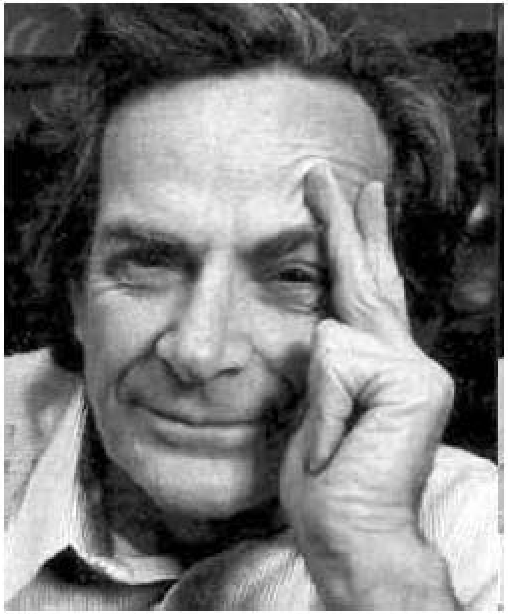
# Brief Introduction to Photonic Crystals

**Byoungho Lee**
**School of Electrical Engineering**
**Seoul National University**

**byoungho@snu.ac.kr**

# Nano



## "There is Plenty of Room at the Bottom"

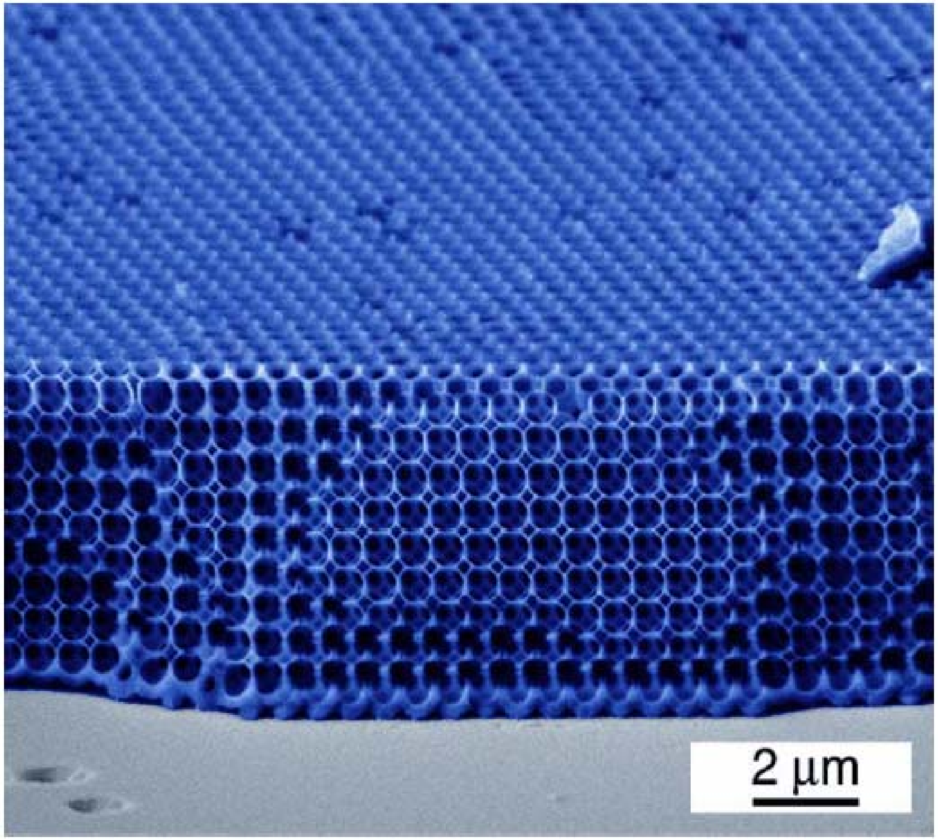
*Prof. Richard P. Feynman*

*December, 1959*

*California Institute of Technology*

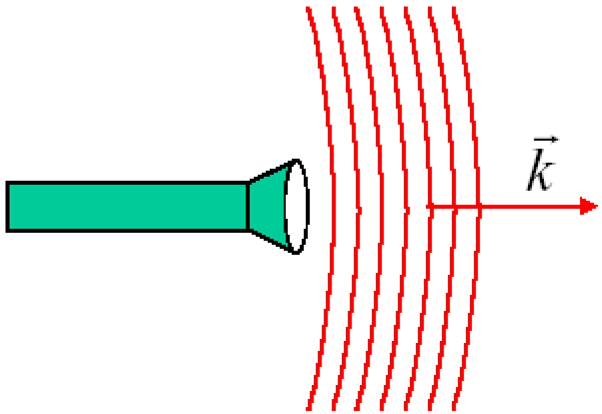
*http://www.zyvex.com/nanotech/feynman.html*

# Photonic crystals




[ Y. A. Vlasov *et al.*, *Nature* **414**, 289 (2001). ]

# Scattering

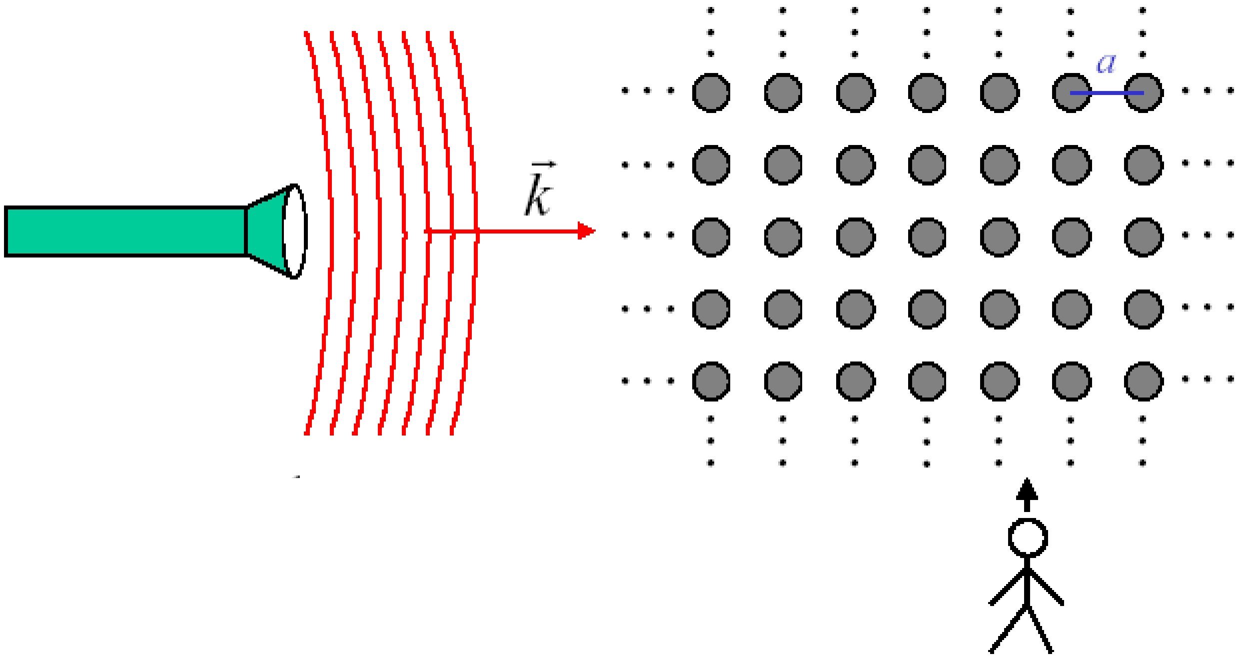



planewave

$$\vec{E}, \vec{H} \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$

# Periodic medium




No light propagation for certain wavelengths:
a photonic bandgap

# Photonic structures in biology

**Pete Vukusic and J. Roy Sambles**

*Thin Film Photonics, School of Physics, Exeter University, Exeter EX4 4QL, UK (e-mail: P.Vukusic@ex.ac.uk)*

**Millions of years before we began to manipulate the flow of light using synthetic structures, biological systems were using nanometre-scale architectures to produce striking optical effects. An astonishing variety of natural photonic structures exists: a species of Brittlestar uses photonic elements composed of calcite to collect light, *Morpho* butterflies use multiple layers of cuticle and air to produce their striking blue colour and some insects use arrays of elements, known as nipple arrays, to reduce reflectivity in their compound eyes. Natural photonic structures are providing inspiration for technological applications.**

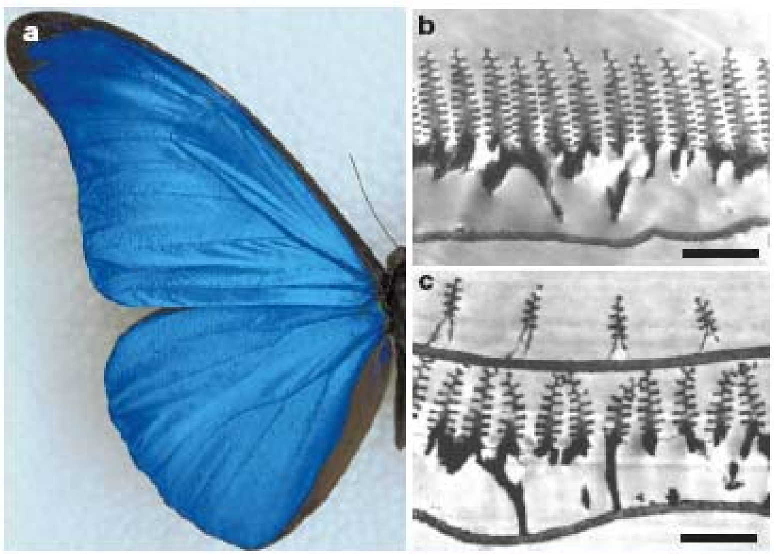



**Figure 3** Iridescence in the butterfly *Morpho rhetenor*. **a**, Real colour image of the blue iridescence from a *M. rhetenor* wing. **b**, Transmission electron micrograph (TEM) images showing wing-scale cross-sections of *M. rhetenor*. **c**, TEM images of a wing-scale cross-section of the related species *M. didius* reveal its discretely configured multilayers. The high occupancy and high layer number of *M. rhetenor* in **b** creates an intense reflectivity that contrasts with the more diffusely coloured appearance of *M. didius*, in which an overlying second layer of scales effects strong diffraction[4]. Bars, **a**, 1 cm; **b**, 1.8 μm; **c**, 1.3 μm.

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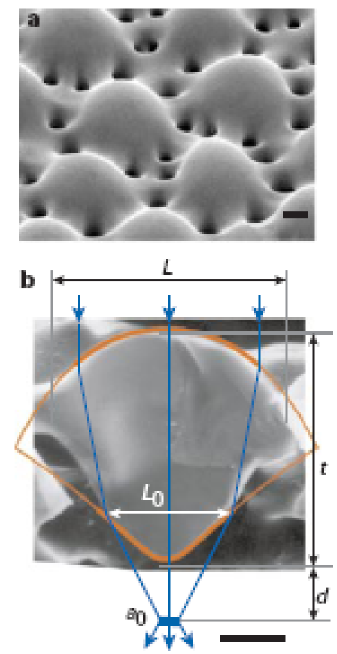



**Figure 1** Peripheral layer of ophiocomid brittlestars. **a**, Scanning electron micrographs (SEM) of the peripheral layer of a dorsal arm plate from the brittlestar *Ophiocoma wendtii* showing the microlens array. **b**, SEM of an individual lens in *O. wendtii*. The functional region of this lens ($L_0$) closely matches the profile of a lens that is compensated for spherical aberration (represented by the red lines). The light paths are shown in blue (images reproduced with permission from J. Aizenberg). Bars, 10 μm.

