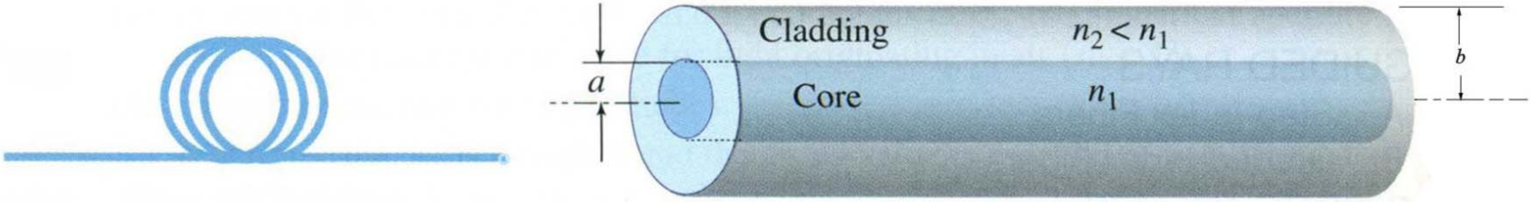
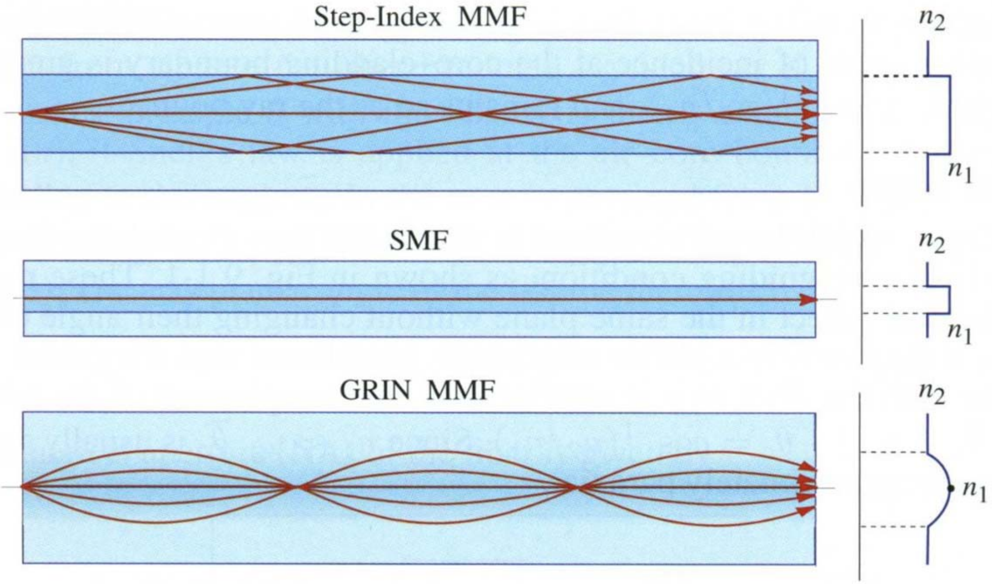
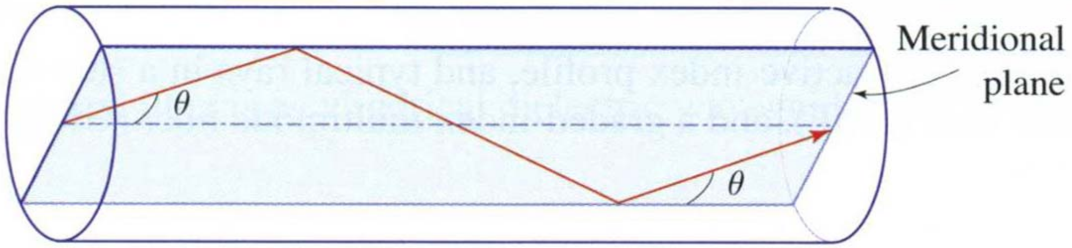
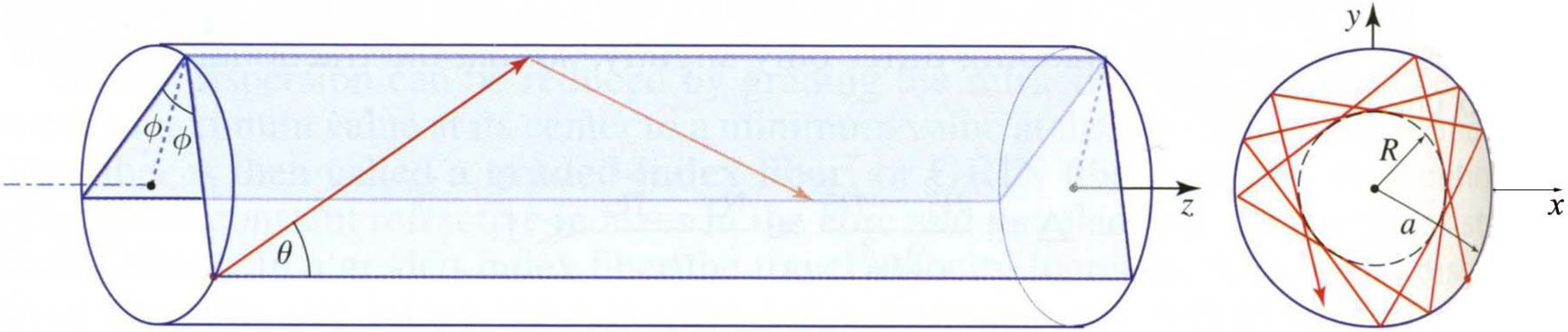
Ch. 9. Fiber Optics



