II. Types of Magnetism

(1) Diamagnetism: no net atomic magnetic moment

(2) Paramagnetism: non-zero net atomic magnetic moment, disordered

(3) Ferromagnetism: non-zero net atomic magnetic moment, ordered (parallel)

(4) Antiferromagnetism: non-zero net atomic magnetic moment, ordered (antiparallel)

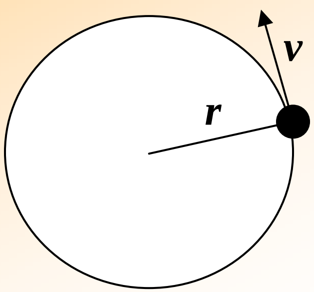
(5) Ferrimagnetism: non-zero net atomic magnetic moment, ordered (antiparallel)

Introduction

▶ Origin of net atomic magnetic moment?

- **electron motion**
  - orbital motion → electronic moment
  - spin motion
- **nulear motion** → nuclear moment : normally, negligibly small

Orbit motion : Bohr model



Electron (charge = e$^-$, mass m_e)

Current, i = $\dfrac{\text{charge passing a given point}}{\text{unit time}} = \dfrac{\dfrac{e}{2\pi r c}}{\upsilon} = \dfrac{e\upsilon}{2\pi r c}$

An additional postulate, $m\upsilon r = n(h/2\pi)$ (quantized angular momentum)

Then, magnetic moment due to orbital motion, $\mu_{\text{orbit}} = iA = \dfrac{e\upsilon}{2\pi r c}\pi r^2 = \dfrac{er\upsilon}{2c} = \dfrac{eh}{4\pi mc}$ for n =1

Spin motion :

1925 yr. fine split in the optical spectrum under a magnetic field :

Anomalous Zeeman Effect

Magnetic moment due to spin motion,

$$\mu_{\text{spin}} = \frac{eh}{4\pi mc}$$

$= 0.927 \times 10^{-20}$ **erg/Oe (or emu) :**

→ **A fundamental quantity called, Bohr magneton, μ_B**

(1) Diamgnetism

No net atomic moment : $\mu_r < 1, \chi < 0$

- **Classical theory, 1905 yr Paul Langevin** (see Cullity)

For an *e*- in orbital motion, no net μ_{orbit} of paired electrons if $H_a = 0$,

A net μ_{orbit} within atom opposing the flux change(Lentz law) due to accelerated or decelerated orbital motion of *e*- if $H_a > 0$.

On the basis of Bohr model, $\triangle\mu_{orbit} = \frac{er}{2c}\Delta\upsilon = -\mu_o \frac{e^2r^2}{4m}H$ since $\Delta\upsilon = -erH/2mc$

If ***H*** is not perpendicular to the orbit plane, $\triangle\mu_{orbit} = -\mu_o \frac{Ze^2r^2}{6m}H$

For an atom with *Z* outer electrons, $\triangle\mu_{orbit} = \Delta M = -(N_o \frac{\rho}{W})(\mu_o \frac{Ze^2r^2}{6m}H)$

For a bulk magnetization,

$$\chi = \frac{M}{H} = \frac{\Delta M}{H} = -(N_o \frac{\rho}{W})(\mu_o \frac{Ze^2r^2}{6m})$$

where, N_o = Abogadro's number, ρ = density, and *W* = atomic number

: ***independent of temperature!***

Diamagnetism in superconductors

- **For $H < H_c$ in type I superconductors and $H < H_{c1}$ in type II superconductors,**

Perfect diamagnetism: $\mu_r = 0, \chi = -1$ since $B = 0$

Origin: shielding supercurrent in the case of zero-field-cooling

Meissner effect in the case of field-cooling

- **For $H_{c1} < H < H_{c2}$ in type II superconductors,**

mixed (or vortex) state : $B > 0$

(1) Diamgnetism (additional)