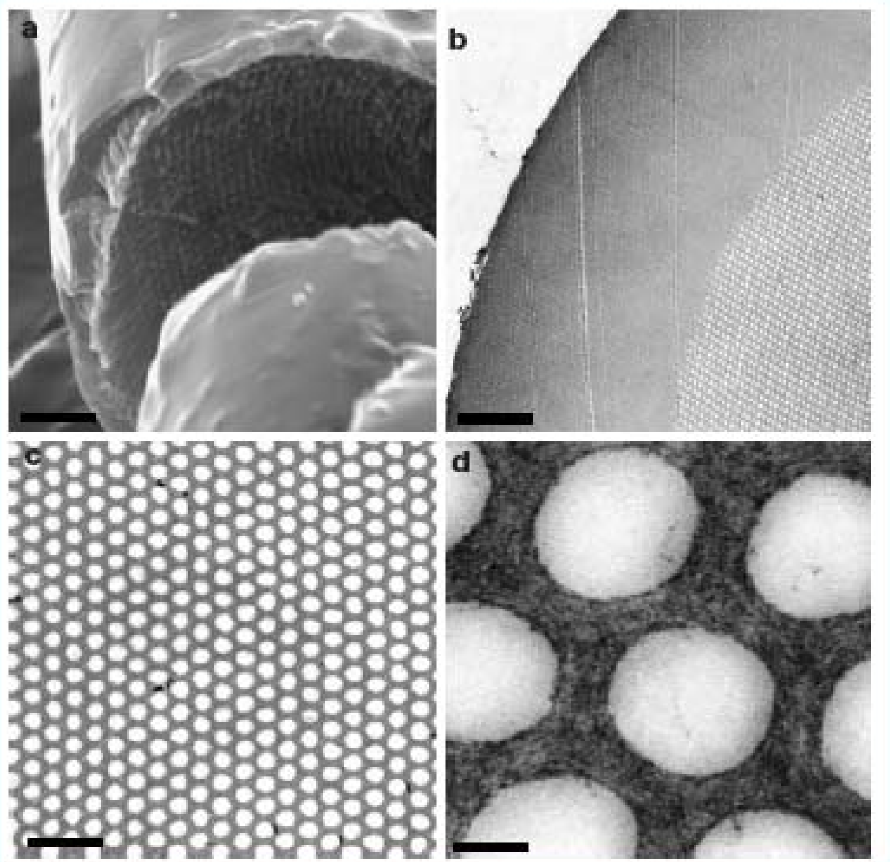



**Figure 2** Iridescent setae from polychaete worms. **a**, Scanning electron micrograph (SEM) and **b–d**, transmission electron micrograph (TEM) images of transverse sections through a single iridescent seta. Bars, **a**, 2 μm; **b**, 5 μm; **c**, 1 μm; **d**, 120 nm.

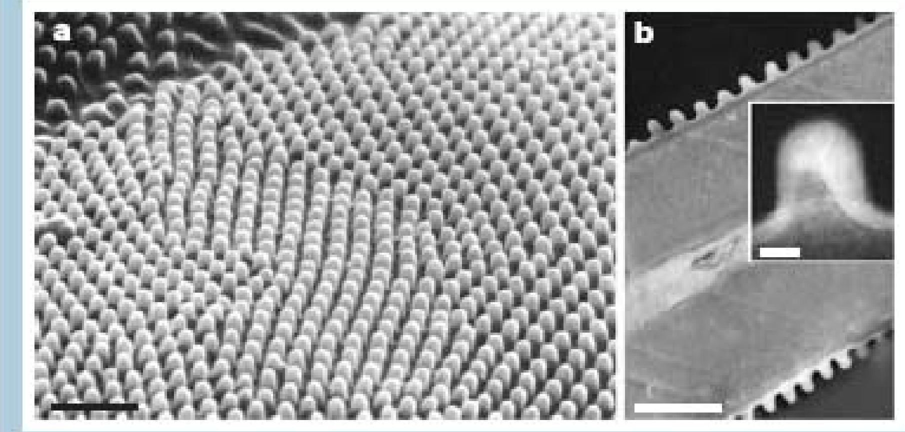



**Figure 7** Anti-reflective nipple arrays. **a**, The anti-reflective nipple arrays on

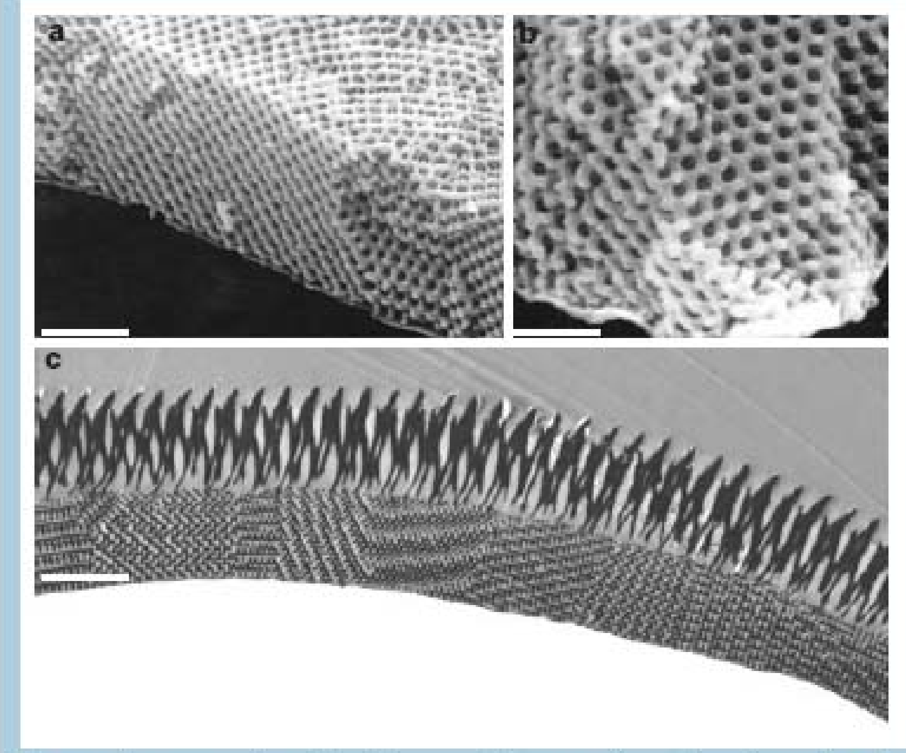
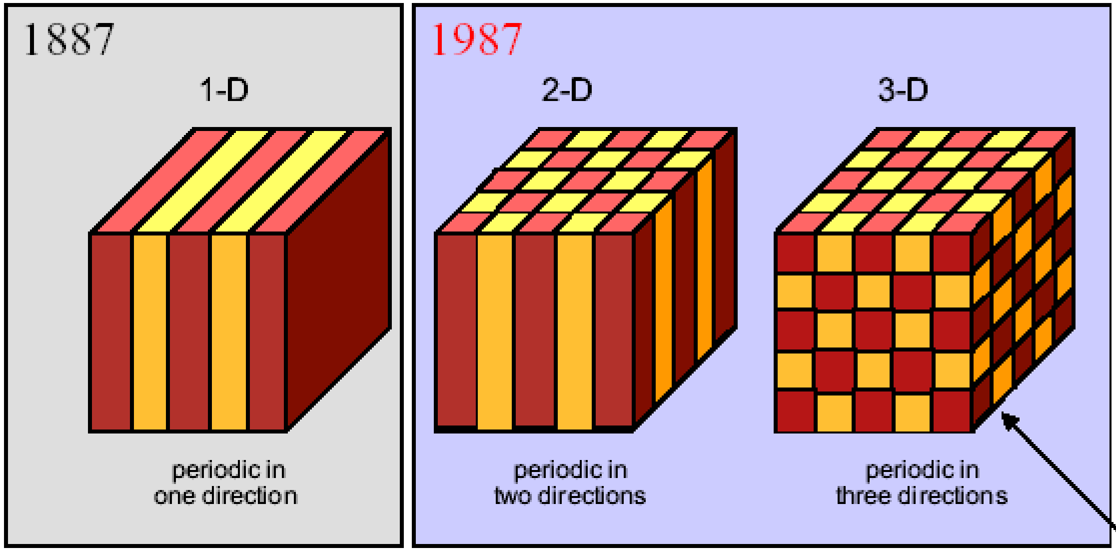



**Figure 6** The green colour of *Parides sesostris* is created by a photonic crystal. **a**, **b**,

# Photonic crystals

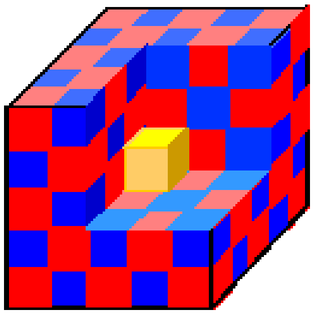



requires complex topology

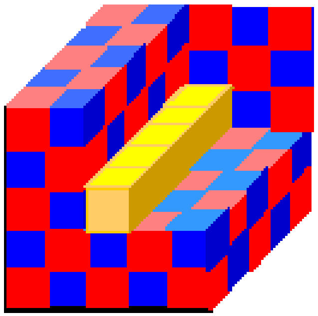
Periodic electromagnetic media

Photonic bandgap: optical insulator

# Photonic crystals



Light cavities

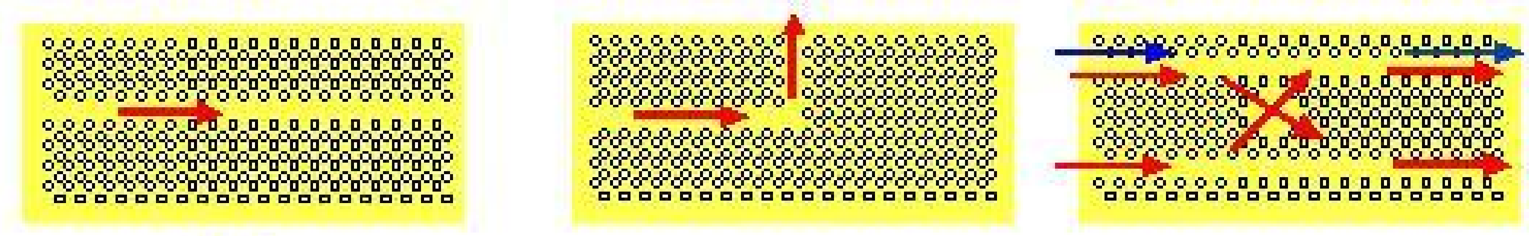


Light waveguides
('wires')

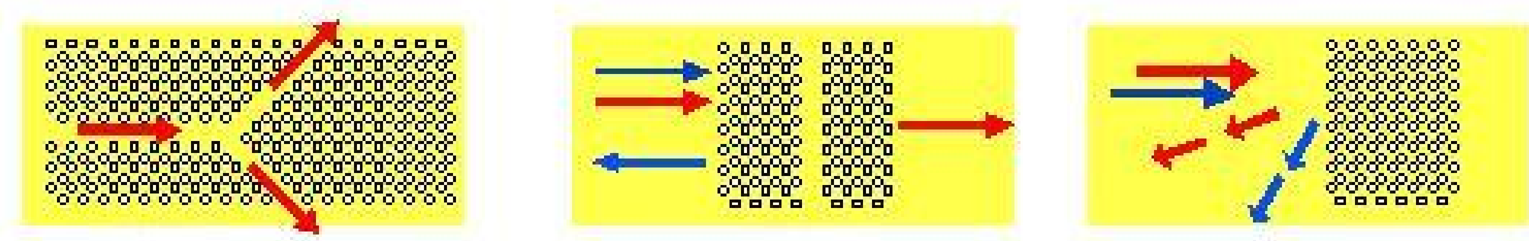
Periodic electromagnetic media

Photonic bandgap: optical insulator

# Photonic bandgap waveguides



(a) (b) (c)

