# Complex Geometries

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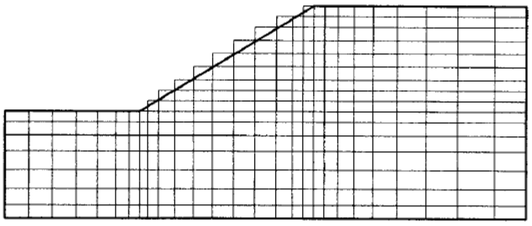
## Grid Choice

- Stepwise approximation using regular grids
    - Simplest approach.
    - To apply such a grid to solution domains with inclined or curved boundaries, the boundaries have to be approximated by staircase-like steps.
    - # of grid points per grid line is not constant. → indirect addressing or special arrays.
    - Steps at boundary introduces errors.



## Grid Choice – Cont.

- Overlapping grids
  - Use of a set of regular grids to cover irregular solution domains. → Combine rectangular, cylindrical, spherical or non-orthogonal grids near bodies with Cartesian grids in the rest of domain.
  - Programming and coupling of the grids can be complicated.
  - Solution method is applied on one grid after another, the interpolated solution from one grid providing BCs for the next iteration on adjacent grids.

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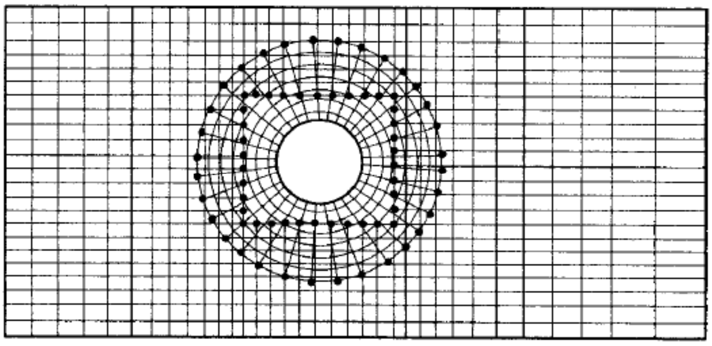
## Grid Choice – Cont.

- Overlapping grids
  - Each grid is attached to one reference frame, including some which move with the bodies.
  - Ex. Chimera grids



## Grid Choice – Cont.

- Boundary-fitted non-orthogonal grids
  - Most often used to calculate flows in complex geometries
  - Can be structured, block-structured, or unstructured
  - BCs are more easily implemented, since the grid lines follow the boundaries.
  - Transformed equations contain more terms thereby increasing both the difficulty of programming and the cost of solving eqns.
  - Grid non-orthogonality may cause unphysical solutions and the arrangement of variables on the grid affects the accuracy and efficiency of the algorithm.

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## Grid Generation

- Reference: Thompson et al. (1985)
- In FV methods, the angle between the cell face surface normal vector and the line connecting the CV centers on either side of it is important.
- If the midpoint rule integral approximation, linear interpolation, and center differences are used to discretize the eqns, then the accuracy will be higher if CVs are quadrilateral in 2D and hexahedral in 3D, than triangles and tetrahedra, respectively.
- Accuracy is improved if one set of grid lines follows the streamlines of the flow, especially for the convective terms.

## Grid Generation – Cont.

- Ratio of sizes of adjacent cells should be kept under control. Especially when block-structured grids are used, one should take care that the cells are of nearly equal size near block interfaces.
- Adaptive grid methods: Start with a coarse grid and later refine it locally according to an estimate of the discretization error.
- Overlapping grid are easier to generate, but there are geometries in which application of this is difficult due to the existence of too many irregular pieces (e.g., automobile under-hood geometries).

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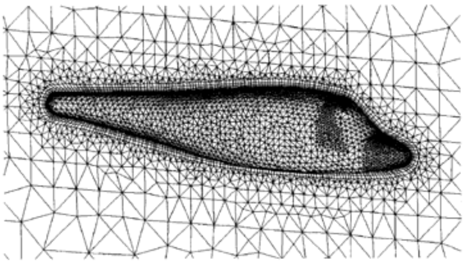
## Grid Generation – Cont.

