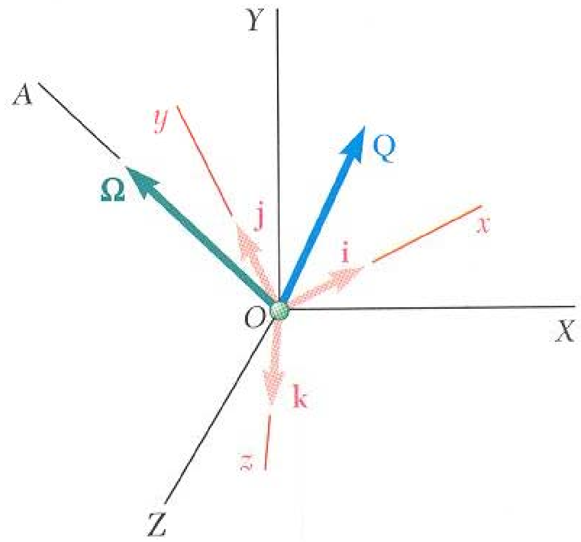
Preview of 15.10- 15.11

15.10 Rate of Change With Respect to a Rotating Frame



- Frame *OXYZ* is fixed.
- Frame *Oxyz* rotates about fixed axis *OA* with angular velocity
- Vector function varies in direction and magnitude.

- With respect to the rotating *Oxyz* frame,

$$\vec{Q} = Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k}$$

$$\left(\dot{\vec{Q}}\right)_{Oxyz} = Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k}$$

- With respect to the fixed *OXYZ* frame,

$$\left(\dot{\vec{Q}}\right)_{OXYZ} = Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k} + Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k}$$

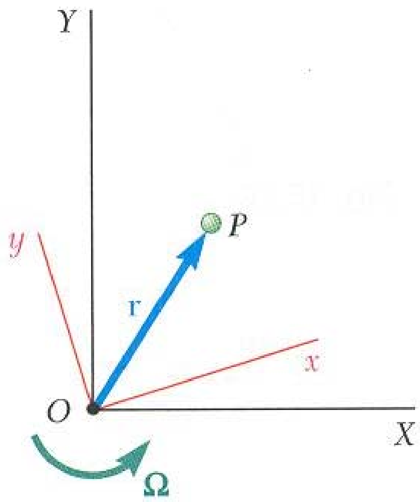
- $\dot{Q}_x\vec{i} + \dot{Q}_y\vec{j} + \dot{Q}_z\vec{k} = \left(\dot{\vec{Q}}\right)_{Oxyz}$ = rate of change with respect to rotating frame.

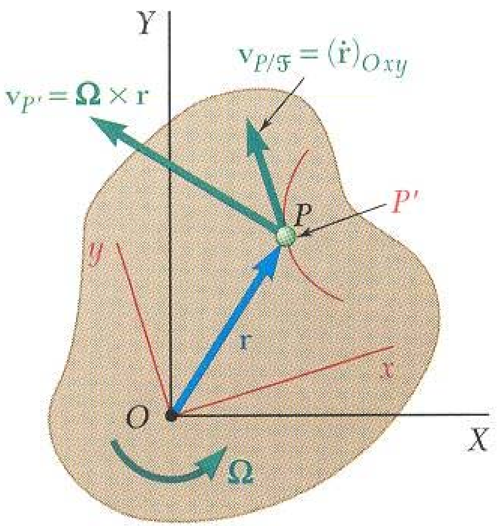
- If $\vec{Q}$ were fixed within *Oxyz* then $\left(\dot{\vec{Q}}\right)_{OXYZ}$ is equivalent to velocity of a point in a rigid body attached to *Oxyz* and $Q_x\dot{\vec{i}} + Q_y\dot{\vec{j}} + Q_z\dot{\vec{k}} =$

- With respect to the fixed *OXYZ* frame,

$$\left(\dot{\vec{Q}}\right)_{OXYZ} = \left(\dot{\vec{Q}}\right)_{Oxyz} +$$

15.11 Coriolis Acceleration





- Frame *OXY* is fixed and frame *Oxy* rotates with angular velocity $\vec{\Omega}$.
- Position vector $\vec{r}_P$ for the particle *P* is the same in both frames but the rate of change depends on the choice of frame.
- The absolute velocity of the particle *P* is

  $$\vec{v}_P = \left(\dot{\vec{r}}\right)_{OXY} = \quad$$

- Imagine a rigid slab attached to the rotating frame *Oxy* or $\mathfrak{F}$ for short. Let *P'* be a point on the slab which corresponds instantaneously to position of particle *P*.