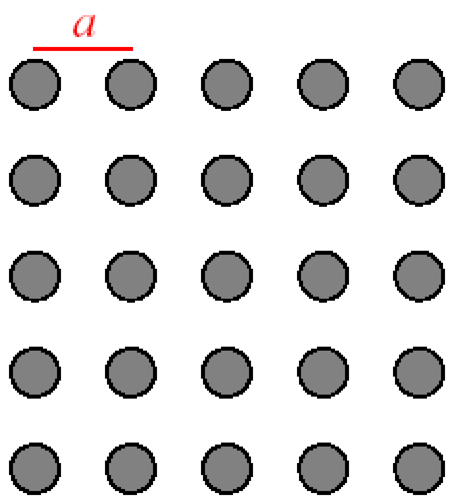


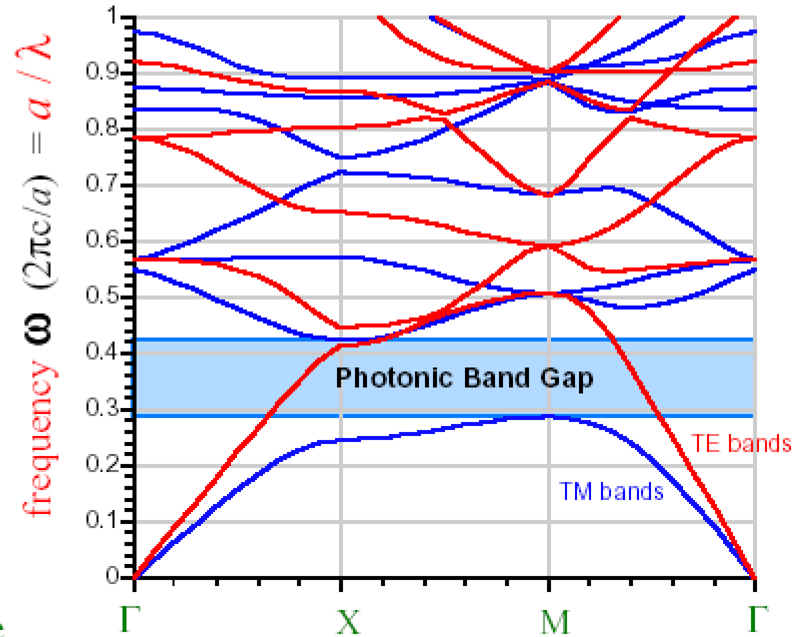
(d) (e) (f)

a : guide b : sharp bend c : add/drop

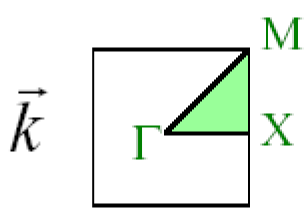
d : Y-coupler e : filter f : dispersive element

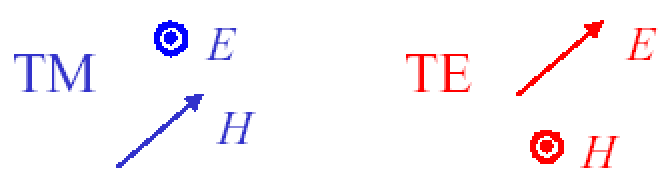
# Photonic bands



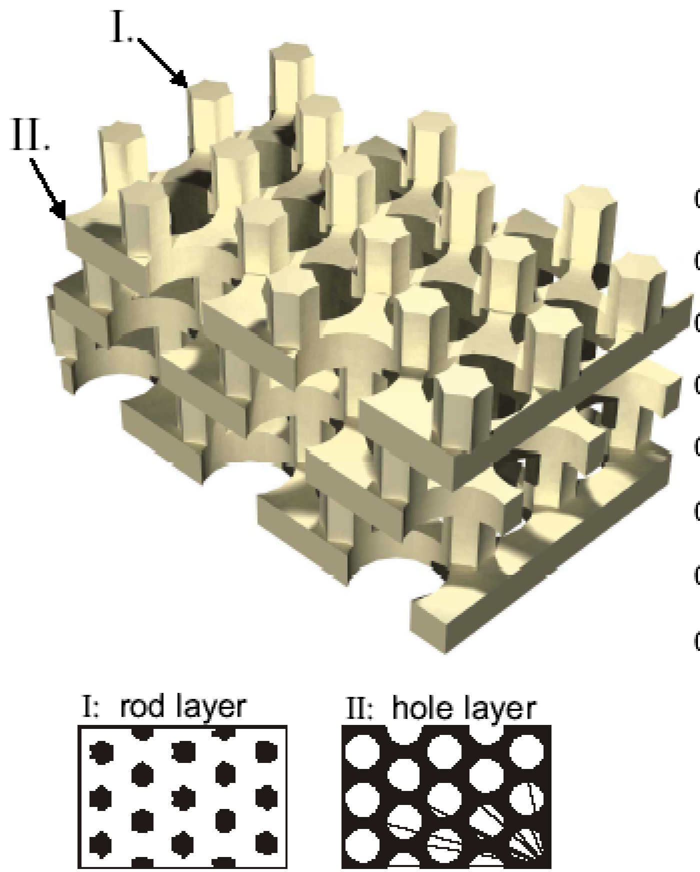



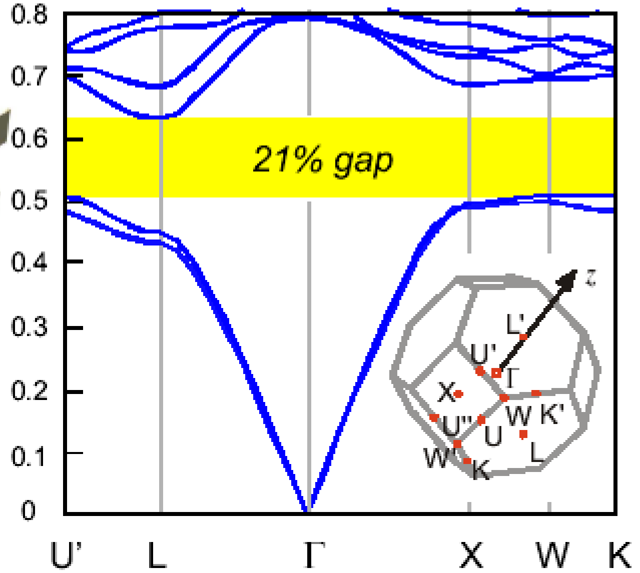

irreducible Brillouin zone



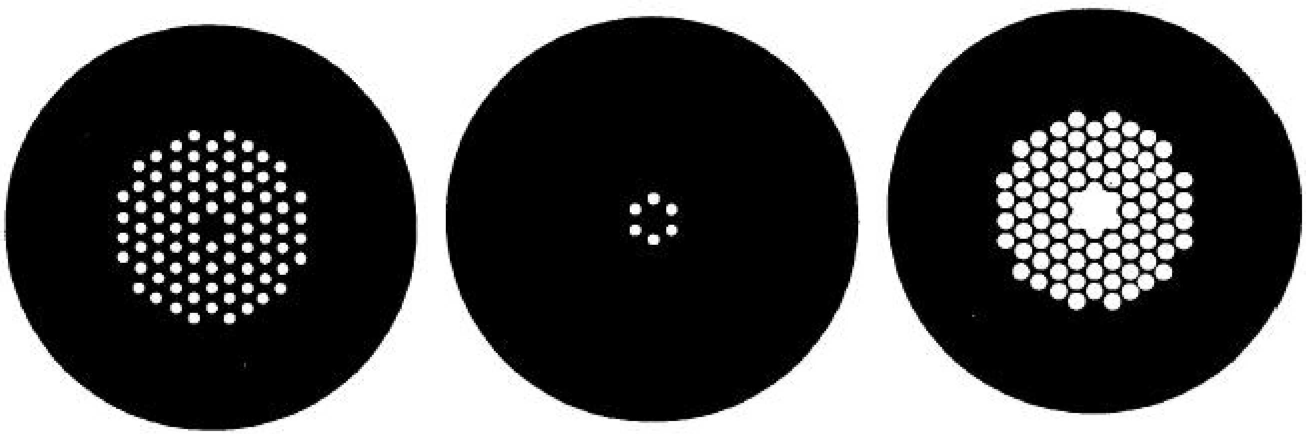
# 3D photonic crystals – photonic bandgap




[ S. G. Johnson *et al.*, *Appl. Phys. Lett.* **77**, 3490 (2000) ]

# Photonic crystal fibers



(a) (b) (c)

a : PCF (TIR) ⇐ very strong core-cladding index contrast enhanced

⇐ (ESM) ⇐ large bending loss

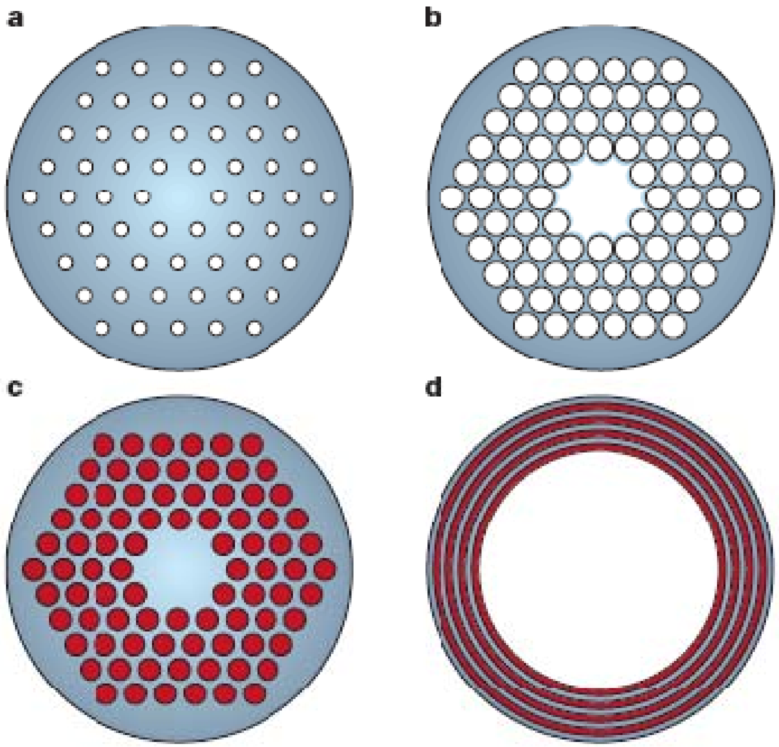
b : Air-clad core ⇐ highly dispersive ⇐ DCF

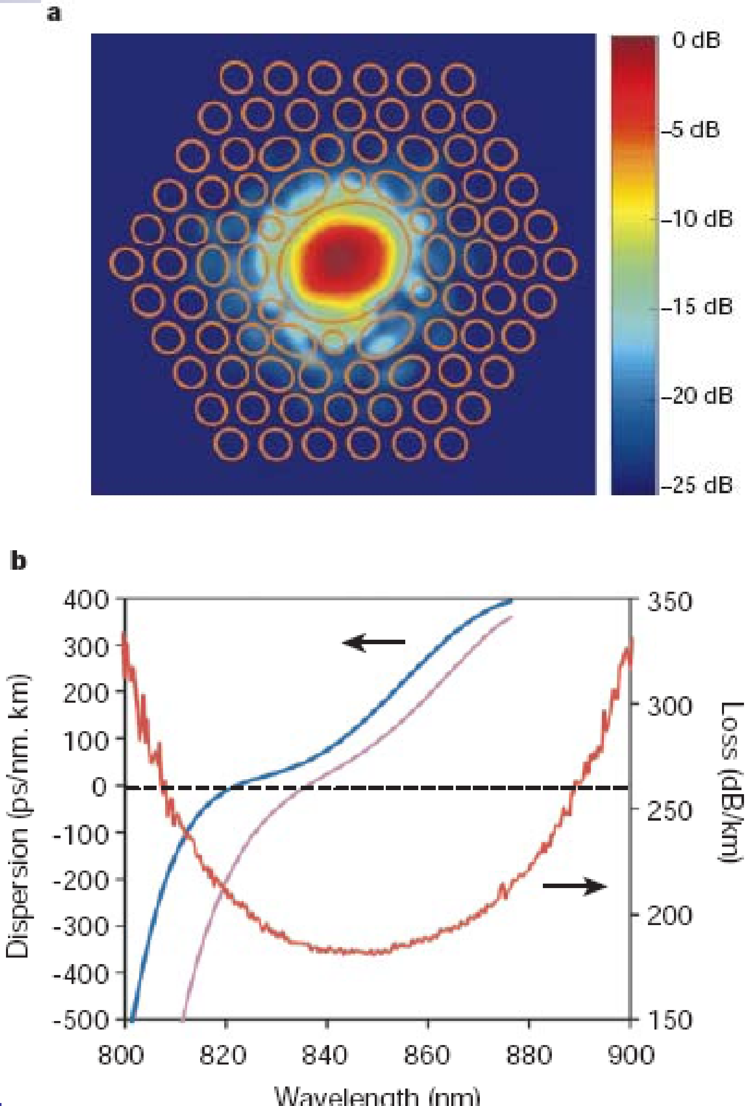
c : PBGF ⇐ too lossy (~1dB/m) ⇐ still has a potential for the future

