Ch. 5. EM Optics

Byoungho Lee
School of EE, Seoul National University
byoungho@snu.ac.kr
Fall Semester, 2008

Maxwell's equations

In source-free medium

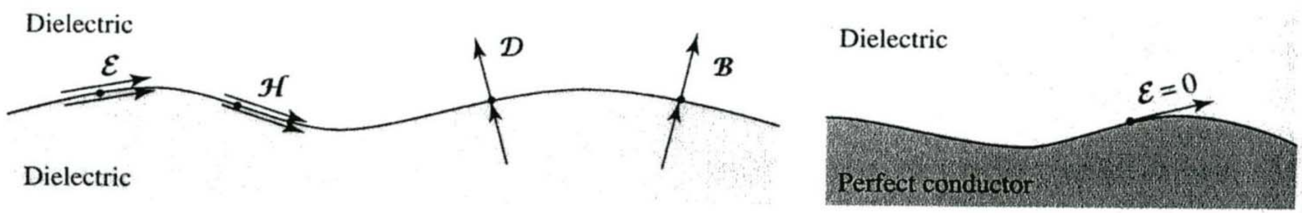
$$\nabla \times \mathcal{H} = \frac{\partial \mathcal{D}}{\partial t} \qquad \mathcal{D} = \epsilon_o \mathcal{E} + \mathcal{P}$$

$$\nabla \times \mathcal{E} = -\frac{\partial \mathcal{B}}{\partial t} \qquad \mathcal{B} = \mu_o \mathcal{H} + \mu_o \mathcal{M}$$

$$\nabla \cdot \mathcal{D} = 0$$

$$\nabla \cdot \mathcal{B} = 0.$$

Boundary conditions

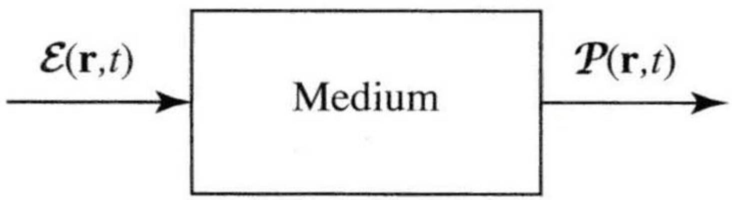


Poynting vector and Poynting theorem

$$\mathcal{S} = \mathcal{E} \times \mathcal{H}$$

$$\nabla \cdot \mathcal{S} = -\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon_o \mathcal{E}^2 + \frac{1}{2}\mu_o \mathcal{H}^2\right) + \mathcal{E} \cdot \frac{\partial \mathcal{P}}{\partial t} + \mu_o \mathcal{H} \cdot \frac{\partial \mathcal{M}}{\partial t}$$

EM waves in media

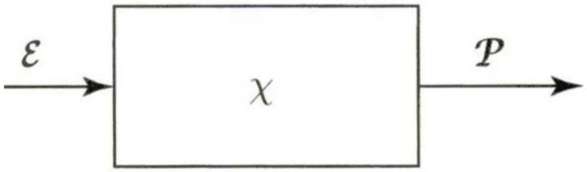


$$\mathcal{P} = \epsilon_o \chi \mathcal{E}$$

Definitions

- A dielectric medium is said to be *linear* if the vector field $\mathcal{P}(\mathbf{r},t)$ is linearly related to the vector field $\mathcal{E}(\mathbf{r},t)$. The principle of superposition then applies.
- The medium is said to be *nondispersive* if its response is instantaneous, i.e., if $\mathcal{P}$ at time t is determined by $\mathcal{E}$ at the same time t and not by prior values of $\mathcal{E}$. Nondispersiveness is clearly an idealization since all physical systems, no matter how rapidly they may respond, do have a response time that is finite.
- The medium is said to be *homogeneous* if the relation between $\mathcal{P}$ and $\mathcal{E}$ is independent of the position $\mathbf{r}$.
- The medium is said to be *isotropic* if the relation between the vectors $\mathcal{P}$ and $\mathcal{E}$ is independent of the direction of the vector $\mathcal{E}$, so that the medium exhibits the same behavior from all directions. The vectors $\mathcal{P}$ and $\mathcal{E}$ must then be parallel.
- The medium is said to be *spatially nondispersive* if the relation between $\mathcal{P}$ and $\mathcal{E}$ is local, i.e., if $\mathcal{P}$ at each position $\mathbf{r}$ is influenced only by $\mathcal{E}$ at the same position $\mathbf{r}$. The medium is assumed to be spatially nondispersive throughout this chapter (optically active media, considered in Sec. 6.4A, are spatially dispersive).

