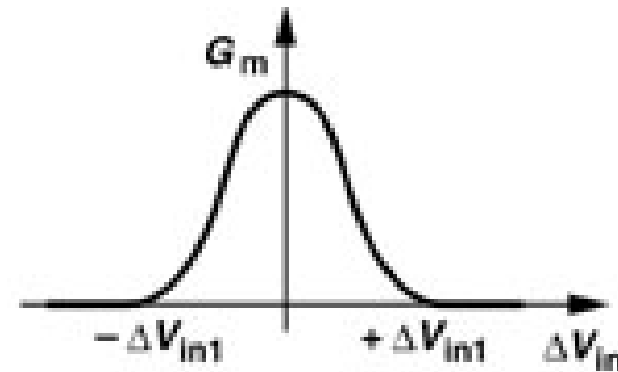
$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{OX} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{OX} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

$\Delta V_{in} = \Delta V_{in1}$  is the maximum differential input that the amplifier can “handle”

$$\Delta V_{in1} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$



(a)



(b)

Also note that at  $\Delta V_{in}=0$  each transistor carries a current of  $I_{SS}/2$  and therefore :

$$(V_{GS} - V_{TH})_{1,2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

Compare this equilibrium overdrive to the maximum differential input permitted:

$$(V_{GS} - V_{TH})_{1,2} = \frac{\Delta V_{in1}}{\sqrt{2}}$$

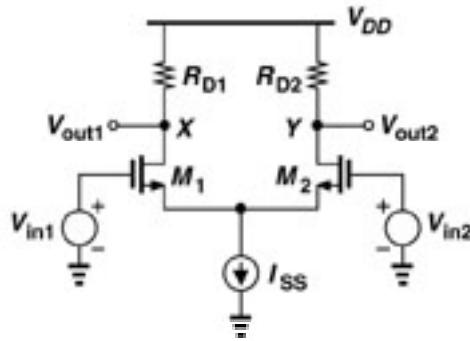
$$\Delta V_{in1} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

It means that if we try to increase  $\Delta V_{in1}$ , for a given  $I_{SS}$ , we need larger overdrive voltage in each transistor, and this is accomplished by reducing  $W/L$

$$(V_{GS} - V_{TH})_{1,2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \quad (V_{GS} - V_{TH})_{1,2} = \frac{\Delta V_{in1}}{\sqrt{2}}$$

$$\Delta V_{in1} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

# Small-Signal Differential Voltage Gain



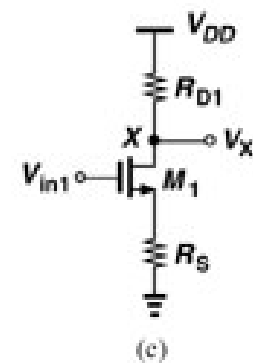
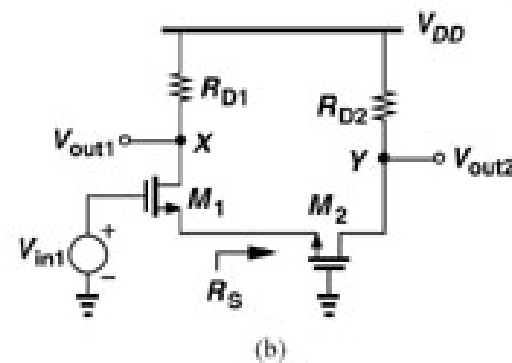
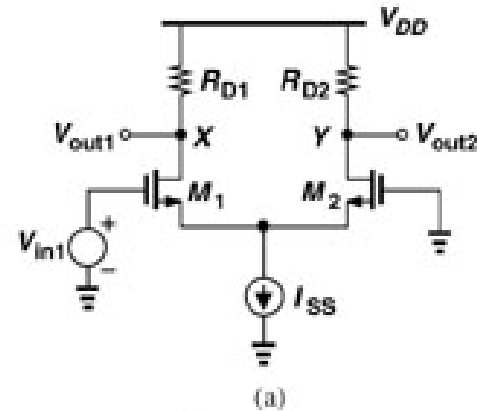
- For  $|\Delta V_{in}| \approx 0$  (sufficiently small) we have:

$$|A_V| = \frac{\Delta V_{out}}{\Delta V_{in}} = G_{m,\max} R_D = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} R_D = g_m R_D$$

Where  $g_m$  is that of a NMOS with a current of  $I_{SS}/2$

# Differential Gain: What does each input “see”?

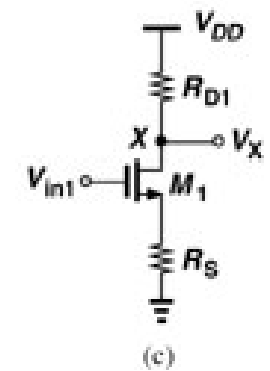
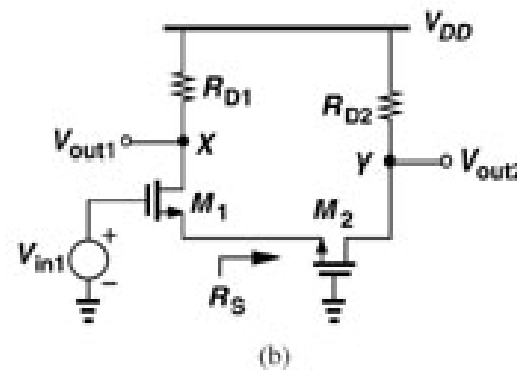
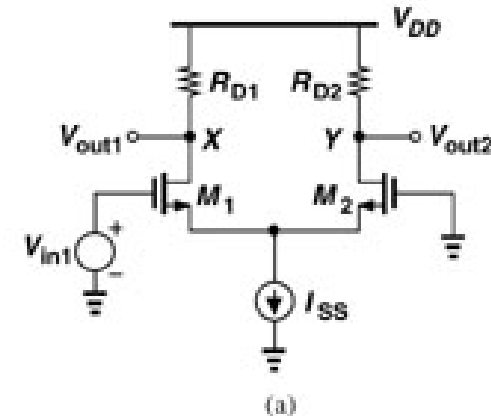
- By superposition let's short  $V_{in2}$  to ground and see the effect of  $V_{in1}$ :
- The effect of  $V_{in1}$  on  $V_X$  is the same as that of a CS amplifier (degenerated by the resistance seen looking into the source of  $M_2$ ) on  $V_D$



# Differential Gain: Effect of $V_{in1}$ on $V_X$

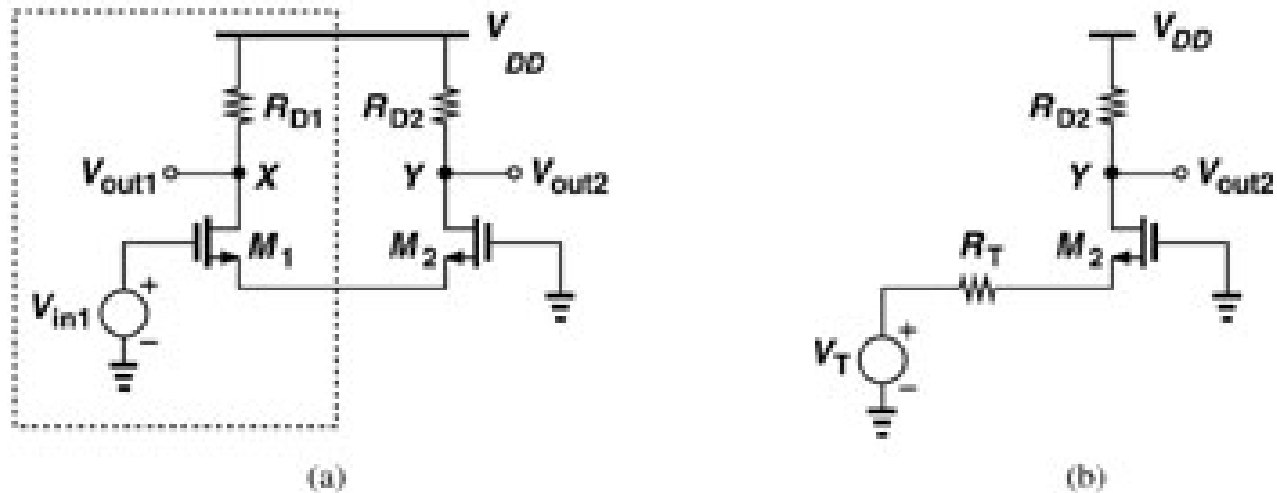
Neglecting  $\lambda_2$  and  $g_{mb2}$   $R_S \approx 1/g_{m2}$

$$\frac{V_X}{V_{in1}} \approx \frac{-R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$





# Differential Gain: Effect of $V_{in1}$ on $V_Y$

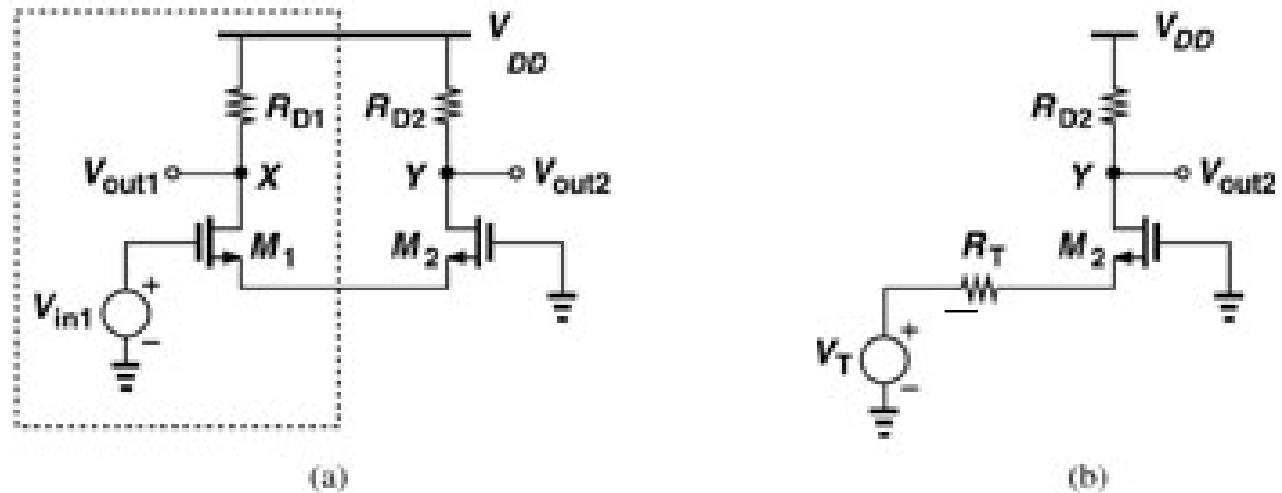


The effect of  $V_{in1}$  on  $V_Y$  is the same as that of a Source Follower ( $M_1$ ) amplifier driving a Common-Gate amplifier ( $M_2$ )

$$V_T = V_{in1}$$

$$R_T = \frac{1}{g_{m1}}$$

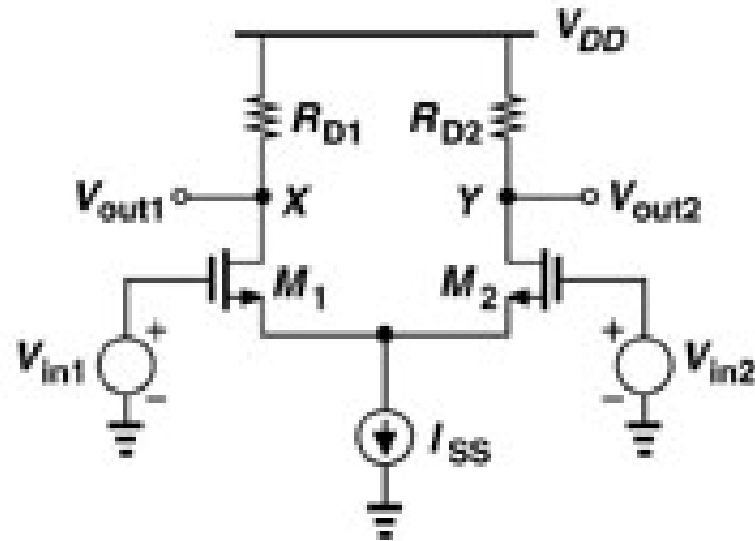
# Differential Gain: Effect of $V_{in1}$ on $V_Y$



The effect of  $V_{in1}$  on  $V_Y$  is the same as that of a Source Follower ( $M_1$ ) amplifier driving a Common-Gate amplifier ( $M_2$ )

$$\frac{V_Y}{V_{in1}} \approx \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

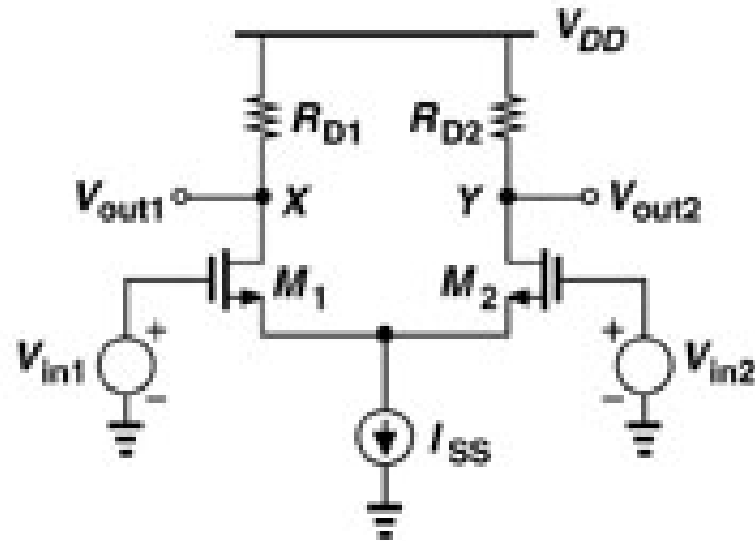
$V_X - V_Y$  as function of  $V_{in1}$  if  $V_{in2} = 0$



$$(V_X - V_Y)_{due\_to\_V_{in1}} = \frac{-2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in1} = -g_m R_D V_{in1}$$

