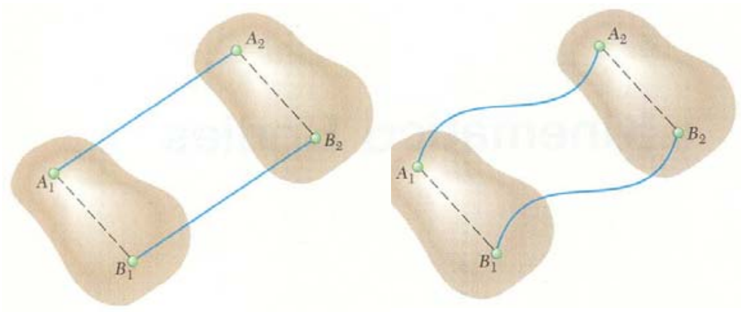
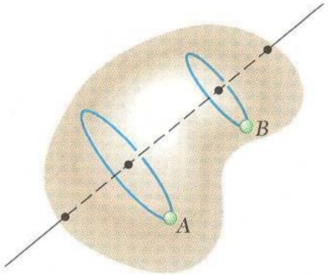
Kinematics of Rigid Bodies

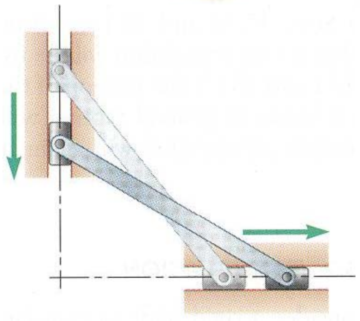
Preview of 15.1- 15.6

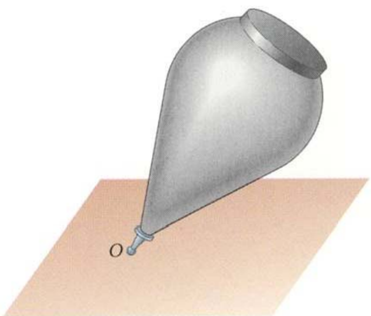
Read p.1011 and 1013 and summarize below. (At least 12 lines)