Linear, nondispersive, homogeneous and isotropic medium



$$\mathcal{P} = \epsilon_o \chi \mathcal{E}$$

$$\mathcal{D} = \epsilon \mathcal{E}$$

$$\epsilon = \epsilon_o (1 + \chi)$$

$$\mathcal{B} = \mu \mathcal{H}$$

$$\nabla \times \mathcal{H} = \epsilon \frac{\partial \mathcal{E}}{\partial t}$$

$$\nabla \times \mathcal{E} = -\mu \frac{\partial \mathcal{H}}{\partial t}$$

$$\nabla \cdot \mathcal{E} = 0$$

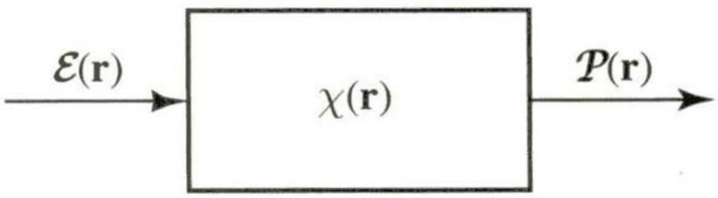
$$\nabla \cdot \mathcal{H} = 0.$$

$$\nabla^2 u - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\epsilon \mu}}$$

$$n = \sqrt{\frac{\epsilon}{\epsilon_o}} = \sqrt{1 + \chi}$$

Inhomogeneous media



$$\frac{\epsilon_o}{\epsilon}\nabla \times (\nabla \times \mathcal{E}) = -\frac{1}{c_o^2}\frac{\partial^2 \mathcal{E}}{\partial t^2}$$

$$\nabla \times \left(\frac{\epsilon_o}{\epsilon}\nabla \times \mathcal{H}\right) = -\frac{1}{c_o^2}\frac{\partial^2 \mathcal{H}}{\partial t^2}$$

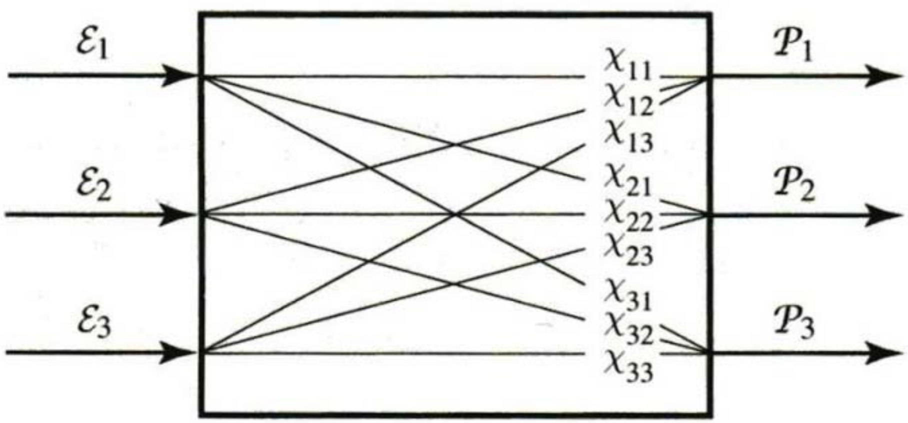
$$\nabla^2 \mathcal{E} + \nabla\left(\frac{1}{\epsilon}\nabla\epsilon \cdot \mathcal{E}\right) - \mu_o \epsilon \frac{\partial^2 \mathcal{E}}{\partial t^2} = 0$$

$$\nabla^2 \mathcal{E} - \frac{1}{c^2(\mathbf{r})}\frac{\partial^2 \mathcal{E}}{\partial t^2} \approx 0$$

