Kinematics of Particles

Announcements

- Recitation time is set to 8am every Monday.
- Participation credit will be given to students who uploads a good question or good answer to the Q&A bulletin board. Suggestions?
- TA s and I will be uploading questions/answers if necessary.
- During class, please try to ask in English.
- Upload your pictures.
- Download working model 2D, play around with it.

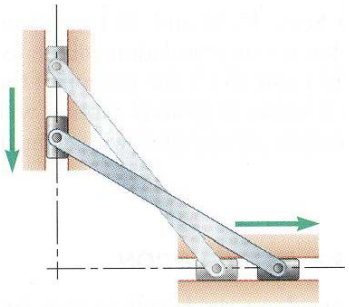
Kinematics of Particles

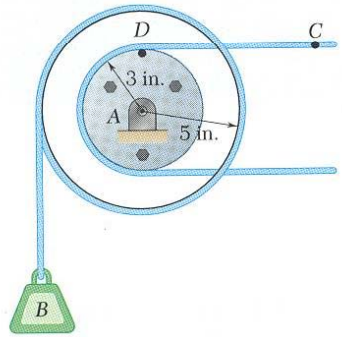
Introduction

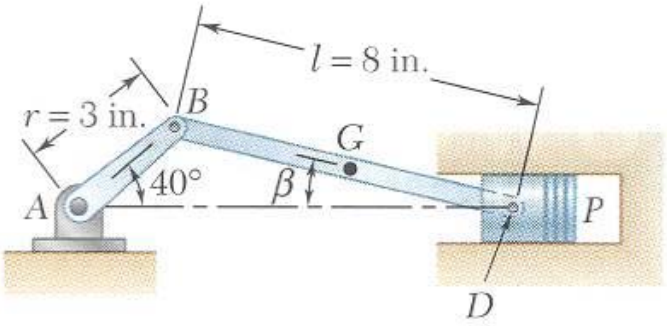
- What are the two parts of Dynamics?
  - *Kinematics*: study of the geometry of motion. Kinematics is used to relate displacement, velocity, acceleration, and time without reference to the cause of motion.
  - *Kinetics*: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.
- *Rectilinear* motion: position, velocity, and acceleration of a particle as it moves along a straight line.
- *Curvilinear* motion: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

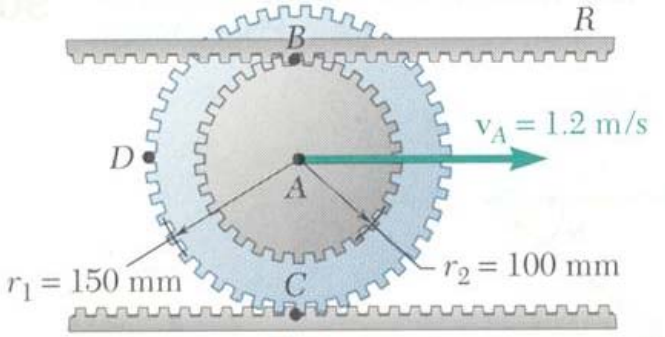
Tools and Mechanisms

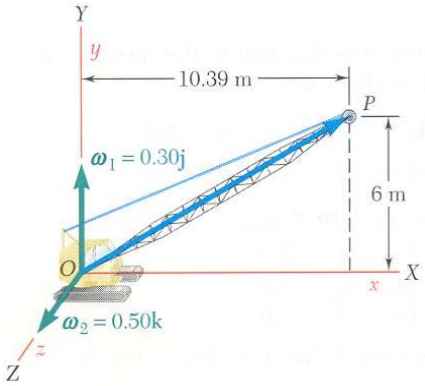
Wheel, Lever, Pulley, Hammer etc….

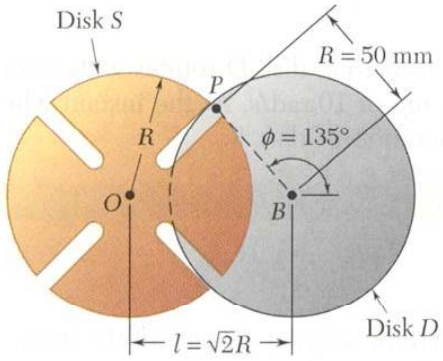












And movies…..