15.1 Introduction







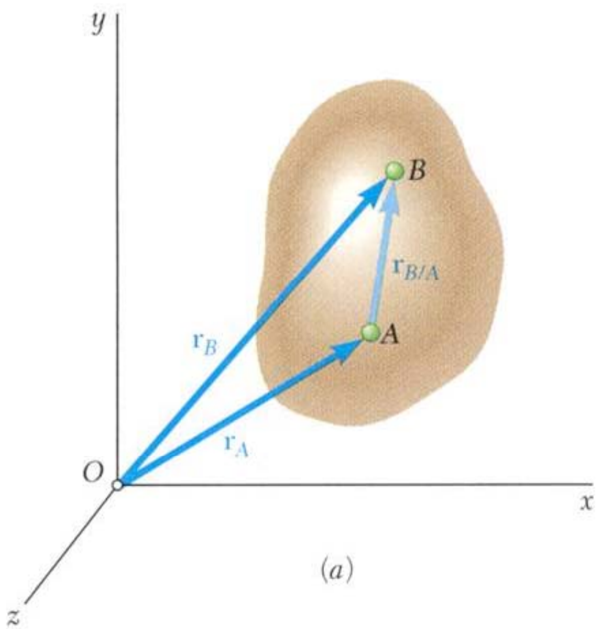


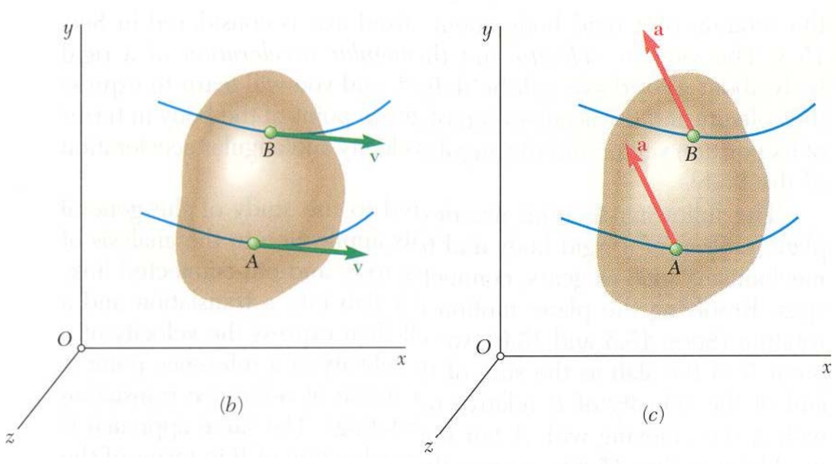
- Kinematics of particles vs. rigid bodies.

- Degrees of Freedom?

- Classification of rigid body motions:
  - translation:
    - rectilinear translation
    - curvilinear translation
  - rotation about a fixed axis
  - general plane motion
  - motion about a fixed point
  - general motion

15.2 Translation





- Consider rigid body in translation:
  - direction of any straight line inside the body is constant,
  - all particles forming the body move in parallel lines.

- For any two particles in the body,

- Differentiating with respect to time,

  $$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$$

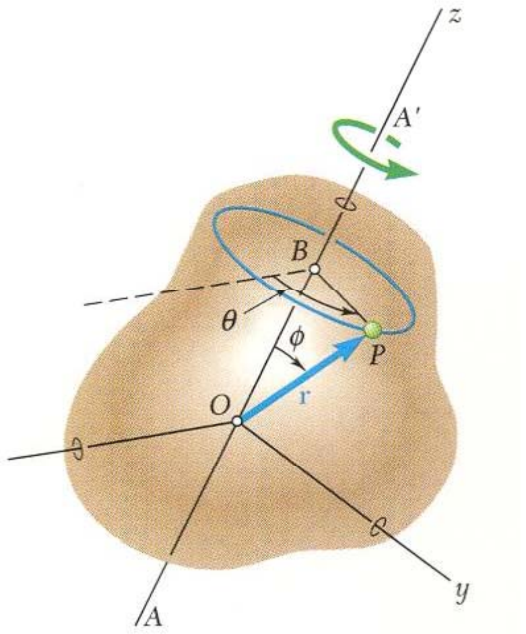
  All particles have the same velocity.

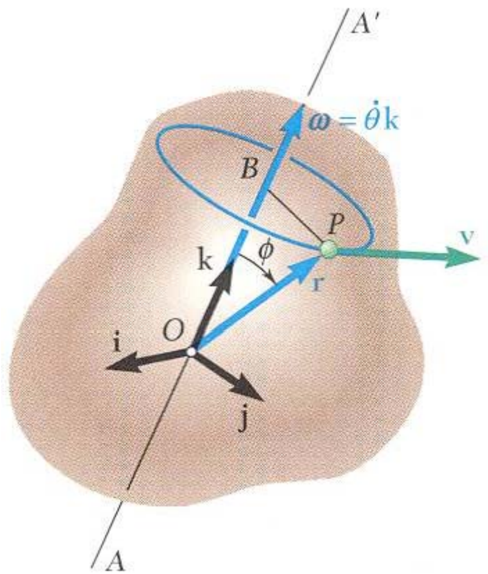
- Differentiating with respect to time again,

  $$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$$

  All particles have the same acceleration.