▶ Classical theory, Lamor frequency ω_L

The torque causes the orientation of the angular momentum vector $\boldsymbol{L}$ (and thus the orbital magnetic dipole moment $\boldsymbol{\mu}_{orb}$) to change by $d\boldsymbol{L}$ perpendicular to both $\boldsymbol{B}$ and $\boldsymbol{\mu}_{orbit}$. (see Fig. 3.3 in O'Handley)

Since $dL = L\sin\theta d\phi = \omega_L L\sin\theta dt$, and the torque $dL/dt = \mu_{orbit}B\sin\theta$ as $\boldsymbol{\tau} = d\boldsymbol{L}/dt = \boldsymbol{\mu}_{orbit}\times\boldsymbol{B}$

Therefore, $\omega_L = \mu_{orbit}B/L = \gamma_o B = eB/2m$

An angular rotation of the orbital magnetic moment vector $\boldsymbol{\mu}_{orbit}$ with ω_L // $\boldsymbol{B}$

An electric current i equivalent to the Lamor precession of an orbit electron

$$i = \text{(charge)(revolution per unit time)} = (-e)/(\omega_L/2\pi) = -e^2B/4\pi m$$

Thus, $\triangle\mu_{orbit} = iA = (-e^2B/4\pi m)(\pi r^2) = -(e^2r^2/4m)B$

For the Lamor precession of Z electrons

$$\triangle\mu_{orbit} = -Ze^2B\langle r^2\rangle/4m$$

If B is not perpendicular to the orbit plane,

$$\triangle\mu_{orbit} = -Ze^2B\langle r^2\rangle/6m$$

Consequently,

$$\chi = N_v\triangle\mu_{orbit}/H = -\mu_o N_v Ze^2\langle r^2\rangle/6m$$

where, N_v = number of atoms per unit volume

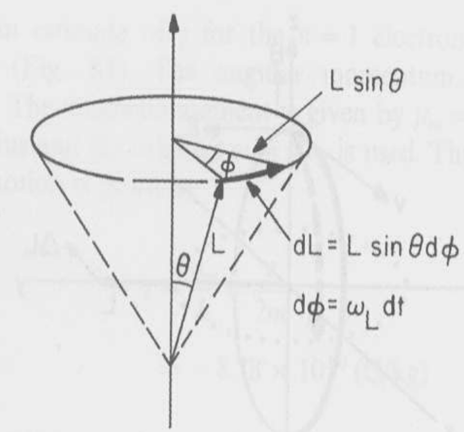


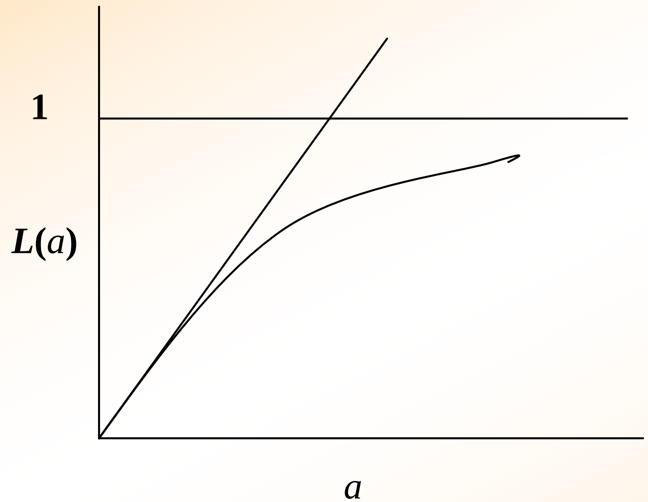
Figure 3.3 Construction for relating the change in angular momentum to the precession frequency.

