Space Group

- 32 point groups- symmetry groups of many molecules
  and of all crystals so long as morphology
  is considered
- space group- symmetry of crystal lattices and
  crystal structures

  14 Bravais lattice

  centered lattices- new symmetry operations

  reflection + translation

  rotation + translation

Space Lattice

- 14 Bravais lattice

	P	C	I	F
Triclinic	$P\bar{1}$			
Monoclinic	$P\,2/m$	$C\,2/m$		
Orthorhombic	$P\,2/m\,2/m\,2/m$	$C\,2/m\,2/m\,2/m$	$I\,2/m\,2/m\,2/m$	$F\,2/m\,2/m\,2/m$
Tetragonal	$P\,4/m\,2/m\,2/m$		$I\,4/m\,2/m\,2/m$	
Trigonal	$P\,6/m\,2/m\,2/m$	$R\,\bar{3}\,2/m$		
Hexagonal				
Cubic	$P\,4/m\,\bar{3}\,2/m$		$I\,4/m\,\bar{3}\,2/m$	$F\,4/m\,\bar{3}\,2/m$

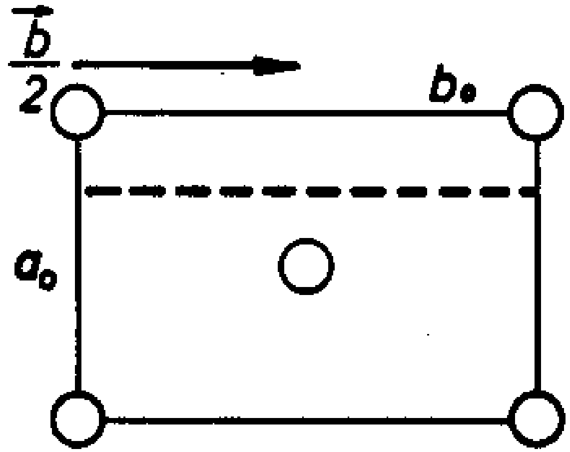
The 14 Bravais lattice represent the 14 and only way in which it is possible to fill space by a three-dimensional periodic array of points.

표 1.1 결정계, 결정축계, Bravais 격자

결정 패밀리	결정계	결정축계	격자 상수	Bravais 격자
입 방 (cubic)	입 방	입 방	$a = b = c$ $\alpha = \beta = \gamma = 90°$	P, I, F
육 방 (hexagonal)	육 방	육 방	$a = b \neq c$ $\alpha = \beta = 90°$, $\gamma = 120°$	P
	삼 방 (trigonal)	능 면 (rhombohedral)	$a = b = c$ $\alpha = \beta = \gamma \neq 90°$	R
정 방 (tetragonal)	정 방	정 방	$a = b \neq c$ $\alpha = \beta = \gamma = 90°$	P, I
사 방 (orthorhombic)	사 방	사 방	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90°$	P, $C(A, B)$, I, F
단 사 (monoclinic)	단 사	단 사	1. c-unique $a \neq b \neq c$ $\alpha = \beta = 90° \neq \gamma$	(P), (A)
			2. b-unique $a \neq b \neq c$ $\alpha = \gamma = 90° \neq \beta$	P, C
삼 사 (triclinic)	삼 사	삼 사	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90°$	P

New Symmetry Operations

i) orthorhombic C-lattice



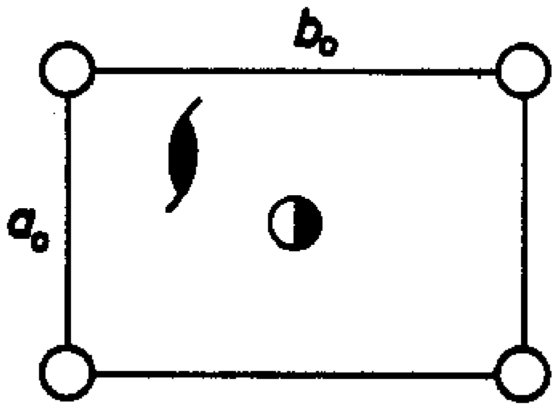
reflection at $\frac{1}{4}, y, z$

+ translation $\frac{\vec{b}}{2}$

$0,0,0 \rightarrow \frac{1}{2}, \frac{1}{2}, 0$

glide reflection
glide plane (b-glide)

ii) orthorhombic I-lattice



rotation about at $\frac{1}{4}, \frac{1}{4}, z$

+ translation $\frac{\vec{c}}{2}$

$0,0,0 \rightarrow \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

screw rotation
screw axis (2_1-screw)

Compound Symmetry Operation

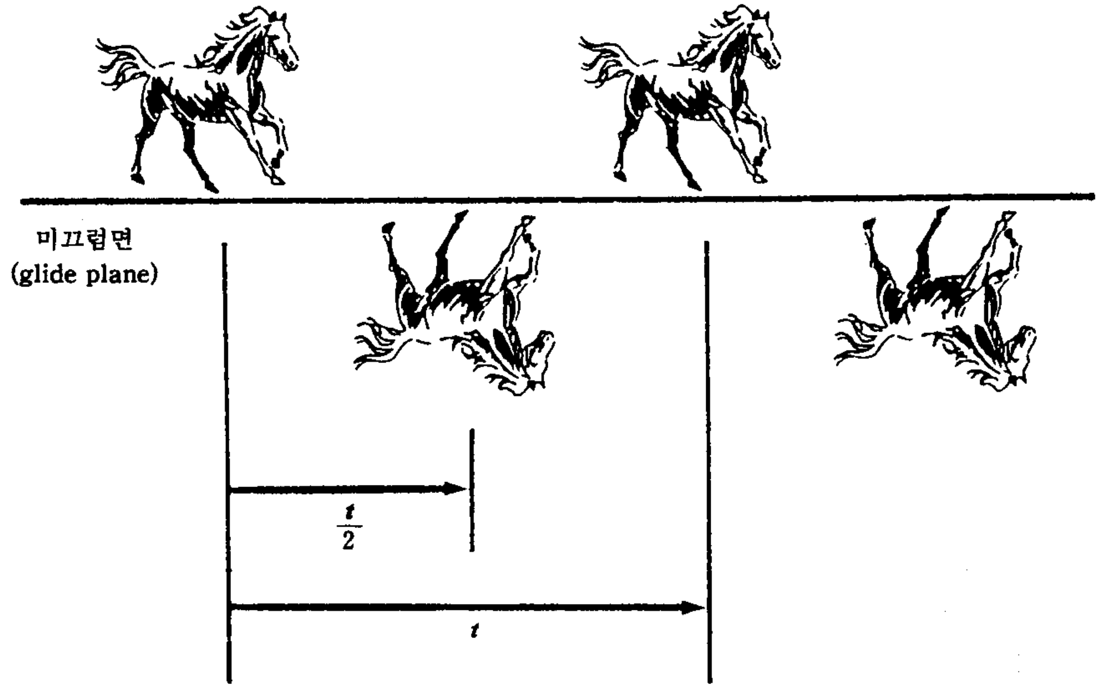
Table 5.1. Compound symmetry operations of simple operations. The corresponding symmetry elements are given in round brackets

	Rotation	Reflection	Inversion	Translation
Rotation	×	Roto-reflection	Roto-inversion	Screw rotation
Reflection	(Roto-reflection axis)	×	2-fold rotation	Glide reflection
Inversion	(Roto-inversion axis)	(2-fold rotation axis)	×	Inversion
Translation	(Screw axis)	(Glide plane)	(Inversion centre)	×

Glide Plane

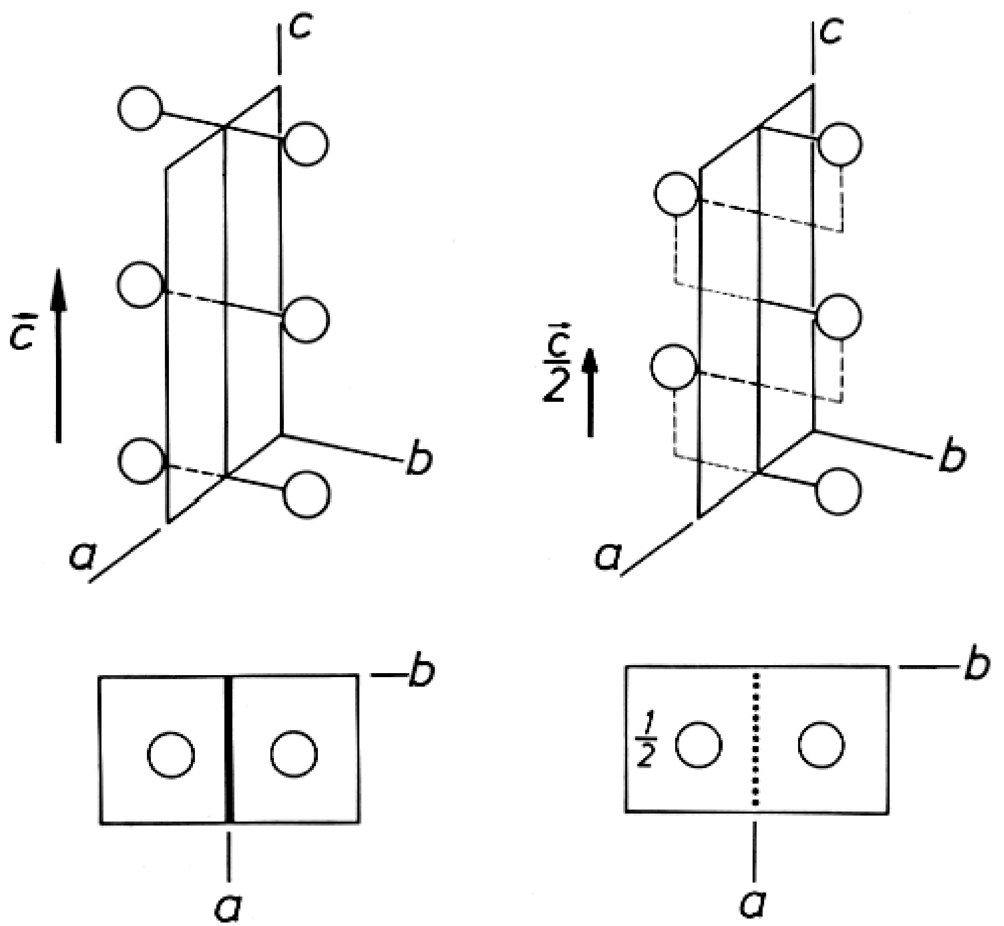
i) reflection

ii) translation by the vector $\vec{g}$ parallel to the plane of glide reflection where $|\vec{g}|$ is called glide component



$\vec{g}$ is one half of a lattice translation parallel to the glide plane $|\vec{g}| = \frac{1}{2}|\vec{t}|$

Mirror Plane vs. Glide Plane



- glide plane can occur in an orientation that is possible for a mirror plane

Glide Plane

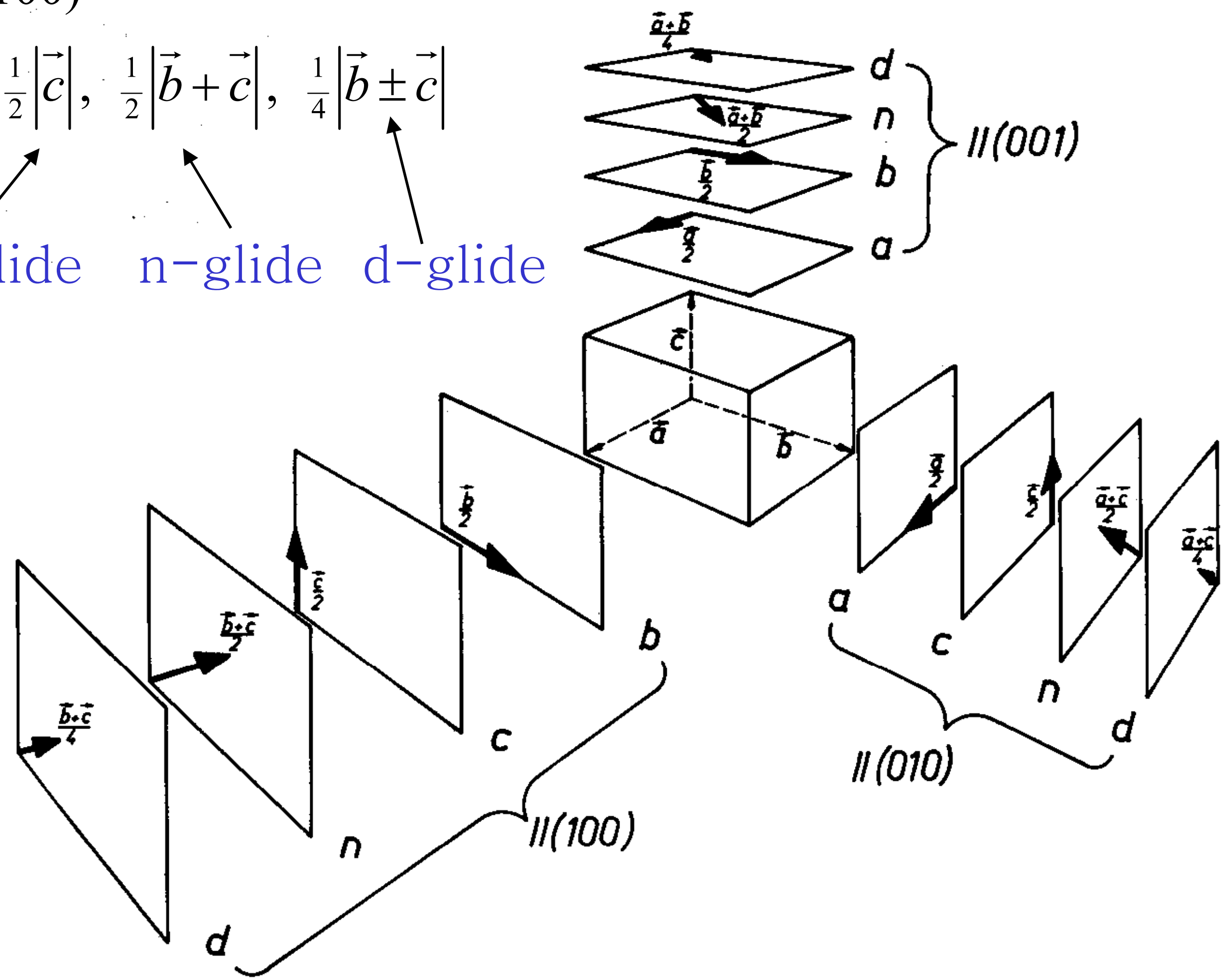
- orthorhombic $P\frac{m}{2}\frac{m}{2}\frac{m}{2}$

(100), (010), (001) possible

glide plane parallel to (100)

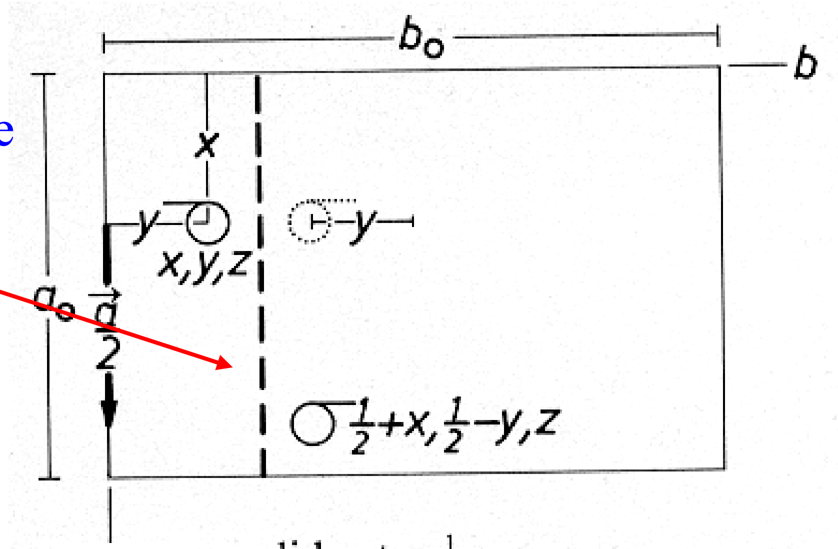
glide component $\frac{1}{2}\left|\vec{b}\right|,\ \frac{1}{2}\left|\vec{c}\right|,\ \frac{1}{2}\left|\vec{b}+\vec{c}\right|,\ \frac{1}{4}\left|\vec{b}\pm\vec{c}\right|$

b-glide c-glide n-glide d-glide



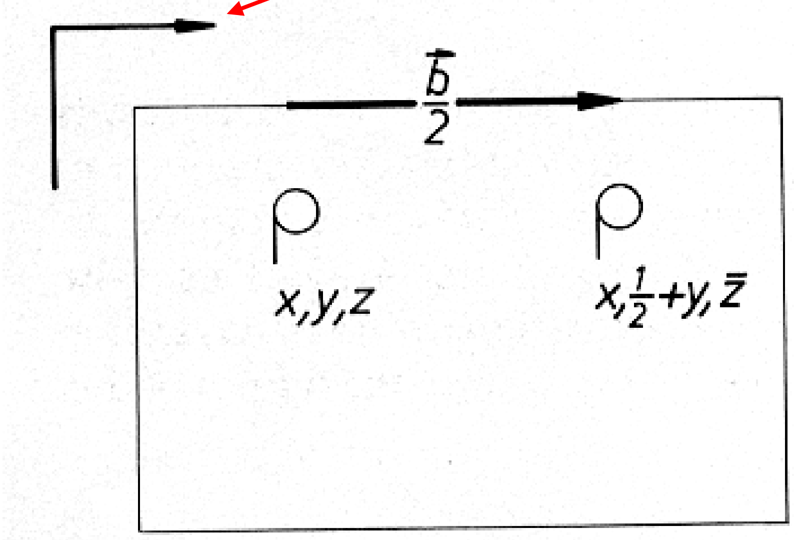
Glide Plane

$\frac{1}{2}$ along the line parallel to the projection plane

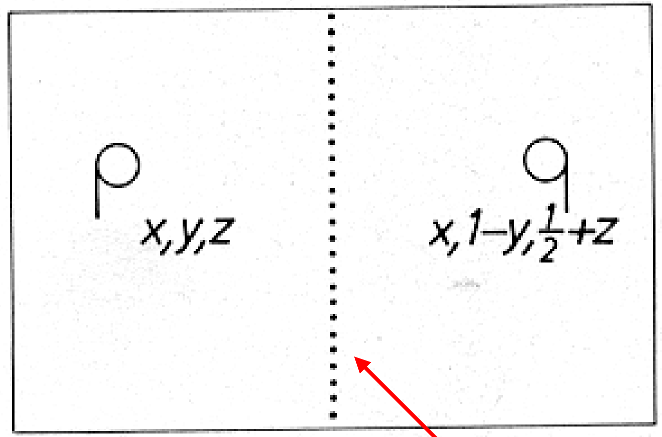


a a-glide at x, $\frac{1}{4}$ z

$\frac{1}{2}$ in the direction of the arrow



b b-glide at x, y, 0



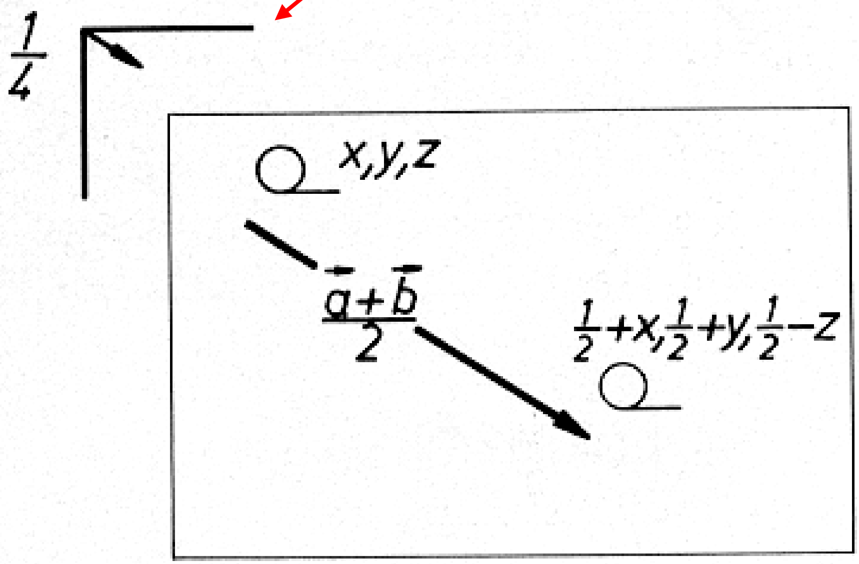
c c-glide at x, $\frac{1}{2}$, z

$\frac{1}{2}$ normal to the projection plane

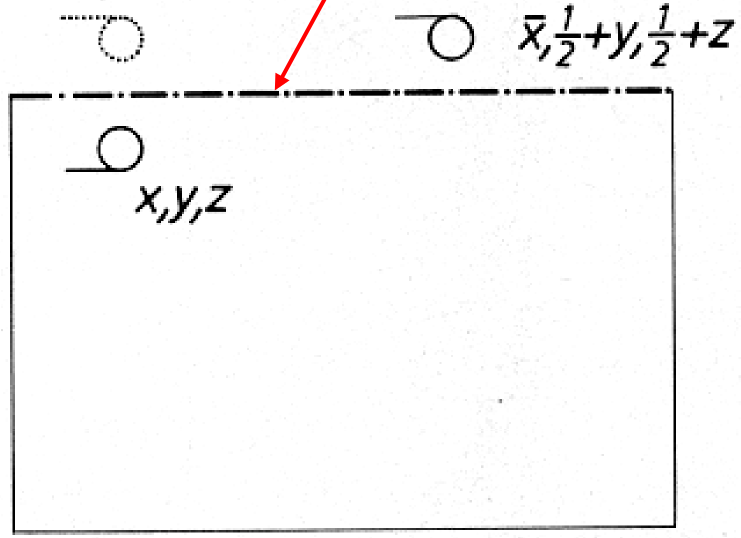
Glide Plane

$\frac{1}{2}$ in the direction of the arrow

$\frac{1}{2}$ along the line parallel to the projection plane combined with $\frac{1}{2}$ normal to the projection plane



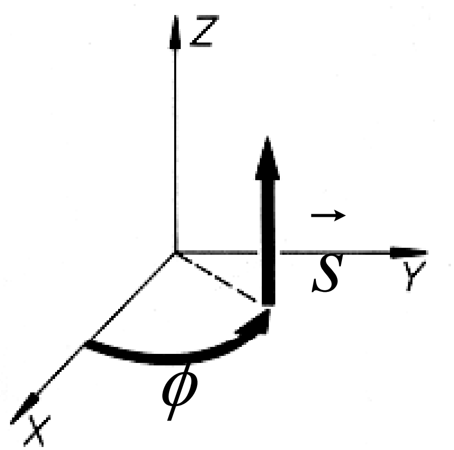
d n-glide at $x, y, \frac{1}{4}$ with glide component $\frac{1}{2}|\vec{a}+\vec{b}|$

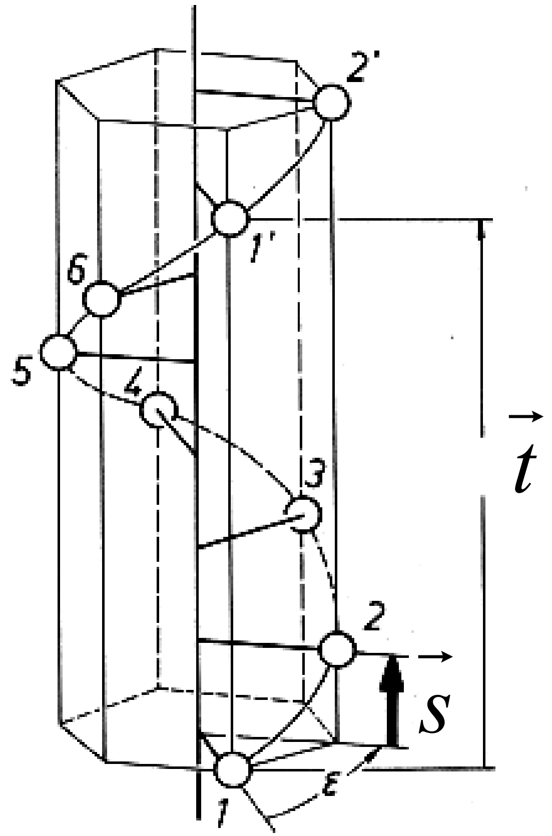


e n-glide at $0, y, z$ with glide component $\frac{1}{2}|\vec{b}+\vec{c}|$

Screw Axis

- rotation $\phi = \dfrac{2\pi}{X}$ $(X=1,2,3,4,6)$
- translation by a vector $\vec{s}$ parallel to the axis where $|\vec{s}|$ is called the screw component