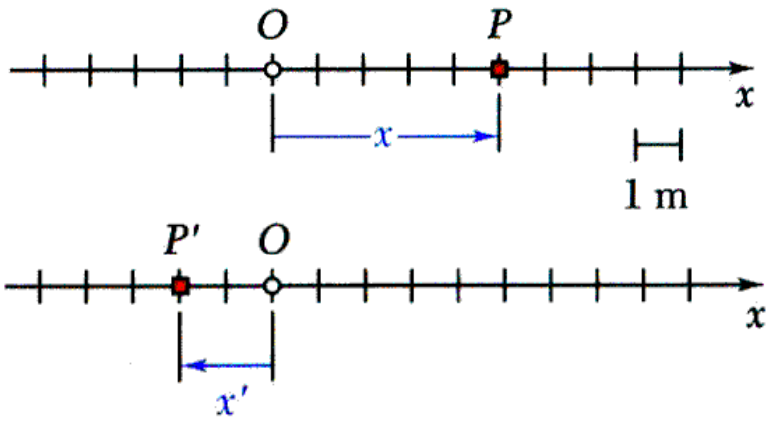
Kinematics of Particles

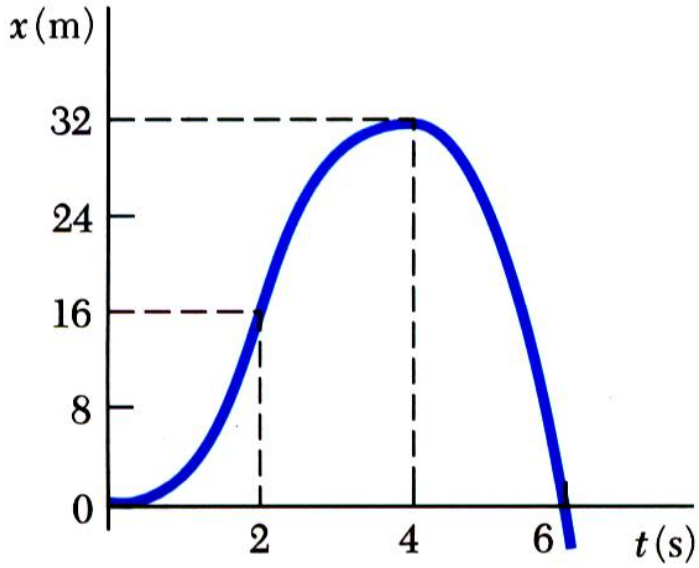
Why do we need dynamics?

- Why do we want to know kinematics? What are we really interested in?
  - Let's understand what we are analyzing before we analyze!
- It solves problems. What kind of problems?
- Limited motion, limited force characteristics.
- Translate one motion to another. Translate one force to another.
- There is always a Desired motion, Desired force vs motion and force that we can generate.
- Mechanisms enable us to generated the desired motion, force!!!
- Need to understand these mechanism's kinematics and kinetics!