(2) Paramagnetism

▶ $\mu_r > 1$, $\chi > 0$
If $H = 0$, $M = 0$
If $H > 0$, $M > 0$

▶ Experimental
Systematic measurements of χ : 1895 yr Pierre Curie
Cuire law
$\chi = \frac{C}{T}$ C: Cuire constant
Curie-Weiss law : a more general law
$\chi = \frac{C}{T-\theta}$

▶ Classical theory : Localized electron model
① **Langevin theory**
Let the net atomic magnetic moment be m
$m = m_o + m_s$
Magnetic potential energy, $E = -\mu_0 m \bullet H$ (or $- m \bullet B$)
Assuming all m are identical and non-interacting,
and m depends on H and thermal agitation,
According to classical Boltzman statistics,
Probability having E, P(E)
$P(E) = \exp\left(\frac{-E}{kT}\right)$ k : Boltzmann constant
$= \exp\left(\frac{-\mu_o mH}{kT}\right)$



For N atoms/unit volume, and let $\frac{\mu_o mH}{kT} = a$

Magnetization, $M = NmL(a)$, where

$L(a) = \coth a - \frac{1}{a}$ Langevin function

Let $Nm = M_0$ (maximum possible magnetization, corresponding to the perfect alignment of all m parallel to H: a state of complete saturation)

Then , $L(a) = \frac{M}{M_o} = \frac{a}{3} - \frac{a^2}{45} + \frac{2a^5}{945} - ....$

$L(a) = \frac{a}{3}$ ← 1st approximation

(2) Paramagnetism (continued)

② Weiss theory

Assuming the moments interact each other, an interacting field H_e (called, molecular field or exchange field)

$H_e = \alpha M$ **: Weiss assumption**

α **is the molecular field constant**

$H_{tot} = H + H_e$

$= H + \alpha M$

Since $\chi = \frac{M}{H_{tot}} = \frac{C}{T}$ **from Curie law**

$H_{tot} = \frac{MT}{C}$

Then, $\chi = \frac{M}{H} = \frac{M}{H_{tot} - \alpha M} = \frac{M}{\frac{MT}{C} - \alpha M} = \frac{C}{T - \theta}$

$\Theta = \alpha C$ **: measure of the strength of the interaction**

$\rightarrow$ **leading to Curie-Weiss law**

▶ Quantum theory

Localized electron model

Non-localized electron model : Pauli paramagnetism

Remember the following;

Net atomic moment, *m* $m = g\mu_B J$

where, $|J| = \sqrt{J(J+1)}\,\hbar$

g**(Lande splitting factor),** $1 \leq g \leq 2$ **: empirical values**

In general, Lande equation

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

If $L = 0,\ J = S \rightarrow g = 2$ **(only spin contribution)**

If $S = 0,\ J = L \rightarrow g = 1$ **(only orbital contribution)**

If $J = \infty$ **: random orientation (classical)**

(2) Paramagnetism (continued)

Localized electron model

Magnetic potential energy,

$E = -\mu_{total} \cdot B$ (μ_{total} **: total magnetic moment of an atom or net atomic magnetic moment)**

$= -g\mu_B(J/\hbar) \cdot B$ ($|J| = \sqrt{J(J+1)}\,\hbar$ **: total angular momentum)**

$= -g\mu_B(J_z/\hbar)B$

$= -g\mu_B M_J B$ **since** $J_z = M_J\hbar$ (M_J **: total magnetic quantum number)**

where, M_J **can have only** $2J+1$ **values**

$M_J = -J, -(J-1), \ldots, (J-1), J$ (J **: total angular momentum quantum number)**

Therefore, the average magnetization in B **is given by**

Applying Boltzman statistics, $N_v = N/V$**,**

$$M = N_v g\mu_B \frac{\sum_{Mj=-J}^{J} M_J \exp(g\mu_B M_J B/kT)}{\sum_{Mj=-J}^{J} \exp(g\mu_B M_J/kT)}$$

$= NgJ\mu_B B_J(x) = M_o B_J(x)$

where

$$x = \frac{gJ\mu_B\mu_O H}{kT}$$

$$B_J(x) = \frac{M}{M_O} = \frac{2J+1}{2J}\coth(\frac{2J+1}{2J})x - \frac{1}{2J}\coth\frac{x}{2J}$$

