(4) Magnetostatics

(a) Magnetostatic Effects

- Dependence of magnetization and generated field on the shape (or aspect ratio) of a permanent magnet (see Fig. 2.1-2 in O'Handley)
- Deferent ***B-H*** response for a ring or toroid of a magnet (see Fig. 2.3 in O'Handley)
  Why? Due to the existence of surface free poles

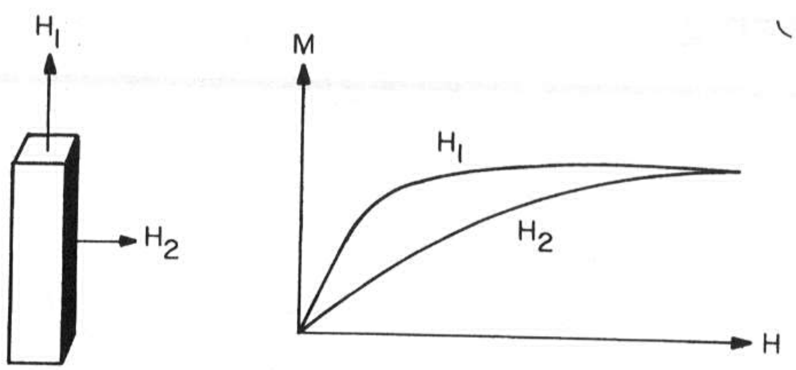


Figure 2.1 Magnetization curves for a polycrystalline ferromagnetic sample with field applied in different directions.

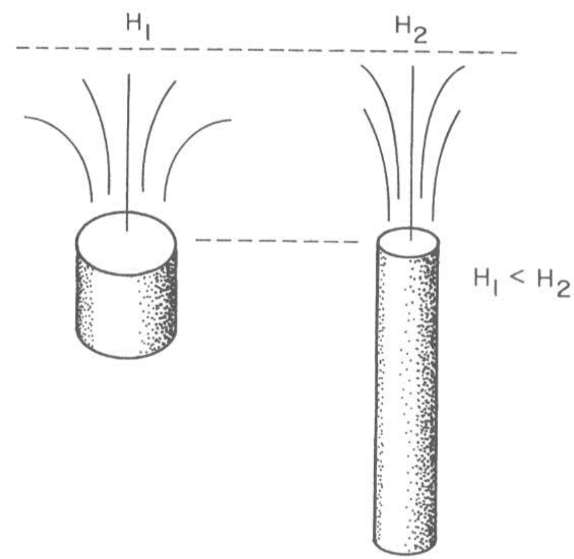


Figure 2.2 Dipole fields at a fixed distance from the ends of two permanent magnets of the same volume but different shapes.

(4) Magnetostatics

(b) Demagnetizing Field, H_d and Factors, N_d

▶ Origin for the demagnetizing field: The field from the surface poles that passes through the interior on the sample

- Relation between the field strength and orientation: Solution of Maxwell's equations

  Boundary conditions

  Flux density $\boldsymbol{B}$: (Gauss's theorem)

  Field Intensity $\boldsymbol{H}$: ($\nabla \times \boldsymbol{H} = \boldsymbol{J}$ & Stokes theorem)

  Results : $\boldsymbol{B}$ is always continuous. However, the tangential component of $\boldsymbol{H}$ is discontinuous by a transverse surface current.

- Demagnetizing fiend $\boldsymbol{H}_d$ and factors N_d

$$\boldsymbol{H} = \boldsymbol{H}_a + \boldsymbol{H}_d$$

$$\boldsymbol{H}_d = -N_d\boldsymbol{M}$$

(4) Magnetostatics

(b) Demagnetizing Field and Factors (continued)

- Quantitative interpretation of $\boldsymbol{B}$-$\boldsymbol{H_a}$ loops (see Fig. 2.9 in O'Handley)

Since $\boldsymbol{B_i} = \mu_o(\boldsymbol{H_i} + \boldsymbol{M})$, where $\boldsymbol{B_i}$ = the induction inside the material, $\boldsymbol{H_i}$ = internal field,

and $\boldsymbol{H_i} = \boldsymbol{H_a} - N_d\boldsymbol{M}$, $\quad \boldsymbol{B_i}/\mu_o = \boldsymbol{H_a}/N_d - (1 - N_d)\boldsymbol{H_i}/N_d$

