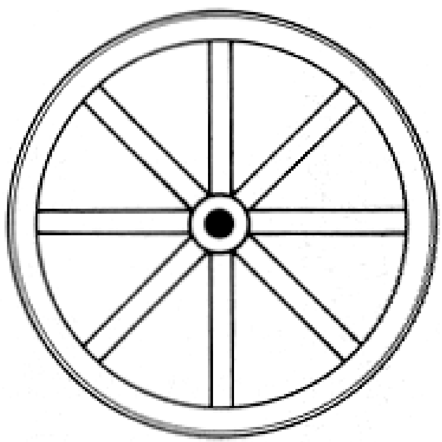
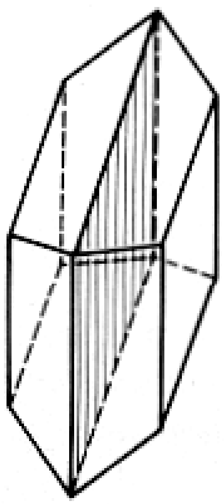
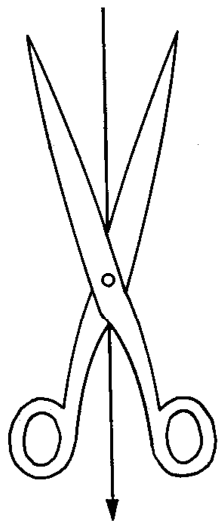
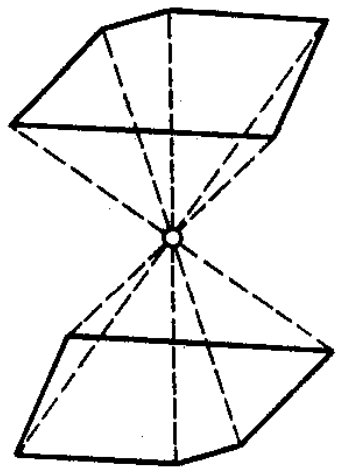
Symmetry

- Repetition

1. Lattice translation- three non-coplanar lattice translation

   space lattice

2. Rotation (회전)
3. Reflection (반사)
4. Inversion (반전)

Symmetry Aspects of M. C. Escher's Periodic Drawings

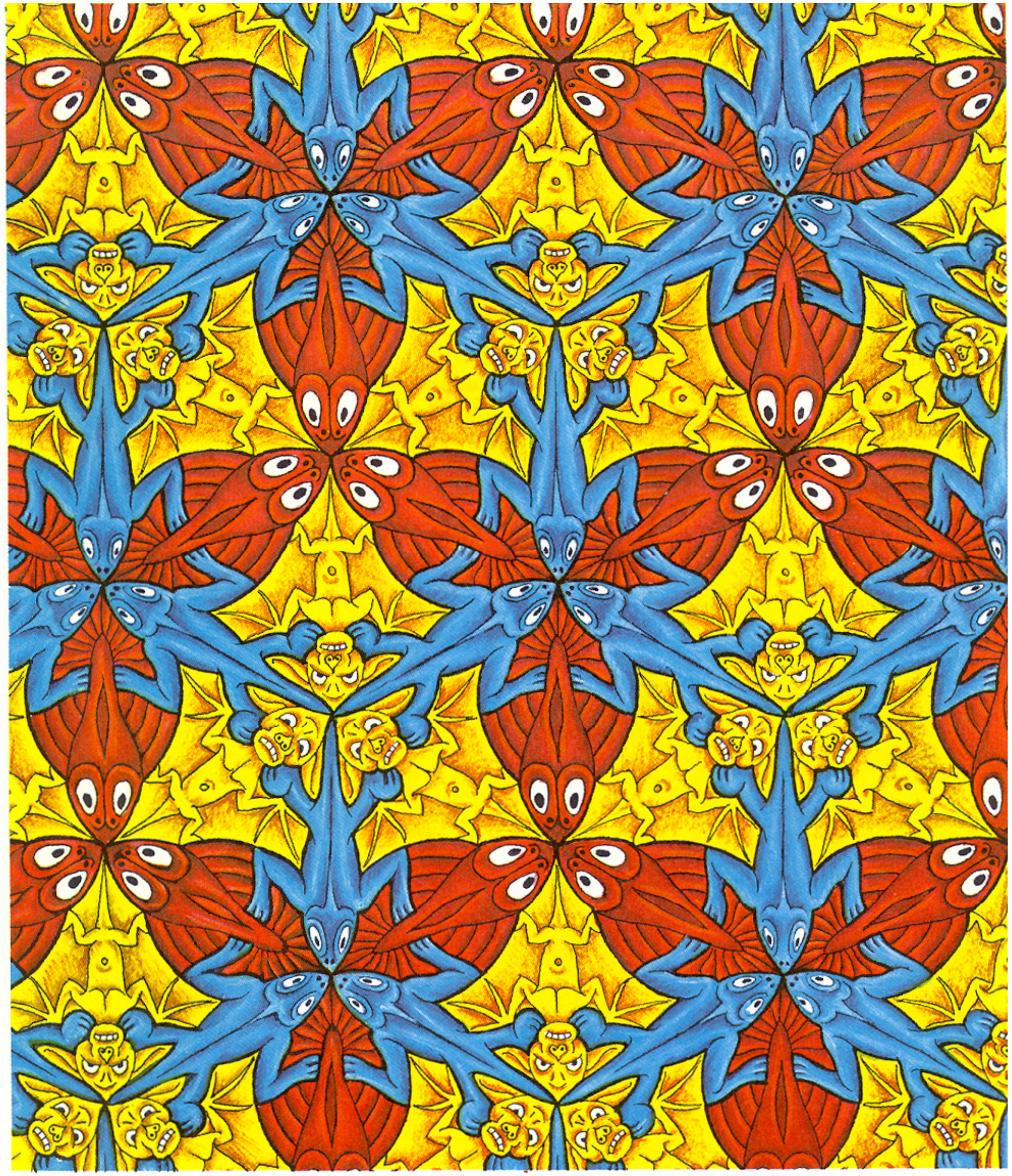
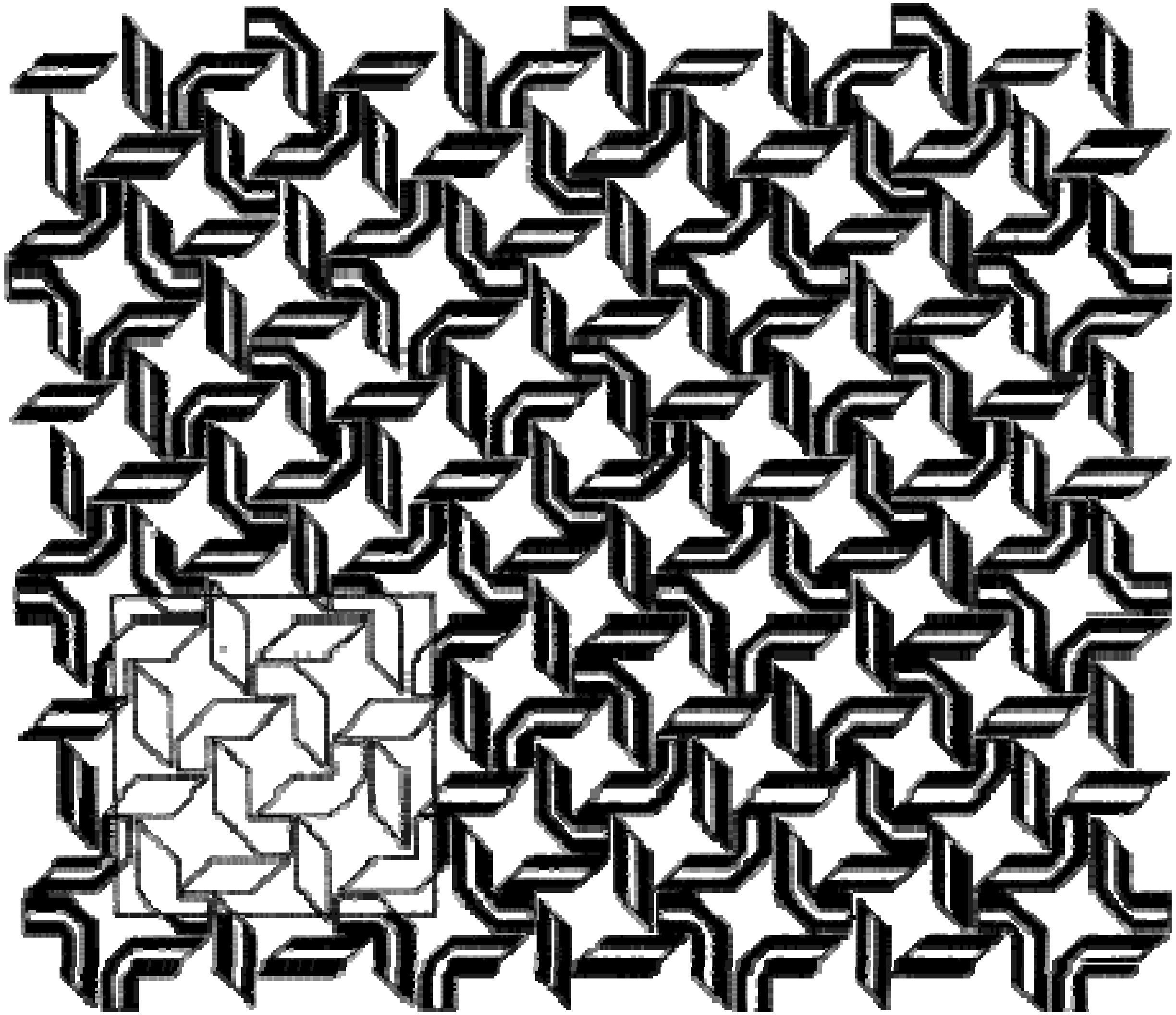


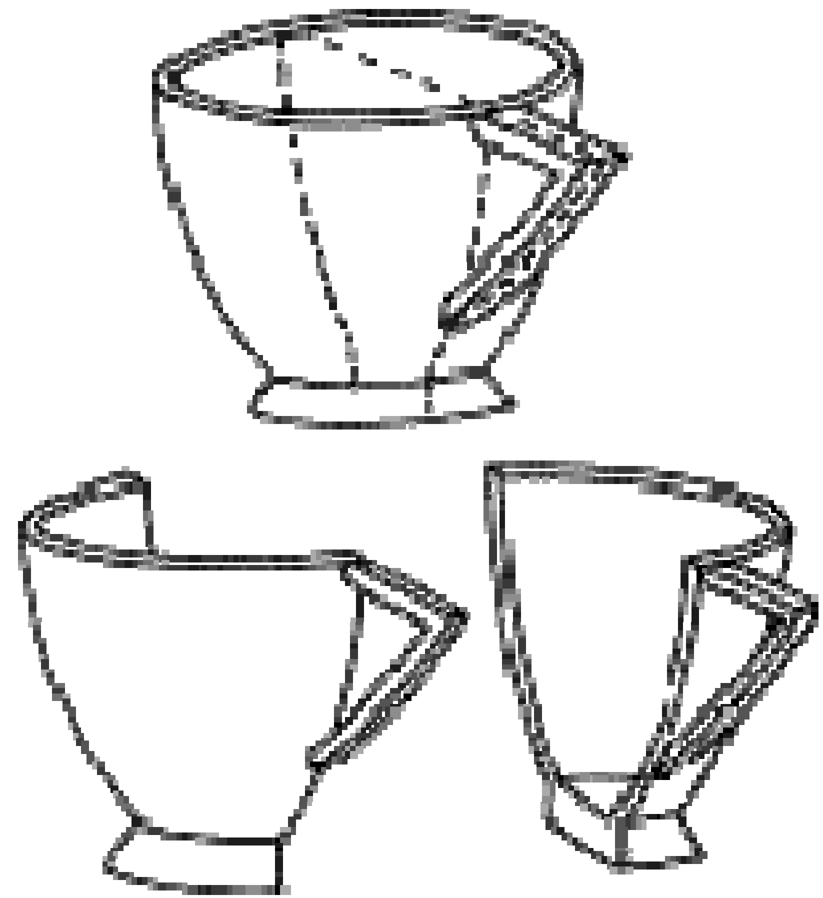
PLATE 9

Fig. 1.1. Pattern based on a fourteenth-century Persian tiling design.



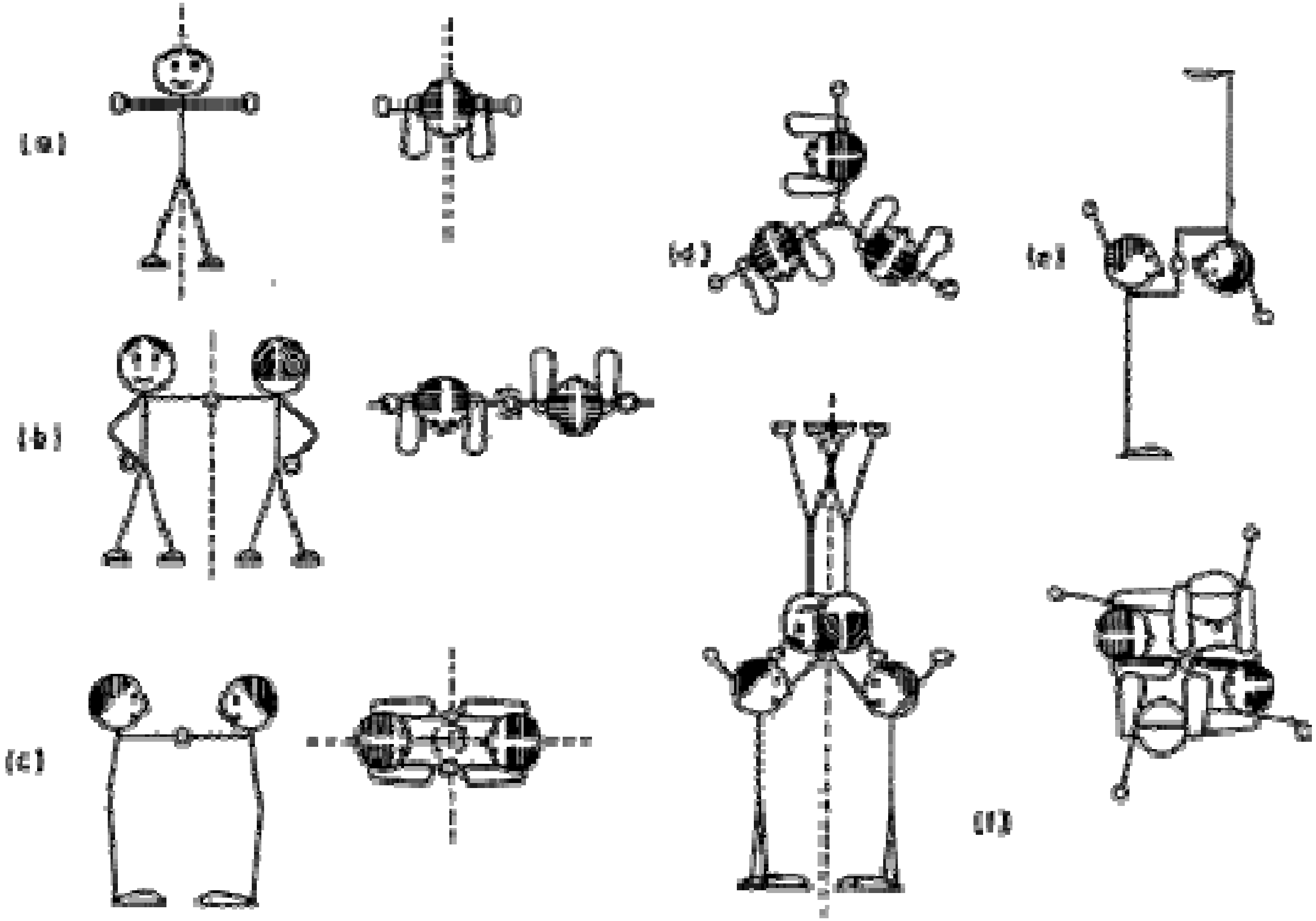
http://www.iucr.ac.uk/

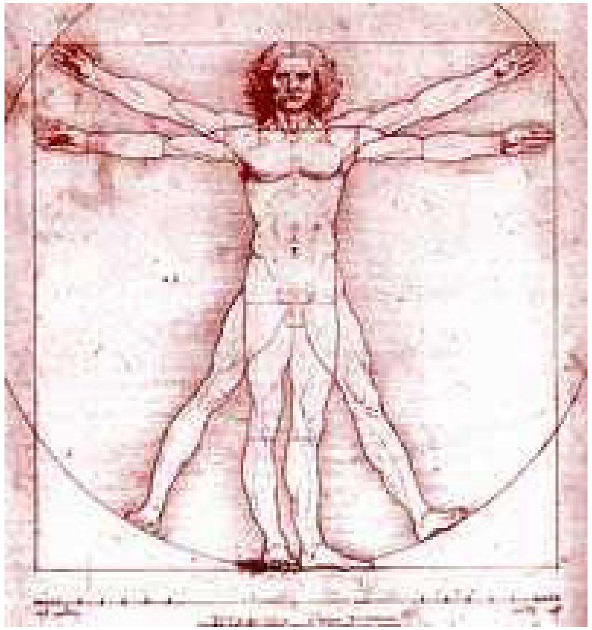
Fig. 1.2. A teacup, showing its mirror plane of symmetry. (After L. S. Dent Glasser, Crystallography & its applications: Van Nostrand Reinhold, 1977.)



http://www.iucr.ac.uk/

Fig. 1.3. Some symmetry elements, represented by human figures. (a) Mirror plane, shown as dashed line, in elevation and plan. (b) Twofold axis, lying along broken line in elevation, passing perpendicularly through clasped hands in plan. (c) Combination of twofold axis with mirror planes; the position of the symmetry elements given only in plan. (d) Threefold axis, shown in plan only. (e) Centre of symmetry (in centre of clasped hands). (f) Fourfold inversion axis, in elevation and plan, running along the dashed line and through the centre of the clasped hands.
(After L. S. Dent Glasser, Chapter 19, The Chemistry of Cements: Academic Press, 1964.)