15.3 Rotation About a Fixed Axis. Velocity





- Consider rotation of rigid body about a fixed axis *AA'*

- Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle *P* is tangent to the path with magnitude $v = ds/dt$

$$\Delta s = (BP)\Delta\theta = \qquad$$

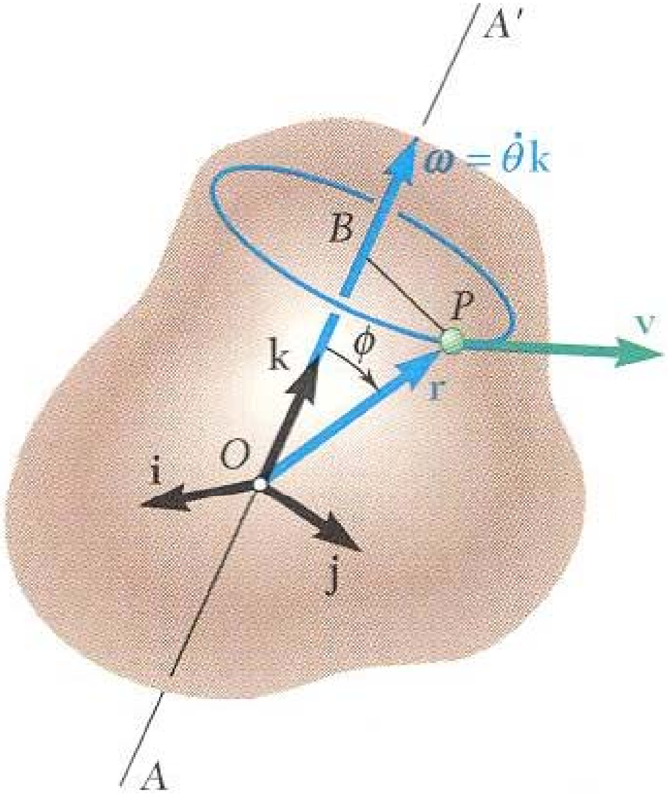
$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0} (r \sin\phi)\frac{\Delta\theta}{\Delta t} = \qquad$$

- The same result is obtained from

$$\vec{v} = \frac{d\vec{r}}{dt} = \qquad$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = angular\ velocity$$

15.3 Rotation About a Fixed Axis. Acceleration



- Differentiating to determine the acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = \boxed{}$$

$$= \boxed{}$$

$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \vec{v}$$

- $$\frac{d\vec{\omega}}{dt} = \vec{\alpha} = angular\ acceleration$$

$$= \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$

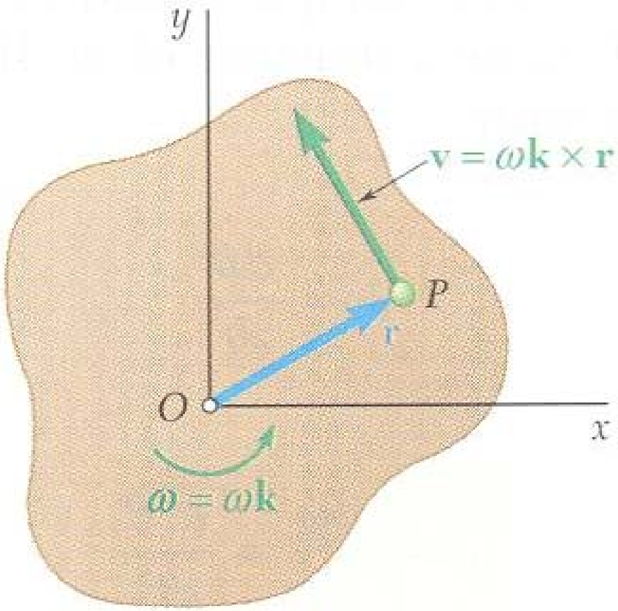
- Acceleration of P is combination of two vectors,