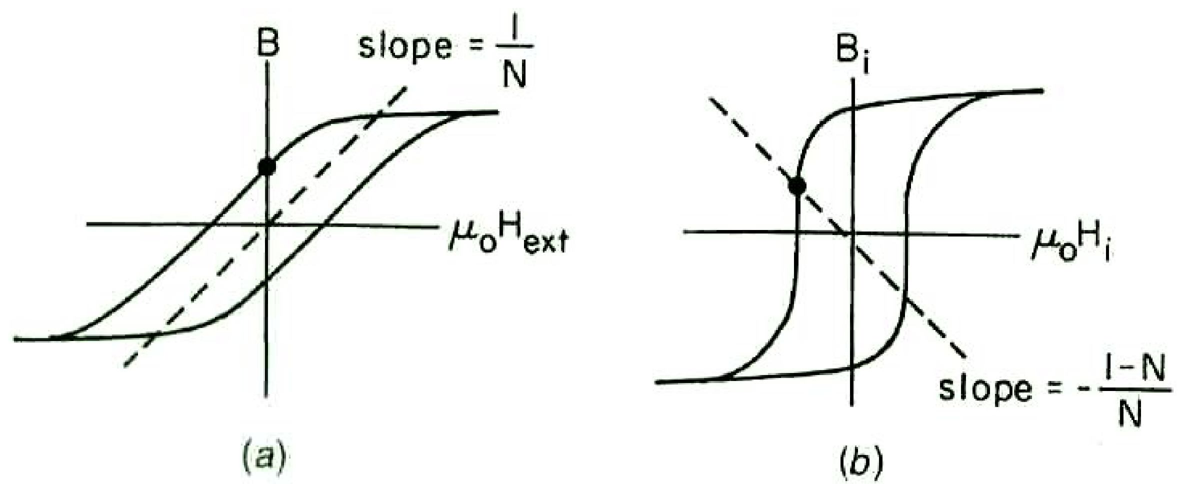


Figure 2.9 Schematic representation of demagnetization effect on B-H_{appl} loops (a) and on $B-H_i$ loops (b). The dashed lines rotated into the vertical axis in each case relate one loop to the other.

(4) Magnetostatics

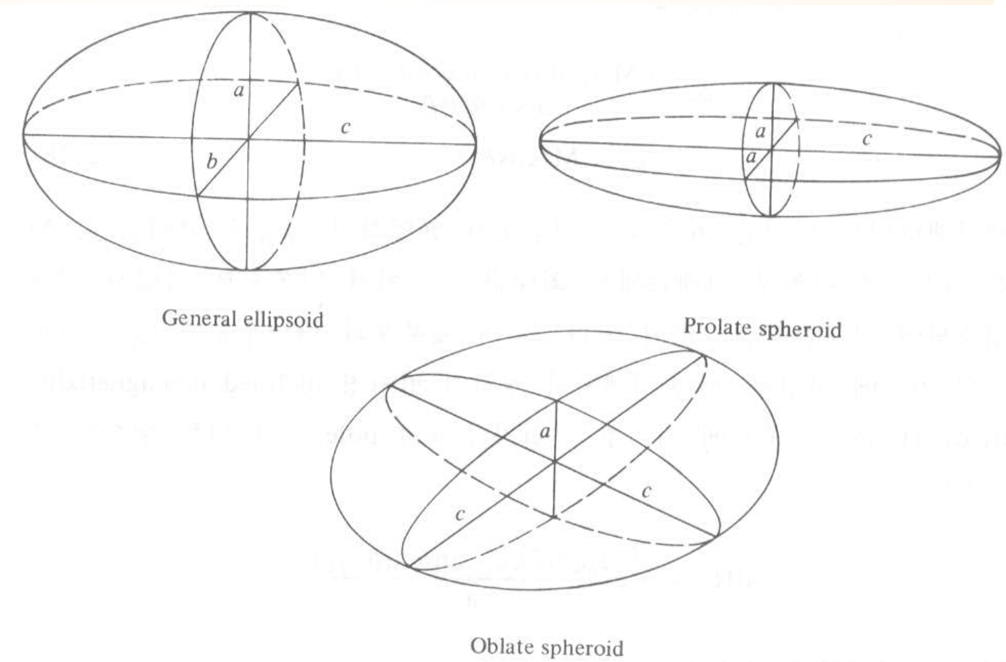
- Calculation of **demagnetization factors *N*** for various ellipsoids (see Fig.2.27 in Cullity)

