※What to be learned :

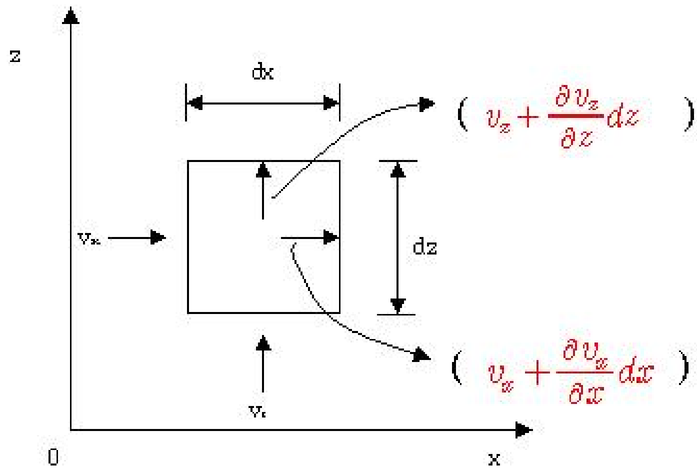
① Derive governing equation

② Get solution (h (x, z)) for various B.C

③ Determine p.w.p / h, gradient / flow quantities

1. Seepage Equation (2-D)

○ Assumptions : homogeneous, isotropic, Darcy's law valid



○ The volume of water entering the element per unit time

= $v_x dy dz + v_z dx dy$

○ The volume of water leaving the element per unit time

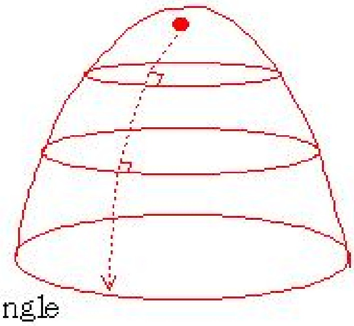
= $\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dy dz + \left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx dy$

○ Since no volume change assumed, i.e., (**steady - state**)

$q_{in} = q_{out}$

$$\rightarrow \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \xrightarrow[\left(v_x = -k \frac{\partial h}{\partial x}\right)]{} \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \right]$$

$\rightarrow \nabla^2 h = 0$ **(Laplace Eq.)**



2. Flow nets

○ Definition : (text p.69)

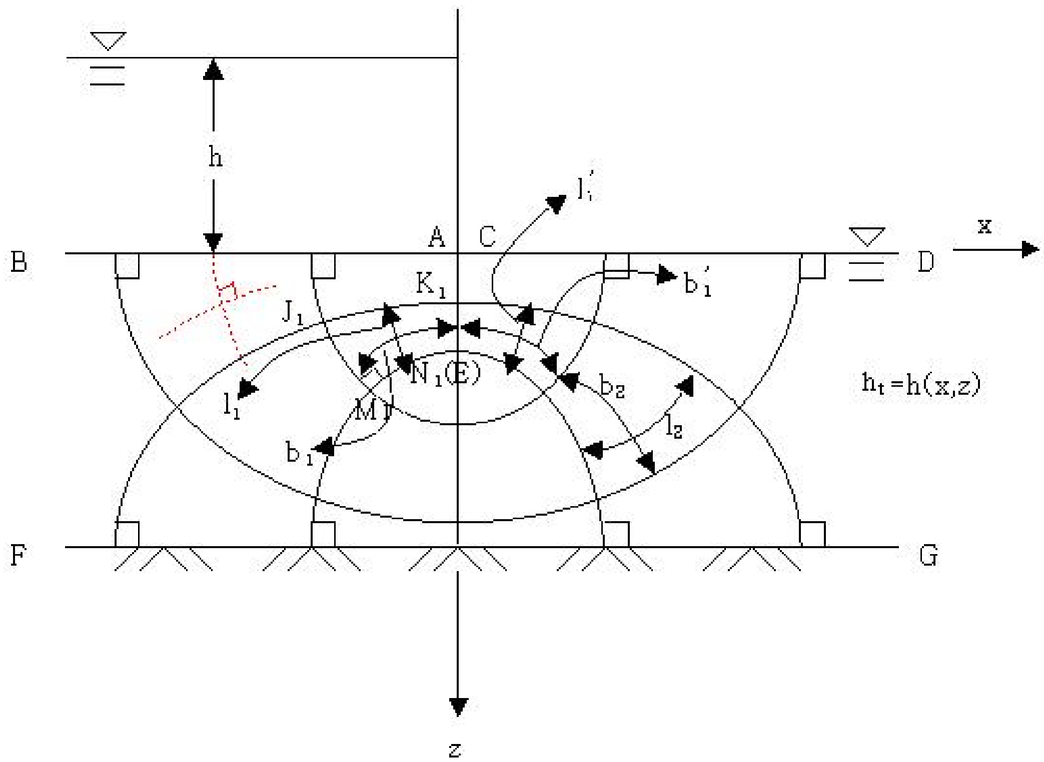
E-P line & Flow line : meet at right angle