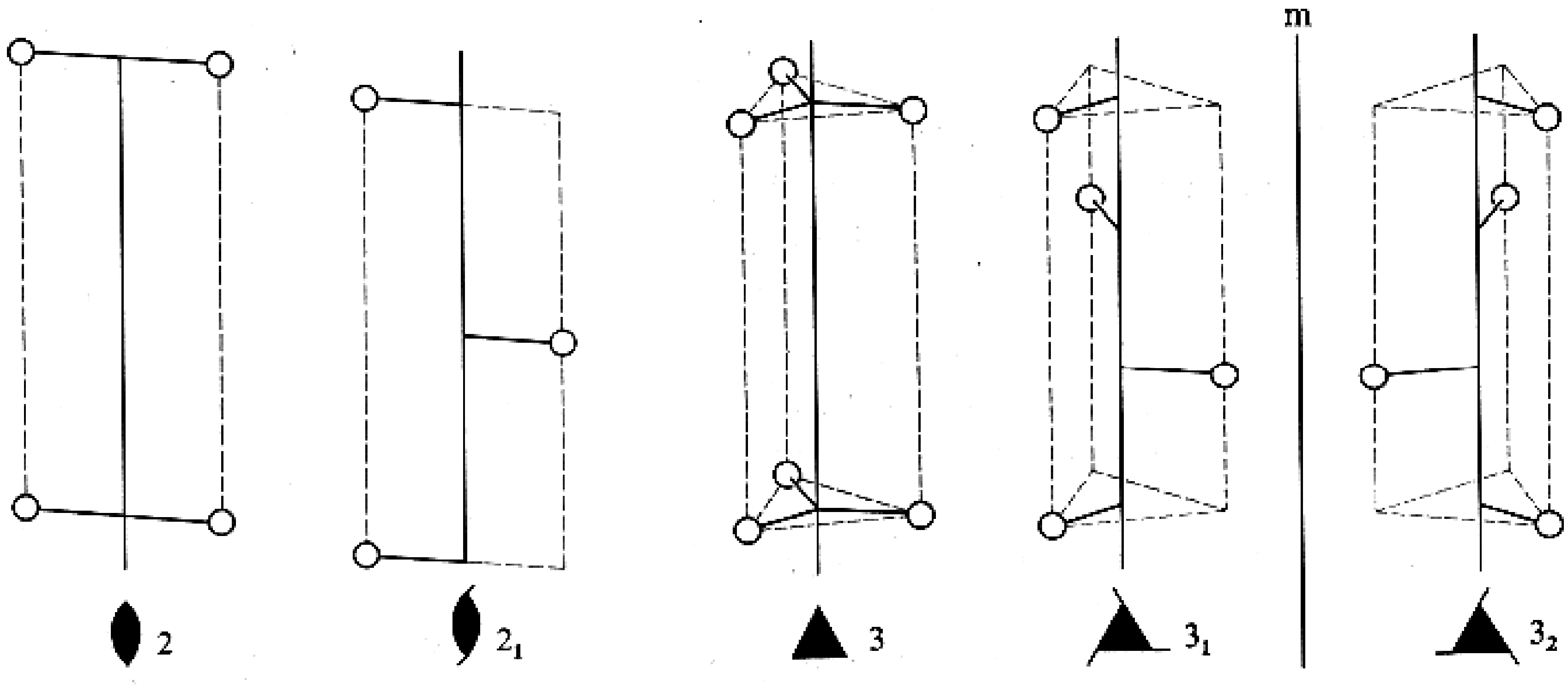




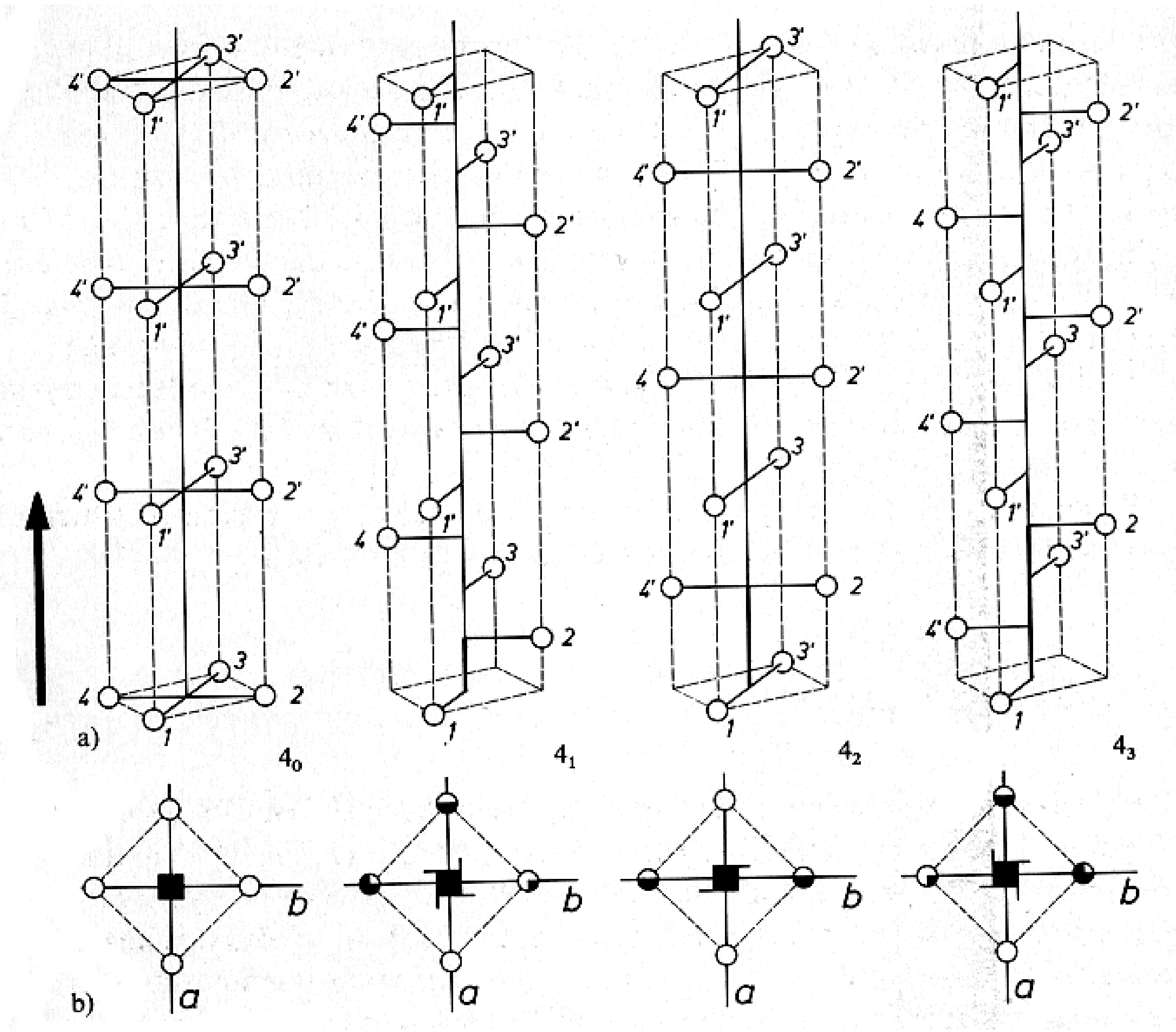
$$|\vec{s}| = \frac{p}{X}|\vec{t}| \quad \text{p=0,1,2...,X-1}$$

$$X_p = X_0, X_1, \ldots.. X_{X-1}$$

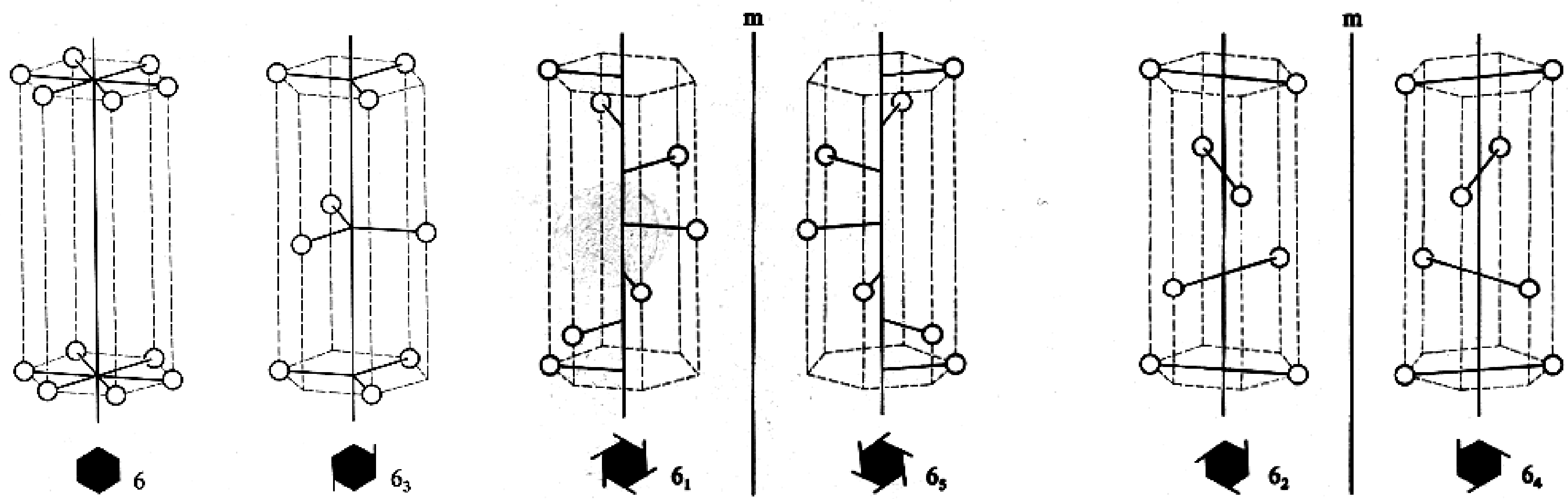
Screw Axis



Screw Axis

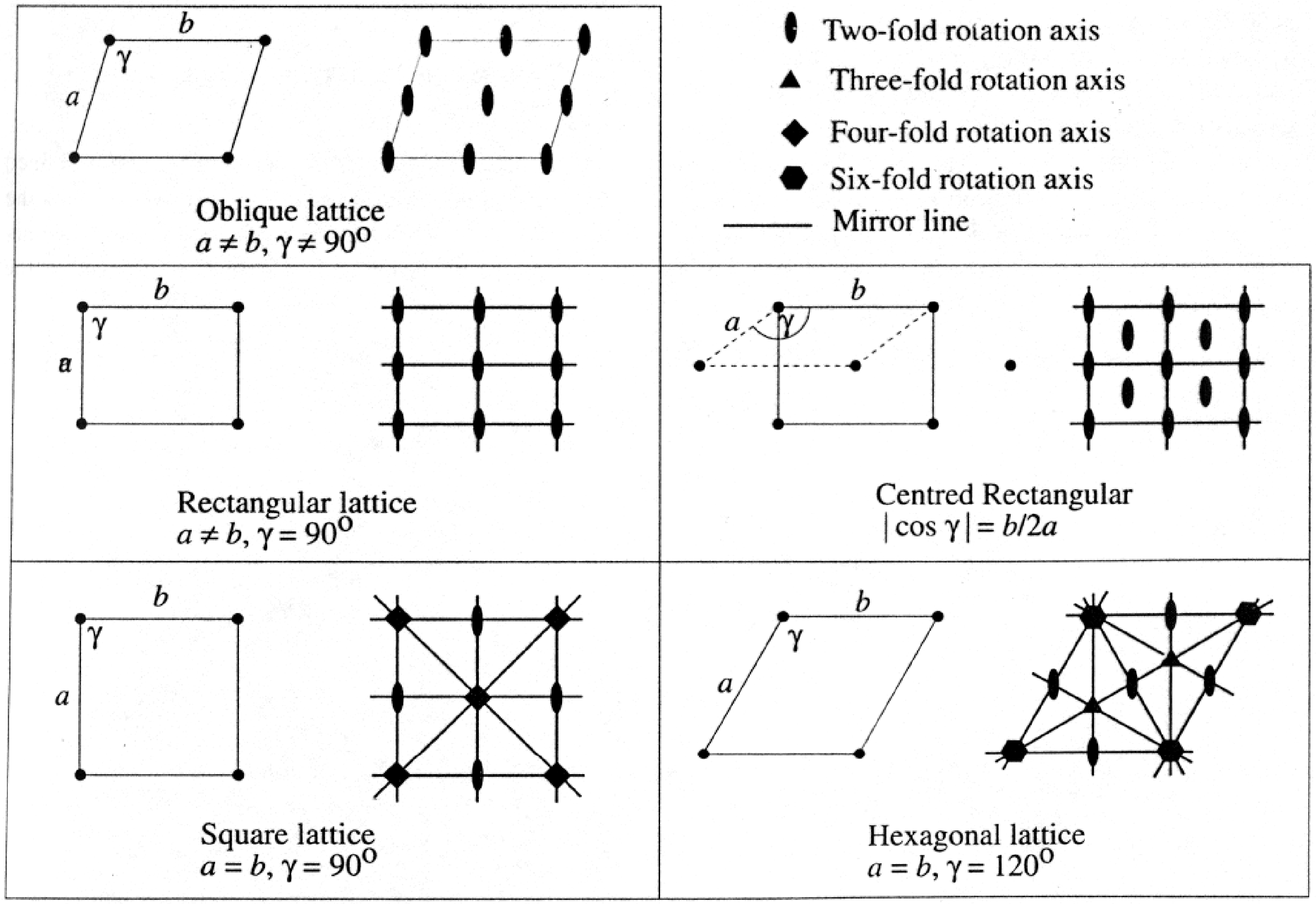


Screw Axis

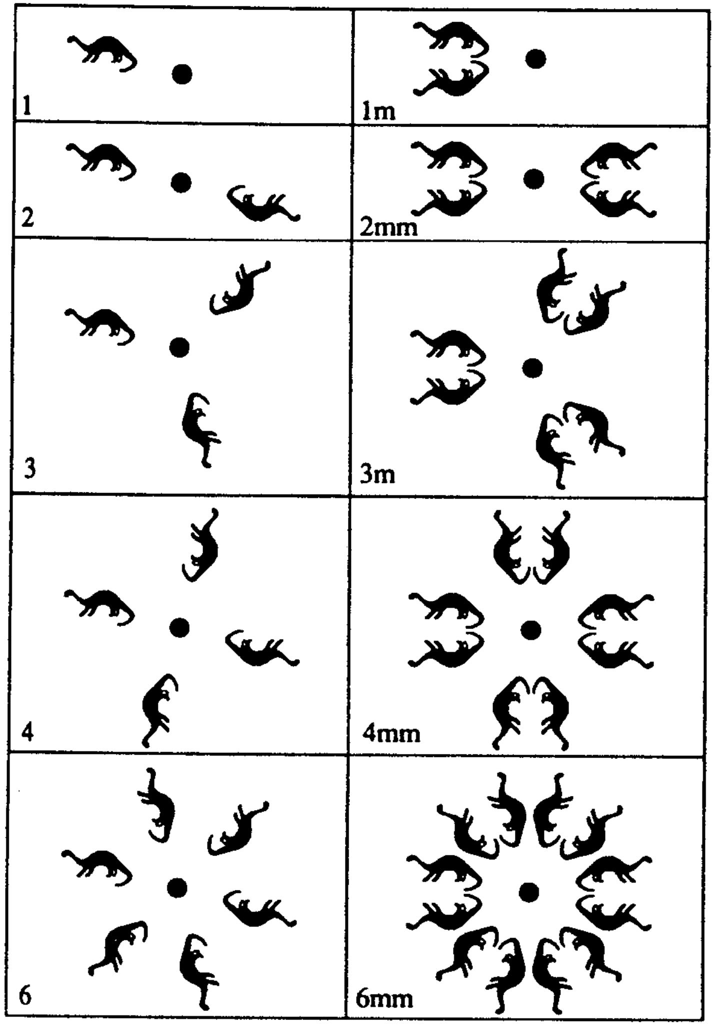


17 Plane Groups

- 5 plane lattices + 10 plane point groups+ glide plane
- criterion: lattice itself must possess at least the symmetry of the motif



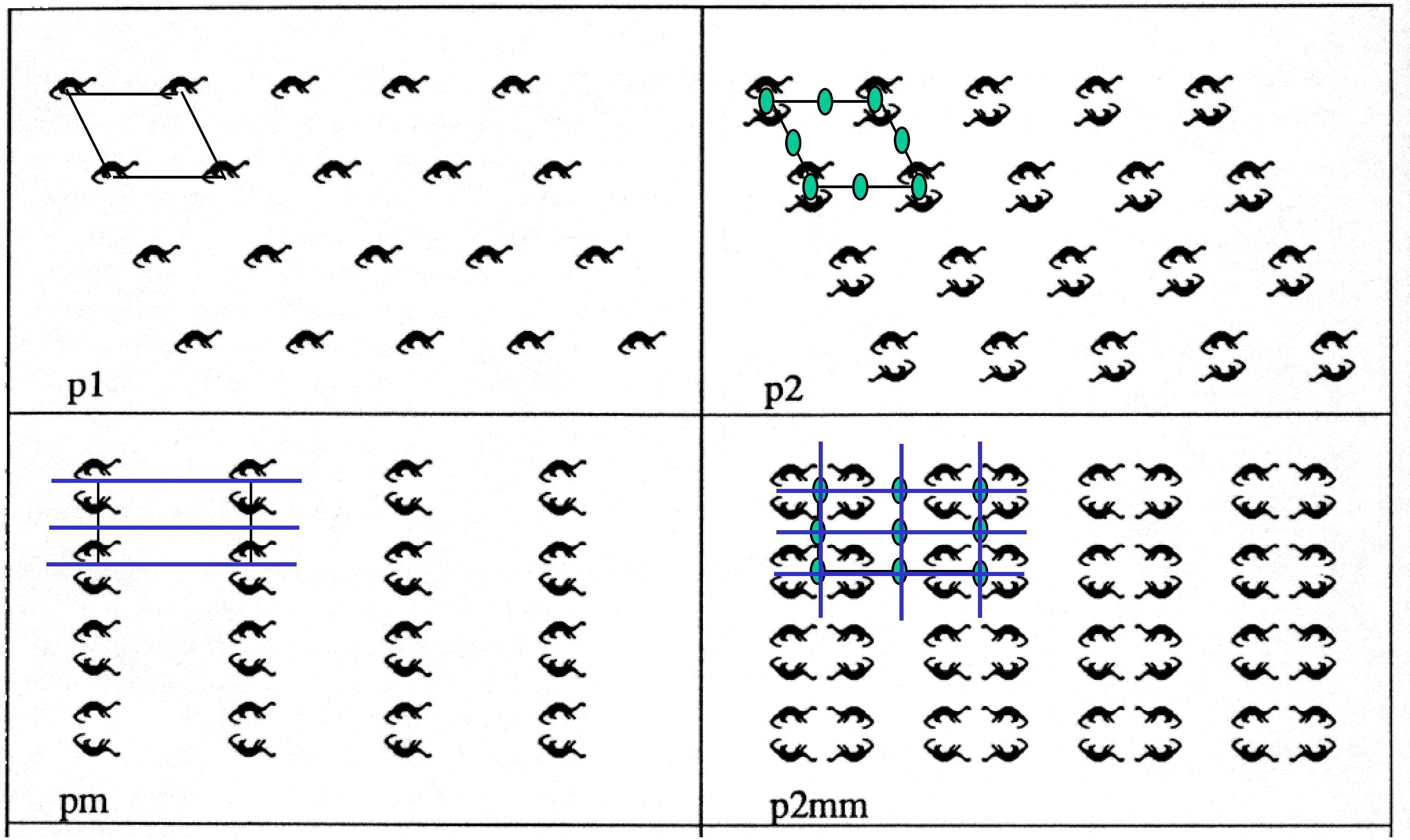
17 Plane Groups



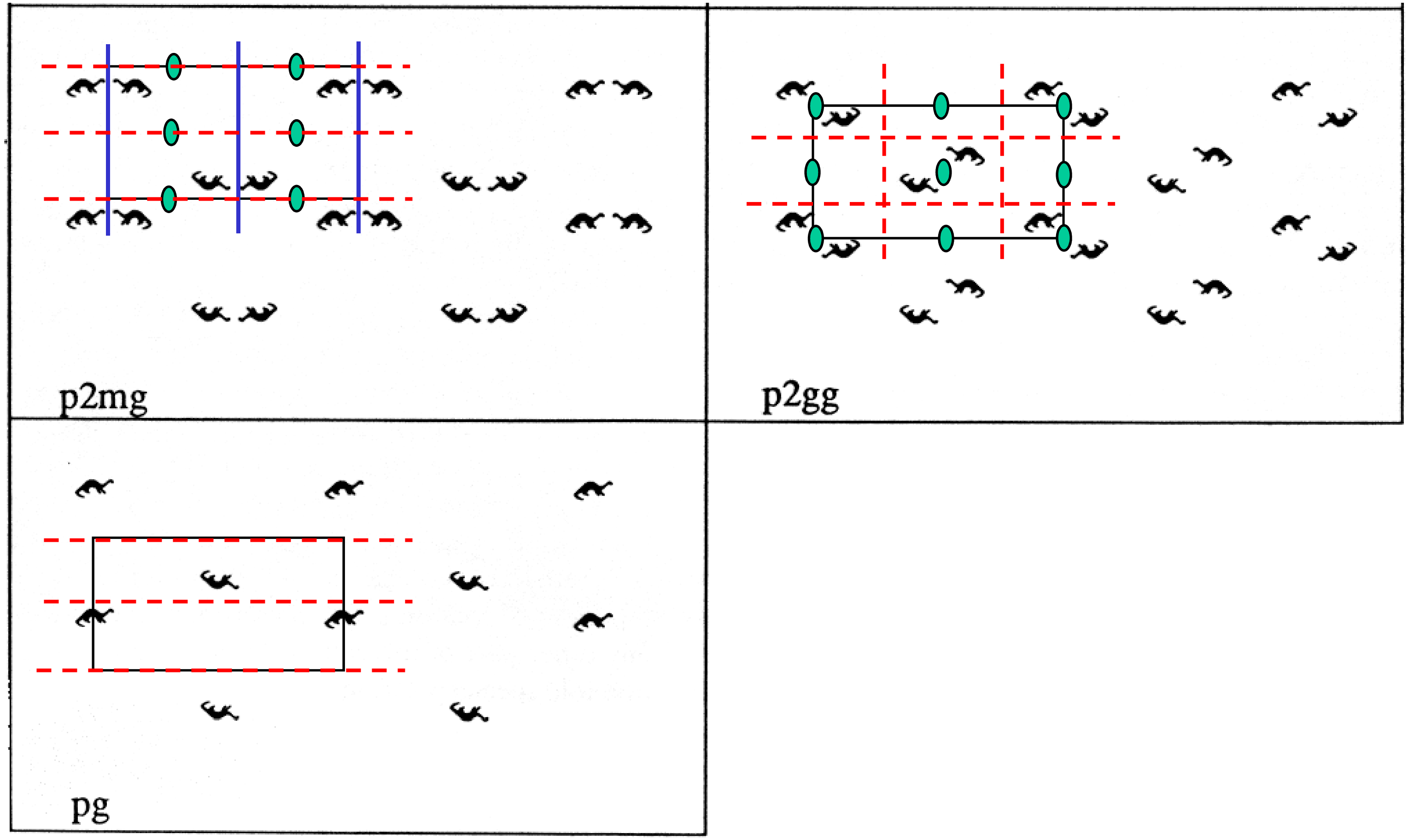
Crystal system	Point groups compatible with crystal system	Lattices in system	Space groups compatible with lattice
Oblique $a \neq b, \gamma \neq 90°$	1, 2	p (primitive)	p1, p2
Rectangular $a \neq b, \gamma = 90°$	1m, 2mm	p (primitive)	pm, p2mm, pg, p2mg p2gg
		c (centred)	cm, c2mm
Square $a = b, \gamma = 90°$	4, 4mm	p (primitive)	p4, p4mm, p4gm
Hexagonal $a = b, \gamma = 120°$	3, 3m, 6, 6mm	p (primitive)	p3, p31m, p3m1 p6, p6mm

Lattice	Symmetry direction (position in Hermann–Mauguin symbol)		
	Primary	Secondary	Tertiary
Two dimensions Oblique	Rotation point in plane		
Rectangular		[10]	[01]
Square		$\left\{\begin{matrix}[10]\\ [01]\end{matrix}\right\}$	$\left\{\begin{matrix}[1\bar{1}]\\ [11]\end{matrix}\right\}$
Hexagonal		$\left\{\begin{matrix}[10]\\ [01]\\ [\bar{1}\bar{1}]\end{matrix}\right\}$	$\left\{\begin{matrix}[1\bar{1}]\\ [12]\\ [\bar{2}\bar{1}]\end{matrix}\right\}$

