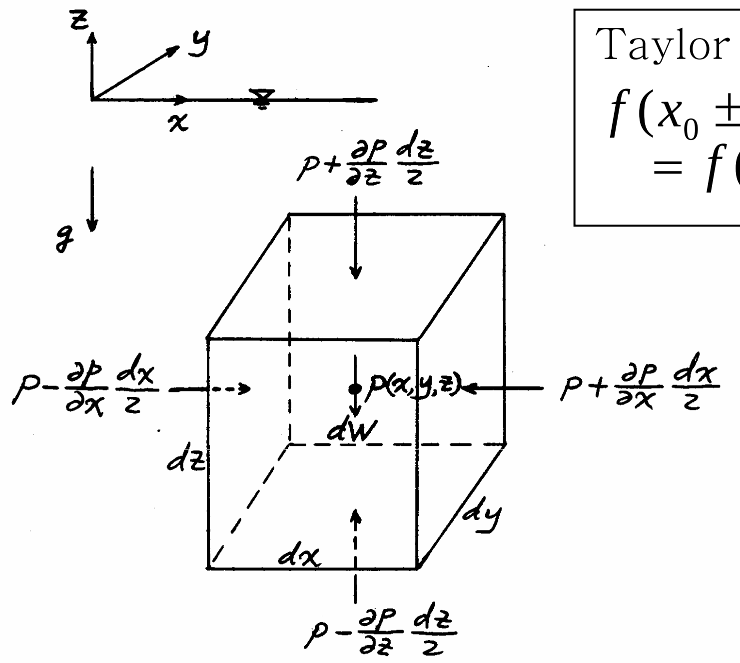
Chap.2 FLUID STATICS

Fluid at rest → No shear stress → Pressure only

2.1 Pressure Variation with Elevation
(relation between $p, \rho,$ and z)



Taylor series:

$$f(x_0 \pm \Delta x) = f(x_0) \pm f'(x_0)\Delta x + \cdots$$

Force balance:

$$\sum F_x = 0 = \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right)dydz - \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right)dydz$$

$$\sum F_y = 0 = \left(p - \frac{\partial p}{\partial y}\frac{dy}{2}\right)dxdz - \left(p + \frac{\partial p}{\partial y}\frac{dy}{2}\right)dxdz$$

$$\sum F_z = 0 = \left(p - \frac{\partial p}{\partial z}\frac{dz}{2}\right)dxdy - \left(p + \frac{\partial p}{\partial z}\frac{dz}{2}\right)dxdy - dW$$

$$dW = \rho g dxdydz$$

$$\left.\begin{array}{l}\sum F_x = 0 \rightarrow \dfrac{\partial p}{\partial x} = 0 \rightarrow p = C_1(y,z) \\ \sum F_y = 0 \rightarrow \dfrac{\partial p}{\partial y} = 0 \rightarrow p = C_2(x,z)\end{array}\right\} p = p(z)$$

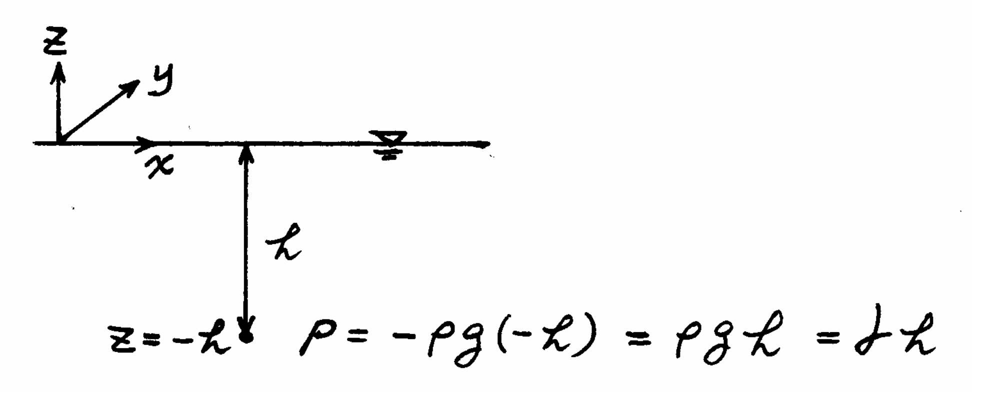
$$\sum F_z = 0 \rightarrow \frac{\partial p}{\partial z} + \rho g = 0$$

$$\rightarrow \frac{\partial p}{\partial z} = -\rho g$$

$$\rightarrow p = -\rho g z + C_3(x,y) \;\rightarrow\; C_3 = \text{const.}$$

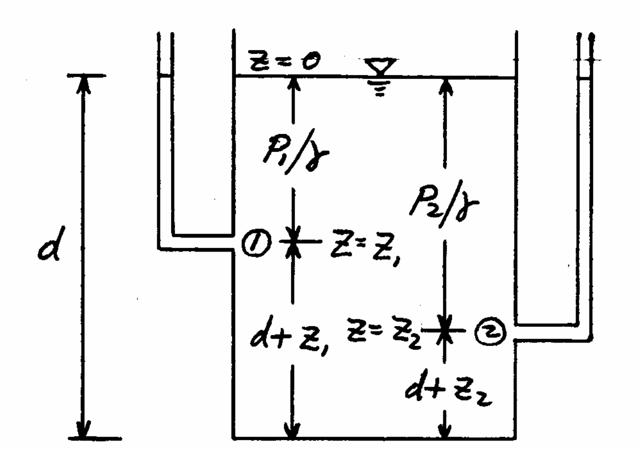
B.C.: $p = 0$ at $z = 0 \rightarrow C_3 = 0$

$$\boxed{p = -\rho g z}$$



Pressure varies linearly from 0 at surface to γh at depth h.