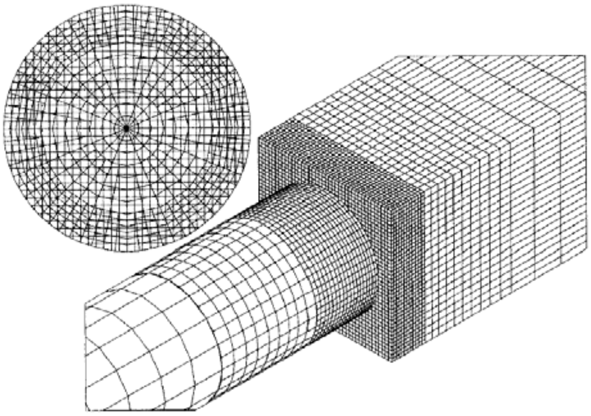
- Tetrahedral cells are not desirable near walls if BL needs to be resolved. → prism layers on wall.



- Cover the solution domain with a coarse Cartesian grid and adjust the cells cut by domain boundaries to fit the boundary. → Cells near boundary are irregular and may require special treatment.

# Grid Generation – Cont.

- Generation of grids with non-matching interfaces is much simpler than creation of a single-block grid fitted to the whole domain. Note the solution method has to allow treatment of polyhedral CVs with arbitrary # of faces.



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# Velocity Components

- Cartesian basis
  - When 3D, no advantage to using any other basis, e.g., grid-oriented, covariant, or contravariant
  - If FDM is used, one has to employ the appropriate forms of the divergence and gradient operators for non-orthogonal coordinates. → increased # of terms.
  - In FVM, no need for coordinate transfomations.

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## Sections 8.4 – 8.6.5

- Read through.

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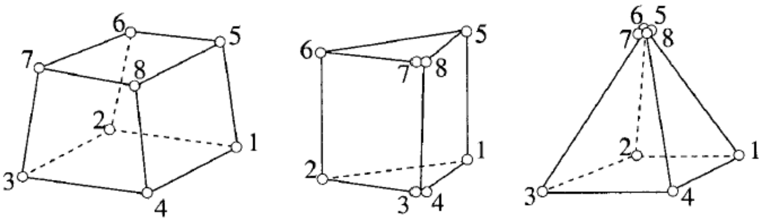
## Finite Volume Methods

- Unstructured grids
  - Data structure depends on CVs used.
  - When a grid is generated, a list of vertices is created.
  - Each CV is defined by 4 or 8 vertices, so the list of CVs also contains a list of associated vertices.
  - The order of vertices in the list represents the relative positions of the cell faces.



## Finite Volume Methods – Cont.

- Unstructured grids – Cont.
  - Another possibility – introduce object oriented data structure and define objects vertex, edge, face, and volume.
  - Edges are defined by vertices.
  - Faces by lists of edges.
  - Volumes by lists of faces.

## Sections 8.7 – 8.9

- Read through.

# Implementation of BCs

- Implementation of BCs on non-orthogonal grids requires special attention because the boundaries are usually not aligned with the Cartesian velocity components.

- Inlet
  - All quantities have to be prescribed.
- Outlet
  - Know little about the flow → these boundaries should be as far downstream of the region of interest as possible.
  - The flow should be directed out of the domain over the entire outlet cross-section and be parallel. → extrapolates along grid lines from the interior to the boundary.

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# Implementation of BCs – Cont.

- Impermeable walls

$$u_i = u_{i,\mathrm{wall}} \; . \tag{8.72}$$

  - Viscous fluids stick to solid boundaries.
  - Convective fluxes of all quantities are zero.
  - For momentum eqns,

$$\boldsymbol{f}_{\mathrm{wall}} = \int_{S_{\mathrm{s}}} \boldsymbol{t}\tau_{nt}\,\mathrm{d}S \approx (\boldsymbol{t}\tau_{nt}S)_{\mathrm{s}} \tag{8.78}$$

$$\boldsymbol{v}_p = \boldsymbol{v} - (\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n} \quad \Rightarrow \quad \boldsymbol{t} = \frac{\boldsymbol{v}_p}{|\boldsymbol{v}_p|} \; . \tag{8.76}$$

$$\tau_{nn} = 2\mu\left(\frac{\partial v_n}{\partial n}\right)_{\mathrm{wall}} = 0\,, \quad \tau_{nt} = \mu\left(\frac{\partial v_t}{\partial n}\right)_{\mathrm{wall}} \; . \tag{8.75}$$

# Implementation of BCs – Cont.

- Symmetry planes
  - Convective fluxes of all quantities are zero.
  - Normal gradients of the velocity components parallel to symmetry plane and of all scalar quantities are zero.

$$\boldsymbol{f}_{\text{sym}} = \int_{S_s} \boldsymbol{n}\tau_{nn}\,\mathrm{d}S \approx (\boldsymbol{n}\tau_{nn}S)_s \; . \qquad (8.81)$$

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