Turbulent Flows

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Instability

- All flows become unstable above a certain Reynolds number.
  - At low Re flows are laminar.
  - For high Re flows are turbulent.
  - The transition occurs anywhere Re between 2,000 and 1M, depending on the flow.
- For turbulent flows, the computational effort involved in solving those for all time and length scales is prohibitive.
- An engineering approach to calculate time-averaged flow fields for turbulent flows will be developed.

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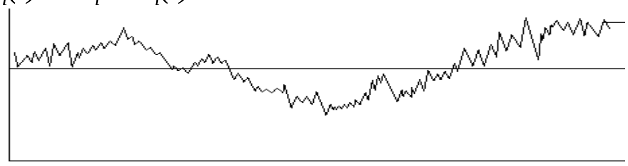
What is Turbulence?

- Unsteady, aperiodic motion in which all three velocity components fluctuate, mixing matter, momentum, and energy.
- Decompose velocity into mean and fluctuating parts:

$$U_i(t) \equiv U_i + u_i(t).$$



Time

- Similar fluctuations for pressure, temperature, and species concentration values.

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What is Turbulence? – Cont.

- **Irregularity** or randomness: A full deterministic approach is very difficult. Turbulent flows are usually described statistically.
- **Chaotic**. But not all chaotic flows are turbulent. Waves in the ocean, for example, can be chaotic but are not necessarily turbulent.
- The **diffusivity** of turbulence causes rapid mixing and increased rates of momentum, heat, and mass transfer. A flow that looks random but does not exhibit the spreading of velocity fluctuations through the surrounding fluid is not turbulent.

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What is Turbulence? – Cont.

- Turbulent flows always occur at **high Reynolds numbers**. They are caused by the complex interaction between the viscous terms and the inertia terms in the momentum equations.
- Turbulent flows are **rotational**; that is, they have non-zero vorticity. Mechanisms such as the stretching of three-dimensional vortices play a key role in turbulence.

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What is Turbulence? – Cont.

- **Dissipative**. Kinetic energy gets converted into heat due to viscous shear stresses. Turbulent flows die out quickly when no energy is supplied. Random motions that have insignificant viscous losses, such as random sound waves, are not turbulent.
- A **continuum** phenomenon. Even the smallest eddies are significantly larger than the molecular scales. Turbulence is governed by the equations of fluid mechanics.
- A **feature of fluid flow**, not of the fluid. When the Reynolds number is high enough, most of the dynamics of turbulence are the same whether the fluid is a liquid or a gas. Most of the dynamics are then independent of the properties of the fluid.

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Is the Flow Turbulent

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External flows:

$Re_x \geq 5\times 10^5$ along a surface

$Re_D \geq 20{,}000$ around an obstacle

Internal flows:

$Re_{D_h} \geq \sim 2{,}200$

where $Re_L \equiv \dfrac{\rho U L}{\mu}$

$L = x, D, D_h$, etc.

Other factors such as free-stream turbulence, surface conditions, and disturbances may cause earlier transition to turbulent flow.

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Turbulence Modeling

- Develop equations that predict the time averaged velocity, pressure, and temperature fields without calculating the complete turbulent flow pattern as a function of time.
  - This saves us a lot of work!
  - Most of the time it is all we need to know.
  - We may also calculate other statistical properties, such as RMS values.

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Turbulence Modeling

- Important to understand: the time averaged flow pattern is a statistical property of the flow.
  - It is not an existing flow pattern!
  - It does not usually satisfy the steady Navier-Stokes equations!
  - The flow never actually looks that way!

Flow Variable Decomposition

- Flow property φ. The mean Φ is defined as:

$$\Phi = \frac{1}{\Delta t} \int_0^{\Delta t} \varphi(t)\, dt$$

Δt should be larger than the time scale of the slowest turbulent fluctuations.

Time dependence: $\varphi(t) = \Phi + \varphi'(t)$

Write shorthand as: $\varphi = \Phi + \varphi'$

$$\overline{\varphi}' = \frac{1}{\Delta t} \int_0^{\Delta t} \varphi'(t)\, dt = 0 \quad \text{by definition}$$

Information regarding the fluctuating part of the flow can be obtained from the root – mean - square (rms) of the fluctuations:

$$\varphi_{rms} = \sqrt{\overline{(\varphi')^2}} = \left[\frac{1}{\Delta t} \int_0^{\Delta t} (\varphi')^2\, dt \right]^{1/2}$$

Flow Variable Decomposition – Cont.