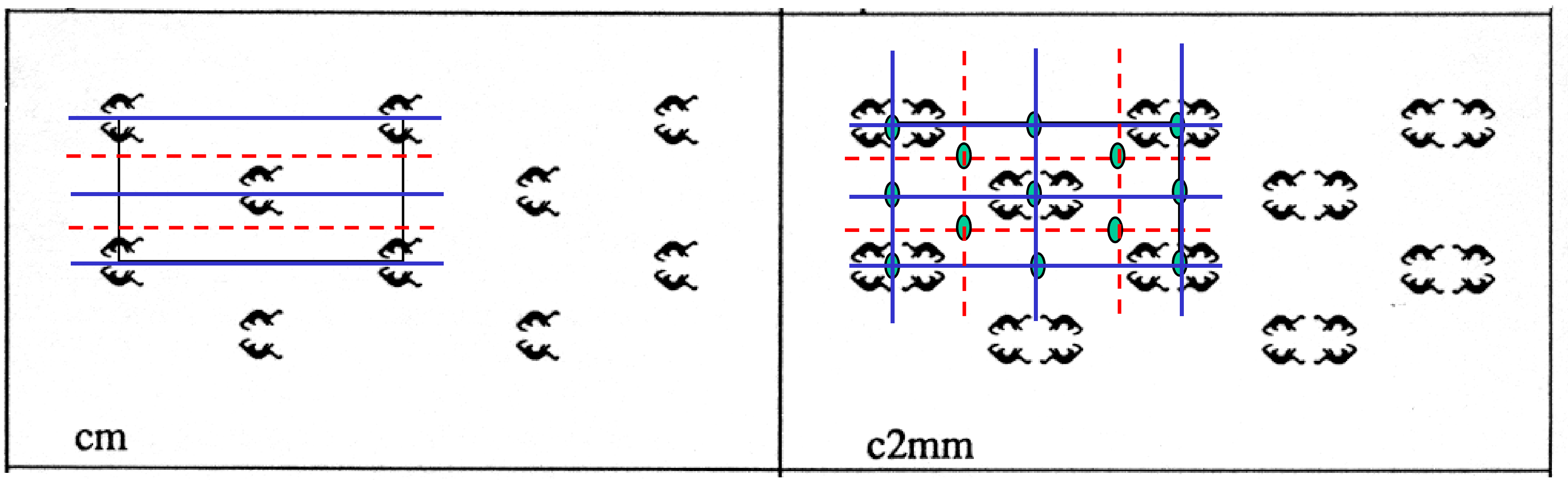
17 Plane Groups



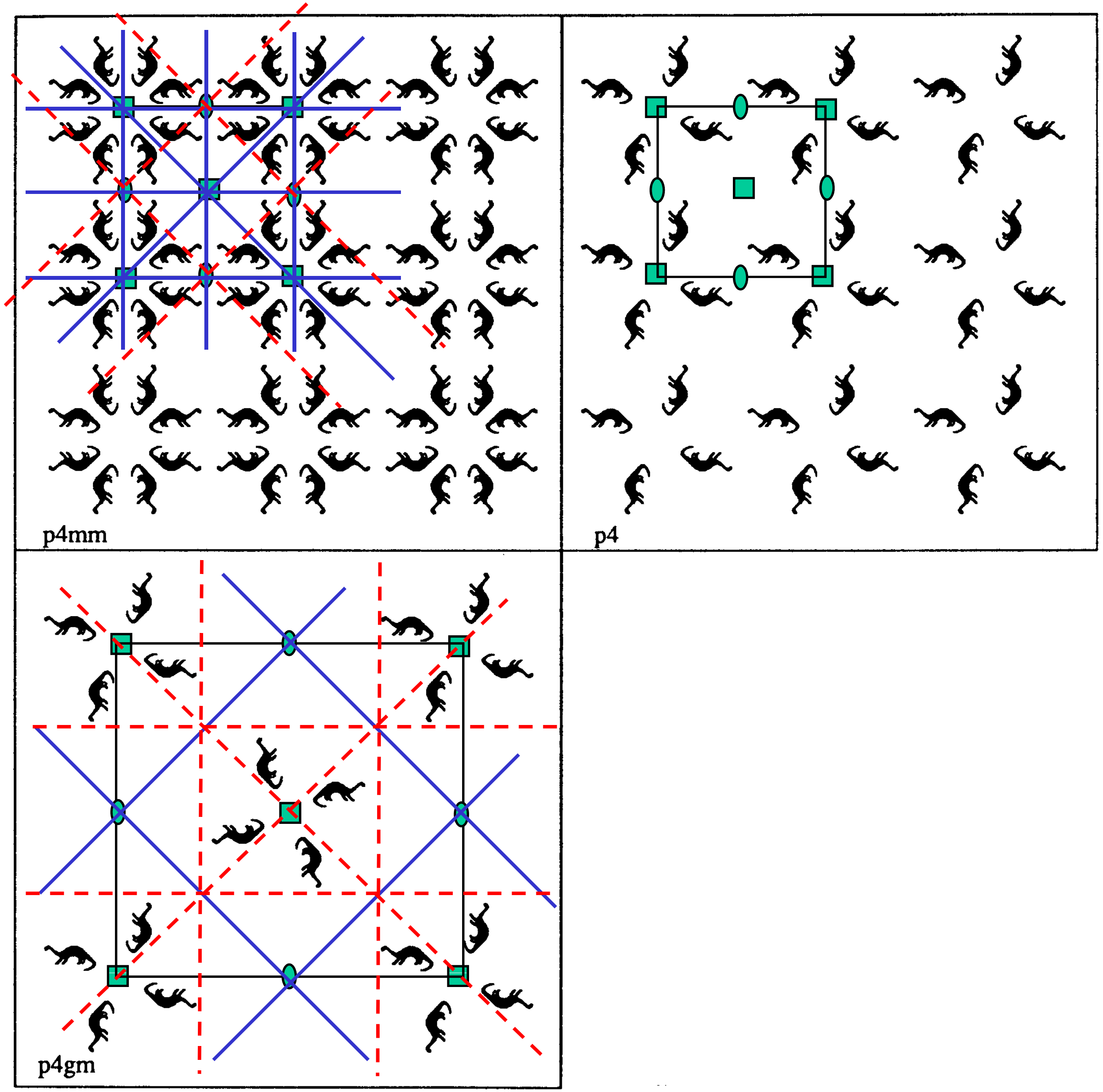
17 Plane Groups



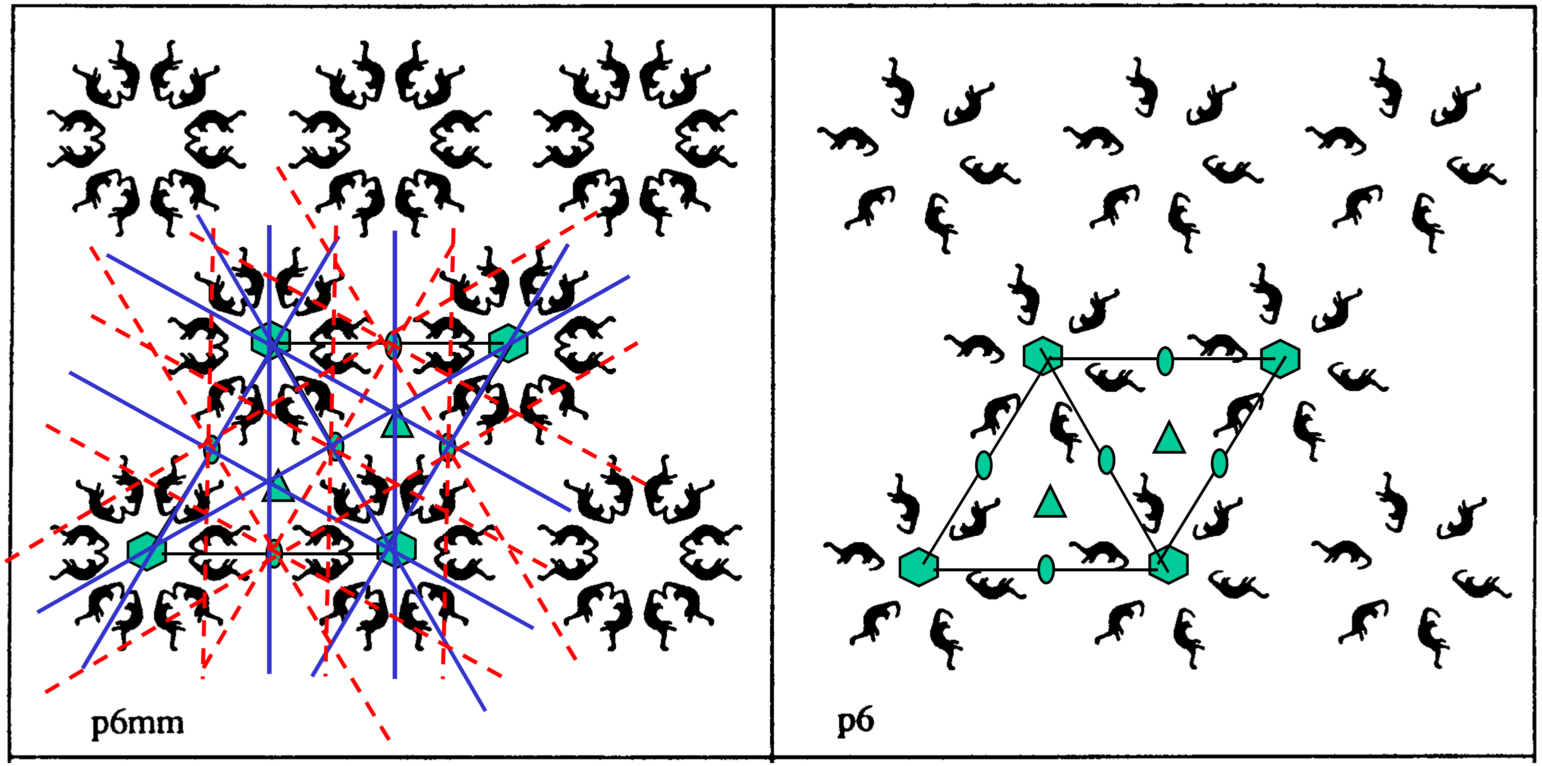
17 Plane Groups



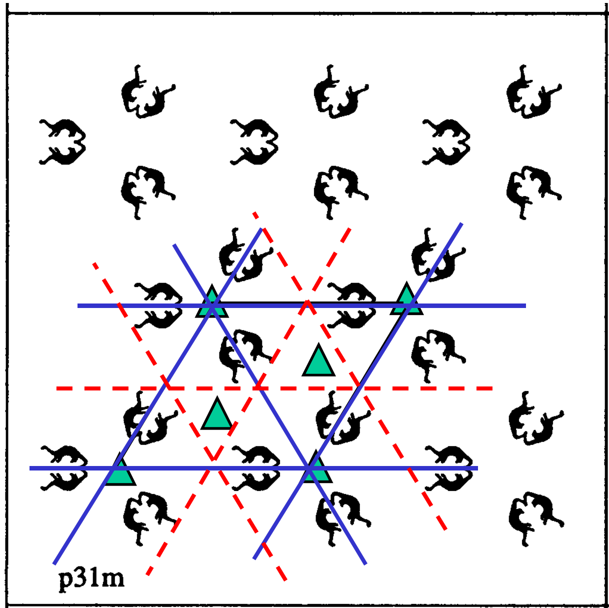
17 Plane Groups

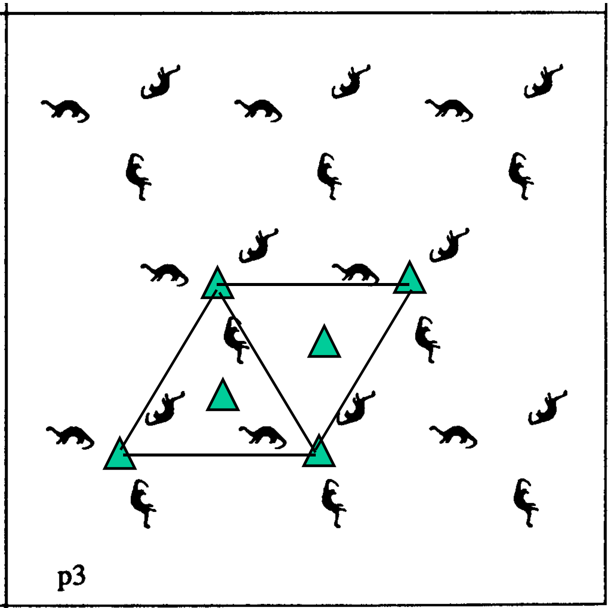


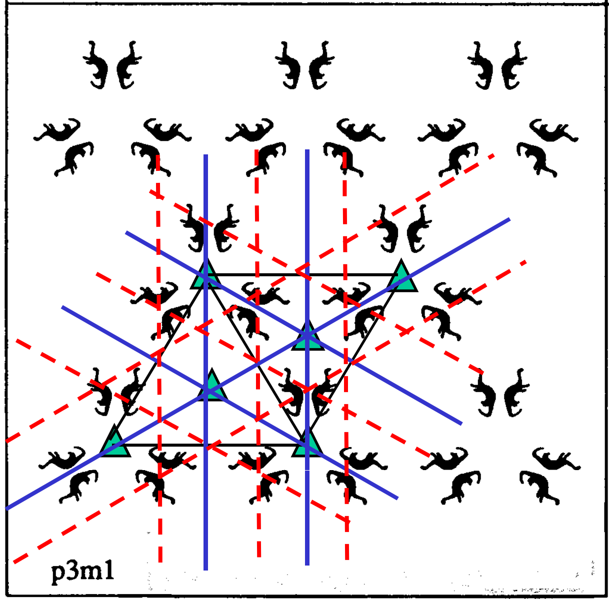
17 Plane Groups



17 Plane Groups







International Tables for X-ray Crystallography

p 1

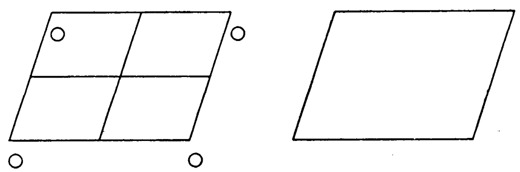
No. 1

1

p 1

Oblique

Patterson symmetry *p* 2



p 2

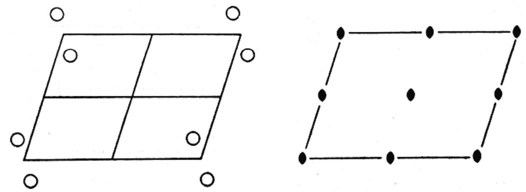
No. 2

Oblique

Patterson symmetry *p* 2

2

p 2



pm

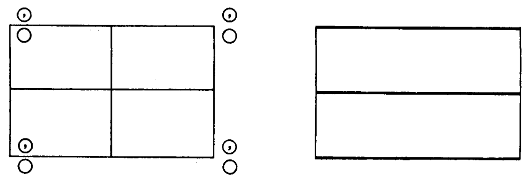
No. 3

m

p 1 *m* 1

Rectangular

Patterson symmetry *p* 2 *m m*



pg

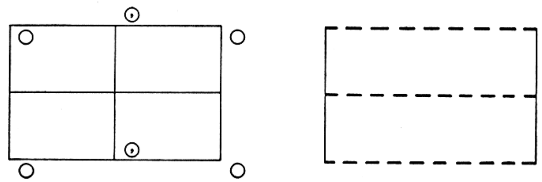
No. 4

Rectangular

Patterson symmetry *p* 2 *m m*

m

p 1 *g* 1



cm

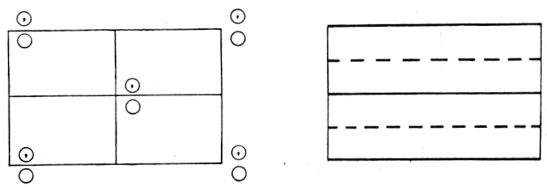
No. 5

m

c 1 *m* 1

Rectangula

Patterson symmetry *c* 2



p 2 *m m*

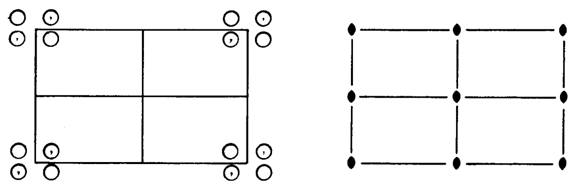
No. 6

Rectangular

Patterson symmetry *p* 2 *m m*

2 *m m*

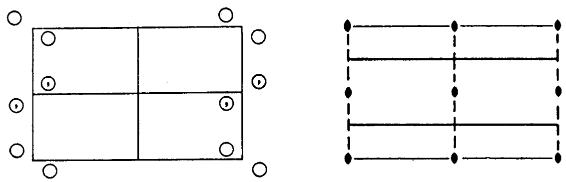
p 2 *m m*



International Tables for X-ray Crystallography

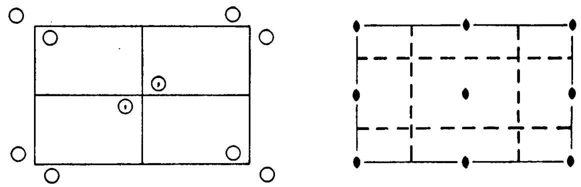
p 2 *m g* | 2 *m m* | Rectangular

No. 7 | *p* 2 *m g* | Patterson symmetry *p* 2 *m m*



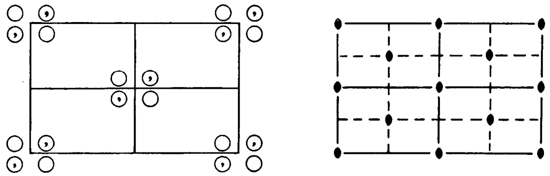
Rectangular | 2 *m m* | *p* 2 *g g*

Patterson symmetry *p* 2 *m m* | *p* 2 *g g* | No. 8



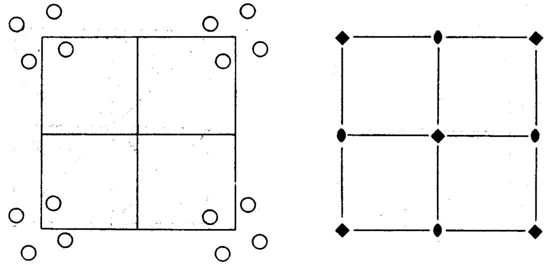
c 2 *m m* | 2 *m m* | Rectangular

No. 9 | *c* 2 *m m* | Patterson symmetry *c* 2 *m m*



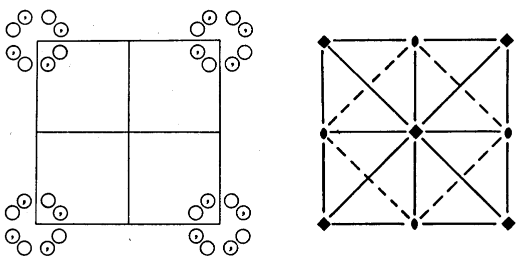
Square | 4 | *p* 4

Patterson symmetry *p* 4 | *p* 4 | No. 1(



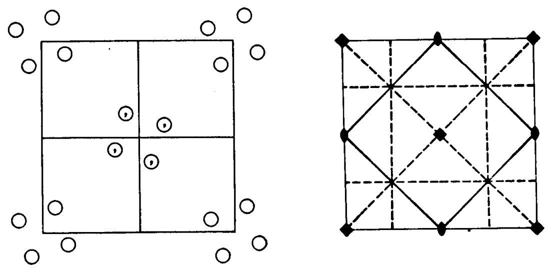
p 4 *m m* | 4 *m m* | Square

No. 11 | *p* 4 *m m* | Patterson symmetry *p* 4 *m m*



Square | 4 *m m* | *p* 4 *g m*

Patterson symmetry *p* 4 *m m* | *p* 4 *g m* | No. 12

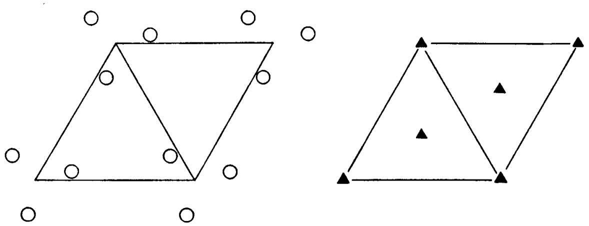


International Tables for X-ray Crystallography

p 3

No. 13 | *p* 3 | 3 | Hexagonal

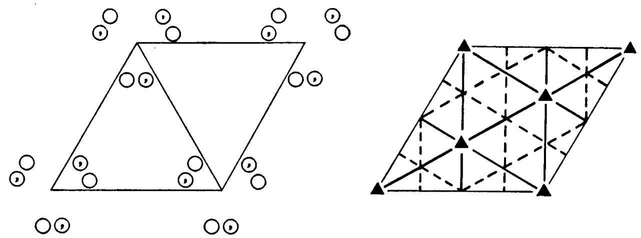
Patterson symmetry *p* 6



p 3 *m* 1

No. 14 | *p* 3 *m* 1 | 3 *m* | Hexagonal

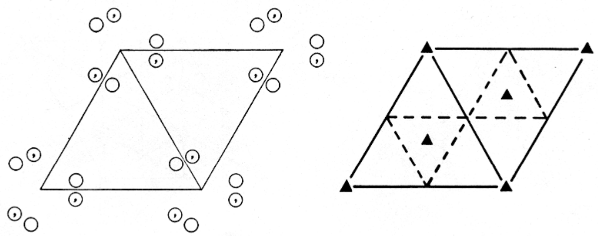
Patterson symmetry *p* 6 *m m*



p 3 1 *m*

No. 15 | *p* 3 1 *m* | 3 *m* | Hexagonal

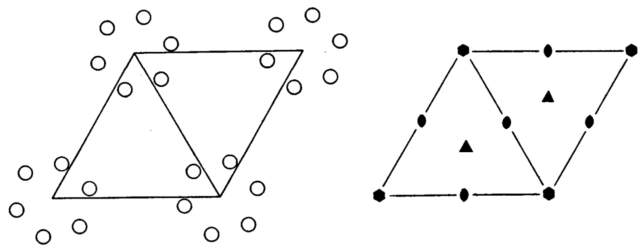
Patterson symmetry *p* 6 *m m*



p 6

No. 16 | *p* 6 | 6 | Hexagonal

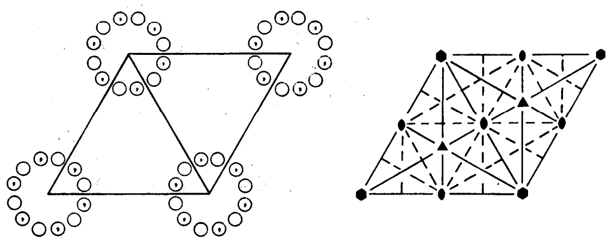
Patterson symmetry *p* 6



p 6 *m m*

No. 17 | *p* 6 *m m* | 6 *m m* | Hexagonal

Patterson symmetry *p* 6 *m m*

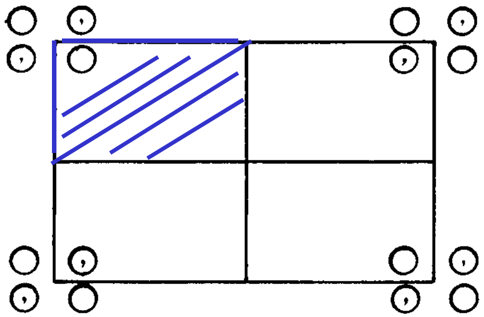


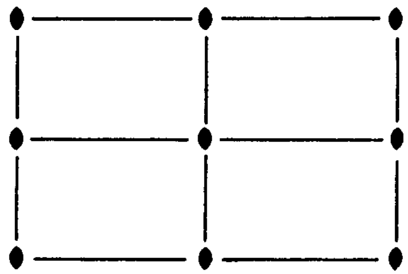
International Tables for X-ray Crystallography

③Rectangular ②$2mm$ ①$p2mm$

⑥Patterson symmetry $p2mm$ ⑤$p2mm$ ④No. 6

⑦





Origin at $2mm$

Asymmetric unit $0 \leq x \leq \frac{1}{2}$; $0 \leq y \leq \frac{1}{2}$

Symmetry operations

(1) 1 (2) 2 0,0 (3) m 0,y (4) m x,0

Generators selected (1); t(1,0); t(0,1); (2); (3)

International Tables for X-ray Crystallography

① short international (Hermann-Mauguin) symbol for the plane group
② short international (Hermann-Mauguin) symbol for the point group
③ crystal system
④ sequential number of plane group
⑤ full international (Hermann-Mauguin) symbol for the plane group
⑥ patterson symmetry
⑦ diagram for the symmetry elements and the general position

International Tables for X-ray Crystallography

Generators selected (1); $t(1,0)$; $t(0,1)$; (2); (3)

Positions

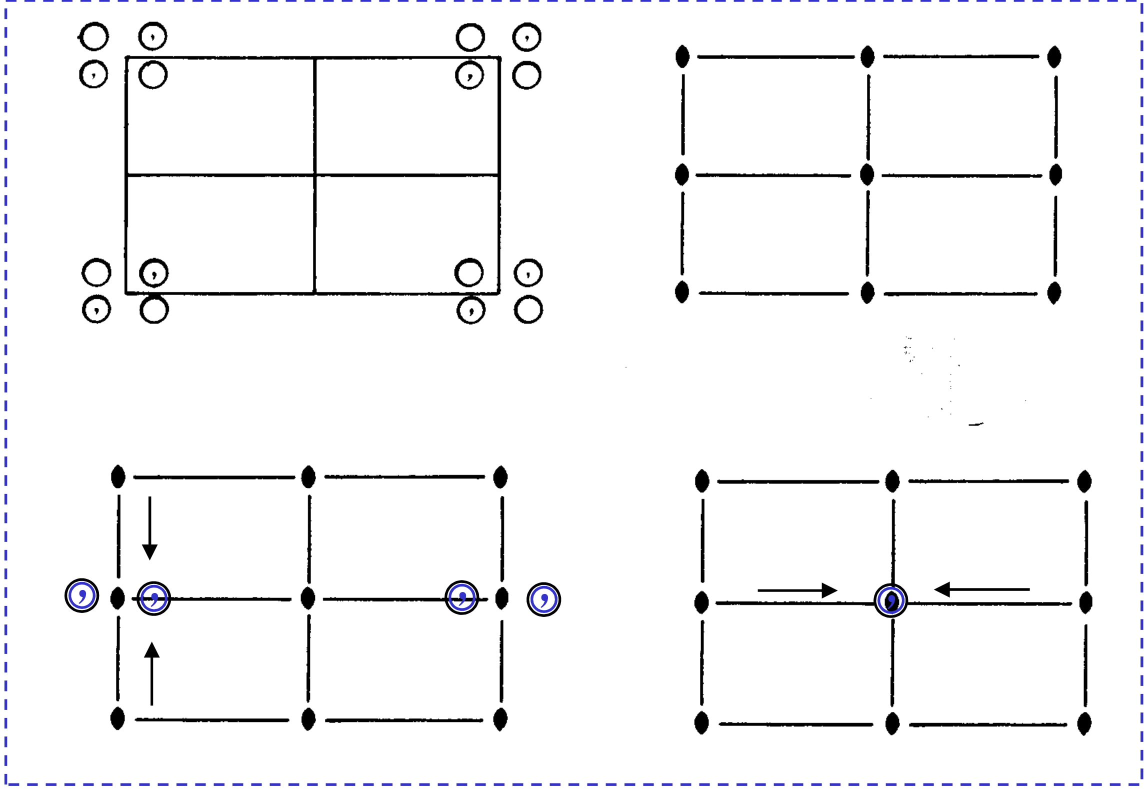
Multiplicity, Wyckoff letter, Site symmetry			Coordinates			
4	*i*	1	(1) x,y	(2) $\bar{x},\bar{y}$	(3) $\bar{x},y$	(4) $x,\bar{y}$
2	*h*	. *m* .	$\frac{1}{2},y$	$\frac{1}{2},\bar{y}$		
2	*g*	. *m* .	$0,y$	$0,\bar{y}$		
2	*f*	. . *m*	$x,\frac{1}{2}$	$\bar{x},\frac{1}{2}$		
2	*e*	. . *m*	$x,0$	$\bar{x},0$		
1	*d*	2 *m m*	$\frac{1}{2},\frac{1}{2}$			
1	*c*	2 *m m*	$\frac{1}{2},0$			
1	*b*	2 *m m*	$0,\frac{1}{2}$			
1	*a*	2 *m m*	$0,0$			

Reflection conditions

General:

no conditions

Special: no extra conditions



International Tables for X-ray Crystallography

Maximal non-isomorphic subgroups

I	[2] $p\,2\,1\,1\,(p\,2)$	1; 2
	[2] $p\,1\,m\,1\,(p\,m)$	1; 3
	[2] $p\,1\,1\,m\,(p\,m)$	1; 4

IIa none

IIb [2] $p\,2m\,g\,(\mathbf{a}'=2\mathbf{a})$; [2] $p\,2g\,m\,(\mathbf{b}'=2\mathbf{b})\,(p2mg)$; [2] $c\,2m\,m\,(\mathbf{a}'=2\mathbf{a}, \mathbf{b}'=2\mathbf{b})$

Maximal isomorphic subgroups of lowest index

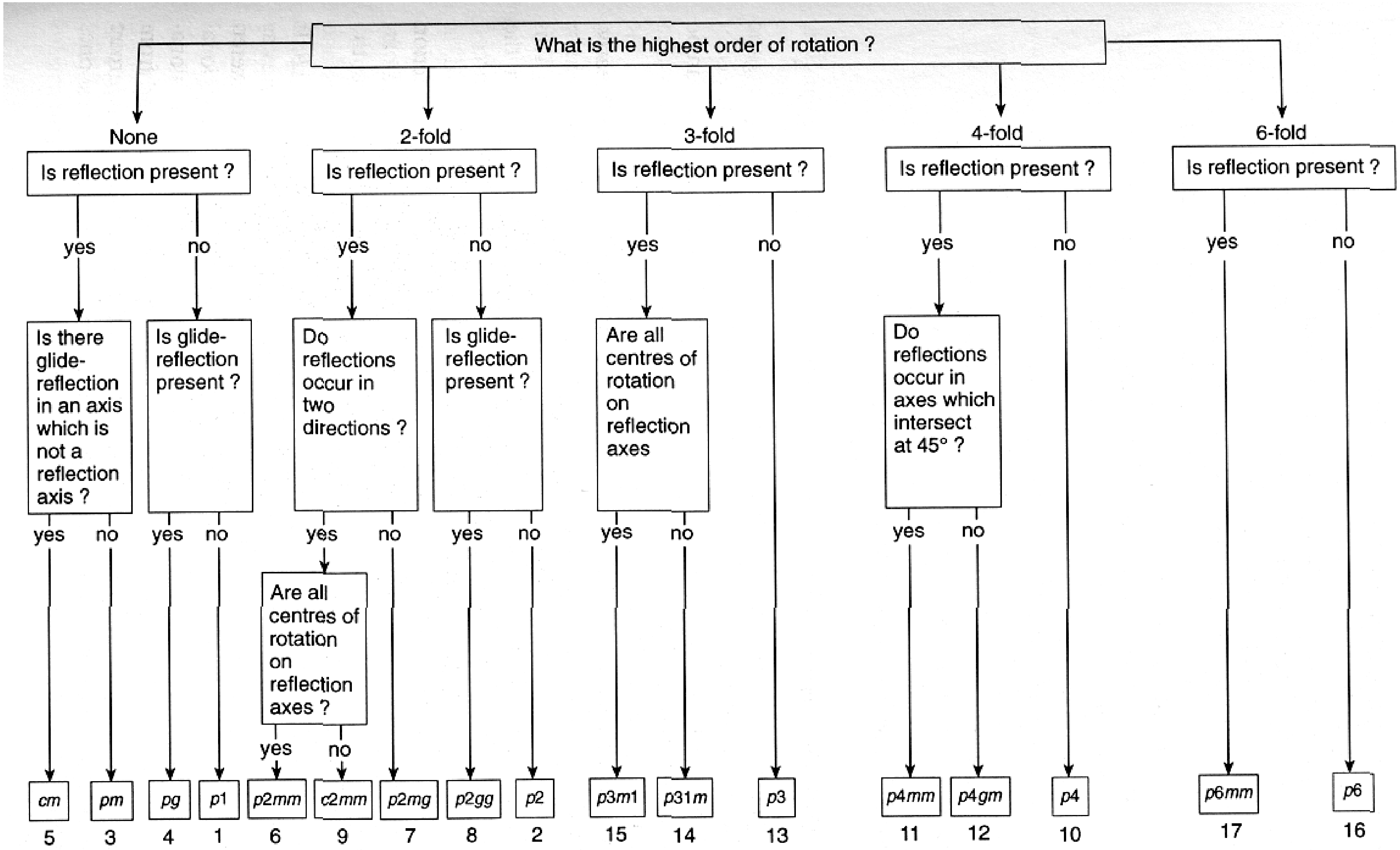
IIc [2] $p\,2m\,m\,(\mathbf{a}'=2\mathbf{a}$ or $\mathbf{b}'=2\mathbf{b})$