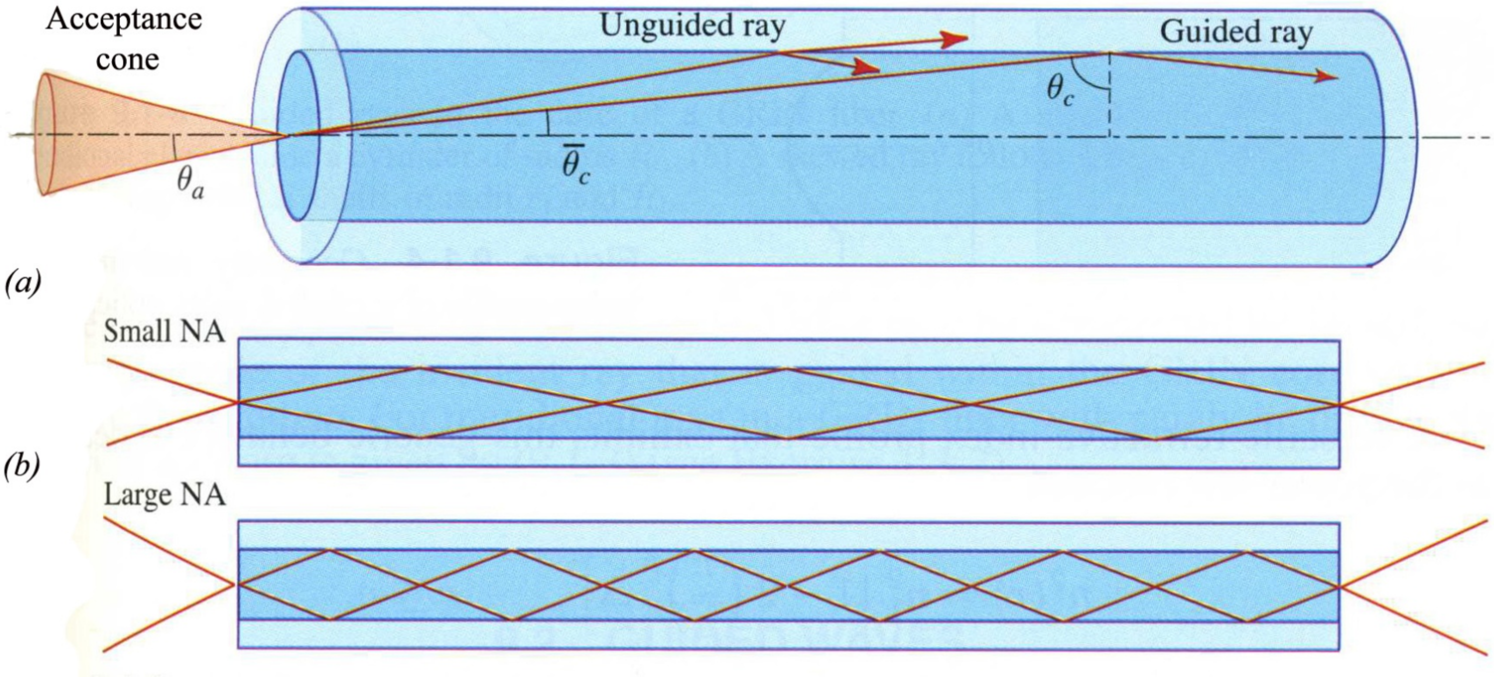


$$\Delta \equiv \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1} \ll 1$$


Meridional
plane
θ
θ


ϕ
ϕ
θ
z
y
x
R
a

$$\theta_a = \sin^{-1} \mathrm{NA} \qquad \mathrm{NA} = \sqrt{n_1^2 - n_2^2} \approx n_1\sqrt{2\Delta}$$

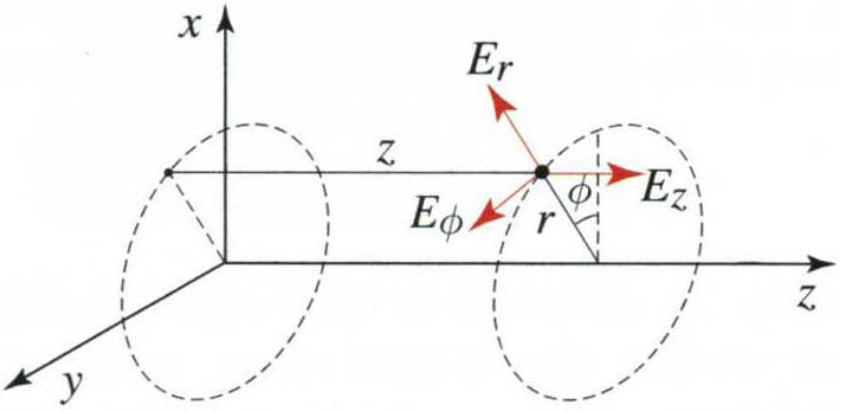


Helmholtz equation in fiber

$$\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}+\frac{1}{r^2}\frac{\partial^2 U}{\partial \phi^2}+\frac{\partial^2 U}{\partial z^2}+n^2k_o^2U=0$$

$$U(r,\phi,z)=u(r)e^{-jl\phi}e^{-j\beta z}, \qquad l=0,\pm1,\pm2,\dots$$