K
K

Symmetry

* All repetition operations are called symmetry operations.
  Symmetry consists of the repetition of a pattern by the application of specific rules.
* When a symmetry operation has a locus, that is a point, or a line, or a plane that is left unchanged by the operation, this locus is referred to as the symmetry element.

*

Symmetry operation	symmetry element
reflection	mirror plane
rotation	rotation axis
inversion	inversion center (center of symmetry)

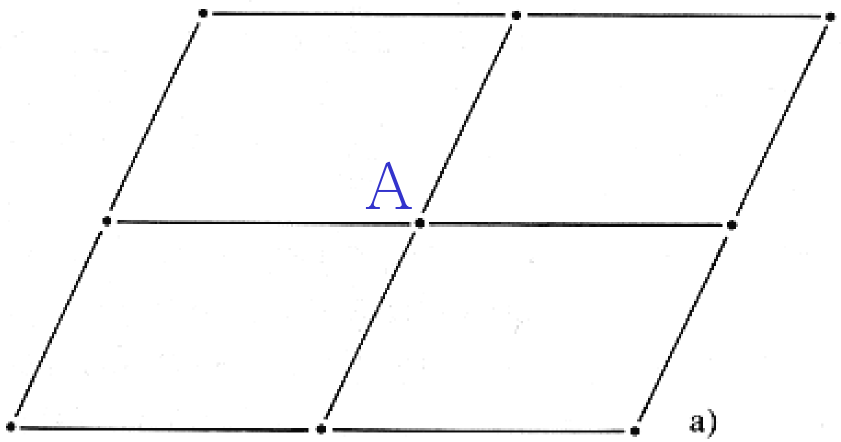
Rotation Axis

- general plane lattice

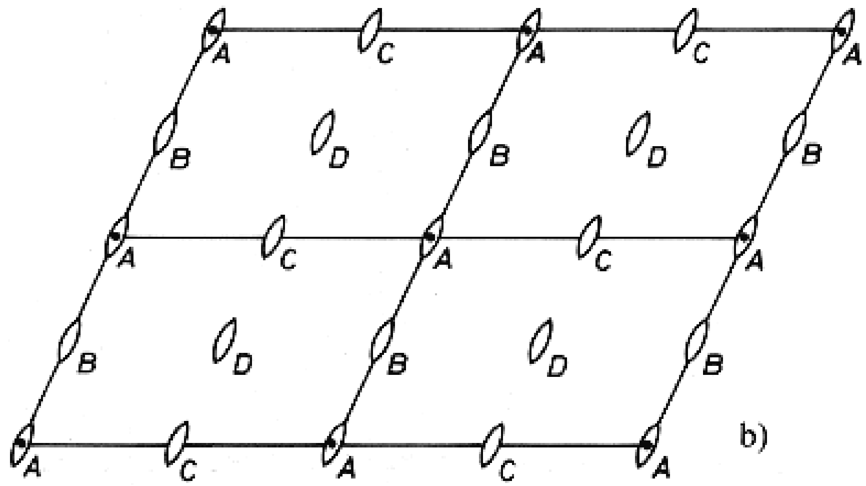
180° rotation about the central lattice point A– coincidence
– 2 fold rotation axis

symbol: 2, ⬮ → normal or parallel to plane of paper

$$n = \frac{360^o}{\phi} = \frac{2\pi}{\phi}$$



a)



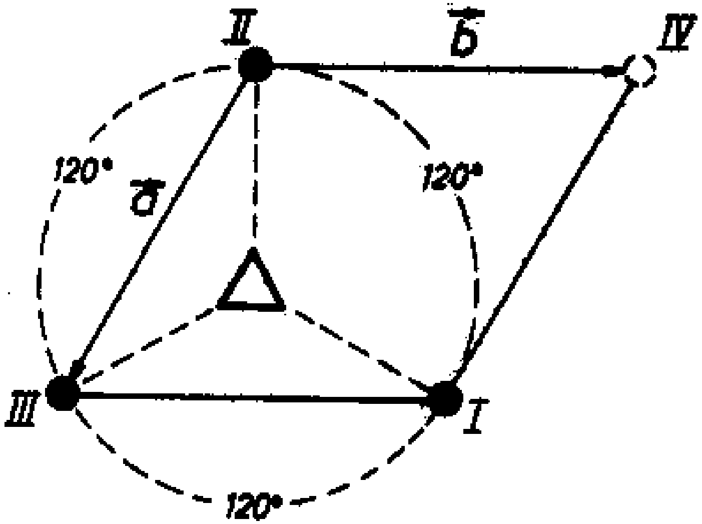
b)

Rotation Axis

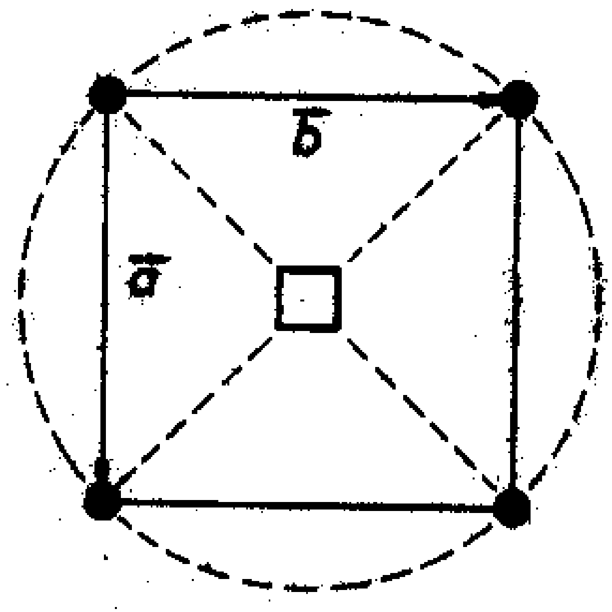
-n-fold axis $\quad n = \dfrac{360^o}{\phi} = \dfrac{2\pi}{\phi}$ $\quad$ ϕ: minimum angle required to reach a position indistinguishable

-n > 2 produce at least two other points lying in a plane normal to it
- three non-colinear points define a lattice plane
- fulfill the conditions for being a lattice plane (translational periodicity)

- 3 fold axis: ϕ= 120°, n=3, ▲

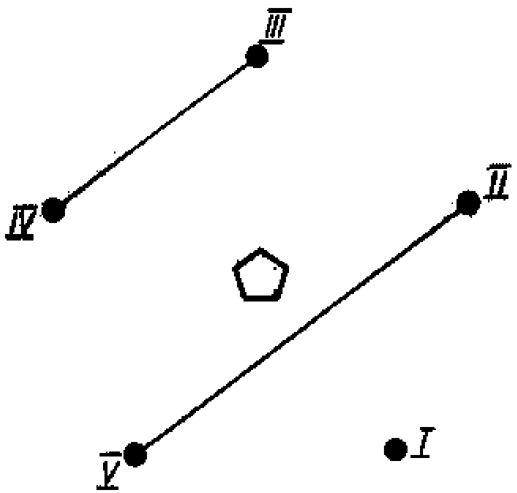


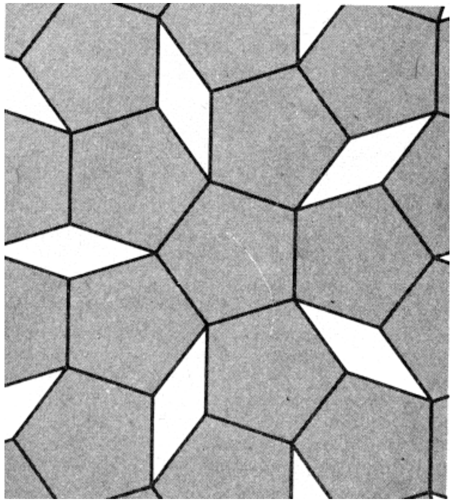
- 4 fold axis: ϕ=90°, n=4, ■



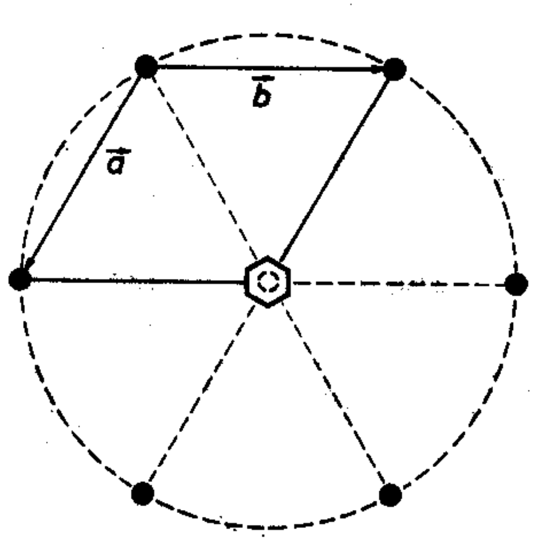
Rotation Axis

-5 fold axis: ϕ= 72^o, n=5,



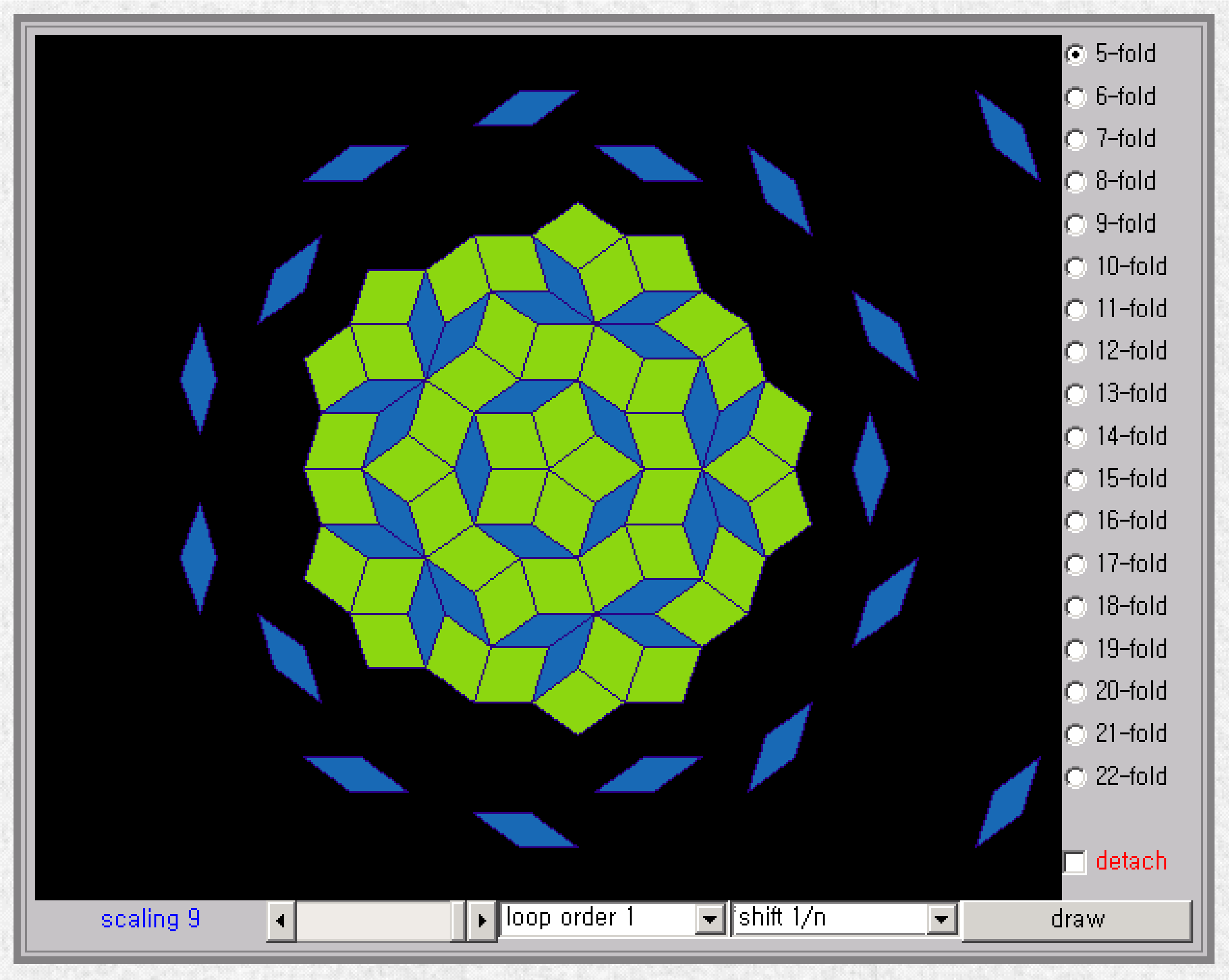


- 6 fold axis: ϕ=60^o, n=6, ⬢

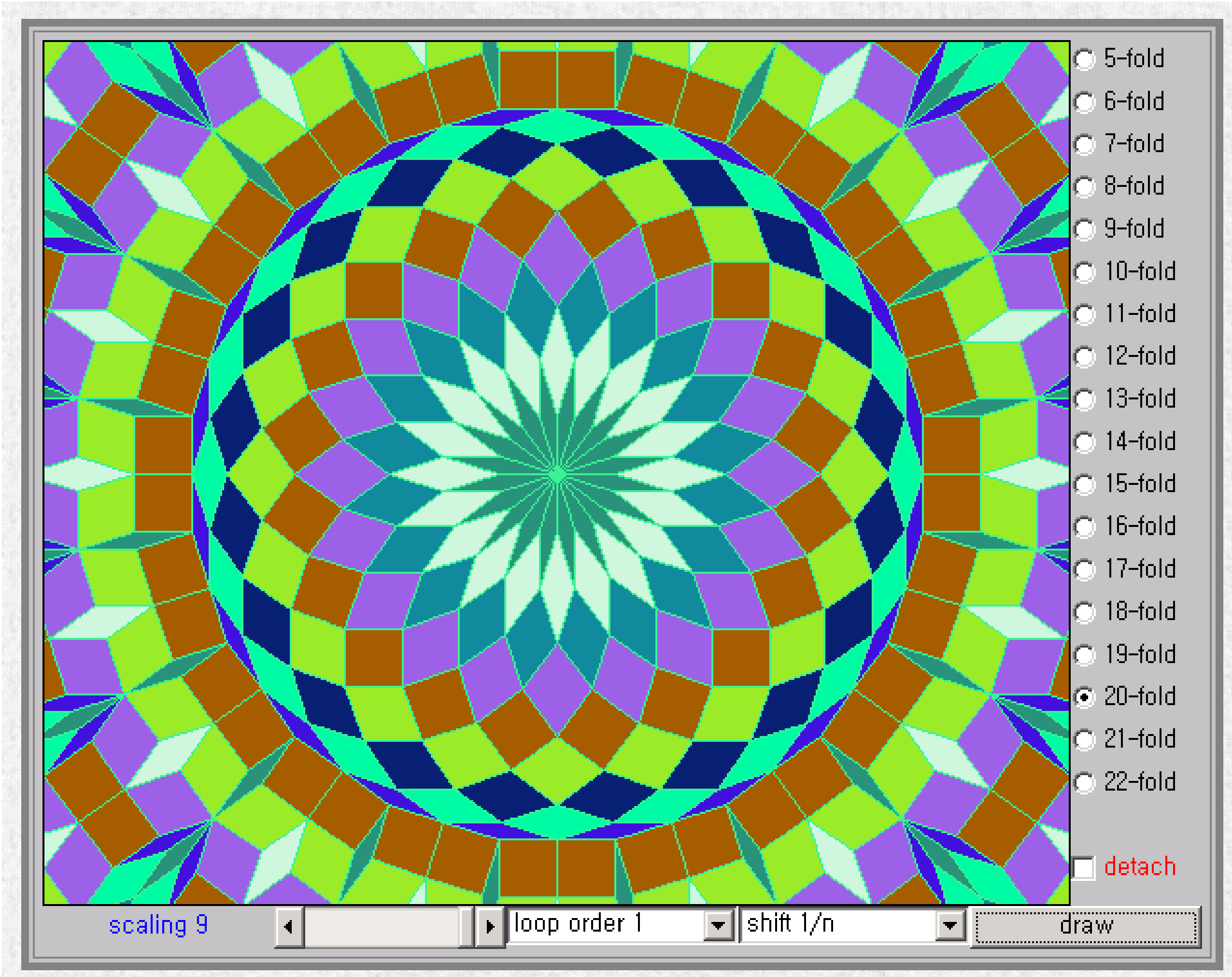


II-V and III-IV parallel but
not equal or integral ratio

* In space lattices and consequently in crystals, only 1-, 2-, 3-, 4-, and 6-fold rotation axes can occur.