Minimal non-isomorphic supergroups

I [2] $p\,4m\,m$

II [2] $c\,2m\,m$

Flow Diagram for Identifying Plane Groups



Example I

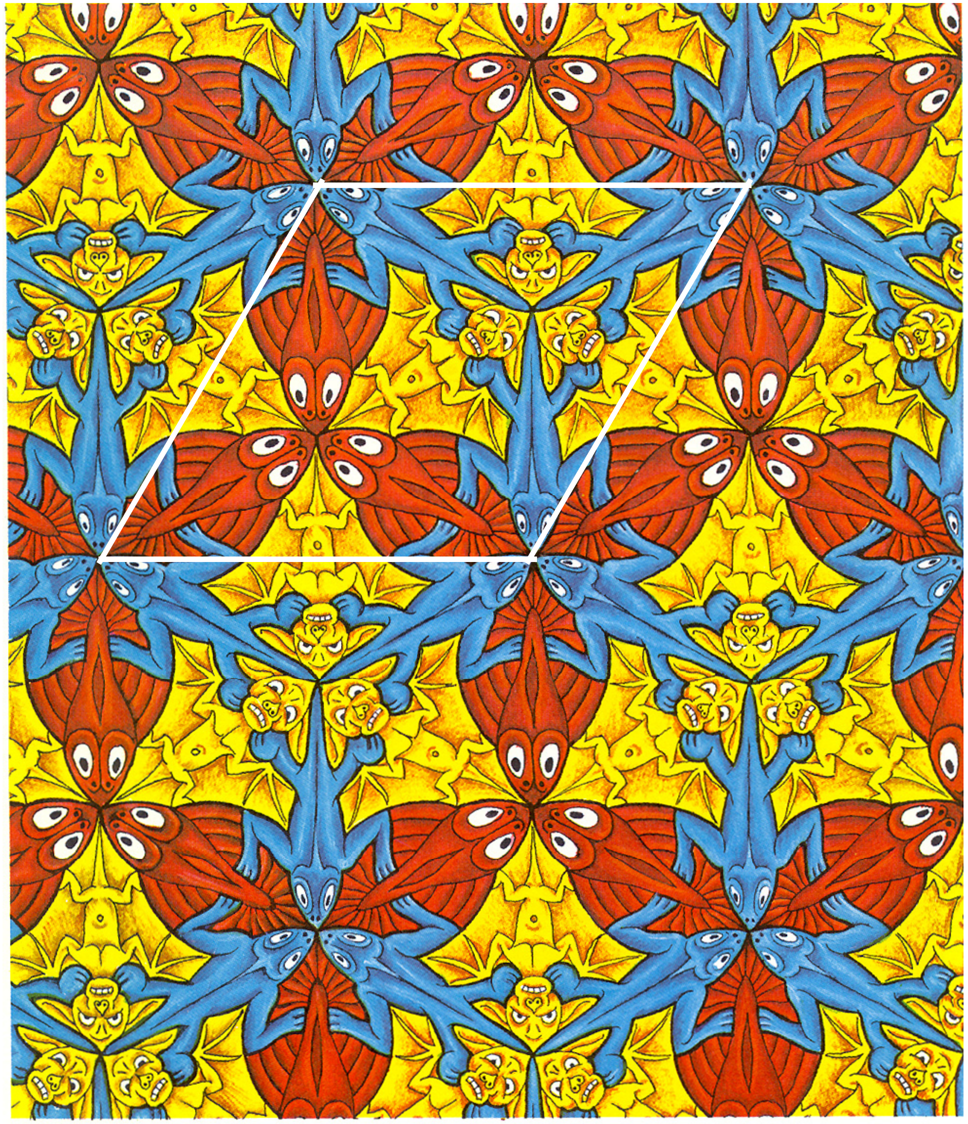
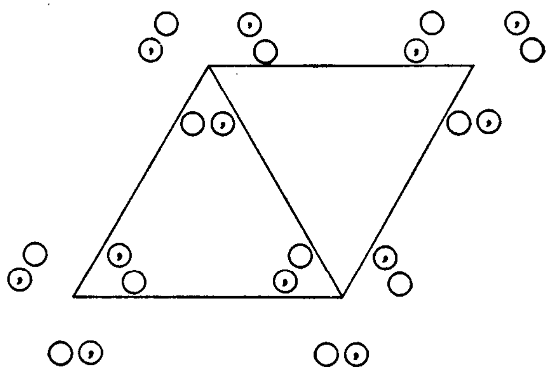
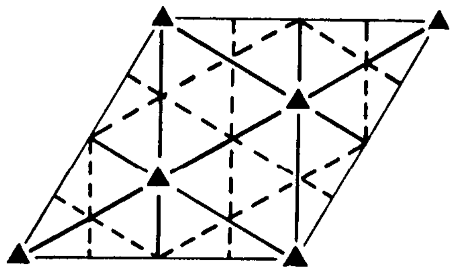


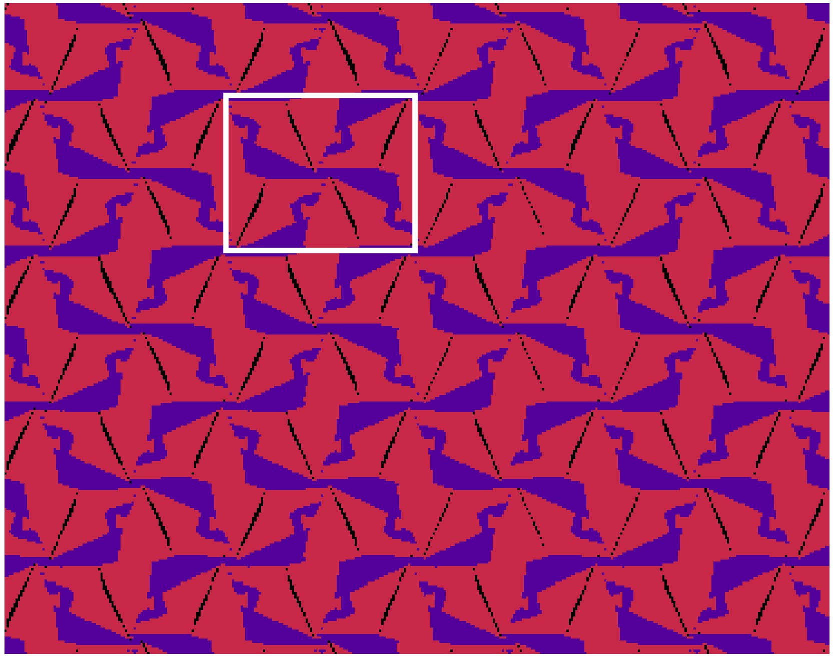
PLATE 9

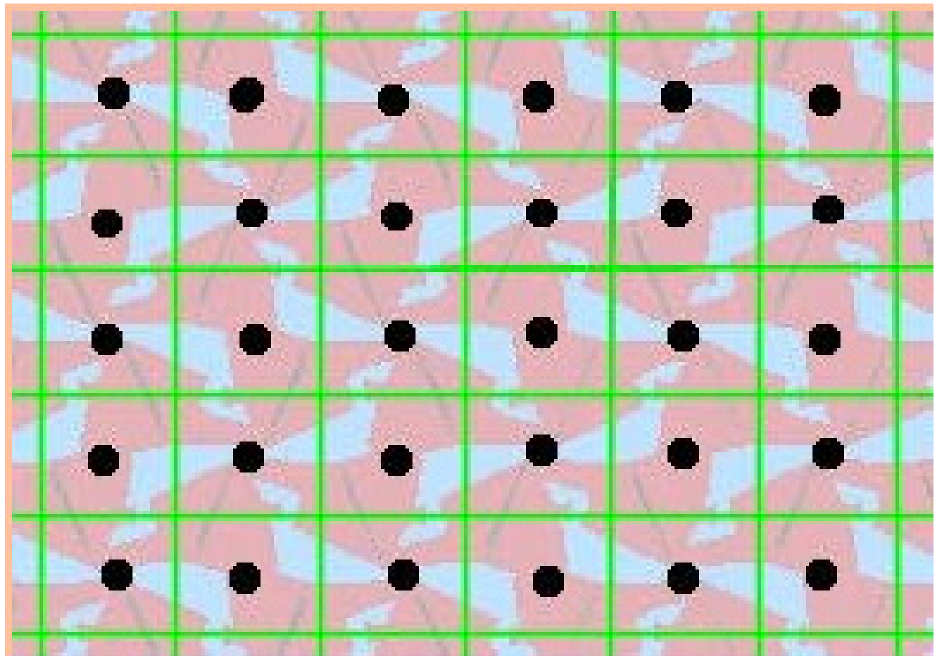
$P3m1$

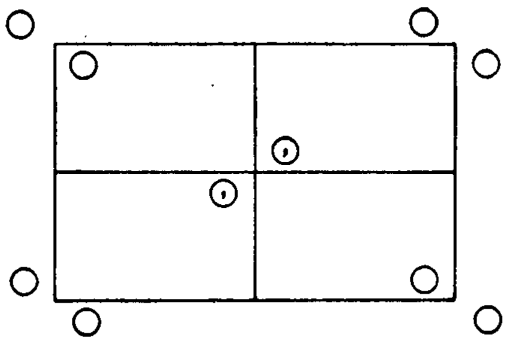


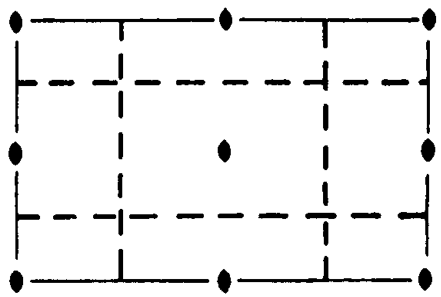


Example II









P_{2gg}

Space Groups

-Bravais lattice + point group $\rightarrow$ 230 space groups
+ screw axis
+ glide plane

- Bravais lattice + point group= 73
- Bravais lattice + screw axis = 41
- Bravais lattice + glide plane = 116

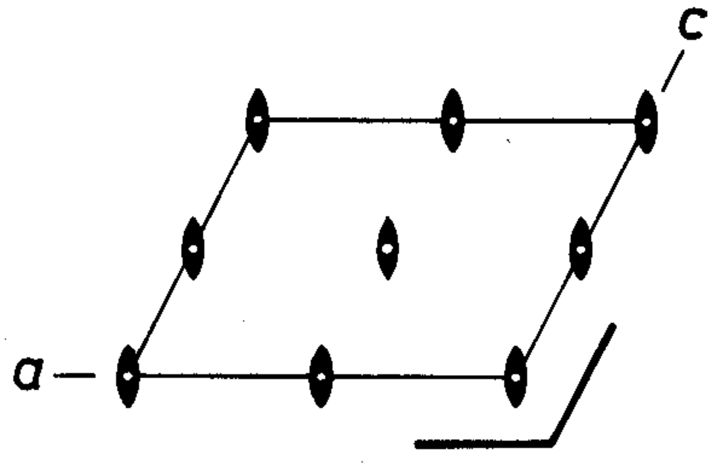
Three dimensions Triclinic	None		
Monoclinic*	[010] ('unique axis *b*') [001] ('unique axis *c*')		
Orthorhombic	[100]	[010]	[001]
Tetragonal	[001]	$\left\{\begin{array}{l}[100]\\ [010]\end{array}\right\}$	$\left\{\begin{array}{l}[1\bar{1}0]\\ [110]\end{array}\right\}$
Hexagonal	[001]	$\left\{\begin{array}{l}[100]\\ [010]\\ [\bar{1}\bar{1}0]\end{array}\right\}$	$\left\{\begin{array}{l}[1\bar{1}0]\\ [120]\\ [\bar{2}\bar{1}0]\end{array}\right\}$
Rhombohedral (hexagonal axes)	[001]	$\left\{\begin{array}{l}[100]\\ [010]\\ [\bar{1}\bar{1}0]\end{array}\right\}$	
Rhombohedral (rhombohedral axes)	[111]	$\left\{\begin{array}{l}[1\bar{1}0]\\ [01\bar{1}]\\ [\bar{1}01]\end{array}\right\}$	
Cubic	$\left\{\begin{array}{l}[100]\\ [010]\\ [001]\end{array}\right\}$	$\left\{\begin{array}{l}[111]\\ [1\bar{1}\bar{1}]\\ [\bar{1}1\bar{1}]\\ [\bar{1}\bar{1}1]\end{array}\right\}$	$\left\{\begin{array}{ll}[1\bar{1}0] & [110]\\ [01\bar{1}] & [011]\\ [\bar{1}01] & [101]\end{array}\right\}$

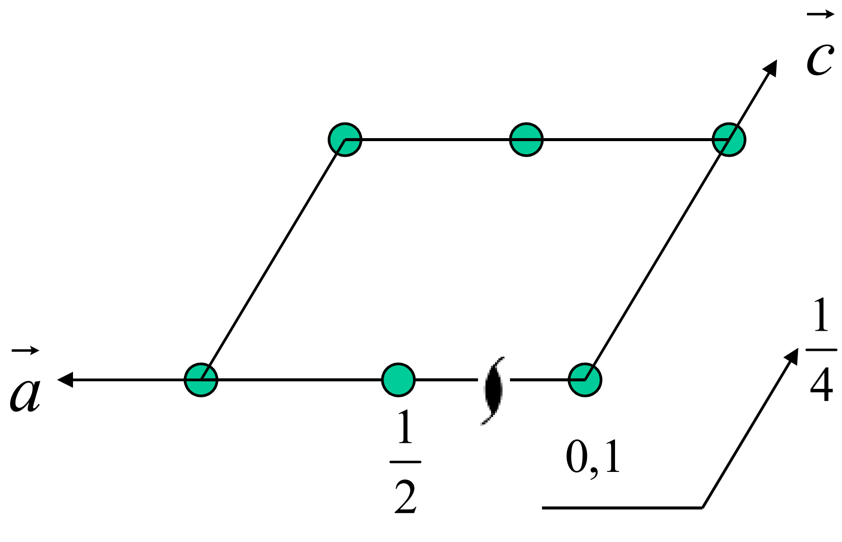
Space Groups-monoclinic

- monoclinic system- highest symmetry

$P\frac{2}{m}$, $\qquad C\frac{2}{m}$ (a-glide at $x, \frac{1}{4}, z$, $\; x, \frac{3}{4}, z$

2_1 screw axis at $\frac{1}{4}, y, 0$, $\frac{1}{4}, y, \frac{1}{2}$, $\frac{3}{4}, y, 0$, $\frac{3}{4}, y, \frac{1}{2}$)

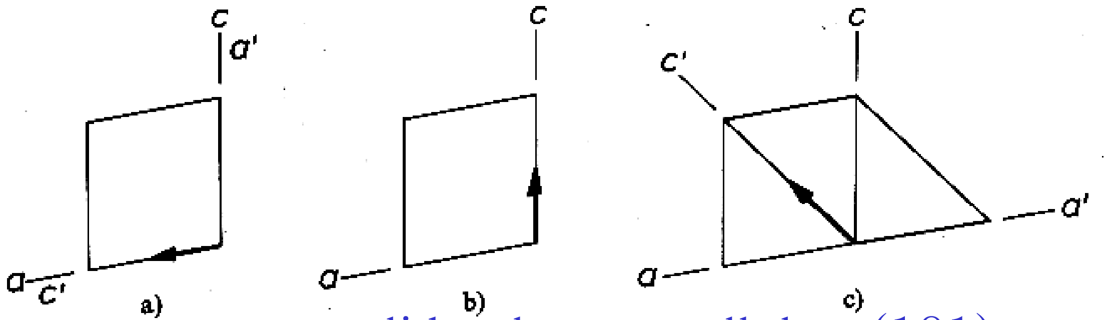




Space Groups-monoclinic

$- \dfrac{2}{m}$ subgroup $2, \quad m$

$2_1 \rightarrow 2$, c-glide $\rightarrow$ m



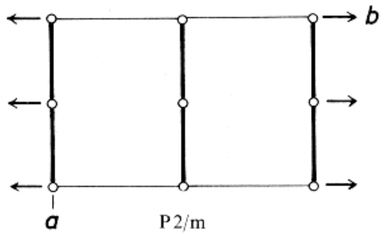
glide plane parallel to (101)

-13 monoclinic space groups

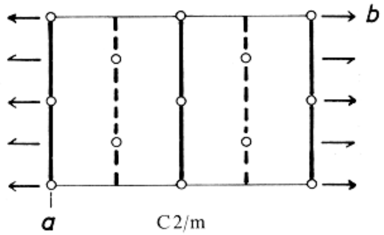
Point groups	Space groups	
2/m	P2/m $P2_1/m$ P2/c $P2_1/c$	C2/m –[a] C2/c –[b]
m	Pm Pc	Cm Cc
2	P2 $P2_1$	C2 –[c]

[a] $C2_1/m \equiv C2/m$, [b] $C2_1/c \equiv C2/c$, [c] $C2_1 \equiv C2$

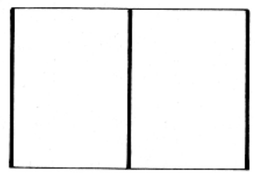
Space Groups-monoclinic



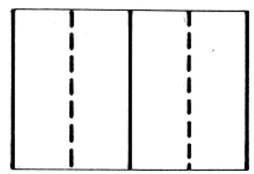
P 2/m



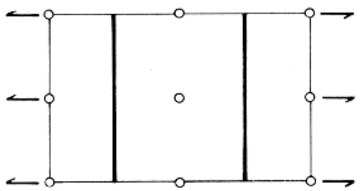
C 2/m



Pm

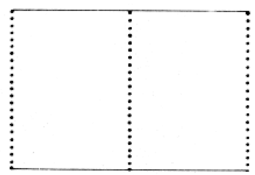


Cm

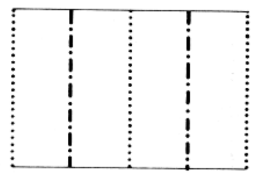


P 2_1/m

—

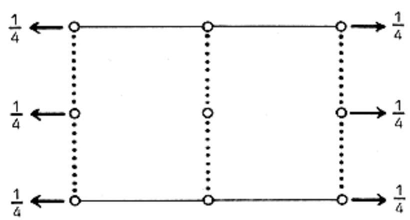


Pc

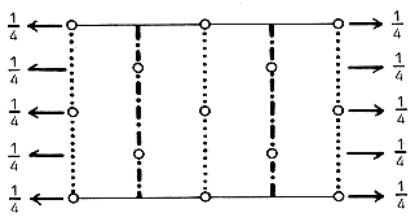


Cc

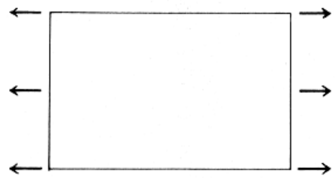
b Space groups of point group m



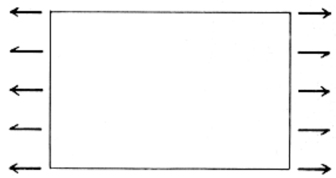
P 2/c



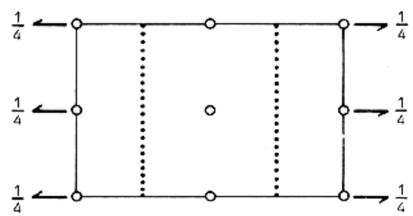
C 2/c



P 2

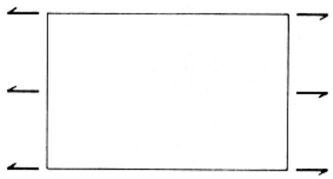


C 2



P 2_1/c

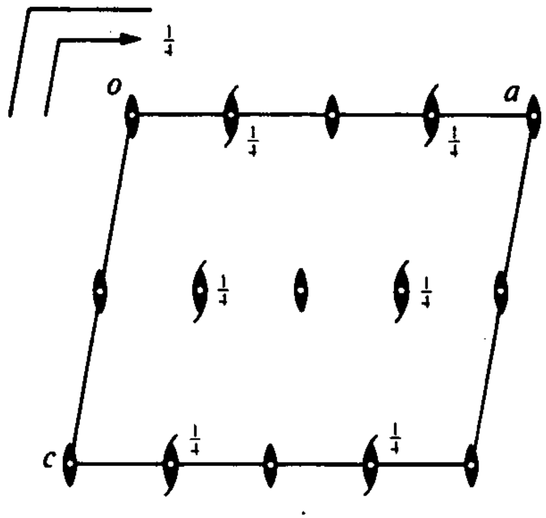
—

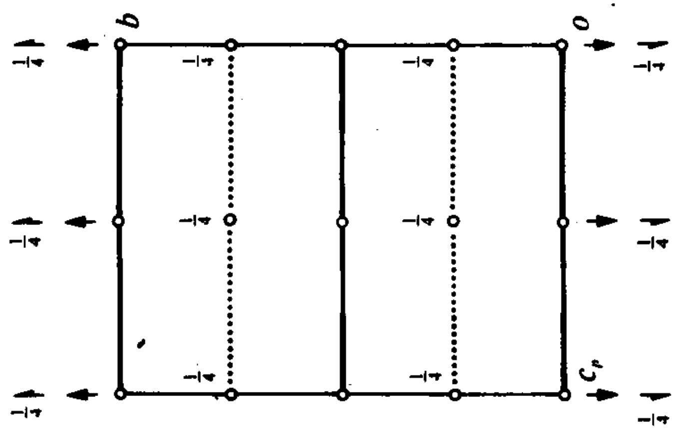


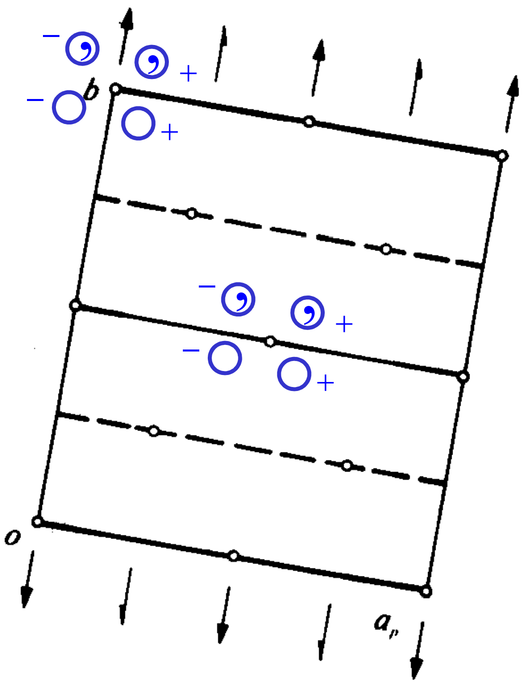
P 2_1

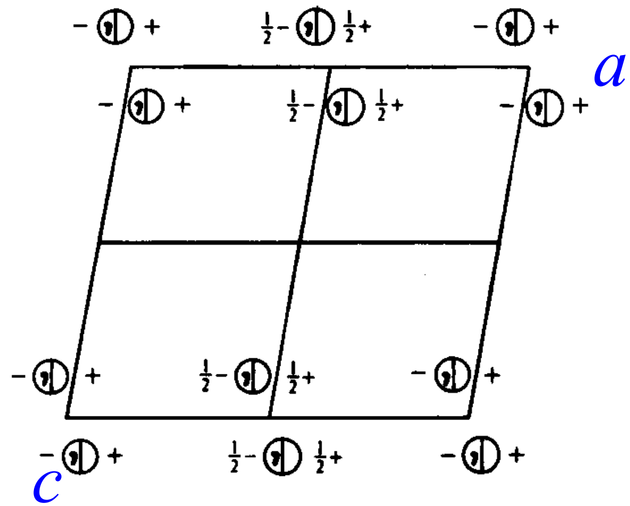
—

$C\frac{2}{m}$









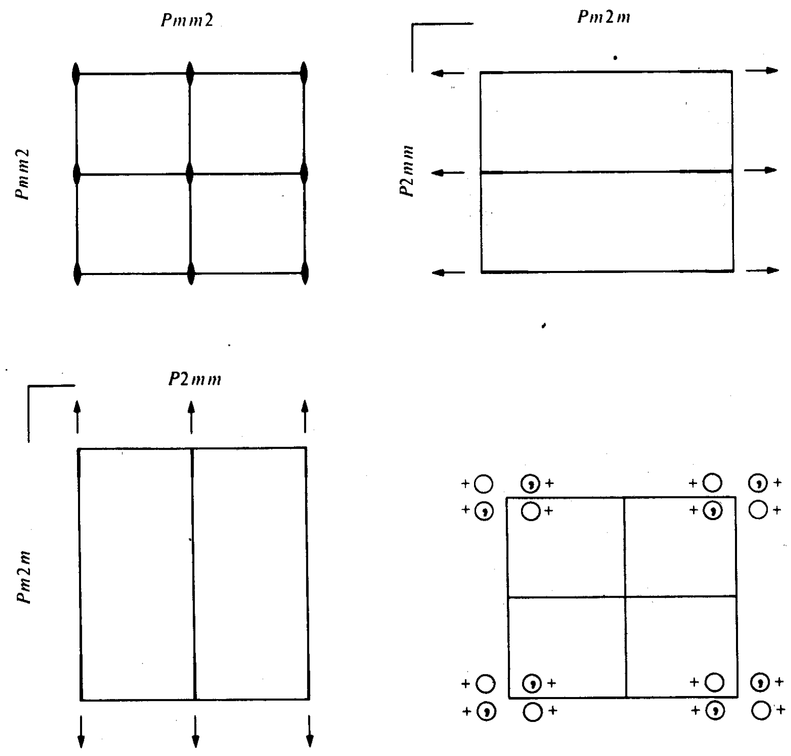
1.4. Graphical symbols for symmetry elements in one, two, and three dimensions

(a) Symmetry planes normal to the plane of projection (three dimensions) and symmetry lines in the plane of the figure (two dimensions)

Symmetry plane or symmetry line	Graphical symbol	Glide vector in units of lattice translation vectors parallel and normal to the projection plane	Printed symbol
Reflection plane, mirror plane Reflection line, mirror line (two dimensions)	———	None	*m*
'Axial' glide plane	– – – – –	$\frac{1}{2}$ along line parallel to projection plane	*a*, *b* or *c*
Glide line (two dimensions)	– – – – –	$\frac{1}{2}$ along line in plane	*g*
'Axial' glide plane	…………	$\frac{1}{2}$ normal to projection plane	*a*, *b* or *c*
'Diagonal' glide plane	—·—·—	$\frac{1}{2}$ along line parallel to projection plane, combined with $\frac{1}{2}$ normal to projection plane	*n*
'Diamond' glide plane (pair of planes; in centred cells only)	—·—·◂·— —·—·▸·—	$\frac{1}{4}$ along line parallel to projection plane, combined with $\frac{1}{4}$ normal to projection plane (arrow indicates direction parallel to the projection plane for which the normal component is positive)	*d*

Space Groups−Pmm2

***Pmm*2**	C_{2v}^{1}	*mm*2	**Orthorhombic**
No. 25	***Pmm*2**		**Patterson symmetry** *Pmmm*



Origin on *mm*2

Asymmetric unit $0 \leq x \leq \frac{1}{2};\quad 0 \leq y \leq \frac{1}{2};\quad 0 \leq z \leq 1$

Symmetry operations

(1) 1 (2) 2 0,0,*z* (3) *m* *x*,0,*z* (4) *m* 0,*y*,*z*

short space group symbol
Schoenflies symbol
point group
crystal system
number of space group
full space group symbol

projection of symmetry
elements
projection of general
position

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (3)

Positions

Multiplicity, Wyckoff letter, Site symmetry			Coordinates				Reflection conditions
							General:
4	i	1	(1) x,y,z	(2) $\bar{x},\bar{y},z$	(3) $x,\bar{y},z$	(4) $\bar{x},y,z$	no conditions
							Special: no extra conditions
2	h	$m\,.\,.$	$\frac{1}{2},y,z$	$\frac{1}{2},\bar{y},z$			
2	g	$m\,.\,.$	$0,y,z$	$0,\bar{y},z$			
2	f	$.\,m\,.$	$x,\frac{1}{2},z$	$\bar{x},\frac{1}{2},z$			
2	e	$.\,m\,.$	$x,0,z$	$\bar{x},0,z$			
1	d	$mm2$	$\frac{1}{2},\frac{1}{2},z$				
1	c	$mm2$	$\frac{1}{2},0,z$				
1	b	$mm2$	$0,\frac{1}{2},z$				
1	a	$mm2$	$0,0,z$				

Space Groups-Pmm2

- origin

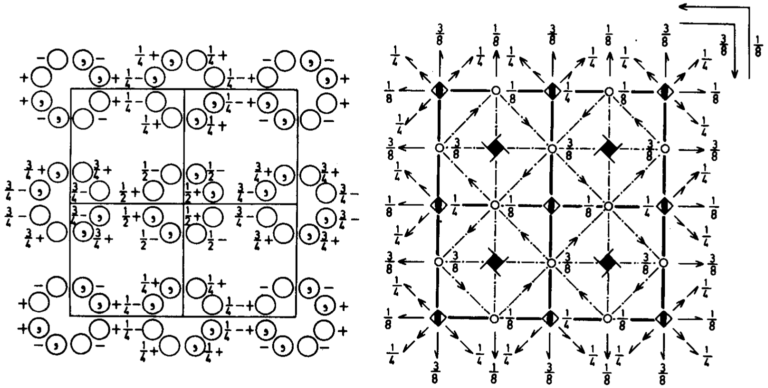
  (i) all centrosymmetric space groups are described with an inversion centers as origin.
  a second description is given if a space group contains points of high site symmetry that do not coincide with a center of symmetry

  (ii) for non-centrosymmetric space groups, the origin is at a point of highest site symmetry.
  if no site symmetry is higher than 1, the origin is placed on a screw axis or a glide plane, or at the intersection of several such symmetry elements