# Photonic crystal fibers

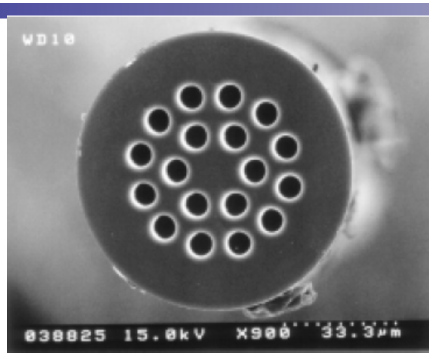
**Photonic crystal fibers**

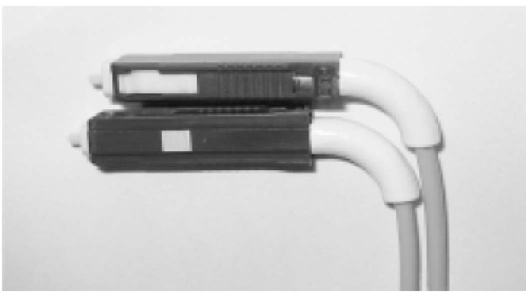
J. C. Knight, *Nature*, 424, pp. 847-851, 2003.



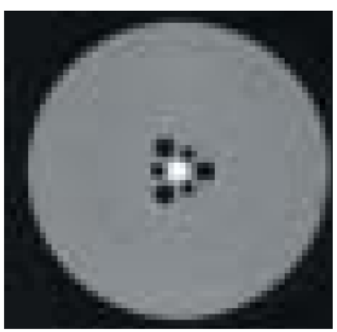
# Holey fibers



## Mitsubishi Cable

| | |
|---|---|
| 클래딩 지름 | 80μm |
| 유효 코어 모드 지름 | 11.5 μm |
| 전송 손실 (1550nm) | 0.35dB/km |
| 허용 곡률 반경 | 7.5mm |
| 벤딩 손실 (1550nm)<br>ϕ 15mm × 10 turn | <0.1dB |



## Fujikura

FutureGuide®−SR15
→허용 곡률 반경 15 mm

FutureGuide®−SR7.5
→ 허용 곡률 반경 7.5 mm

# Vectorial diffractive optics

- Concepts of Maxwell equation analysis
  FDTD, RCWA, PFMA
- Surface plasmon analysis
  Surface plasmon resonance, excitation by finite beam and pulses, 3D metallic structures
- Photonic crystal analysis

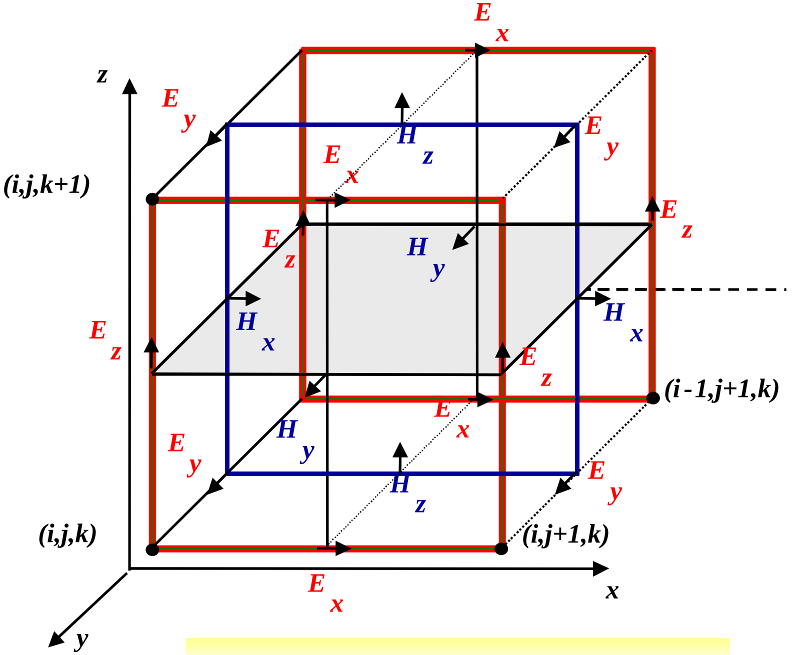
# Finite-difference time-domain (FDTD) method

FDTD is a rigorous analysis method with widely applications from nano- to all length-scales using the Maxwell's equations and curl equations

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_x}\left(\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial y} - \sigma_x E_x\right) \qquad \frac{\partial H_x}{\partial t} = \frac{1}{\mu_x}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \sigma_x^* H_x\right)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon_y}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} - \sigma_y E_y\right) \qquad \frac{\partial H_y}{\partial t} = \frac{1}{\mu_y}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \sigma_y^* H_y\right)$$

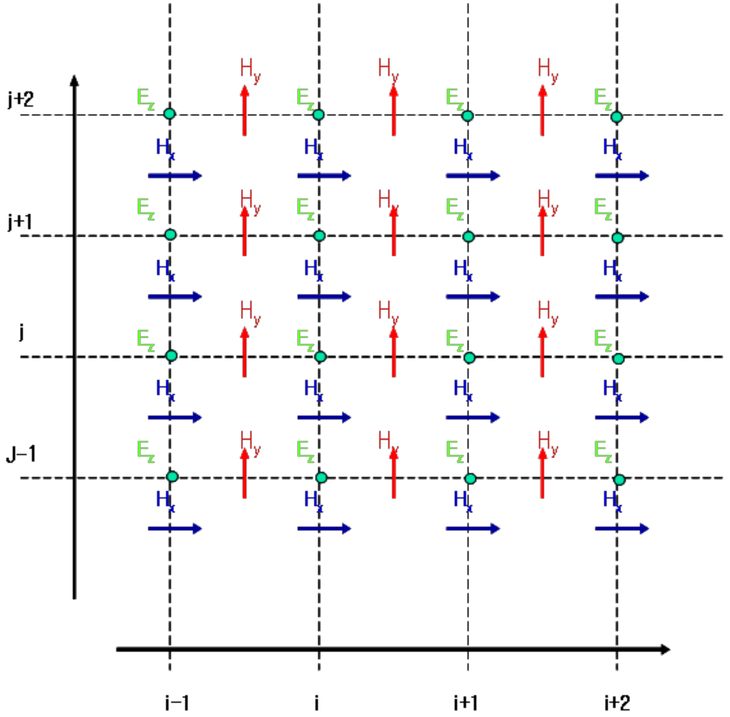
$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_z}\left(\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x} - \sigma_z E_z\right) \qquad \frac{\partial H_z}{\partial t} = \frac{1}{\mu_z}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \sigma_z^* H_z\right)$$




*3D Yee's cell geometry*

# Finite-difference time-domain (FDTD) method

2D Yee's cell : interleaving mesh




$$\frac{D_z^{n+0.5}(i,j)-D_z^{n-0.5}(i,j)}{\Delta t}=c_0\left(\frac{H_y^{n}(i+0.5,j)-H_y^{n}(i-0.5,j)}{\Delta x}\right)-c_0\left(\frac{H_x^{n}(i,j+0.5)-H_x^{n}(i,j-0.5)}{\Delta x}\right)$$

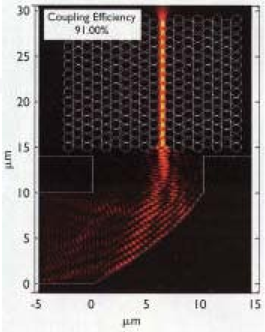
$$\frac{H_x^{n+1}(i,j+0.5)-H_x^{n}(i,j+0.5)}{\Delta t}=-c_0\left(\frac{E_z^{n+0.5}(i,j+1)-E_z^{n+0.5}(i,j)}{\Delta x}\right)$$

# Finite-difference time-domain (FDTD) method

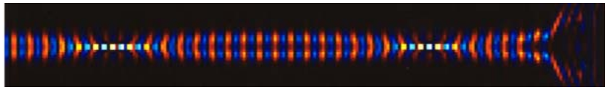
## Numerical methods for EM wave solution

- Numerical methods for solving the electromagnetic wave propagation problems
  - FEM, MOM, BPM, TLM, FDTD (which is the post popular)
  - Yee used centered finite-difference expressions for the space and time derivatives that are both simply programmed and have second-order accuracy in the space and time increments.
- Advantages of FDTD
  - Problems exhibiting complex structures
  - Only one computation is required to get the frequency domain results over a large frequency spectrum.
  - FDTD method does not need to solve any integral and matrix equations.
  - It is conceptually simple and simple to code.
  - It is applicable to a large class of problems.
  - However, it needs very large computer memories.
- FDTD conditions and properties
  - Stability, dispersion, source conditions, absorbing boundary conditions

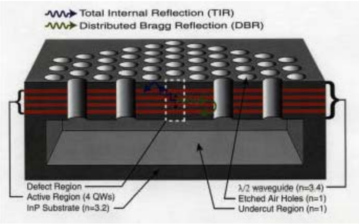
# Finite-difference time-domain (FDTD) method

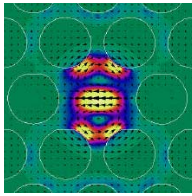



- **FDTD applications**

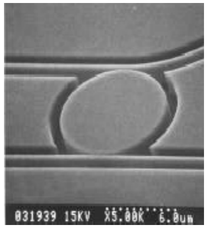


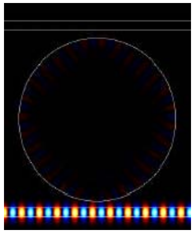
  - ✓ Waveguides and nano-photonics
    - – Solitons
      - Mingaleev and Kivshar, *Optics and Photonics News*, July 2002.
      - Prather, *Optics and Photonics News*, June 2002.

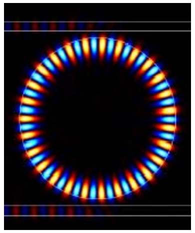


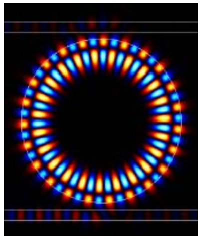



    - – Defect cavities and defect mode lasers
      - Painter et al., *Science*, June 11, 1999.

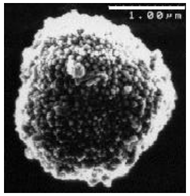


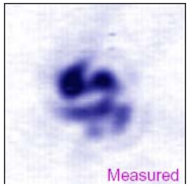


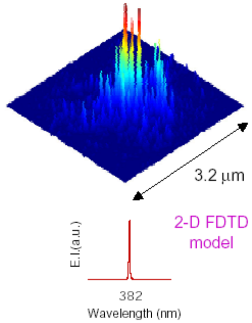




    - – Waveguides coupled to disks and rings
      - S. C. Hagness, D. Rafizadeh, S. T. Ho, and A. Taflove, *IEEE J. Lightwave Tech.*, 1997.