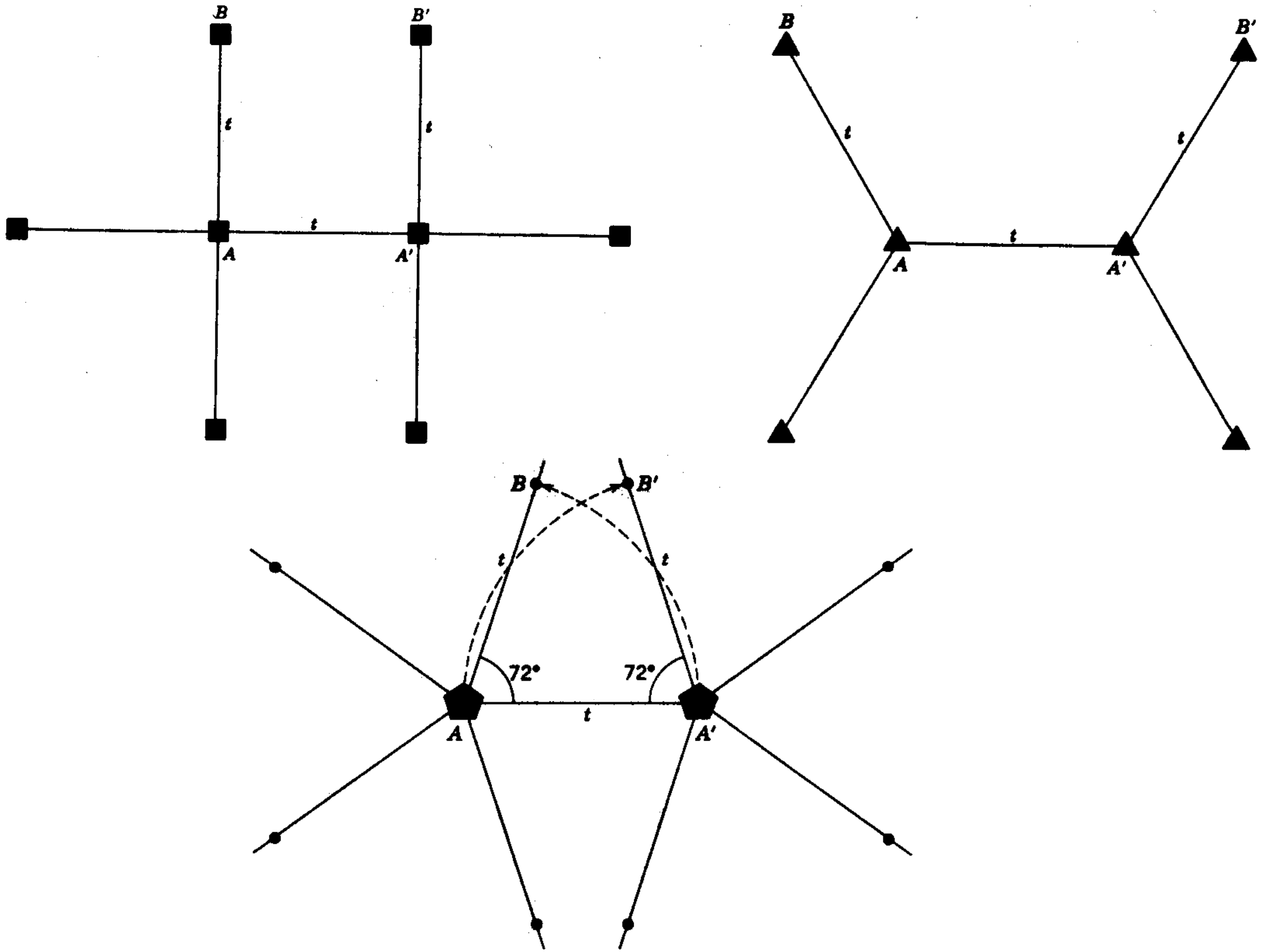


http://jcrystal.com/steffenweber/JAVA/jtiling/jtiling.html



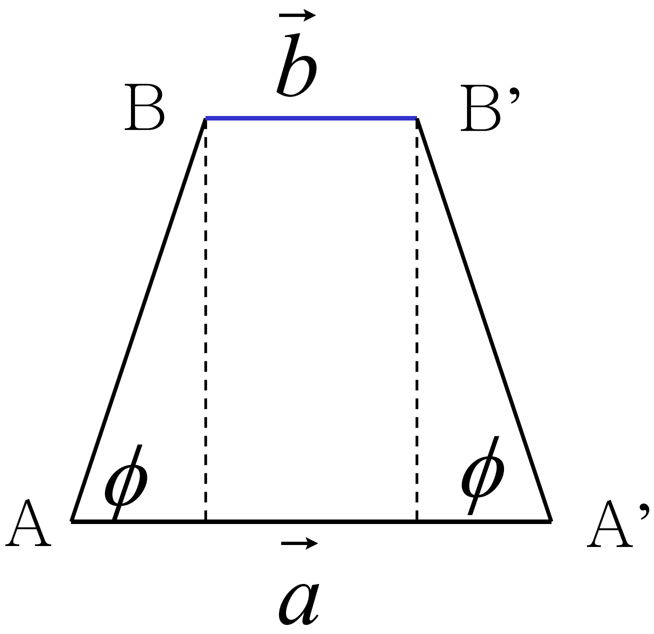
http://jcrystal.com/steffenweber/JAVA/jtiling/jtiling.html

Rotation Axis



Rotation Axis

- limitation of ϕ set by translation periodicity



$\vec{b} = m\vec{a}$ where m is an integer

$$ma = a - 2a\cos\phi$$

$$m = 1 - 2\cos\phi$$

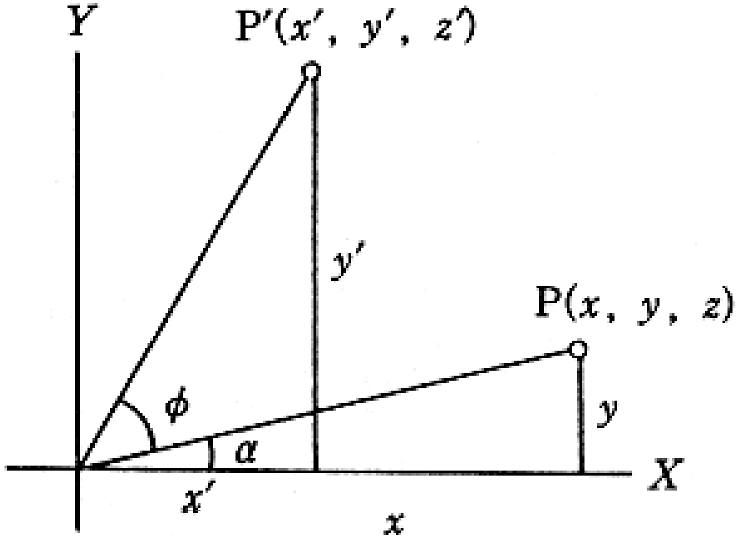
$$\cos\phi = \frac{1-m}{2}$$

m	$\cos\phi$	ϕ	n
−1	1	2π	1
0	½	$\pi/3$	6
1	0	$\pi/2$	4
2	−½	$2\pi/3$	3
3	−1	π	2

$\boxed{1.\ 2,\ 3,\ 4,\ 6}$

Rotation Axis

- Matrix representation of rotation in Cartesian coordinate



직교축계에서 z축을 회전축으로 하고
φ각만큼 회전시킨 회전조작

$$R(n_z^1) = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(2_z^1) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(3_z^1) = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(3_z^2) = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotation Axis

$$R(4_z^1) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R(4_z^2) = R(2_z^1) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R(4_z^3) = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(4_z^1) \bullet R(4_z^2) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = R(4_z^3)$$

$$R(6_z^1) = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(6_z^2) = R(3_z^1)$$

$$R(6_z^3) = R(2_z^1)$$

$$R(6_z^4) = R(3_z^2)$$

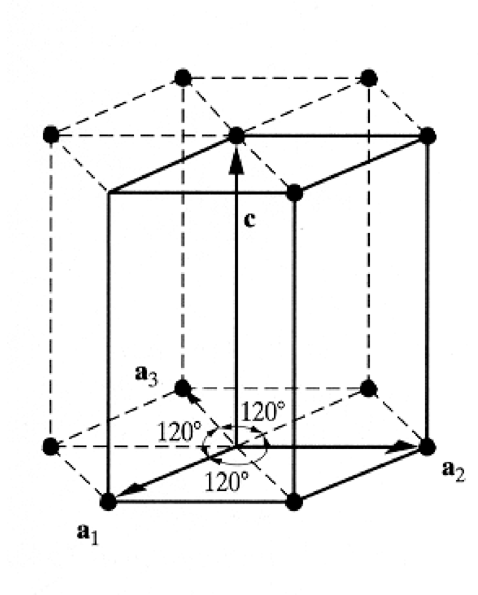
$$R(6_z^5) = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

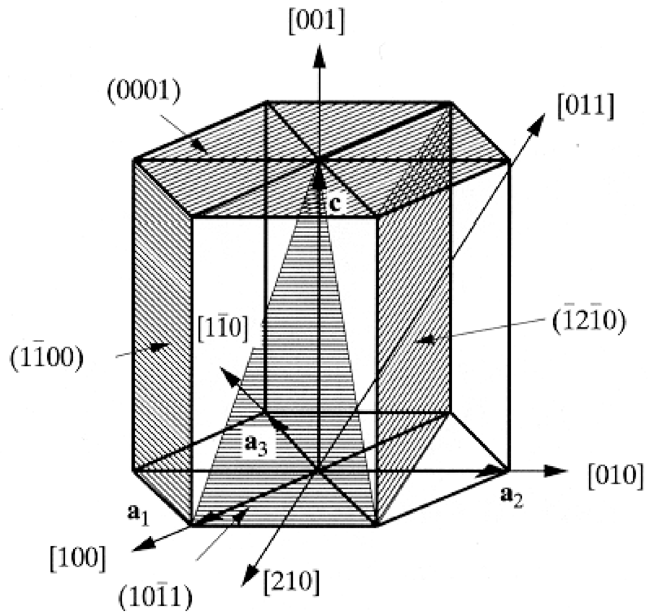
Rotation Axis

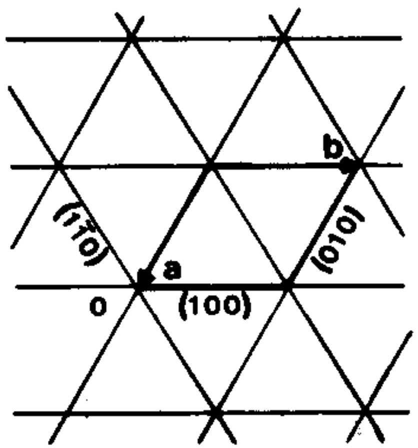
- hexagonal coordinate

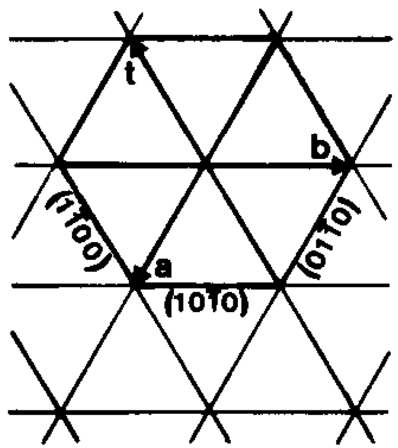
  a_1, a_2, a_3, c

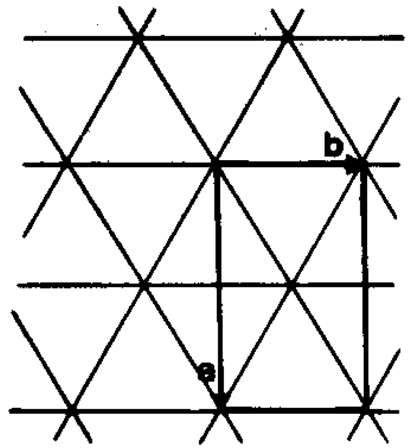
  Miller-Bravais indices (h k i l) $i=-(h+k)$



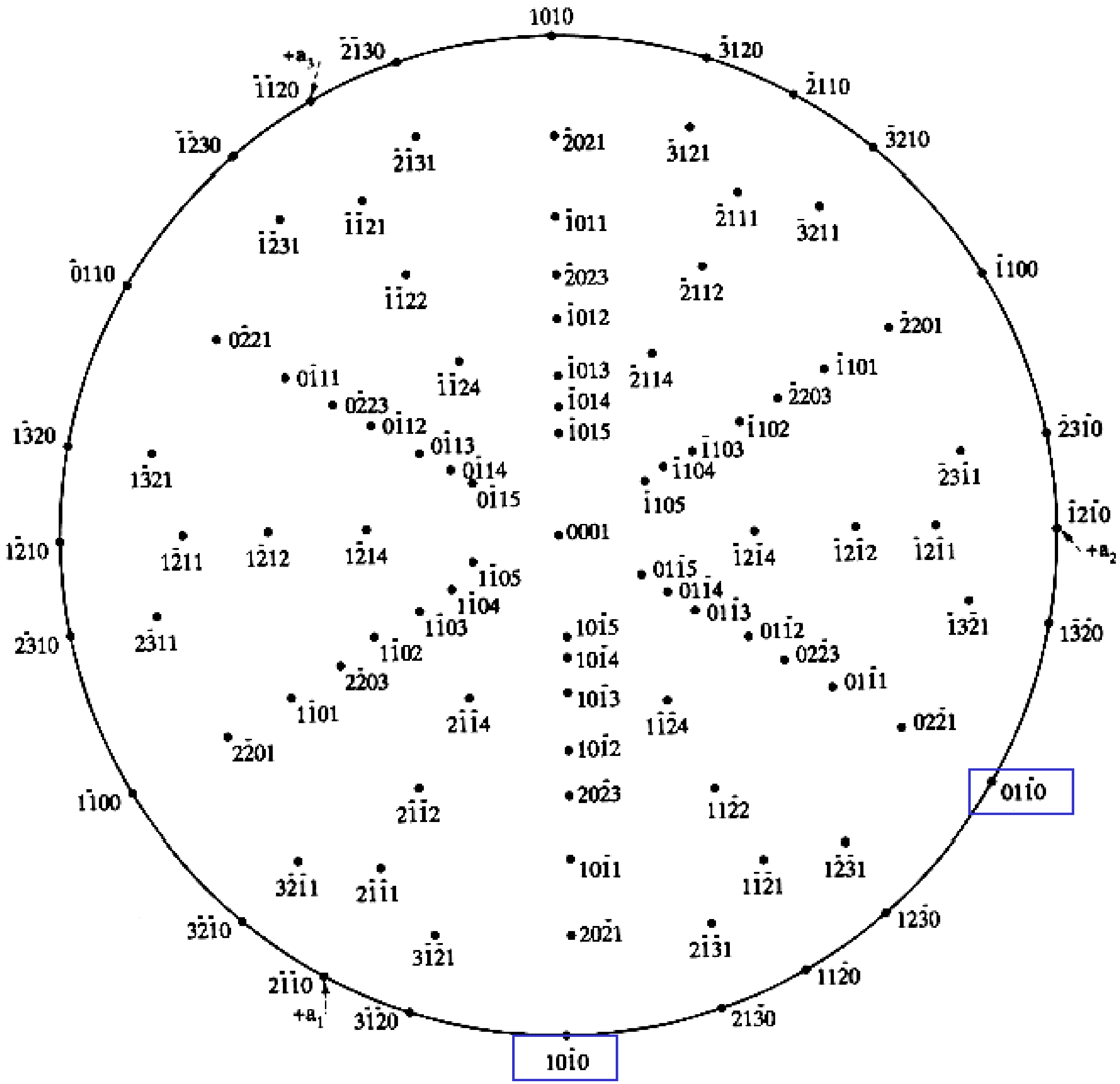








Rotation Axis

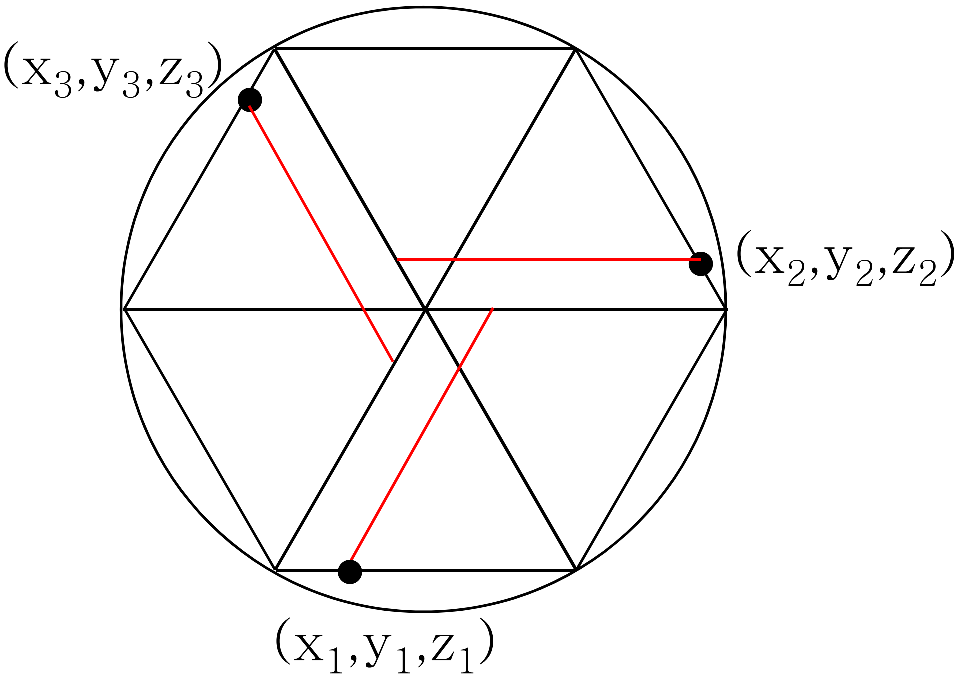


Standard (0001) projection (hexagonal, c/a=1.86)

