# Chapter 5. Consolidation (outlines)

\# 14

1. General
   - Definition
   - Spring analogy

2. Theory of Consolidation
   - Derivation of governing eq.
   - Assumptions in 1-D consolidation eq.

\# 15

3. Solution of Consolidation Eq.
   - Solution in non-dimensional terms
   - Degree of consolidation(Settlement times)

\# 16

4. Oedometer test
   - Test set-up
   - Load applications & dial gage readings
   - Test results & interpretations
     i ) Dial gage readgings vs time : *calculates the rate of consolidation($C_v$)*
     ii ) Void ratio vs stress : *Determines the magnitude of consolidation settlements*
     iii ) Determination of pre-consolidation stress

\# 17

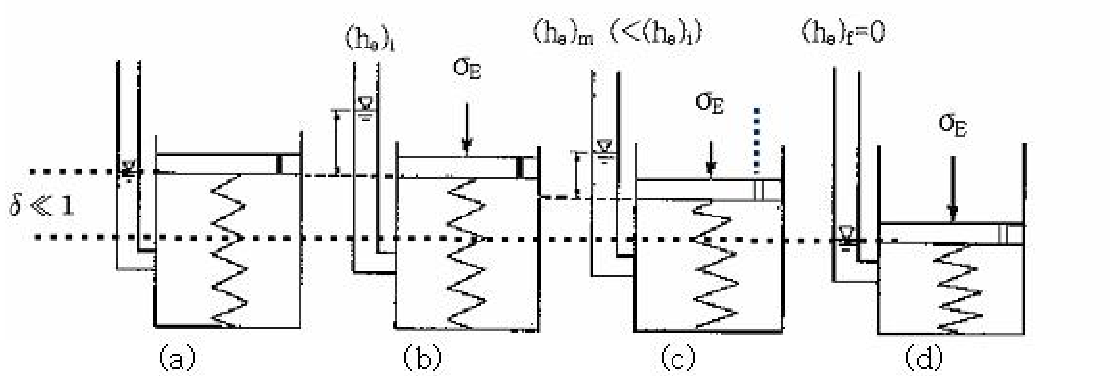
5. Calculation of Consolidation settlements
   - Magnitude of settlement
   - Settlement times(Degree of consolidation)

보조자료로 대치

6. Fast draining methods
   - Concept
   - Settlement times
   - Types of drains
   - Preloading

# Chapter 5. Consolidation

## 1. Spring analogy (Fig. 2.17, p.57)



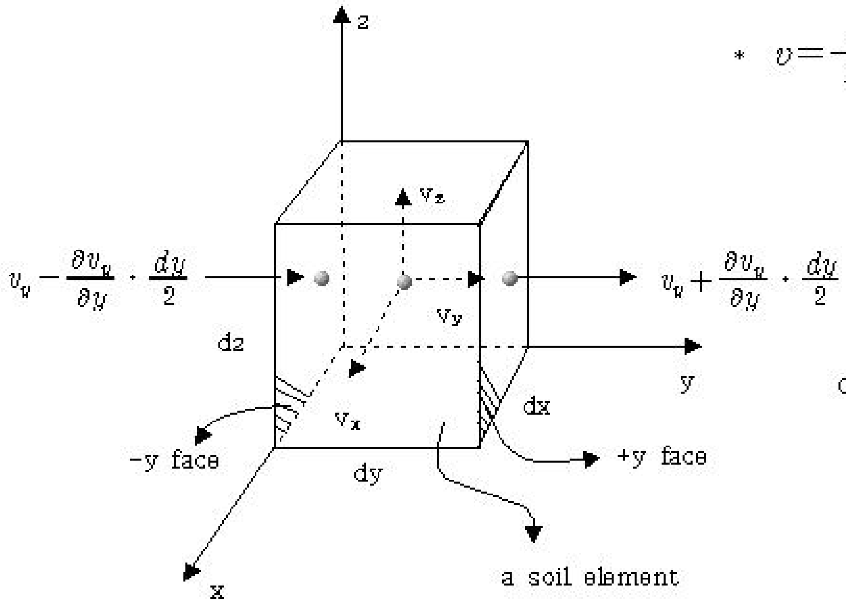

| (a) | (b) | (c) | (d) |
|---|---|---|---|
| $\sigma_E=0$ | $\sigma_E>0$ | $\sigma_E>0$ | $\sigma_E>0$ |
| $\Delta\sigma=0$ | $\Delta\sigma=\sigma_E$ | $\Delta\sigma=\sigma_E$ | $\Delta\sigma=\sigma_E$ |
| $\Delta\bar{\sigma}=0$ | $\Delta\bar{\sigma}=0$ | $\Delta\bar{\sigma}>0$ | $\Delta\bar{\sigma}=\sigma_E$ |
| $\Delta u_e=0$ | $\Delta u_e=\sigma_E$ | $\Delta u_e<\sigma_E$ | $\Delta u_e=0$ |

* Consolidation ?