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}+\left(n^2(r)k_o^2-\beta^2-\frac{l^2}{r^2}\right)u=0$$



Step-index fibers

$$k_T^2 = n_1^2 k_o^2 - \beta^2$$

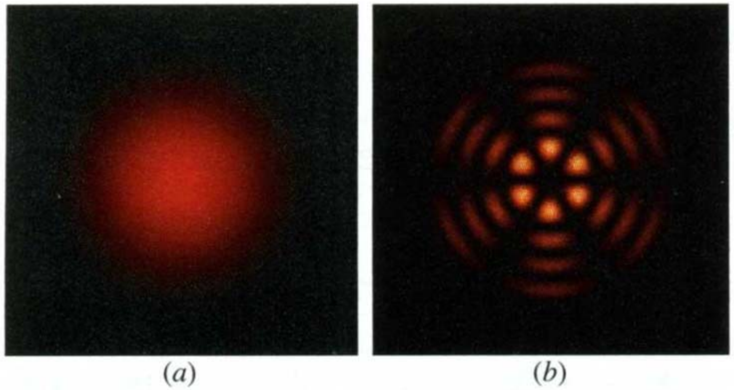
$$\gamma^2 = \beta^2 - n_2^2 k_o^2$$

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + \left(k_T^2 - \frac{l^2}{r^2}\right)u = 0, \qquad r < a \qquad \text{(core)}$$

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \left(\gamma^2 + \frac{l^2}{r^2}\right)u = 0, \qquad r > a \qquad \text{(cladding)}$$

$$u(r) \propto \begin{cases} J_l(k_T r), & r < a \qquad \text{(core)} \\ K_l(\gamma r), & r > a \qquad \text{(cladding)} \end{cases}$$

