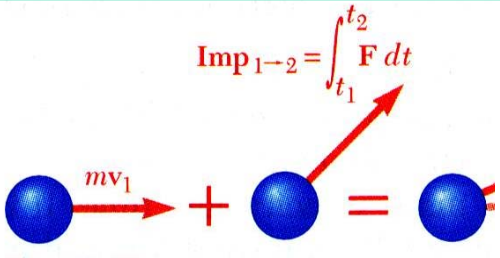
Kinetics of Particles: Energy and Momentum Methods

Preview of 13.10- 13.15

Read p.845 and 847 and summarize below. (At least 12 lines)

13.10 Principle of Impulse and Momentum

- What do we want to know?
- Definition of Impulse of a force?
- Relationship of the impulse with the motion?



- Units/Dimension for the impulse of a force are

$$\mathrm{N\cdot s} = \left(\mathrm{kg\cdot m/s^2}\right)\cdot \mathrm{s} = \mathrm{kg\cdot m/s}$$

- From Newton's second law,

$$\vec{F} = \boxed{\quad\quad} \qquad m\vec{v} = \text{linear momentum}$$

$$= \boxed{\quad\quad}$$

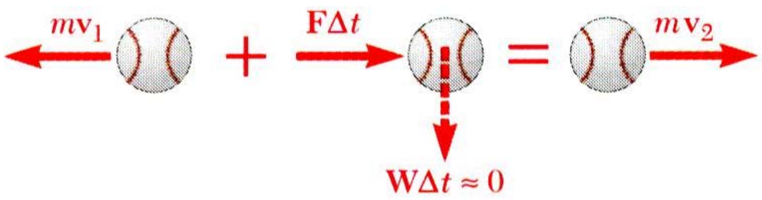
$$\boxed{\quad\quad} = \boxed{\quad\quad}$$

$$\int_{t_1}^{t_2} \vec{F} dt = \boxed{\quad\quad} = \text{impulse of the force } \vec{F}$$

- The final momentum of the particle → Initial momentum + Impulse of the force during the time interval.

$$\boxed{\quad\quad\quad\quad}$$

13.11 Impulsive Motion



Impulsive force:

- Force acting on a particle during a very short time interval that is large enough to cause a significant change in momentum.

$$m\vec{v}_1 + \sum \vec{F}\,\Delta t = m\vec{v}_2$$

- *Nonimpulsive forces* are forces for which $\vec{F}\Delta t$ is small and therefore, may be neglected.

- Examples?

Direct Newton's Method:

Work and Energy Method:

Impulse and Momentum Method:

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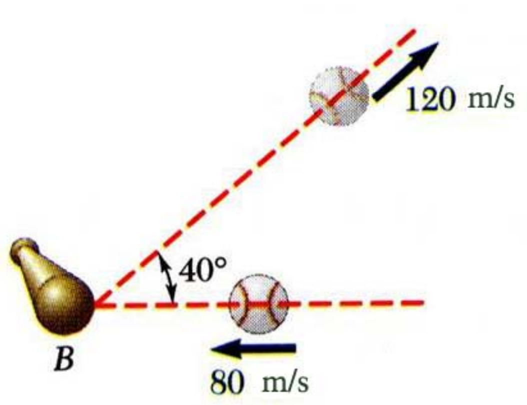
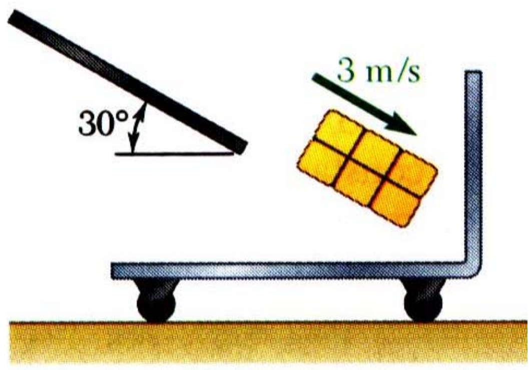
Sample Problem 13.11



A 0.5 kg baseball is pitched with a velocity of 80 m/s. After the ball is hit by the bat, it has a velocity of 120 m/s in the direction shown. If the bat and ball are in contact for 0.015 s, determine the average impulsive force exerted on the ball during the impact.

