Surface in topology and its application

- Topology: the branch of geometry that deals with the properties of a figure that remain unchanged even when the figure is bent, stretched, or otherwise distorted

- Topology in ESRI software: (1) the arrangement that constrains how point, line, and polygon features share geometry (2) the spatial relationship between connecting or adjacent features in a geographic data layer

(from A to Z GIS, Wade and Sommer (eds.), 2006, ESRI Press)

Surface in topology and its application

- Terrain: a smooth, doubly continuous function of the form $z = f(x,y)$, where z is the height associated with equal point (x,y)

- Surface data structure: a format storing the geometric and topological information (e.g. point heights and adjacency relationships) in a single construction

- Surface data model: an extended version of the surface data structure in which additional metadata information characterizing the surface (e.g. valleys, ridges, i.e. characteristic properties of surface) is also incorporated to produce a representation of the surface. (simple term) value-added product of the surface data structure

Surface in topology and its application

- Simplification of surface using topology: use of MIPs (Morphologically Important Points), e.g. corners

- Surface network (Pfaltz, 1976), Reeb graph (Reeb, 1946), Contour tree (Morse, 1968): critical points + lines (Figs. in p.34,40,56 Rana,2004)

- Critical points: have local zero slope ($dz/dx = dz/dy = 0$)
- - peaks (local maxima, $\partial^2 z/\partial x^2 > 0$, $\partial^2 z/\partial y^2 > 0$)
- - pits (local minima, $\partial^2 z/\partial x^2 < 0$, $\partial^2 z/\partial y^2 < 0$)
- - passes (=saddles, $\partial^2 z/\partial x^2 > 0$, $\partial^2 z/\partial y^2 < 0$ or $\partial^2 z/\partial x^2 < 0$, $\partial^2 z/\partial y^2 > 0$)
- - Mountaineer’s eqn. or Euler-Poincaré formula (Griffiths, 1981)
- peaks + pits – passes = 2

- Critical lines: special pair of slope lines that originate and terminate at critical points
- -ridge line: originates from a peak and terminates at a pass
- -channel line: originates from a pass and terminates at a pit

Surface in topology and its application

- Advantages of using MIPs to represent surface
- (1) saving memory: e.g. 90% in volumetric surface data
- (2) much more efficient way of accessing a spatial database of the surface, esp. good for clustering
- (3) provide unified representation of the global structure , e.g. good for erosion modeling
- (4) useful for the visualization of the structure of the surface, esp. (i) for multi-dimensional surfaces, and (ii) rendering of surface

Surface in topology and its application

- MIPs (Morphologically Important Points)

- Landform elements
- Critical points and lines
- Surface-specific features
- Symbolic surface features
- Surface patches
- Specific geomorphological elements

Surface in topology and its application

- Surface network: topographic surface represented with critical points and critical lines (Fig.2.1, Rana, 2004)

- Graph with Edge weights (height differences) (Fig.2.2, Rana, 2004): Weighted surface networks
- Po: the set of all pits
- P1: the set of all passes
- P2: the set of all peaks

- e.g. 1 surrounding pit, 5 passes, 6 peaks in Fig.2.1

Surface in topology and its application: Surface network Generalization

- I (Importnace): maximum of differences in altitude between a peak (pit) and all of its adjacent saddles, shown in ascending order e.g. z4(450), x1(550), z2(850), z1(950), z6(950), z3(1150), Z5(1250)
- *other I used: minimum or sum of differences

- Select the pit x^o or peak z^o whose importance is minimal
- Find the highest adjacent saddle y^o
- Apply (x^o , y^o)-w-contraction or (y^o , z^o)-w-contraction
- Repeat the above process if needed
- e.g. generalized map of Fig.2.1: z4, y3 (Fig.2.4)
- Other form of disply: Fig.2.5 and Fig.2.6

Surface in topology and its application: Reeb graph

- Reeb graph: one of the critical point graphs and represents the topological transitions in cross-sectional contours as the corresponding height value changes (Fig.3.8, Rana,2004)

- Advantage of Reeb graph: good display of the height relationship

- Algorithm for converting the surface network to the Reeb graph (Fig.3.12, Rana, 2004)

Surface in topology and its application: contour tree

Contour tree of a scalar field is the graph obtained by contracting all the connected components of the level sets of the field into points

-A powerful abstraction for representing the structure of the field with explicit description of the topological changes of its level sets

- proven effective as a data-structure for fast extraction of isosurfaces

Surface in topology and its application: contour tree

Example of algorithm: (1) Top down (Join tree)

Surface in topology and its application: contour tree

Example of algorithm: (2) Bottom up (Split tree)

Surface in topology and its application:

- Application

- -Structural similarity for surfaces
- -Structural similarity for urban population surfaces (Chapter 7, Rana, 2004, esp. p.118)
- -Visualization (Chapters 8, 9, 10, Rana, 2004)
- -Analysis of surface texture (Chapter 11, Rana, 2004)
- -Fast analysis of visibility (Chapter 12, Rana, 2004)