Rotation Axis

$$x_2 = -y_1 \qquad x_3 = -x_1 + y_1$$
$$y_2 = x_1 - y_1 \qquad y_3 = -x_1$$
$$z_2 = z_1 \qquad x_3 = z_1$$

$$R(3_z^1) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R(3_z^2) = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

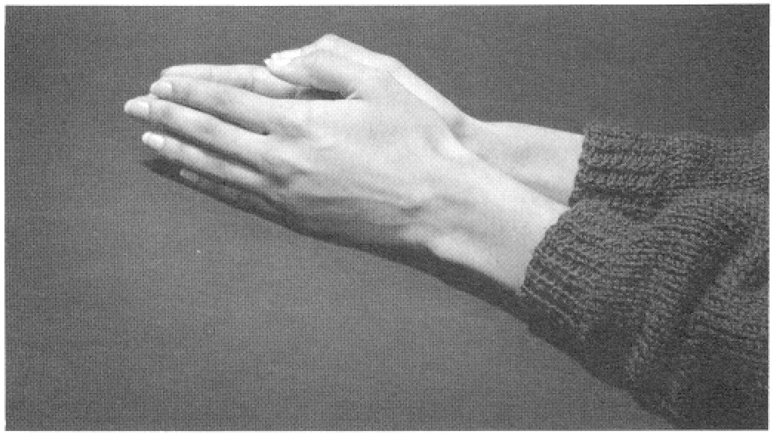


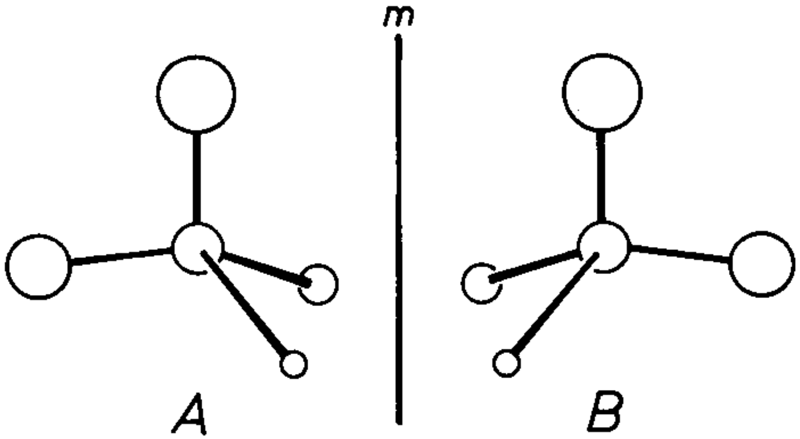
* Cartesian coordinate

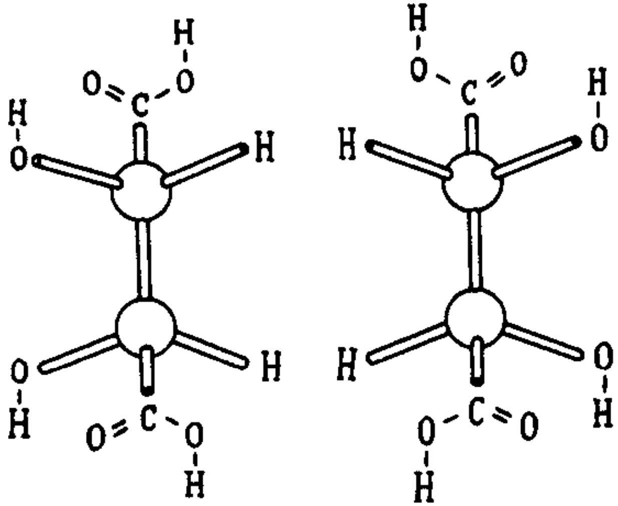
$$R(3_z^1) = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Reflection

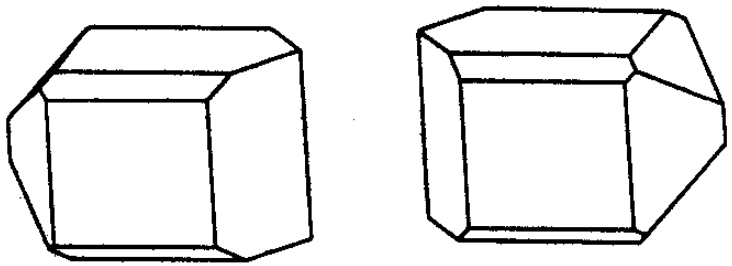
- reflection, a plane of symmetry or a mirror plane, m, |, Γ







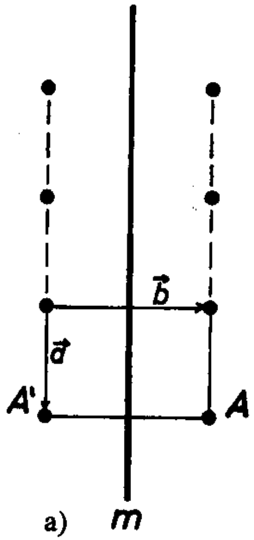
(a)



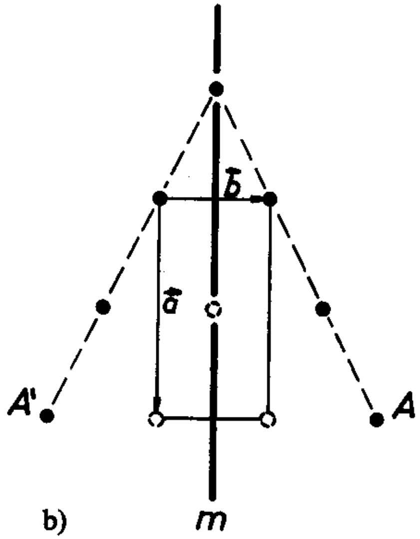
(b)

Tartaric acid

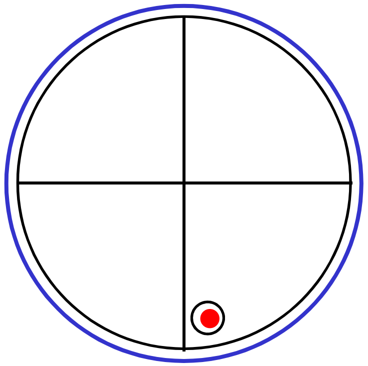
Reflection



rectangular



centered rectangular



m_{xy} (m_z)

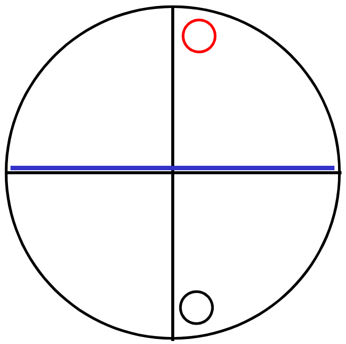
$$x_2 = x_1$$
$$y_2 = y_1$$
$$z_2 = -z_1$$

$$R(m_z) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\left| R(m_z) \right| = -1$$

enantiomorph

대장상



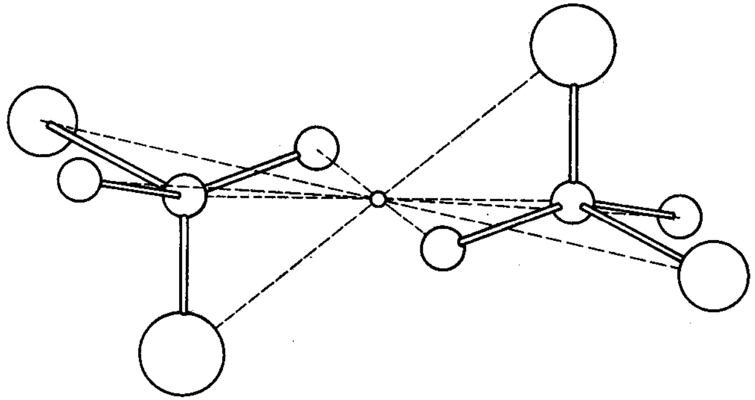
m_{yz} (m_x)

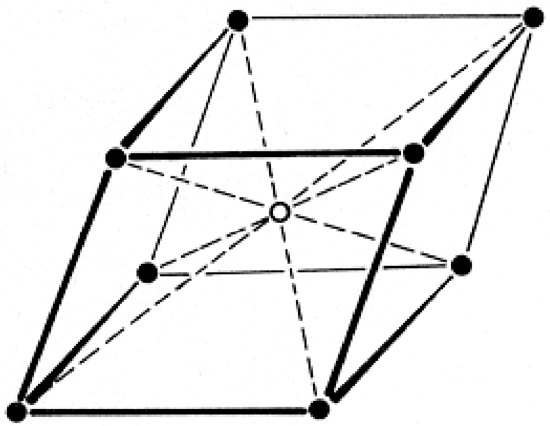
$$x_2 = -x_1$$
$$y_2 = y_1$$
$$z_2 = z_1$$

$$R(m_x) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

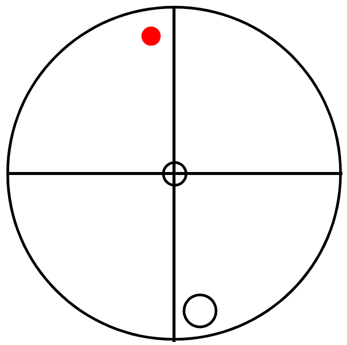
Inversion

- inversion, center of symmetry or inversion center, $\overline{1}$ ○





All lattices are centrosymmteric.

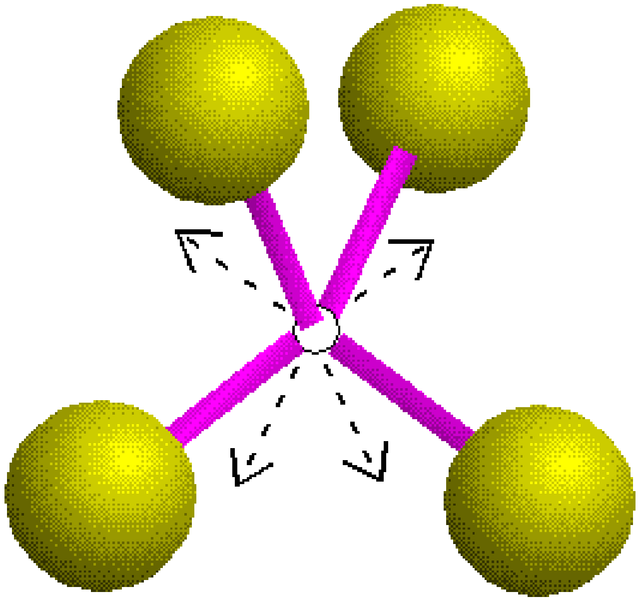


$$\left|R(\overline{1})\right| = -1$$

$$x_2 = -x_1$$
$$y_2 = -y_1$$
$$z_2 = -z_1$$

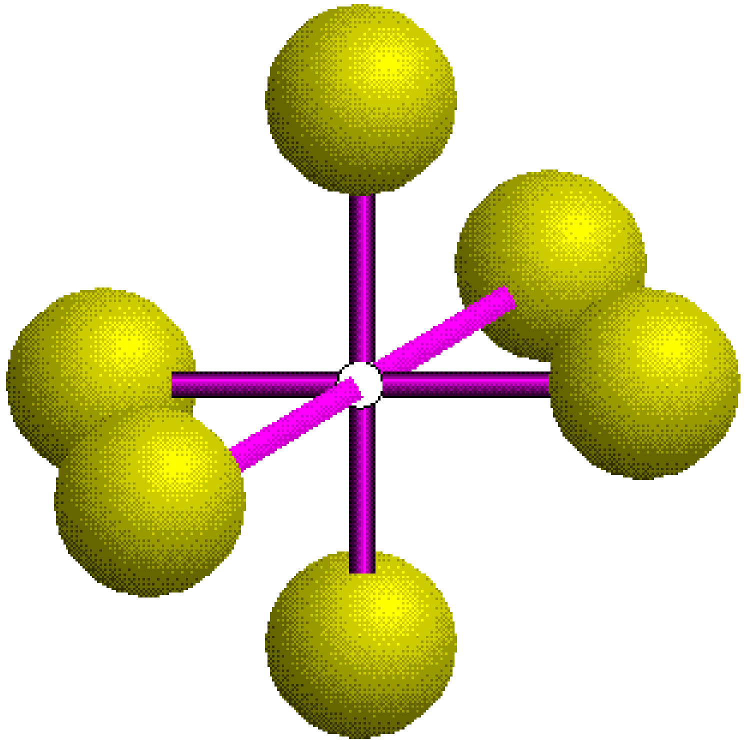
$$R(\overline{1}) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Inversion



No inversion centre

Inversion centre



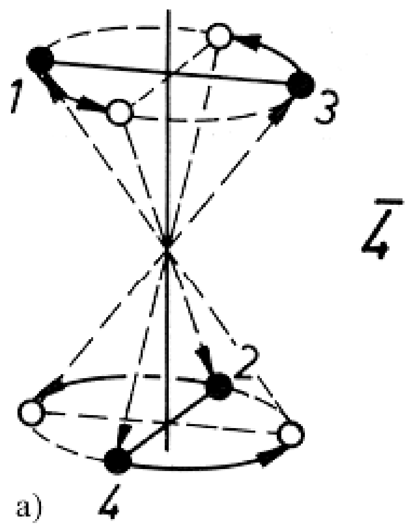
http://www.gh.wits.ac.za/craig/diagrams/

Compound Symmetry Operation

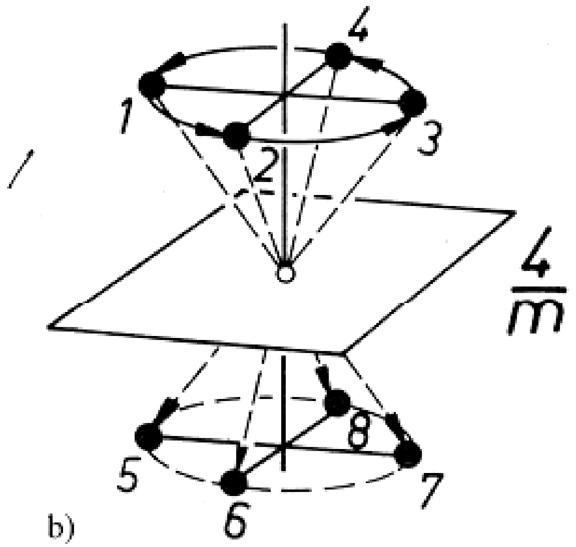
- link of translation, rotation, reflection, and inversion operation

-compound symmetry operation
  two symmetry operation in sequence as a single event

- combination of symmetry operations
  two or more individual symmetry operations are combined
  which are themselves symmetry operations



compound



combination

Compound Symmetry Operation

Table 5.1. Compound symmetry operations of simple operations. The corresponding symmetry elements are given in round brackets

	Rotation	Reflection	Inversion	Translation
Rotation	×	Roto-reflection	Roto-inversion	Screw rotation
Reflection	(Roto-reflection axis)	×	2-fold rotation	Glide reflection
Inversion	(Roto-inversion axis)	(2-fold rotation axis)	×	Inversion
Translation	(Screw axis)	(Glide plane)	(Inversion centre)	×

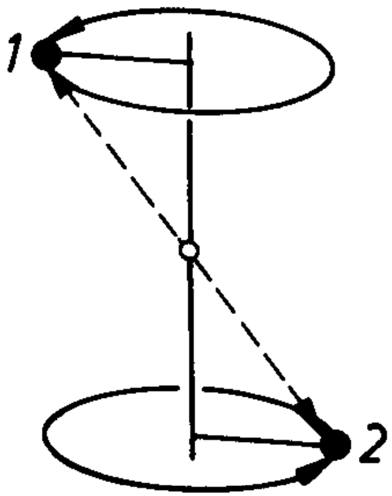
Rotoinversion

-compound symmetry operation of rotation and inversion

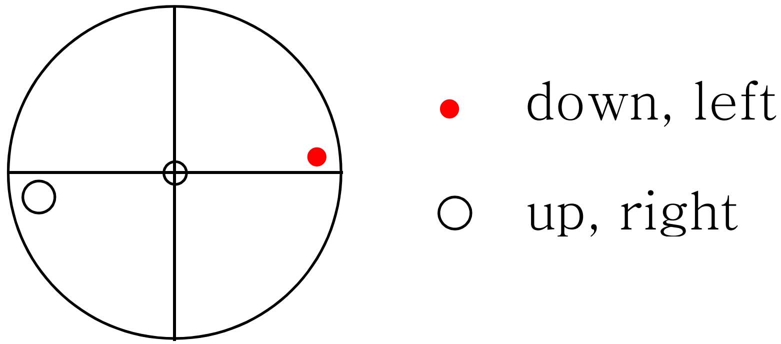
-rotoinversion axis $\bar{n}$

* 1, 2, 3, 4, 6 → $\bar{1}$, $\bar{2}$, $\bar{3}$, $\bar{4}$, $\bar{6}$

* $\bar{1}$

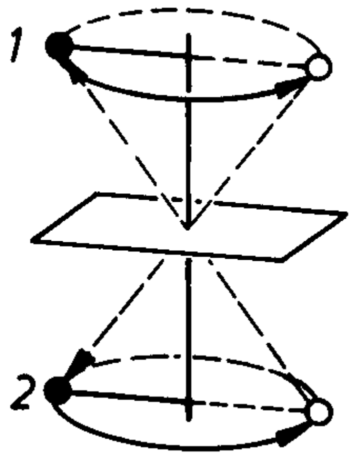


$\bar{1} \equiv$ **inversion centre**

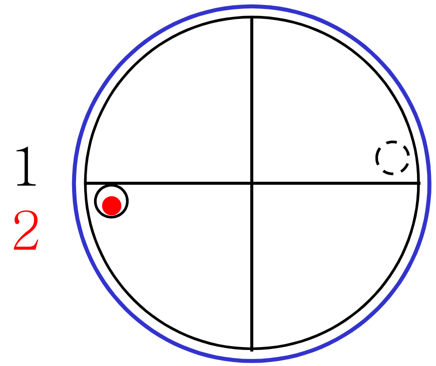


Rotoinversion

$\bar{2}(\equiv m)$

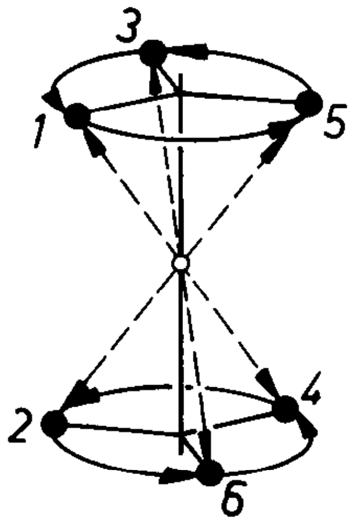


$\bar{2} \equiv m$

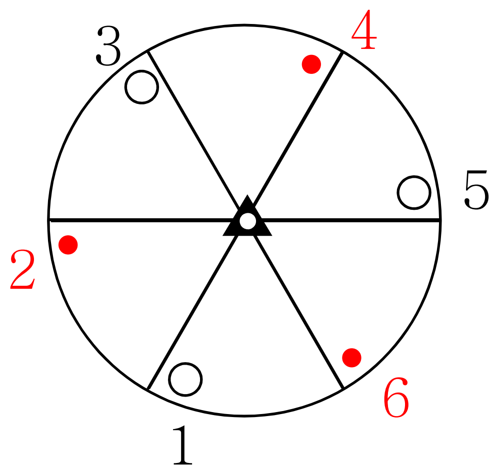


m_{xy} (m_z)