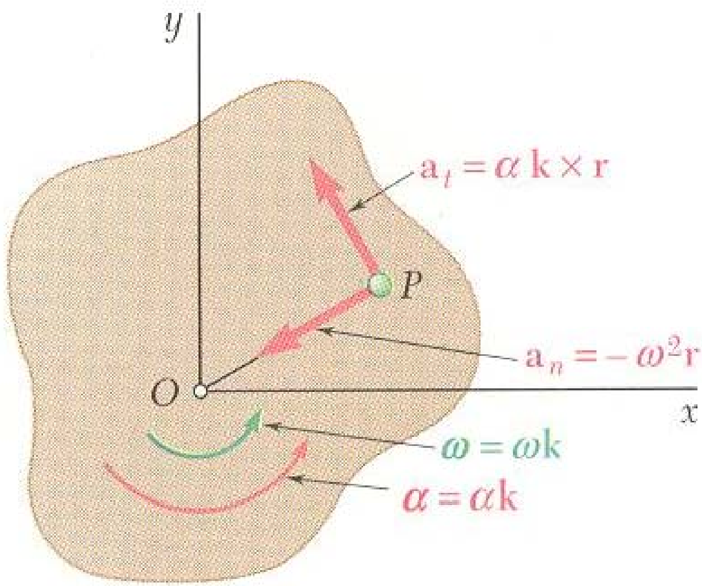
$$\vec{a} = \boxed{}$$

$\boxed{}$ = tangential acceleration component

$\boxed{}$ = radial acceleration component

15.3 Rotation About a Fixed Axis. Representative Slab





- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.

- Velocity of any point P of the slab,

  $\vec{v} = \square = \square$

  $v = \square$

- Acceleration of any point P of the slab,

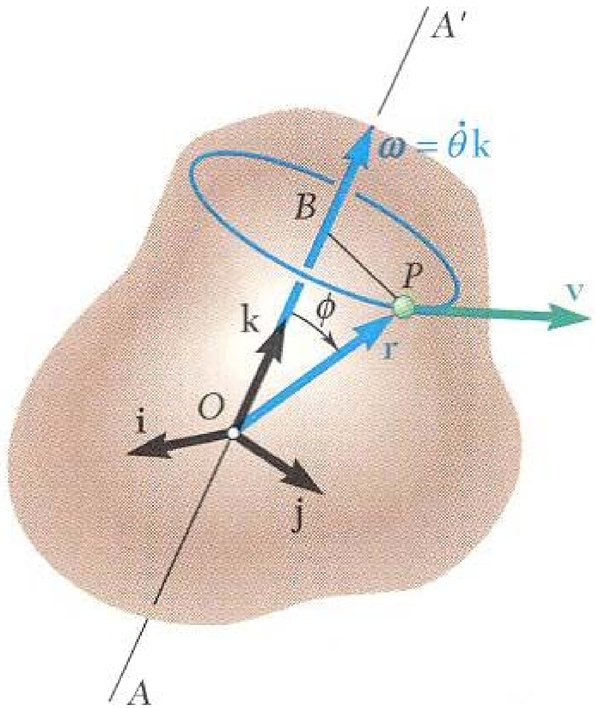
  $\vec{a} = \square$

  $= \square$

- Resolving the acceleration into tangential and normal components,

$$\vec{a}_t = \alpha\vec{k} \times \vec{r} \qquad a_t = r\alpha$$
$$\vec{a}_n = -\omega^2\vec{r} \qquad a_n = r\omega^2$$

15.4 Equations Defining the Rotation of a Rigid Body About a Fixed Axis



- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

- Recall $\omega = \boxed{}$ or $dt = \dfrac{d\theta}{\omega}$

$$\alpha = \boxed{} = \frac{d^2\theta}{dt^2} = \boxed{}$$

- *Uniform Rotation*, $\alpha = 0$:

$$\theta = \boxed{}$$

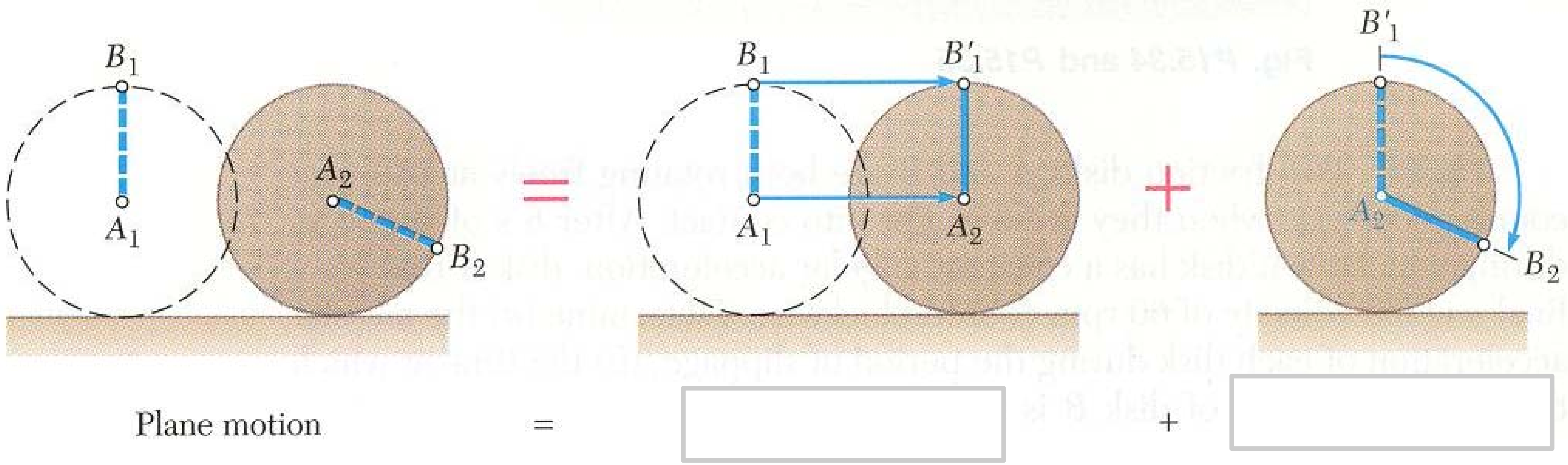
- *Uniformly Accelerated Rotation*, α = constant:

$$\omega = \boxed{}$$

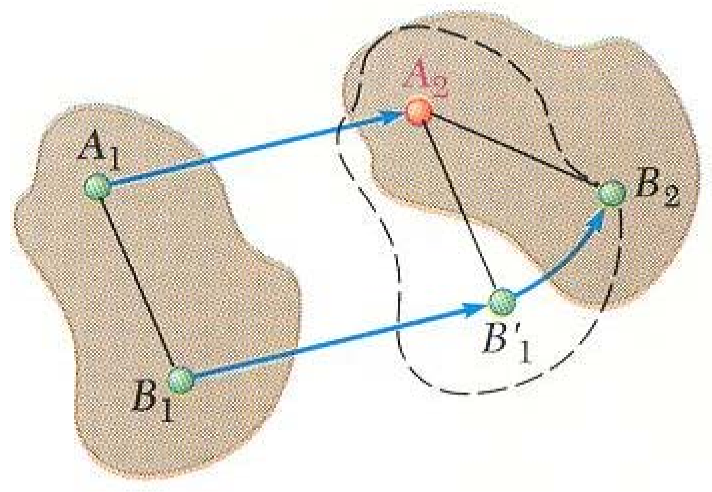
$$\theta = \boxed{}$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