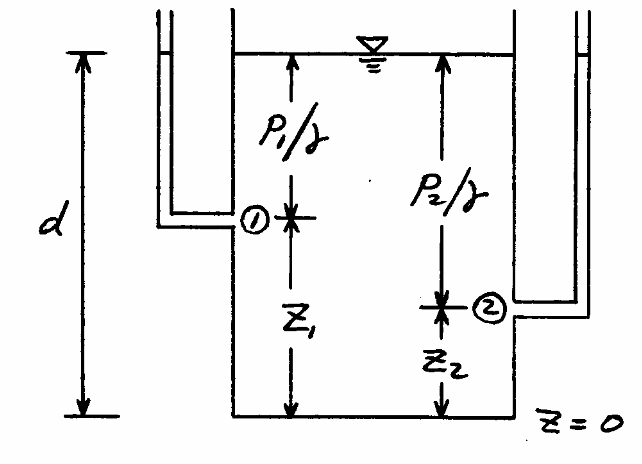
Pressure head (壓力水頭): $h = \frac{p}{\rho g} = \frac{p}{\gamma}$



$$\frac{p_1}{\gamma} + d + z_1 = \frac{p_2}{\gamma} + d + z_2$$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 = \text{const}$$

z_1, z_2 different



$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 = \text{const}$$

$$\boxed{\frac{p}{\gamma} + z = \text{const}}$$

이 식은 z의 원점을 어디에 잡던지 성립.

z = elevation head (位置水頭)

2.2 Absolute and Gauge Pressure

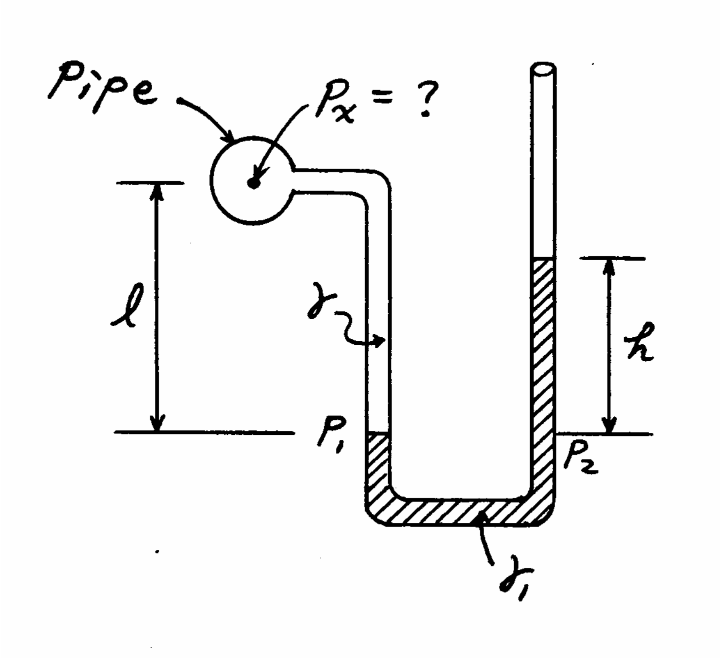
- Absolute pressure (絶對壓力):
  진공상태를 기준으로("0"으로) 하여 측정한 압력
  예: 대기압 = 101.3 kPa

- Gauge pressure (計器壓力):
  대기압을 기준으로("0"으로) 하여 측정한 압력
  예: 타이어 압력 = 35 psi

- Gauge pressure = Absolute – Atmospheric

- Note: 공학적으로 사용되는 pressure는
  통상 gauge pressure를 의미함.