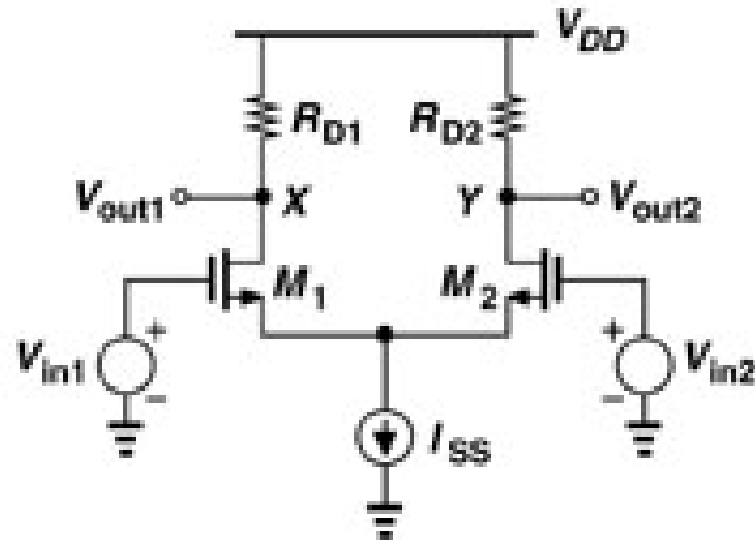
Because  $g_{m1} = g_{m2} = g_m$

By Symmetry:  $V_X - V_Y$  as function of  $V_{in2}$  if  
 $V_{in1} = 0$



$$(V_X - V_Y)_{due\_to\_V_{in2}} = \frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in2} = g_m R_D V_{in2}$$

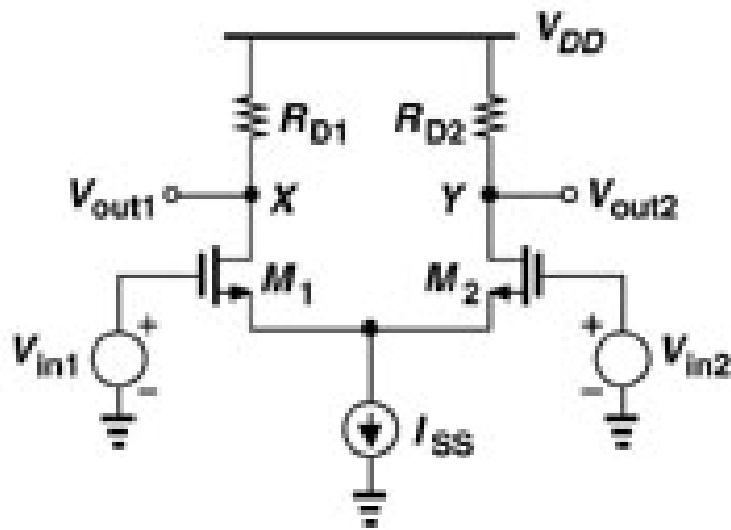
$V_X - V_Y$  as function of  $V_{in2}$  and  $V_{in1}$



$$(V_X - V_Y)_{due\_to\_both} = g_m R_D V_{in2} - g_m R_D V_{in1}$$

yielding the gain  $g_m R_D$  regardless where and how the inputs are applied

# Single-ended Differential Voltage Gain



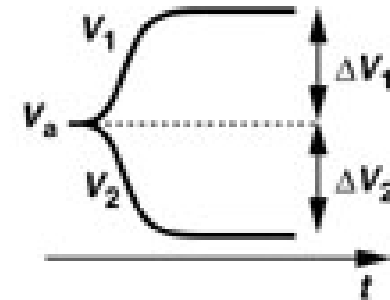
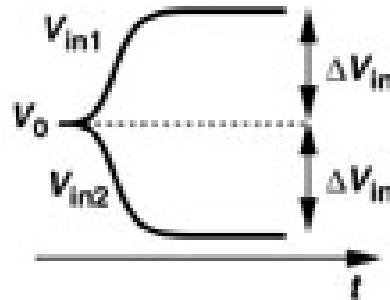
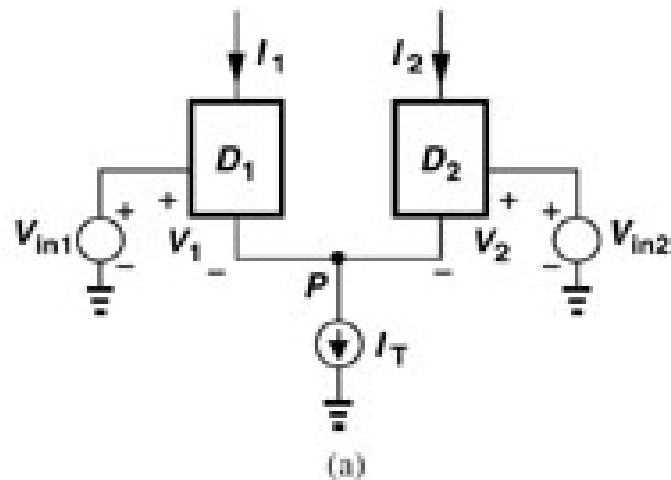
$$A_{V,SE} = \frac{V_X}{V_{in1} - V_{in2}} = -\frac{g_m}{2} R_D$$

$$A_{V,SE} = \frac{V_Y}{V_{in1} - V_{in2}} = \frac{g_m}{2} R_D$$

## Comparison: Differential voltage gain of a differential amplifier vs. voltage gain of a CS amplifier

- If the same current source  $I_{SS}$  drives the differential amplifier and the CS, each transistor of the differential amplifier has  $g_m$  which is  $1/\sqrt{2}$  of that of the CS transistor. Differential gain reduces by a factor of  $1/\sqrt{2}$ .
- If both amplifiers have the same  $W/L$  in each transistor and the same load, and we want the gain to be the same, then if we use  $I_{SS}$  at CS, we need to use  $2I_{SS}$  at the differential amplifier.

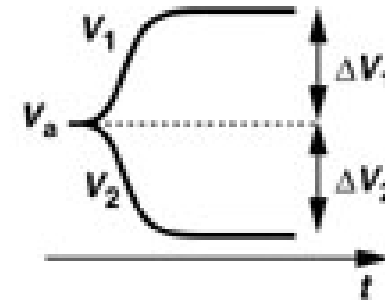
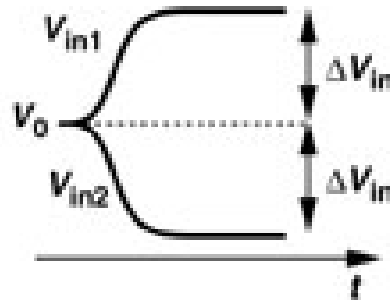
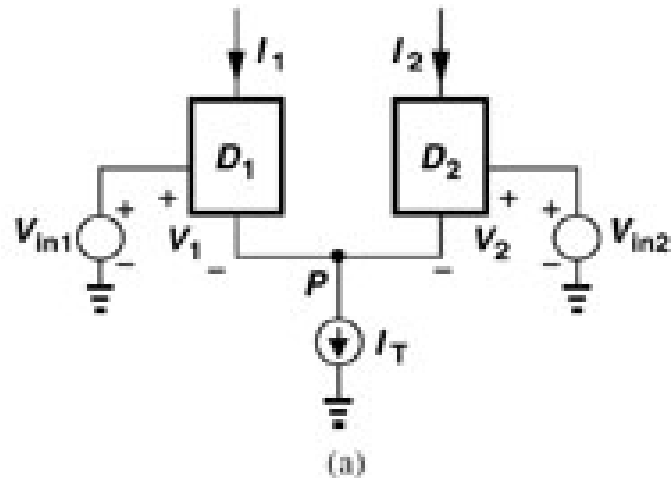
# The “Virtual Ground” Concept



- In a symmetric device (as above), if inputs change anti-symmetrically (one goes up by a certain amount, and the other goes down by the same amount), then  $V_P$  does not change.
- For small-signal analysis point  $P$  becomes “virtual ground”.

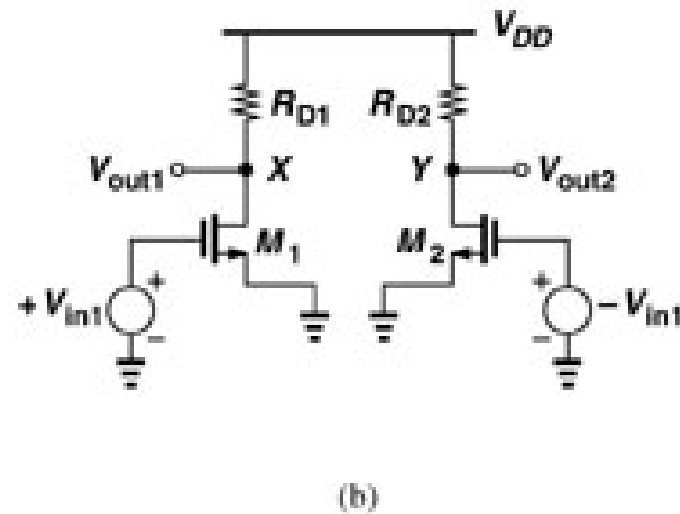
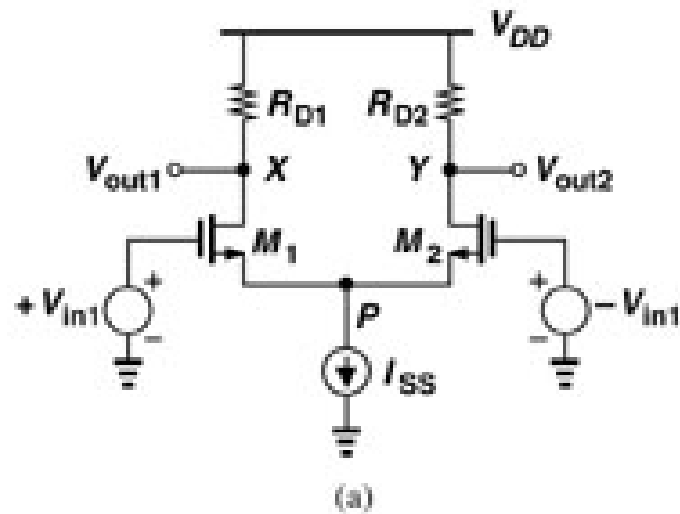


# The “Virtual Ground” Concept



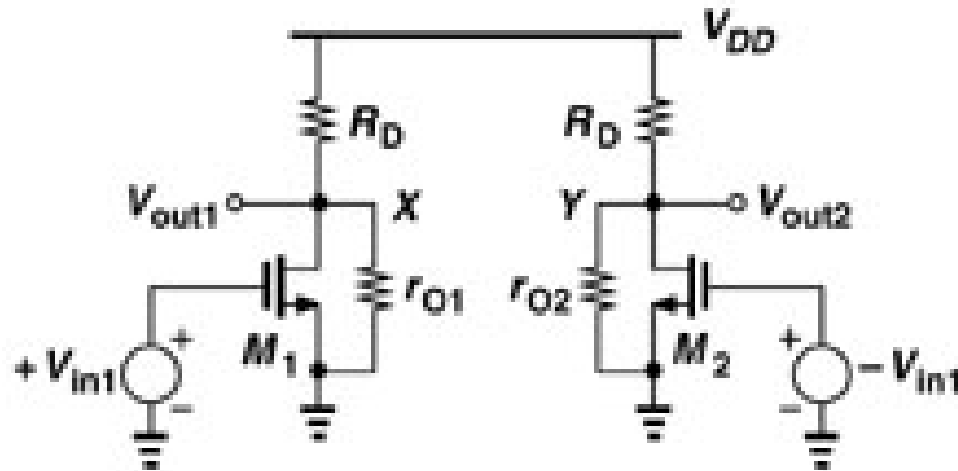
- Explanation: Consider the complete “Kirchhoff path”  $V_{in1} \rightarrow D_1 \rightarrow D_2 \rightarrow V_{in2}$ .
- By symmetry of the devices, and anti-symmetry of sources, using voltage-division argument,  $V_p$  stays constant.
- See book (pp. 114-115) for 2-3 more explanations.

# The “Half-Circuit” Concept



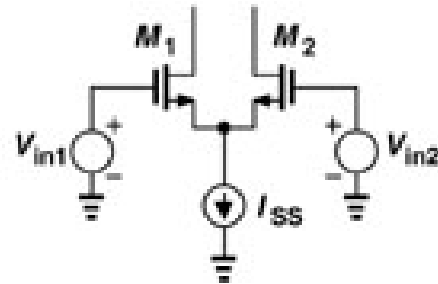
- For small-signal analysis, because point P is “ground” (this is valid only if inputs are anti-symmetric and devices are symmetric!), we can analyze each CS “half-circuit” separately.

# Differential Gain with $\lambda$ effect

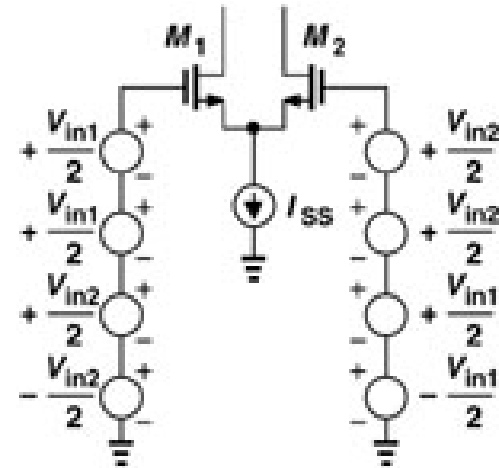


- Based on the half-circuit concept, gain calculation is highly simplified. We get:
- $V_X/V_{in1} = -g_m(R_D || r_o) = V_Y/(-V_{in1})$
- $(V_X - V_Y)/2V_{in1} = -g_m(R_D || r_o)$

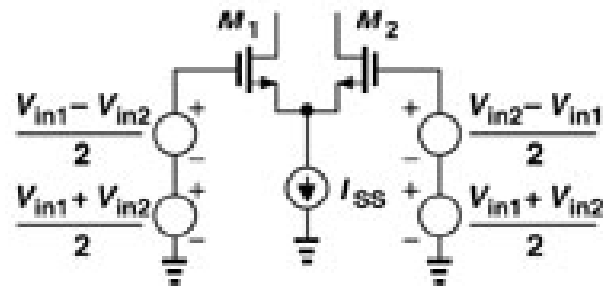
In general  $V_{in1}$  and  $V_{in2}$  are arbitrary (not necessarily anti-symmetric): What do we do?