$B_J(x)$ **is Brillioiun function**

If $J = \infty \rightarrow B_J(x) = L(a)$ **: classical distribition**

If $J = \frac{1}{2}$ **: one spin /atom, = tanh**x

(2) Paramagnetism (continued)

Comments

Comparison of quantum picture with classical one

(see Fig. 3.15 in O’Handley)

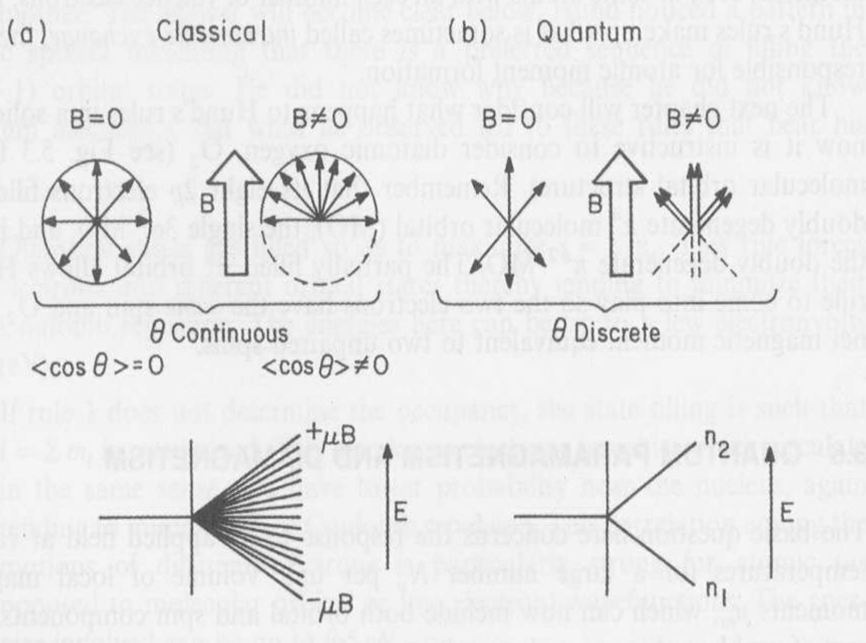


Figure 3.15 (*a*), Classical picture of a continuous distribution of spin orientations in zero field and the effect of a magnetic field that drives the spins to lower energy states in the field-split manifold; (*b*) the quantum picture in which only certain spin orientations are allowed.

Brillouin function vs Lagevin fun

(see Fig. 3.16 in O’Handley)

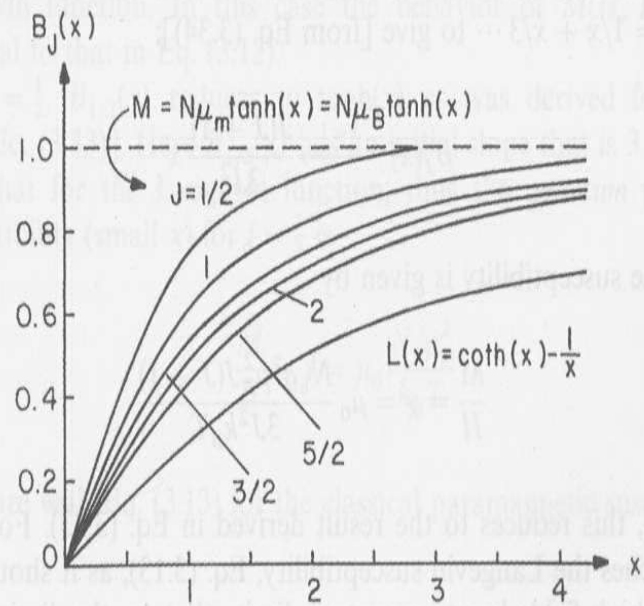


Figure 3.16 Brillouin function versus $x = \mu_m B/k_B T$ for various values of j. The spin $\frac{1}{2}$ limit is given by Eq. (3.33), and the infinite spin limit by the classical Langevin function is derived in Eq. (3.11).