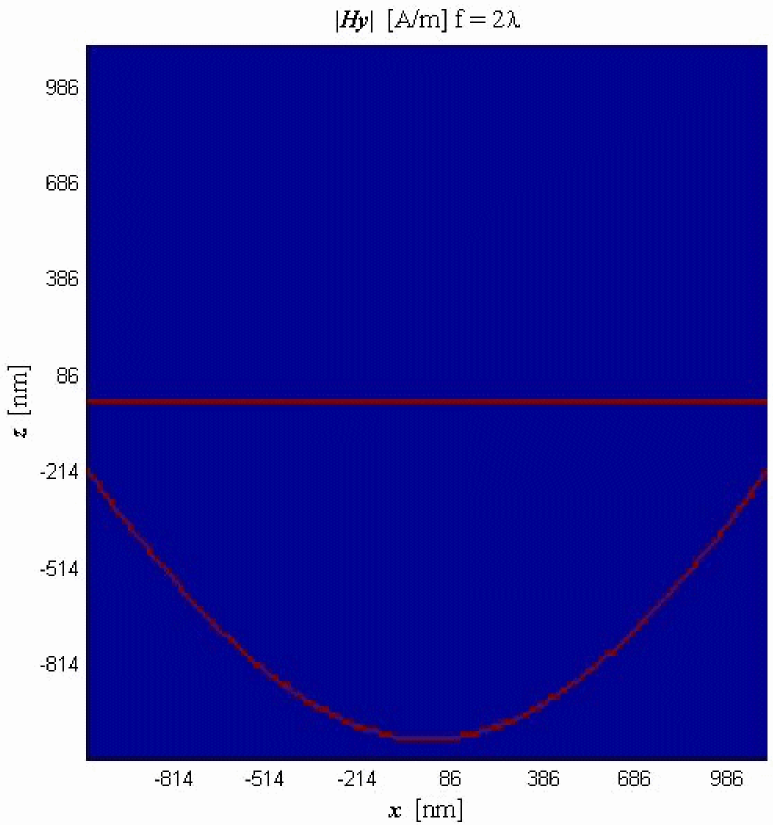


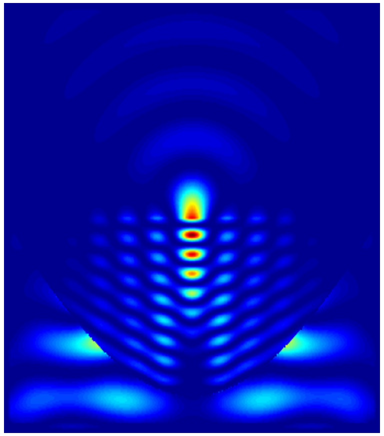


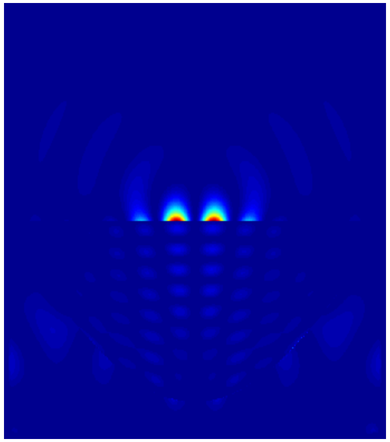

    - – Lasing in a random clump of ZnO particles
      - H. Cao et al., *Phys. Rev. Lett.*, 2000.

# Solid immersion lens simulation (FDTD)



Ex – component

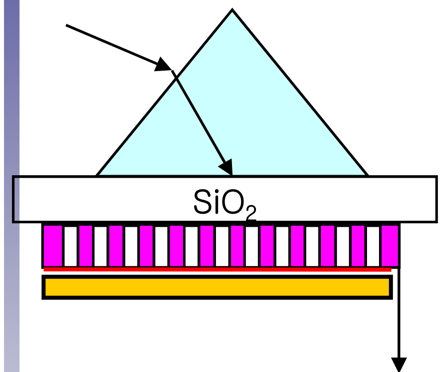


Ey – component

Extension to three-dimensional solid immersion lens nano-focusing for high density optical memory

# FDTD example

**Nd- YAG (532nm)**




(FS) N=1.457

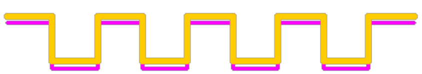
PMMA with Grating

Cr – 1nm

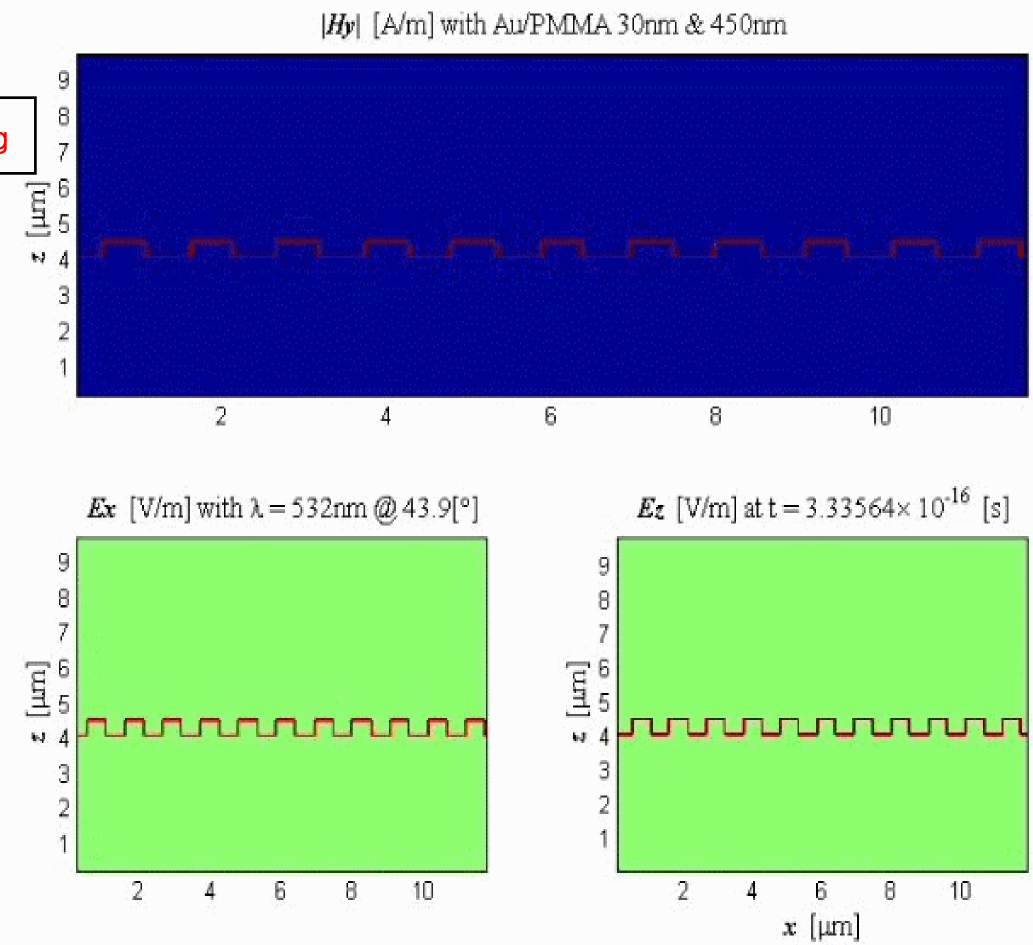
Au – 30nm

**Lamella grating**

**CASE 1**



**CASE 2**



# Frequency domain method for the Maxwell equations

### Homogeneous Maxwell equations in the spatial domain

$$\nabla \times \underline{E} = jw\mu_0\mu(x,y,z)\left(\hat{x}H_x + \hat{y}H_y + \hat{z}H_z\right)$$

$$\nabla \times \underline{H} = -jw\varepsilon_0\varepsilon(x,y,z)\left(\hat{x}E_x + \hat{y}E_y + \hat{z}E_z\right)$$

Fourier transform

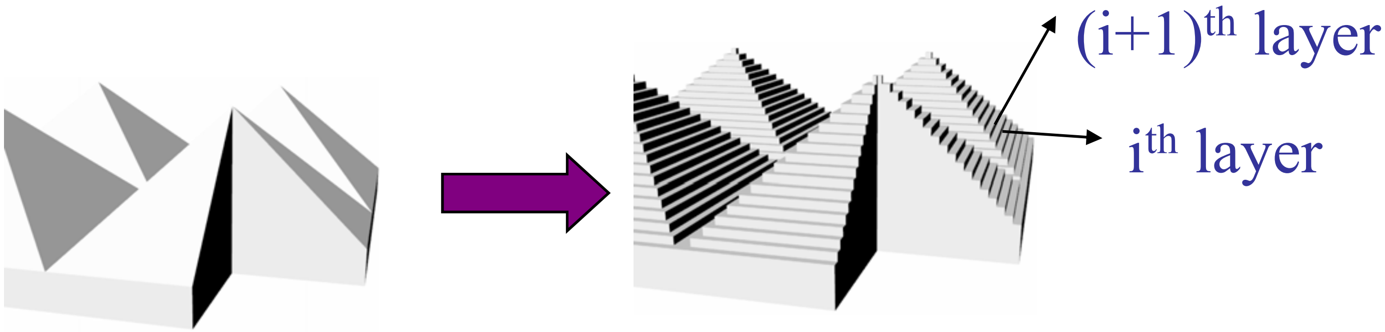
### Homogeneous Maxwell equations in the (spatial) frequency domain

$$k_0 \begin{bmatrix} \underline{\underline{\varepsilon}}_{(x)} & 0 & 0 & 0 & 0 & 0 \\ 0 & \underline{\underline{\varepsilon}}_{(y)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{\underline{\varepsilon}}_{(z)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{\underline{\mu}}_{(x)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \underline{\underline{\mu}}_{(y)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{\underline{\mu}}_{(z)} \end{bmatrix} \begin{bmatrix} \underline{E}_x \\ \underline{E}_y \\ \underline{E}_z \\ \underline{H}_x \\ \underline{H}_y \\ \underline{H}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & j\underline{\underline{K}}_z & -j\underline{\underline{K}}_y \\ 0 & 0 & 0 & -j\underline{\underline{K}}_z & 0 & j\underline{\underline{K}}_x \\ 0 & 0 & 0 & j\underline{\underline{K}}_y & -j\underline{\underline{K}}_x & 0 \\ 0 & j\underline{\underline{K}}_z & -j\underline{\underline{K}}_y & 0 & 0 & 0 \\ -j\underline{\underline{K}}_z & 0 & j\underline{\underline{K}}_x & 0 & 0 & 0 \\ j\underline{\underline{K}}_y & -j\underline{\underline{K}}_x & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{E}_x \\ \underline{E}_y \\ \underline{E}_z \\ \underline{H}_x \\ \underline{H}_y \\ \underline{H}_z \end{bmatrix}$$

The Maxwell equations in the spatial domain have partial differential operators. But Maxwell equations in the frequency domain are described by several algebraic equations.