Step-index fibers

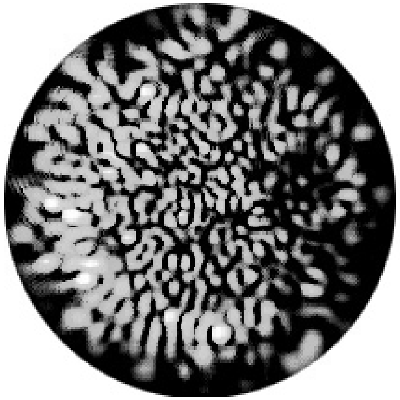

(a) (b)

$$V = 2\pi \frac{a}{\lambda_o} \mathrm{NA}$$

V parameter

$$V < 2.405$$

Single-mode condition

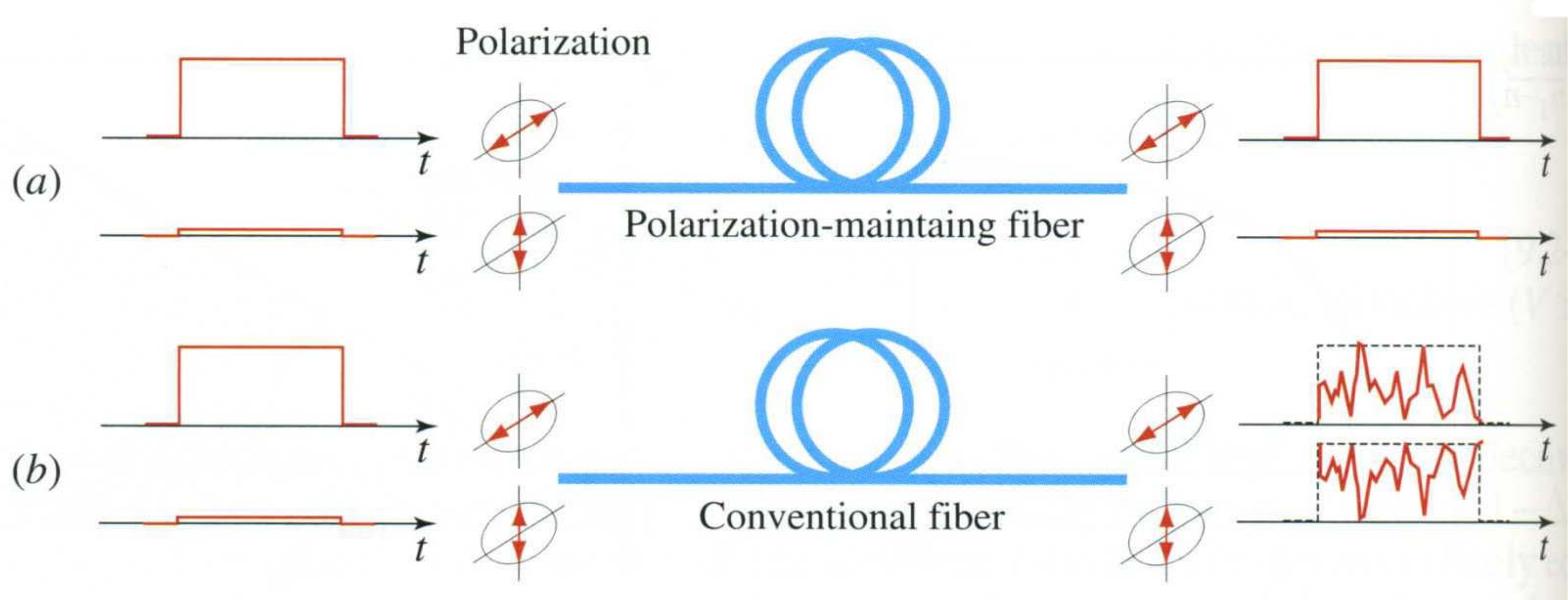


$$M \approx \frac{4}{\pi^2} V^2$$

Number of modes (V>>1)