$I\,4_1/a\,m\,d$ $\quad D_{4h}^{19}$ $\quad 4/m\,m\,m$ $\quad$ Tetragonal

No. 141 $\quad I\,4_1/a\,2/m\,2/d$ $\quad$ Patterson symmetry $I\,4/m\,m\,m$

ORIGIN CHOICE 1

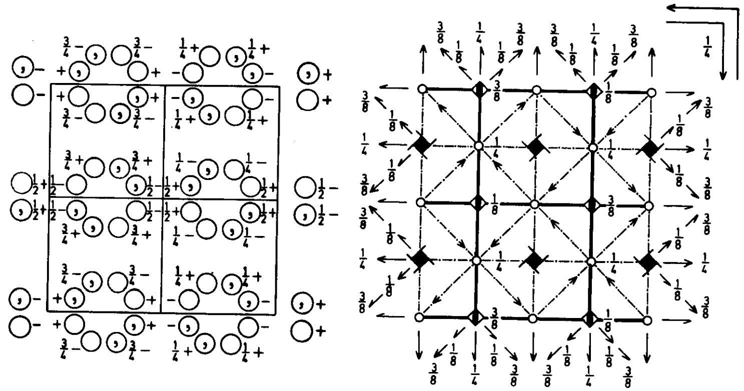


Origin at $\bar{4}m2$, at $0,\frac{1}{4},-\frac{1}{8}$ from centre ($2/m$)

$I\,4_1/a\,m\,d$ $\quad D_{4h}^{19}$ $\quad 4/m\,m\,m$ $\quad$ Tetragonal

No. 141 $\quad I\,4_1/a\,2/m\,2/d$ $\quad$ Patterson symmetry $I\,4/m\,m\,m$

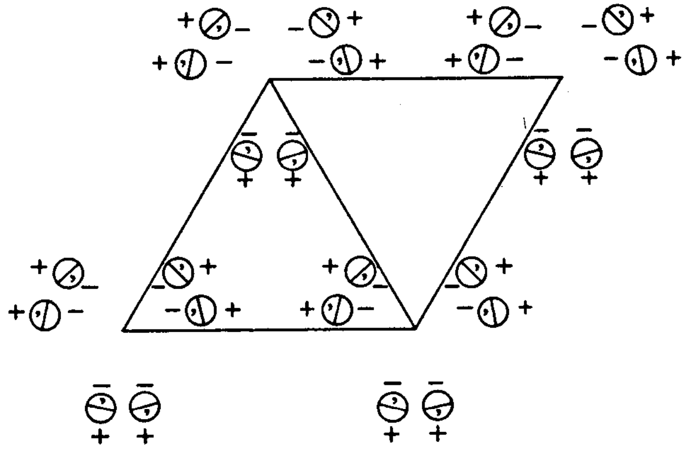
ORIGIN CHOICE 2

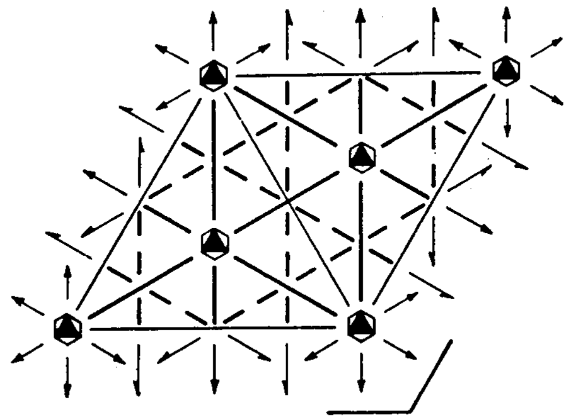


Origin at centre ($2/m$) at $b(2/m,2_1/n)d$, at $0,-\frac{1}{4},\frac{1}{8}$ from $\bar{4}m2$

$P\bar{6}m2$ $\quad D_{3h}^{1}$ $\quad \bar{6}m2$ $\quad$ Hexagonal

No. 187 $\quad P\bar{6}m2$ $\quad$ Patterson symmetry $P6/mmm$





Origin at $\bar{6}m2$

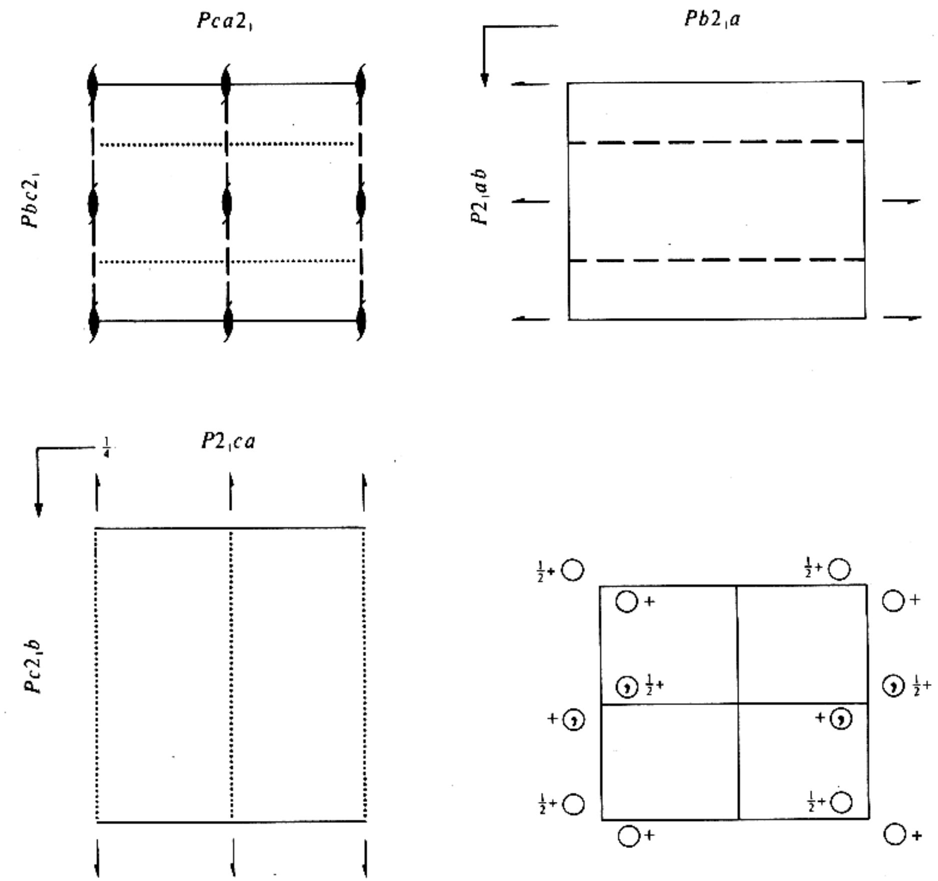
Asymmetric unit $\quad 0 \le x \le \frac{2}{3}$; $\; 0 \le y \le \frac{2}{3}$; $\; 0 \le z \le \frac{1}{2}$; $\; x \le 2y$; $\; y \le \min(1-x, 2x)$

Vertices $\quad 0,0,0 \quad \frac{2}{3},\frac{1}{3},0 \quad \frac{1}{3},\frac{2}{3},0$

$0,0,\frac{1}{2} \quad \frac{2}{3},\frac{1}{3},\frac{1}{2} \quad \frac{1}{3},\frac{2}{3},\frac{1}{2}$

$Pca2_1$ C_{2v}^5 $mm2$ Orthorhombic

No. 29 $Pca2_1$ Patterson symmetry $Pmmm$



Origin on $1a2_1$

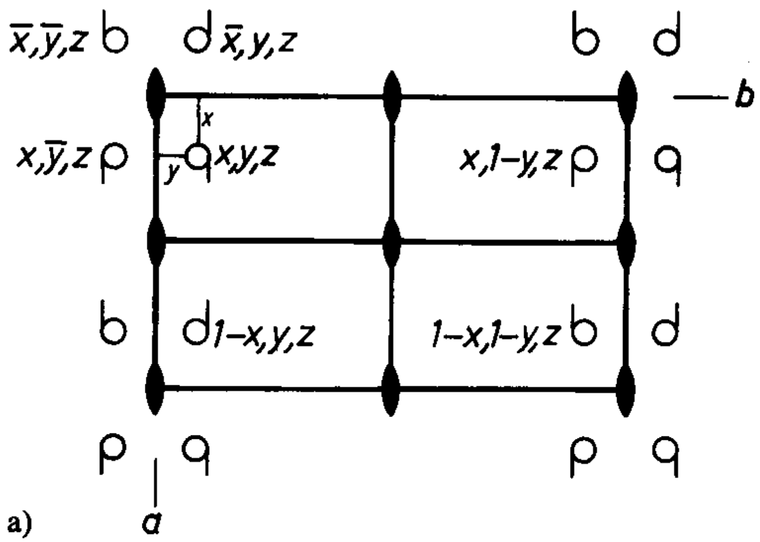
Asymmetric unit $0 \leq x \leq \frac{1}{4}$; $0 \leq y \leq 1$; $0 \leq z \leq 1$

Symmetry operations

(1) 1 (2) $2(0,0,\frac{1}{2})$ $0,0,z$ (3) a $x,0,z$ (4) c $\frac{1}{4},y,z$

Space Groups-Pmm2

- Pmm2- for a point x, y, z (general point) symmetry element generates $\bar{x}, y, z;\ x, \bar{y}, z;\ \bar{x}, \bar{y}, z$
  $x, y, z;\ \bar{x}, y, z;\ x, \bar{y}, z;\ \bar{x}, \bar{y}, z$ are equivalent (multiplicity of 4)
- The number of equivalent points in the unit cell is called its multiplicity.
- A general position is a set of equivalent points with point symmetry (site symmetry) 1.



Space Groups-Pmm2

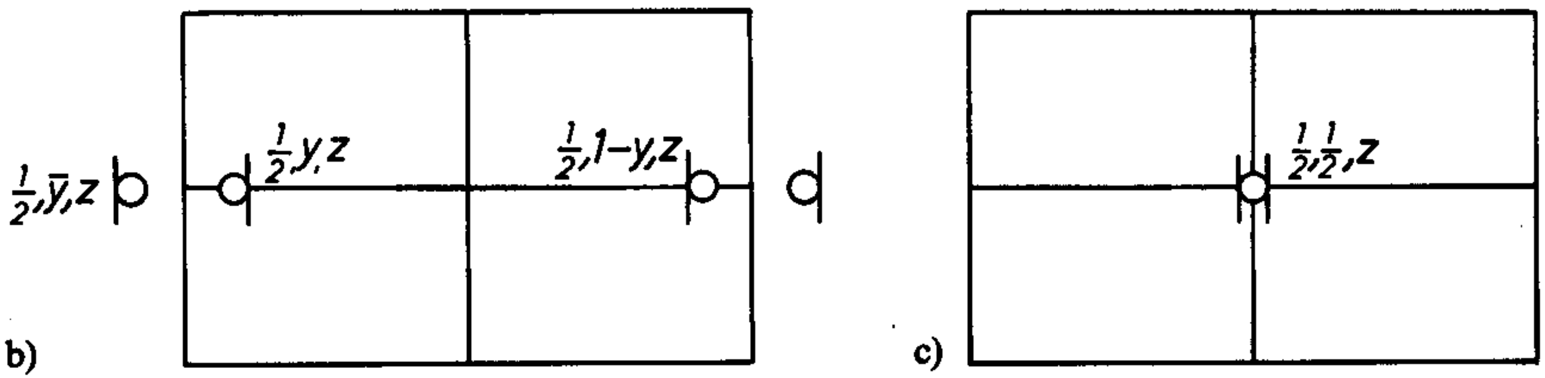
- move a point x, y, z on to mirror plane at $\frac{1}{2}, y, z$

  x, y, z and $1 - x, y, z$ coalesce to $\frac{1}{2}, y, z$

  $x, 1 - y, z$ and $1 - x, 1 - y, z$ coalesce to $\frac{1}{2}, 1 - y, z$

  multiplicity of 2

  as long as the point remains on the mirror plane, its multiplicity is unchanged- degree of freedom 2
- A special position is a set of equivalent points with point symmetry (site symmetry) higher than 1.



Space Groups-Pmm2

Position	Degrees of freedom	Multiplicity	Site symmetry	Coordinates of equivalent points
general	3	4	1	$x,y,z;\ \bar{x},\bar{y},z;$ $x,\bar{y},z;\ \bar{x},y,z$
special	2	2	m	$\frac{1}{2},y,z;\ \frac{1}{2},\bar{y},z$
		2	m	$0,y,z;\ 0,\bar{y},z$
		2	m	$x,\frac{1}{2},z;\ \bar{x},\frac{1}{2},z$
		2	m	$x,0,z;\ \bar{x},0,z$
	1	1	mm2	$\frac{1}{2},\frac{1}{2},z$
		1	mm2	$\frac{1}{2},0,z$
		1	mm2	$0,\frac{1}{2},z$
		1	mm2	$0,0,z$

Space Groups-multiplicity

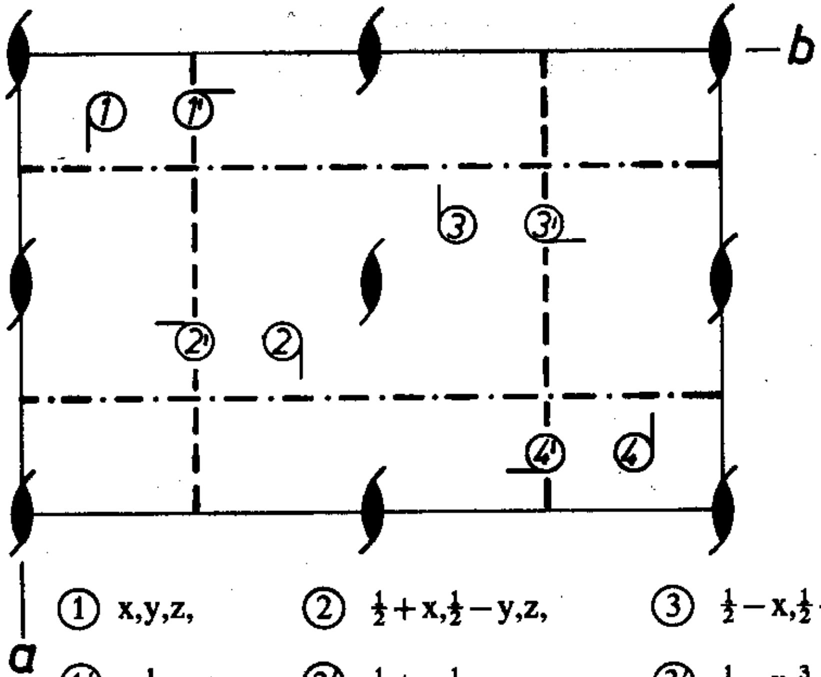
- screw axis and glide plane do not alter the multiplicity of a point

-Pna2_1: orthorhombic

n-glide normal to a-axis

a-glide normal to b-axis

2_1 screw axis along c-axis



(1) x, y, z, (2) $\frac{1}{2}+x, \frac{1}{2}-y, z$, (3) $\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$, (4) $1-x, 1-y, \frac{1}{2}+z$

(1') $x, \frac{1}{4}, z$, (2') $\frac{1}{2}+x, \frac{1}{4}, z$, (3') $\frac{1}{2}-x, \frac{3}{4}, \frac{1}{2}+z$, (4') $1-x, \frac{3}{4}, \frac{1}{2}+z$

no special position

Space Groups-asymmetric unit

-The asymmetric unit of a space group is the smallest part of the unit cell from which the whole cell may be filled by the operation of all the symmetry operations. It volume is given by:

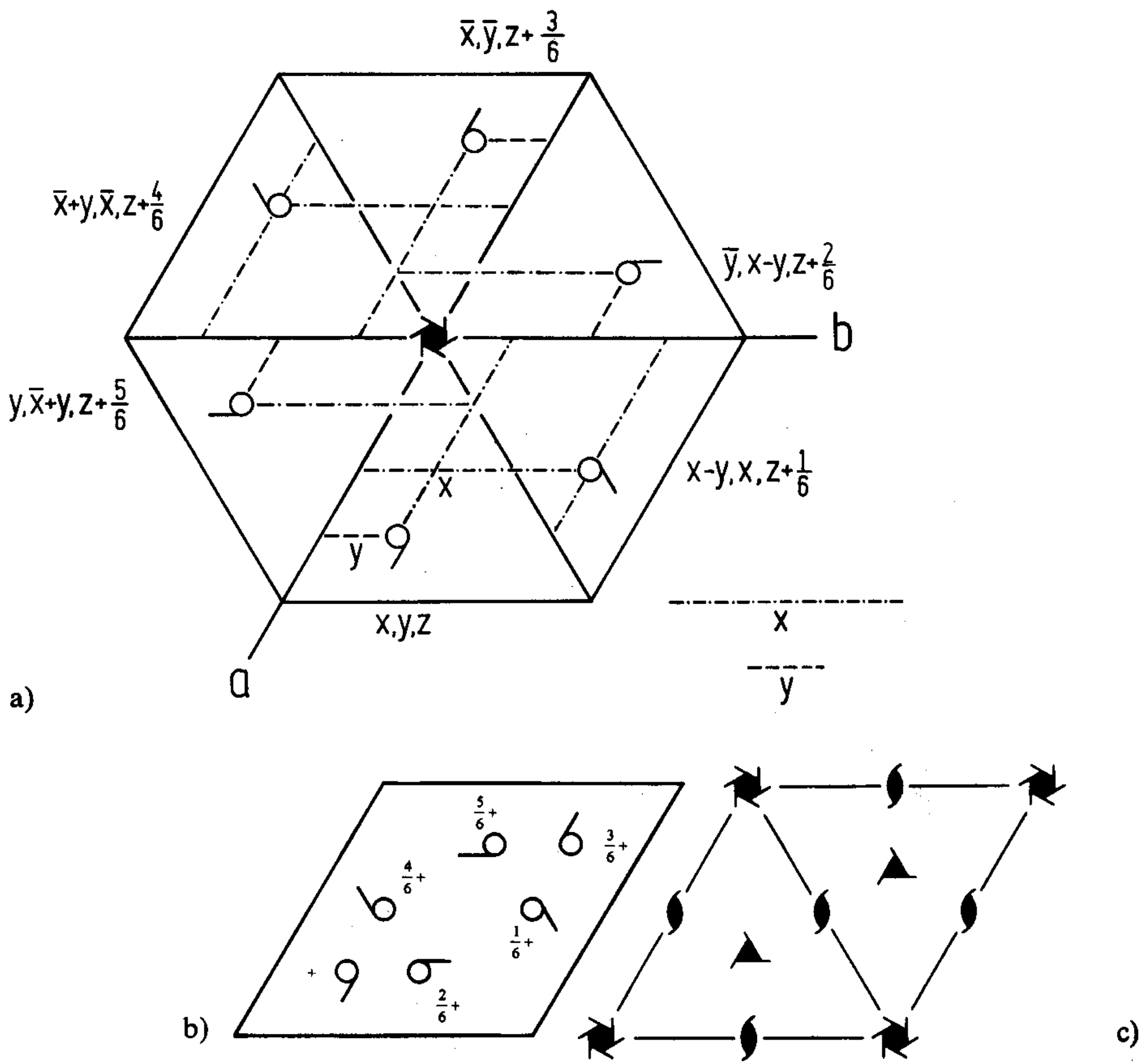
$$V_{\text{asymm.unit}} = \frac{V_{\text{unit cell}}}{\textbf{multiplicity of general position}}$$

ex) Pmm2- multiplicity of 4, vol. of asymm=1/4 unit cell

$$0 \leq x \leq \tfrac{1}{2},\ 0 \leq y \leq \tfrac{1}{2},\ 0 \leq z \leq 1$$

-An asymmetric unit contains all the information necessary for the complete descript of a crystal structure.

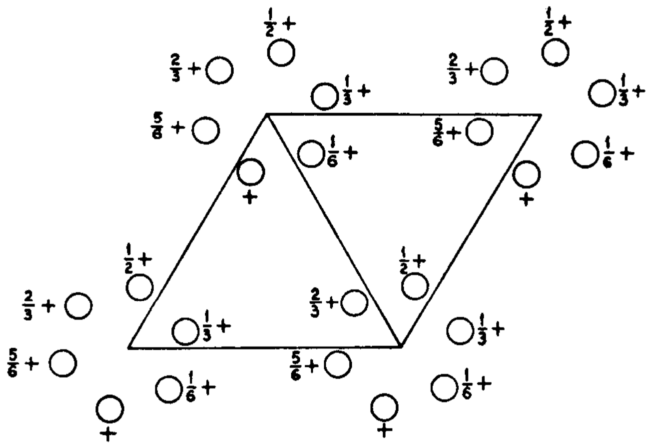
Space Groups–$P6_1$

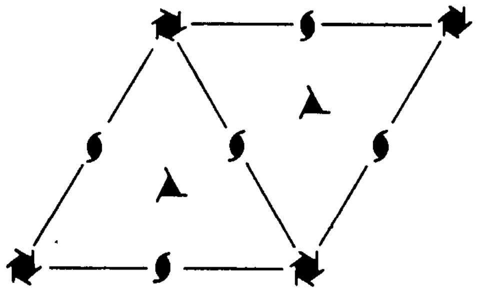


3_1

2_1

$P\,6_1$	C_6^2	6	Hexagonal
No. 169	$P\,6_1$		Patterson symmetry $P\,6/m$





Origin on 6_1

Asymmetric unit $0 \le x \le 1;\quad 0 \le y \le 1;\quad 0 \le z \le \frac{1}{6}$

Vertices $0,0,0 \quad 1,0,0 \quad 1,1,0 \quad 0,1,0$

$0,0,\frac{1}{6} \quad 1,0,\frac{1}{6} \quad 1,1,\frac{1}{6} \quad 0,1,\frac{1}{6}$

Symmetry operations

(1) 1	(2) $3^{+}(0,0,\frac{1}{3})\ \ 0,0,z$	(3) $3^{-}(0,0,\frac{2}{3})\ \ 0,0,z$
(4) $2(0,0,\frac{1}{2})\ \ 0,0,z$	(5) $6^{-}(0,0,\frac{5}{6})\ \ 0,0,z$	(6) $6^{+}(0,0,\frac{1}{6})\ \ 0,0,z$

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (4)

Positions

Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

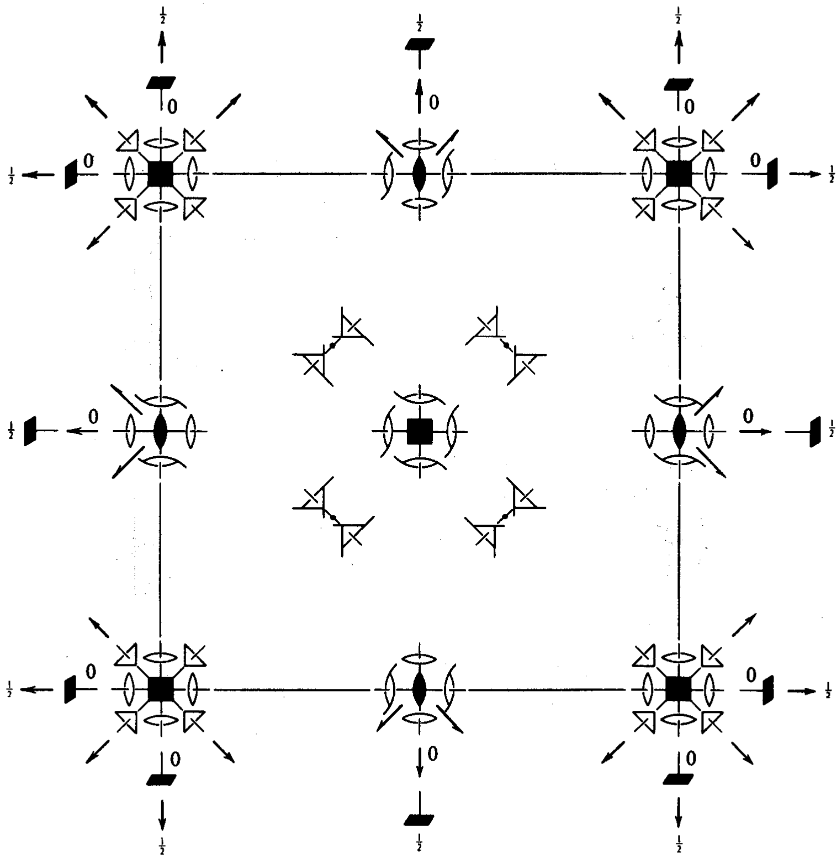
Reflection conditions

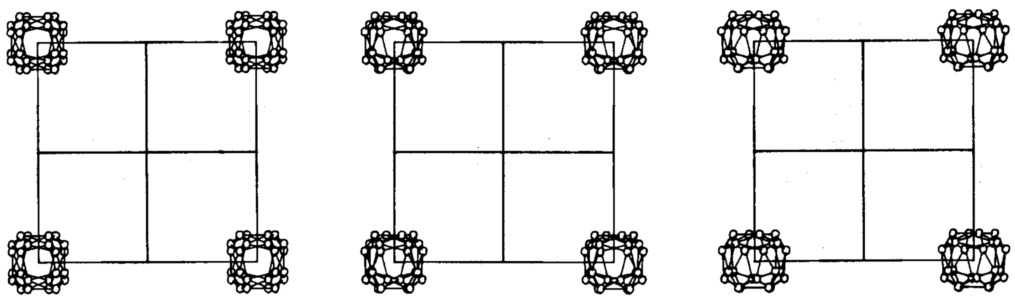
General:

6	*a*	1	(1) x,y,z	(2) $\bar{y},x-y,z+\frac{1}{3}$	(3) $\bar{x}+y,\bar{x},z+\frac{2}{3}$	$000l:\ l=6n$
			(4) $\bar{x},\bar{y},z+\frac{1}{2}$	(5) $y,\bar{x}+y,z+\frac{5}{6}$	(6) $x-y,x,z+\frac{1}{6}$	

$P\,4\,3\,2$ $\quad$ O^1 $\quad$ $4\,3\,2$ $\quad$ Cubic

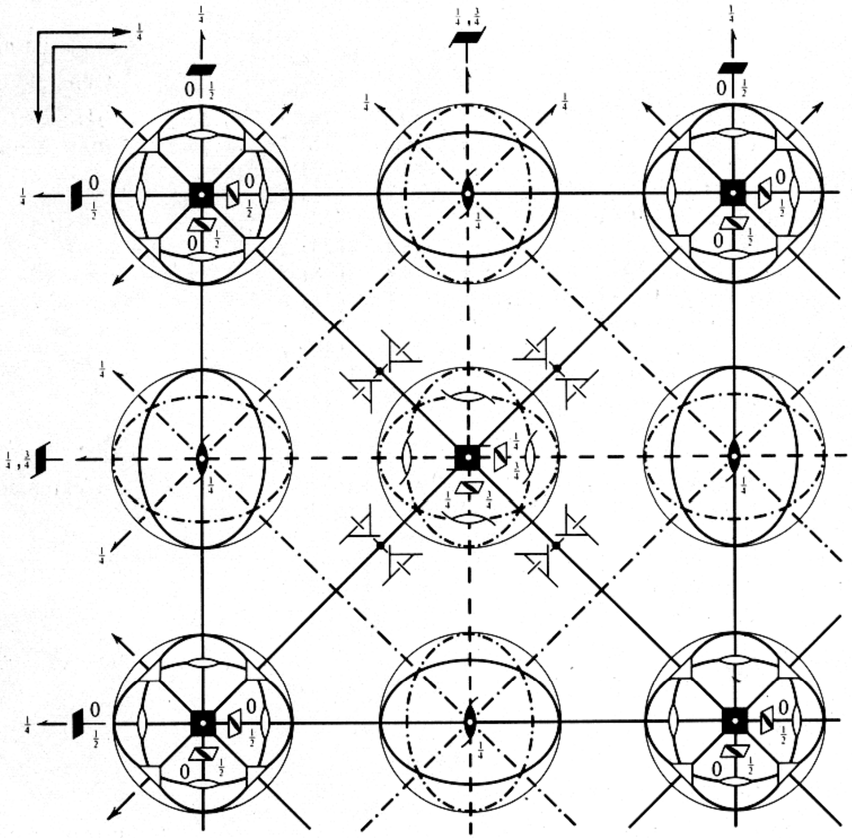
No. 207 $\quad$ $P\,4\,3\,2$ $\quad$ Patterson symmetry $Pm\bar{3}m$