Sample Problem 13.11

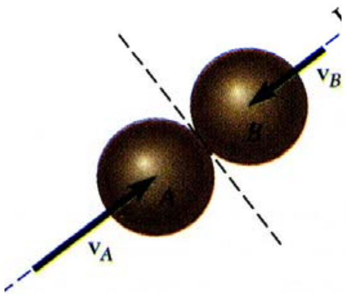
Sample Problem 13.12



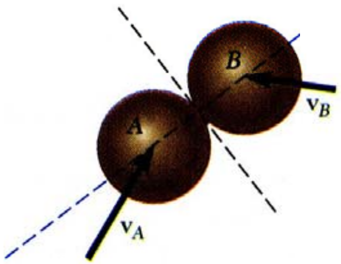
A 10 kg package drops from a chute into a 24 kg cart with a velocity of 3 m/s. Knowing that the cart is initially at rest and can roll freely, determine *(a)* the final velocity of the cart, *(b)* the impulse exerted by the cart on the package, and *(c)* the fraction of the initial energy lost in the impact.

Sample Problem 13.12

13.12 Impact



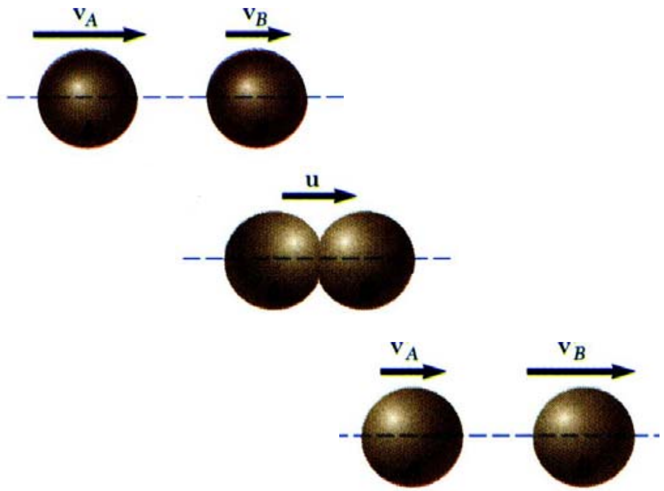
- ***Impact:*** Collision between two bodies which occurs during a small time interval and during which the bodies exert large forces on each other.
- ***Line of Impact:*** Common normal to the surfaces in contact during impact.
- ***Central Impact:*** Impact for which the mass centers of the two bodies lie on the line of impact; otherwise, it is an ***eccentric impact.***



- ***Direct Impact:*** Impact for which the velocities of the two bodies are directed along the line of impact.
- ***Oblique Impact:*** Impact for which one or both of the bodies move along a line other than the line of impact.

13. 13 Direct Central Impact

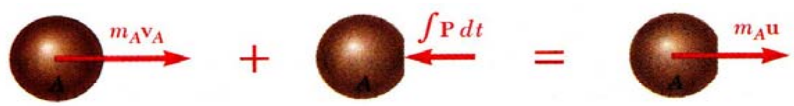
Bodies moving in the same straight line, $v_A > v_B$.



- Wish to determine the final velocities of the two bodies. The total momentum of the two body system is preserved,

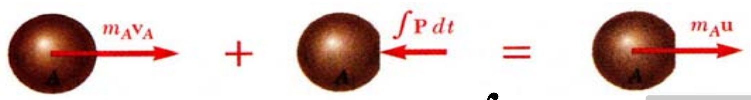
- Two unknowns!: A second relation between the final velocities is required.

- Upon impact the bodies undergo a ***period of deformation***, at the end of which, they are in contact and moving at a common velocity.

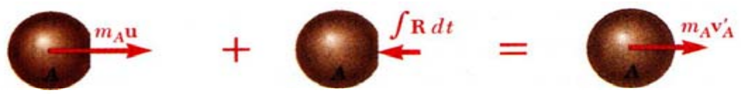


- A ***period of restitution*** follows during which the bodies either regain their original shape or remain permanently deformed.

13.13 Direct Central Impact



- Period of deformation: $\int P dt =$



- Period of restitution: $\int R dt =$

- Coefficient of restitution

$$e = \text{coefficient of restitution}$$

$$= \qquad = \qquad$$

$$0 \le e \le 1$$

- A similar analysis of particle *B* yields
  (Change of sign in the impulsive force)

$$e = \frac{v'_B - u}{u - v_B}$$

- Combine the relations → Second relation between the final velocities.