Kinematics of Particles

Rectilinear Motion: Position, Velocity & Acceleration





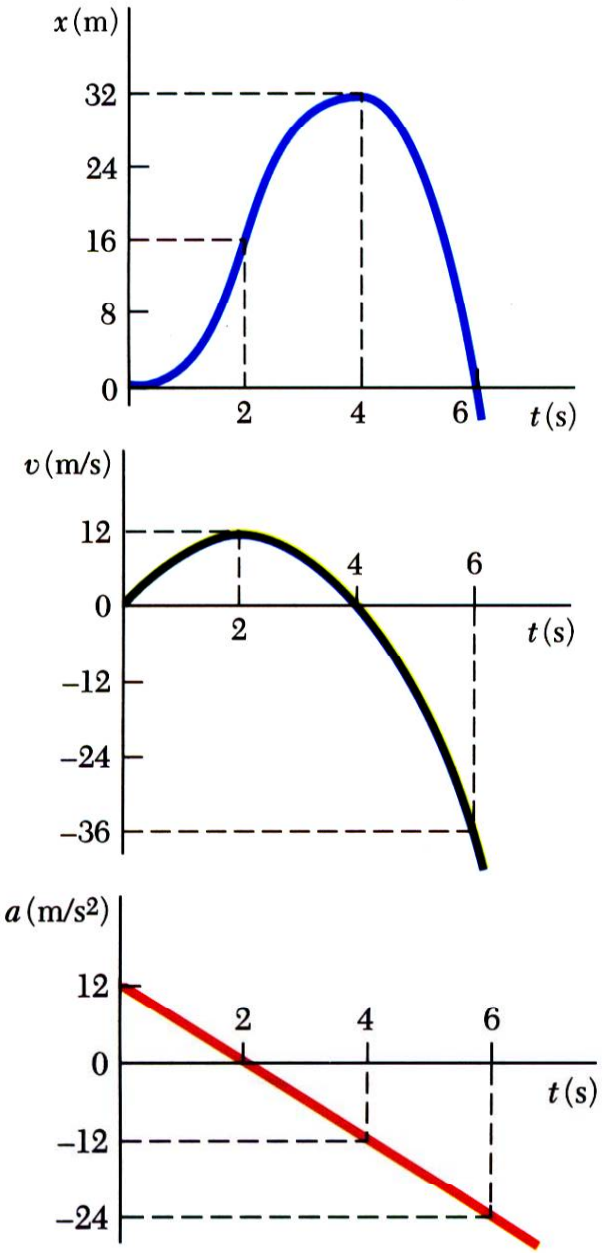
- Particle moving along a straight line is said to be in *rectilinear motion*.
- *Position coordinate* of a particle is defined by positive or negative distance of particle from a fixed origin on the line.
- The *motion* of a particle is known if the position coordinate for particle is known for every value of time t. Motion of the particle may be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$

or in the form of a graph x vs. t.

Kinematics of Particles

Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle with motion given by

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$$

- at $t = 0$, $x = 0$, $v = 0$, $a = 12$ m/s^2
- at $t = 2$ s, $x = 16$ m, $v = v_{max} = 12$ m/s, $a = 0$
- at $t = 4$ s, $x = x_{max} = 32$ m, $v = 0$, $a = -12$ m/s^2
- at $t = 6$ s, $x = 0$, $v = -36$ m/s, $a = 24$ m/s^2

Determination of the Motion of a Particle

- Recall, *motion* of a particle is known if position is known for all time t.
- Typically, conditions of motion are specified by the type of acceleration experienced by the particle. Determination of velocity and position requires two successive integrations.
- Three classes of motion may be defined for:
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = f(x)$
  - acceleration given as a function of *velocity*, $a = f(v)$

Determination of the Motion of a Particle

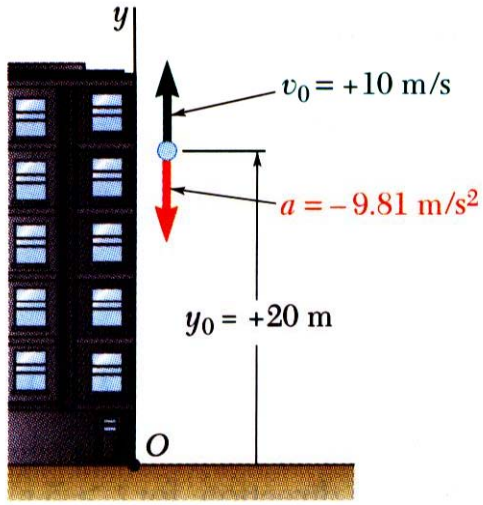
- Acceleration given as a function of *time*, $a = f(t)$:

- Acceleration given as a function of *position*, $a = f(x)$:

Determination of the Motion of a Particle

- Acceleration given as a function of velocity, $a = f(v)$:

Sample Problem 11.2



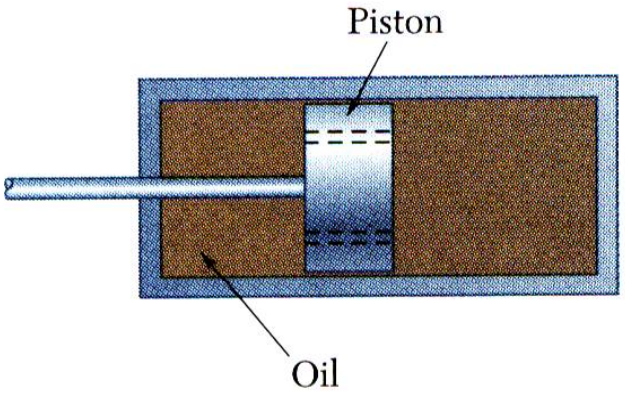
SOLUTION:

Ball tossed with 10 m/s vertical velocity from window 20 m above ground.