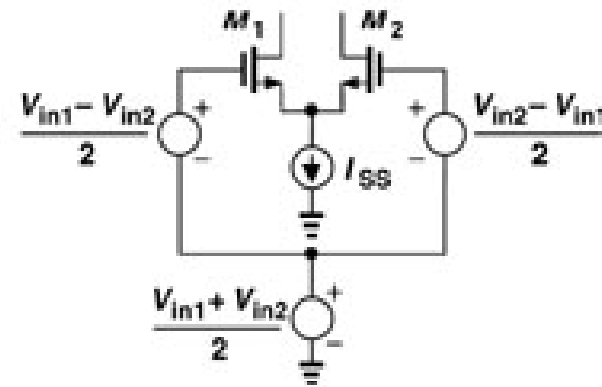
(a)



(b)



(c)



(d)

# Analysis of Differential Amplifiers for arbitrary inputs

- As long as circuit operates more or less linearly, we use superposition of two analyses:
- Differential input analysis, using anti-symmetric inputs derived from the difference between the inputs.
- Common-mode analysis, where an input equals to the average of both inputs is applied to both transistors.

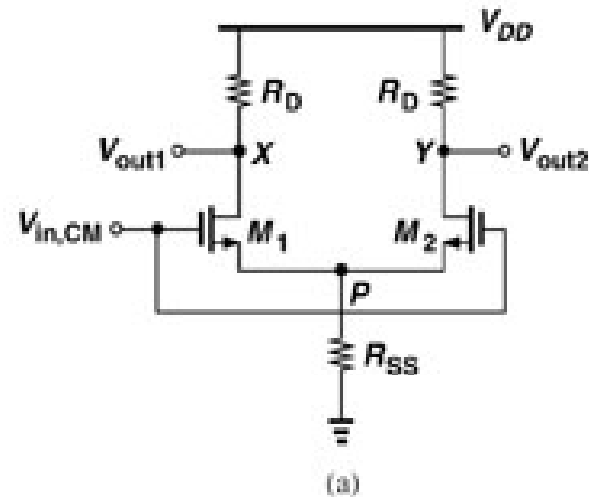
# CMOS Differential Amplifiers

Common-Mode Analysis –L17

# Goal: No common-mode signal at amplifier's output

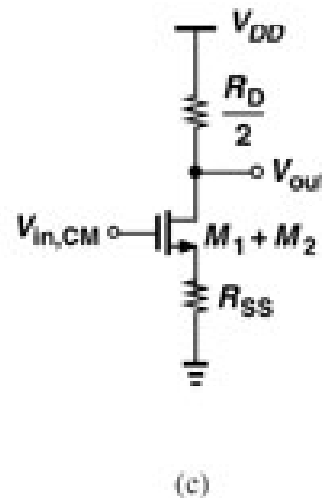
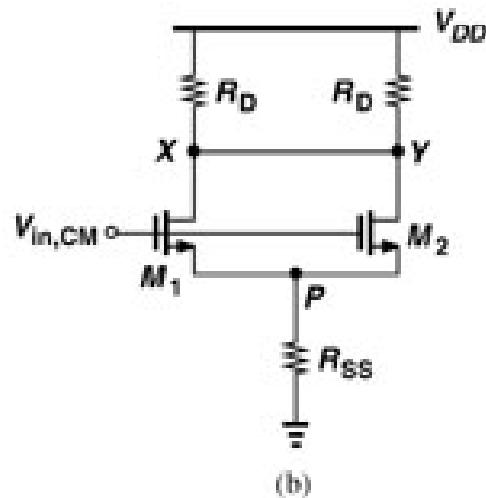
- If amplifier is not 100% symmetric, CM signals will not be fully cancelled out.
- If the DC current source (biasing the amplifier) is not ideal (that is, has a finite output resistance) then CM signal appears at each single-ended output.

# Single-ended Common-Mode Response of a symmetric amplifier



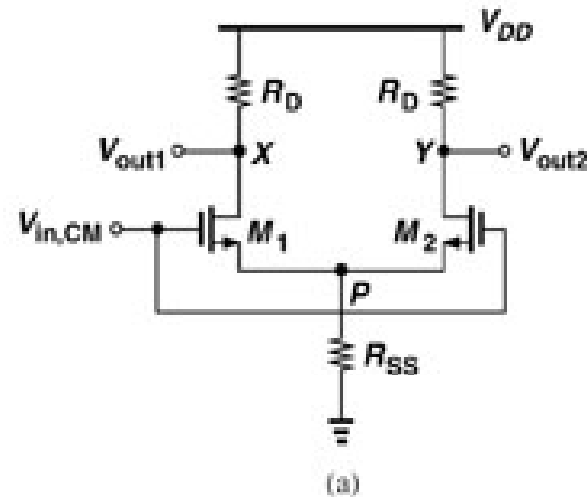
Consider a small-signal analysis for the common-mode signal.

Current source is represented by its output resistance  $R_{SS}$

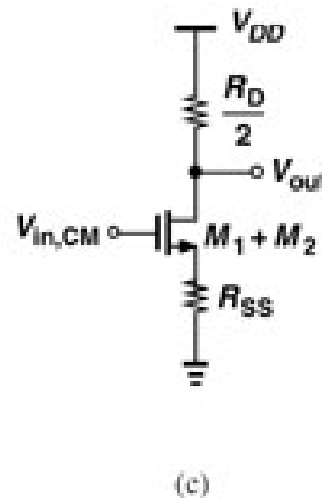
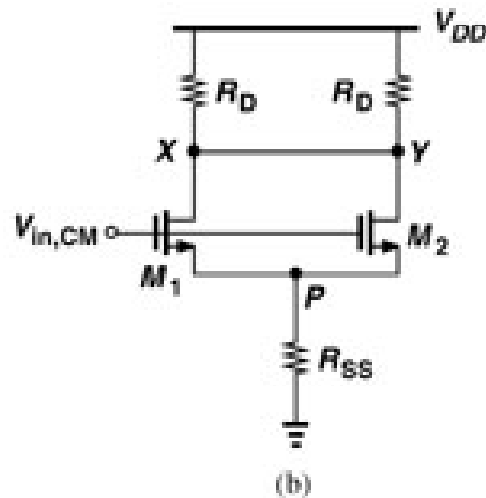




# Single-ended Common-Mode Response of a symmetric amplifier

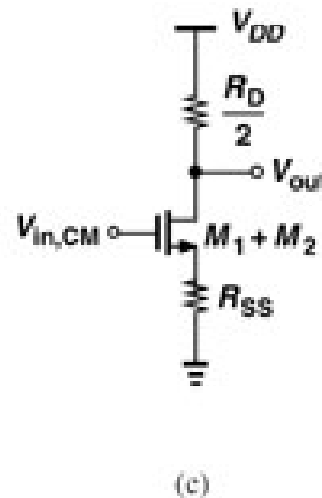
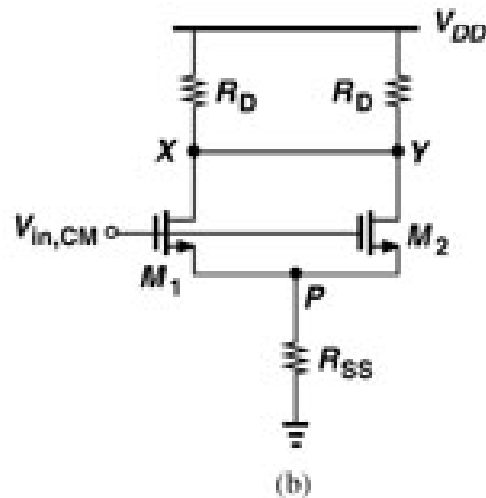
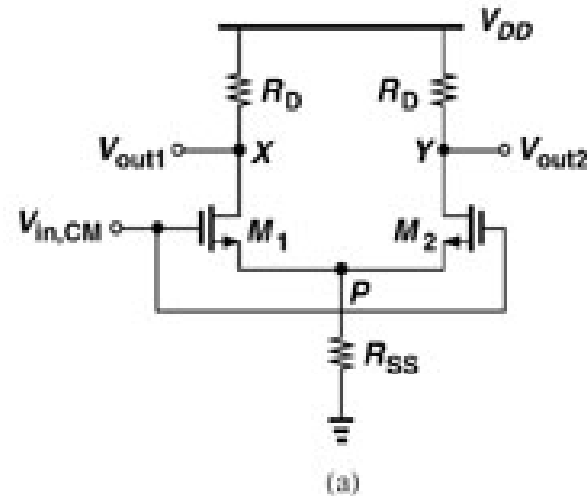


As  $V_{in,CM}$  changes so does  $V_P$ . As a result,  $I_D$  currents change, and  $V_X$  and  $V_Y$  change.



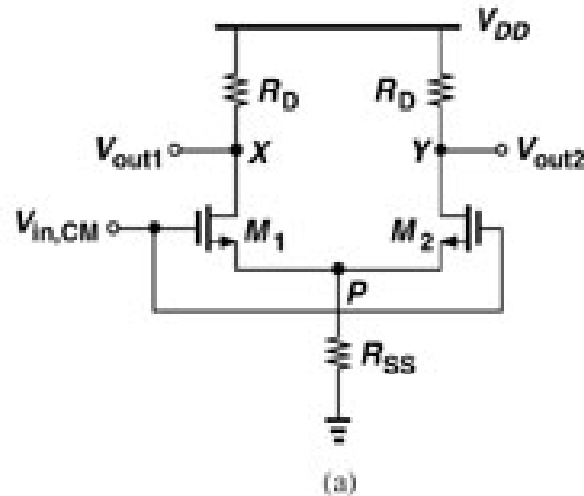
$V_X - V_Y$  continues to be zero.

# Single-ended Common-Mode Response of a symmetric amplifier

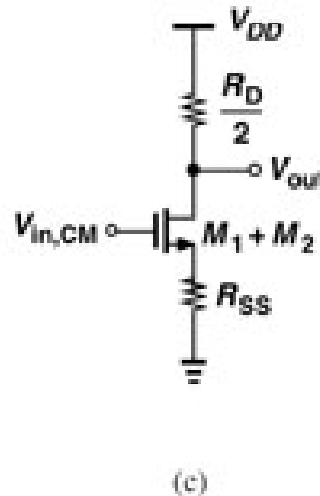
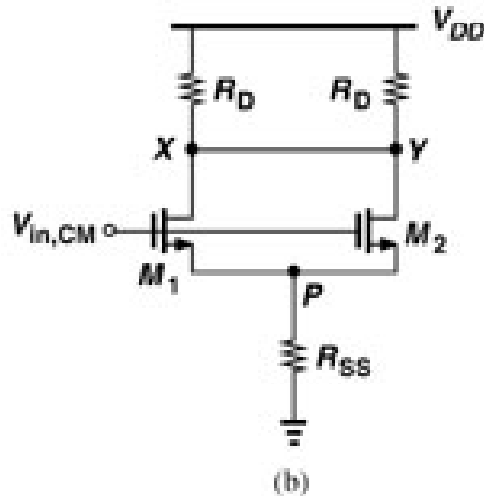


To find  $V_X$  as function of  $V_{in,CM}$  we may do a “half circuit analysis”, splitting  $R_{SS}$  into two parallel  $2R_{SS}$  resistors, or equivalently, connect transistors in parallel (as shown).

# Single-ended Common-Mode Gain of a symmetric amplifier assuming $\lambda=0$ and $\gamma=0$

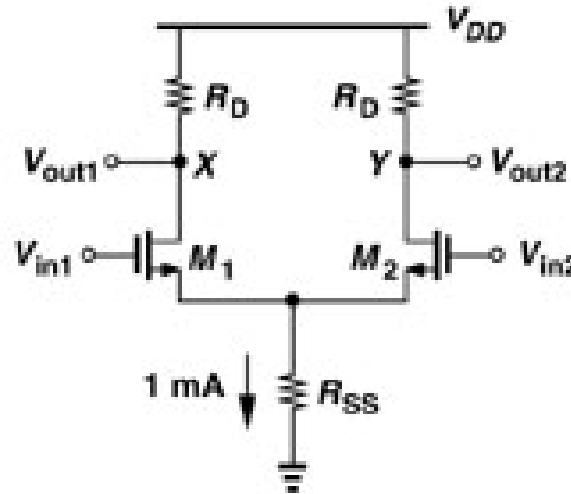


$$\begin{aligned}
 V_{,CM} &= \frac{V_{out}}{V_{in,CM}} = \frac{V_X}{V_{in,CM}} = \frac{V_Y}{V_{in,CM}} \\
 &= - \frac{R_D / 2}{1/(2g_m) + R_{SS}}
 \end{aligned}$$



$M_1+M_2$  has twice the width and bias current, therefore  $g_m$  is doubled.

# Most Primitive Differential Amplifier: Resistor in place of current source

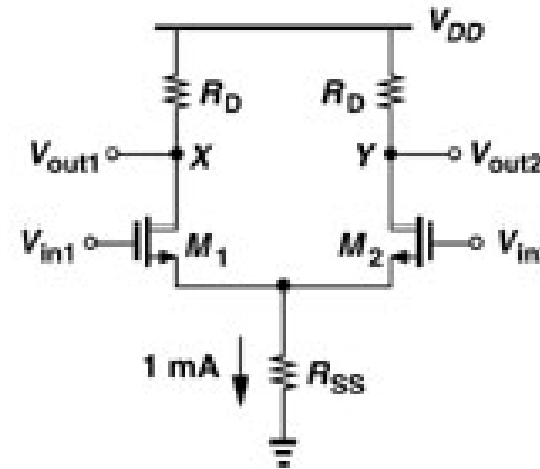


- Example: Let  $V_{DD}=3V$ ,  $(W/L)_1=(W/L)_2=25/0.5$
- $\mu_n C_{OX}=50\mu A/V^2$ ,  $V_{TH}=0.6V$ ,  $\lambda=0$ ,  $\gamma=0$ ,  $R_{SS}=500\Omega$
- Because  $I_{D1}=I_{D2}=0.5mA$ , we have:

$$V_{GS1} = V_{GS2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{OX} \frac{W}{L}}} + V_{TH} = 1.23V$$

# “Resistor current source” example

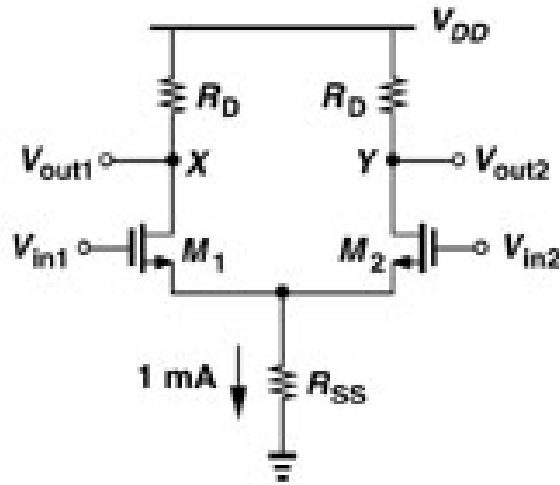
$$V_{GS1} = V_{GS2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{OX} \frac{W}{L}}} + V_{TH} = 1.23V$$



- Also:  $V_S = I_{SS} R_{SS} = 0.5V$
- Bias voltage at gates  $V_{in,CM} = V_{GS1} + V_S = 1.73V$
- This voltage creates the necessary  $0.5\text{mA}$  current in each of the transistors.