The gradual reduction in volume of a fully saturated soil of low permeability due to drainage of some of the ( *pore pressure* ), (the process) continuing until the ( *excess pore water pressure* ) set up by an increase in total stress has completely dissipated : The simplest case is that of one-dimensional consolidation in which a condition of ( *no lateral strain* ) is implicit.

## 2. Theory of Consolidation

( 5.2 , Terzaghi's one-dimensional consolidation equation )




* $v = \frac{q}{A}$ : Apparent Velocity

*cf. Seepage velocity*

q : rate of flow thru A

(= $vA = -kiA$ )

- Rate of flow <u>out of</u> the element thru the +y face

$$q_{out} = (v_y + \frac{\partial v_y}{\partial y}\frac{dy}{2})\, dx \cdot dz \quad \cdots\cdots\cdots \quad (1)$$

- Rate of flow <u>into</u> the element thru the -y face

$$q_{in} = (v_y - \frac{\partial v_y}{\partial y}\frac{dy}{2})\, dx \cdot dz \quad \cdots\cdots\cdots \quad (2)$$

- net rate of flow <u>into</u> the element in the y direction

$$q_{in} - q_{out} = (2) - (1) = (-\frac{\partial v_y}{\partial y})\, dx\, dy\, dz$$

[check (-) sign]

flow <u>quantity</u> per time

- Total weight change of flux due to 3-D flow into the element

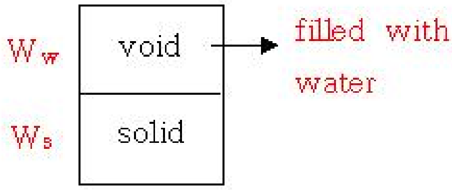
$$= -\gamma_w(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z})dx \cdot dy\, dz$$

- For a soil element,

$$W_T = W_s + W_w$$

per unit volume of soil

$$W_T/V = W_s/V + W_w/V$$




- Let $W_w/V = W_w^*$ : *wt. of water per unit vol. of soil (흙의 단위 부피당 물의 무게)*

- Then,

$$-\gamma_w\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)\overbrace{dxdydz/V}^{\text{one}} = \frac{\partial W_w^*}{\partial t}$$

↓

[ *Eq. of continuity* ]

- If ( *steady-state* ) condition exists,

$$\frac{\partial W_w^*}{\partial t} = 0$$ : i.e., ( *no accumulation or no dissipation of water* )

$$\rightarrow \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_y}{\partial z} = 0$$ : *steady-state seepage equation*

- If ( *transient* ) case,

$$-\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = \frac{1}{\gamma_w}\frac{\partial W_w^*}{\partial t} \quad \cdots\cdots \quad Eq.A$$

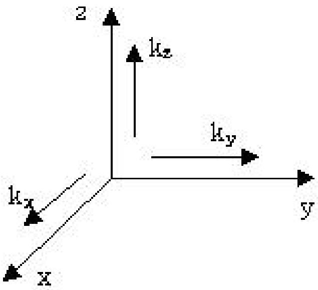
*Assumption #1*

- Darcy's law - holds for water flow(laminar) in soil

$$v \propto i \quad \rightarrow \quad v = -ki$$

$$i = \frac{dh}{dl}$$ : hydraulic gradient

- In three-dimensional & anisotropic (general case)




$$k : \; k_x, \; k_y, \; k_z$$

$$v_x = -k_x \frac{\partial h}{\partial x}, \quad v_y = -k_y \frac{\partial h}{\partial y}, \quad v_z = -k_z \frac{\partial h}{\partial z}$$

- Substitute these into Eq. Ⓐ

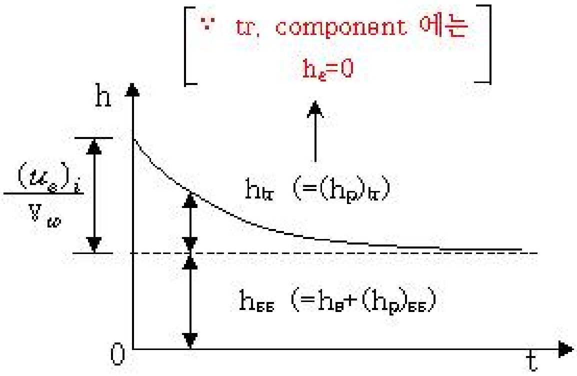
$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_z \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial h}{\partial z}\right) = \frac{1}{\gamma_w} \frac{\partial W_w^*}{\partial t}$$