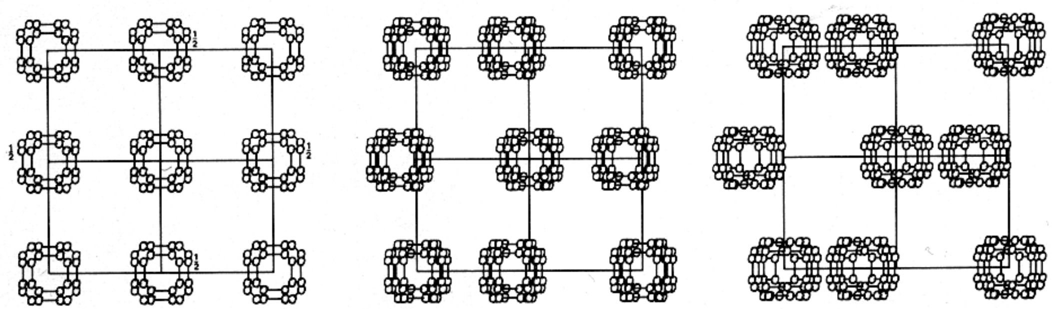
$Fm\bar{3}m$ O_h^5 $m\bar{3}m$ Cubic

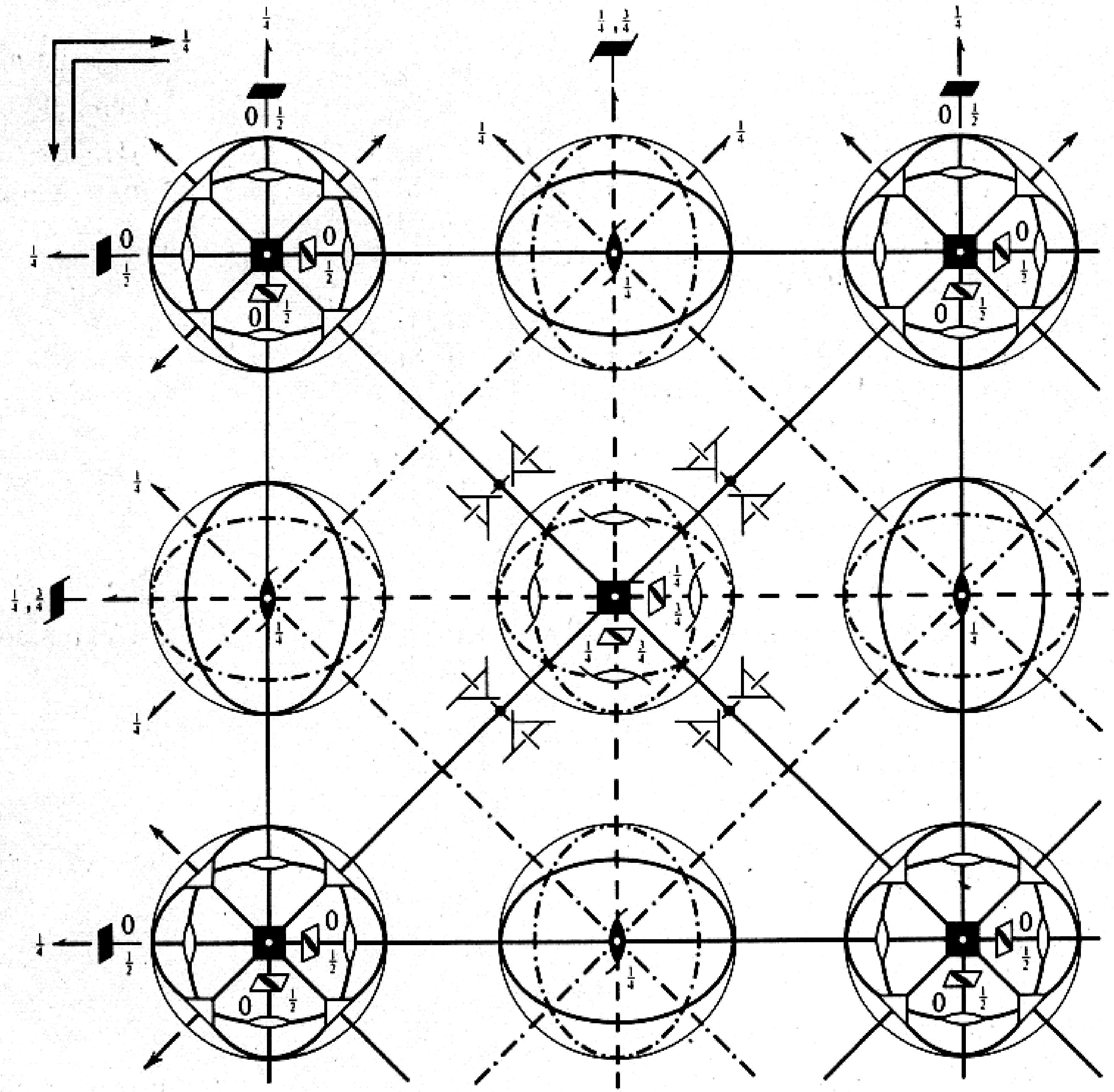
No. 225 $F4/m\bar{3}2/m$ Patterson symmetry $Fm\bar{3}m$



orthographic representation

Upper left quadrant only



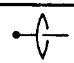
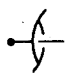


Upper left quadrant only

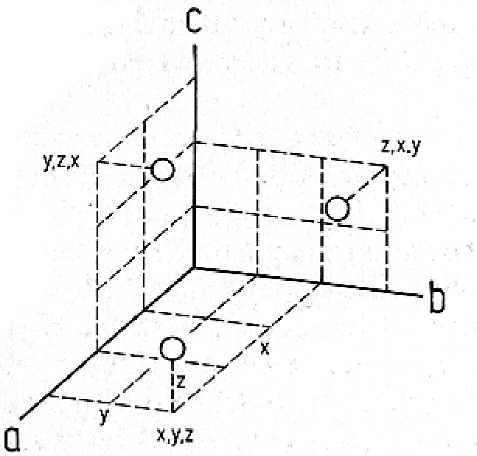
(e) Symmetry axes parallel to the plane of projection

Symmetry axis	Graphical symbol	Screw vector of a right-handed screw rotation in units of the shortest lattice translation vector parallel to the axis	Printed symbol
Twofold rotation axis		None	2
Twofold screw axis: '2 sub 1'		$\frac{1}{2}$	2_1
Fourfold rotation axis		None	4
Fourfold screw axis: '4 sub 1'		$\frac{1}{4}$	4_1
Fourfold screw axis: '4 sub 2'		$\frac{1}{2}$	4_2
Fourfold screw axis: '4 sub 3'		$\frac{3}{4}$	4_3
Inversion axis: '4 bar'		None	$\bar{4}$

(f) Symmetry axes inclined to the plane of projection (in cubic space groups only)

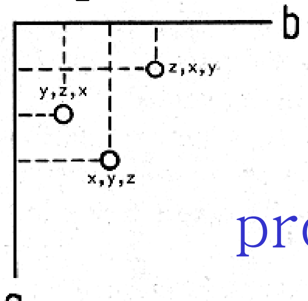
Symmetry axis	Graphical symbol	Screw vector of a right-handed screw rotation in units of the shortest lattice translation vector parallel to the axis	Printed symb
Twofold rotation axis		None	2
Twofold screw axis: '2 sub 1'		$\frac{1}{2}$	2_1
Threefold rotation axis		None	3
Threefold screw axis: '3 sub 1'		$\frac{1}{3}$	3_1
Threefold screw axis: '3 sub 2'		$\frac{2}{3}$	3_2
Inversion axis: '3 bar'		None	$\bar{3}$

48-fold general position



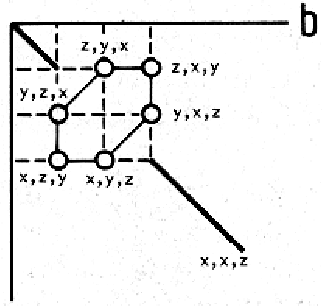
a)

3-fold rotation



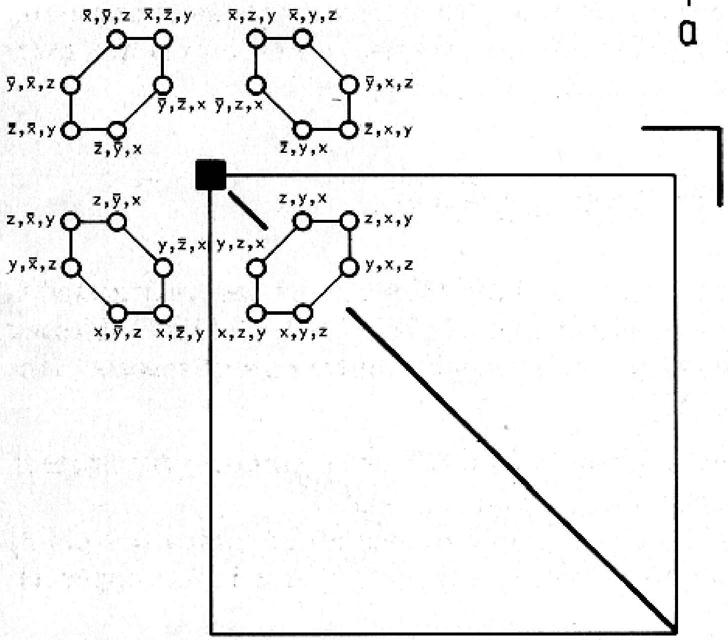
projection on x,y,0

b)



c)

mirror plane at x,x,z

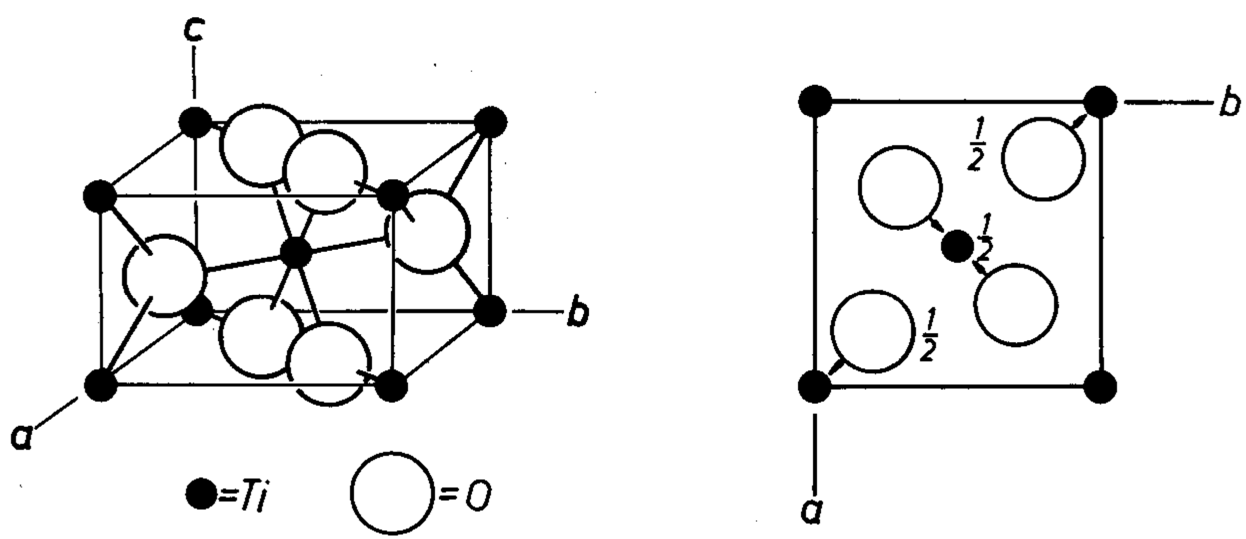


4-fold rotation at 0,0,z

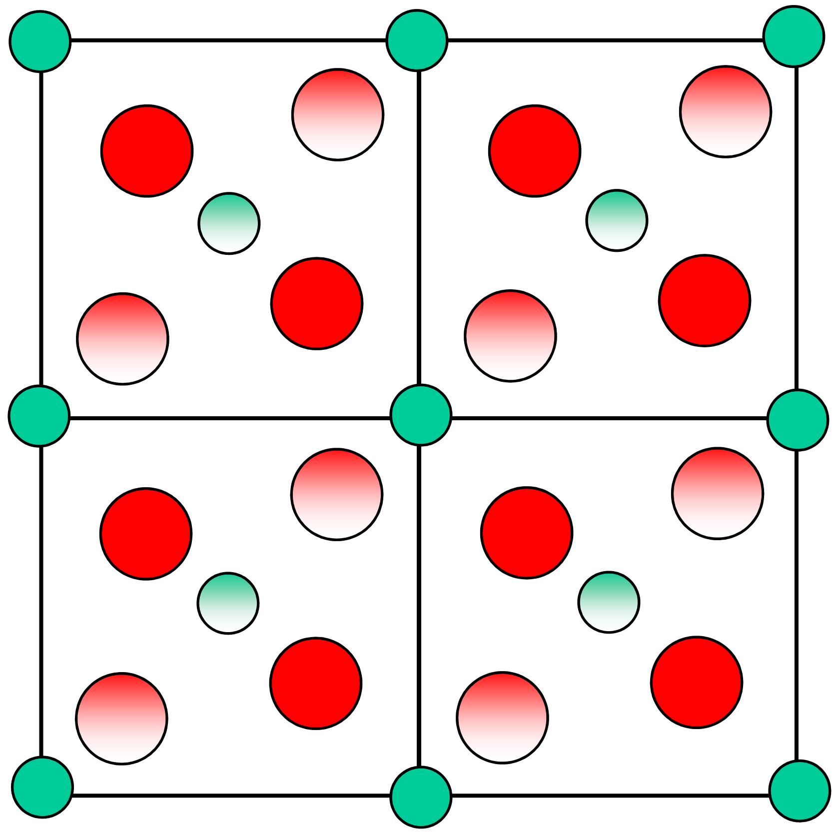
d)

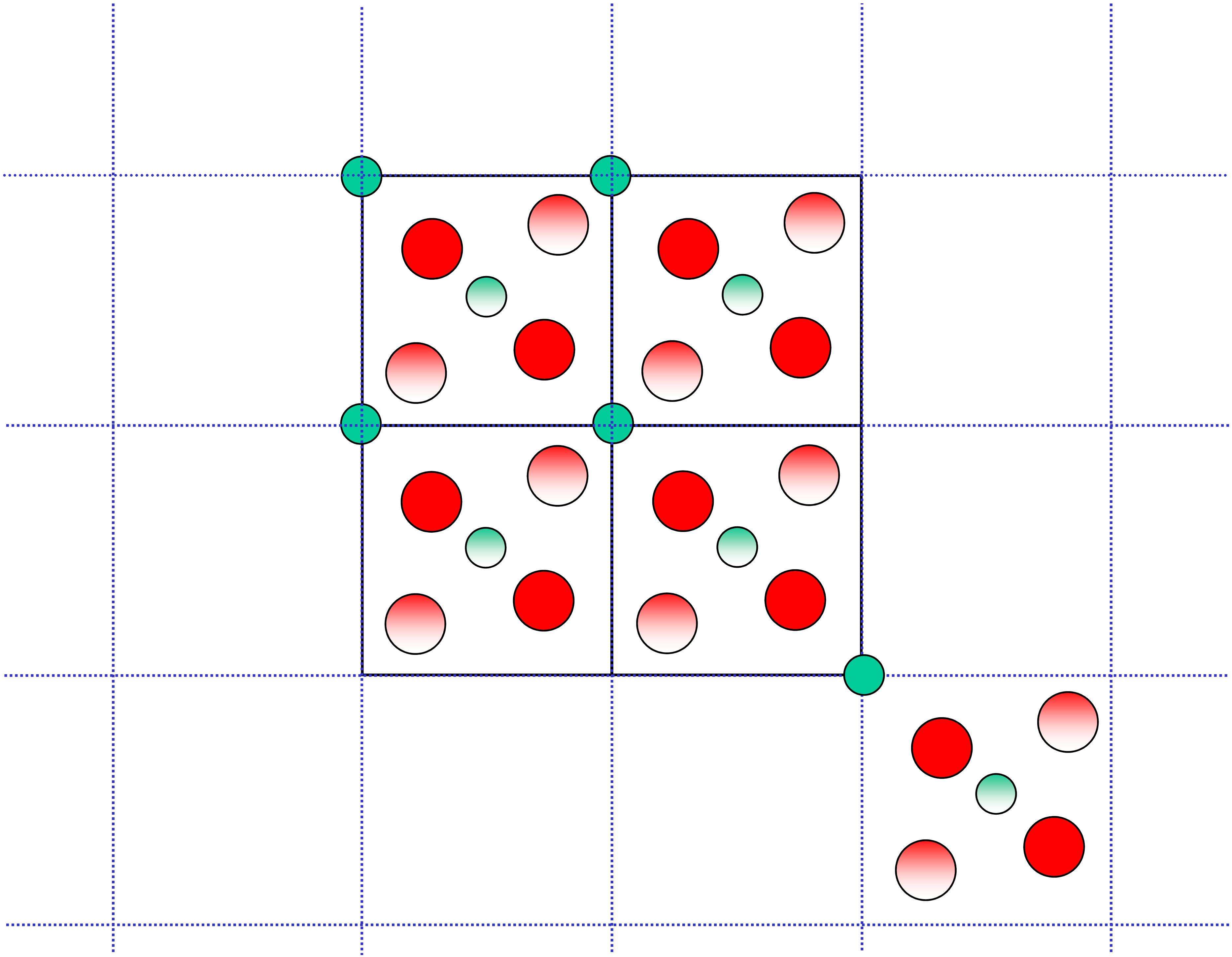
Rutile, TiO_2

A		B		
Lattice	Basis	Space group	Positions of the atoms	
tetragonal P	Ti: 0,0,0 $\frac{1}{2},\frac{1}{2},\frac{1}{2}$	P 4_2/mnm	a	Ti: 0,0,0 $\frac{1}{2},\frac{1}{2},\frac{1}{2}$
$a_0 = 4.59$ Å $c_0 = 2.96$ Å	O: 0.3, 0.3, 0 0.8, 0.2, $\frac{1}{2}$ 0.2, 0.8, $\frac{1}{2}$ 0.7, 0.7, 0	$a_0 = 4.59$ Å $c_0 = 2.96$ Å	f	O: x,x,0 $\frac{1}{2}+x, \frac{1}{2}-x, \frac{1}{2}$ $\frac{1}{2}-x, \frac{1}{2}+x, \frac{1}{2}$ $\bar{x}, \bar{x}, 0$ x = 0.3



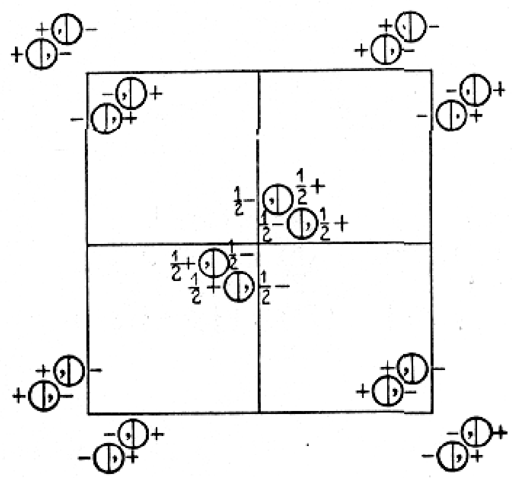
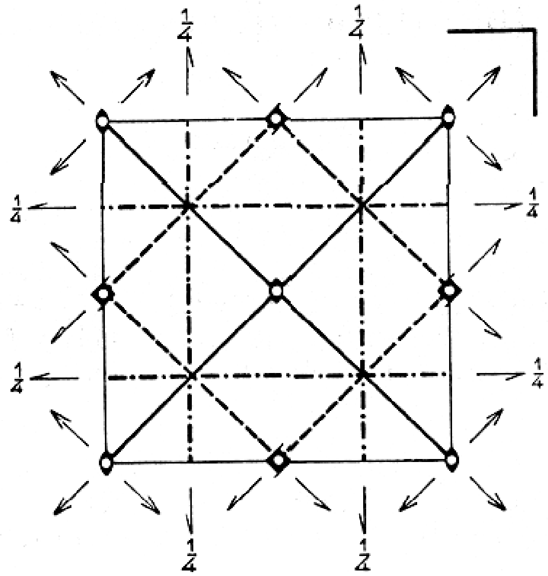
Rutile, TiO_2





$P\,4_2/m\,n\,m$ $\quad D_{4h}^{14}$ $\quad 4/m\,m\,m$ $\quad$ Tetragonal

No. 136 $\quad P\,4_2/m\,2_1/n\,2/m$ $\quad$ Patterson symmetry $P\,4/m\,m\,m$

Origin at centre ($m\,m\,m$) at $2/m\,1\,2/m$

Asymmetric unit $\quad 0 \le x \le \frac{1}{2};\quad 0 \le y \le \frac{1}{2};\quad 0 \le z \le \frac{1}{2};\quad x \le y$

Symmetry operations

(1) 1	(2) $2\quad 0,0,z$	(3) $4^+(0,0,\frac{1}{2})\quad 0,\frac{1}{2},z$	(4) $4^-(0,0,\frac{1}{2})\quad \frac{1}{2},0,z$
(5) $2(0,\frac{1}{2},0)\quad \frac{1}{4},y,\frac{1}{4}$	(6) $2(\frac{1}{2},0,0)\quad x,\frac{1}{4},\frac{1}{4}$	(7) $2\quad x,x,0$	(8) $2\quad x,\bar{x},0$
(9) $\bar{1}\quad 0,0,0$	(10) $m\quad x,y,0$	(11) $\bar{4}^+\quad \frac{1}{2},0,z;\ \frac{1}{2},0,\frac{1}{4}$	(12) $\bar{4}^-\quad 0,\frac{1}{2},z;\ 0,\frac{1}{2},\frac{1}{4}$
(13) $n(\frac{1}{2},0,\frac{1}{2})\quad x,\frac{1}{4},z$	(14) $n(0,\frac{1}{2},\frac{1}{2})\quad \frac{1}{4},y,z$	(15) $m\quad x,\bar{x},z$	(16) $m\quad x,x,z$

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (3); (5); (9)

Positions

Multiplicity, Wyckoff letter, Site symmetry			Coordinates				Reflection conditions
							General:
16	k	1	(1) x,y,z	(2) $\bar{x},\bar{y},z$	(3) $\bar{y}+\frac{1}{2},x+\frac{1}{2},z+\frac{1}{2}$	(4) $y+\frac{1}{2},\bar{x}+\frac{1}{2},z+\frac{1}{2}$	$0kl$: $k+l=2n$
			(5) $\bar{x}+\frac{1}{2},y+\frac{1}{2},\bar{z}+\frac{1}{2}$	(6) $x+\frac{1}{2},\bar{y}+\frac{1}{2},\bar{z}+\frac{1}{2}$	(7) $y,x,\bar{z}$	(8) $\bar{y},\bar{x},\bar{z}$	$00l$: $l=2n$
			(9) $\bar{x},\bar{y},\bar{z}$	(10) $x,y,\bar{z}$	(11) $y+\frac{1}{2},\bar{x}+\frac{1}{2},\bar{z}+\frac{1}{2}$	(12) $\bar{y}+\frac{1}{2},x+\frac{1}{2},\bar{z}+\frac{1}{2}$	$h00$: $h=2n$
			(13) $x+\frac{1}{2},\bar{y}+\frac{1}{2},z+\frac{1}{2}$	(14) $\bar{x}+\frac{1}{2},y+\frac{1}{2},z+\frac{1}{2}$	(15) $\bar{y},\bar{x},z$	(16) y,x,z	
							Special: as above, plus
8	j	$..m$	x,x,z	$\bar{x},\bar{x},z$	$\bar{x}+\frac{1}{2},x+\frac{1}{2},z+\frac{1}{2}$	$x+\frac{1}{2},\bar{x}+\frac{1}{2},z+\frac{1}{2}$	no extra conditions
			$\bar{x}+\frac{1}{2},x+\frac{1}{2},\bar{z}+\frac{1}{2}$	$x+\frac{1}{2},\bar{x}+\frac{1}{2},\bar{z}+\frac{1}{2}$	$x,x,\bar{z}$	$\bar{x},\bar{x},\bar{z}$	
8	i	$m..$	$x,y,0$	$\bar{x},\bar{y},0$	$\bar{y}+\frac{1}{2},x+\frac{1}{2},\frac{1}{2}$	$y+\frac{1}{2},\bar{x}+\frac{1}{2},\frac{1}{2}$	no extra conditions
			$\bar{x}+\frac{1}{2},y+\frac{1}{2},\frac{1}{2}$	$x+\frac{1}{2},\bar{y}+\frac{1}{2},\frac{1}{2}$	$y,x,0$	$\bar{y},\bar{x},0$	
8	h	$2..$	$0,\frac{1}{2},z$ $\quad$ $0,\frac{1}{2},z+\frac{1}{2}$	$\frac{1}{2},0,\bar{z}+\frac{1}{2}$ $\quad$ $\frac{1}{2},0,\bar{z}$			hkl : $h+k,l=2n$
			$0,\frac{1}{2},\bar{z}$ $\quad$ $0,\frac{1}{2},\bar{z}+\frac{1}{2}$	$\frac{1}{2},0,z+\frac{1}{2}$ $\quad$ $\frac{1}{2},0,z$			
4	g	$m.2m$	$x,\bar{x},0$ $\quad$ $\bar{x},x,0$	$x+\frac{1}{2},x+\frac{1}{2},\frac{1}{2}$ $\quad$ $\bar{x}+\frac{1}{2},\bar{x}+\frac{1}{2},\frac{1}{2}$			no extra conditions
4	f	$m.2m$	$x,x,0$ $\quad$ $\bar{x},\bar{x},0$	$\bar{x}+\frac{1}{2},x+\frac{1}{2},\frac{1}{2}$ $\quad$ $x+\frac{1}{2},\bar{x}+\frac{1}{2},\frac{1}{2}$			no extra conditions
4	e	$2.mm$	$0,0,z$ $\quad$ $\frac{1}{2},\frac{1}{2},z+\frac{1}{2}$	$\frac{1}{2},\frac{1}{2},\bar{z}+\frac{1}{2}$ $\quad$ $0,0,\bar{z}$			hkl : $h+k+l=2n$
4	d	$\bar{4}..$	$0,\frac{1}{2},\frac{1}{4}$ $\quad$ $0,\frac{1}{2},\frac{3}{4}$	$\frac{1}{2},0,\frac{1}{4}$ $\quad$ $\frac{1}{2},0,\frac{3}{4}$			hkl : $h+k,l=2n$
4	c	$2/m..$	$0,\frac{1}{2},0$ $\quad$ $0,\frac{1}{2},\frac{1}{2}$	$\frac{1}{2},0,\frac{1}{2}$ $\quad$ $\frac{1}{2},0,0$			hkl : $h+k,l=2n$
2	b	$m.mm$	$0,0,\frac{1}{2}$ $\quad$ $\frac{1}{2},\frac{1}{2},0$				hkl : $h+k+l=2n$
2	a	$m.mm$	$0,0,0$ $\quad$ $\frac{1}{2},\frac{1}{2},\frac{1}{2}$				hkl : $h+k+l=2n$

Perovskite, $CaTiO_3$

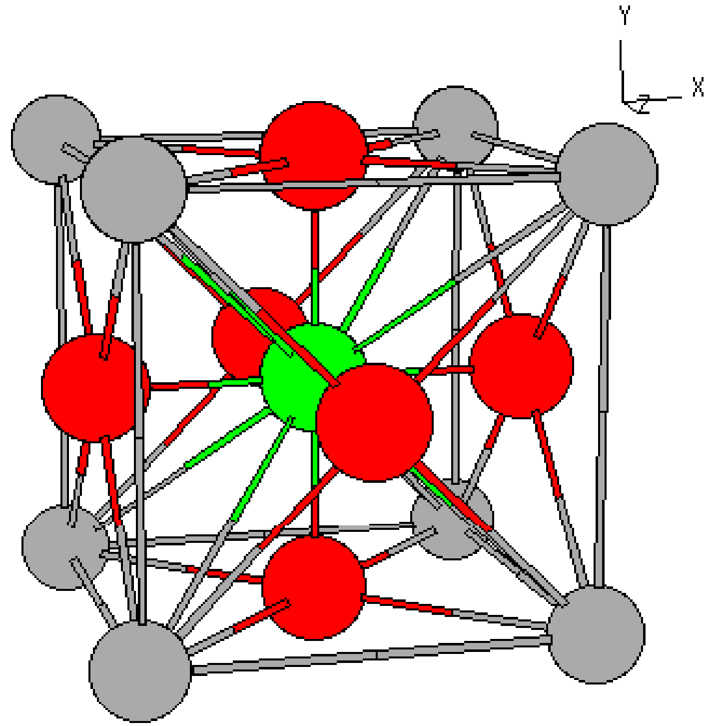
- Ca-corner
  O- face centered
  Ti- body centered
- high temperature- cubic
  Pm3m (No.221)