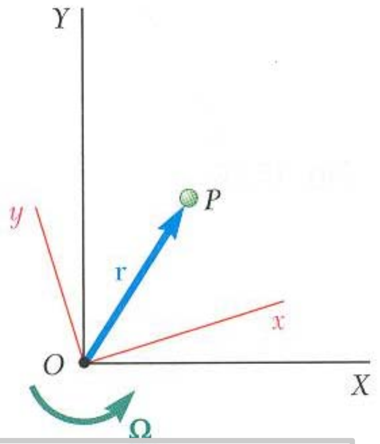
  $\vec{v}_{P/\mathfrak{F}} = \left(\dot{\vec{r}}\right)_{Oxy}$ = velocity of *P* along its path on the slab

  $\vec{v}_{P'}$ = absolute velocity of point *P'* on the slab

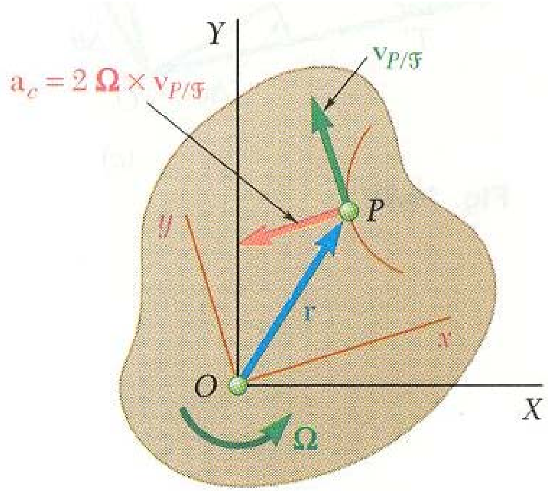
- Absolute velocity for the particle P may be written as

  $$\vec{v}_P = \quad$$

15.11 Coriolis Acceleration



$$\vec{v}_P = \vec{\Omega} \times \vec{r} + (\dot{r})_{Oxy}$$
$$= \vec{v}_{P'} + \vec{v}_{P/\mathfrak{F}}$$



- Absolute acceleration for the particle P is

$$\vec{a}_P = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\dot{\vec{r}})_{OXY} + \frac{d}{dt}\left[(\dot{\vec{r}})_{Oxy}\right]$$

but, $(\dot{\vec{r}})_{OXY} = \vec{\Omega} \times \vec{r} + (\dot{\vec{r}})_{Oxy}$

$$\frac{d}{dt}\left[(\dot{\vec{r}})_{Oxy}\right] = (\ddot{\vec{r}})_{Oxy} + \vec{\Omega} \times (\dot{\vec{r}})_{Oxy}$$

$$\vec{a}_P = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right) + 2\vec{\Omega} \times (\dot{\vec{r}})_{Oxy} + (\ddot{\vec{r}})_{Oxy}$$

- Utilizing the conceptual point P' on the slab,

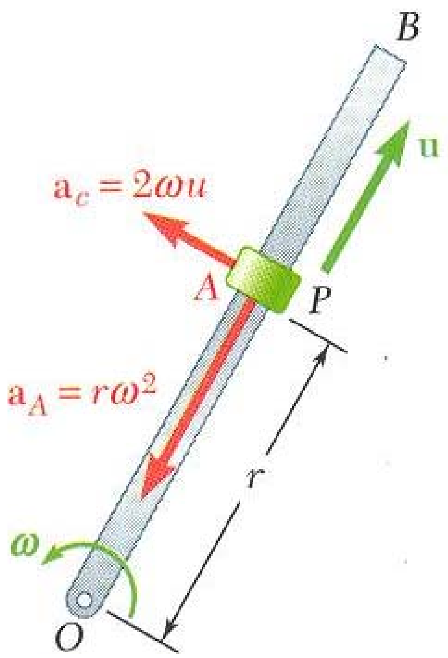
$$\vec{a}_{P'} = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right)$$
$$\vec{a}_{P/\mathfrak{F}} = (\ddot{\vec{r}})_{Oxy}$$