2.3 Manometer (液柱壓力計)



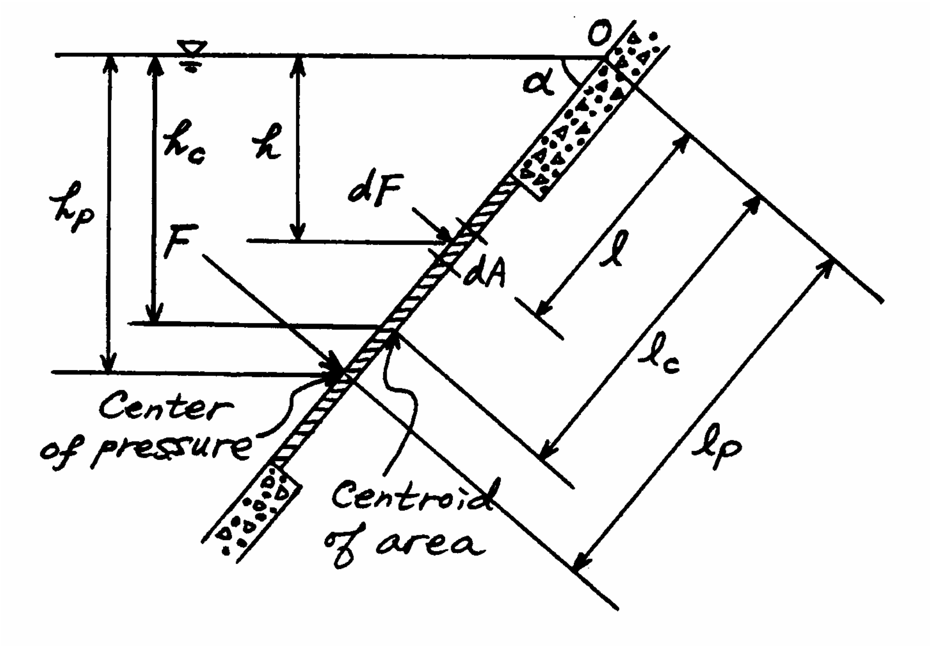
$$p_1 = p_2$$

$$\left.\begin{aligned} p_1 &= p_x + \gamma\ell \\ p_2 &= \gamma_1 h \end{aligned}\right\} \rightarrow p_x = \gamma_1 h - \gamma\ell$$

Read text for other types of manometers.

2.4 Pressure Forces on Plane Surfaces

$$\begin{cases} \text{Magnitude } (F)? \\ \text{Direction (Normal to surface)} \\ \text{Point of action?} \end{cases}$$



$$dF = pdA = \gamma h dA = \gamma l \sin\alpha dA$$

$$F = \int dF = \gamma \sin\alpha \int l dA = \gamma \sin\alpha l_c A = \gamma h_c A$$

where $l_c = \frac{1}{A}\int l dA$: Centroid of area

(depending on the shape of the area)

$$\boxed{F = \gamma h_c A}$$

Total force (F) = pressure at centroid of area(γh_c) × Area(A)

- Point of action ($l_p = ?$)

Consider moment about O:

$$l_p F = \int l dF = \gamma \sin\alpha \underbrace{\int l^2 dA}_{I_o}$$

$dF = \gamma l \sin\alpha dA$

2nd moment about O

$= I_c + l_c^2 A$

moment of inertia (Appendix 3)
= 2nd moment about centroid
of area

$$l_p F = \gamma \sin\alpha (I_c + l_c^2 A)$$

$$l_p \gamma \sin\alpha l_c A = \gamma \sin\alpha (I_c + l_c^2 A)$$

$$\boxed{l_p = \frac{I_c}{l_c A} + l_c}$$

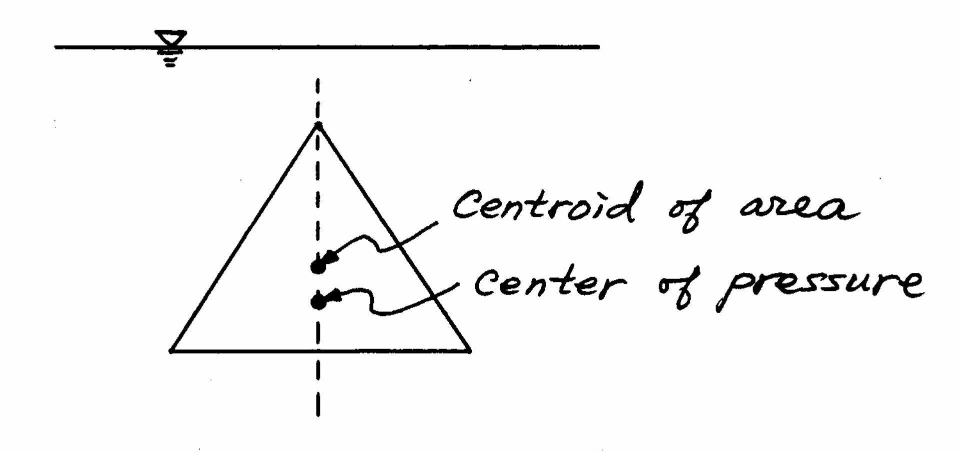
$\therefore \quad l_c, I_c, A \rightarrow F, l_p$

$$I_o = \int l^2 dA$$

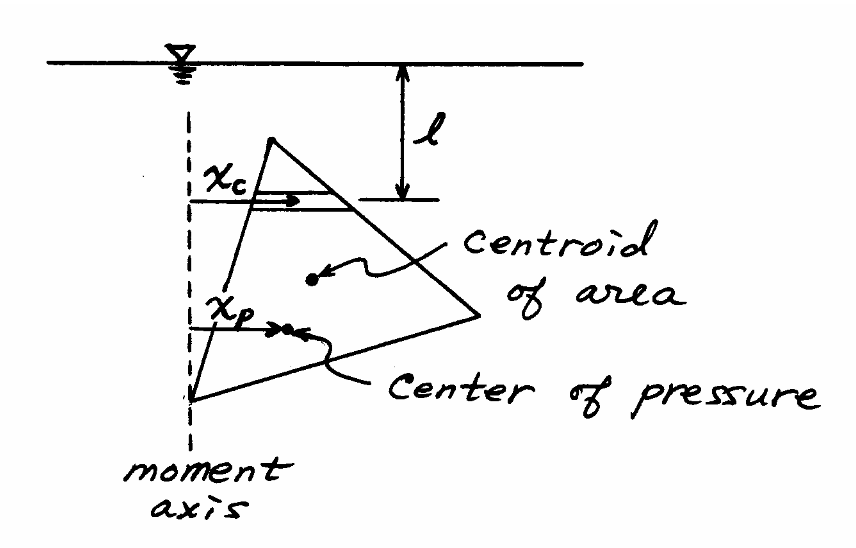
$$I_c = \int (l_c - l)^2 dA = l_c^2 \underbrace{\int dA}_{=A} - 2l_c \underbrace{\int l dA}_{=l_c A} + \underbrace{\int l^2 dA}_{=I_o}$$