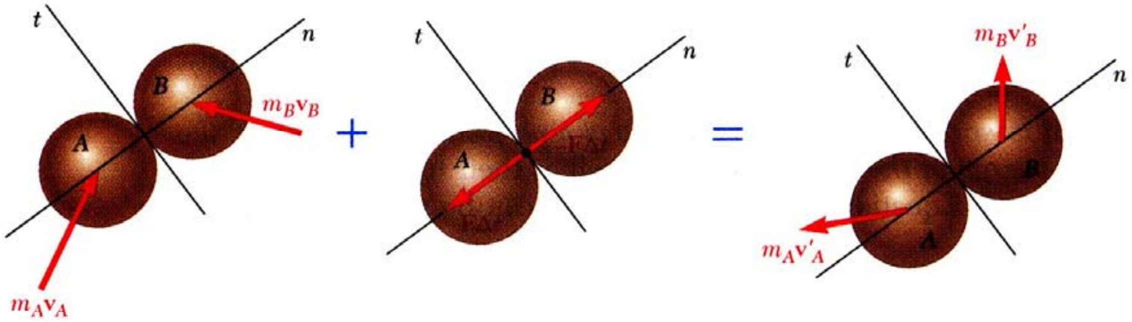
- *Perfectly plastic impact, e* = 0:

$$m_A v_A + m_B v_B =$$

- *Perfectly elastic impact, e* = 1:
  Total energy and total momentum conserved.

$$v'_B - v'_A =$$

13.14 Oblique Central Impact



- Final velocities are unknown in magnitude and direction. → Four equations are required.

- No tangential impulse component; tangential component of momentum for each particle is conserved.

$$(v_A)_t =$$

$$(v_B)_t =$$

- Normal component of total momentum of the two particles is conserved.

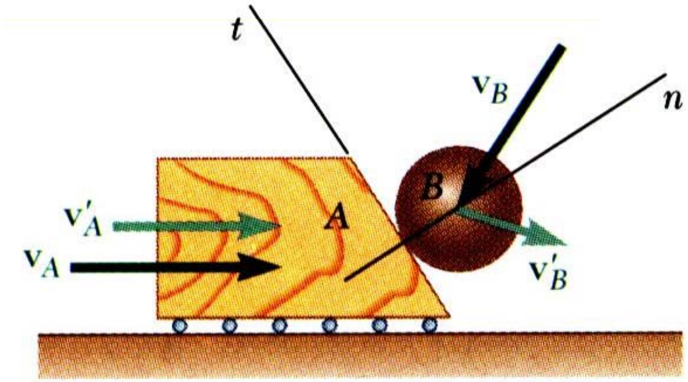
$$m_A(v_A)_n + m_B(v_B)_n =$$

- Normal components of relative velocities before and after impact are related by the