For a prolate, $a = b < c$, $N_a = N_b = (4\pi - N_c)/2$, $N_c \rightarrow 0$ as $c/a \rightarrow \infty$

For a oblate ellipsoid, $a < b = c$, $N_b = N_c = (4\pi - N_a)/2$, $N_c \rightarrow 4\pi$ as $c/a \rightarrow \infty$

For a more general ellipsoid, $a >> b, c$

$$N_a \sim \left(\frac{bc}{a^2}\right)\left\{\ln\left(\frac{4a}{b+c}\right) - 1\right\}$$



(4) Magnetostatics

- Demagnetization factors for ellipsoids and cylinders

(see Table 2.1 Fig. 2.10 in O'Handley, Fig. 2.28 in Cullity)

TABLE 2.1 Demagnetizing Factors *N* for Rods and Ellipsoids Magnetized Parallel to Long Axis

Dimensional Ratio (Length/Diameter)	Rod	Prolate Ellipsoid	Oblate Ellipsoid
0	1.0	1.0	1.0
1	0.27	0.3333	0.3333
1.5	—	0.233	0.329
2	0.14	0.1735	0.2364
5	0.040	0.558	0.1248
10	0.0172	0.0203	0.0696
20	0.00617	0.00675	0.0369
50	0.00129	0.00144	0.01472
100	0.00036	0.000430	0.00776
200	0.00090	0.000125	0.00390
500	0.000014	0.0000236	0.001567
1000	0.0000036	0.0000066	0.000784
2000	0.0000009	0.0000019	0.000392

Source: Bozorth, IEEE Press, 1993, p. 849.

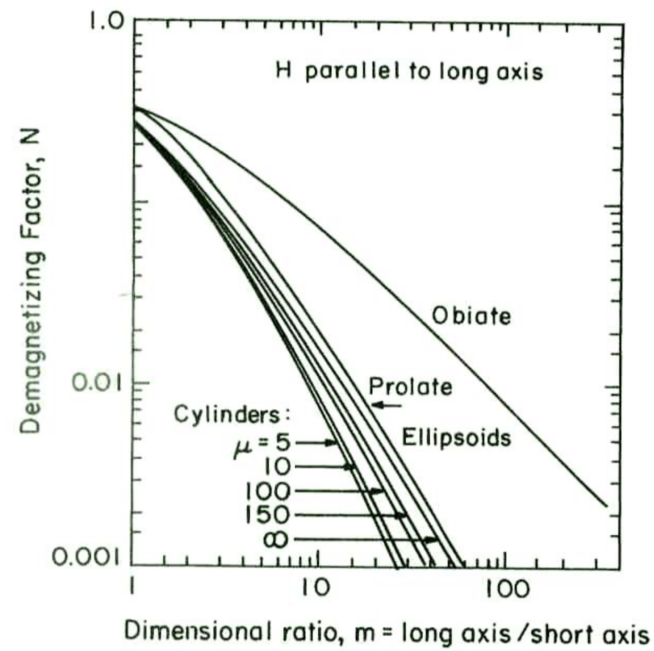


Figure 2.10 Demagnetization factors for ellipsoids and cylinders with field applied parallel to long axis, with aspect ratios closer to unity. [Bozorth, © IEEE Press (1993)].

(4) Magnetostatics

(c) Magnetization Curves

Calculation of the $\boldsymbol{M}(\boldsymbol{H})$ curve for magnetizing a single-domain sample

i) $\boldsymbol{H}$ in a hard direction (against its demagnetizing field) using a known N_d

Total magnetostatic energy density, $u = u_{ms} + u_H$

- Magnetostatic energy density u_{ms}

$$u_{ms} = -\,\boldsymbol{M}\bullet\boldsymbol{H_d} = -\,MH_d\cos\theta = +(N_d/2)M_s^2\cos^2\theta$$

- The Zeeman energy density u_H

$$u_H = -\,\boldsymbol{M_s}\bullet\boldsymbol{H} = -\,M_sH\cos\theta$$

Minimum total energy when $du/d\theta = 0$

$$du/d\theta = -\,(N_dM_s^2\cos\theta - M_sH)\sin\theta$$

Solutions

For $H > H_s$ $\sin\theta = 0$, $\theta = 0$

For $H < H_s$ $M(H) = M_s\cos\theta = H/N_d$

since $N_dM_s\cos\theta = H$

Stability condition, $u'' > 0$

$$u'' = -\,N_dM_s^2\cos^2\theta - M_sH\cos\theta > 0$$

Applications (see Fig 2.12-13 in O'Handley)