$$\therefore I_o = I_c + l_c^2 A$$

Symmetric:



Non-symmetric:

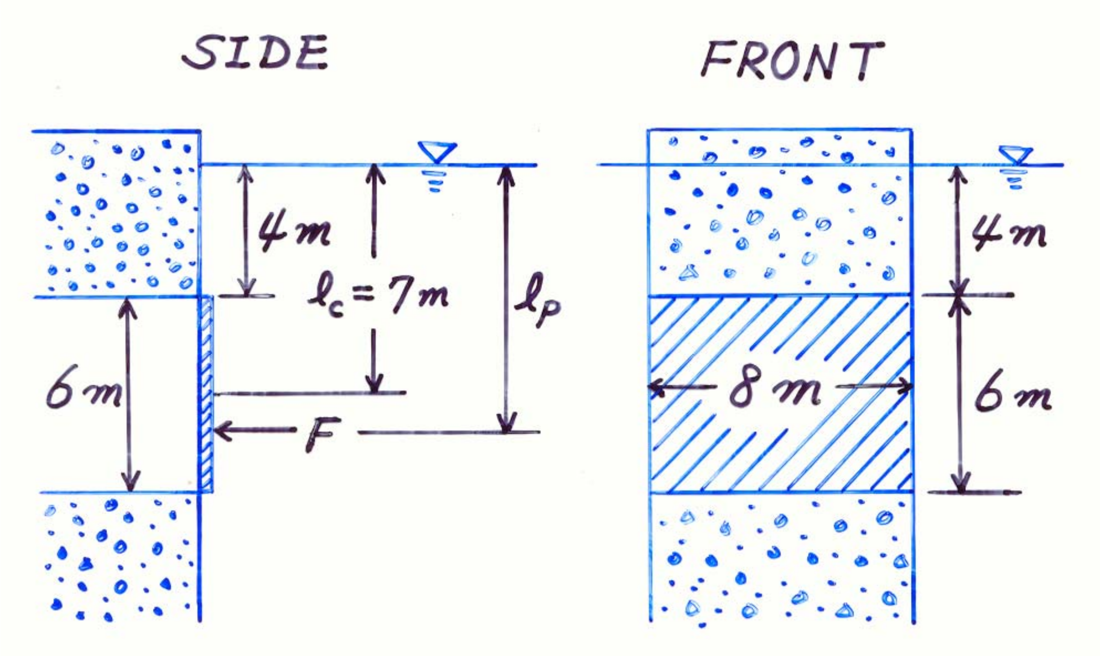


F and l_p can be calculated as previous.

Lateral position?

$$x_p = \frac{\gamma \sin\alpha \int x_c l dA}{F}$$ (Read text and IP2.9)

<u>Example</u>



Vertical gate: $\alpha = 90° \rightarrow \sin\alpha = 1.0$

1) Direct integration

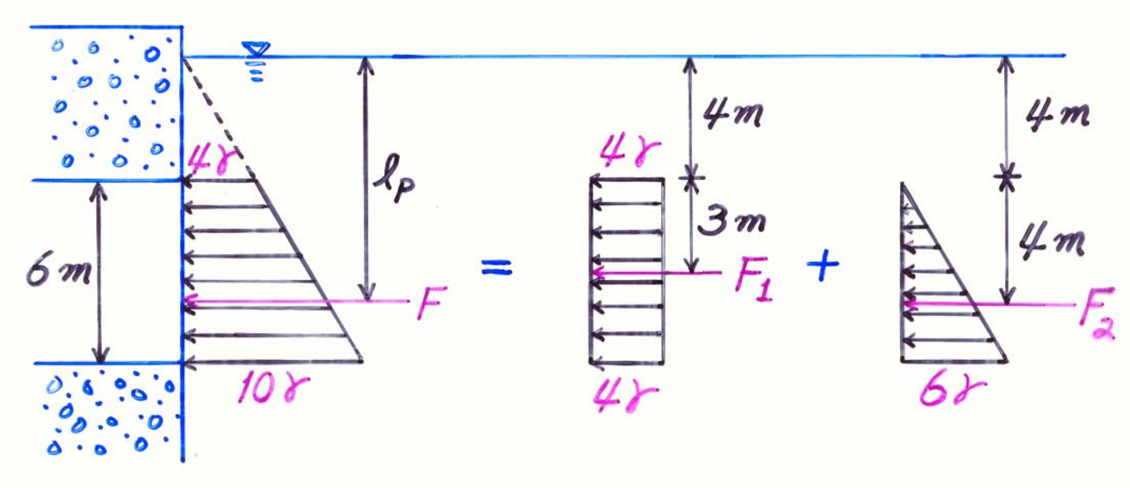
$$F = \gamma \sin\alpha \int l dA = \gamma \int_4^{10} l \cdot 8dl = 8\gamma \left.\frac{l^2}{2}\right|_4^{10} = 336\gamma$$