15.5 General Plane Motion

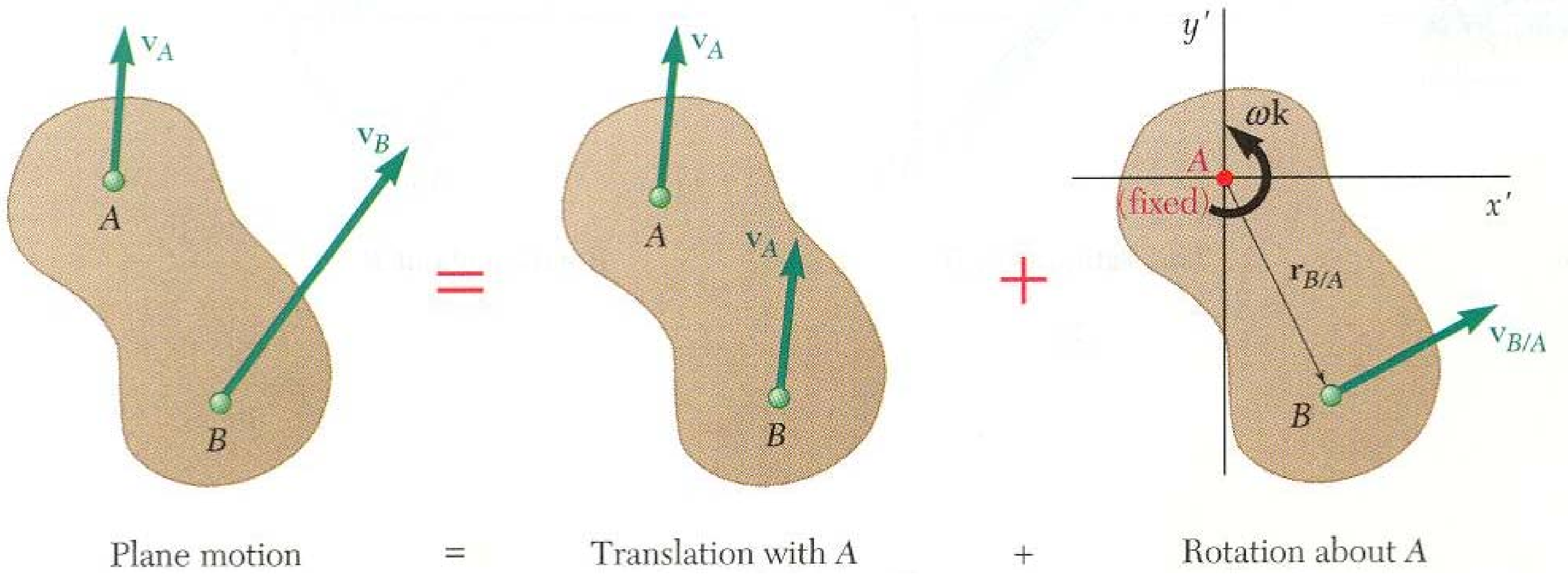


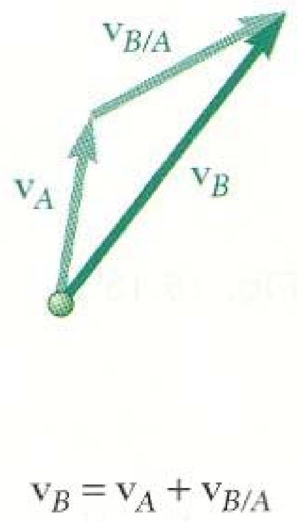
- General plane motion can be considered as the *sum* of a translation and rotation.



- Displacement of particles A and B to A_2 and B_2 can be divided into two parts:
  - translation to A_2 and B_1'
  - rotation of B_1' about A_2 to B_2