- Velocity and pressure decomposition:

$$\text{Velocity}: \ \mathbf{u} = \mathbf{U} + \mathbf{u}'$$

$$\text{Pressure}: \ p = P + p'$$

- Turbulent kinetic energy k (per unit mass) is defined as: $k = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$

$$\text{Turbulence intensity}: \ T_i = \frac{\left(\frac{2}{3}k\right)^{1/2}}{U_{ref}}$$

- Continuity equation:

$$div\ \mathbf{u} = 0; \ \ Time\ average: \overline{div\ \mathbf{u}} = div\ \mathbf{U} = 0$$

$$\Rightarrow \text{continuity equation for the mean flow}: \ div\ \mathbf{U} = 0$$

- Next step, time average the momentum equation. This results in the Reynolds equations..

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RANS Equations

- $$x - momentum: \quad \frac{\partial(\rho U)}{\partial t} + div(\rho U\mathbf{U}) = -\frac{\partial P}{\partial x} + div(\mu\ grad\ U) + S_{Mx} + \left[-\frac{\partial(\rho\overline{u'^2})}{\partial x} - \frac{\partial(\rho\overline{u'v'})}{\partial y} - \frac{\partial(\rho\overline{u'w'})}{\partial z}\right]$$

$$y - momentum: \quad \frac{\partial(\rho V)}{\partial t} + div(\rho V\mathbf{U}) = -\frac{\partial P}{\partial y} + div(\mu\ grad\ V) + S_{My} + \left[-\frac{\partial(\rho\overline{u'v'})}{\partial x} - \frac{\partial(\rho\overline{v'^2})}{\partial y} - \frac{\partial(\rho\overline{v'w'})}{\partial z}\right]$$

$$z - momentum: \quad \frac{\partial(\rho W)}{\partial t} + div(\rho W\mathbf{U}) = -\frac{\partial P}{\partial z} + div(\mu\ grad\ W) + S_{Mz} + \left[-\frac{\partial(\rho\overline{u'w'})}{\partial x} - \frac{\partial(\rho\overline{v'w'})}{\partial y} - \frac{\partial(\rho\overline{w'^2})}{\partial z}\right]$$

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Reynolds Stresses

- These equations contain an additional stress tensor – Reynolds stresses.

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} = \begin{pmatrix} -\rho\overline{u'^2} & -\rho\overline{u'v'} & -\rho\overline{u'w'} \\ -\rho\overline{u'v'} & -\rho\overline{v'^2} & -\rho\overline{v'w'} \\ -\rho\overline{u'w'} & -\rho\overline{v'w'} & -\rho\overline{w'^2} \end{pmatrix}$$

- In turbulent flow, the Reynolds stresses are usually large compared to the viscous stresses.
- The normal stresses are always non-zero because they contain squared velocity fluctuations. The shear stresses would be zero if the fluctuations were statistically independent. However, they are correlated and the shear stresses are therefore usually also non-zero.

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Other Equations

- Continuity: $\frac{\partial \rho}{\partial t} + div(\rho\,\mathbf{U}) = 0$
- Scalar transport equation:

$$\frac{\partial(\rho\Phi)}{\partial t} + div(\rho\Phi\mathbf{U}) = div(\Gamma_\Phi \; grad\ \Phi) + S_\Phi$$

$$+\left[-\frac{\partial(\rho\overline{u'\varphi'})}{\partial x} - \frac{\partial(\rho\overline{v'\varphi'})}{\partial y} - \frac{\partial(\rho\overline{w'\varphi'})}{\partial z}\right]$$

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Turbulence Models

- A turbulence model is a computational procedure to close the system of *mean flow* equations.
- For most engineering applications it is unnecessary to resolve the details of the turbulent fluctuations.
- Turbulence models allow the calculation of the mean flow without first calculating the full time-dependent flow field.
- We only need to know how turbulence affected the mean flow.

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Turbulence Models – Cont.

- In particular we need expressions for the Reynolds stresses.
- For a turbulence model to be useful it:
  - must have wide applicability,
  - be accurate,
  - simple,
  - and economical to run.