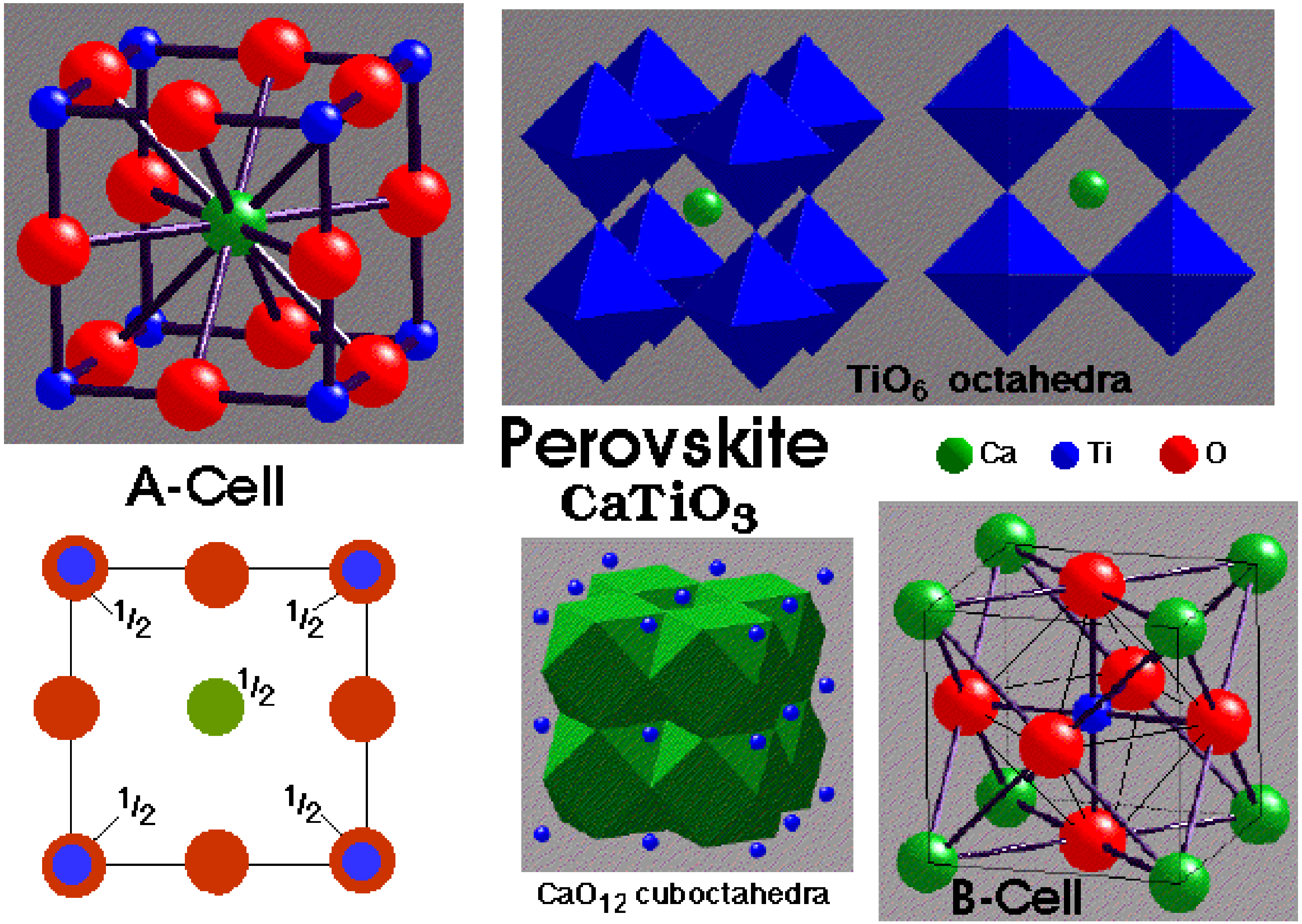
$$Ca:\ 1a,\ m\bar{3}m,\quad 0,0,0$$

$$Ti:\ 1b,\ m\bar{3}m,\quad \frac{1}{2},\frac{1}{2},\frac{1}{2}$$

$$O:\ 3c,\ 4/mmm,\quad 0,\frac{1}{2},\frac{1}{2}\quad \frac{1}{2},0,\frac{1}{2}\quad \frac{1}{2},\frac{1}{2},0$$

- Ti- 6O
  O- 4Ca+ 2Ti
  Ca- 12O

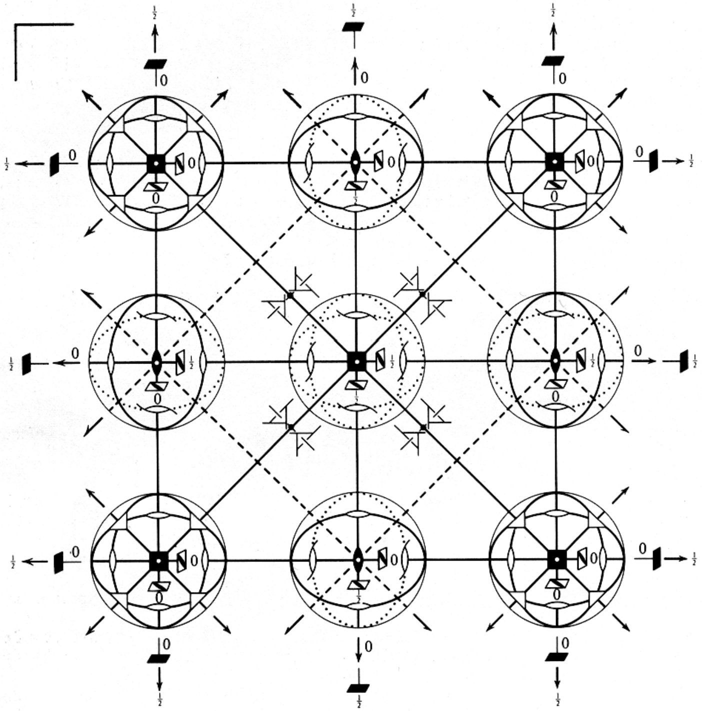


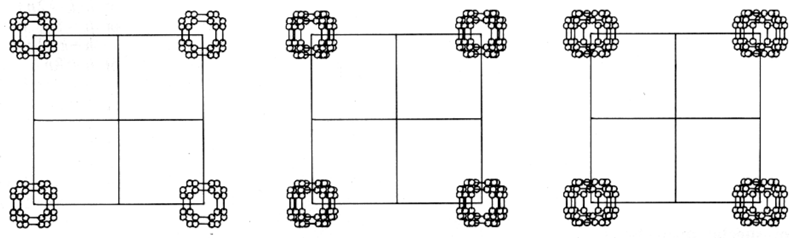


Perovskite
$CaTiO_3$
A-Cell
B-Cell
TiO_6 octahedra
CaO_{12} cuboctahedra
Ca
Ti
O
1/2
1/2
1/2
1/2
1/2

$Pm\bar{3}m$ O_h^1 $m\bar{3}m$ Cubic

No. 221 $P\,4/m\,\bar{3}\,2/m$ Patterson symmetry $Pm\bar{3}m$





Origin at centre ($m\bar{3}m$)

Asymmetric unit $0 \le x \le \frac{1}{2}$; $0 \le y \le \frac{1}{2}$; $0 \le z \le \frac{1}{2}$; $y \le x$; $z \le y$

Vertices $0,0,0$ $\frac{1}{2},0,0$ $\frac{1}{2},\frac{1}{2},0$ $\frac{1}{2},\frac{1}{2},\frac{1}{2}$

Symmetry operations
(*given on page* 664)

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (3); (5); (13); (25)

Positions

Multiplicity,	Wyckoff letter,	Site symmetry	Coordinates					
48	n	1	(1) x,y,z	(2) $\bar{x},\bar{y},z$	(3) $\bar{x},y,\bar{z}$	(4) $x,\bar{y},\bar{z}$		
			(5) z,x,y	(6) $z,\bar{x},\bar{y}$	(7) $\bar{z},\bar{x},y$	(8) $\bar{z},x,\bar{y}$		
			(9) y,z,x	(10) $\bar{y},z,\bar{x}$	(11) $y,\bar{z},\bar{x}$	(12) $\bar{y},\bar{z},x$		
			(13) $y,x,\bar{z}$	(14) $\bar{y},\bar{x},\bar{z}$	(15) $y,\bar{x},z$	(16) $\bar{y},x,z$		
			(17) $x,z,\bar{y}$	(18) $\bar{x},z,y$	(19) $\bar{x},\bar{z},\bar{y}$	(20) $x,\bar{z},y$		
			(21) $z,y,\bar{x}$	(22) $z,\bar{y},x$	(23) $\bar{z},y,x$	(24) $\bar{z},\bar{y},\bar{x}$		
			(25) $\bar{x},\bar{y},\bar{z}$	(26) $x,y,\bar{z}$	(27) $x,\bar{y},z$	(28) $\bar{x},y,z$		
			(29) $\bar{z},\bar{x},\bar{y}$	(30) $\bar{z},x,y$	(31) $z,x,\bar{y}$	(32) $z,\bar{x},y$		
			(33) $\bar{y},\bar{z},\bar{x}$	(34) $y,\bar{z},x$	(35) $\bar{y},z,x$	(36) $y,z,\bar{x}$		
			(37) $\bar{y},\bar{x},z$	(38) y,x,z	(39) $\bar{y},x,\bar{z}$	(40) $y,\bar{x},\bar{z}$		
			(41) $\bar{x},\bar{z},y$	(42) $x,\bar{z},\bar{y}$	(43) x,z,y	(44) $\bar{x},z,\bar{y}$		
			(45) $\bar{z},\bar{y},x$	(46) $\bar{z},y,\bar{x}$	(47) $z,\bar{y},\bar{x}$	(48) z,y,x		
24	m	$..m$	x,x,z	$\bar{x},\bar{x},z$	$\bar{x},x,\bar{z}$	$x,\bar{x},\bar{z}$	z,x,x	$z,\bar{x},\bar{x}$
			$\bar{z},\bar{x},x$	$\bar{z},x,\bar{x}$	x,z,x	$\bar{x},z,\bar{x}$	$x,\bar{z},\bar{x}$	$\bar{x},\bar{z},x$
			$x,x,\bar{z}$	$\bar{x},\bar{x},\bar{z}$	$x,\bar{x},z$	$\bar{x},x,z$	$x,z,\bar{x}$	$\bar{x},z,x$
			$\bar{x},\bar{z},\bar{x}$	$x,\bar{z},x$	$z,x,\bar{x}$	$z,\bar{x},x$	$\bar{z},x,x$	$\bar{z},\bar{x},\bar{x}$
24	l	$m..$	$\frac12,y,z$	$\frac12,\bar{y},z$	$\frac12,y,\bar{z}$	$\frac12,\bar{y},\bar{z}$	$z,\frac12,y$	$z,\frac12,\bar{y}$
			$\bar{z},\frac12,y$	$\bar{z},\frac12,\bar{y}$	$y,z,\frac12$	$\bar{y},z,\frac12$	$y,\bar{z},\frac12$	$\bar{y},\bar{z},\frac12$
			$y,\frac12,\bar{z}$	$\bar{y},\frac12,\bar{z}$	$y,\frac12,z$	$\bar{y},\frac12,z$	$\frac12,z,\bar{y}$	$\frac12,z,y$
			$\frac12,\bar{z},\bar{y}$	$\frac12,\bar{z},y$	$z,y,\frac12$	$z,\bar{y},\frac12$	$\bar{z},y,\frac12$	$\bar{z},\bar{y},\frac12$
24	k	$m..$	$0,y,z$	$0,\bar{y},z$	$0,y,\bar{z}$	$0,\bar{y},\bar{z}$	$z,0,y$	$z,0,\bar{y}$
			$\bar{z},0,y$	$\bar{z},0,\bar{y}$	$y,z,0$	$\bar{y},z,0$	$y,\bar{z},0$	$\bar{y},\bar{z},0$
			$y,0,\bar{z}$	$\bar{y},0,\bar{z}$	$y,0,z$	$\bar{y},0,z$	$0,z,\bar{y}$	$0,z,y$
			$0,\bar{z},\bar{y}$	$0,\bar{z},y$	$z,y,0$	$z,\bar{y},0$	$\bar{z},y,0$	$\bar{z},\bar{y},0$
12	j	$m.m2$	$\frac12,y,y$	$\frac12,\bar{y},y$	$\frac12,y,\bar{y}$	$\frac12,\bar{y},\bar{y}$	$y,\frac12,y$	$y,\frac12,\bar{y}$
			$\bar{y},\frac12,y$	$\bar{y},\frac12,\bar{y}$	$y,y,\frac12$	$\bar{y},y,\frac12$	$y,\bar{y},\frac12$	$\bar{y},\bar{y},\frac12$
12	i	$m.m2$	$0,y,y$	$0,\bar{y},y$	$0,y,\bar{y}$	$0,\bar{y},\bar{y}$	$y,0,y$	$y,0,\bar{y}$
			$\bar{y},0,y$	$\bar{y},0,\bar{y}$	$y,y,0$	$\bar{y},y,0$	$y,\bar{y},0$	$\bar{y},\bar{y},0$
12	h	$mm2..$	$x,\frac12,0$	$\bar{x},\frac12,0$	$0,x,\frac12$	$0,\bar{x},\frac12$	$\frac12,0,x$	$\frac12,0,\bar{x}$
			$\frac12,x,0$	$\frac12,\bar{x},0$	$x,0,\frac12$	$\bar{x},0,\frac12$	$0,\frac12,\bar{x}$	$0,\frac12,x$
8	g	$.3m$	x,x,x	$\bar{x},\bar{x},x$	$\bar{x},x,\bar{x}$	$x,\bar{x},\bar{x}$		
			$x,x,\bar{x}$	$\bar{x},\bar{x},\bar{x}$	$x,\bar{x},x$	$\bar{x},x,x$		
6	f	$4m.m$	$x,\frac12,\frac12$	$\bar{x},\frac12,\frac12$	$\frac12,x,\frac12$	$\frac12,\bar{x},\frac12$	$\frac12,\frac12,x$	$\frac12,\frac12,\bar{x}$
6	e	$4m.m$	$x,0,0$	$\bar{x},0,0$	$0,x,0$	$0,\bar{x},0$	$0,0,x$	$0,0,\bar{x}$
3	d	$4/mm.m$	$\frac12,0,0$	$0,\frac12,0$	$0,0,\frac12$			
3	c	$4/mm.m$	$0,\frac12,\frac12$	$\frac12,0,\frac12$	$\frac12,\frac12,0$			
1	b	$m\bar{3}m$	$\frac12,\frac12,\frac12$					
1	a	$m\bar{3}m$	$0,0,0$					

Reflection conditions

h,k,l permutable
General:
no conditions

Special: no extra conditions

3	d	$4/mm.m$	$\frac12,0,0$	$0,\frac12,0$	$0,0,\frac12$
3	c	$4/mm.m$	$0,\frac12,\frac12$	$\frac12,0,\frac12$	$\frac12,\frac12,0$
1	b	$m\bar{3}m$	$\frac12,\frac12,\frac12$		
1	a	$m\bar{3}m$	$0,0,0$		

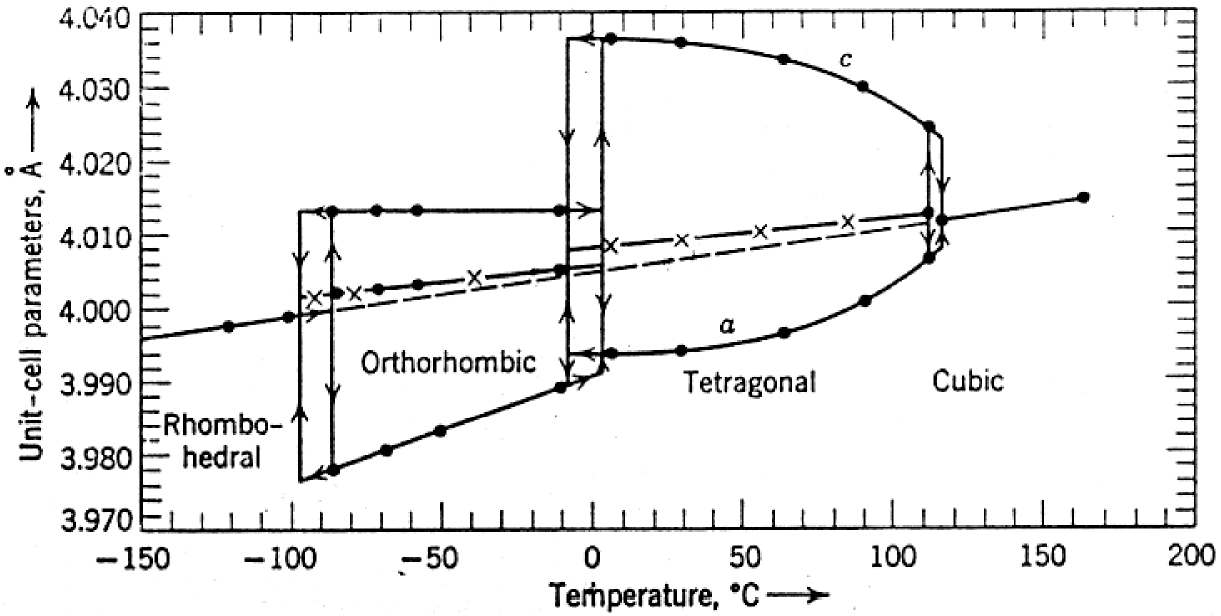
Symmetry of special projections

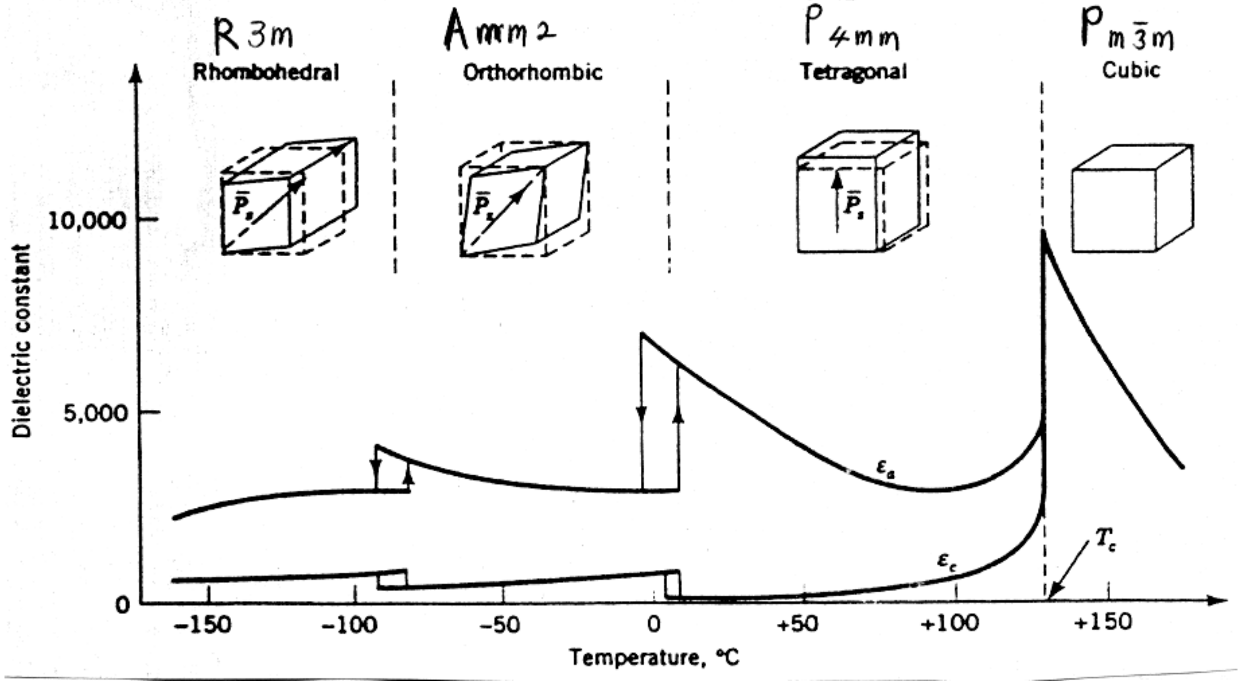
Along [001] $p4mm$
$\mathbf{a}'=\mathbf{a}$ $\mathbf{b}'=\mathbf{b}$
Origin at $0,0,z$

Along [111] $p6mm$
$\mathbf{a}'=\frac13(2\mathbf{a}-\mathbf{b}-\mathbf{c})$ $\mathbf{b}'=\frac13(-\mathbf{a}+2\mathbf{b}-\mathbf{c})$
Origin at x,x,x

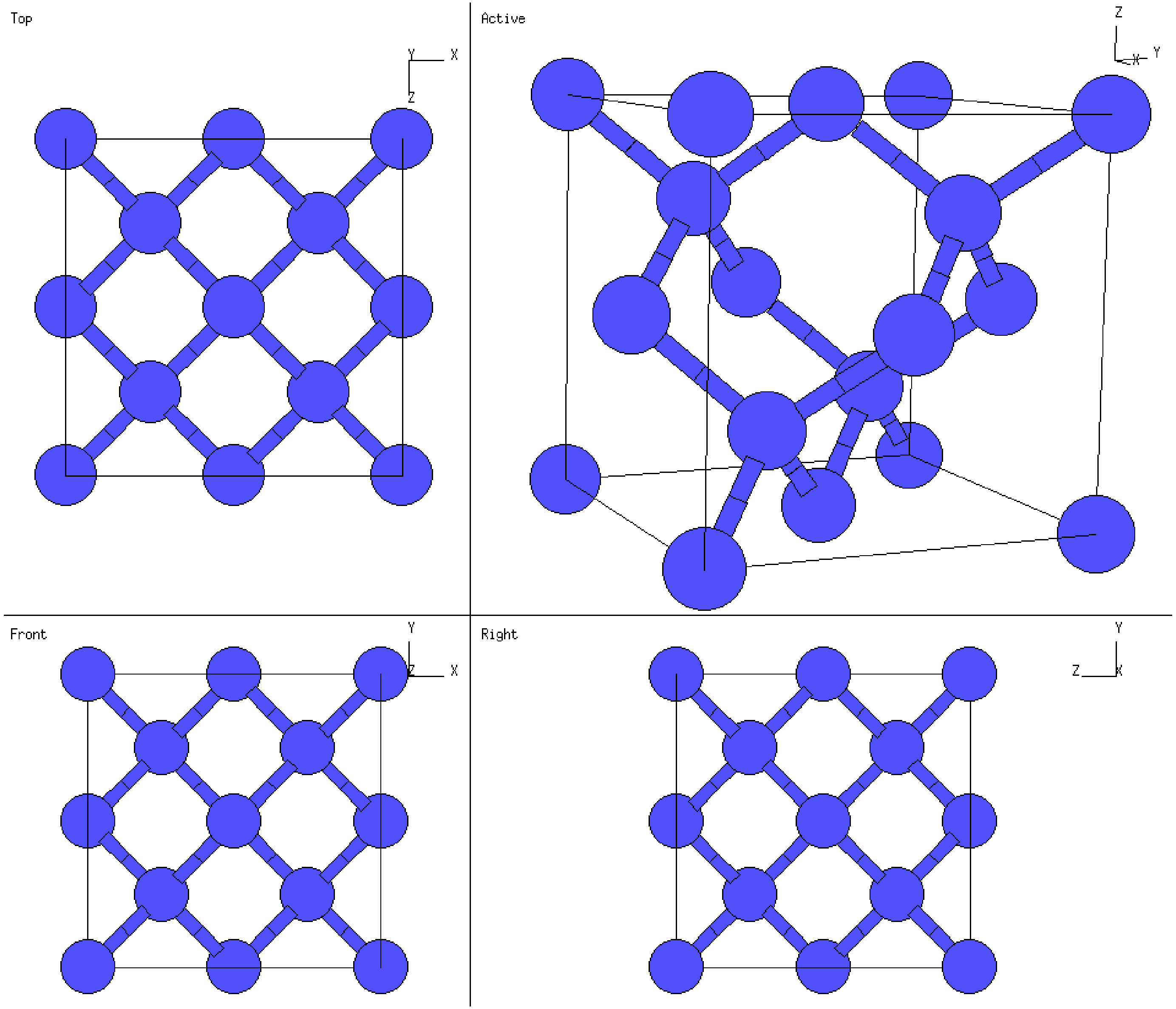
Along [110] $p2mm$
$\mathbf{a}'=\frac12(-\mathbf{a}+\mathbf{b})$ $\mathbf{b}'=\mathbf{c}$
Origin at $x,x,0$

Perovskite, $BaTiO_3$





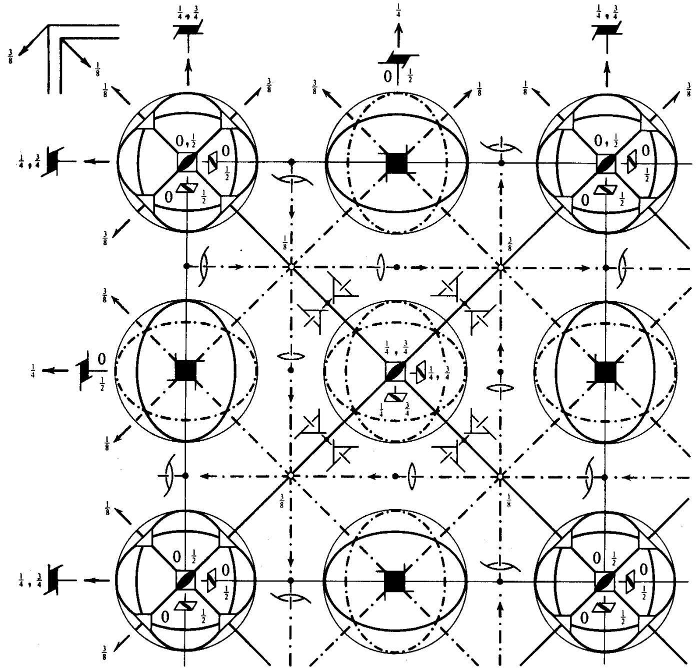
Diamond, C $\quad F\,\frac{4_1}{d}\,\bar{3}\,\frac{2}{m}$ (No.227)



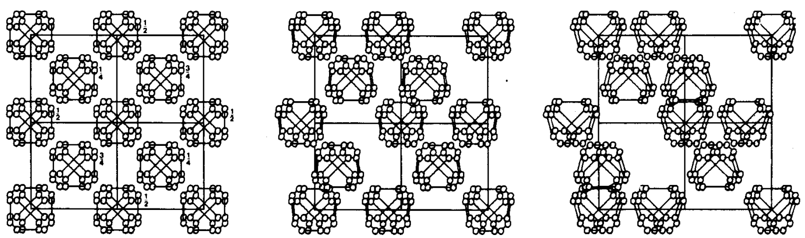
$Fd\bar{3}m$ O_h^7 $m\bar{3}m$ Cubic

No. 227 $F\,4_1/d\,\bar{3}\,2/m$ Patterson symmetry $Fm\bar{3}m$

ORIGIN CHOICE 1



Upper left quadrant only



Origin at $\bar{4}3m$, at $-\frac{1}{8},-\frac{1}{8},-\frac{1}{8}$ from centre ($\bar{3}m$)

Asymmetric unit $0\leq x\leq\frac{1}{2}$; $0\leq y\leq\frac{1}{8}$; $-\frac{1}{8}\leq z\leq\frac{1}{8}$; $y\leq\min(\frac{1}{2}-x,x)$; $-y\leq z\leq y$