Multimode Fiber

Polarization-maintaining fiber

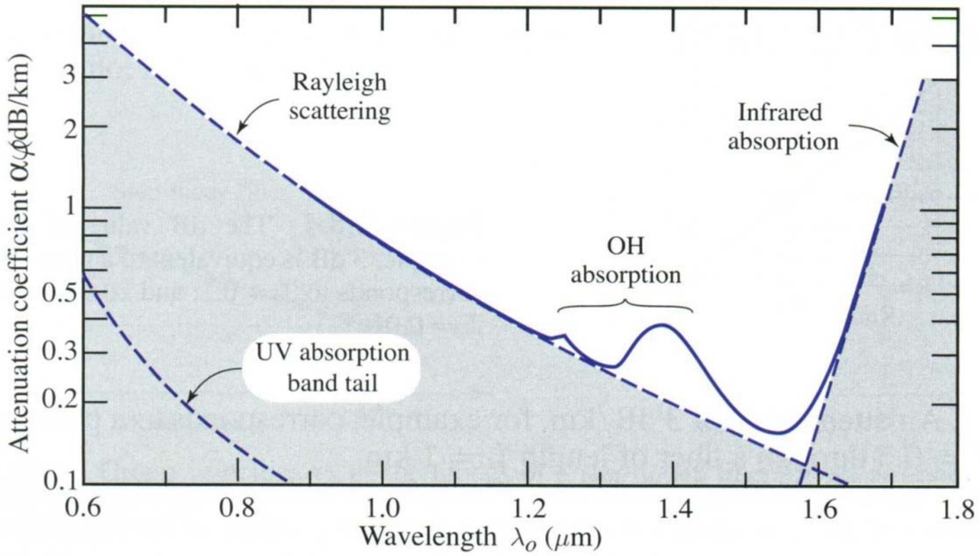


Optical fiber - attenuation

$$\alpha = \frac{1}{L} 10 \log_{10} \frac{1}{\mathcal{T}}$$

$$\frac{P(z)}{P(0)} = 10^{-\alpha z/10} \approx e^{-0.23\,\alpha z}, \qquad \alpha \text{ in dB/km}$$

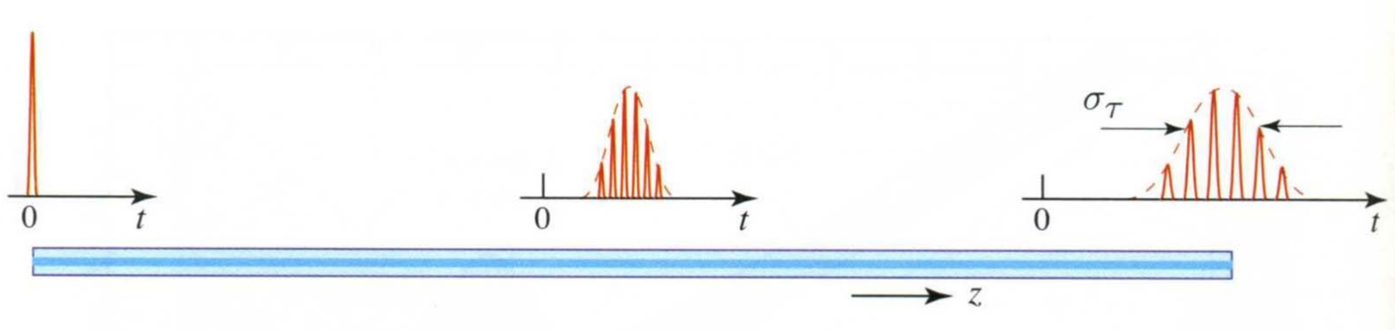
$$P(z)/P(0) = e^{-\alpha z}, \qquad \alpha \text{ in km}^{-1}$$



Optical fiber - dispersion

- Modal dispersion
- Material dispersion
- Waveguide dispersion
- Polarization-mode dispersion
- Nonlinear dispersion