$$l_p F = \gamma \sin\alpha \int l^2 dA = \gamma \int_4^{10} l^2 \cdot 8dl = 8\gamma \left.\frac{l^3}{3}\right|_4^{10}$$

$$= \frac{8}{3}(1000 - 64)\gamma = \frac{8 \times 936}{3}\gamma$$

$$l_p = \frac{\frac{8 \times 936}{3}\gamma}{336\gamma} = 7.43 \text{ m}$$

2) Pressure prism approach



$$\left.\begin{aligned} F_1 &= 4\gamma \times (6\times 8) = 192\gamma \\ F_2 &= \frac{6\gamma\times 6}{2}\times 8 = 144\gamma \end{aligned}\right\} \rightarrow F = F_1 + F_2 = 336\gamma$$

$$l_p F = 7F_1 + 8F_2 \rightarrow l_p = \frac{7F_1 + 8F_2}{F} = 7.43 \text{ m}$$

3) Formula method

$$l_c = h_c = 4 + 3 = 7 \text{ m}$$

$$F = \gamma h_c A = \gamma \times 7 \times (6\times 8) = 336\gamma$$

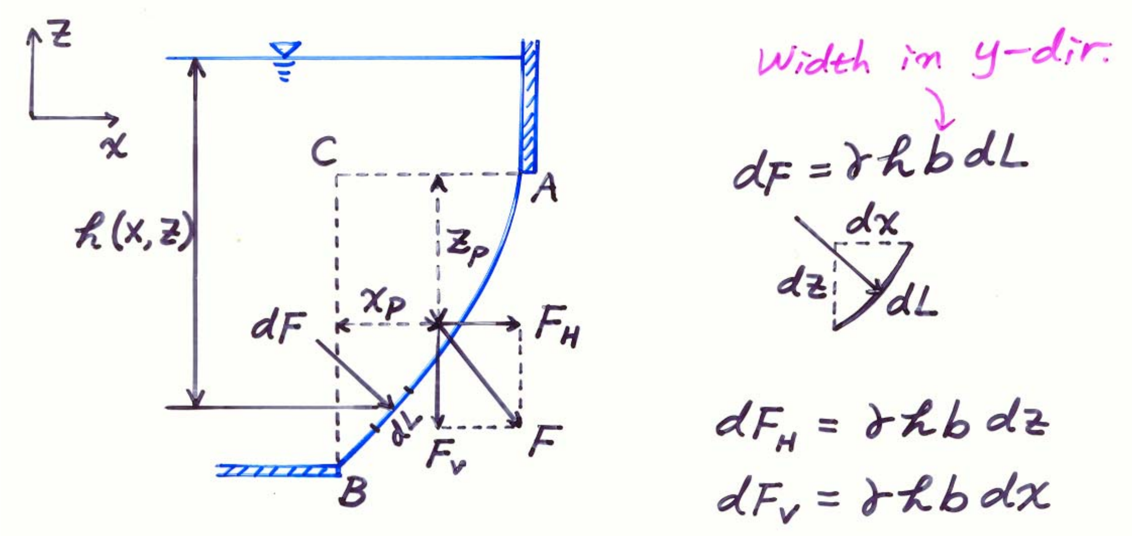
$$I_c = \frac{8\times 6^3}{12} = 144$$

$$l_p = \frac{I_c}{l_c A} + l_c = \frac{144}{7\times(6\times 8)} + 7 = 7.43 \text{ m}$$

2.5 Pressure Forces on Curved Surfaces

Direction is not constant → difficulty!

1) Direct integration method



- Curved surface = $h(x,z) \leftarrow$ No y variation
- Find F_H, F_V and x_p, z_p separately
- Vector sum of F_H and F_V $\rightarrow$ F