# Rigorous coupled wave analysis (RCWA)

**Structure modeling (staircase approximation)**




**Fourier representation of the permittivity of the ith layer**

$$\varepsilon^{(i)}(x,y)=\sum_{g,h}\varepsilon_{gh}^{(i)}\exp\left[j(G_{x,g}x+G_{x,h}y)\right]$$

**Fourier representation of the EM fields in the ith layer**

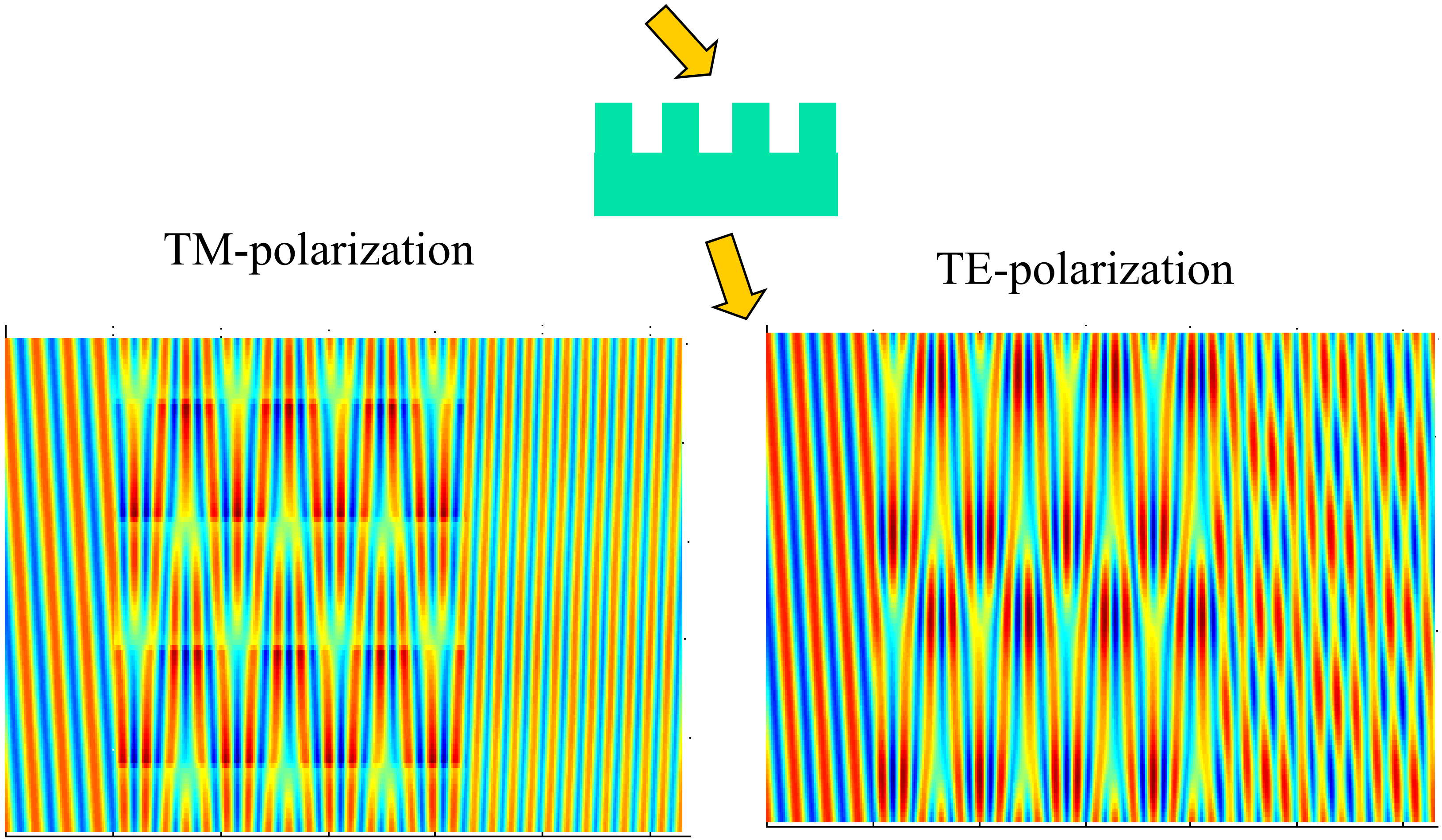
$$\underline{E}^{(i)}=\sum_{m=-M}^{M}\sum_{n=-N}^{N}\left[\underline{x}S_{x,mn}^{(i)}(z)+\underline{y}S_{y,mn}^{(i)}(z)+\underline{z}S_{z,mn}^{(i)}(z)\right]\exp\left[j\left(k_{x,m}x+k_{y,n}y\right)\right]$$

$$\underline{H}^{(i)}=j\sqrt{\frac{\varepsilon_0}{\mu_0}}\sum_{m=-M}^{M}\sum_{n=-N}^{N}\left[\underline{x}U_{x,mn}^{(i)}(z)+\underline{y}U_{y,mn}^{(i)}(z)+\underline{z}U_{z,mn}^{(i)}(z)\right]\exp\left[j\left(k_{x,m}x+k_{y,n}y\right)\right]$$

**The boundary conditions between adjacent layers are satisfied by the S-matrix method.**

# RCWA examples

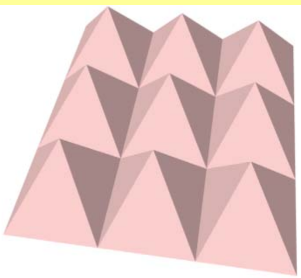
**2D-binary dielectric grating showing polarization-dependent diffraction**

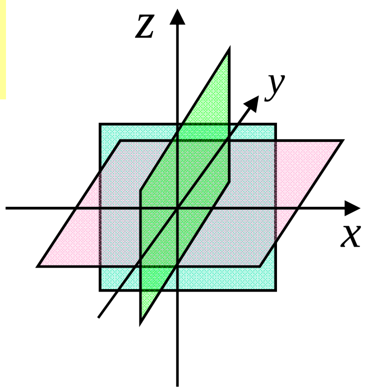


# RCWA examples

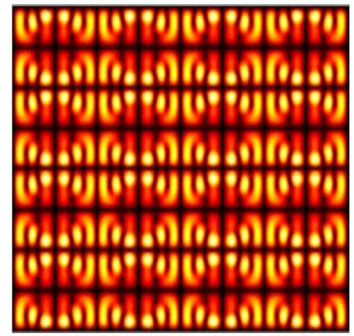
## 3D micro-pyramid structure (15 level staircase approximation)

Wavelength(λ)=532nm
Period=3λ, Height=6λ
Normal incidence



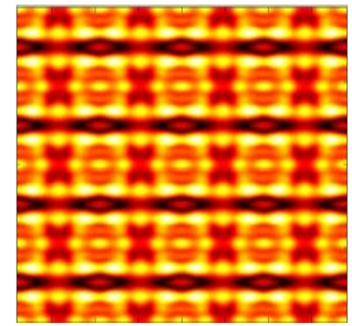



$|E_y|$



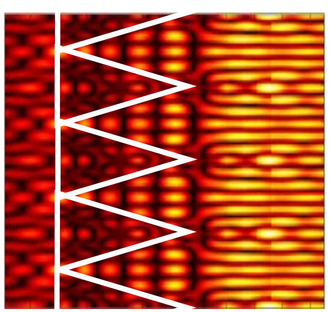
x-y crosssection

$|E_x|$



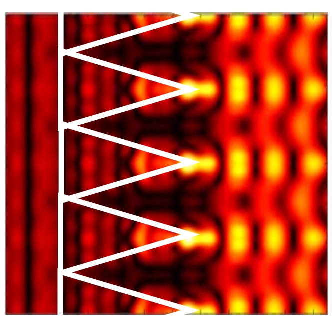
x-y crosssection

$|E_y|$



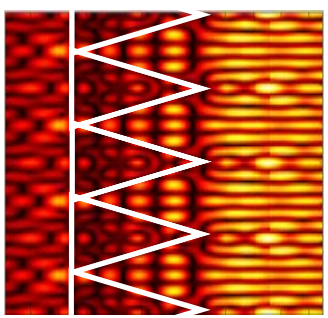
x-z crosssection

$|E_x|$



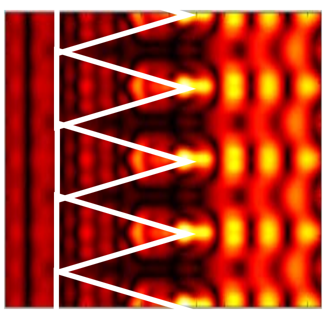
x-z crosssection

$|E_y|$



y-z crosssection

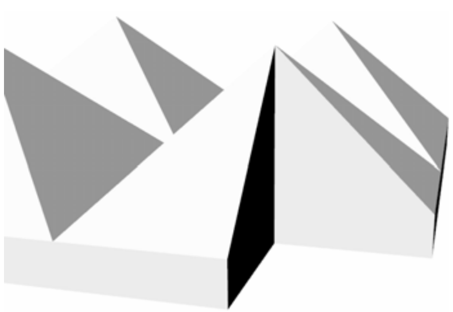
$|E_x|$



y-z crosssection

# Pseudo-Fourier modal analysis (PFMA)

## Structure modeling (3D Fourier series)



$$\varepsilon(x,y,z)=\sum_{s=-2M}^{2M}\sum_{t=-2N}^{2N}\sum_{p=-2H}^{2H}\tilde{\varepsilon}_{stp}\exp\left(j\left(k_{x,stp}x+k_{y,stp}y+k_{z,stp}z\right)\right)$$

## Pseudo-Fourier representation of the E-M field

$$\tilde{\mathbf{E}}_{\mathbf{k}}=\mathrm{e}^{j\left(k_{x,0}x+k_{y,0}y+k_{z,0}z\right)}\sum_{m,n,q}\left(E_{x,m,n,q}\underline{x}+E_{y,m,n,q}\underline{y}+E_{z,m,n,q}\underline{z}\right)\exp\left(j\left(k_{x,mnq}x+k_{y,mnq}y+k_{z,mnq}z\right)\right)$$

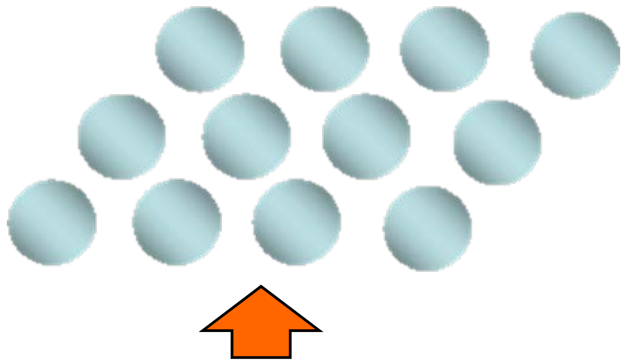
## Maxwell equation in the PFMA

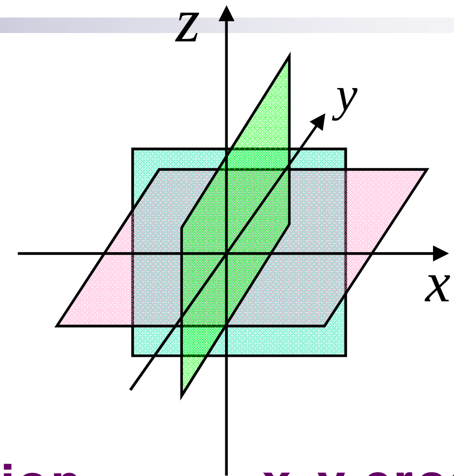
Eigenvalue

$$\begin{bmatrix} -j\underline{\underline{G}}_z & 0 & \underline{\underline{K}}_y\underline{\underline{\varepsilon}}_{(z)}^{-1}\underline{\underline{K}}_x & \underline{\underline{\mu}}_{(x)}-\underline{\underline{K}}_y\underline{\underline{\varepsilon}}_{(z)}^{-1}\underline{\underline{K}}_y \\ 0 & -j\underline{\underline{G}}_z & -\underline{\underline{\mu}}_{(y)}+\underline{\underline{K}}_x\underline{\underline{\varepsilon}}_{(z)}^{-1}\underline{\underline{K}}_x & -\underline{\underline{K}}_x\underline{\underline{\varepsilon}}_{(z)}^{-1}\underline{\underline{K}}_y \\ \underline{\underline{K}}_y\underline{\underline{\mu}}_{(z)}^{-1}\underline{\underline{K}}_x & \underline{\underline{\varepsilon}}_{(x)}-\underline{\underline{K}}_y\underline{\underline{\mu}}_{(z)}^{-1}\underline{\underline{K}}_y & -j\underline{\underline{G}}_z & 0 \\ -\underline{\underline{\varepsilon}}_{(y)}+\underline{\underline{K}}_x\underline{\underline{\mu}}_{(z)}^{-1}\underline{\underline{K}}_x & -\underline{\underline{K}}_x\underline{\underline{\mu}}_{(z)}^{-1}\underline{\underline{K}}_y & 0 & -j\underline{\underline{G}}_z \end{bmatrix}\begin{bmatrix}\underline{E}_y\\ \underline{E}_x\\ \underline{H}_y\\ \underline{H}_x\end{bmatrix}=\frac{jk_{z,0}}{k_0}\begin{bmatrix}\underline{E}_y\\ \underline{E}_x\\ \underline{H}_y\\ \underline{H}_x\end{bmatrix}$$