Vertices   0,0,0 $\frac{1}{2}$,0,0 $\frac{3}{8},\frac{1}{8},\frac{1}{8}$ $\frac{1}{8},\frac{1}{8},\frac{1}{8}$ $\frac{3}{8},\frac{1}{8},-\frac{1}{8}$ $\frac{1}{8},\frac{1}{8},-\frac{1}{8}$

Symmetry operations

(*given on page* 689)

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; $t(0,\frac{1}{2},\frac{1}{2})$; $t(\frac{1}{2},0,\frac{1}{2})$; (2); (3); (5); (13); (25)

Positions

Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

$(0,0,0)+$ $(0,\frac{1}{2},\frac{1}{2})+$ $(\frac{1}{2},0,\frac{1}{2})+$ $(\frac{1}{2},\frac{1}{2},0)+$

Reflection conditions

h,k,l permutable

General:

192	i	1	(1) x,y,z	(2) $\bar{x},\bar{y}+\frac{1}{2},z+\frac{1}{2}$	(3) $\bar{x}+\frac{1}{2},y+\frac{1}{2},\bar{z}$	(4) $x+\frac{1}{2},\bar{y},\bar{z}+\frac{1}{2}$	hkl : $h+k=2n$ and
			(5) z,x,y	(6) $z+\frac{1}{2},\bar{x},\bar{y}+\frac{1}{2}$	(7) $\bar{z},\bar{x}+\frac{1}{2},y+\frac{1}{2}$	(8) $\bar{z}+\frac{1}{2},x+\frac{1}{2},\bar{y}$	$h+l,k+l=2n$
			(9) y,z,x	(10) $\bar{y}+\frac{1}{2},z+\frac{1}{2},\bar{x}$	(11) $y+\frac{1}{2},\bar{z},\bar{x}+\frac{1}{2}$	(12) $\bar{y},\bar{z}+\frac{1}{2},x+\frac{1}{2}$	$0kl$: $k+l=4n$ and
			(13) $y+\frac{3}{4},x+\frac{1}{4},\bar{z}+\frac{3}{4}$	(14) $\bar{y}+\frac{1}{4},\bar{x}+\frac{1}{4},\bar{z}+\frac{1}{4}$	(15) $y+\frac{1}{4},\bar{x}+\frac{3}{4},z+\frac{3}{4}$	(16) $\bar{y}+\frac{3}{4},x+\frac{3}{4},z+\frac{1}{4}$	$k,l=2n$
			(17) $x+\frac{3}{4},z+\frac{1}{4},\bar{y}+\frac{3}{4}$	(18) $\bar{x}+\frac{3}{4},z+\frac{3}{4},y+\frac{1}{4}$	(19) $\bar{x}+\frac{1}{4},\bar{z}+\frac{1}{4},\bar{y}+\frac{1}{4}$	(20) $x+\frac{1}{4},\bar{z}+\frac{3}{4},y+\frac{3}{4}$	hhl : $h+l=2n$
			(21) $z+\frac{3}{4},y+\frac{1}{4},\bar{x}+\frac{3}{4}$	(22) $z+\frac{1}{4},\bar{y}+\frac{3}{4},x+\frac{3}{4}$	(23) $\bar{z}+\frac{3}{4},y+\frac{3}{4},x+\frac{1}{4}$	(24) $\bar{z}+\frac{1}{4},\bar{y}+\frac{1}{4},\bar{x}+\frac{1}{4}$	$h00$: $h=4n$
			(25) $\bar{x}+\frac{1}{4},\bar{y}+\frac{1}{4},\bar{z}+\frac{1}{4}$	(26) $x+\frac{1}{4},y+\frac{3}{4},\bar{z}+\frac{3}{4}$	(27) $x+\frac{3}{4},\bar{y}+\frac{3}{4},z+\frac{1}{4}$	(28) $\bar{x}+\frac{3}{4},y+\frac{1}{4},z+\frac{3}{4}$	
			(29) $\bar{z}+\frac{1}{4},\bar{x}+\frac{1}{4},\bar{y}+\frac{1}{4}$	(30) $\bar{z}+\frac{3}{4},x+\frac{1}{4},y+\frac{3}{4}$	(31) $z+\frac{1}{4},x+\frac{3}{4},\bar{y}+\frac{3}{4}$	(32) $z+\frac{3}{4},\bar{x}+\frac{3}{4},y+\frac{1}{4}$	
			(33) $\bar{y}+\frac{1}{4},\bar{z}+\frac{1}{4},\bar{x}+\frac{1}{4}$	(34) $y+\frac{3}{4},\bar{z}+\frac{3}{4},x+\frac{1}{4}$	(35) $\bar{y}+\frac{3}{4},z+\frac{1}{4},x+\frac{3}{4}$	(36) $y+\frac{1}{4},z+\frac{3}{4},\bar{x}+\frac{3}{4}$	
			(37) $\bar{y}+\frac{1}{2},\bar{x},z+\frac{1}{2}$	(38) y,x,z	(39) $\bar{y},x+\frac{1}{2},\bar{z}+\frac{1}{2}$	(40) $y+\frac{1}{2},\bar{x}+\frac{1}{2},\bar{z}$	
			(41) $\bar{x}+\frac{1}{2},\bar{z},y+\frac{1}{2}$	(42) $x+\frac{1}{2},\bar{z}+\frac{1}{2},\bar{y}$	(43) x,z,y	(44) $\bar{x},z+\frac{1}{2},\bar{y}+\frac{1}{2}$	
			(45) $\bar{z}+\frac{1}{2},\bar{y},x+\frac{1}{2}$	(46) $\bar{z},y+\frac{1}{2},\bar{x}+\frac{1}{2}$	(47) $z+\frac{1}{2},\bar{y}+\frac{1}{2},\bar{x}$	(48) z,y,x	

Special: as above, plus

96	h	$..2$	$\frac{1}{8},y,\bar{y}+\frac{1}{4}$	$\frac{7}{8},\bar{y}+\frac{1}{2},\bar{y}+\frac{3}{4}$	$\frac{3}{8},y+\frac{1}{2},y+\frac{3}{4}$	$\frac{5}{8},\bar{y},y+\frac{1}{4}$	no extra conditions
			$\bar{y}+\frac{1}{4},\frac{1}{8},y$	$\bar{y}+\frac{3}{4},\frac{7}{8},\bar{y}+\frac{1}{2}$	$y+\frac{3}{4},\frac{3}{8},y+\frac{1}{2}$	$y+\frac{1}{4},\frac{5}{8},\bar{y}$	
			$y,\bar{y}+\frac{1}{4},\frac{1}{8}$	$\bar{y}+\frac{1}{2},\bar{y}+\frac{3}{4},\frac{7}{8}$	$y+\frac{1}{2},y+\frac{3}{4},\frac{3}{8}$	$\bar{y},y+\frac{1}{4},\frac{5}{8}$	
			$\frac{1}{8},\bar{y}+\frac{1}{4},y$	$\frac{3}{8},y+\frac{3}{4},y+\frac{1}{2}$	$\frac{7}{8},\bar{y}+\frac{3}{4},\bar{y}+\frac{1}{2}$	$\frac{5}{8},y+\frac{1}{4},\bar{y}$	
			$y,\frac{1}{8},\bar{y}+\frac{1}{4}$	$y+\frac{1}{2},\frac{3}{8},y+\frac{3}{4}$	$\bar{y}+\frac{1}{2},\frac{7}{8},\bar{y}+\frac{3}{4}$	$\bar{y},\frac{5}{8},y+\frac{1}{4}$	
			$\bar{y}+\frac{1}{4},y,\frac{1}{8}$	$y+\frac{3}{4},y+\frac{1}{2},\frac{3}{8}$	$\bar{y}+\frac{3}{4},\bar{y}+\frac{1}{2},\frac{7}{8}$	$y+\frac{1}{4},\bar{y},\frac{5}{8}$	
96	g	$..m$	x,x,z	$\bar{x},\bar{x}+\frac{1}{2},z+\frac{1}{2}$	$\bar{x}+\frac{1}{2},x+\frac{1}{2},\bar{z}$	$x+\frac{1}{2},\bar{x},\bar{z}+\frac{1}{2}$	no extra conditions
			z,x,x	$z+\frac{1}{2},\bar{x},\bar{x}+\frac{1}{2}$	$\bar{z},\bar{x}+\frac{1}{2},x+\frac{1}{2}$	$\bar{z}+\frac{1}{2},x+\frac{1}{2},\bar{x}$	
			x,z,x	$\bar{x}+\frac{1}{2},z+\frac{1}{2},\bar{x}$	$x+\frac{1}{2},\bar{z},\bar{x}+\frac{1}{2}$	$\bar{x},\bar{z}+\frac{1}{2},x+\frac{1}{2}$	
			$x+\frac{3}{4},x+\frac{1}{4},\bar{z}+\frac{3}{4}$	$\bar{x}+\frac{1}{4},\bar{x}+\frac{1}{4},\bar{z}+\frac{1}{4}$	$x+\frac{1}{4},\bar{x}+\frac{3}{4},z+\frac{3}{4}$	$\bar{x}+\frac{3}{4},x+\frac{3}{4},z+\frac{1}{4}$	
			$x+\frac{3}{4},z+\frac{1}{4},\bar{x}+\frac{3}{4}$	$\bar{x}+\frac{3}{4},z+\frac{3}{4},x+\frac{1}{4}$	$\bar{x}+\frac{1}{4},\bar{z}+\frac{1}{4}.\bar{x}+\frac{1}{4}$	$x+\frac{1}{4},\bar{z}+\frac{3}{4},x+\frac{3}{4}$	
			$z+\frac{3}{4},x+\frac{1}{4},\bar{x}+\frac{3}{4}$	$z+\frac{1}{4},\bar{x}+\frac{3}{4},x+\frac{3}{4}$	$\bar{z}+\frac{3}{4},x+\frac{3}{4},x+\frac{1}{4}$	$\bar{z}+\frac{1}{4},\bar{x}+\frac{1}{4},\bar{x}+\frac{1}{4}$	

48	f	$2.mm$	$x,0,0$	$\bar{x},\frac{1}{2},\frac{1}{2}$	$0,x,0$	$\frac{1}{2},\bar{x},\frac{1}{2}$	$0,0,x$	$\frac{1}{2},\frac{1}{2},\bar{x}$	hkl : $h=2n+1$
			$\frac{3}{4},x+\frac{1}{4},\frac{3}{4}$	$\frac{1}{4},\bar{x}+\frac{1}{4},\frac{1}{4}$	$x+\frac{3}{4},\frac{1}{4},\frac{3}{4}$	$\bar{x}+\frac{3}{4},\frac{3}{4},\frac{1}{4}$	$\frac{3}{4},\frac{1}{4},\bar{x}+\frac{3}{4}$	$\frac{1}{4},\frac{3}{4},x+\frac{3}{4}$	or $h+k+l=4n$

32	e	$.3m$	x,x,x	$\bar{x},\bar{x}+\frac{1}{2},x+\frac{1}{2}$			no extra conditions
			$\bar{x}+\frac{1}{2},x+\frac{1}{2},\bar{x}$	$x+\frac{1}{2},\bar{x},\bar{x}+\frac{1}{2}$			
			$x+\frac{3}{4},x+\frac{1}{4},\bar{x}+\frac{3}{4}$	$\bar{x}+\frac{1}{4},\bar{x}+\frac{1}{4},\bar{x}+\frac{1}{4}$			
			$x+\frac{1}{4},\bar{x}+\frac{3}{4},x+\frac{3}{4}$	$\bar{x}+\frac{3}{4},x+\frac{3}{4},x+\frac{1}{4}$			
16	d	$.\bar{3}m$	$\frac{5}{8},\frac{5}{8},\frac{5}{8}$	$\frac{3}{8},\frac{7}{8},\frac{1}{8}$	$\frac{7}{8},\frac{1}{8},\frac{3}{8}$	$\frac{1}{8},\frac{3}{8},\frac{7}{8}$	hkl : $h=2n+1$ or $h,k,l=4n+2$ or $h,k,l=4n$
16	c	$.\bar{3}m$	$\frac{1}{8},\frac{1}{8},\frac{1}{8}$	$\frac{7}{8},\frac{3}{8},\frac{5}{8}$	$\frac{3}{8},\frac{5}{8},\frac{7}{8}$	$\frac{5}{8},\frac{7}{8},\frac{3}{8}$	
8	b	$\bar{4}3m$	$\frac{1}{2},\frac{1}{2},\frac{1}{2}$	$\frac{1}{4},\frac{3}{4},\frac{1}{4}$			hkl : $h=2n+1$ or $h+k+l=4n$
8	a	$\bar{4}3m$	$0,0,0$	$\frac{3}{4},\frac{1}{4},\frac{3}{4}$			

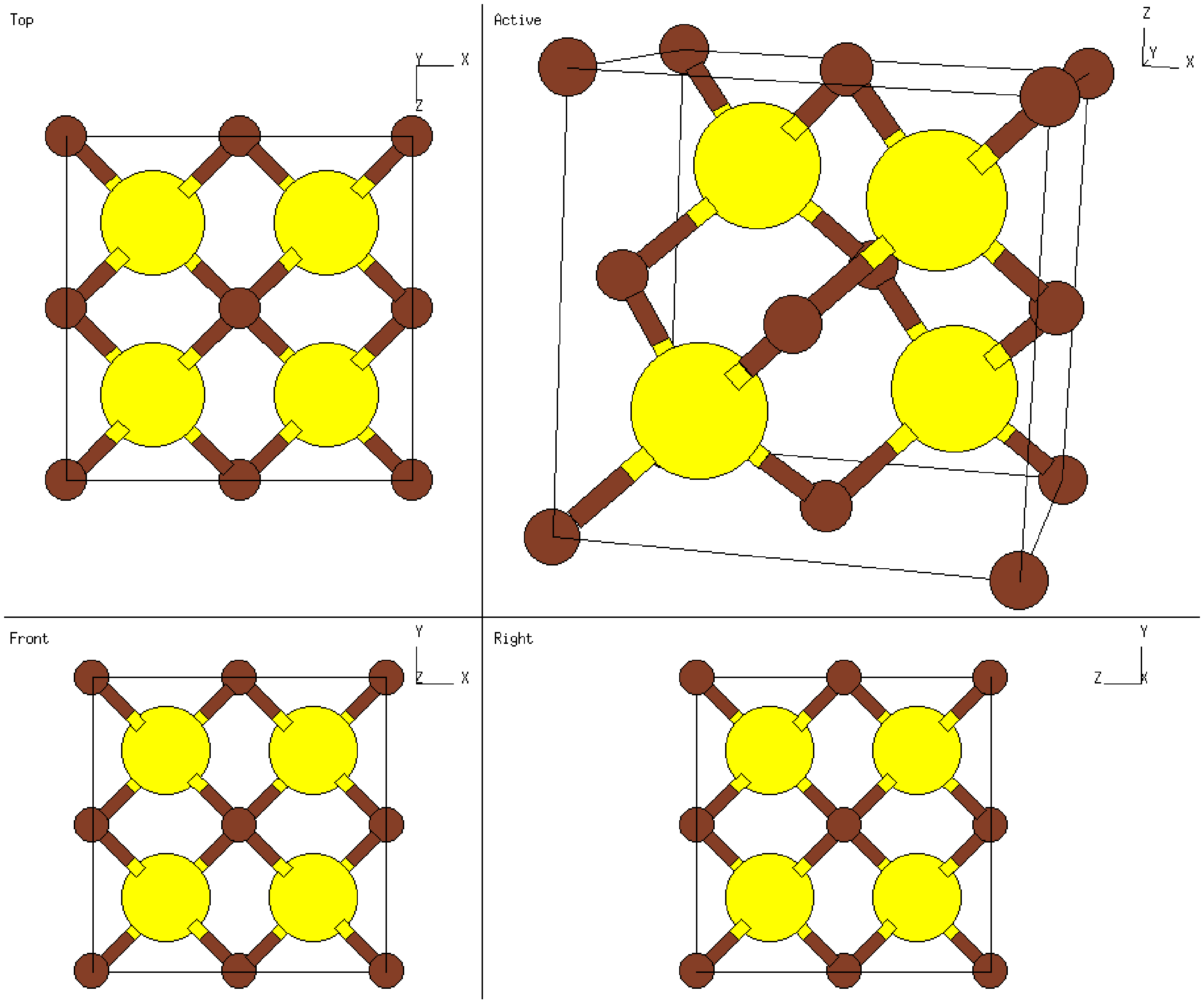
Symmetry of special projections

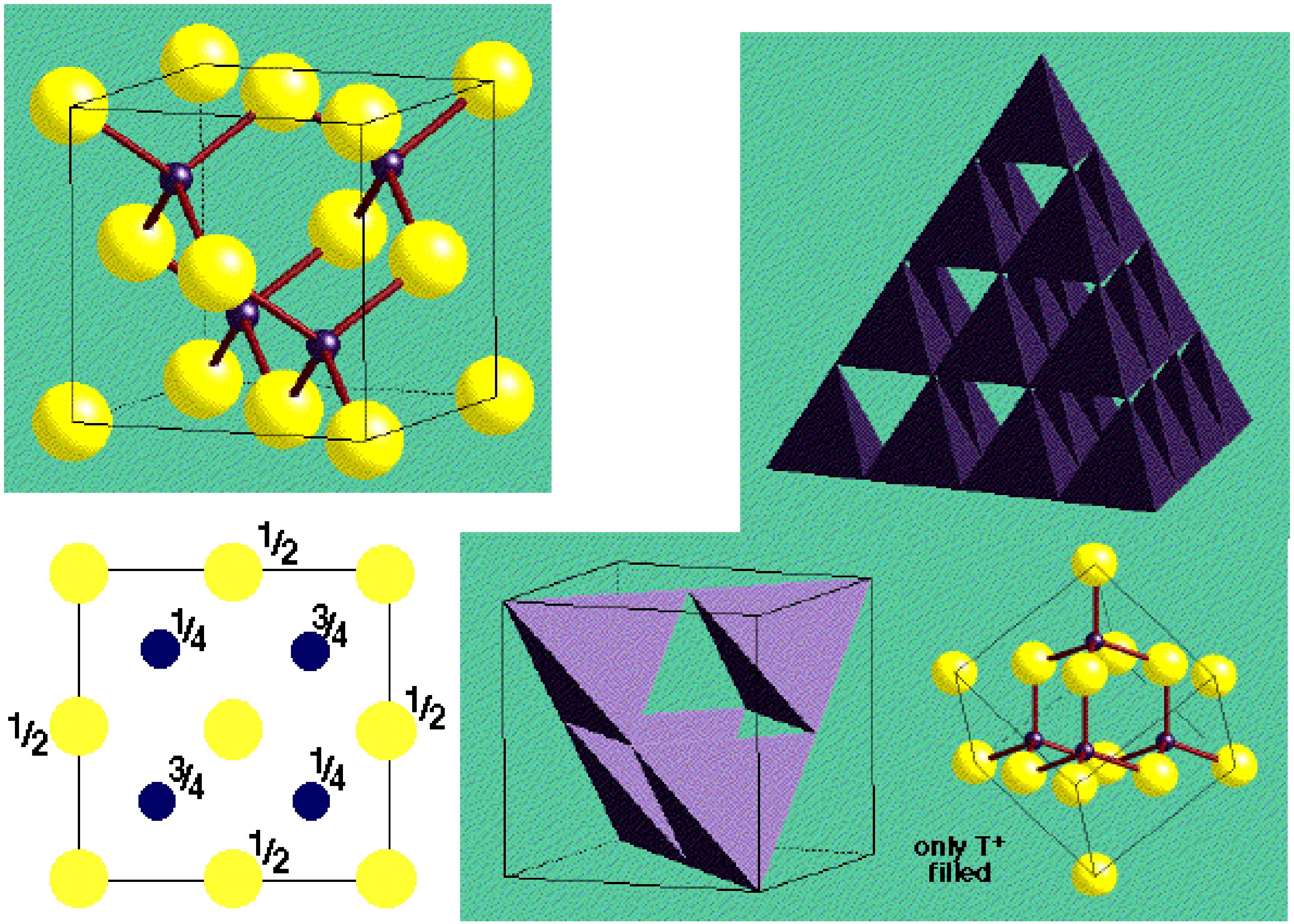
Along [001] $p4mm$
$\mathbf{a}'=\frac{1}{2}(\mathbf{a}-\mathbf{b})$ $\mathbf{b}'=\frac{1}{2}(\mathbf{a}+\mathbf{b})$
Origin at $0,0,z$

Along [111] $p6mm$
$\mathbf{a}'=\frac{1}{6}(2\mathbf{a}-\mathbf{b}-\mathbf{c})$ $\mathbf{b}'=\frac{1}{6}(-\mathbf{a}+2\mathbf{b}-\mathbf{c})$
Origin at x,x,x

Along [110] $c2mm$
$\mathbf{a}'=\frac{1}{2}(-\mathbf{a}+\mathbf{b})$ $\mathbf{b}'=\mathbf{c}$
Origin at $x,x,\frac{1}{8}$

Zinc Blende, ZnS




1/2
1/4
3/4
1/2
1/2
3/4
1/4
1/2
only T+
filled

Zinc Blende, ZnS

- diamond derivative structure
- Zn and S replace the C atoms
- Zn cubic close packing
  S ½ tetrahedral site
- Zn and S cubic close packing displaced by $\frac{1}{4},\frac{1}{4},\frac{1}{4}$
- Space group

$$F\bar{4}3m \text{ (No.216)}$$

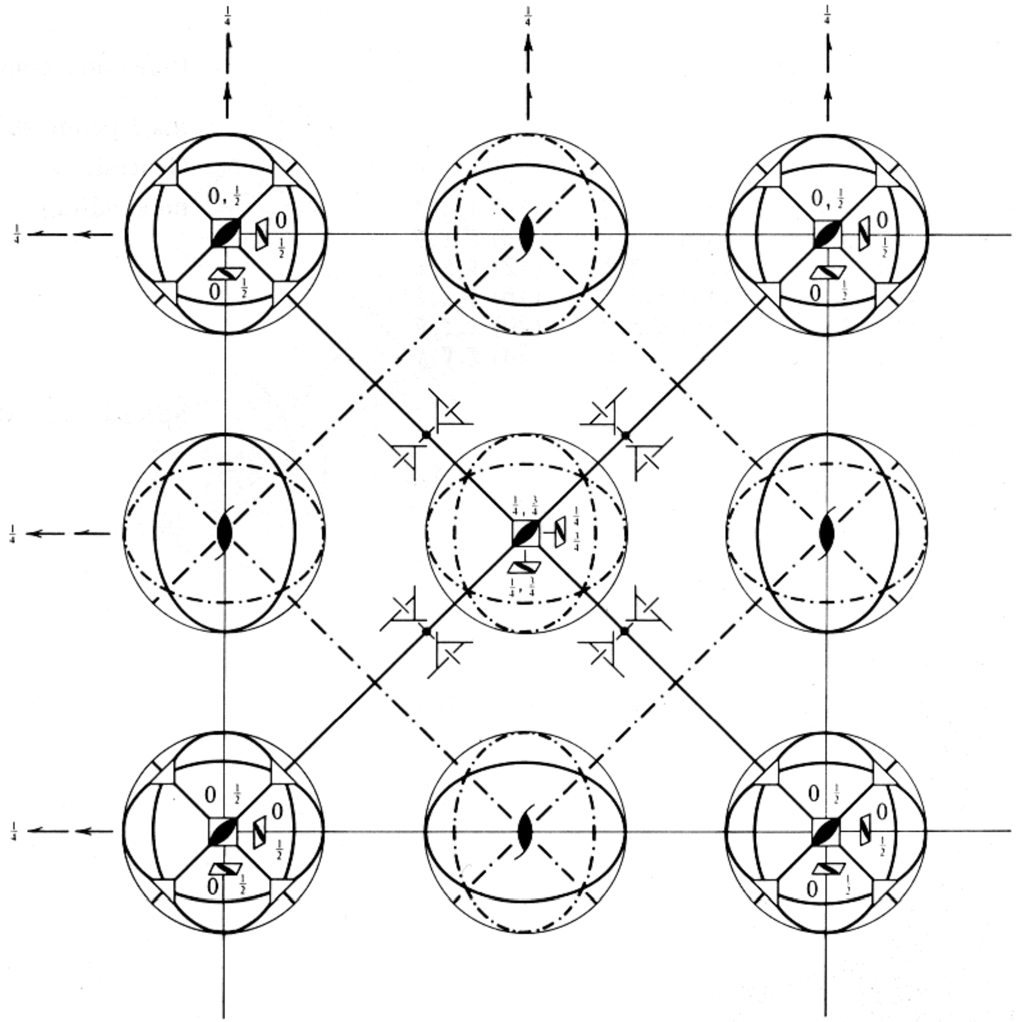
$$\text{Zn: 4a, } \bar{4}3m,\ 0,0,0$$

$$\text{Zn: 4c, } \bar{4}3m,\ \frac{1}{4},\frac{1}{4},\frac{1}{4}$$

Zinc Blende, ZnS

$F\bar{4}3m$ | T_d^2 | $\bar{4}3m$ | Cubic

No. 216 | $F\bar{4}3m$ | Patterson symmetry $Fm\bar{3}m$



Upper left quadrant only

Zinc Blende, ZnS

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; $t(0,\frac{1}{2},\frac{1}{2})$; $t(\frac{1}{2},0,\frac{1}{2})$; (2); (3); (5); (13)

Positions

Multiplicity, Wyckoff letter, Site symmetry			Coordinates			
			$(0,0,0)+$	$(0,\frac{1}{2},\frac{1}{2})+$	$(\frac{1}{2},0,\frac{1}{2})+$	$(\frac{1}{2},\frac{1}{2},0)+$
96	i	1	(1) x,y,z	(2) $\bar{x},\bar{y},z$	(3) $\bar{x},y,\bar{z}$	(4) $x,\bar{y},\bar{z}$
			(5) z,x,y	(6) $z,\bar{x},\bar{y}$	(7) $\bar{z},\bar{x},y$	(8) $\bar{z},x,\bar{y}$
			(9) y,z,x	(10) $\bar{y},z,\bar{x}$	(11) $y,\bar{z},\bar{x}$	(12) $\bar{y},\bar{z},x$
			(13) y,x,z	(14) $\bar{y},\bar{x},z$	(15) $y,\bar{x},\bar{z}$	(16) $\bar{y},x,\bar{z}$
			(17) x,z,y	(18) $\bar{x},z,\bar{y}$	(19) $\bar{x},\bar{z},y$	(20) $x,\bar{z},\bar{y}$
			(21) z,y,x	(22) $z,\bar{y},\bar{x}$	(23) $\bar{z},y,\bar{x}$	(24) $\bar{z},\bar{y},x$

48	h	$..m$	x,x,z	$\bar{x},\bar{x},z$	$\bar{x},x,\bar{z}$	$x,\bar{x},\bar{z}$	z,x,x	$z,\bar{x},\bar{x}$
			$\bar{z},\bar{x},x$	$\bar{z},x,\bar{x}$	x,z,x	$\bar{x},z,\bar{x}$	$x,\bar{z},\bar{x}$	$\bar{x},\bar{z},x$
24	g	$2.mm$	$x,\frac{1}{4},\frac{1}{4}$	$\bar{x},\frac{3}{4},\frac{1}{4}$	$\frac{1}{4},x,\frac{1}{4}$	$\frac{1}{4},\bar{x},\frac{3}{4}$	$\frac{1}{4},\frac{1}{4},x$	$\frac{3}{4},\frac{1}{4},\bar{x}$
24	f	$2.mm$	$x,0,0$	$\bar{x},0,0$	$0,x,0$	$0,\bar{x},0$	$0,0,x$	$0,0,\bar{x}$
16	e	$.3m$	x,x,x	$\bar{x},\bar{x},x$	$\bar{x},x,\bar{x}$	$x,\bar{x},\bar{x}$		
4	d	$\bar{4}3m$	$\frac{3}{4},\frac{3}{4},\frac{3}{4}$					
4	c	$\bar{4}3m$	$\frac{1}{4},\frac{1}{4},\frac{1}{4}$					
4	b	$\bar{4}3m$	$\frac{1}{2},\frac{1}{2},\frac{1}{2}$					
4	a	$\bar{4}3m$	$0,0,0$					

Reflection conditions

h,k,l permutable

General:

hkl : $h+k, h+l, k+l = 2n$
$0kl$: $k,l = 2n$
hhl : $h+l = 2n$
$h00$: $h = 2n$

Special: no extra conditions