# “Resistor current source” example

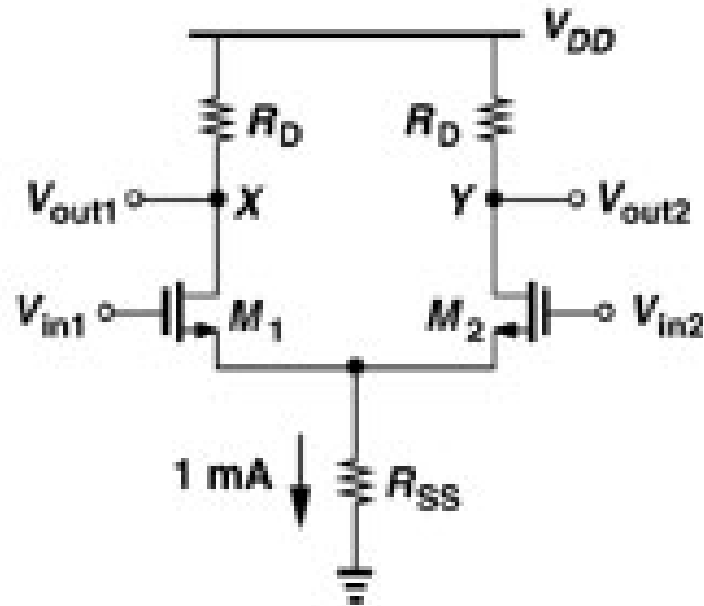
– differential gain design



$$g_m = \sqrt{2\mu_n C_{OX} \frac{W}{L} I_{D1}} = \frac{1}{632\Omega}$$

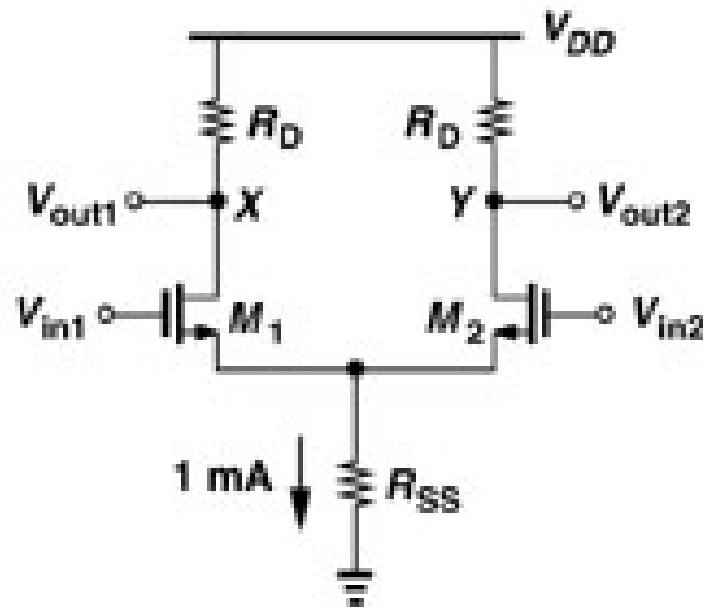
If  $R_D = 3.16\text{K}\Omega$  then differential voltage gain =  
 $g_m R_D = 5$

“Resistor current source” example – with such  $R_D$  are transistors in Saturation?



$$V_{out1} = V_{out2} = V_{DD} - I_D R_D = 1.42\text{V} > V_{in,CM} - V_{TH} = 1.73 - 0.6 = 1.13\text{V} \text{ by } 290\text{mV} \text{ (the overdrive)}$$

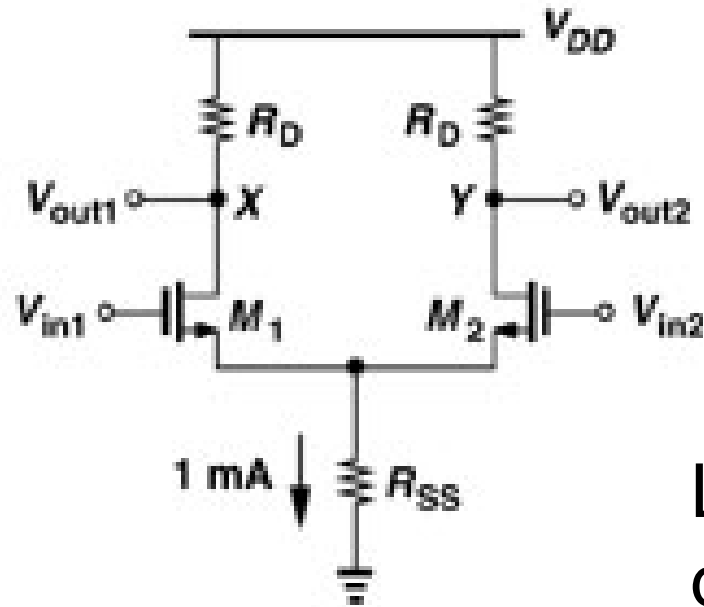
# “Resistor current source” example – Common-Mode Response



If  $V_{in,CM}$  increases by 50mV, what will happen to each output?



# “Resistor current source” example – Common-Mode Response

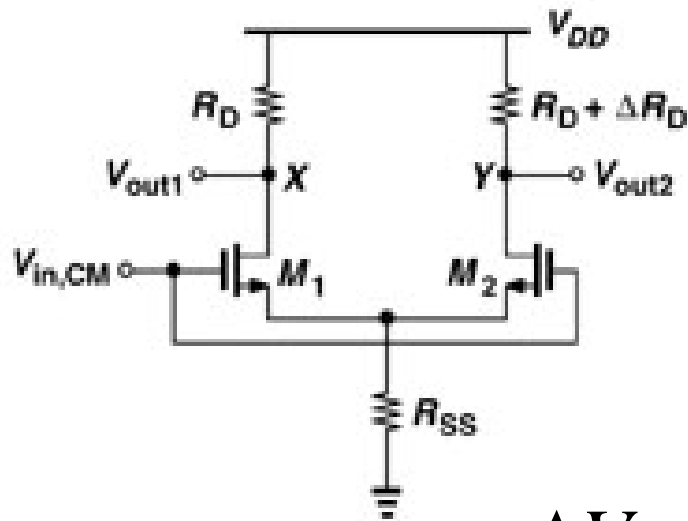


Large CM  
gain of 1.94  
is due to the  
small  $R_{SS}$

$$A_{V,CM} = \frac{\Delta V_{out}}{\Delta V_{in,CM}} = -\frac{R_D / 2}{1/(2g_m) + R_{SS}}$$

$$\Rightarrow |\Delta V_X| = \Delta V_{in,CM} \frac{R_D / 2}{1/(2g_m) + R_{SS}} = (50mV) \cdot 1.94 = 96.8mV$$

# Common-Mode Response with asymmetric $R_D$ assuming $\lambda=0$



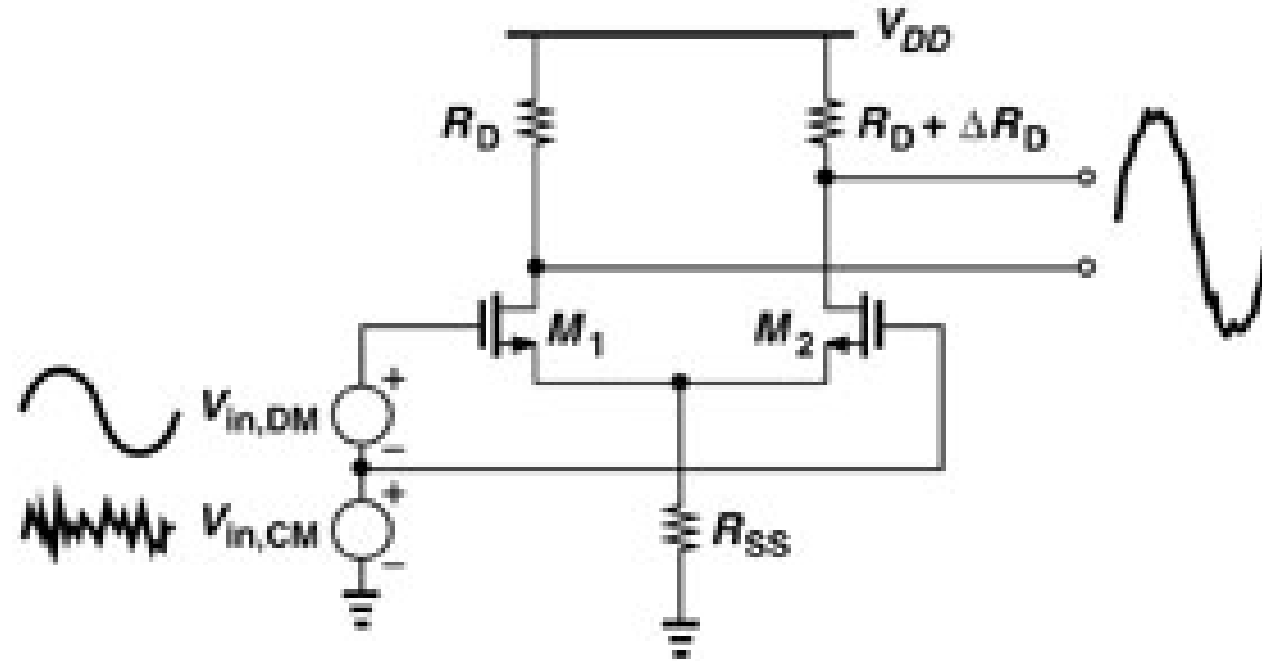
$$\frac{\Delta V_X}{\Delta V_{in,CM}} = -\frac{g_m}{1 + 2g_m R_{SS}} R_D$$

$$\frac{\Delta V_Y}{\Delta V_{in,CM}} = -\frac{g_m}{1 + 2g_m R_{SS}} (R_D + \Delta R_D)$$

$$\frac{\Delta V_X - \Delta V_Y}{\Delta V_{in,CM}} = \frac{g_m}{1 + 2g_m R_{SS}} \Delta R_D$$

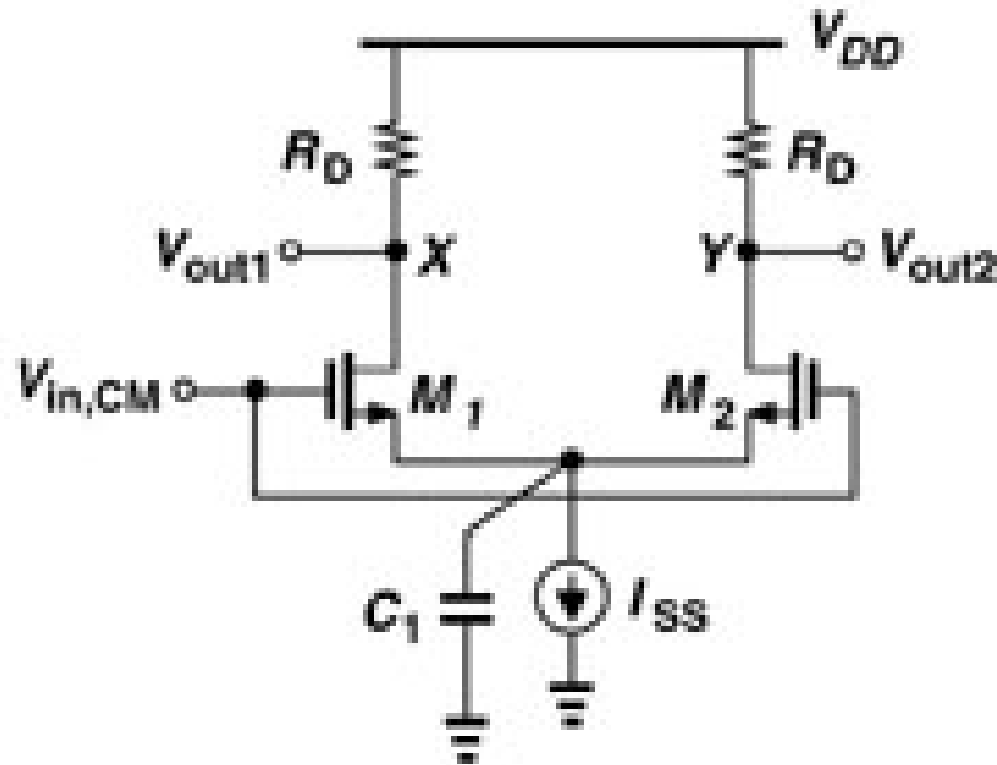
$R_{SS}$  above represents the current source – need large  $R_{SS}$