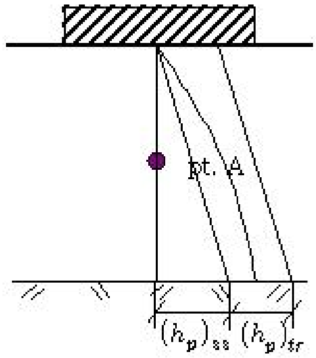
- Assuming $k_x$, $k_y$, $k_z$ are constant w.r.t. x, y, z coordinates

[ i.e., <u>*homogeneous*</u> ].

*Assumption #2*

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = \frac{1}{\gamma_w} \frac{\partial W_w^*}{\partial t} \quad \cdots\cdots \; Eq.B$$




<at pt. A>

- Since $h_t = h_e + h_p = \overbrace{h_e + ((h_p)_{ss}}^{h_{ss}}) + ((h_p)_{tr})$

$$= (h_{ss}) + ((h_p)_{tr}) = h_{ss} + \left(\frac{u_e}{\gamma_w}\right)$$

where $u_e$ : excess pore water pressure

$\frac{\partial^2}{\partial x^2}$ ↓ linear operator

- Sub. this into Eq. B, then L.H.S. becomes

$$k_x \frac{\partial^2 h_{SS}}{\partial x^2} + k_y \frac{\partial^2 h_{SS}}{\partial y^2} + k_z \frac{\partial^2 h_{SS}}{\partial z^2}$$

isotropic Assumption #3

$$+ \frac{k_x}{\gamma_w}\frac{\partial^2 u_e}{\partial x^2} + \frac{k_y}{\gamma_w}\frac{\partial^2 u_e}{\partial y^2} + \frac{k_z}{\gamma_w}\frac{\partial^2 u_e}{\partial z^2} = \frac{k}{\gamma_w}\nabla^2 u_e, \quad k_x = k_y = k_z$$

where $\nabla^2$ : Laplacian operator $\left(= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$

- R.H.S $\left(= \frac{1}{\gamma_w}\frac{\partial W_w^*}{\partial t}\right)$

$$W_w^* = \frac{\gamma_w \cdot V_W}{V} = \gamma_w \cdot S \cdot V_v / V \quad \left(\because S = \frac{V_w}{V_v}\right)$$

$$W_S^* = \gamma_w \cdot G_s \cdot V_s / V$$

$$\frac{W_w^*}{W_S^*} = \frac{S \cdot V_v}{G_s \cdot V_s} = \frac{S \cdot e}{G_s} \quad \left(\because e = \frac{V_v}{V_s}\right)$$

- Thus,

$$W_w^* = \left( \frac{S \cdot e}{G_s} \right) W_S^*$$

$$= \left( \frac{e}{G_s} \right) W_S^* \leftarrow (\text{Assume } S=1, \text{ fully saturated})$$

Assumption # 4

and, we get

$$\frac{1}{V_w} \frac{\partial W_w^*}{\partial t} = \frac{1}{V_w} \cdot \frac{W_s^*}{G_s} \frac{\partial e}{\partial t}$$

here,

$$\frac{W_s^*}{V_w \cdot G_s} = \frac{\text{wt. of solid / vol. of soil}}{\text{unit wt. of solid}}$$

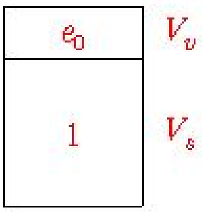
$$= \frac{\text{wt. of solid}}{(\text{unit wt. of solid})(\text{vol. of soil})}$$