15.6 Absolute and Relative Velocity in Plane Motion





- Any plane motion can be replaced by

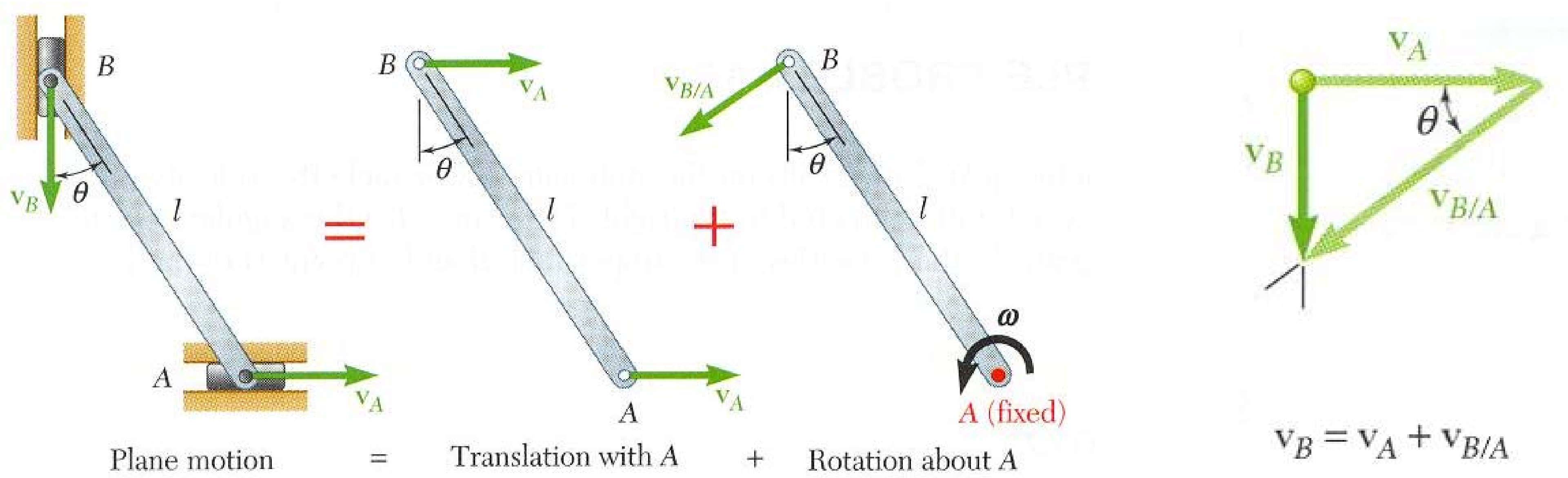
  a translation of an arbitrary reference point A
  + a simultaneous rotation about A.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \boxed{} \qquad v_{B/A} = r\omega$$

$$\vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$

15.6 Absolute and Relative Velocity in Plane Motion



- Assuming that the velocity v_A of end A is known, wish to determine the velocity v_B of end B and the angular velocity ω in terms of v_A, l, and θ.
- The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