↑ Constant head

○ Types of seepage flow :

- Confined flow : **Phreatic line known** (Top-flow)
- Unconfined flow : **not known**

3. Seepage underneath cofferdams (confined flow)



○ Boundary conditions for the confined flow

i) Line $\overline{AB}$: **e-p line** $(h_t = h)$

ii) From pt. Ⓐ → pt. Ⓔ → pt Ⓒ : **flow line** (top)

iii) Line $\overline{CD}$: **e-p line** $(h_t = 0)$

iv) Line $\overline{FG}$: **flow line** $(bottom)$

○ Square figures (⊡ $J_1K_1M_1N_1$)

- A : whole x-section, $A/y = B$
- a : x-section between flow lines, $a/y = b$

- From Darcy's law,

- $q/y = kia/y$, y: the dimension normal to the section

- let $\overline{q} = q/y$, $a/y = b$

  then, $\overline{q} = kib$, $\overline{q}$: the flow rate per running foot

  (i.e., **pet foot normal to the section**)

  b : the width of a flow line

- $\overline{q} = kib = k\frac{\Delta h}{l}b = k\Delta h\frac{b}{l}$

  Δh, l : the head loss and the length in accrossing the figure

Constant thru the section

○ $\Delta \overline{q_1} = k \Delta h_1 \dfrac{b_1}{l_1}$ (for sub$_1$ figure)

$\Delta \overline{q_1}' = k \Delta h'_1 \dfrac{b_1'}{l_1'}$ (for sub$_{1'}$ figure)

$\Delta \overline{q_2} = k \Delta h_2 \dfrac{b_2}{l_2}$ (for sub$_2$ figure)

$= 1$

○ If all the 3 figures are squares, i.e.,

$$\frac{b_1}{l_1} = \frac{b_1'}{l_1'} = \frac{b_2}{l_2} = 1$$

○ and $\Delta \overline{q_1} = \Delta \overline{q_1}'$ ($\because$ Same flow line boundaries)

then, $\Delta h_1 = \Delta h_1'$

○ and $\Delta h_1' = \Delta h_2$ ($\because$ Same e-p line boundaries)

then, $\Delta \overline{q_1'} = \Delta \overline{q_2}$

& $\Delta h_1 = \Delta h_2$

○ Thus, if all figures are squares

i) the same quantity of flow thru each figure

ii) the same head drop in crossing each figure

○ In fact, these are trues if $\dfrac{b}{l}$ = constant ($\neq$ 1)

○ Graphical determination of flow nets

i) Trial and error

ii) Remember boundary conditions

iii) 4~5 flow channels sufficient

iv) All figures should resemble squares

v) Size of the squares change gradually

vi) One trial flow line (or e-p line) near a boundary line drawn first

○ Example (text Fig 3.7, p 72)

Fig (c) should be symmetric to the bottom (i.e., mirror image)

○ Ref.s : "Seepage, Drainage & Flow Nets" by H. R. Cedergen (Wiley Interscience)