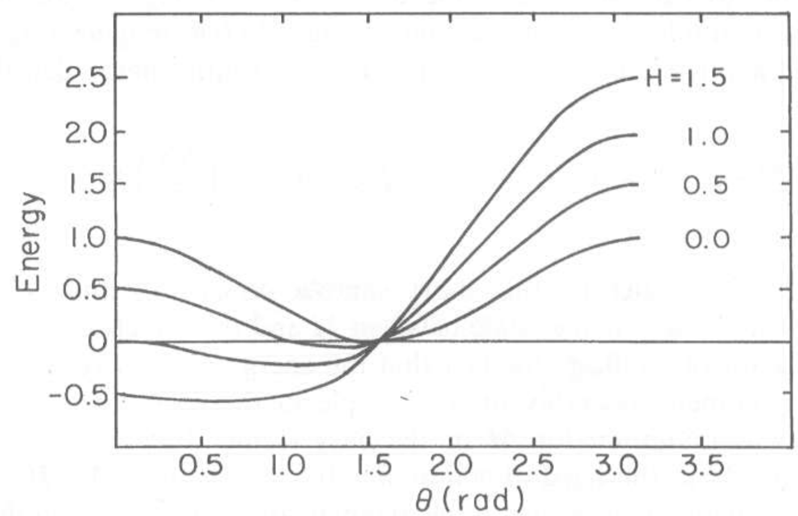


Figure 2.12 Variation of magnetostatic plus Zeeman energy density with θ for increasing values of applied field (arbitrary units). Note how the stable energy minimum moves from $\pi/2$ toward zero as applied field increases.

(4) Magnetostatics

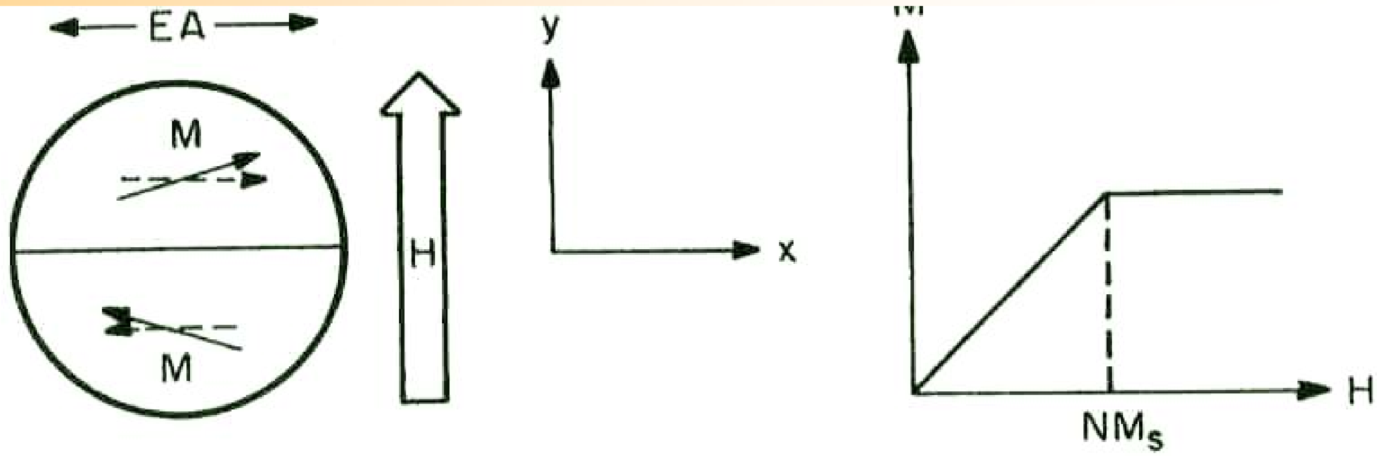


Figure 2.13 Schematic representation of a magnetic material having purely uniaxial anisotropy in the direction of the easy axis (EA). Dashed lines indicate magnetization configurations for $H = 0$. Application of a field H transverse to the EA results in rotation of the domain magnetizations but no wall motion.