# Common-Mode Response with asymmetric $R_D$

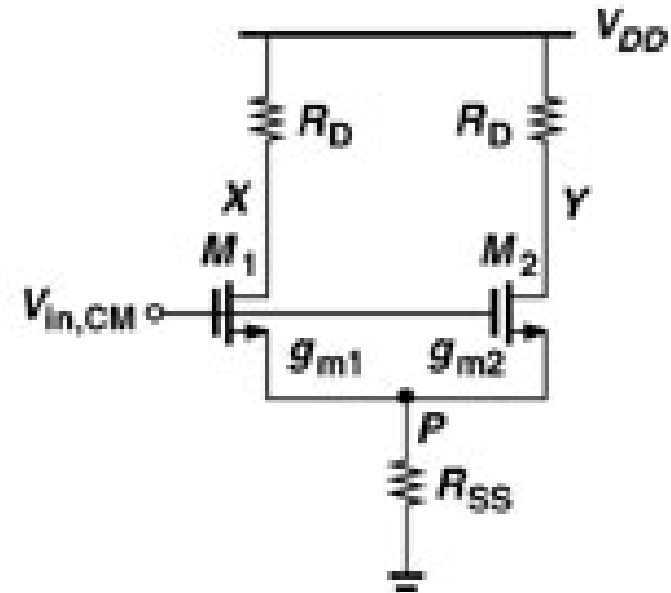
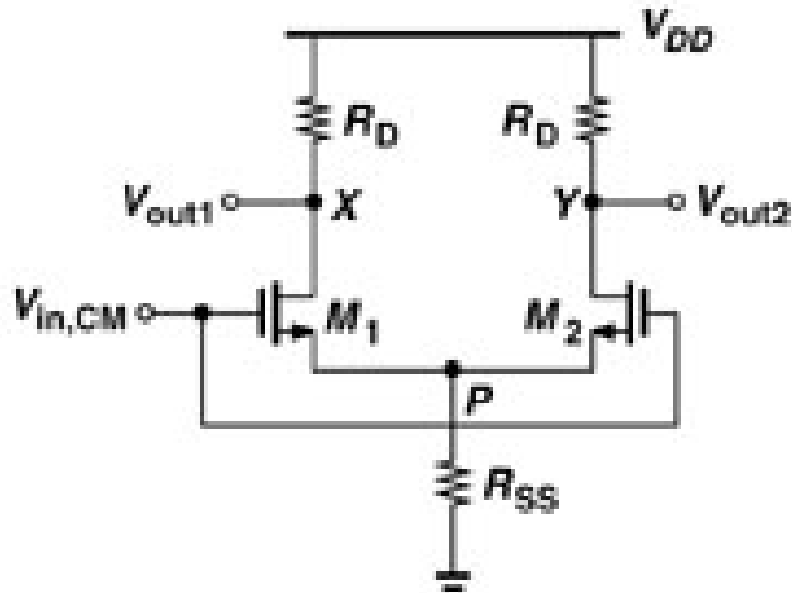


CM input noise corrupts the amplified differential signal, because of the asymmetry

For high-frequency Common-Mode input need to take into account parasitic capacitance effects, even if  $R_{SS}$  is large.  $C_1$  is contributed by  $M_1, M_2$  and  $I_{SS}$  and contributes to the impedance “seen” by the CM signal.



Mismatches in  $W/L$ ,  $V_{TH}$  and other transistor parameters all translate to mismatches in  $g_m$



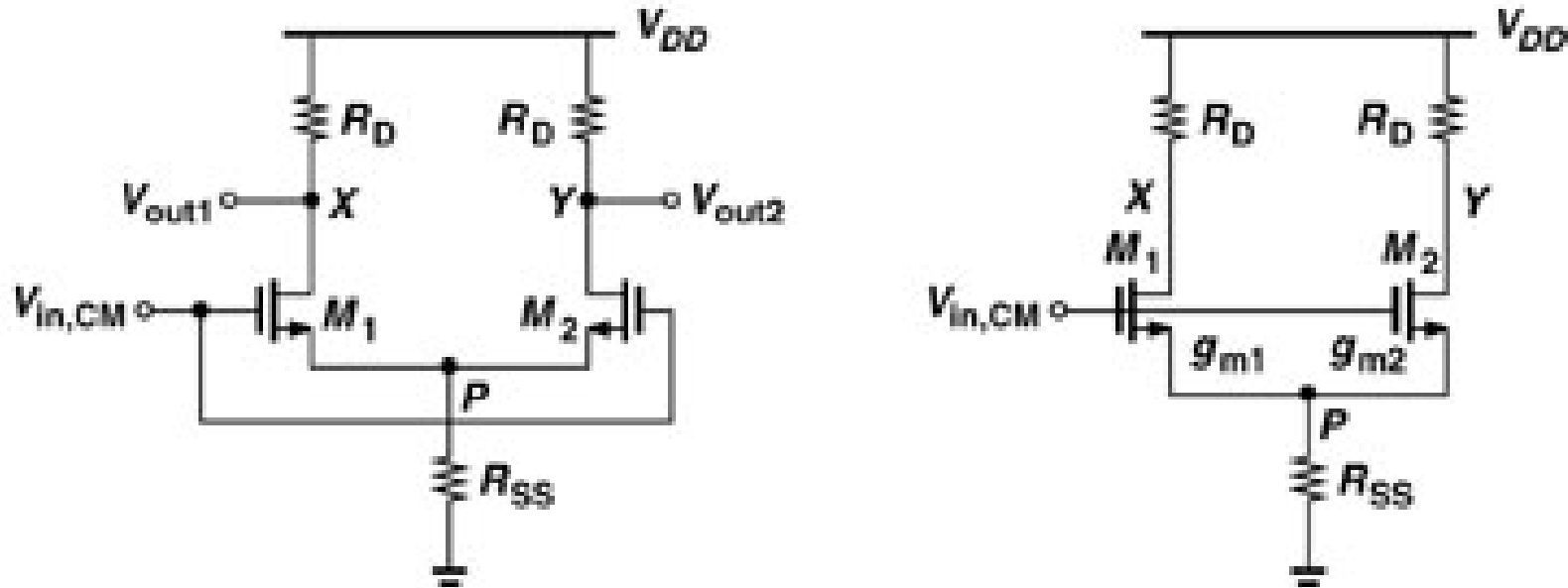
$$I_{D1} = g_{m1}^{(a)} (V_{in,CM} - V_P)$$

$$I_{D2} = g_{m2} (V_{in,CM} - V_P)$$

$$\Rightarrow (g_{m1} + g_{m2})(V_{in,CM} - V_P)R_{SS} = V_P$$

(b)  
Small-signal  
analysis

Mismatches in  $W/L$ ,  $V_{TH}$  and other transistor parameters all translate to mismatches in  $g_m$



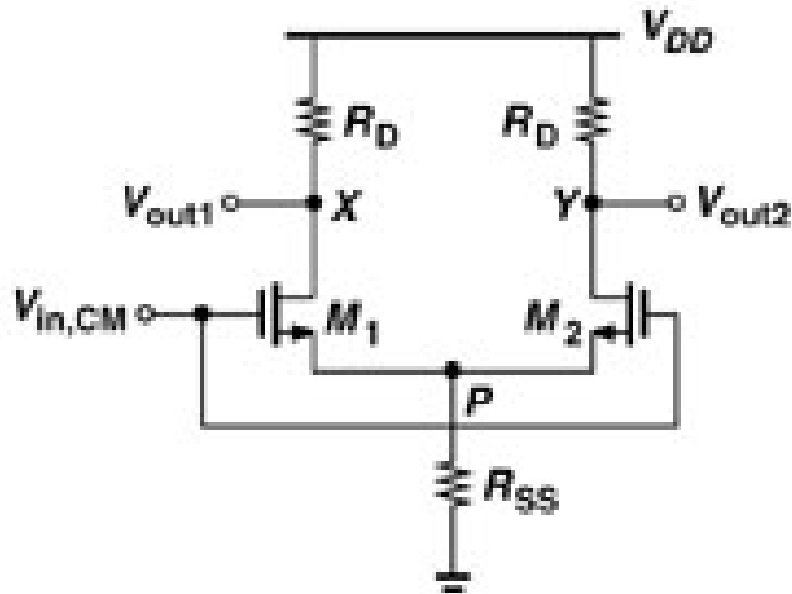
$$(g_{m1} + {}^{(a)}g_{m2})(V_{in,CM} - V_P)R_{SS} \stackrel{(b)}{=} V_P$$

$$\Rightarrow V_P = \frac{(g_{m1} + g_{m2})R_{SS}}{(g_{m1} + g_{m2})R_{SS} + 1} V_{in,CM}$$

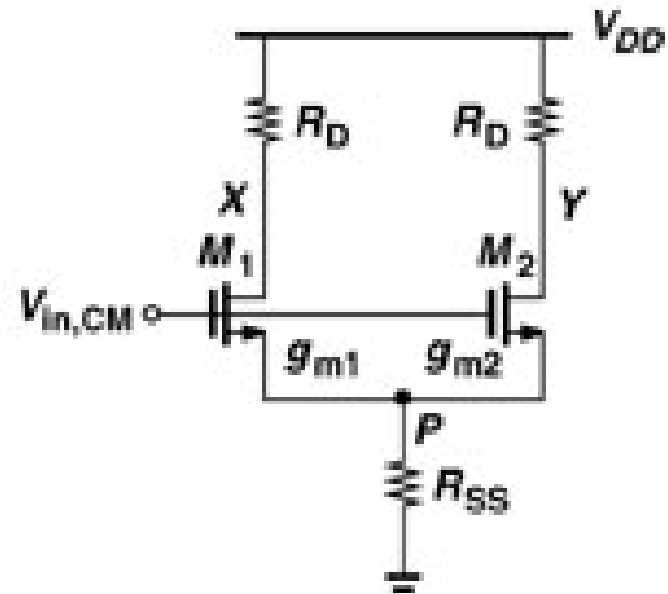
$$V_X = -g_{m1}(V_{in,CM} - V_P)R_D$$

$$V_Y = -g_{m2}(V_{in,CM} - V_P)R_D$$

Mismatches in  $W/L$ ,  $V_{TH}$  and other transistor parameters all translate to mismatches in  $g_m$



(a)



(b)

$$A_{V,CM-DM} = \frac{V_X - V_Y}{V_{in,CM}} = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D$$



# Common-Mode Gains

- We have seen two types of common-mode gain:
- $A_{V,CM}$  : Single-ended output due to CM signal.
- $A_{V,CM-DM}$  : Differential output due to CM signal.

$$A_{V,CM} = \frac{V_X}{V_{in,CM}} = \frac{V_Y}{V_{in,CM}}$$

$$A_{V,CM-DM} = \frac{V_X - V_Y}{V_{in,CM}}$$

# Common-Mode Rejection Ratio (CMRR) Definitions

$$CMRR = CMRR_{SE} = \left| \frac{A_{DM}}{A_{CM}} \right|$$

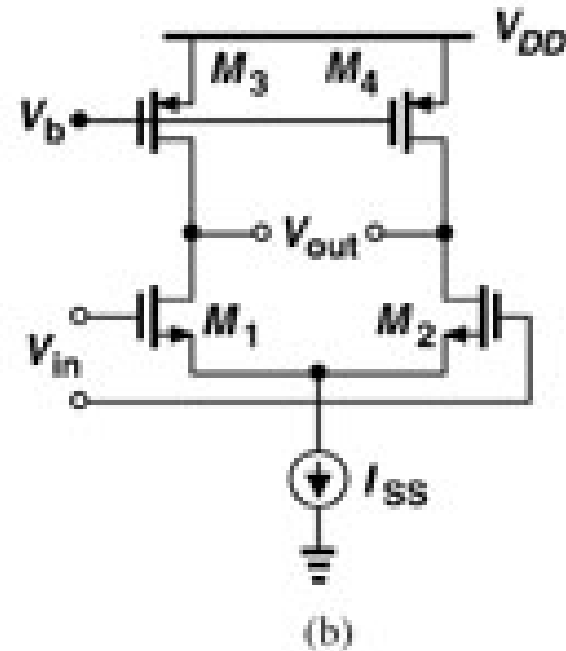
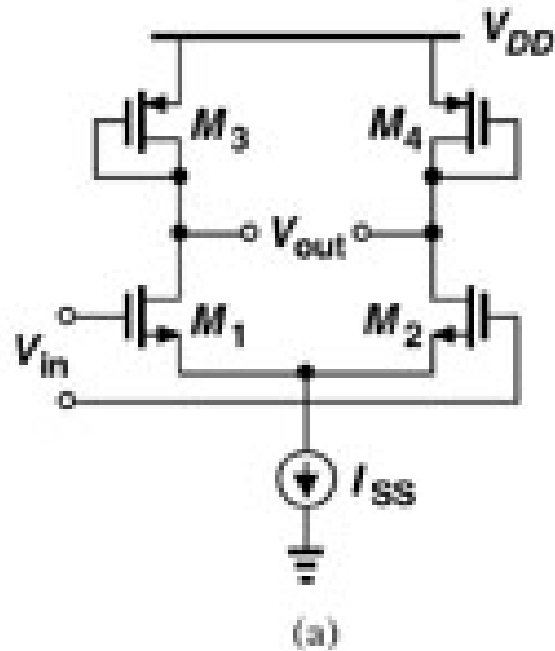
$$CMRR = CMRR_{diff} = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

In both cases we want CMRR to be as large as possible, and it translates into small matching errors and  $R_{SS}$  as large as possible

# CMOS Differential Amplifiers

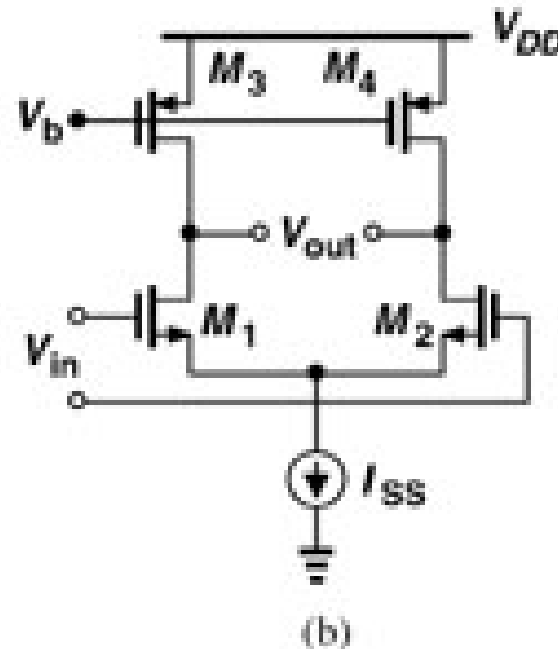
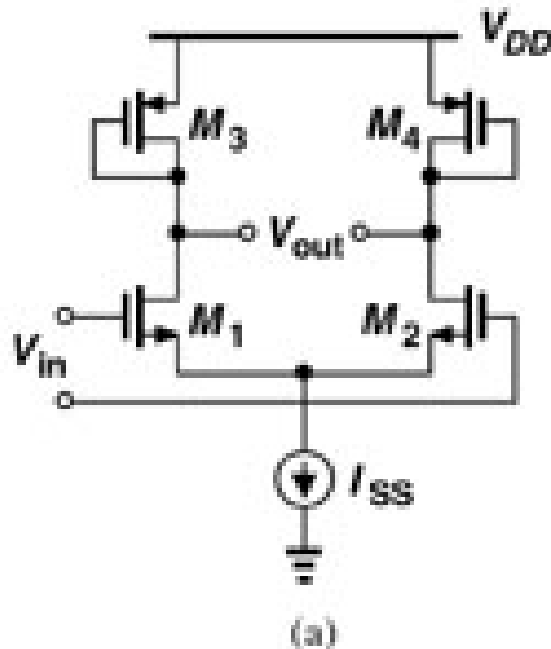
MOS Loads – L18