○ Flow quantity calculation

$$\overline{q_B} = k\, h\, \frac{N_f}{N_d},$$

$$= k\, \frac{h}{N_d}\, N_f$$

$$= k\, \frac{h}{N_d}\, \frac{1}{l}\, b\, N_f$$

$$= k\, \frac{h/N_d}{l}\, b\, N_f$$

$$= k\, \frac{\Delta h}{l}\, b\, N_f$$

$$= k\, i \cdot B(=A/y)$$

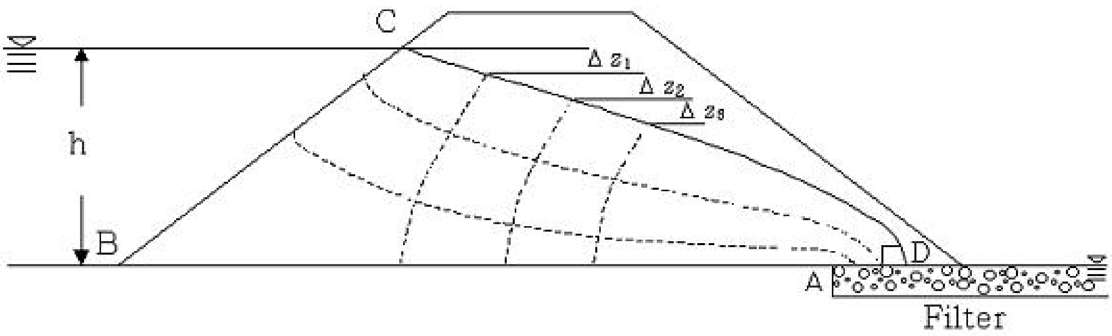
$$= \overline{q_B}$$

N_f : number of flow channel

N_d : number of e.p. drops

h : total head loss

4. Seepage thru earth dams (unconfined flow)



○ Boundary conditions

i) $\overline{BC}$: e-p line, $h_t = h$

ii) $\overline{AD}$: e-p line, $h_t = 0$

iii) $\overline{CD}$: top-flow line $(h_p = 0)$: initially undetermined

iv) $\overline{BA}$: bottom-flow line, $h_e = 0$

○ h_p at every pt. on the top flow line = 0

thus,

$(h_t)_{t.f.l} = (h_e)_{t.f.l} \rightarrow \Delta z_1 = \Delta z_2 = \Delta z_3$

∵ between each e-p line, head drops are the same, & all Δh are equal to Δh_E

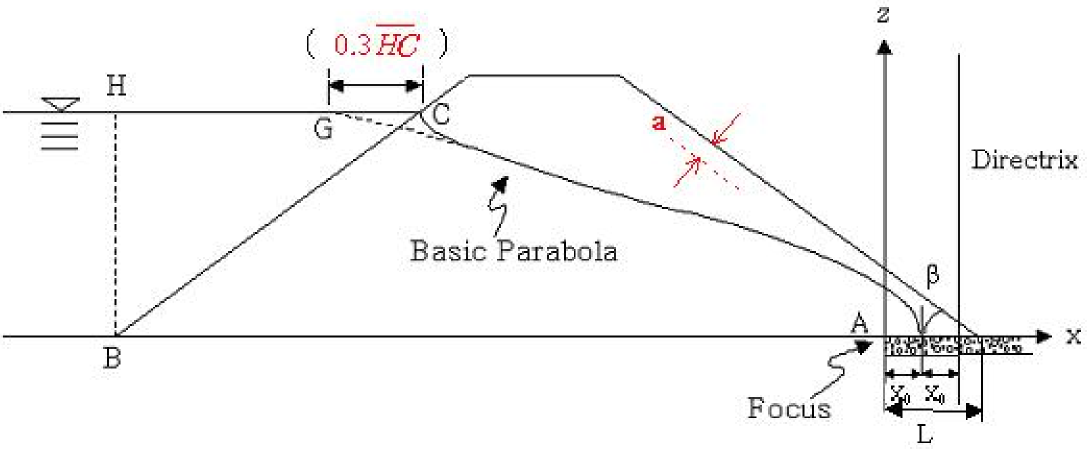
○ Filter

- purpose : to keep the seepage entirely within the dam
- requirements : piping/permeability
- piping : pores be small enough to prevent particles being carried away $\left(\frac{(D_{15})_f}{(D_{85})_s} < 4 \sim 5 \right)$
- permeability : high enough for rapid drain $\left(\frac{(D_{15})_f}{(D_{15})_s} > 4 \sim 5 \right)$

COE
$\frac{(D_{50})_f}{(D_{50})_s} < 25$

USBR
$(D_{max})_f < 3''$

○ Flow net construction for earth dam



$* \; L > x_0(1 + \cot^2\beta)$

for capillary allowance

$L > x_0(1 + \cot^2\beta) + \alpha$

$\uparrow$

$\alpha = f(a)$

- The equation of the basic parabola

$$x = x_0 - \frac{z^2}{4x_0}$$

- By Casagrande, it is recommended to take the initial pt. of the parabola at G where $\overline{GC}$ = [$0.3\overline{HC}$]

to determine the unknown constant, x_0