$\bar{3} \equiv 3 + \bar{1}$

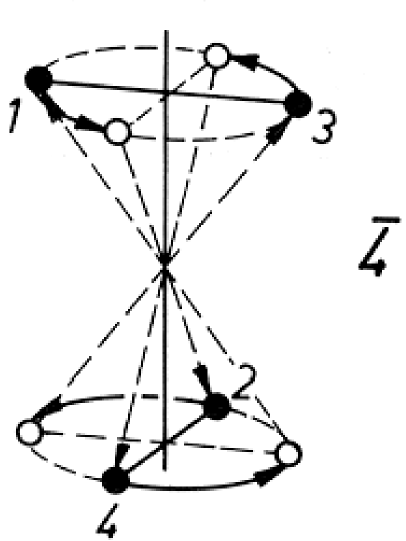


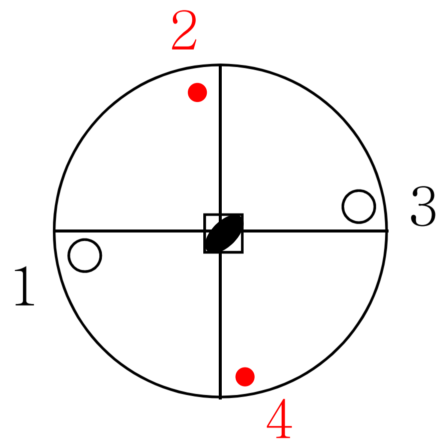
$\bar{3} \equiv 3 + \bar{1}$



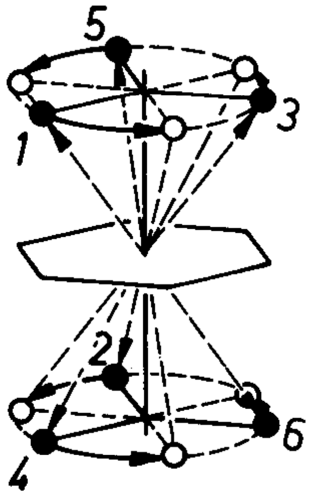
Rotoinversion

$\bar{4}$

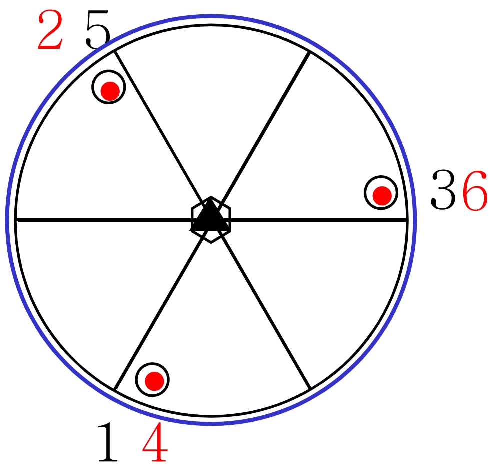




$\bar{6}$



$\bar{6} \equiv 3 \perp m$



Rotoinversion

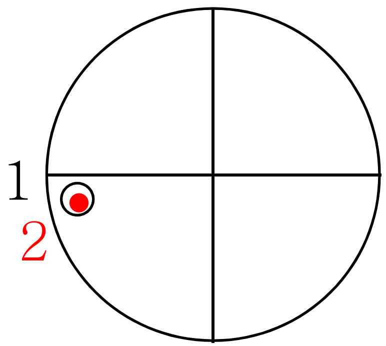
$\bar{1} \equiv$ inversion center, $\bar{2} \equiv m$, $\bar{3} \equiv 3 + \bar{1}$, $\bar{4}$ implies 2, $\bar{6} \equiv 3 \perp m$,

only rotoinversion axes of odd order imply the presence of an inversion center

Rotoreflection

$S_1 = m$ $S_2 = \bar{1}$ $S_3 = \bar{6}$ $S_4 = \bar{4}$ $S_6 = \bar{3}$

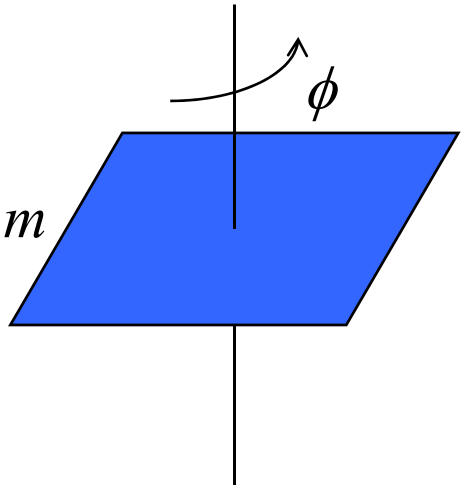
$S_1 = m$



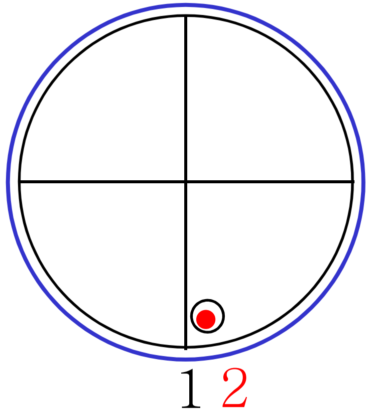
- The axes n and $\bar{n}$, including $\bar{1}$ and m, are called point-symmetry element, since their operations always leave at least one point unmoved

Combination

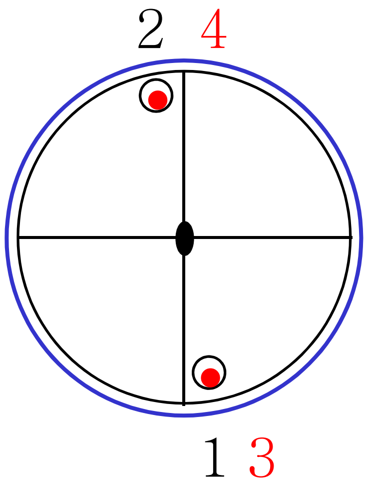
– a mirror plane is added normal to the rotation axis, $\dfrac{X}{m}$



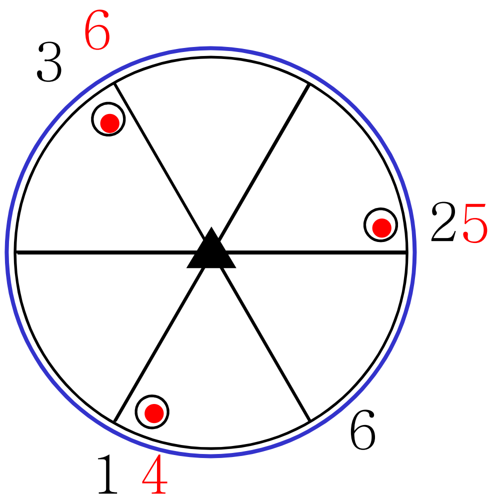
$\dfrac{1}{m}(\equiv m)$



$\dfrac{2}{m}(\equiv m)$



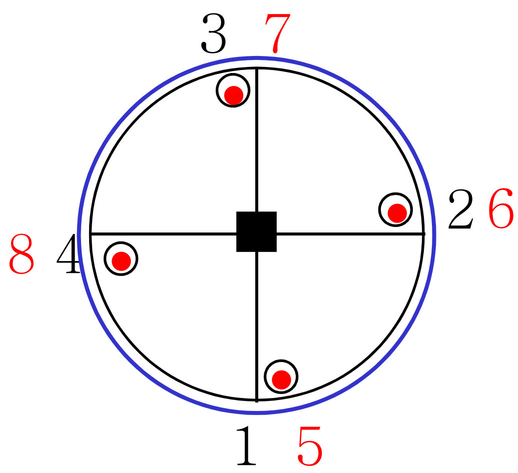
$\dfrac{3}{m}(\equiv \bar{6})$



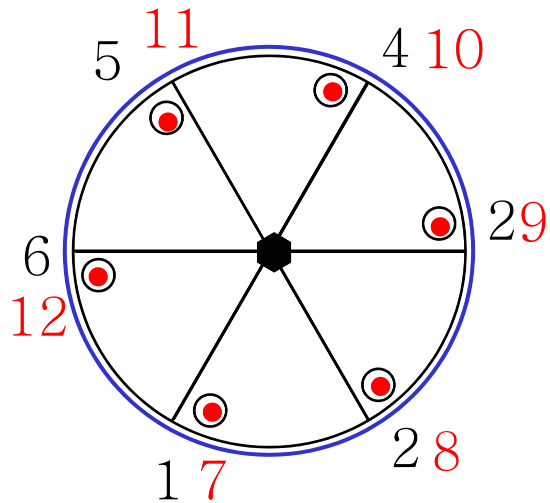
Combination

- a mirror plane is added normal to the rotation axis, $\frac{X}{m}$

$\frac{4}{m}$

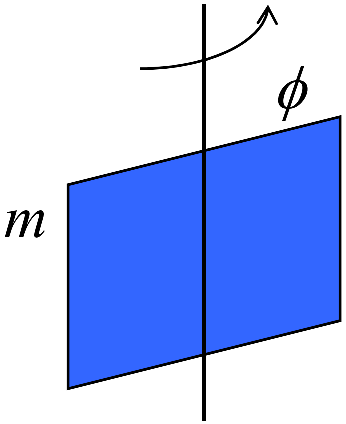


$\frac{6}{m}$

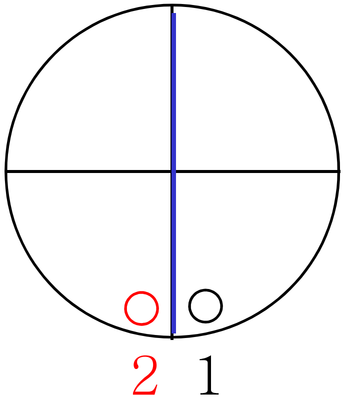


Combination

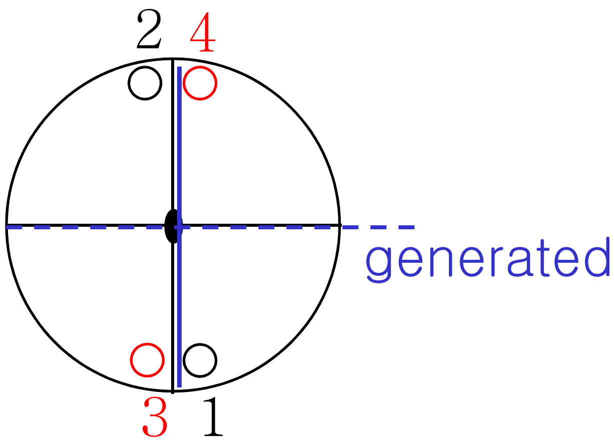
– a mirror plane is added normal to the rotation axis, Xm



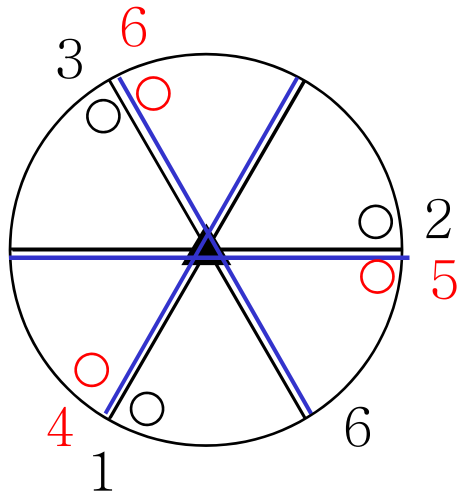
$1m$



$2m(\equiv 2mm, mm2)$



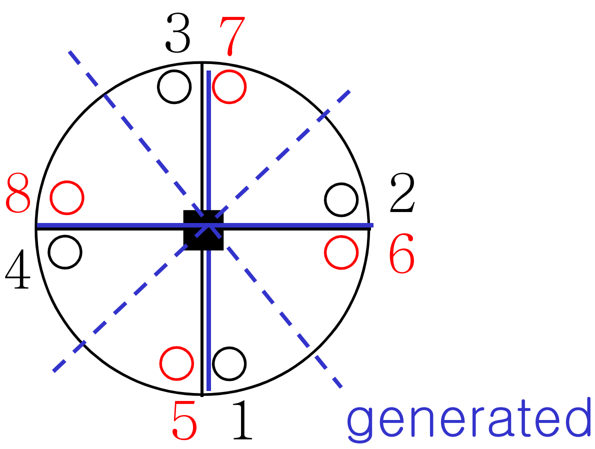
$3m$



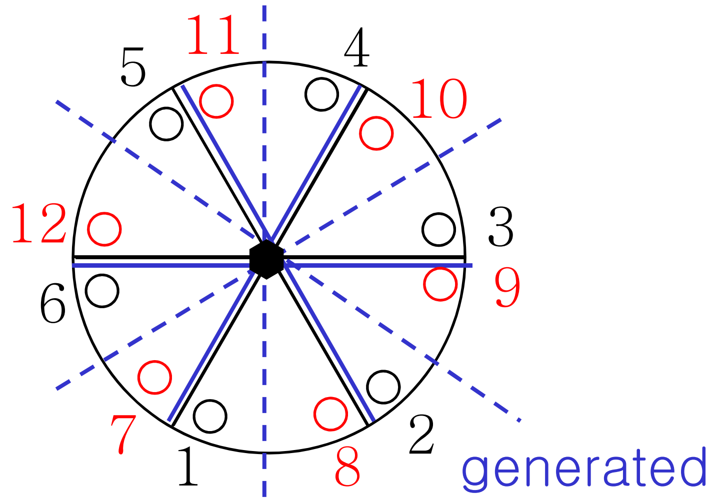
Combination

- a mirror plane is added normal to the rotation axis, Xm

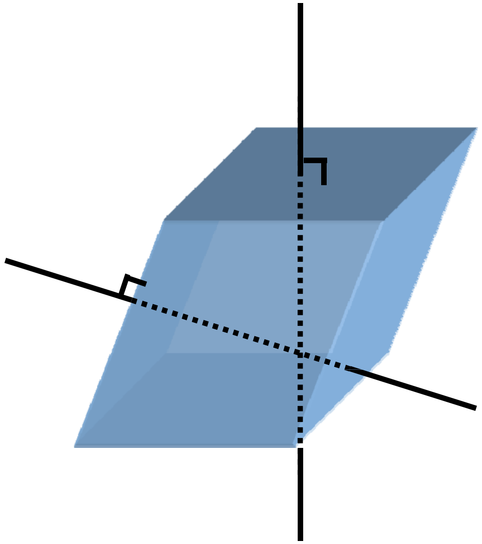
$4m(\equiv 4mm)$



$6m(\equiv 6mm)$



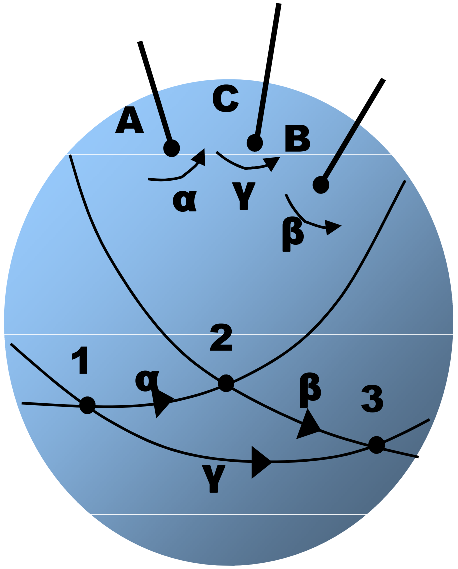
Simultaneous Rotational Symmetry



❖ Crystal could, conceivably, be symmetrical with respect to many different intersection n-fold axis