(2) Paramagnetism (continued)

Example (see Fig. 3.17 in O'Handley)

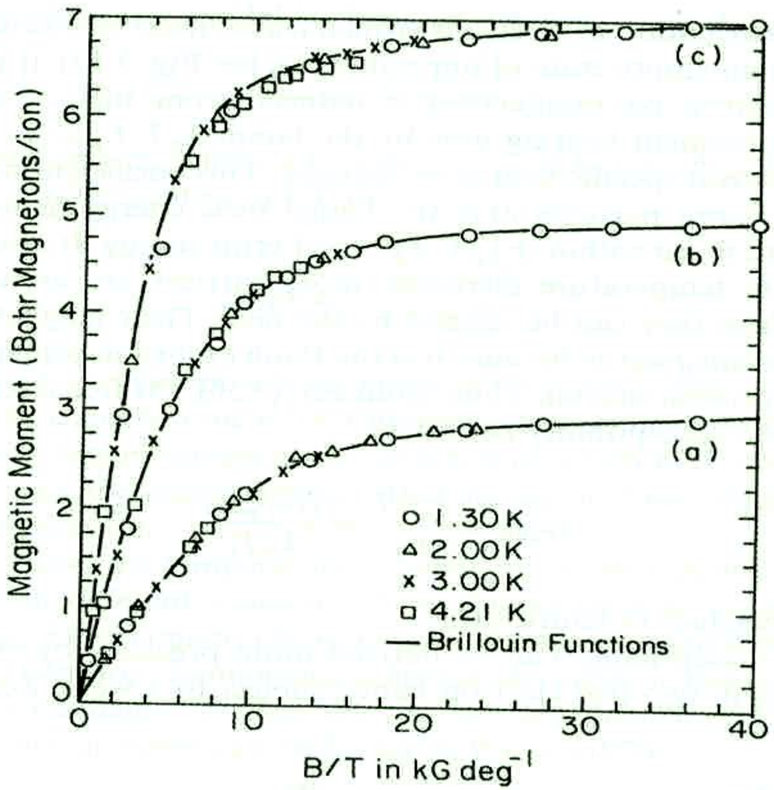


Figure 3.17 Magnetic moment versus B/T for (a) potassium chromium alum (Cr^{3+}, $s = \frac{3}{2}$), (b) ferric ammonium alum (Fe^{3+}, $s = \frac{5}{2}$) and (c) gadolinium sulfate octahydrate (Gd^{3+}, $s = \frac{7}{2}$), (After Henry, 1952).

(2) Paramagnetism (continued)

Pauli Paramagnetism (examples: see Fig. 11 in Chap. 4 of Kittel)

- Conduction electron paramagnetism for indistinguishable free electron spins in a metal

- Only Fermi particles within $\pm kT/2$ of Fermi energy E_F can change their energy(i.e., orientation) in response to an allpiled field.

- As T increases, more carriers are excited above the Fermi level and excited carriers are able to be aligned by the field.

For free electrons in weak fields, $x(\mu_m B/kT) \ll 1$ in localized electron theory, the susceptibility χ_{Pauli}

$$\chi_{\text{Pauli}} \approx \frac{\chi kT}{E_F} = \frac{N_v \mu_m^2 \mu_0}{kT} \quad : \text{independent of } T$$

More precisely, by considering the spin imbalance in two free electron bands subject to a weak Zeeman splitting ($\mu_m B \ll E_F$) :

$$\chi_{\text{Pauli}} = \frac{3N_v \mu_m^2 \mu_0}{2kT_F} \quad : \text{independent of } T$$

Temperature independence of χ_{Pauli} : with increasing temperature, increased free electron spins able to align with an external field and increased thermal disordering of those aligned spins are cancelled.