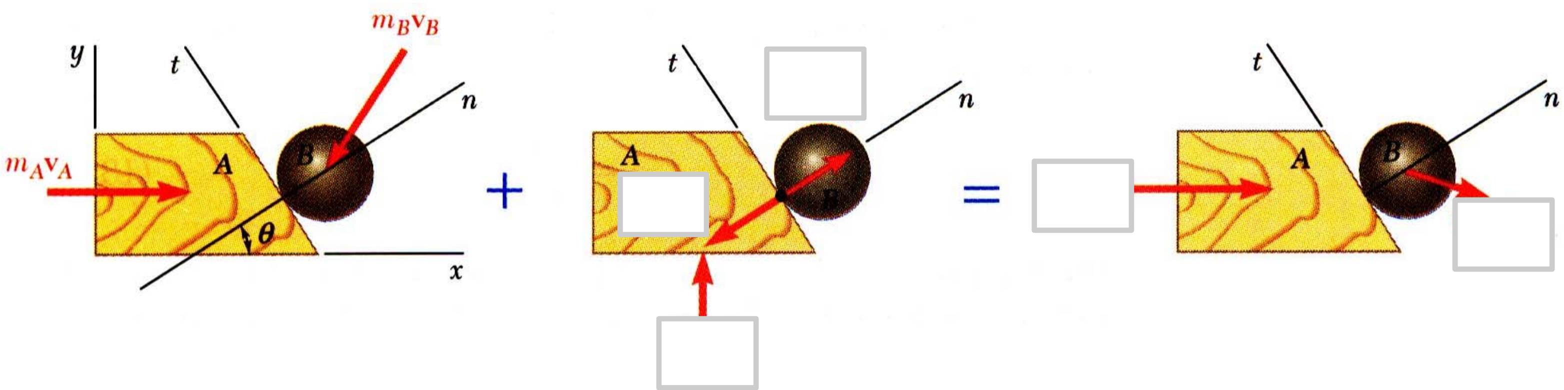
$$=$$

13.14 Oblique Central Impact

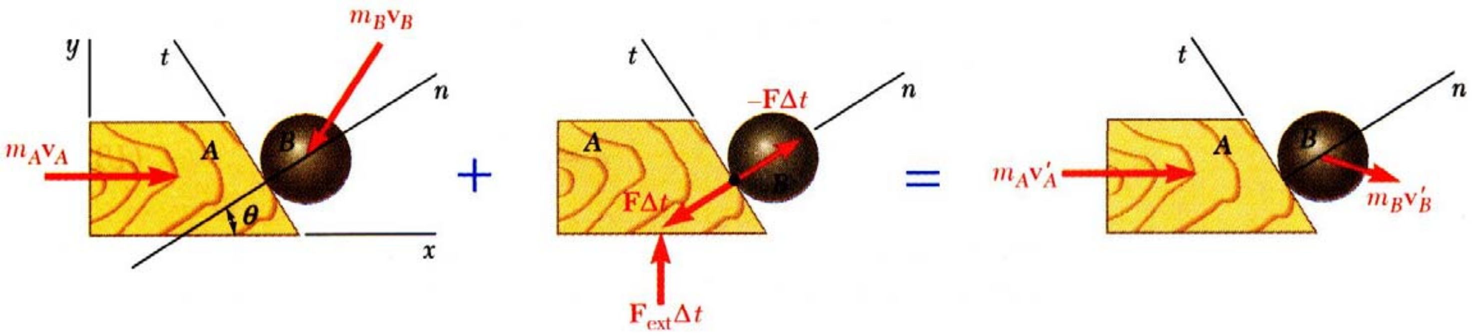
Block constrained to move along horizontal surface.



- Impulses?
  Internal forces $\vec{F}$ and $-\vec{F}$ along the n axis
  External force $\vec{F}_{ext}$ exerted by horizontal surface and directed along the vertical to the surface.
- Final velocity of ball unknown in direction and magnitude and unknown final block velocity magnitude. → [ ] equations required.



13.14 Oblique Central Impact

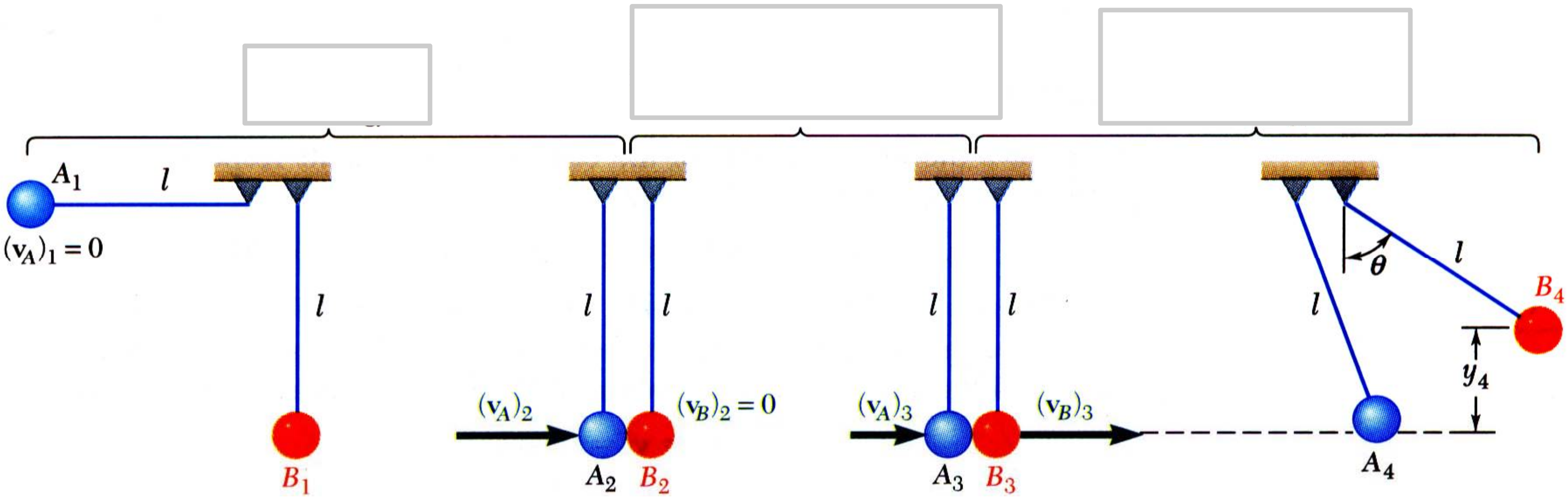


- Tangential momentum of ball is conserved.
- Total horizontal momentum of block and ball is conserved.
- Normal component of relative velocities of block and ball are related by coefficient of restitution.