Two rotations about intersecting axes

→ inevitably create a third rotation equivalent to the combination



$$A_\alpha \cdot B_\beta = C_\gamma$$

Rotation around A axis to α

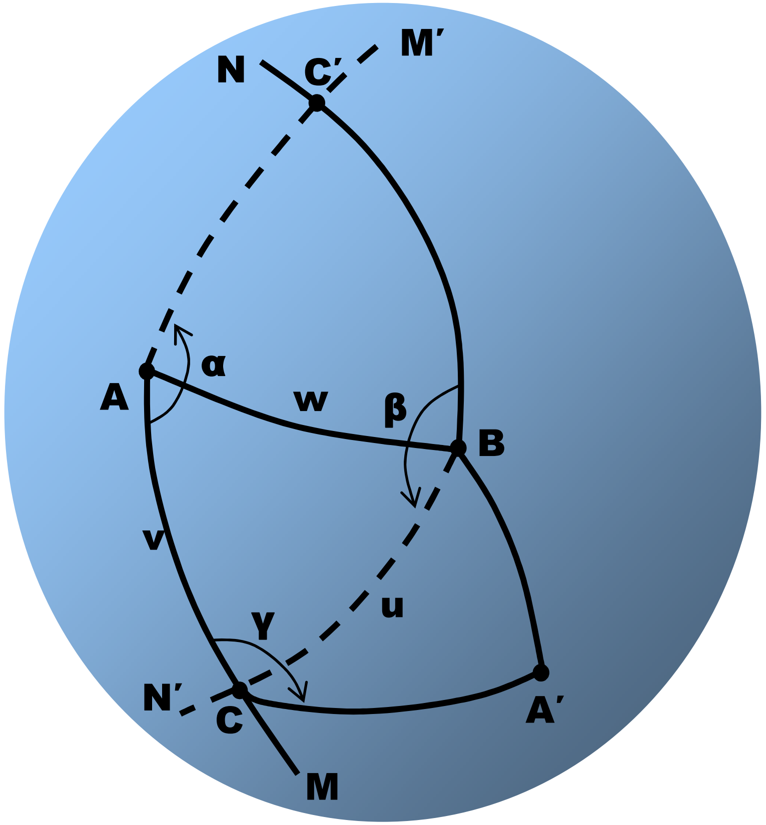
By the vector notation ;

$$\vec{s} + \vec{t} = \vec{R}$$

$$or \quad \vec{s} + \vec{t} - \vec{R} = 0$$

$$A_\alpha \cdot B_\beta \cdot C_{-\gamma} = 1 \quad (identical\ operation)$$

Euler's construction



The combined motions of Aα and Bβ have the following effect :

1 : Aα brings C to C′

2 : Bβ restores C′ to C

Thus, the combination of rotations Aα and Bβ leaves C unmoved.

Therefore, if there is a motion of points on the sphere due to Aα and Bβ, it must be a rotation about an axis OC

To calculate ;

1 : Aα leaves A unmoved

2 : Bβ moves A to A

Now consider $\Delta\, BA'C$

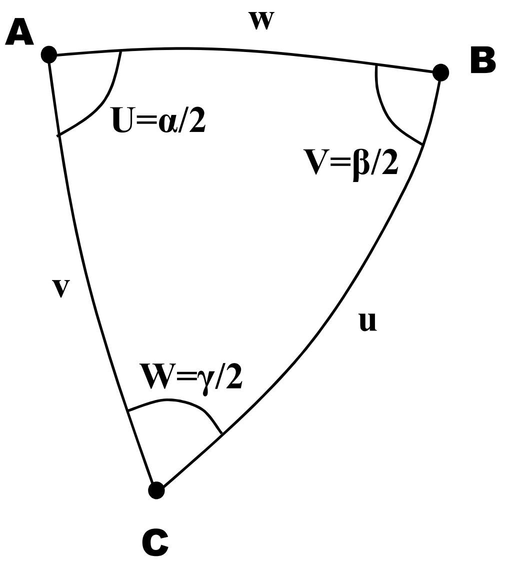
$$\angle ABC = \angle A'BC = \beta/2$$

$$AB = A'B$$

$$\therefore \Delta\, ABC = \Delta\, A'BC$$

$$\therefore \angle ACB = \angle A'CB = \gamma/2$$

Euler's construction



From the law of cosines

$$\cos w = \cos u \cos v + \sin u \sin v \cos W$$

since

$$u = 180^\circ - U \qquad U = 180^\circ - u$$
$$v = 180^\circ - V \qquad V = 180^\circ - v$$
$$w = 180^\circ - W \qquad W = 180^\circ - w$$

$$\cos(180^\circ - W) = \cos(180^\circ - U)\cos(180^\circ - V)$$
$$+ \sin(180^\circ - U)\sin(180^\circ - V)\cos(180^\circ - w)$$
$$\therefore -\cos W = \cos U \cos V - \sin U \sin V \cos w$$
$$or \quad \cos W = -\cos U \cos V + \sin U \sin V \cos W$$

$$\boxed{\therefore \cos w = \frac{\cos W + \cos U \cdot \cos V}{\sin U \sin V}}$$

Euler's construction

Data for crystallographic solutions of

$$\cos w = \frac{\cos W + \cos U \cos V}{\sin U \sin V}$$

Axis at A, B, or C	Throw of axis, α, β, or ɣ	U (=α/2) V(= β/2) or W(= ɣ/2)	cos U, V, W	sin U, V, W
1-fold	360°	180°	−1	0
2-fold	180°	90°	0	1
3-fold	120°	60°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
4-fold	90°	45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
6-fold	60°	30°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

Euler's construction

Combination	Form for first two rotations		cos w	w
222	$\frac{\cos W + 0\cdot 0}{1\cdot 1}$	$\frac{0+0}{1}$	$=0$	90°
223		$\frac{\frac{1}{2}+0}{1}$	$=\frac{1}{2}$	60°
224		$\frac{\frac{1}{\sqrt{2}}+0}{1}$	$=\frac{1}{\sqrt{2}}$	45°
226		$\frac{\frac{\sqrt{3}}{2}+0}{1}$	$=\frac{\sqrt{3}}{2}$	30°
233	$\frac{\cos W + 0\cdot\frac{1}{2}}{1\cdot\frac{\sqrt{3}}{2}}$	$\frac{\frac{1}{2}+0}{\frac{\sqrt{3}}{2}}$	$=\frac{1}{\sqrt{3}}$	54° 44′
234		$\frac{\frac{1}{\sqrt{2}}+0}{\frac{\sqrt{3}}{2}}$	$=\frac{2}{\sqrt{6}}$	35° 16′
236		$\frac{\frac{\sqrt{3}}{2}+0}{\frac{\sqrt{3}}{2}}$	$=1$	0°

nbination	Form for first two rotations		cos w	w
244	$\frac{\cos W + 0\cdot\frac{1}{\sqrt{2}}}{1\cdot\frac{1}{\sqrt{2}}}$	$\frac{\frac{1}{\sqrt{2}}+0}{\frac{1}{\sqrt{2}}}$	$=1$	0°
246		$\frac{\frac{\sqrt{3}}{2}+0}{\frac{1}{\sqrt{2}}}$	$=\frac{\sqrt{6}}{2}>1$	—
266	$\frac{\cos W + 0\cdot\frac{\sqrt{3}}{2}}{1\cdot\frac{1}{2}}$	$\frac{\frac{\sqrt{3}}{2}+0}{\frac{1}{2}}$	$=\sqrt{3}>1$	—
333	$\frac{\cos W + \frac{1}{2}\cdot\frac{1}{2}}{\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}}$	$\frac{\frac{1}{2}+\frac{1}{4}}{\frac{3}{4}}$	$=1$	0°
334		$\frac{\frac{1}{\sqrt{2}}+\frac{1}{4}}{\frac{3}{4}}$	$=\frac{2\sqrt{2}+1}{3}>1$	—
336		$\frac{\frac{\sqrt{3}}{2}+\frac{1}{4}}{\frac{3}{4}}$	$=\frac{2\sqrt{3}+1}{3}>1$	—
344	$\frac{\cos W + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}}$	$\frac{\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}}{\frac{\sqrt{3}}{2\sqrt{2}}}$	$=\frac{3}{\sqrt{3}}>1$	—
346		$\frac{\frac{\sqrt{3}}{2}+\frac{1}{2\sqrt{2}}}{\frac{\sqrt{3}}{2\sqrt{2}}}$	$=\frac{\sqrt{6}+1}{\sqrt{3}}>1$	—

Euler's construction

Combination	Form for first two rotations	$\cos w$	w
366	$\dfrac{\cos W + \frac{1}{2}\cdot\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}\cdot\frac{1}{2}}$	$\dfrac{\frac{\sqrt{3}}{2}+\frac{1}{2}\cdot\frac{\sqrt{3}}{2}}{\frac{1}{2}\cdot\frac{\sqrt{3}}{2}} = \dfrac{1+\frac{1}{2}}{\frac{1}{2}} = 3$	—
444	$\dfrac{\cos W + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}}$	$\dfrac{\frac{1}{\sqrt{2}}+\frac{1}{2}}{\frac{1}{2}} = \dfrac{2}{\sqrt{2}}+1>1$	—
446		$\dfrac{\frac{\sqrt{3}}{2}+\frac{1}{2}}{\frac{1}{2}} = \sqrt{3}+1>1$	—
466	$\dfrac{\cos W + \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}\cdot\frac{1}{2}}$	$\dfrac{\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}\cdot\frac{1}{2}} = \sqrt{3}(\sqrt{2}+1)>1$	—
666	$\dfrac{\cos W + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}}{1 \quad 1}$	$\dfrac{\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}}{1 \quad 1} = 2\sqrt{3}+3>1$	—

Euler's construction

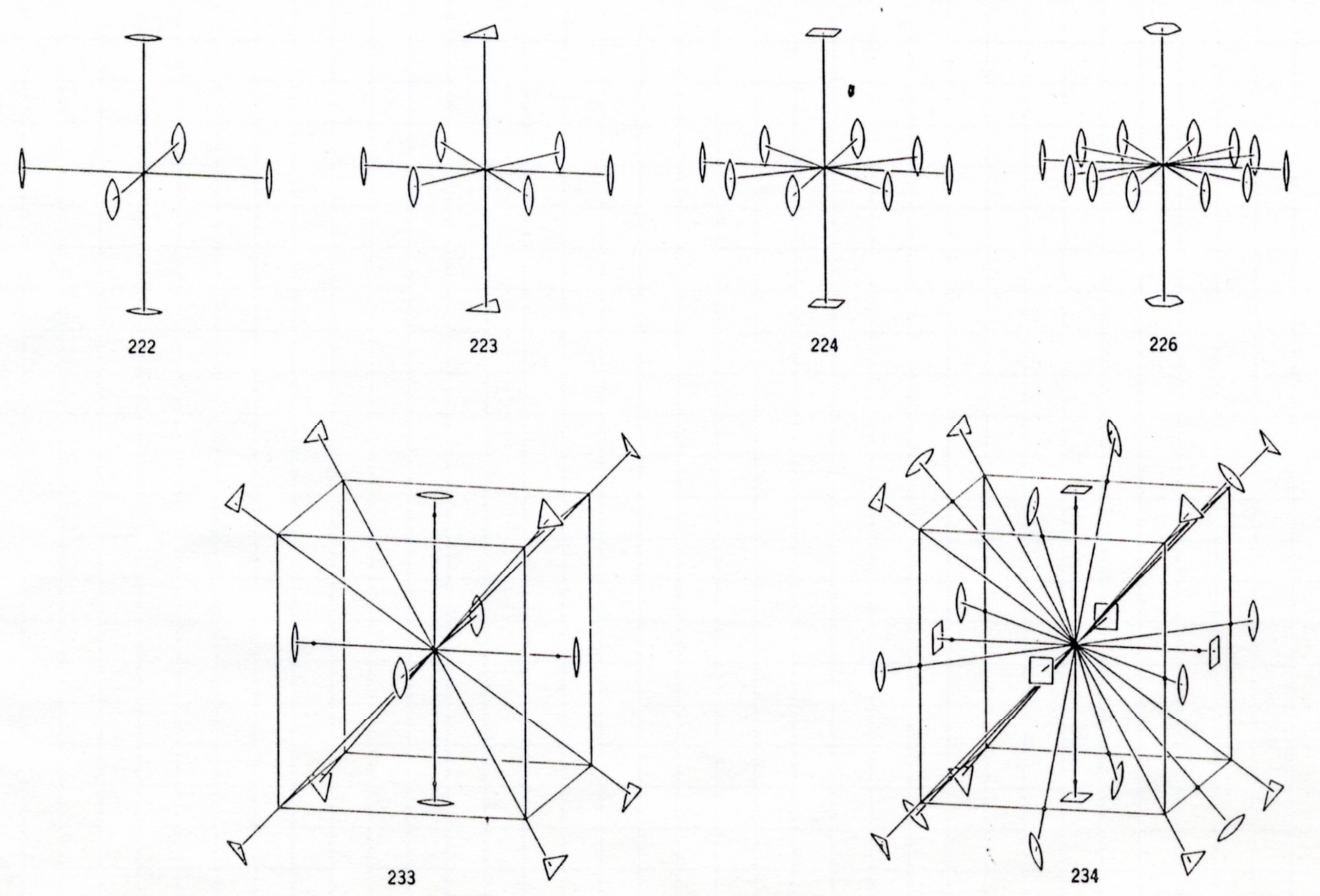
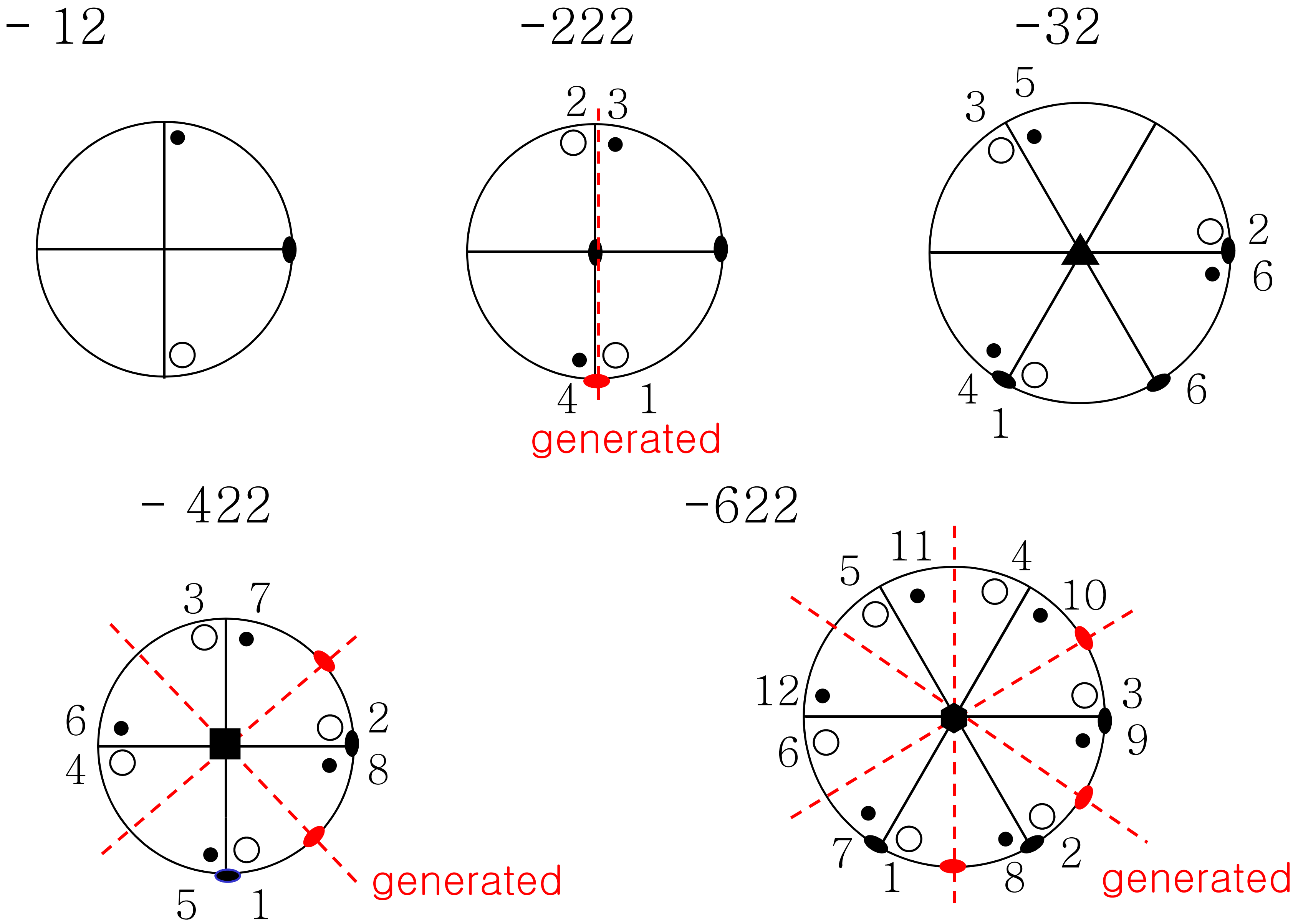


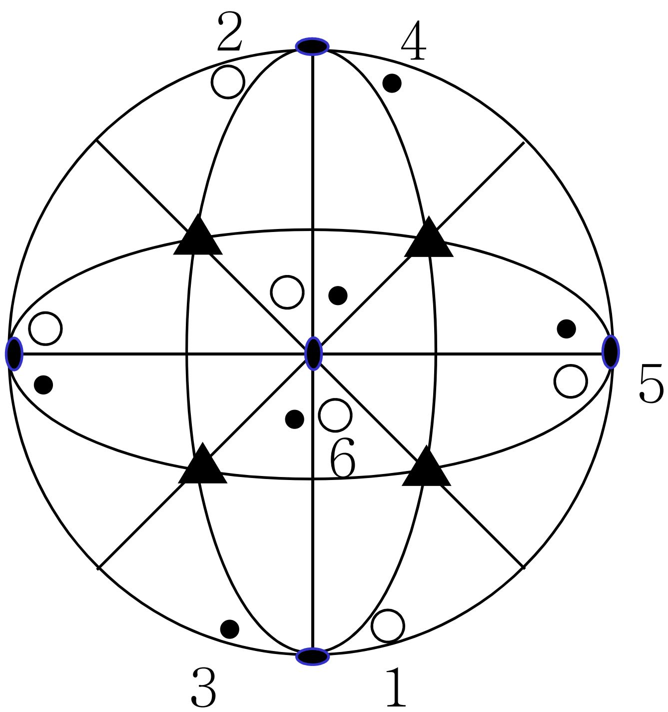
FIG. 14. The six crystallographic axial symmetries based upon the combinations in Fig. 13.

