Bernoulli's Theorem Experiment

Objectives

Chapter 4. Bernoulli's Theorem Experiment

To investigate the validity of Bernoulli's Theorem as applied to the flow of water in a tapering circular duct.

\[ \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + Z_2 + h_L = H \]

Bernoulli Theorem

\[ \frac{d^2}{dx^2} \left( EI \frac{d^2 w}{dx^2} \right) = q \]

Bernoulli Theorem

Daniel Bernoulli (1700-1782)

Born in Netherland
Mathematician, physicist
Hydrodynamique (1738)
Conservation of Energy
Exposition of a New Theory on the Measurement of Risk (1738)
St. Petersburg Paradox
Beam theory

The Bernoulli family

Nicolaus I 1624-1703
Jacob 1654-1705
Nicolaus II 1662-1716
Johann 1667-1748
Nicolaus (I) 1687-1759
Daniel 1695-1726
Johann (II) 1710-1790
Nicolaus (II) 1744-1807
Daniel (II) 1751-1854
Jacob (I) 1759-1799
Bernoulli Theorem

Considering flow at two sections in a pipe

Bernoulli’s equation

\[ \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + Z_2 = H \]

- \( V \) = velocity head \( (m/s)^2 = m \)
- \( P \) = pressure head \( \frac{kg \cdot m/s^2}{m^2} \)
- \( Z \) = potential(elevation) head
- \( H \) = total head

\[ V = velocity \quad g = gravitational \text{ acceleration} \]
\[ P = pressure \quad \gamma = specific \text{ weight} \]
\[ Z = height \quad h_f = head \text{ loss} \]

Piezometer

Small diameter observation well to measure the hydraulic head

Derivation of Theorem

Apply Newton’s 2nd law to the motion of fluid particles
Consider a streamline and select a small cylindrical fluid system
Derivation of Theorem

Apply Newton’s 2nd law to the motion of fluid particles

\[ \sum F = ma \]

\[ dF = \rho dA - (p + dp) dA - dW \sin \theta \]

\[ = -dp \cdot dA = -\rho gdA \cdot \frac{dz}{ds} \]

\[ = -dp \cdot dA - \rho g \cdot dA \cdot dz \]

\[ dm = \rho ds \cdot dA \quad \text{(density \times \ volume)} \]

\[ a = \frac{dV}{dt} = \frac{dV}{ds} \cdot \frac{ds}{dt} = V \frac{dV}{ds} \]

\[ \therefore -dpdA - \rho g \cdot dA \cdot dz = \rho \cdot ds \cdot dA \cdot V \frac{dV}{ds} \]

Derivation of Theorem

Bernoulli’s equation?

Integrate Euler’s equation

\[ d\left( \frac{P}{\gamma} + \frac{V^2}{2g} + z \right) = 0 \]

\[ \frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{const} \]

\[ \frac{V^2}{2g} + \frac{P}{\gamma} + Z_i = \frac{V^2}{2g} + \frac{P}{\gamma} + Z_2 = H \]

Application of Theorem

Air lift? Not the main reason

Lower pressure is caused by the increased speed of the air over the wing.

Since the pressure is higher beneath the wing the wing is pushed upwards.
Experimental Apparatus

1. Obtain the area of cross sections of the duct point connected to the manometer.
2. Calculate the flowrate with a stopwatch and the volumetric tank level.
3. Calculate mean velocity of each cross section with flowrate and area of cross sections.
4. Compute Reynolds number and velocity head using mean velocity.
5. Measure the pressure head by reading Manometer level.
6. The sum of velocity head(4) and pressure head(5) and potential head is the total head.
(Potential head is zero, we assumed that the centerline of the duct is datum)
7. Measure the total head of each cross section using Pitot tube.
8. Compare the computed total head(6) with measured total head(7).
9. Repeat process (2-5) 5 times with each other flowrate.

Results

<table>
<thead>
<tr>
<th>Point</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (mm)</td>
<td>25</td>
<td>13.9</td>
<td>11.8</td>
<td>10.7</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Area (mm²)</td>
<td>490.874</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed (mm/s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity Head (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure Head (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Potential Head (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated Total Head</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured Total Head</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Procedure

1. Using Bernoulli Theorem, discuss relations of diameter of duct, mean velocity, pressure.
2. Compare the computed total head with measured total head, discuss why does the difference occurs.