13.15 Problems Involving Energy and Momentum

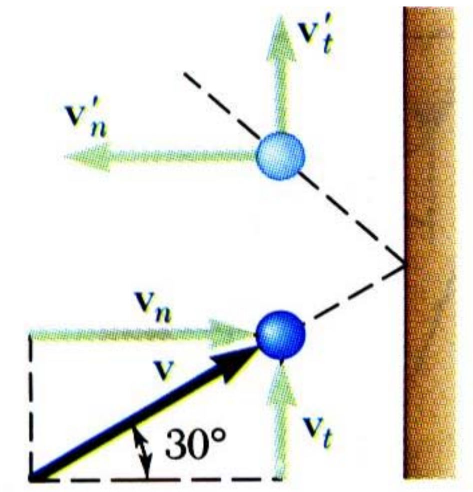
- Three methods for the analysis of kinetics problems:
  - Direct application of Newton's second law
  - Method of work and energy
  - Method of impulse and momentum

- Select the method best suited for the problem or part of a problem under consideration.



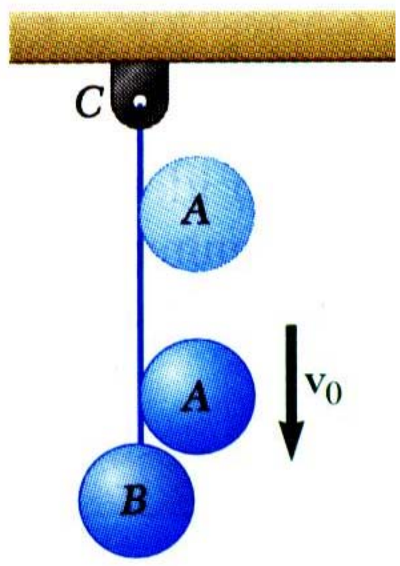
Kinetics of Particles: Energy and Momentum Methods

Sample Problem 13.14



A ball is thrown against a frictionless, vertical wall. Immediately before the ball strikes the wall, its velocity has a magnitude *v* and forms angle of 30° with the horizontal. Knowing that $e = 0.90$, determine the magnitude and direction of the velocity of the ball as it rebounds from the wall.

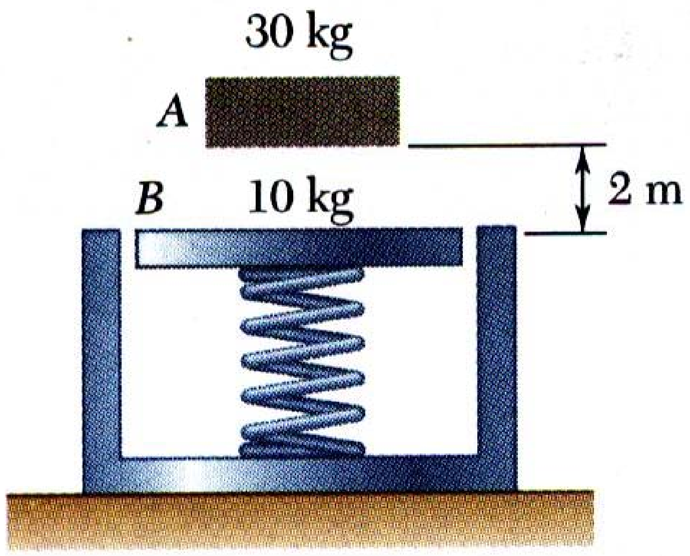
Sample Problem 13.16



Ball *B* is hanging from an inextensible cord. An identical ball *A* is released from rest when it is just touching the cord and acquires a velocity v_0 before striking ball *B*. Assuming perfectly elastic impact (e = 1) and no friction, determine the velocity of each ball immediately after impact.

Sample Problem 13.16

Sample Problem 13.17



A 30 kg block is dropped from a height of 2 m onto the the 10 kg pan of a spring scale. Assuming the impact to be perfectly plastic, determine the maximum deflection of the pan. The constant of the spring is k = 20 kN/m.

Sample Problem 13.17