# MOS Loads



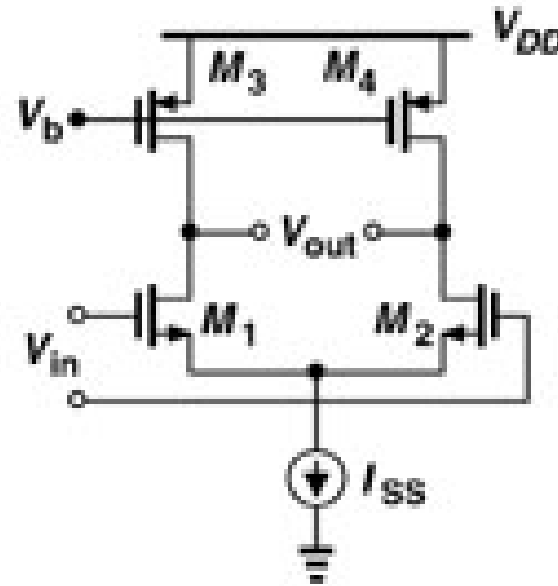
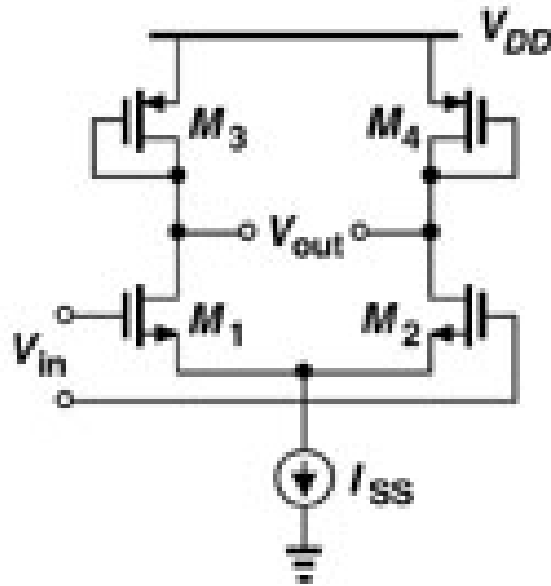
- (a) Diode-connected load
- (b) Current-Source load

# MOS Loads: Analysis Method



- Differential Analysis: Use half-circuit method, with source node at virtual ground.
- Common-Mode Analysis: Again use half-circuit method, with appropriate accommodation for parallel transistors, and for  $R_{SS}$ .

# MOS Loads: Differential Gain Formulas



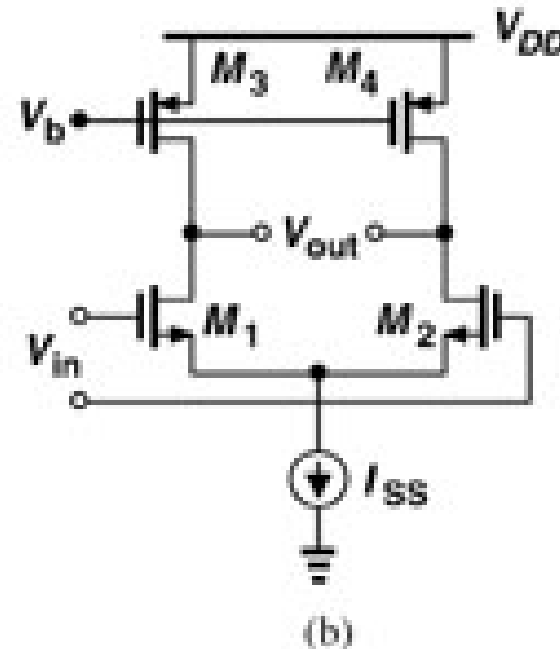
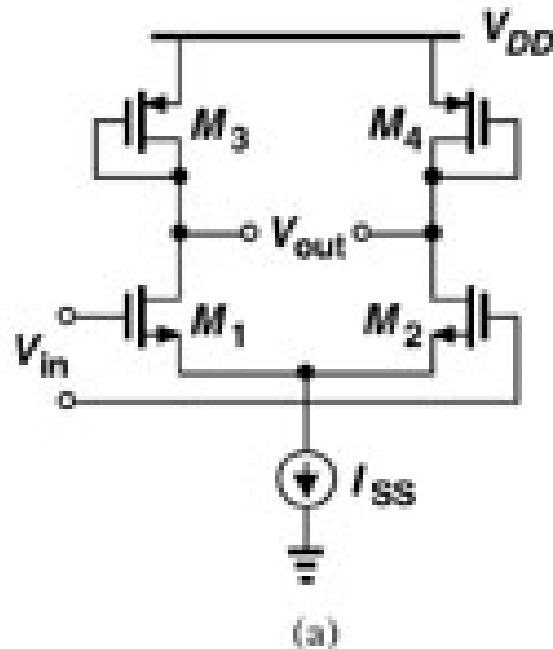
(b)

$$A_{V,diff} = -g_{mN} (g_{mP}^{-1} \parallel r_{oN} \parallel r_{oP})$$

$$\approx -\frac{g_{mN}}{g_{mP}} = -\sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}}$$

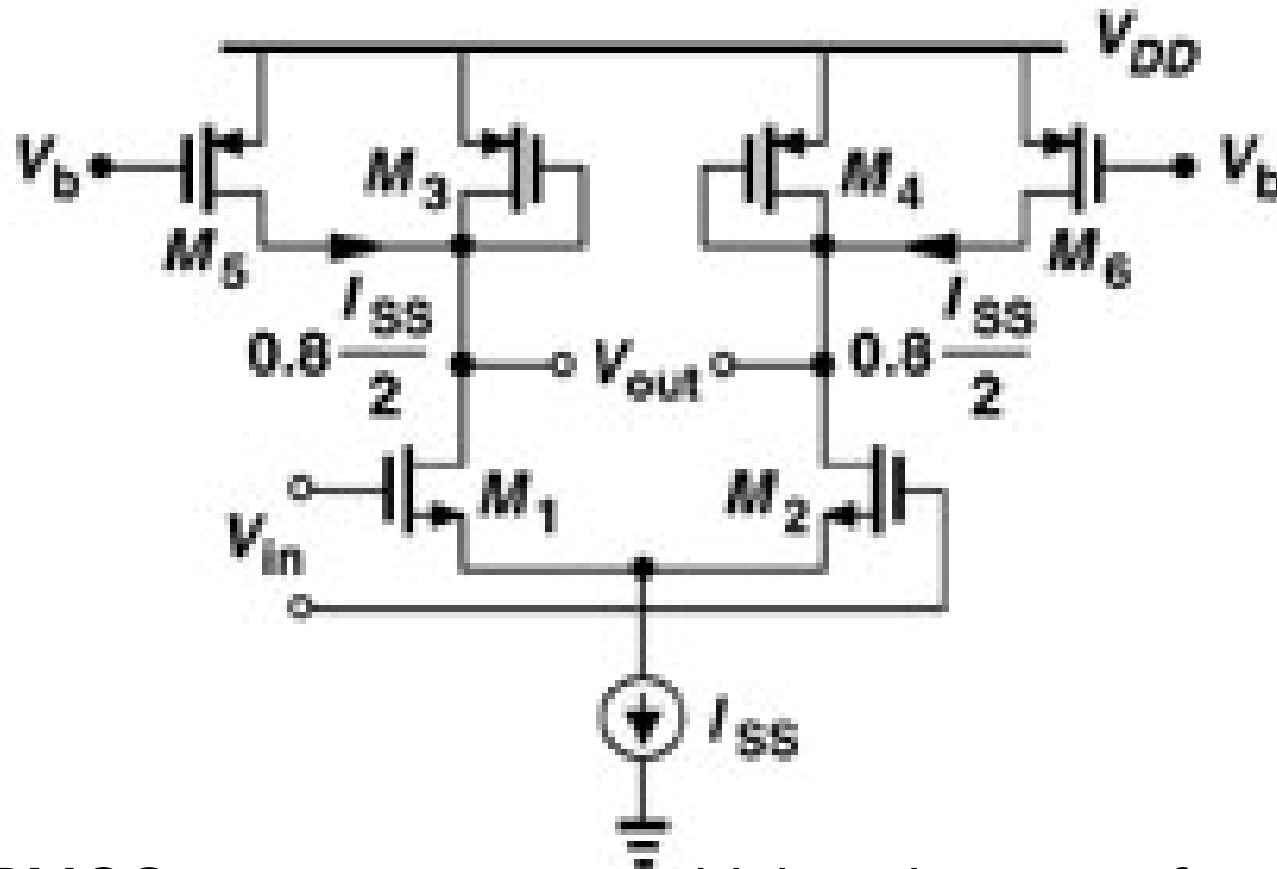
$$A_{V,diff} = -g_{mN} (r_{oN} \parallel r_{oP})$$

# Problems with Diode-connected MOS Loads



- Tradeoff among output voltage swing, voltage gain and CM input range:
- In order to achieve high gain,  $(W/L)_p$  must be decrease, thereby increasing  $|V_{GSP} - V_{THP}|$  and lowering the CM level at nodes X and Y.

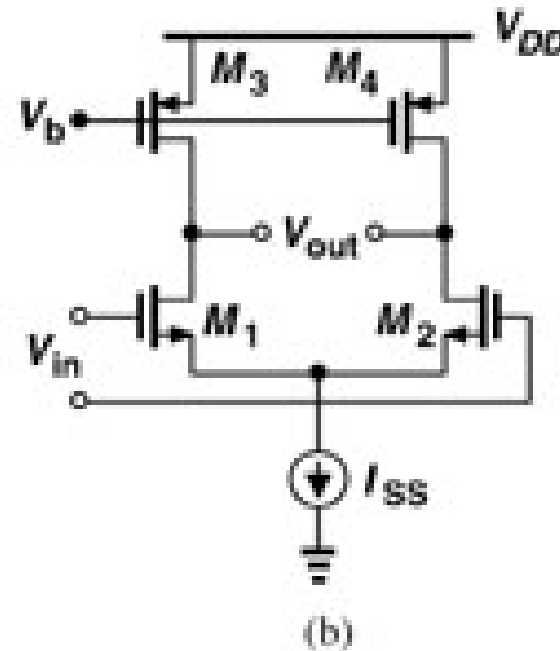
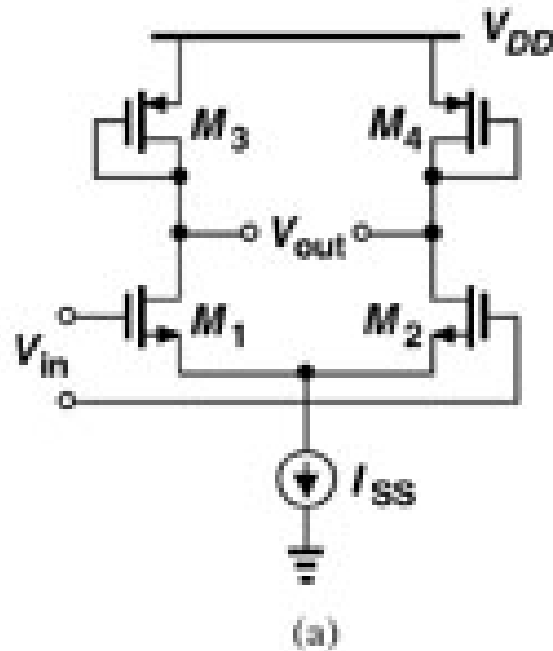
# Overcoming Diode-connected Load swing problem for higher gains:



Use PMOS current sources which reduce  $g_m$  of diode-connected MOS, instead of lowering  $(W/L)_P$  of load. Gain can be increased by factor of 5.