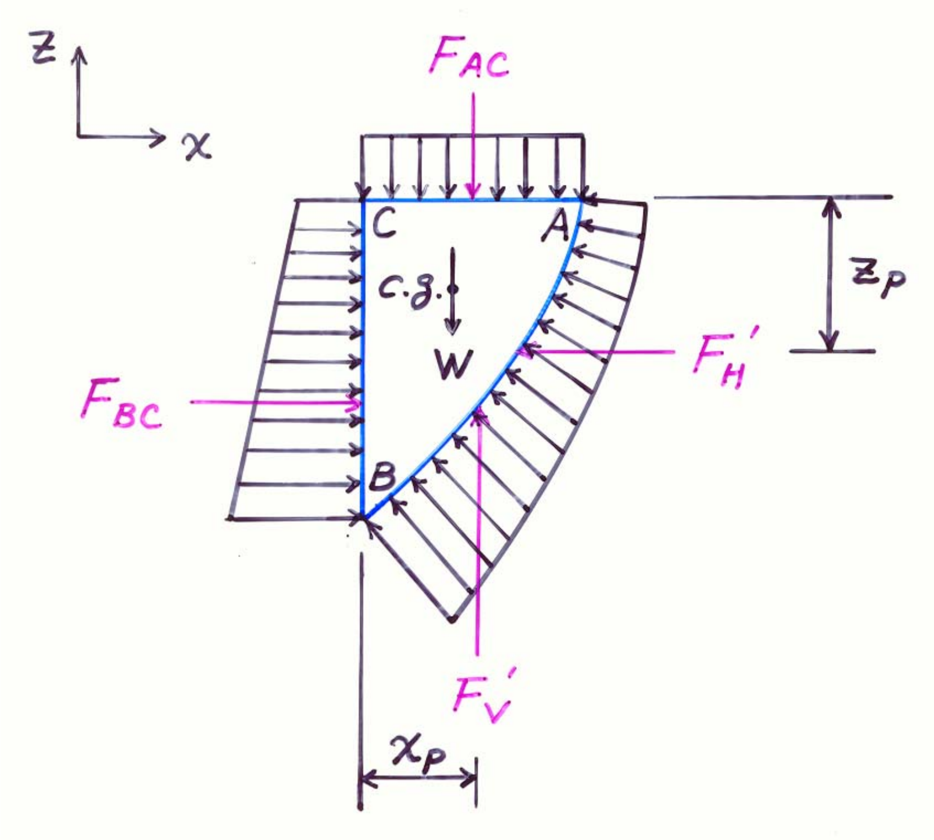
  $F_H = \int dF_H = \int \gamma h b dz$, $F_V = \int dF_V = \int \gamma h b dx$
- Consider moment about C:

  $F_H z_p = \int z dF_H = \int z \gamma h b dz \rightarrow z_p$

  $F_V x_p = \int x dF_V = \int x \gamma h b dx \rightarrow x_p$

2) Basic mechanics method

- Isolate a fluid mass → free body
- Force balance on the free body



$$\sum F_x = F_{BC} - F_H' = 0 \quad \rightarrow F_H' = F_{BC}$$

$$\sum F_z = F_V' - F_{AC} - W = 0 \quad \rightarrow F_V' = F_{AC} + W$$

F_{AC}, F_{BC} (크기 및 작용점) ← Plane surface

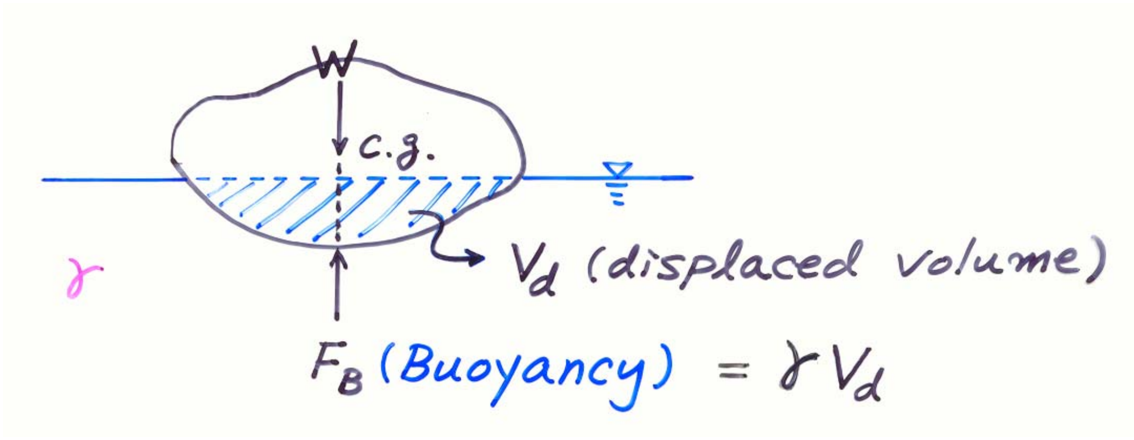
W at c.g. of the free body

Horizontal moment about C → z_p

Vertical moment about C → x_p

2.6 Buoyancy and Stability of Floating Bodies

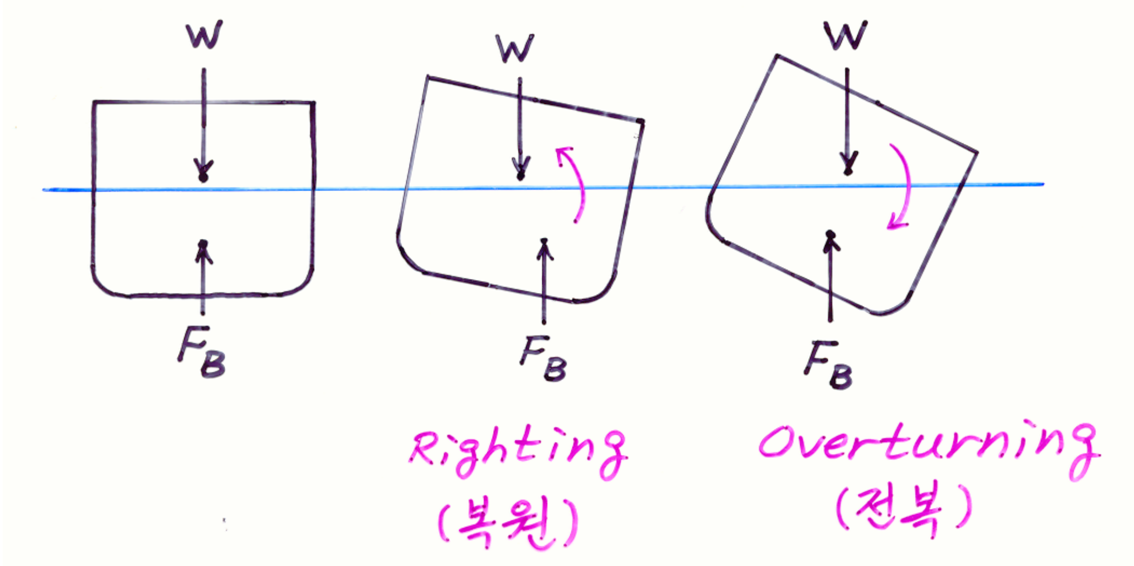
- Archimedes의 원리



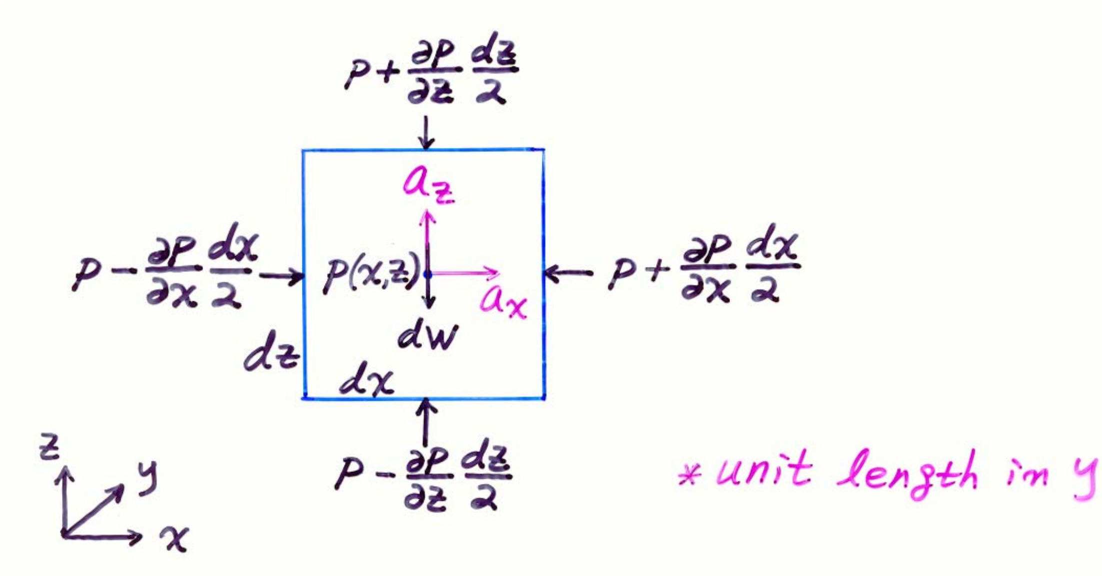
- Stability of floating bodies

W at c.g. of the whole body

F_B at c.g. of V_d



2.7 Fluid Masses Subjected to Acceleration



- Newton's 2nd law:

$$F_x = \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right)dz - \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right)dz = \rho dxdza_x$$

$$\boxed{-\frac{\partial p}{\partial x} = \rho a_x}$$

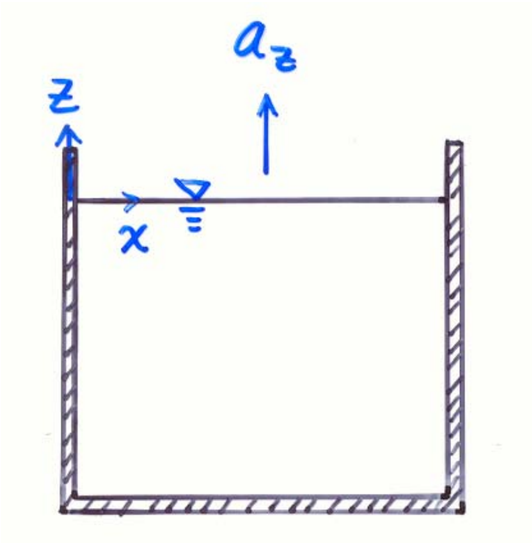
$$F_z = \left(p - \frac{\partial p}{\partial z}\frac{dz}{2}\right)dx - \left(p + \frac{\partial p}{\partial z}\frac{dz}{2}\right)dx - \rho g dxdz = \rho dxdza_z$$

$$\boxed{-\frac{\partial p}{\partial z} = \rho(a_z + g)}$$

Line of constant pressure

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial z}dz = -\rho a_x dx - \rho(a_z + g)dz$$

$$dp = 0 \text{ (const. pressure)} \rightarrow \boxed{\frac{dz}{dx} = -\frac{a_x}{a_z + g}}$$