Determine:

- velocity and elevation above ground at time *t*,
- highest elevation reached by ball and corresponding time, and
- time when ball will hit the ground and corresponding velocity.

Sample Problem 11.3



$a = -kv$

SOLUTION:

Brake mechanism used to reduce gun recoil consists of piston attached to barrel moving in fixed cylinder filled with oil. As barrel recoils with initial velocity v_0, piston moves and oil is forced through orifices in piston, causing piston and cylinder to decelerate at rate proportional to their velocity.

Determine $v(t)$, $x(t)$, and $v(x)$.

Uniform Rectilinear Motion

For particle in uniform rectilinear motion, the acceleration is zero and the velocity is constant.

$$\frac{dx}{dt} = v = \text{constant}$$

$$\int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt$$

Uniformly Accelerated Rectilinear Motion

For particle in uniformly accelerated rectilinear motion, the acceleration of the particle is constant.

$$\frac{dv}{dt} = a = \text{constant} \qquad \int_{v_0}^{v} dv = a\int_{0}^{t} dt \qquad v - v_0 = at$$

$$v = v_0 + at$$

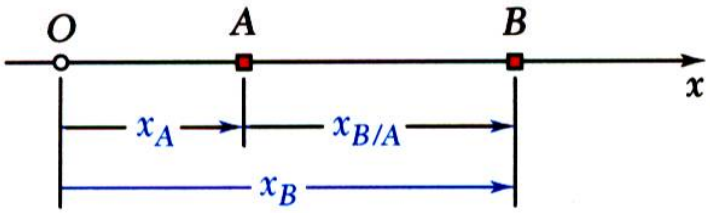
$$\frac{dx}{dt} = v_0 + at \qquad \int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at)dt \qquad x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v\frac{dv}{dx} = a = \text{constant} \qquad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \qquad \tfrac{1}{2}\left(v^2 - v_0^2\right) = a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Motion of Several Particles: Relative Motion



- For particles moving along the same line, time should be recorded from the same starting instant and displacements should be measured from the same origin in the same direction.

$$x_{B/A} = x_B - x_A = \text{relative position of } B \text{ with respect to } A$$

$$x_B = x_A + x_{B/A}$$

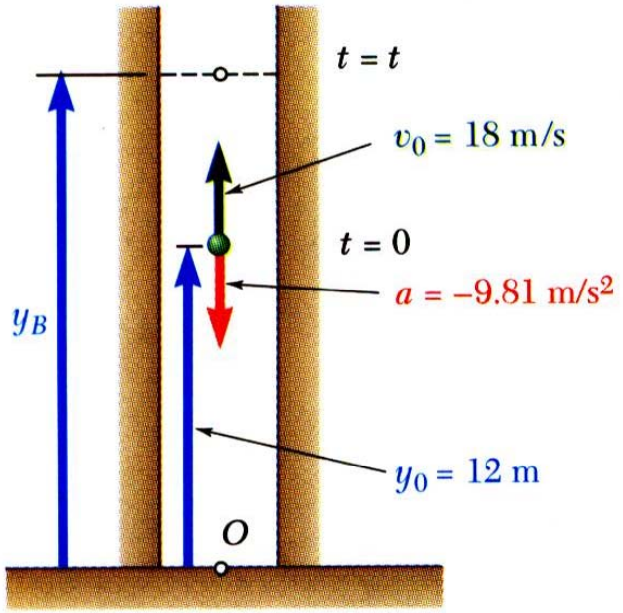
$$v_{B/A} = v_B - v_A = \text{relative velocity of } B \text{ with respect to } A$$

$$v_B = v_A + v_{B/A}$$

$$a_{B/A} = a_B - a_A = \text{relative acceleration of } B \text{ with respect to } A$$

$$a_B = a_A + a_{B/A}$$

Sample Problem 11.4



Ball thrown vertically from 12 m level in elevator shaft with initial velocity of 18 m/s. At same instant, open-platform elevator passes 5 m level moving upward at 2 m/s.

Determine *(a)* when and where ball hits elevator and *(b)* relative velocity of ball and elevator at contact.