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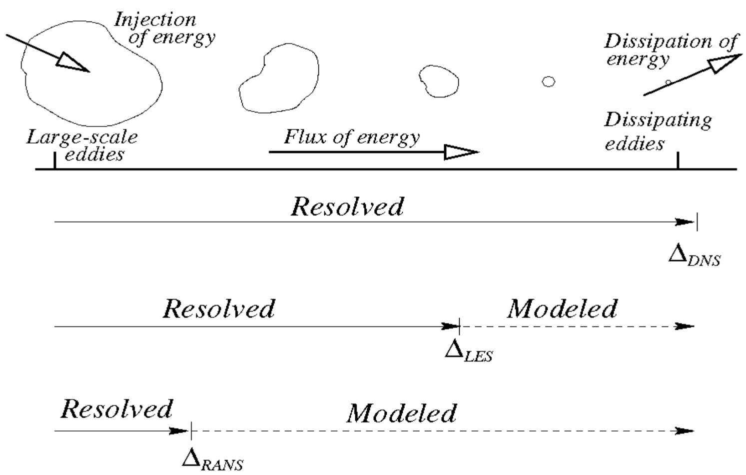
Common Models

- Classical models. Based on Reynolds Averaged Navier-Stokes (RANS) equations (time averaged):
  - 1. Zero equation model: mixing length model.
  - 2. One equation model: Spalart-Almaras.
  - 3. Two equation models: k-ε style models (standard, RNG, realizable), k-ω model, and ASM.
  - 4. Seven equation model: Reynolds stress model.

Common Models – Cont.

- The number of equations denotes the number of additional PDEs that are being solved.
- Large eddy simulation. Based on space-filtered equations. Time dependent calculations are performed. Large eddies are explicitly calculated. For small eddies, their effect on the flow pattern is taken into account with a “subgrid model” of which many styles are available.

Prediction Methods



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Boussinesq Hypothesis

- Many turbulence models are based upon the Boussinesq hypothesis.
  - Experimentally observed that turbulence decays unless there is shear in isothermal incompressible flows.
  - Turbulence was found to increase as the mean rate of deformation increases.
  - Boussinesq proposed in 1877 that the Reynolds stresses could be linked to the mean rate of deformation.

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Boussinesq Hypothesis – Cont.

- Using the suffix notation where i, j, and k denote the x-, y-, and z-directions respectively, viscous stresses are given by:

$$\tau_{ij} = \mu\, e_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

- Similarly, link Reynolds stresses to the mean rate of deformation:

$$\tau_{ij} = -\rho\overline{u_i' u_j'} = \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

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Turbulent Viscosity

- A new quantity appears: the turbulent viscosity μ_t.

$$\tau_{ij} = -\rho\overline{u_i' u_j'} = \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

- Its unit is the same as that of the molecular viscosity: Pa.s.
- It is also called the eddy viscosity.

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Turbulent Viscosity – Cont.

- We can also define a kinematic turbulent viscosity: $\nu_t = \mu_t/\rho$. Its unit is m^2/s.
- The turbulent viscosity is not homogeneous, i.e., it varies in space.
- It is, however, assumed to be isotropic. It is the same in all directions. This assumption is valid for many flows, but not for all (e.g. flows with strong separation or swirl).

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Predicting Turbulent Viscosity

- The following models can be used to predict the turbulent viscosity:
  - Mixing length model
  - Spalart-Allmaras model
  - Standard k-ε model
  - k-ε RNG model
  - Realizable k-ε model
  - k-ω model

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Mixing Length Model

- On dimensional grounds, one can express the kinematic turbulent viscosity (m^2/s) as the product of a velocity scale (m/s) and a length scale (m)
- If we then assume that the velocity scale is proportional to the length scale and the gradients in the velocity (shear rate, which has dimension 1/s):

$$\vartheta \propto \ell \left| \frac{\partial U}{\partial y} \right|$$

we can derive Prandtl's (1925) mixing length model:

$$\nu_t = \ell_m^2 \left| \frac{\partial U}{\partial y} \right|$$

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Mixing Length Model Discussion

- Advantages:
  - Easy to implement.
  - Fast calculation times.
  - Good predictions for simple flows where experimental correlations for the mixing length exist.
- Disadvantages:
  - Completely incapable of describing flows where the turbulent length scale varies: anything with separation or circulation.
  - Only calculates mean flow properties and turbulent shear stress.
- Ignored in commercial CFD programs today.

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Spalart-Allmaras Model Discussion

- Solves a single conservation equation (PDE) for the turbulent viscosity:
  - This conservation equation contains convective and diffusive transport terms, as well as expressions for the production and dissipation of ν_t.
- Economical and accurate for:
  - Attached wall-bounded flows.
  - Flows with mild separation and recirculation.
- Weak for:
  - Massively separated flows.
  - Free shear flows.
  - Decaying turbulence.

k-ε Model Discussion

- Advantages:
  - Relatively simple to implement.
  - Leads to stable calculations that converge relatively easily.
  - Reasonable predictions for many flows.
- Disadvantages:
  - Poor predictions for:
    - swirling and rotating flows, flows with strong separation, axisymmetric jets, certain unconfined flows, and fully developed flows in non-circular ducts.
  - Valid only for fully turbulent flows.
  - Simplistic ε equation.