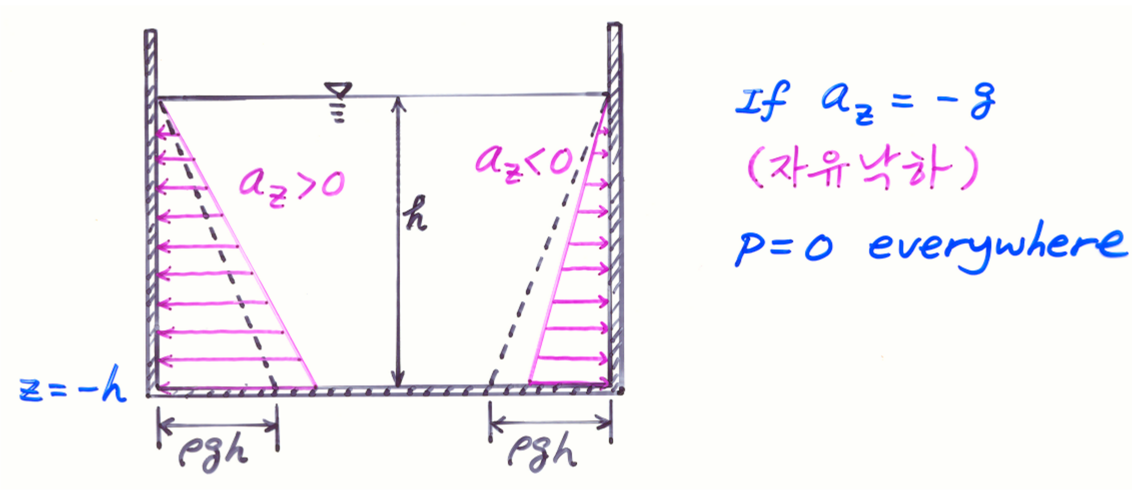
$$a_x = 0 \quad \rightarrow \frac{\partial p}{\partial x} = 0$$

$$\frac{dp}{dz} = -\rho(a_z + g)$$

$$p = -\rho(a_z + g)z + C$$

B.C. $p = 0 \quad \text{at} \quad z = 0 \;\rightarrow\; C = 0$

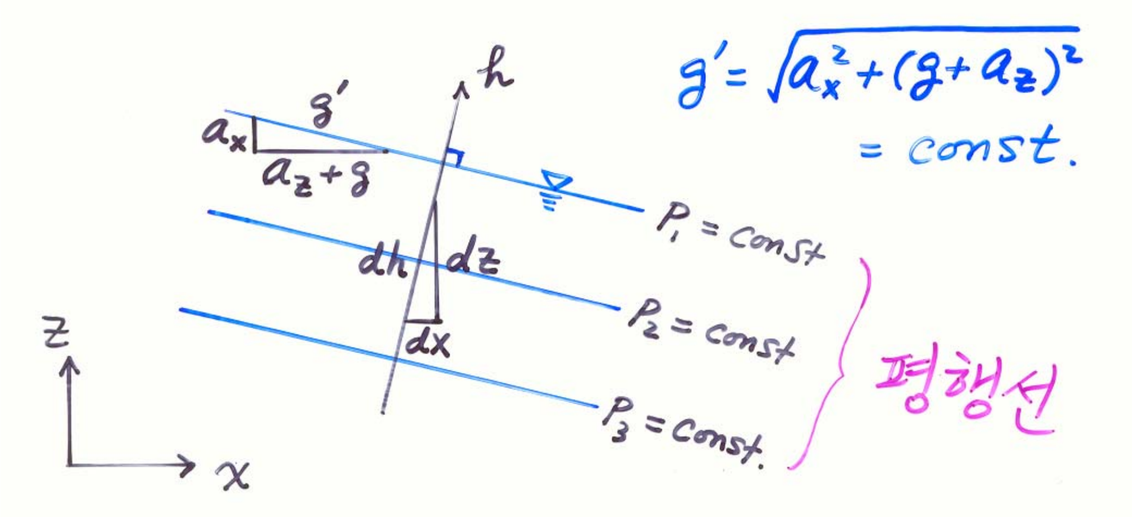
$$\therefore\; p = -\rho(a_z + g)z$$



Constant linear acceleration

Line of constant pressure:

$$\frac{dz}{dx} = -\frac{a_x}{a_z + g} = \text{const} \quad (\because a_x, a_z \text{ const.})$$



$$\frac{dx}{dh} = \frac{a_x}{g'} \quad ; \quad \frac{dz}{dh} = \frac{a_z + g}{g'}$$

$$\begin{aligned}\frac{dp}{dh} &= \frac{\partial p}{\partial x}\frac{dx}{dh} + \frac{\partial p}{\partial z}\frac{dz}{dh} \\ &= -\rho a_x \frac{a_x}{g'} - \rho(a_z + g)\frac{a_z + g}{g'} \\ &= -\frac{\rho}{g'}\left(a_x^2 + (a_z + g)^2\right) \\ &= -\frac{\rho}{g'}(g')^2 \\ &= -\rho g'\end{aligned}$$

Pressure varies linearly with h, but is proportional to $\rho g'$ instead of ρg.