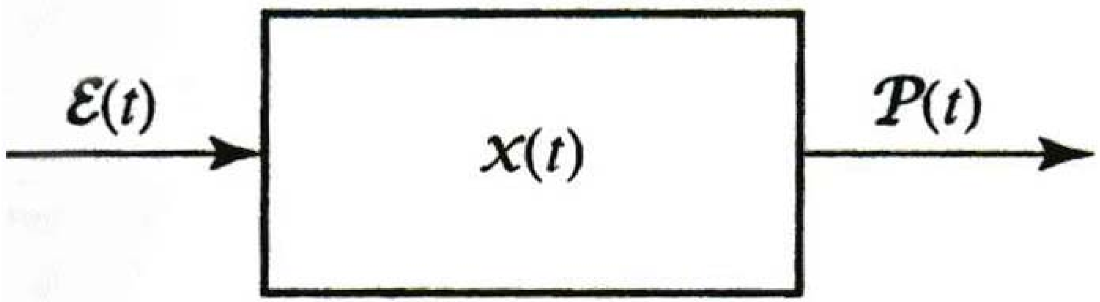
Anisotropic media

$$\mathcal{P}_i = \sum_j \epsilon_o \chi_{ij} \mathcal{E}_j$$

$$\mathcal{D}_i = \sum_j \epsilon_{ij} \mathcal{E}_j$$



Dispersive media



$$\mathcal{P}(t) = \epsilon_o \int_{-\infty}^{\infty} \mathrm{x}(t-t')\,\mathcal{E}(t')\,dt'$$

Nonlinear media

$$\nabla^2 \mathcal{E} - \frac{1}{c_o^2}\frac{\partial^2 \mathcal{E}}{\partial t^2} = \mu_o \frac{\partial^2 \mathcal{P}}{\partial t^2}$$

$$P = \Psi(E)$$

$$\nabla^2 \mathcal{E} - \frac{1}{c_o^2}\frac{\partial^2 \mathcal{E}}{\partial t^2} = \mu_o \frac{\partial^2 \Psi(\mathcal{E})}{\partial t^2}$$

Monochromatic EM waves

$$\mathcal{E}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{E}(\mathbf{r})\exp(j\omega t)\}$$

$$\mathcal{H}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{H}(\mathbf{r})\exp(j\omega t)\}$$

$$\nabla \times \mathbf{H} = j\omega\mathbf{D}$$

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B}$$

$$\nabla \cdot \mathbf{D} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{D} = \epsilon_o\mathbf{E} + \mathbf{P}$$

$$\mathbf{B} = \mu_o\mathbf{H} + \mu_o\mathbf{M}$$

$$\mathbf{S} = \tfrac{1}{2}\mathbf{E} \times \mathbf{H}^*$$

$$\mathbf{D} = \epsilon\mathbf{E} \quad \text{and} \quad \mathbf{B} = \mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$$

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla^2 U + k^2 U = 0$$

$$k = nk_0 = \omega\sqrt{\varepsilon\mu}$$

Dispersive media

$$\mathbf{P} = \epsilon_o \chi(\nu) \mathbf{E}$$

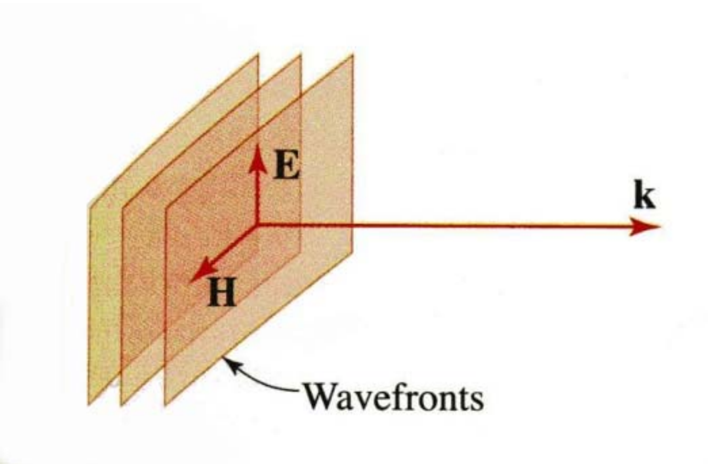
$$\chi(\nu) = \int_{-\infty}^{\infty} \mathrm{x}(t) \exp(-j2\pi\nu t)\, dt$$

$$\mathbf{D} = \epsilon(\nu) \mathbf{E}$$

$$\epsilon(\nu) = \epsilon_o [1 + \chi(\nu)]$$

$$k = \omega \sqrt{\epsilon(\nu)\, \mu_o}$$

TEM wave



$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_0 \exp(-j\mathbf{k}\cdot\mathbf{r})$$

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 \exp(-j\mathbf{k}\cdot\mathbf{r})$$

$$\mathbf{k} \times \mathbf{H}_0 = -\omega\,\epsilon\,\mathbf{E}_0$$

$$\mathbf{k} \times \mathbf{E}_0 = \omega\,\mu\,\mathbf{H}_0$$

$$\eta = \frac{E_0}{H_0} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta = \frac{\eta_o}{n}$$

$$I = \frac{|E_0|^2}{2\eta}$$

$$W = \tfrac{1}{2}\epsilon|E_0|^2 \qquad I = cW$$