(4) Magnetostatics

(c) Magnetization Curves

ii) ***H*** along the easy axis of a multidomain sample (see Fig. 2.14-15 in O'Handley)

Apparent (or effective) susceptibility, χ_e

$$\boldsymbol{M} = \chi \boldsymbol{H_i} = \chi(H_a - N_d M)$$

$$\chi_e = M/H_a = \chi/(1 + N_d)$$

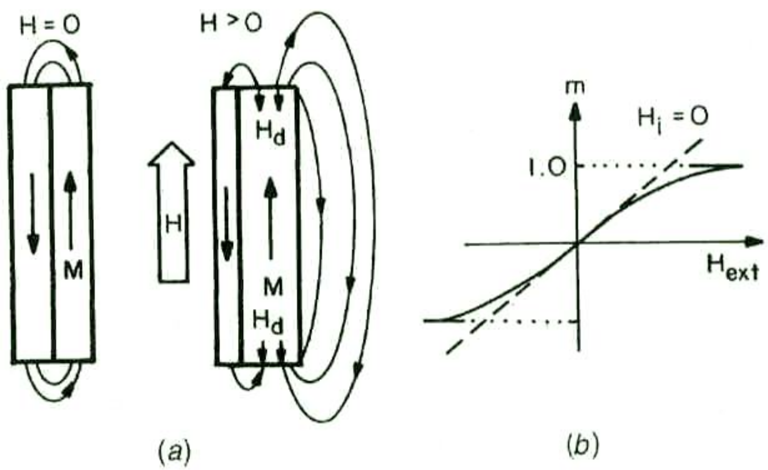


Figure 2.14 (*a*) A demagnetized sample for which shape is a factor responds to an applied field at the cost of increased demagnetization factor and increased magnetostatic energy; (*b*), the changing demagnetization factor causes the $M-H$ loop to be less than linear in the external field.

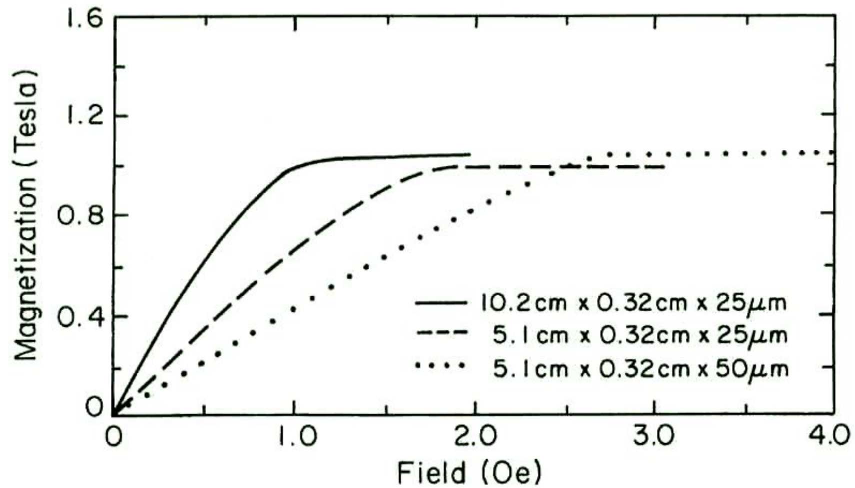


Figure 2.15 Magnetization for transversely annealed amorphous alloy ribbons of various aspect ratios. [After Clark and Wun-Fogel, 1989.]

(4) Magnetostatics

(d) Magnetostatic Energy and Thermodynamics

- The potential energy U of interaction of a magnetic dipole μ_m with an external field $\boldsymbol{B} = \mu_o \boldsymbol{H}$,

$$U = -\,\mu_m \bullet \boldsymbol{B}$$

Extension to a rigid assembly of dipoles (*i.e.*, a macroscopic sample)

Potential energy u per unit volume for magnetization $\boldsymbol{M}$,

$$u = -\,\boldsymbol{M} \bullet \boldsymbol{B}$$

When $\boldsymbol{H}_a = 0$, $\boldsymbol{H} = \boldsymbol{H}_d$,

$$u = -(1/2)\mu_o \boldsymbol{M} \bullet \boldsymbol{H}_d = (\mu_o/2) N_d M^2 \qquad \text{(in cgs, } 2\pi N_d M^2\text{)}$$

This energy represents the work done in assembling a given state of magnetization in a sample.