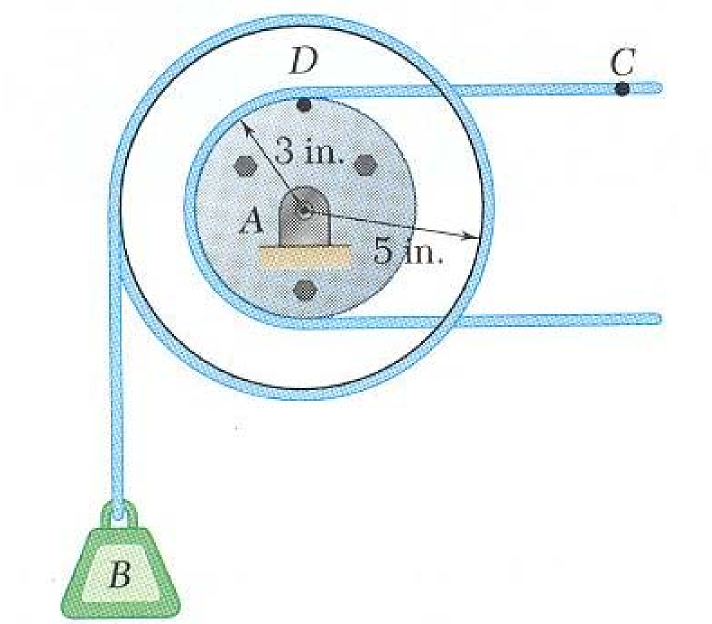
$$\frac{v_B}{v_A} = \boxed{}$$

$$v_B = v_A \tan\theta$$

$$\frac{v_A}{v_{B/A}} = \boxed{} = \boxed{}$$

$$\omega = \boxed{}$$

Sample Problem 5.1

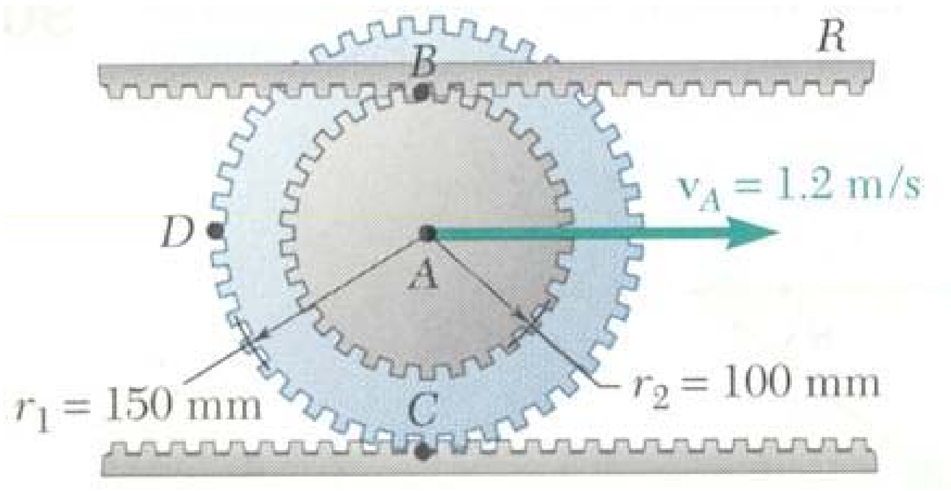


Cable *C* has a constant acceleration of 9 in/s^2 and an initial velocity of 12 in/s, both directed to the right.

Determine *(a)* the number of revolutions of the pulley in 2 s, *(b)* the velocity and change in position of the load *B* after 2 s, and *(c)* the acceleration of the point *D* on the rim of the inner pulley at $t = 0$.

Sample Problem 5.1

Sample Problem 15.2

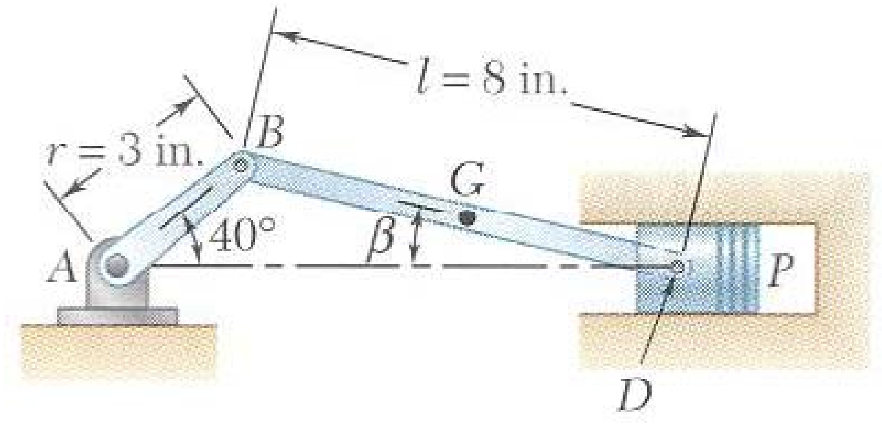


The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s.

Determine *(a)* the angular velocity of the gear, and *(b)* the velocities of the upper rack *R* and point *D* of the gear.

Sample Problem 15.2

Sample Problem 15.3



The crank *AB* has a constant clockwise angular velocity of 2000 rpm.

For the crank position indicated, determine *(a)* the angular velocity of the connecting rod *BD*, and *(b)* the velocity of the piston *P*.

Sample Problem 15.3