RNG k-ε Model Discussion

- Derived from the application of a rigorous statistical technique (Renormalization Group Method) to the instantaneous Navier-Stokes equations.
- Similar in form to the standard *k*-ε equations but includes:
  - Additional term in ε equation for interaction between turbulence dissipation and mean shear.
  - The effect of swirl on turbulence.
  - Analytical formula for turbulent Prandtl number.
  - Differential formula for effective viscosity.
- Improved predictions for:
  - High streamline curvature and strain rate.
  - Transitional flows.
  - Wall heat and mass transfer.
- But still does not predict the spreading of a round jet correctly.

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Realizable k-ε Model Discussion

- Shares the same turbulent kinetic energy equation as the standard *k*-ε model.
- Improved equation for ε.
- Variable C_μ instead of constant.
- Improved performance for flows involving:
  - Planar and round jets (predicts round jet spreading correctly).
  - Boundary layers under strong adverse pressure gradients or separation.
  - Rotation, recirculation.
  - Strong streamline curvature.

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k-ω Model Discussion

- In this model ω is an inverse time scale that is associated with the turbulence.
- Solves two additional PDEs:
  - A modified version of the *k* equation used in the *k*-ε model.
  - A transport equation for ω.
- The turbulent viscosity is then calculated as follows:

$$\mu_t = \rho \frac{k}{\omega}$$

- Its numerical behavior is similar to that of the *k*-ε models.
- It suffers from some of the same drawbacks, such as the assumption that μ_t is isotropic.

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Reynolds Stress Model Discussion

- RSM closes the Reynolds-Averaged Navier-Stokes equations by solving additional transport equations for the six independent Reynolds stresses.
  - Closure also requires one equation for turbulent dissipation.
  - Isotropic eddy viscosity assumption is avoided.
- Resulting equations contain terms that need to be modeled.
- RSM is good for accurately predicting complex flows.
  - Accounts for streamline curvature, swirl, rotation and high strain rates.

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Comparison RANS Turbulence Models

Model	Strengths	Weaknesses
Spalart-Allmaras	Economical (1-eq.); good track record for mildly complex B.L. type of flows.	Not very widely tested yet; lack of submodels (e.g. combustion, buoyancy).
STD k-ε	Robust, economical, reasonably accurate; long accumulated performance data.	Mediocre results for complex flows with severe pressure gradients, strong streamline curvature, swirl and rotation. Predicts that round jets spread 15% faster than planar jets whereas in actuality they spread 15% slower.
RNG k-ε	Good for moderately complex behavior like jet impingement, separating flows, swirling flows, and secondary flows.	Subjected to limitations due to isotropic eddy viscosity assumption. Same problem with round jets as standard k-ε.
Realizable k-ε	Offers largely the same benefits as RNG but also resolves the round-jet anomaly.	Subjected to limitations due to isotropic eddy viscosity assumption.
Reynolds Stress Model	Physically most complete model (history, transport, and anisotropy of turbulent stresses are all accounted for).	Requires more cpu effort (2-3x); tightly coupled momentum and turbulence equations.

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Recommendation

- Start calculations by performing 100 iterations or so with standard *k*-ε model and first order upwind differencing. For very simple flows (no swirl or separation) converge with second order upwind and *k*-ε model.
- If the flow involves jets, separation, or moderate swirl, converge solution with the realizable *k*-ε model and second order differencing.
- If the flow is dominated by swirl (e.g. a cyclone or unbaffled stirred vessel) converge solution deeply using RSM and a second order differencing scheme. If the solution will not converge, use first order differencing instead.
- Ignore the existence of mixing length models and the algebraic stress model.
- Only use the other models if you know from other sources that somehow these are especially suitable for your particular problem (e.g. Spalart-Allmaras for certain external flows, *k*-ε RNG for certain transitional flows, or *k*-ω).