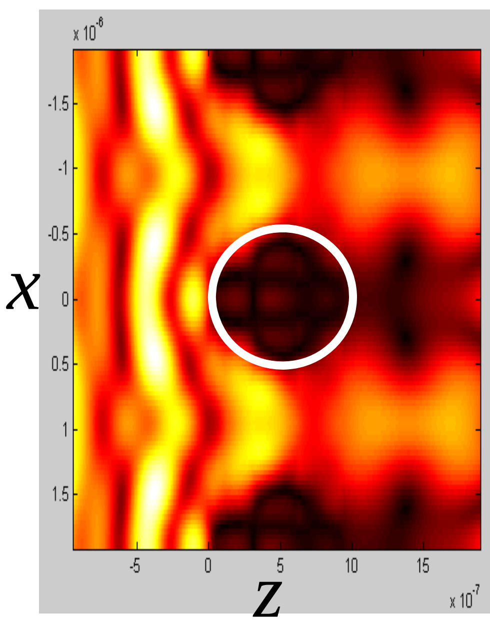
Eigenmode profile

# Metallic micro structures

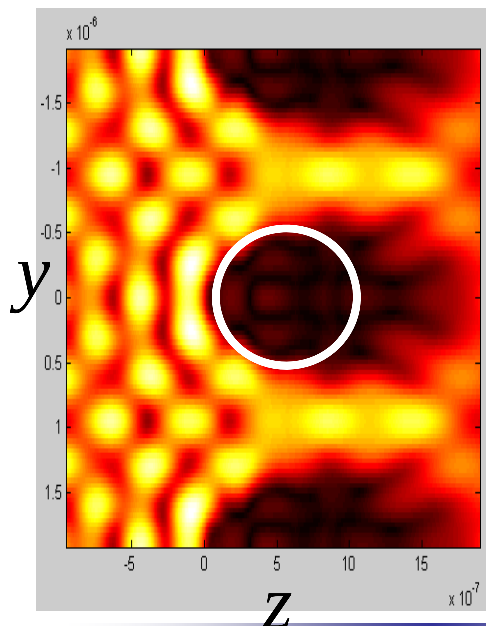




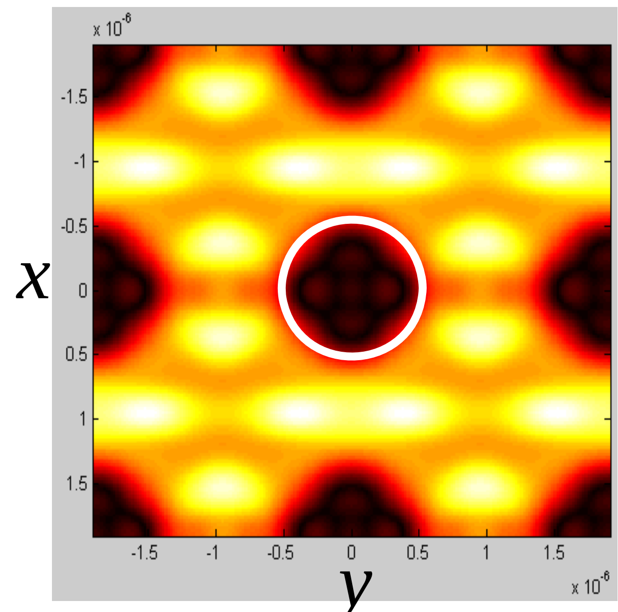

**x-z crosssection**




**y-z crosssection**




**x-y crosssection**

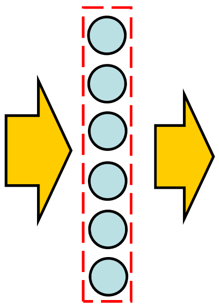


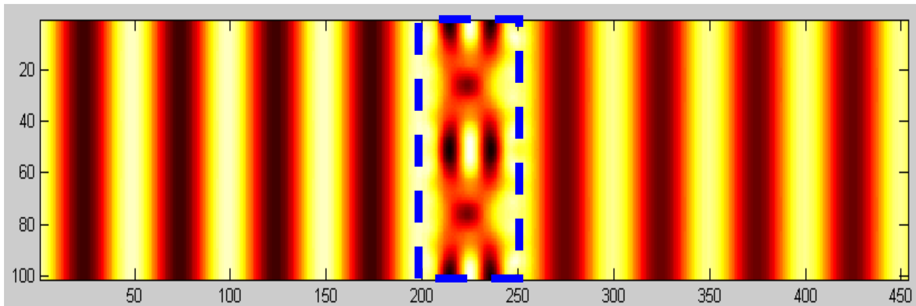
# PFMA example

**Longitudinally periodic and finite 2D photonic crystals**

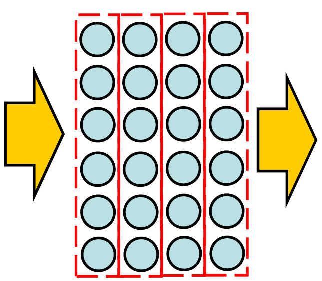
$$\mathbf{E} = \sum_{g} C_g \tilde{\mathbf{E}}_{\mathbf{k}}$$

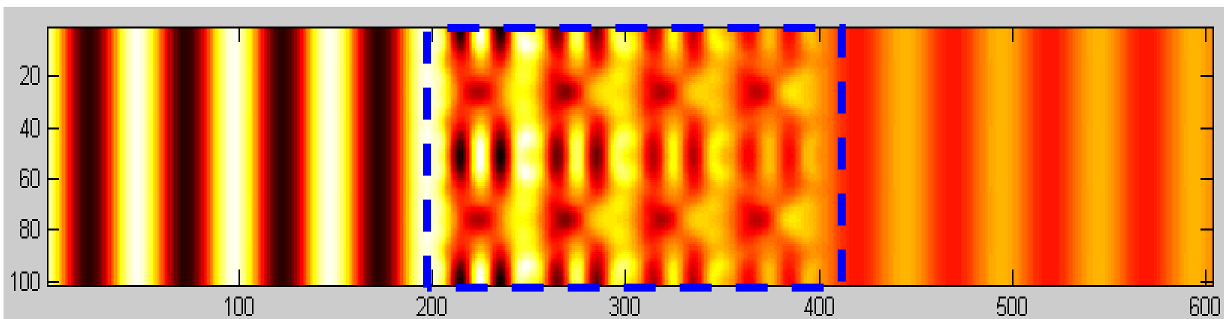
Total field distribution is a superposition of pseudo-Fourier eigenmodes with appropriate coupling coefficients.





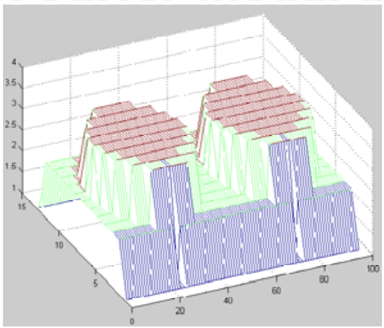

Wavelength(λ)=532nm
Period=1.2λ, radius=0.4 λ
Normal incidence



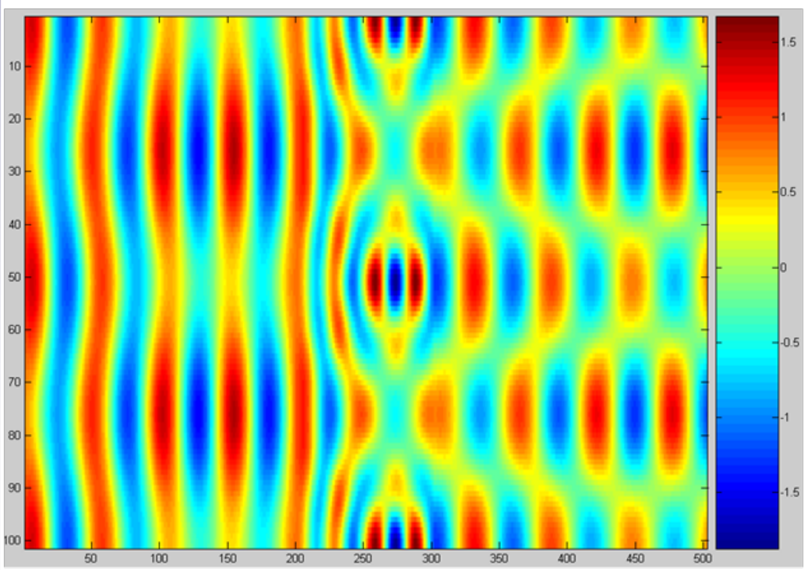


# Bloch-eigen mode extraction in the PFMA

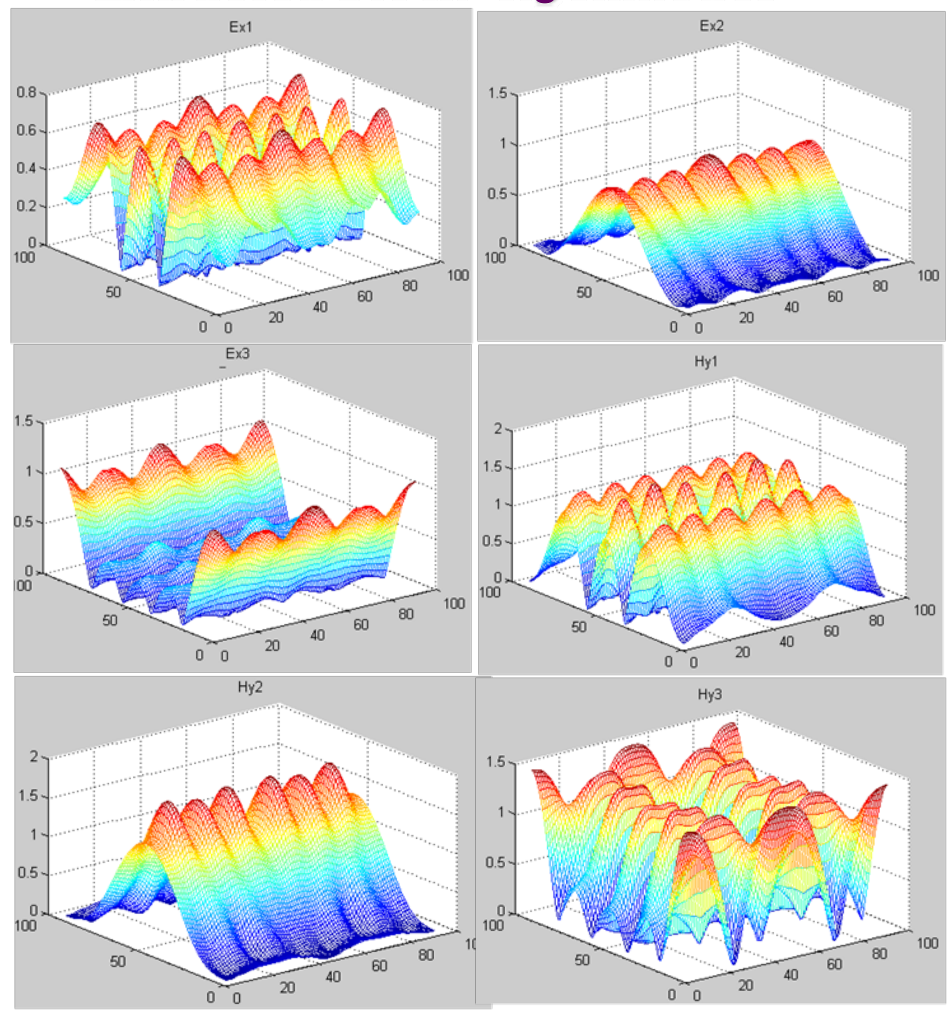
**Analysis method:**
**Pseudo-Fourier modal analysis**