Combination of rotation axes, n2

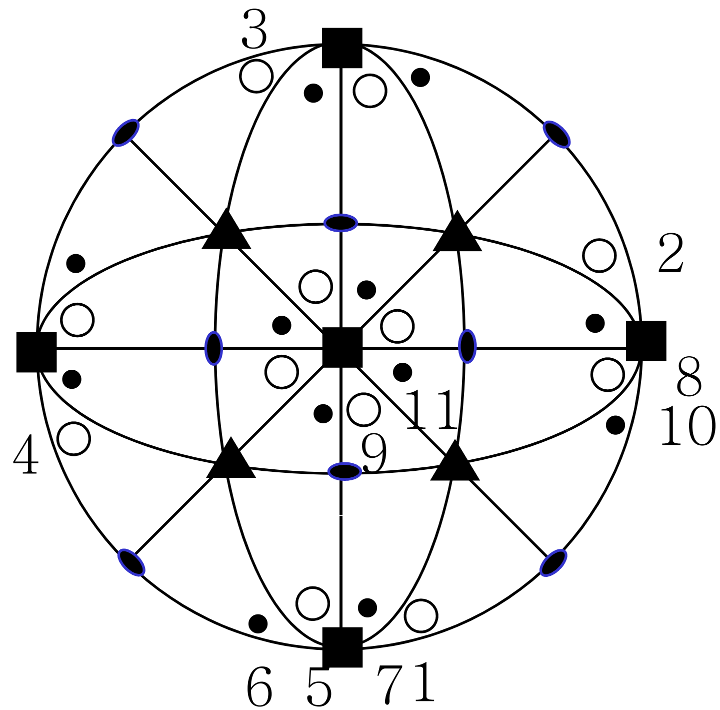


Combination of rotation axes

- 23

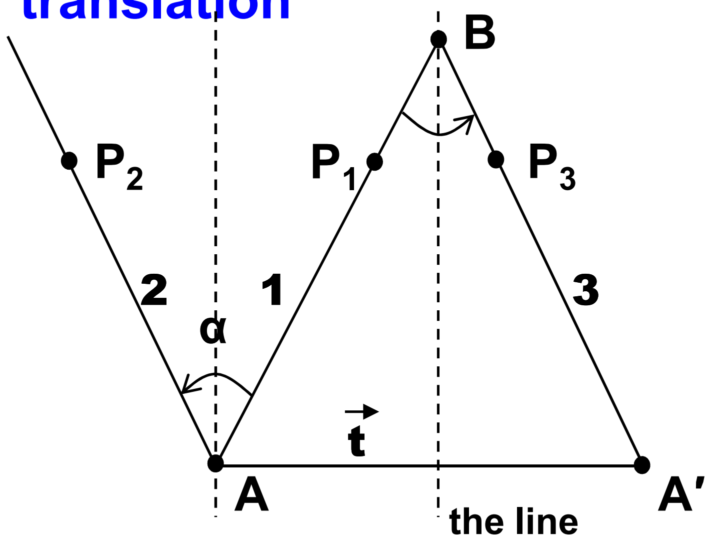


-432



The Symmetrical Plane Lattices

Combination of a rotation and a perpendicular translation



Aα causes $P_1 \rightarrow P_2$

then $\vec{t}$ causes $P_2 \rightarrow P_3$

$\therefore$ A$\alpha \cdot \vec{t}$ causes $P_1 \rightarrow P_3$

$\Rightarrow$ The net motion of the line is therefore equivalent to a rotation about B.

Furthermore, since the line is embedded in space, and since the operations Aα and t action all spaces,

$\Rightarrow$ All space must also be rotated about B by this combination of operations.

Point B : lines on the bisector of AA′

distance (AA′/2)cot(α /2) from AA′

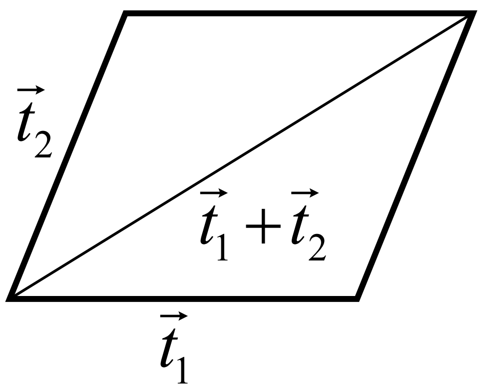
This can be expressed analytically as

$$A\alpha \cdot t_{\perp} = B\alpha$$

The Symmetrical Plane Lattices

Combination of the rotation axes with a plane lattice :

General principles

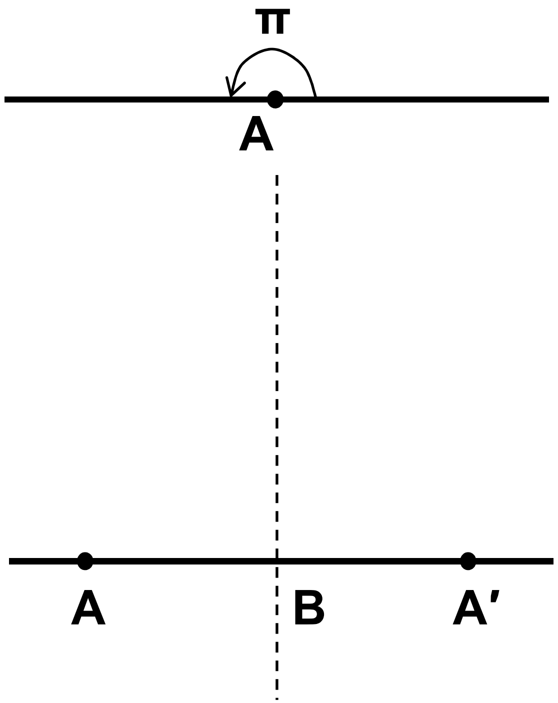


1) A rotation axis implies, in general, several related rotations

- **When the symmetry axis is n-fold, the smallest rotation is α = 2π/n**
- **The rotation axis then implies the rotations α, 2α, … , nα, where nα = 2π**
- **Each of these rotations is to be combined with the translation**

2) Each rotation must be combined with the various translations of the plane lattice such as t_1, t_2, t_1+t_2

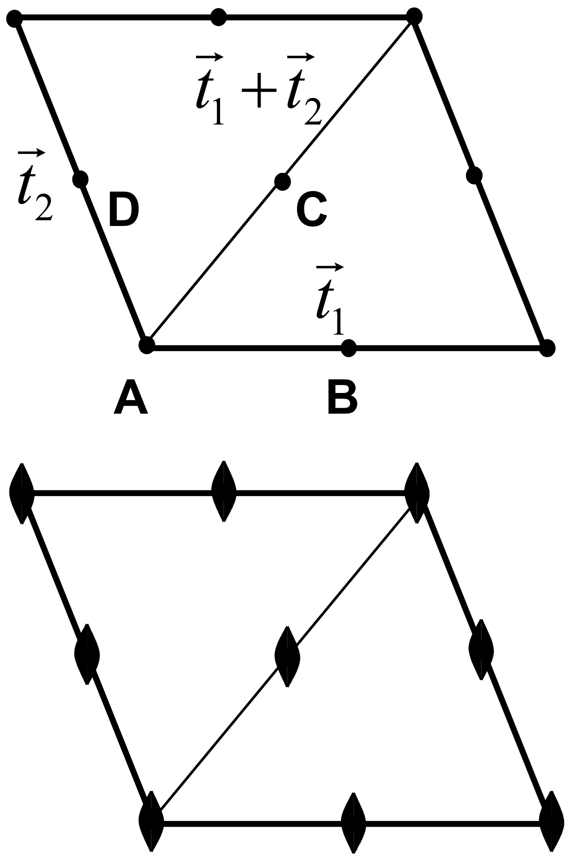
A) 2-fold axis : rotation A_π



$\therefore$ **Aπ · t = Bπ**

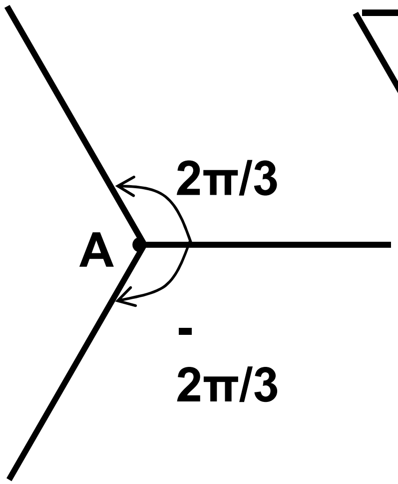
$\therefore$ **B lies in the bisector**

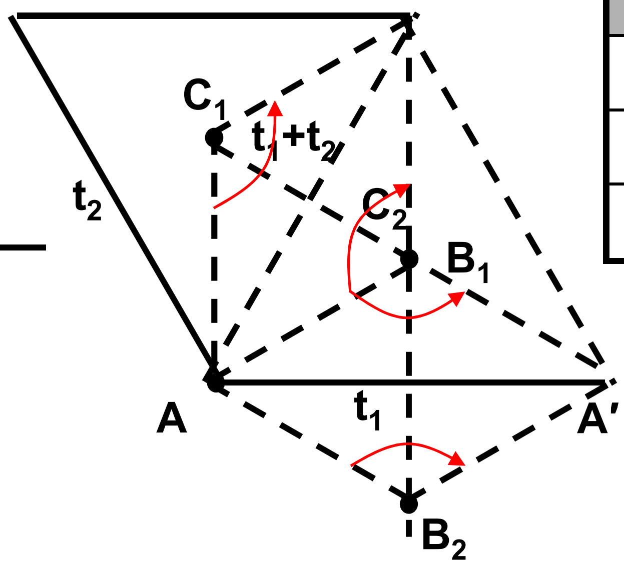
(AA'/2) cot (α/2) =0



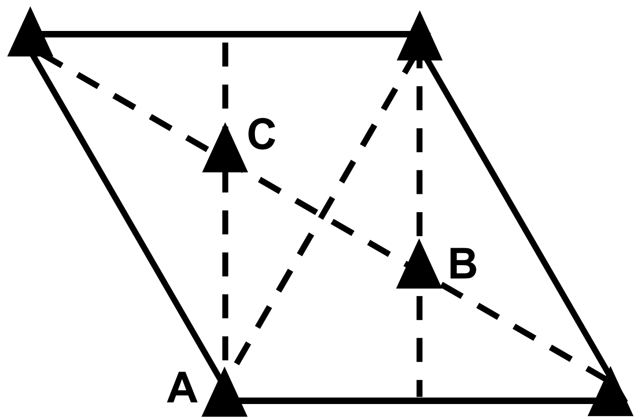
Has 4 non-equivalent 2-fold axis

B) 3-fold axis





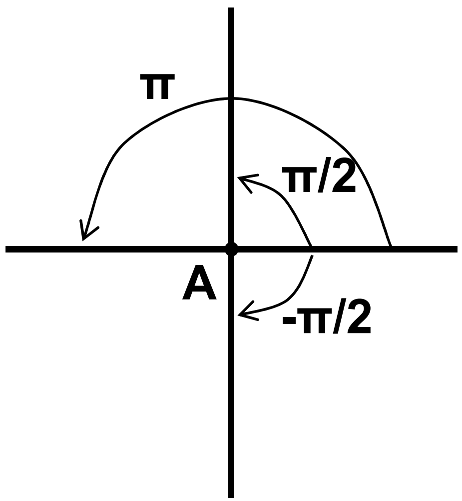
Rotation at A	Translation	
	t_1	t_1+t_2
1	1	1
$2\pi/3$	$2\pi/3$ at B_1	$2\pi/3$ at C_1
$-2\pi/3$	$-2\pi/3$ at $B_2(C_1)$	$-2\pi/3$ at $C_2(B_1)$

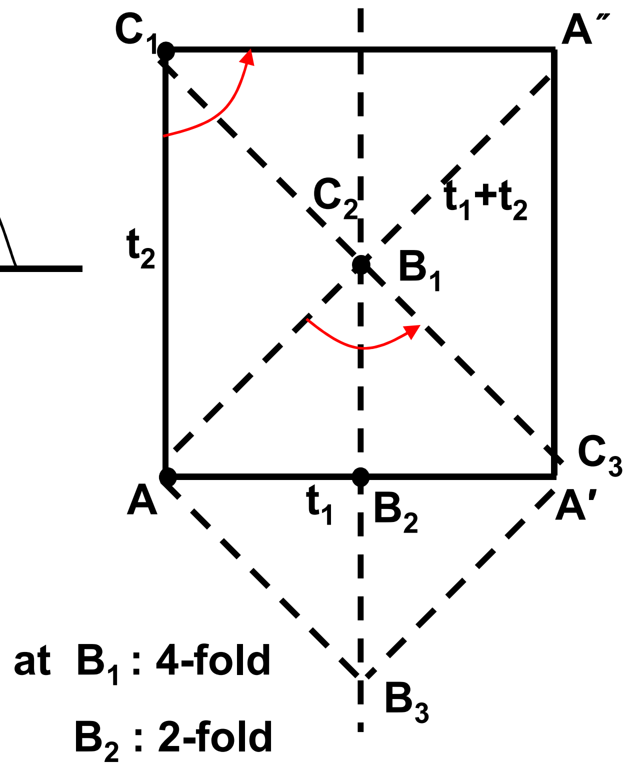


It has three non-equivalent 3-fold axis located in the cell at A, B, C.

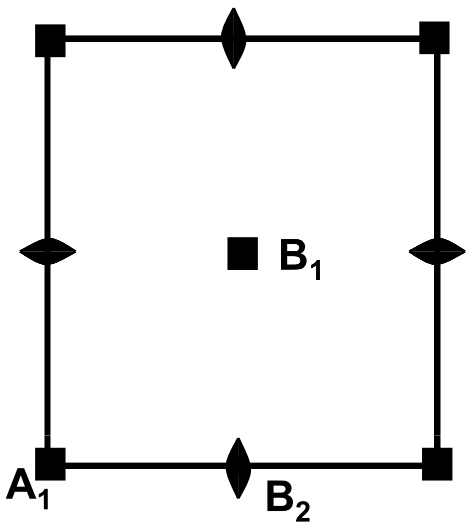
C) 4-fold axis

The operations of the 4-fold axis are rotations of 1, π/2, π, 3π/2(= -π/2)





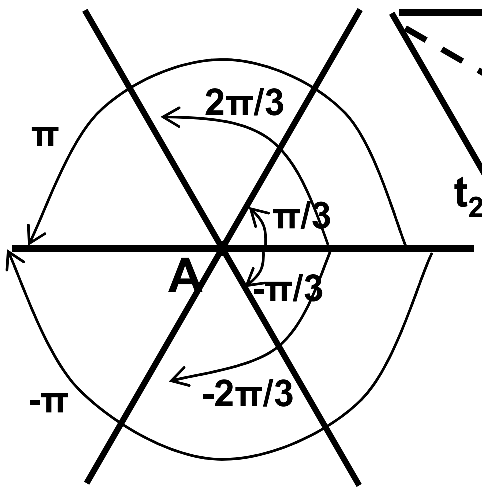
Rotation at A	translations	
	t_1	t_1+t_2
1	1	1
$\pi/2$	$\pi/2$ at B_1	$\pi/2$ at C_1
π	π at B_2	π at C_2 (B_1)
$-\pi/2$	$-\pi/2$ at B_3(B_1)	$-\pi/2$ at C_3 (A')