$$= \frac{\text{vol. of solid}}{\text{vol. of soil}} = \frac{1}{1+e_0}$$

↑ initial void ratio

↑ small strain assumed

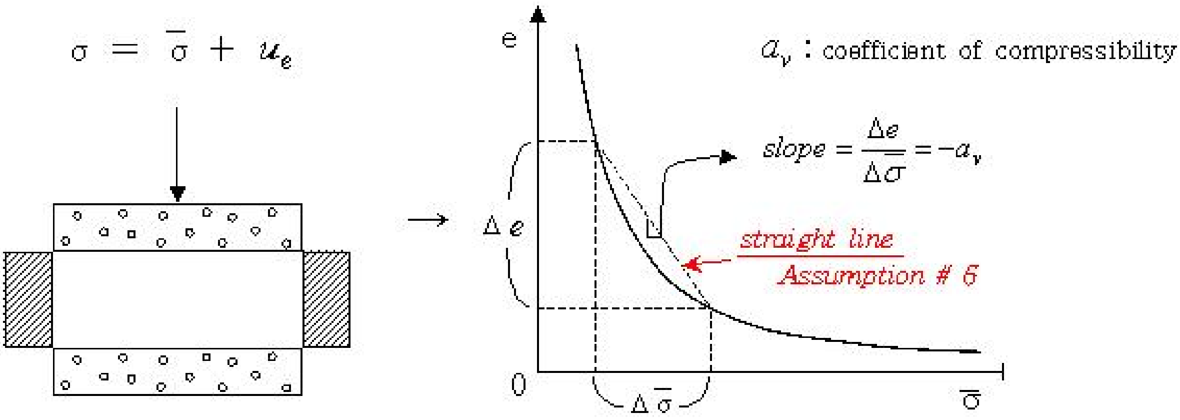
Assumption # 5



* $\frac{\partial e}{\partial t} = ?$

- from ( *Oedometer* ) test

$\sigma = \bar{\sigma} + u_e$




- $\Delta e = -a_v \Delta \bar{\sigma}$

$$\frac{\partial e}{\partial t} = -a_v \frac{\partial \bar{\sigma}}{\partial t} = -a_v \frac{\partial (\sigma - u_e)}{\partial t}$$

$$= +a_v \frac{\partial u_e}{\partial t} - a_v \frac{\partial \sigma}{\partial t}$$

$$= a_v \left( \frac{\partial u_e}{\partial t} - \frac{\partial \sigma}{\partial t} \right)$$

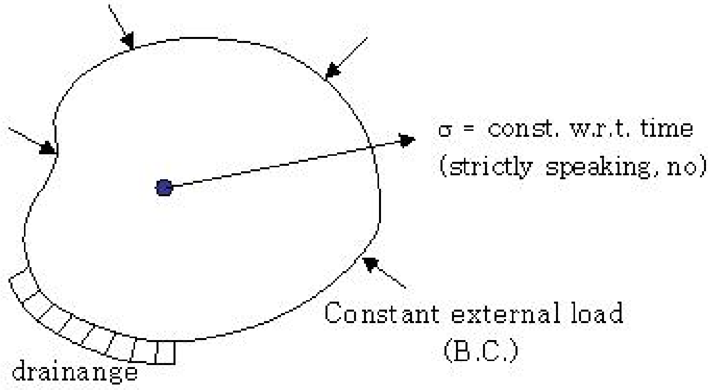
- Finally,

$$\frac{k}{\gamma_w} \cdot \nabla^2 u_e = \frac{a_v}{1+e_0} \left( \frac{\partial u_e}{\partial t} - \frac{\partial \sigma}{\partial t} \right)$$

or

$$\nabla^2 u_e = \frac{\gamma_w \cdot a_v}{k(1+e_0)} \left( \frac{\partial u_e}{\partial t} - \frac{\partial \sigma}{\partial t} \right) \quad \text{.......................} \quad Eq.C$$

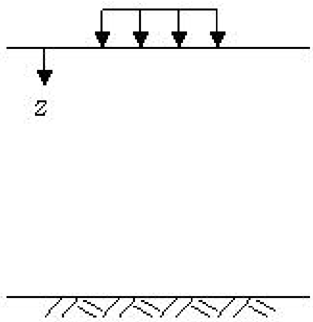
- $\frac{\partial \sigma}{\partial t}$ = ?




- Assume $\frac{\partial \sigma}{\partial t} = 0$ , i.e., total stress σ = constant for constant boundary loading
Assumption # 7

- Assume *one dimensional flow* - *Assumption # 8*

then, Eq. C becomes,




$$\frac{\partial^2 u_e}{\partial z^2} = \frac{1}{C_v}\frac{\partial u_e}{\partial t}$$ .... 1-dim. consolidation Eq.

$u_e$ : excessive pore water pressure

$C_v = \frac{k(1+e_0)}{a_v \cdot \gamma_w}$ : coefficient of consolidation ($L^2T^{-1}$)

assumed to be ( constant ) for a certain soil
*Assumption # 9*

$m_v = \frac{a_v}{1+e_0}$ : coefficient of Volume compressibility

$$\rightarrow C_v = \frac{k}{m_v \gamma_w}$$