Modal dispersion



$$\sigma_\tau \approx \frac{L}{c_1} \cdot \frac{\Delta}{2}$$

Response time

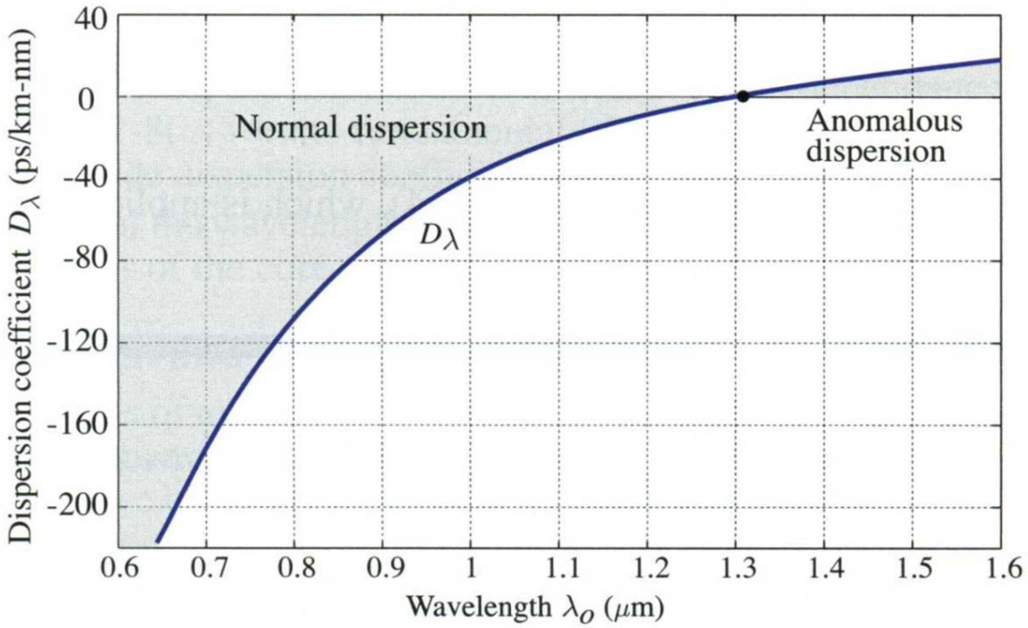
(Multi-mode step index)

$$\sigma_\tau \approx \frac{L}{c_1} \cdot \frac{\Delta^2}{4}$$

Response time

(Graded index)