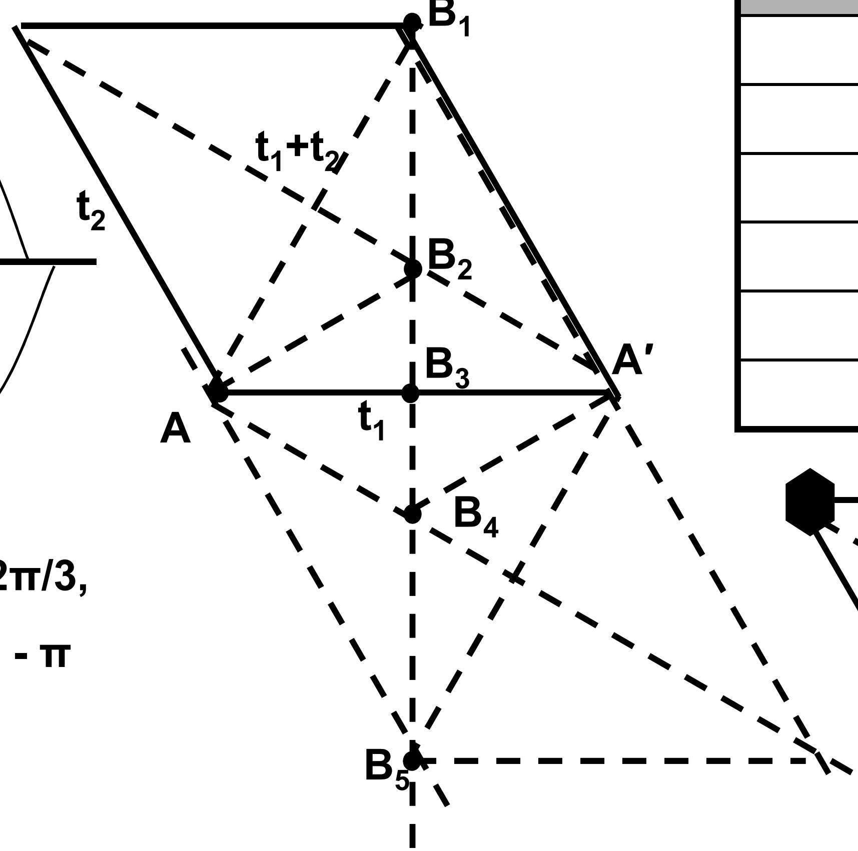


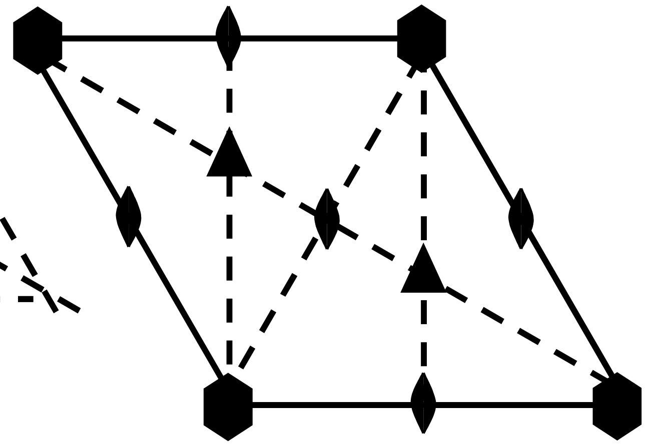
C) 6-fold axis

When a lattice plane has 6-fold axis, then $t_1 = t_2 = t_1+t_2 \rightarrow$ Thus, only consider t_1

Rotation at A	Translations (t_1)
1	1
$\pi/3$	$\pi/3$ at B_1 (A)
$2\pi/3$	$2\pi/3$ at B_2
π	π at B_3
$-2\pi/3$	$-2\pi/3$ at B_4 (B_2)
$-\pi/3$	$-\pi/3$ at B_5







Rotation : 1, π /3, 2π/3,

π, -2π/3, - π
/3

Lattice types consistent with plane symmetries of the 2nd sort

Lemma : For n>2, the shape of a plane lattice consistent with pure axial symmetry *n* is also consistent with corresponding symmetry *nmm* containing reflection planes *m*

→ Therefore, new lattice-plane mesh types can only be found by causing a general lattice type to be consistent either with symmetry m (or 2mm)

Fig. A

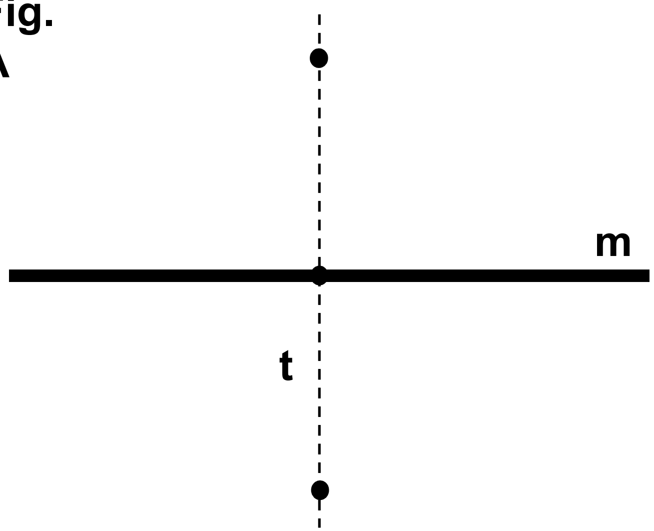
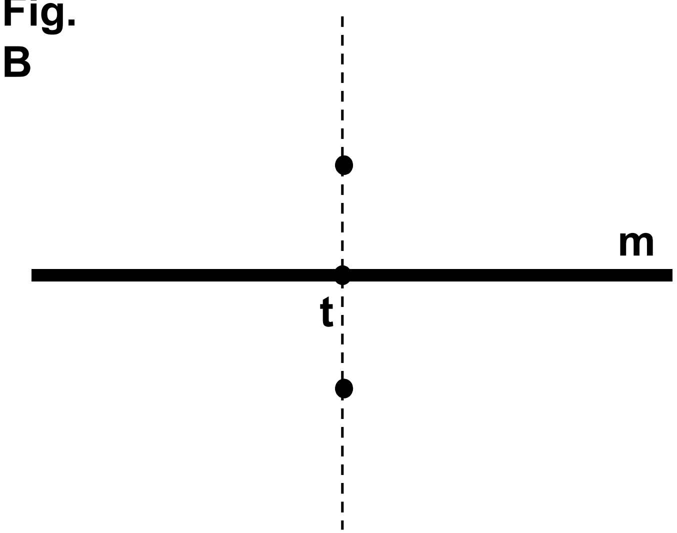


Fig. B



Lattice types consistent with plane symmetries of the 2nd sort

Fig. C

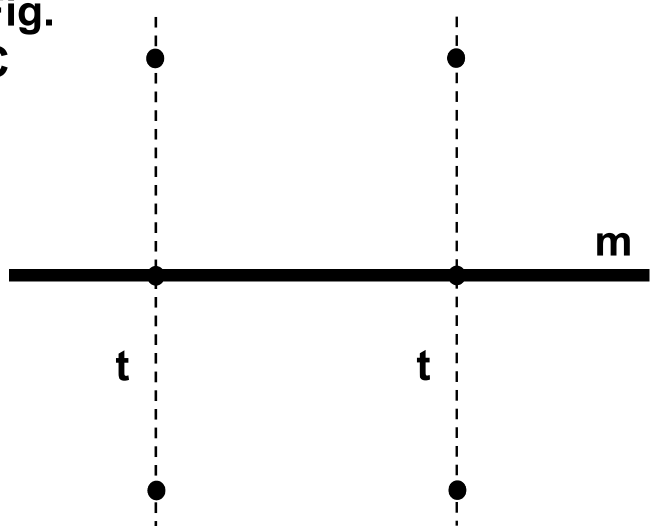


Fig. D

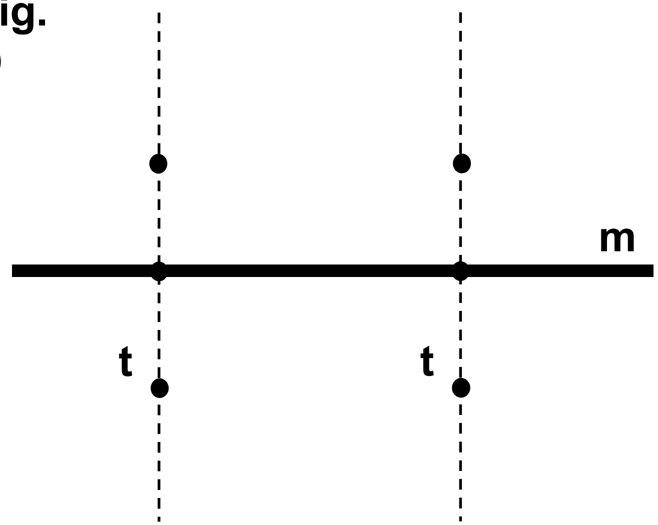
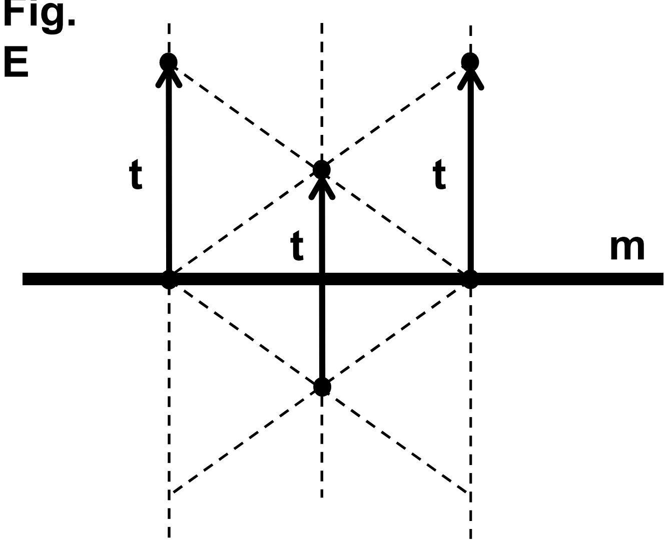


Fig. E



Lattice types consistent with plane symmetries of the 2nd sort

Fig. F

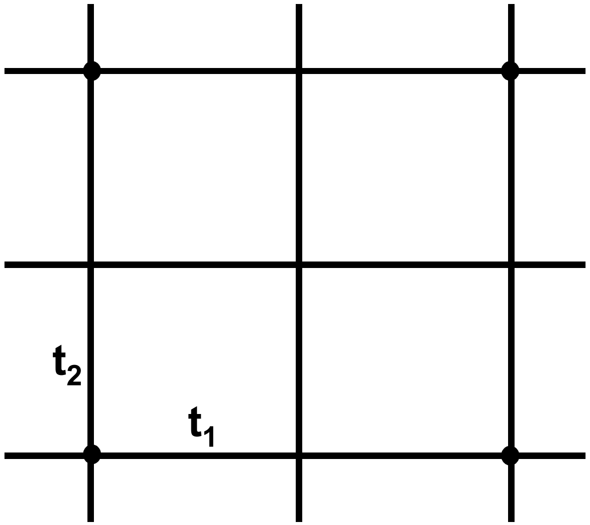
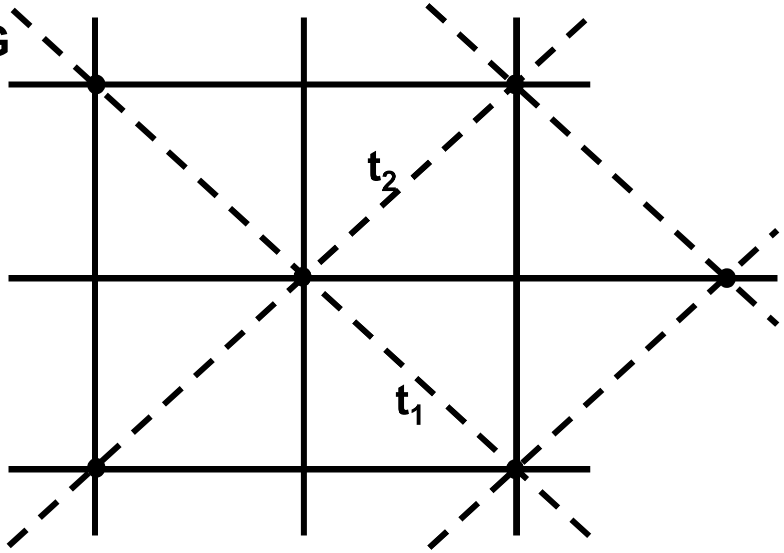
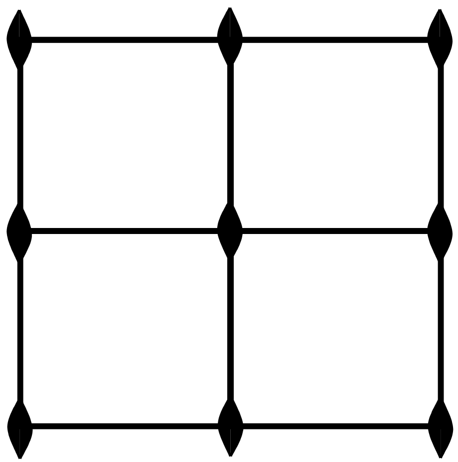
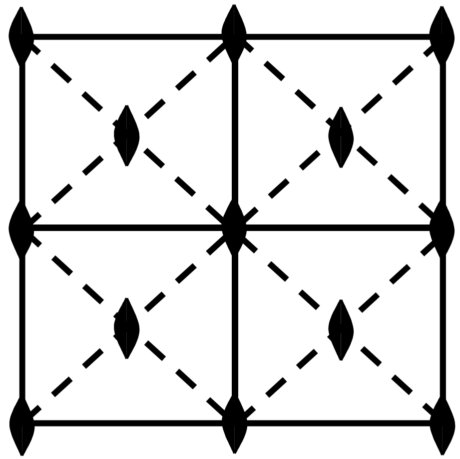


Fig.G



(Diamond Rhombous)



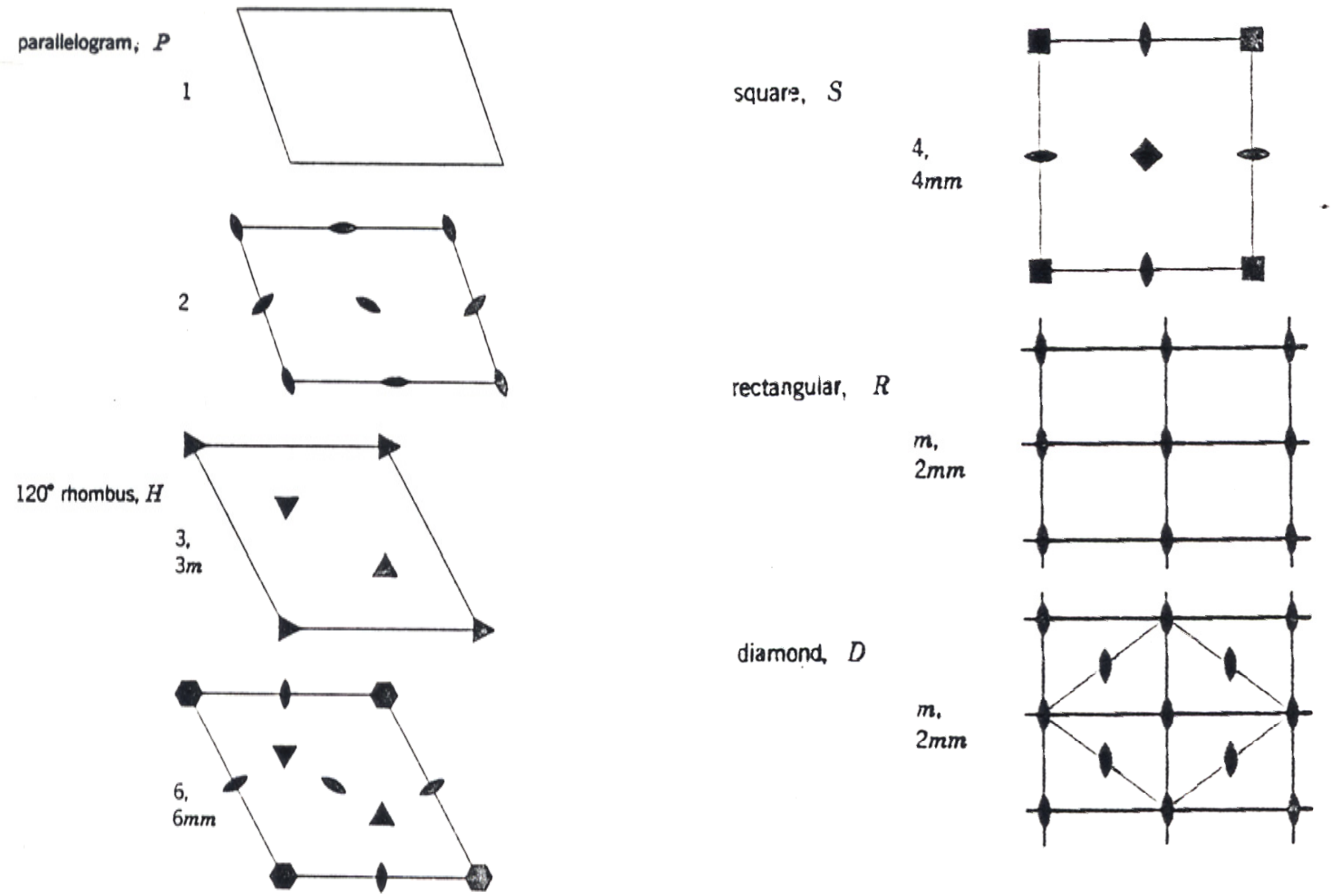


Lattice types consistent with plane symmetries of the 2nd sort

Table 5. The plane-lattice types consistent with the 10 plane point groups

Mesh shape	Symbol	Consistent with plane symmetries
Parallelogram	*P*	1, 2
120°-rhombus	*H*	3, 3*m*, 6, 6*mm*
Square	*S*	4, 4*mm*
Rectangle	*R*	*m*, 2*mm*
Diamond (rhombus)	*D*	*m*, 2*mm*

Lattice types consistent with plane symmetries of the 2nd sort



The distribution of rotation axes and mirrors in the five plane lattice types