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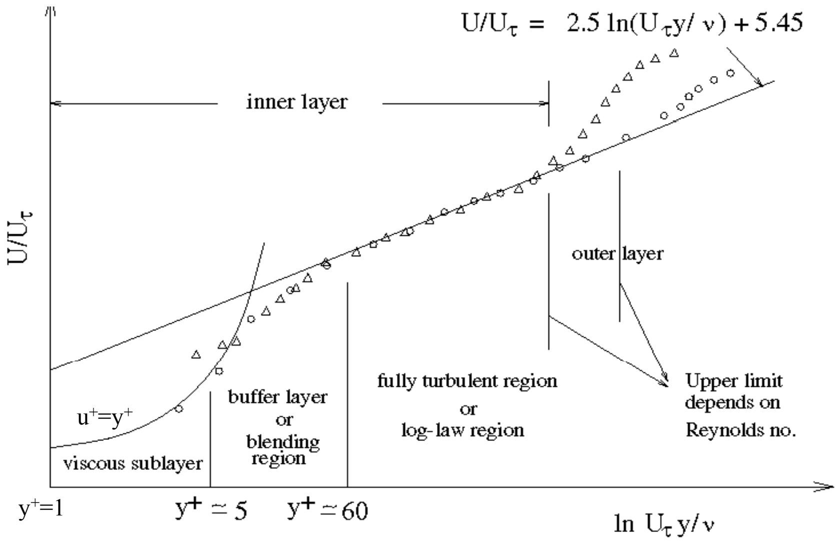
Turbulent Boundary Layer

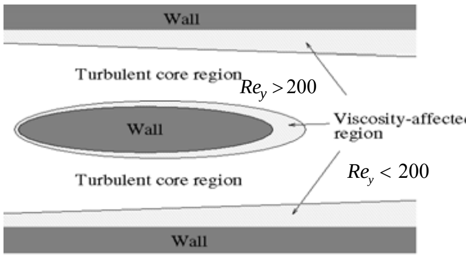


Wall Function Approach

- The objective is to take the effects of the boundary layer correctly into account without having to use a mesh that is so fine that the flow pattern in the layer can be calculated explicitly.
- Using the no-slip boundary condition at wall, velocities at the nodes at the wall equal those of the wall.
- The shear stress in the cell adjacent to the wall is calculated using the correlations shown in the previous slide.
- This allows the first grid point to be placed away from the wall, typically at $50 < y^+ < 500$, and the flow in the viscous sublayer and buffer layer does not have to be resolved.
- This approach is called the "standard wall function" approach.
- Improvements, "non-equilibrium wall functions," are available that can give improved predictions for flows with strong separation and large adverse pressure gradients..

Two-Layer Zonal Approach

- A disadvantage of the wall-function approach is that it relies on empirical correlations.
- The two-layer zonal model does not. It is used for low-Re flows or flows with complex near-wall phenomena.
- Zones distinguished by a wall-distance-based turbulent Reynolds number:



- The flow pattern in the boundary layer is calculated explicitly.
- Regular turbulence models are used in the turbulent core region.
- Only *k* equation is solved in the viscosity-affected region.
- ε is computed using a correlation for the turbulent length scale.
- Zoning is dynamic and solution adaptive.

Computational Grid Guidelines

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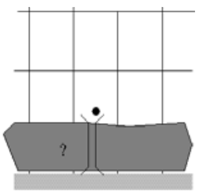
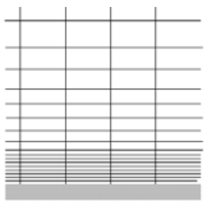
Wall Function Approach	Two-Layer Zonal Model Approach





- First grid point in log-law region:

$$50 \le y^+ \le 500$$

- Gradual expansion in cell size away from the wall.
- Better to use stretched quad/hex cells for economy.

- First grid point at $y^+ \approx 1$.
- At least ten grid points within buffer and sublayers.
- Better to use stretched quad/hex cells for economy.

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Comparison of Near-Wall Treatment

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Approach	Strengths	Weaknesses
Standard wall-functions	Robust, economical, reasonably accurate	Empirically based on simple high- Re flows; poor for low-Re effects, massive transpiration, PGs, strong body forces, highly 3D flows
Non-equilibrium wall-functions	Accounts for pressure gradient (PG) effects. Improved predictions for separation, reattachment, impingement	Poor for low-Re effects, massive transpiration (blowing, suction), severe PGs, strong body forces, highly 3D flows
Two-layer zonal model	Does not rely on empirical law-of-the-wall relations, good for complex flows, applicable to low-Re flows	Requires finer mesh resolution and therefore larger cpu and memory resources

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Large Eddy Simulation

- LES is midway between DNS and RANS in terms of:
  - Rigor.
  - Computational requirement.
- Spectrum of turbulent eddies in the Navier-Stokes equations is “filtered”:
  - The filter is a function of the grid size.
  - Small eddies are removed, and modeled using a *subgrid-scale (SGS)* model.
  - Large eddies are retained, and solved for directly using a transient calculation.

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Prediction Methods