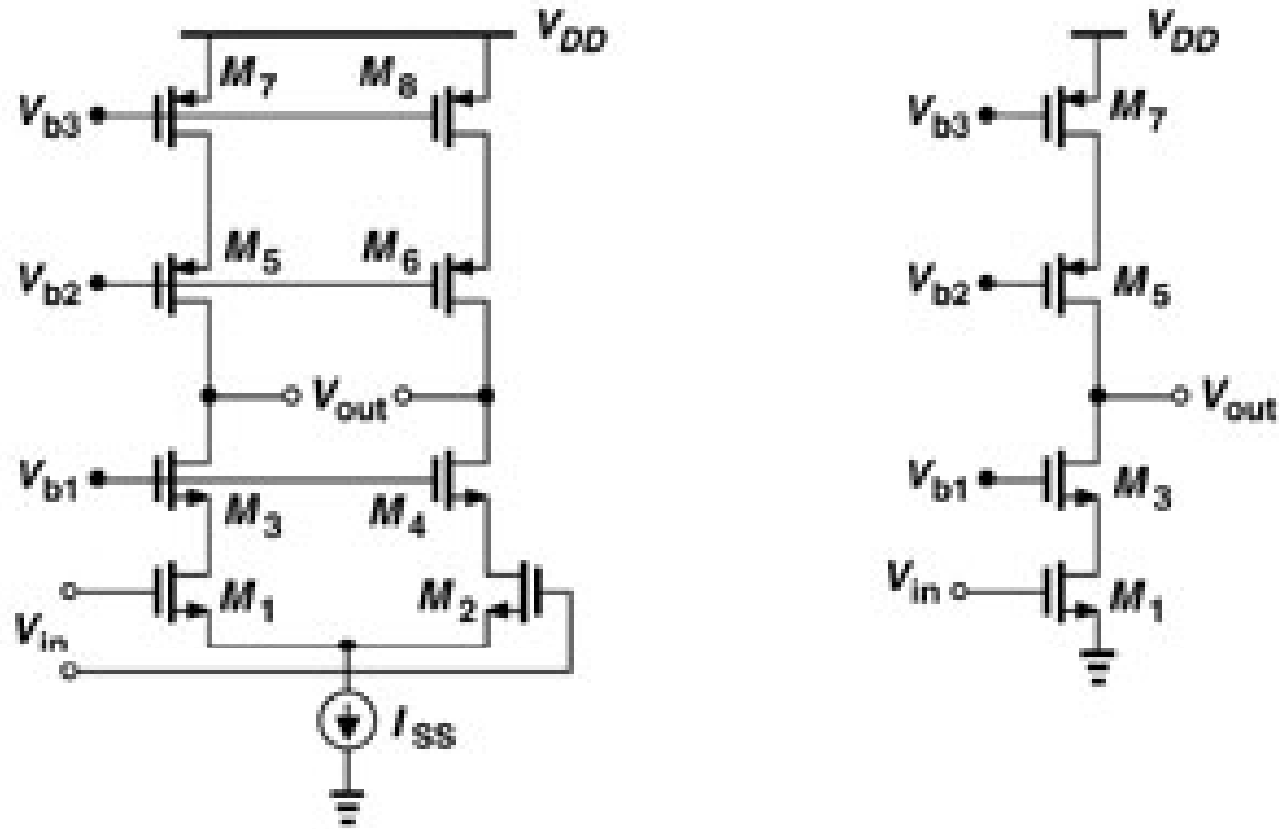


# Problems with Current-Source MOS Loads



- In sub-micron technologies, it's hard to obtain differential gains higher than 10-20.

## Solution to low-gain problem: Cascoding

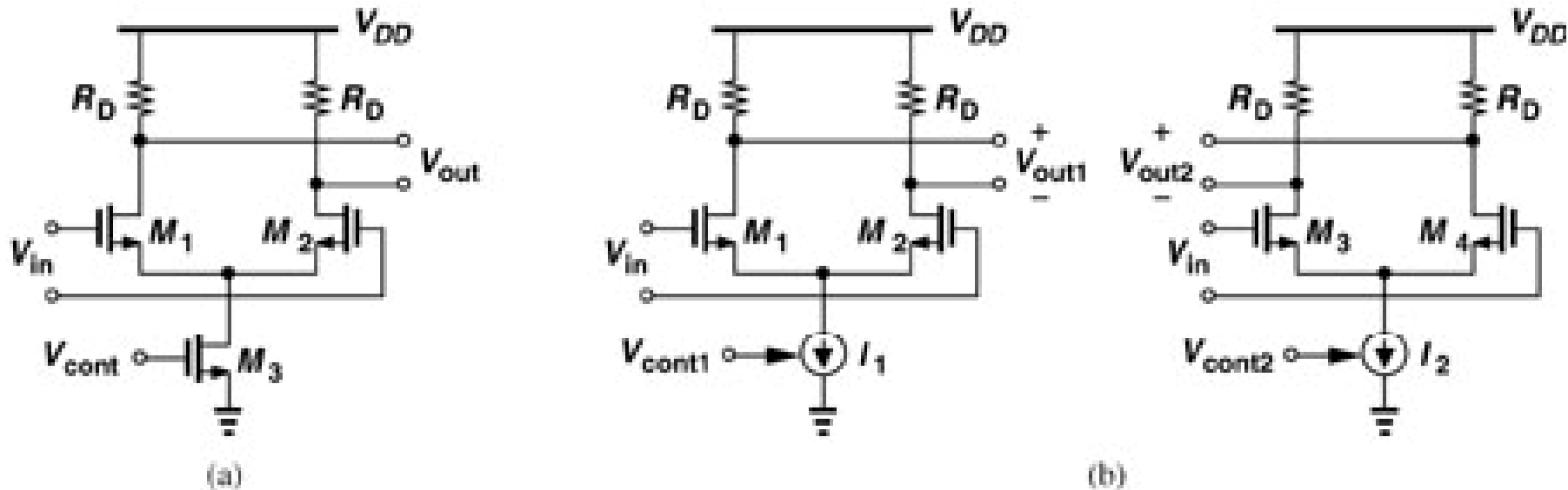


$$A_{V,diff} \approx g_{m1} [(g_{m3} r_{o3} r_{o1}) \parallel (g_{m5} r_{o5} r_{o7})]$$

# CMOS Differential Amplifiers

Gilbert Cell-L19

# Can we create a differential amplifier with voltage-controlled differential gain?



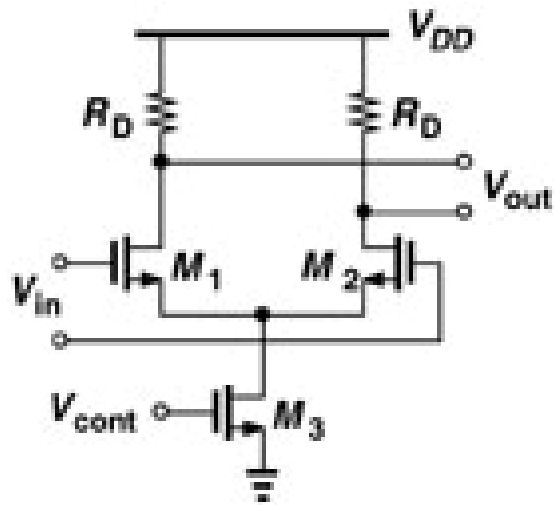
(a) is a VGA (Variable-Gain Amplifier).  $V_{cont}$  determines  $I_{SS}$ , which determines the differential gain.

Gain may be varied from 0 to some maximum value.

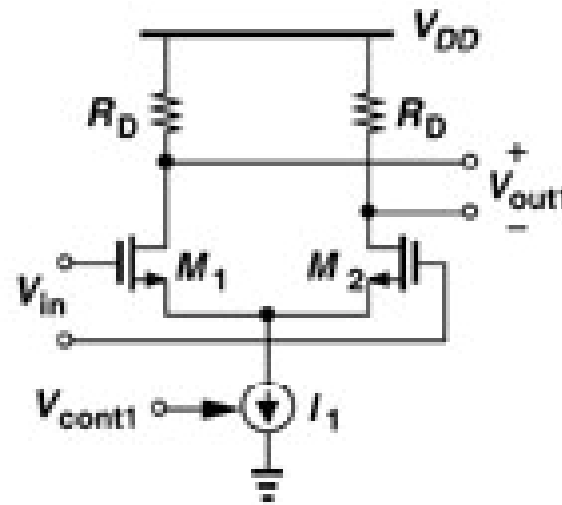
Can we create a differential amplifier with voltage-controlled differential gain?

What do we do if we want to vary the gain continuously from some negative value to some positive value?

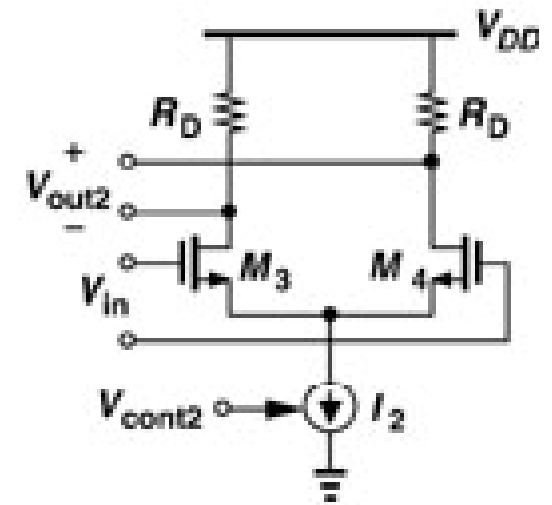
Can we create a differential amplifier with voltage-controlled differential gain?



(a)



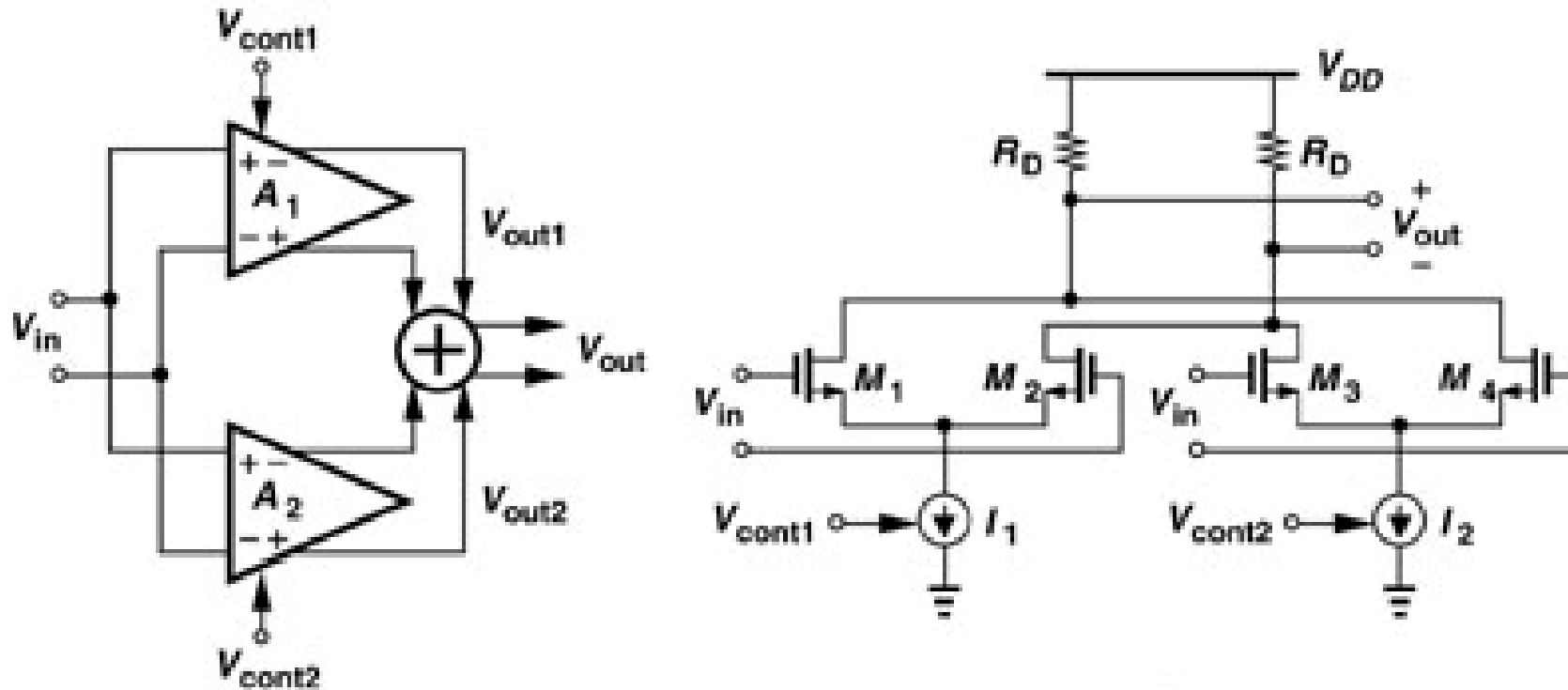
(b)



In circuits (b) the two amplifiers have opposite differential gain simply by interchanging the order of output subtraction)



# How to combine the differential outputs?



On the left, we conceptually see the two voltages ADDED UP.

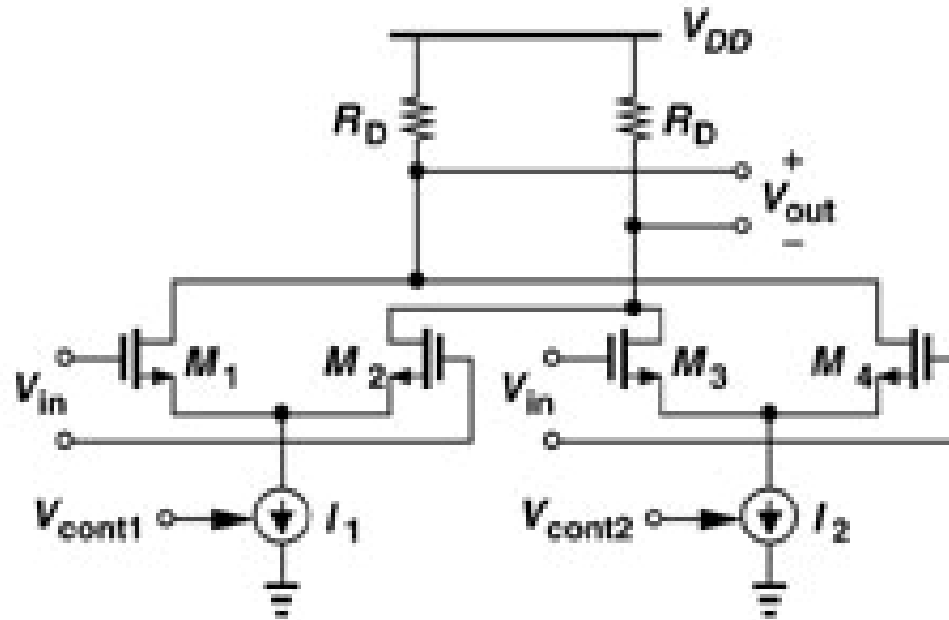
Gains  $A_1$  and  $A_2$  are voltage controlled.







Does it work as intended? Can gain be varied positively as well as negatively?