**Total E-field distribution**



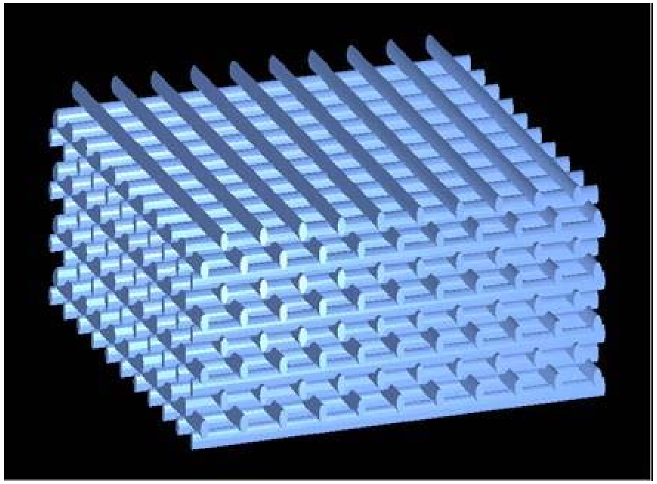
**Extracted Bloch-eigenmodes**



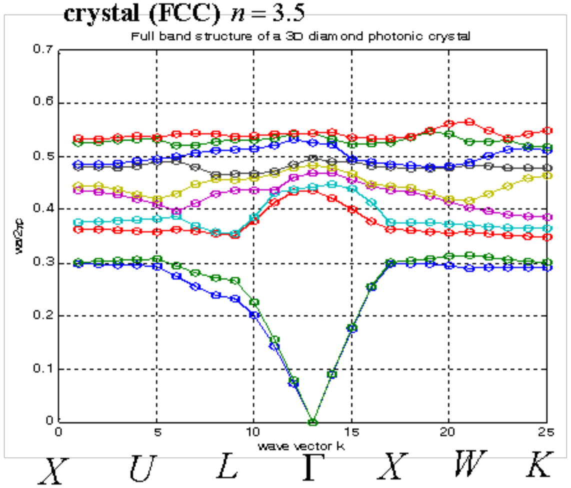
# Comparison

| | FDTD | RCWA | PFMA |
|---|---|---|---|
| Domain | Space | Frequency | Frequency |
| Field representation | Finite-difference method | Piles of truncated 2D-pseudo-Fourier series | Truncated 3D-pseudo-Fourier series |
| Structure modeling | Mesh-structure | Staircase approximation & piles of 2D-Fourier series | 3D-Fourier series (no staircase approximation) |
| Aperiodic structure Analysis | Yes | No (If using PML, yes) | No (If using PML, yes) |
| Evanescent field analysis | No (Cannot separate) | Yes | Yes |
| Modal analysis | No | No | Yes |
| Computation cost | Very huge | Large | Huge |

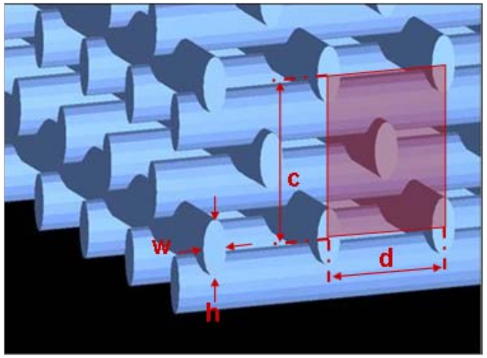
# 3D Photonic crystals



Full band structure of a woodpile photonic crystal (FCC) $n = 3.5$

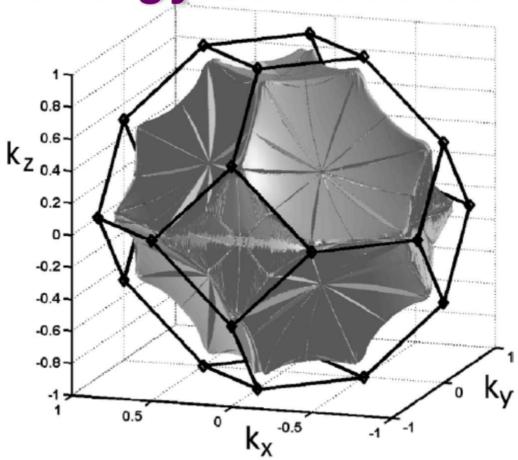



**Photonic band structure calculation**



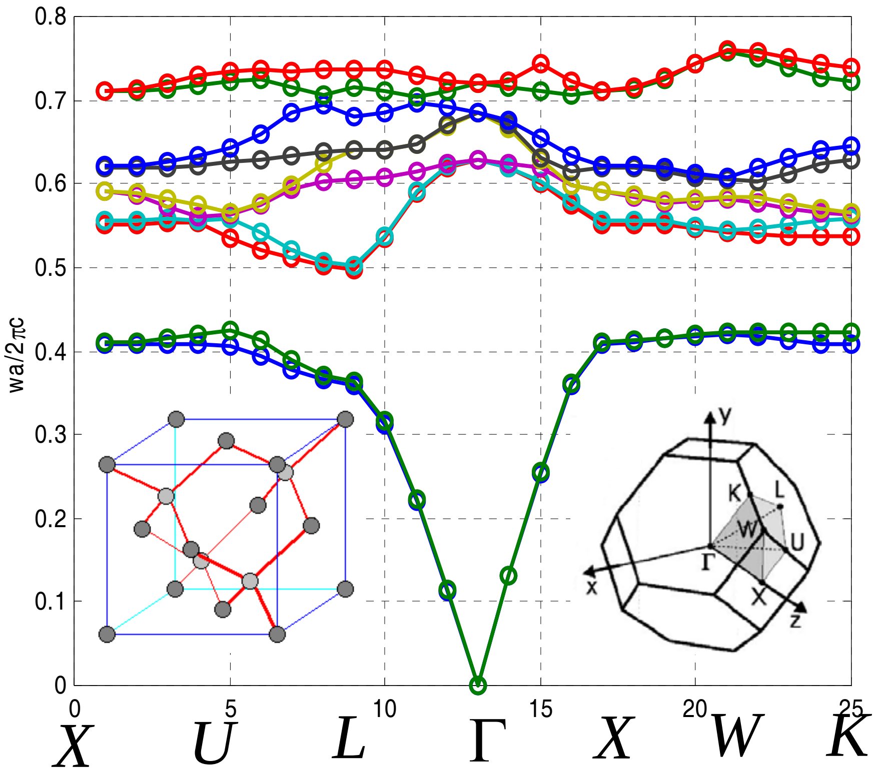

n=1.5
w=0.3d
h=0.7d
c=2^0.5d

**Iso-energy surface calculation**

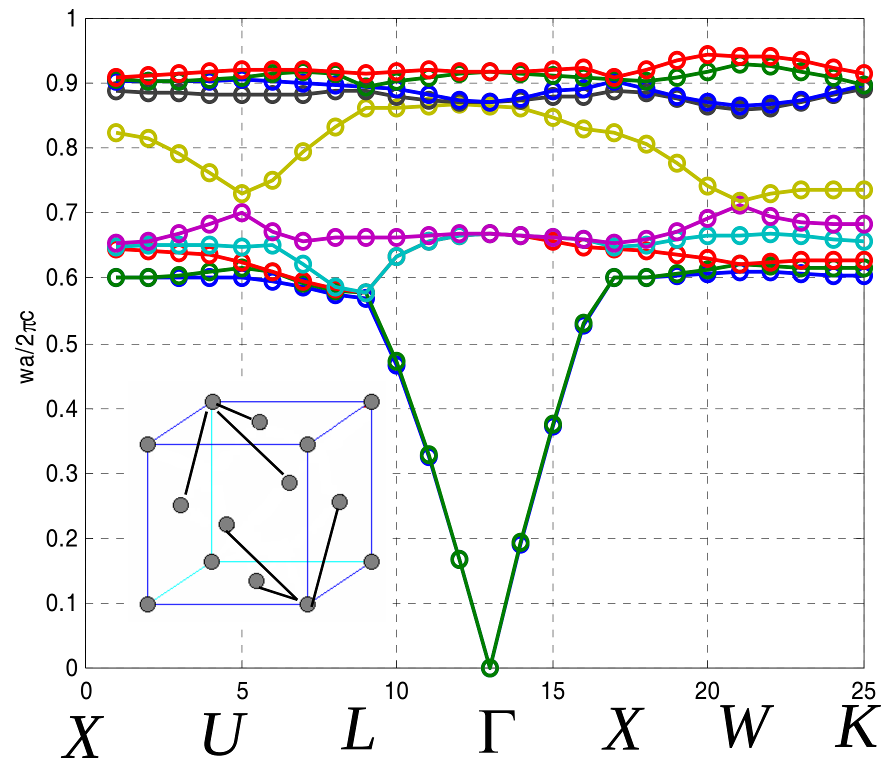


# Examples

**Full band structure of a 3D diamond photonic crystal (FCC)** $n = 3.5$

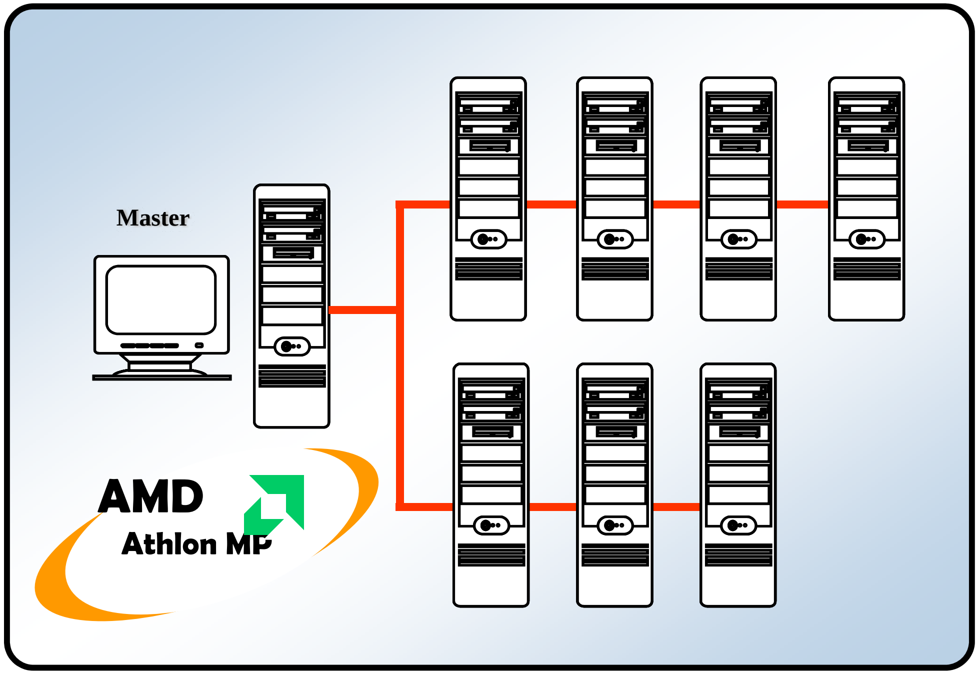



**Full band structure of a 3D simple FCC photonic crystal** $n = 3.5$



# Perspectives : parallel computing

## Parallel computing system




- ❑ Linux MPI 기반의 병렬 컴퓨터
- ❑ 16 × AMD MP 2000+ CPU의 병렬 시스템
- ❑ 16G RAM의 대용량 처리 가능
- ❑ 1Gbps의 초고속 분산 연결
- ❑ AMCL library for parallel matrix computation