For an infinite sheet magnetized perpendicular to its plane,

$$u = (\mu_o/2)M^2 \;(= 2\pi M^2 \text{ in cgs) since } N_d = 1 \text{ (}4\pi \text{ in cgs)}$$

(4) Magnetostatics

(d) Magnetostatic Energy and Thermodynamics (continued)

- Consider the magnetization process from the demagnetized state to any point (M_1, B_1).

Energy densities can be defined in three different ways;

$u = -\mathbf{M} \bullet \mathbf{B} = -M_1 B_1$: the potential energy

$A_1 = \int_0^{M1} B(M)dM$: the work done *on the sample by the field*

$A_2 = \int_0^{B1} M(B)dB$: the work done *by the sample*

The internal energy of a magnetic sample is decreased in the presence of a field; the magnetic sample can do work when exposed to a magnetic field.

If a sample is already magnetized in the absence of a field (*i.e.*, $A_1 = 0$) and properly aligned,

$$u = -M_1 B_1 = -A_2$$

(4) Magnetostatics

(d) Magnetostatic Energy and Thermodynamics (continued)

The internal energy = its potential energy + the work done on the sample

$$u = -M_1 B_1 + A_1 = -A_2 = -\int_0^{B1} M(B)dB = -\mu_o \int_0^{H1} M(H)dH$$

The second law of thermodynamics

$$dQ = TdS = dU + PdV$$

where, dQ = an amount of heat added to a system from the environment,

For a magnetic material, $TdS = dU + PdV - \mu_o HVdM \rightarrow dU = TdS - PdV + \mu_o HVdM$

The internal energy of the material increases as it is magnetized by a field (A_1).

When T, V, and M are the independent variables,

Helmholtz free energy, $F = U - TS$

$$dF = -SdT - PdV + \mu_o HVdM$$

When T, P, and H are the independent variables,

Gibbs free energy, $G = F + PV - \mu_o MHV$

$$dG = -SdT + VdP - \mu_o HVdM$$

Gibbs free energy decreases when a sample of magnetization M is placed in a field (A_2).

(e) Analytic Magnetostatics

i) Magnetostatic Potential

From Maxwell equations,

$$\nabla \times \boldsymbol{B} = 0 \text{ if } \boldsymbol{J} = 0 \text{ (no macroscopic current),}$$

Then $\boldsymbol{B} = -\nabla \phi_{m'}$ or $\boldsymbol{H} = -\nabla \phi_m$

Since $\nabla \bullet \boldsymbol{B} = \mu_o \nabla \bullet (\boldsymbol{H} + \boldsymbol{M}) = 0$,

$$\nabla^2 \phi_m = \nabla \bullet \boldsymbol{M} \text{ (Poisson's equation)}$$

Where $\nabla \bullet \boldsymbol{M}$ defines a volume magnetic charge density ρ_m (associated with a divergence of magnetization)

Thus, the scalar magnetic potential is given by $\phi_m(r) = -(1/4\pi) \iiint \frac{\rho_m}{r - r'} dV'$

In regions of no magnetic charges present, $\nabla^2 \phi_m = 0$ (Laplace's equation)

(4) Magnetostatics

(e) Analytic Magnetostatics

i) Magnetostatic Potential (continued)

Since surface charges ***M•n*** (the sources of demagnetization fields) also contribute to the magnetic potential,

$$H(r) = -\nabla_r \phi_m(r, r') = -(1/4\pi)\iiint d^3r' \nabla M(r') \frac{(r-r')}{|r-r'|^3} + (1/4\pi)\iint d_2 r' \mathbf{n}' \bullet \mathbf{M} \frac{(r-r')}{|r-r'|^3}$$

In 2-D, the field components parallel and perpendicular to the charged surface

$$H_{//} = (\sigma/2\pi)\ln(r_2/r_1),\ H_{\perp} = (\sigma/2\pi)\Theta \iiint \frac{\rho_m}{r-r'} dV'$$

where, σ(= ***M•n***) is the magnetic pole strength per unit surface area

The energy of a given pole distribution, $U_{m,} = \iiint \rho_m \phi_m dV$

(4) Magnetostatics

(e) Analytic Magnetostatics (continued)

ii) Applications of Laplace equation : $\nabla^2 \phi_m = 0$

- Uniformly Magnetized Sphere

Inside the sphere,

$\boldsymbol{B}^{\mathbf{in}} = (2\mu_o/3)\boldsymbol{M}$, (or $8\pi\boldsymbol{M}/3$ in cgs)