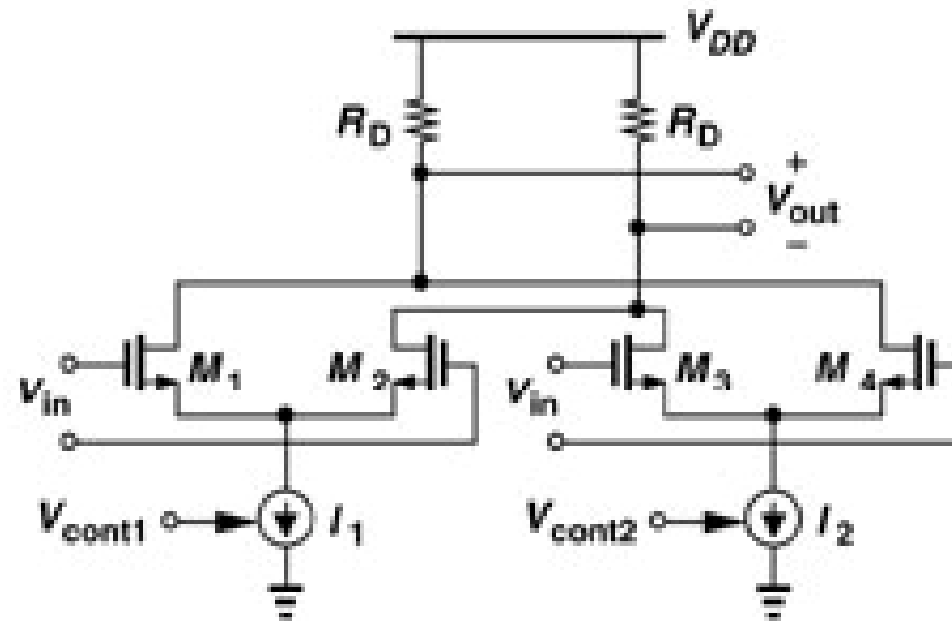


If  $I_1=0$  then  $V_{out}=g_m R_D$ .

If  $I_2=0$  then  $V_{out}=-g_m R_D$ .

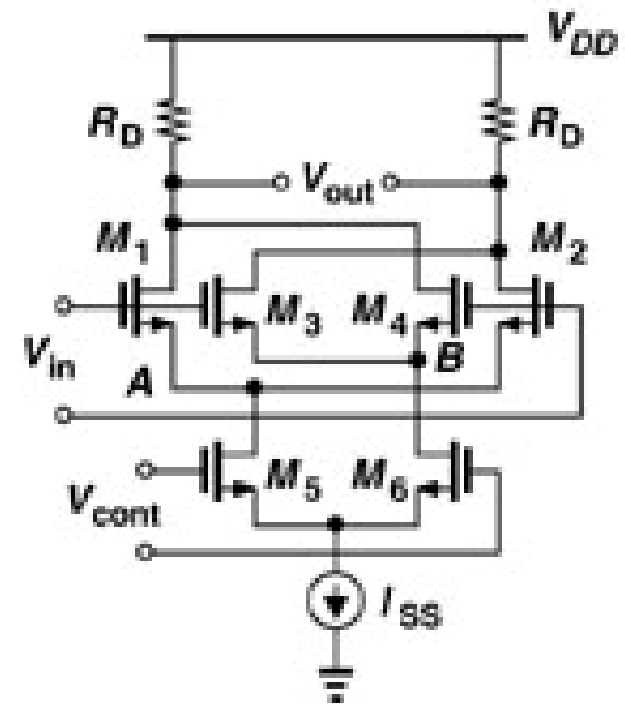
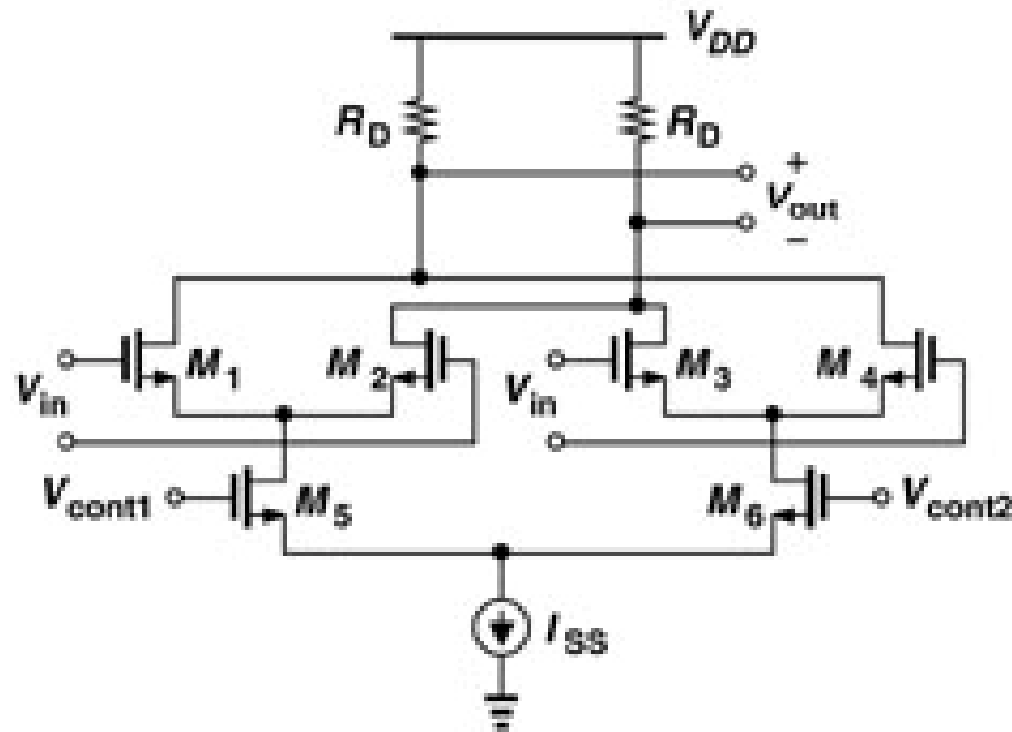
If  $I_1=I_2$  then  $V_{out}=0$  !!

Next phase in the conceptual development:  
Do we really need two separate control  
signals??



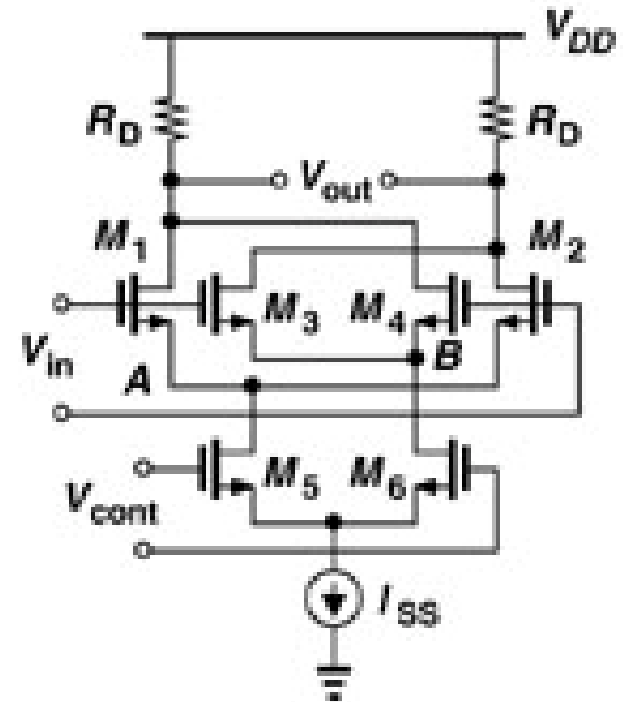
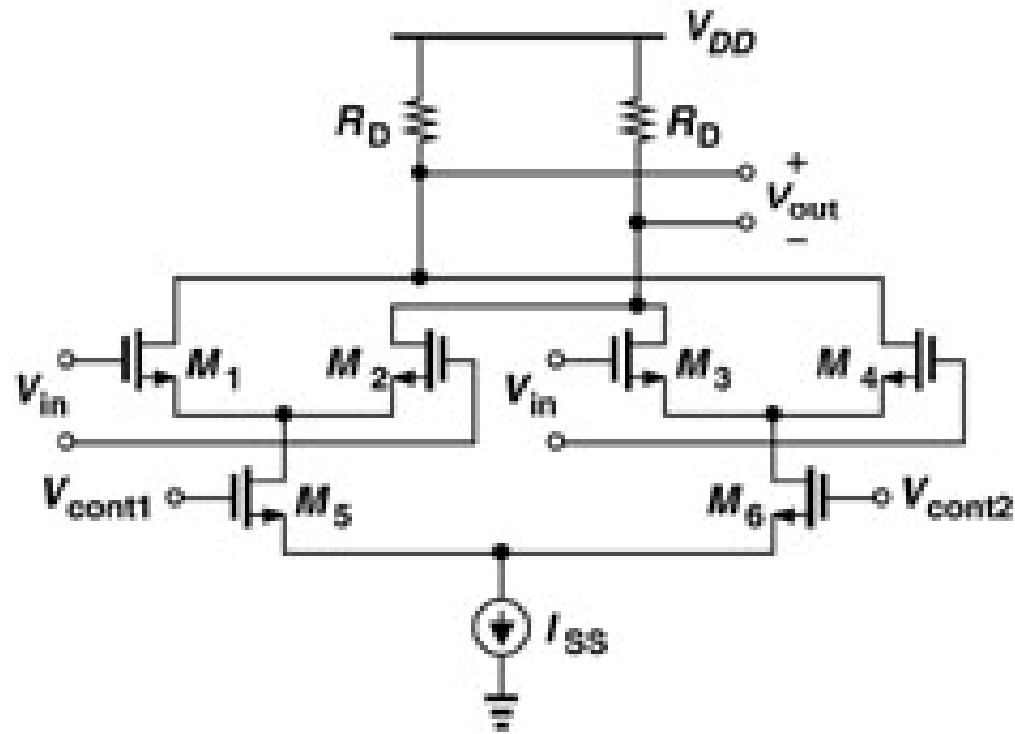
Recall the basic structure of a differential amplifier: The inputs difference cause one of the MOS currents to go up, and at the same time the current of the opposite MOS goes down by the same amount  $\rightarrow$  Let  $I_1$ ,  $I_2$  come from diff. amp.

# Gilbert Cell



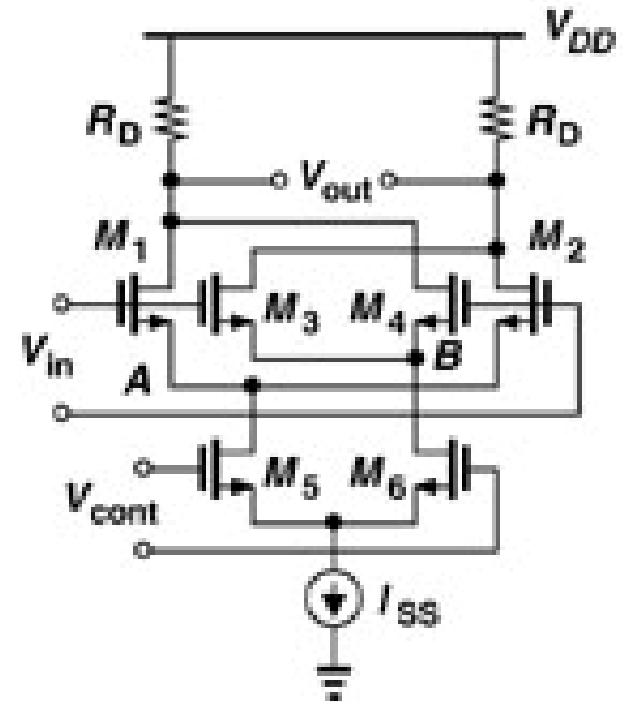
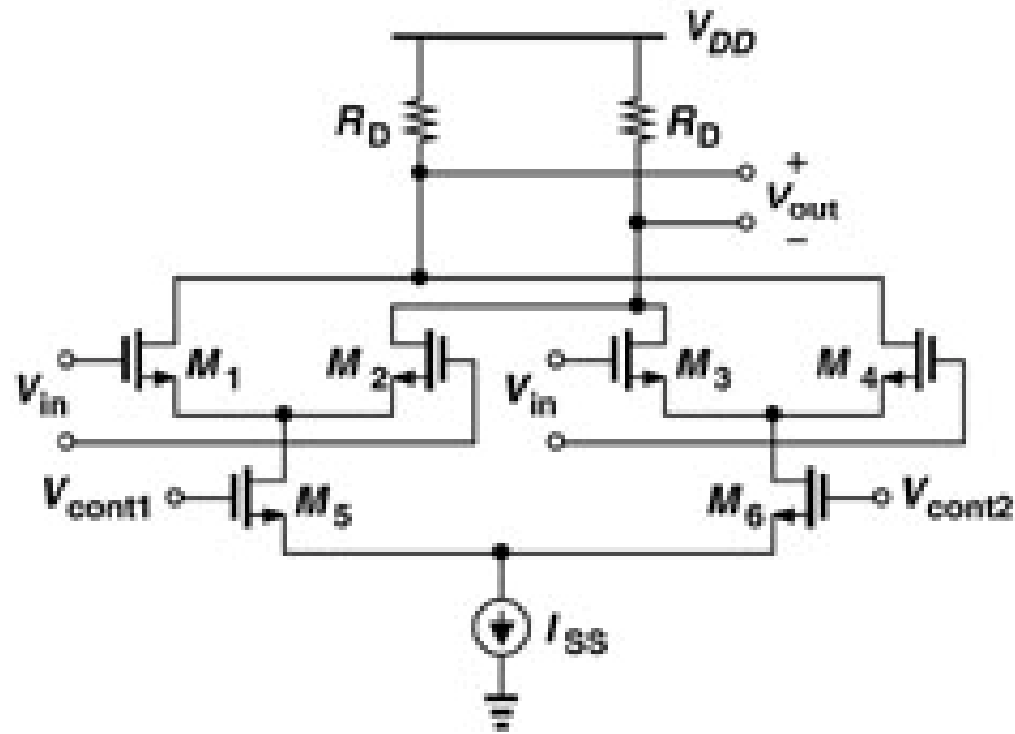
Left: If  $|V_{cont1} - V_{cont2}|$  is large we have “current stealing”, and then gain is either most positive or most negative.

# Gilbert Cell



Right: Actual Implementation

# Gilbert Cell



$$V_{OUT} = kV_{in} V_{cont}$$

# Applications of Gilbert Cell

- Analog Multiplier
  - Mixer
  - Phase Detector
  - AM Modulation
- VGA: Variable-gain amplifier
  - AGC (Automatic Gain Control)

It's a versatile general-purpose tool



# Can switch input and control signals!