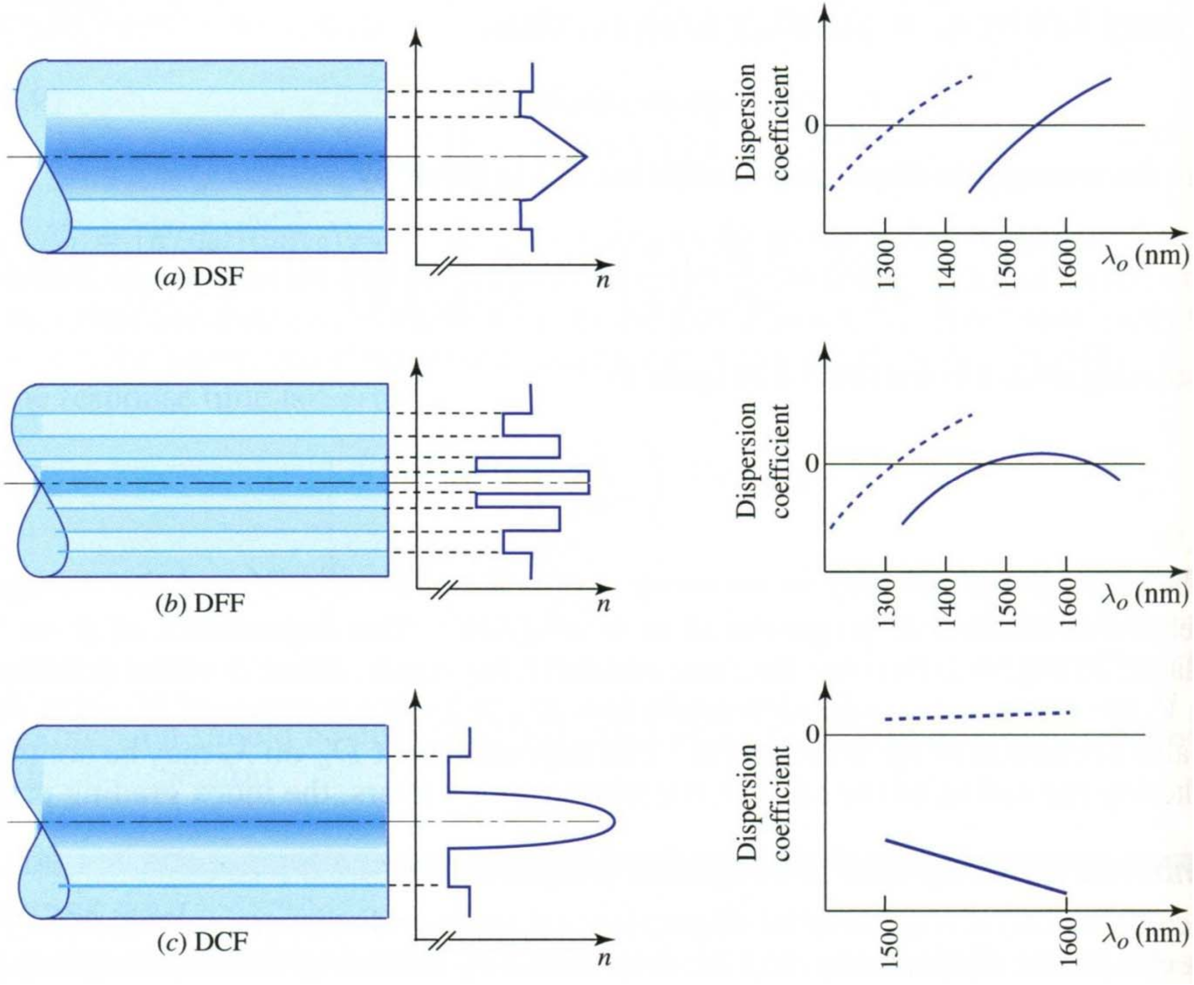
Material dispersion



$$\sigma_\tau = |D_\lambda|\sigma_\lambda L$$

$$D_\lambda = -\frac{\lambda_o}{c_o}\frac{d^2n}{d\lambda_o^2}$$

Waveguide dispersion



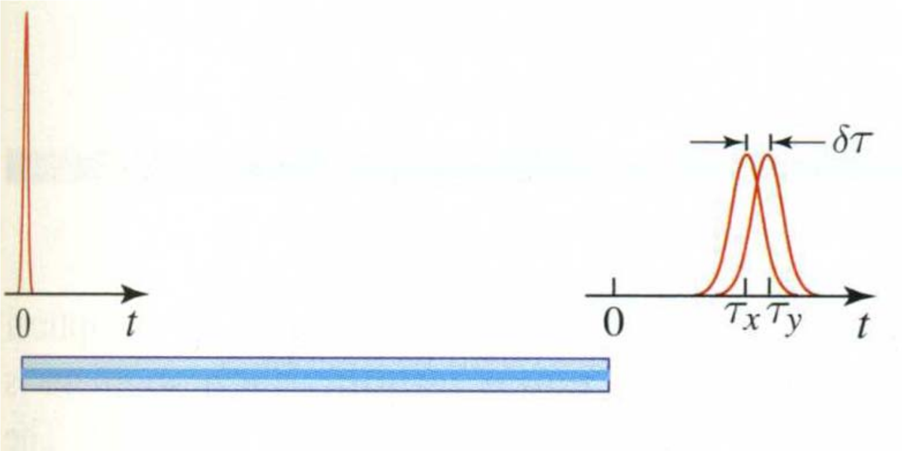
Dispersion-shifted fiber

Dispersion-flattened fiber

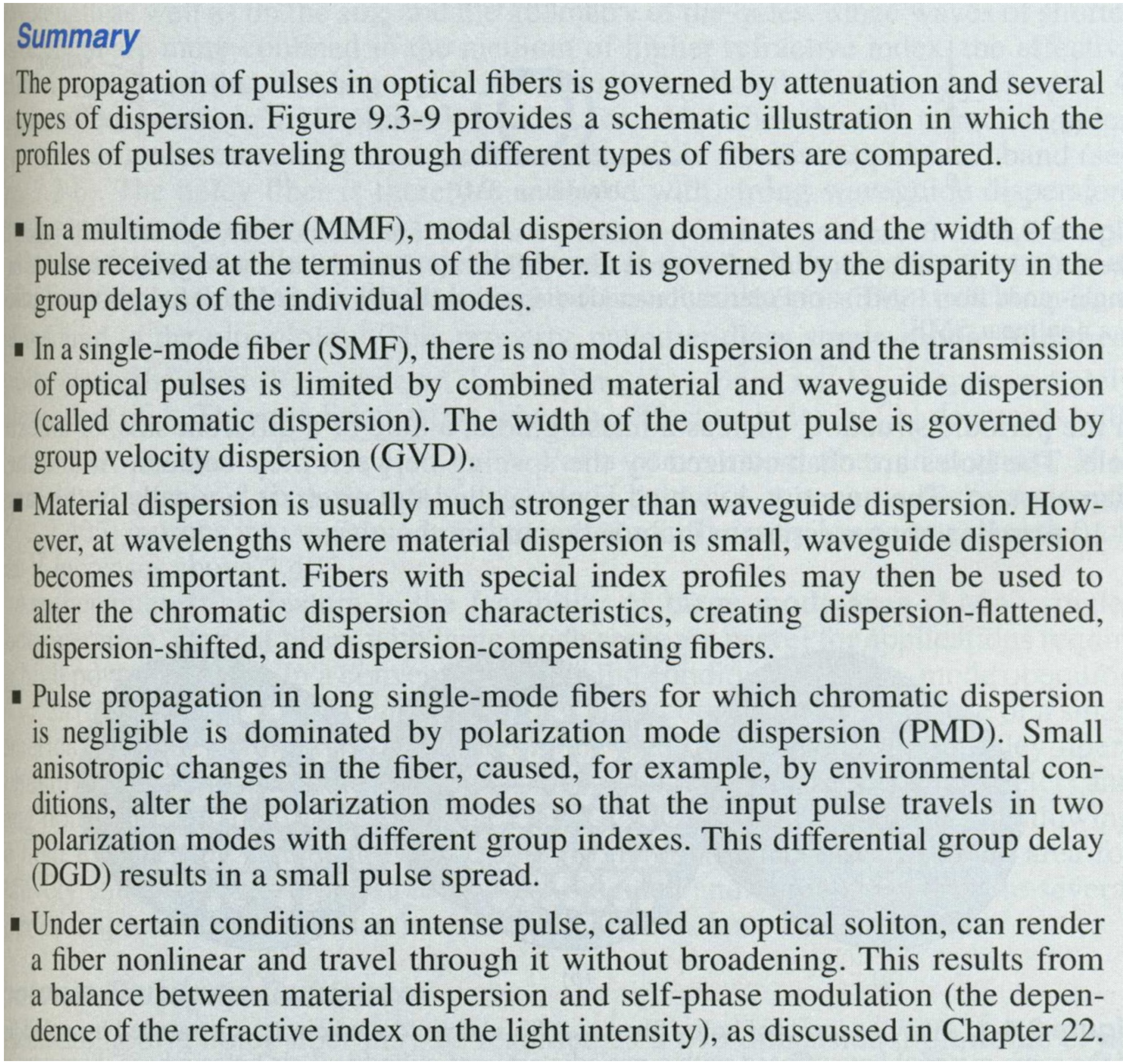
Dispersion-compensating fiber

Polarization Mode Dispersion (PMD)

$$\delta\tau = \Delta N L / c_o$$



$$\sigma_{\text{PMD}} = D_{\text{PMD}} \sqrt{L}$$



Summary

The propagation of pulses in optical fibers is governed by attenuation and several types of dispersion. Figure 9.3-9 provides a schematic illustration in which the profiles of pulses traveling through different types of fibers are compared.

- In a multimode fiber (MMF), modal dispersion dominates and the width of the pulse received at the terminus of the fiber. It is governed by the disparity in the group delays of the individual modes.
- In a single-mode fiber (SMF), there is no modal dispersion and the transmission of optical pulses is limited by combined material and waveguide dispersion (called chromatic dispersion). The width of the output pulse is governed by group velocity dispersion (GVD).
- Material dispersion is usually much stronger than waveguide dispersion. However, at wavelengths where material dispersion is small, waveguide dispersion becomes important. Fibers with special index profiles may then be used to alter the chromatic dispersion characteristics, creating dispersion-flattened, dispersion-shifted, and dispersion-compensating fibers.
- Pulse propagation in long single-mode fibers for which chromatic dispersion is negligible is dominated by polarization mode dispersion (PMD). Small anisotropic changes in the fiber, caused, for example, by environmental conditions, alter the polarization modes so that the input pulse travels in two polarization modes with different group indexes. This differential group delay (DGD) results in a small pulse spread.
- Under certain conditions an intense pulse, called an optical soliton, can render a fiber nonlinear and travel through it without broadening. This results from a balance between material dispersion and self-phase modulation (the dependence of the refractive index on the light intensity), as discussed in Chapter 22.