Since $\boldsymbol{B} = \mu_o(\boldsymbol{H} + \boldsymbol{M})$ (or $\boldsymbol{B} = \boldsymbol{H} + 4\pi\boldsymbol{M}$ in cgs)

Thus, $\boldsymbol{H}_{\boldsymbol{in}} = \boldsymbol{H}_{\boldsymbol{d}} = -\dfrac{M}{3}$ (or $-\dfrac{4\pi M}{3}$), where the demagnetizing factor $N_d = 1/3$ (or $4\pi/3$ in cgs)

Outside the sphere,

$\boldsymbol{H}^{\mathbf{out}} = (a^3\boldsymbol{M}/3)(2\cos\theta e_r + \sin\theta e_\theta)/r^3$

A dipole field for a magnetic moment $\mu_m = a^3M/3$ (see Fig. 2.19 in O'Handley)

- Field Due to Periodic Surface Poles (see Fig. 2.20 in O'Handley)

The magnetostatic energy per unit surface area, $U = 2.13M_s^2d/\pi^2$

(4) Magnetostatics

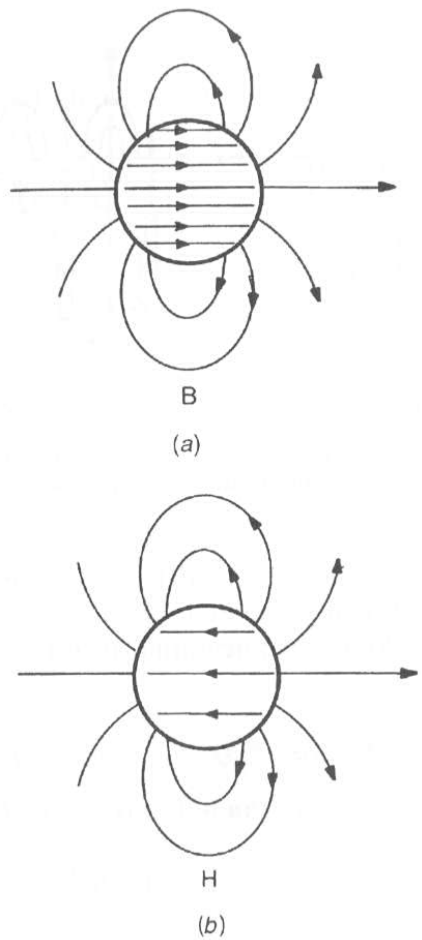


Figure 2.19 Fields inside and outside a uniformly magnetized sphere: (*a*) ***B*** field whose lines form continuous loops inside and outside the material; (*b*) ***H*** field, whose lines are not continuous; some may terminate at the surface poles.

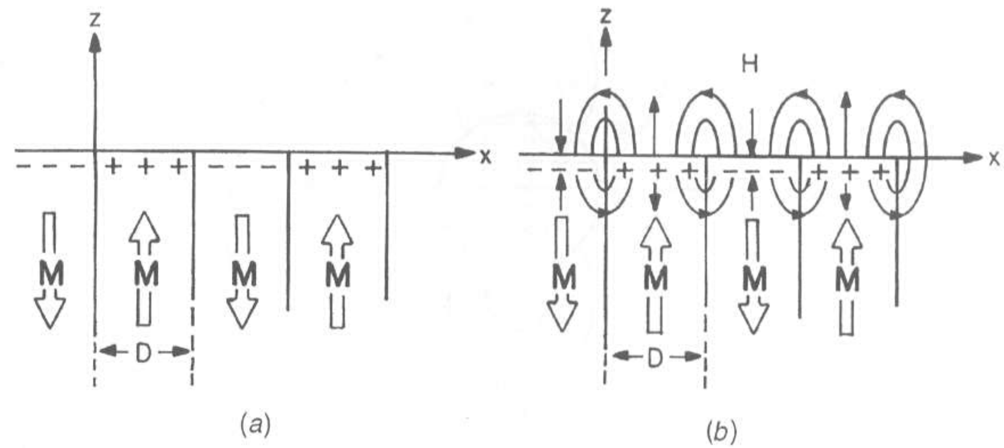


Figure 2.20 (*a*), Schematic of cross section of a semi-infinite sample with a periodic domain structure; (*b*), field distribution due to magnetic surface charges.