- Absolute acceleration for the particle P becomes

$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/\mathfrak{F}} + 2\vec{\Omega} \times (\dot{\vec{r}})_{Oxy}$$
$$= \vec{a}_{P'} + \vec{a}_{P/\mathfrak{F}} + \vec{a}_c$$

$$\vec{a}_c = 2\vec{\Omega} \times (\dot{\vec{r}})_{Oxy} = 2\vec{\Omega} \times \vec{v}_{P/\mathfrak{F}} = \text{Coriolis acceleration}$$

15.11 Coriolis Acceleration



- Consider a collar P which is made to slide at constant relative velocity u along rod OB. The rod is rotating at a constant angular velocity ω. The point A on the rod corresponds to the instantaneous position of P.

- Absolute acceleration of the collar is

$$\vec{a}_P = \vec{a}_A + \vec{a}_{P/\mathfrak{F}} + \vec{a}_c$$

where

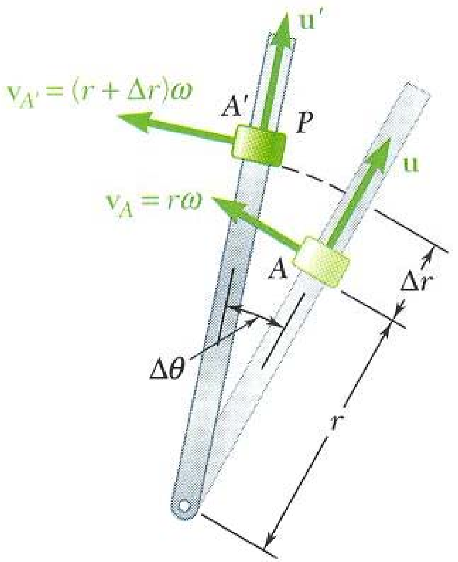
$$\vec{a}_A = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right) \qquad a_A = r\omega^2$$

$$\vec{a}_{P/\mathfrak{F}} = \left(\ddot{\vec{r}}\right)_{Oxy} = 0$$

$$\vec{a}_c = 2\vec{\Omega} \times \vec{v}_{P/\mathfrak{F}} \qquad a_c = 2\omega u$$

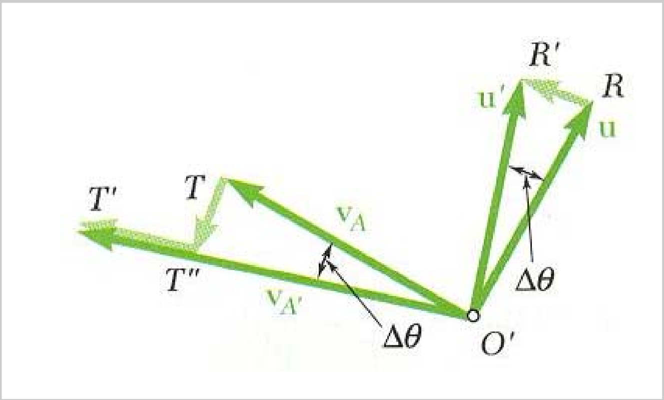
- The absolute acceleration consists of the radial and tangential vectors shown.

15.11 Coriolis Acceleration



at t, $\vec{v} = \vec{v}_A + \vec{u}$

at $t + \Delta t$, $\vec{v}' = \vec{v}_{A'} + \vec{u}'$



- Change in velocity over Δt is represented by the sum of three vectors

$$\Delta\vec{v} = \overrightarrow{RR'} + \overrightarrow{TT''} + \overrightarrow{T''T'}$$

- $\overrightarrow{TT''}$ is due to change in direction of the velocity of point A on the rod,

$$\lim_{\Delta t \to 0} \frac{TT''}{\Delta t} = \lim_{\Delta t \to 0} v_A \frac{\Delta\theta}{\Delta t} = r\omega\omega = r\omega^2 = a_A$$

$$\text{recall,} \quad \vec{a}_A = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \qquad a_A = r\omega^2$$

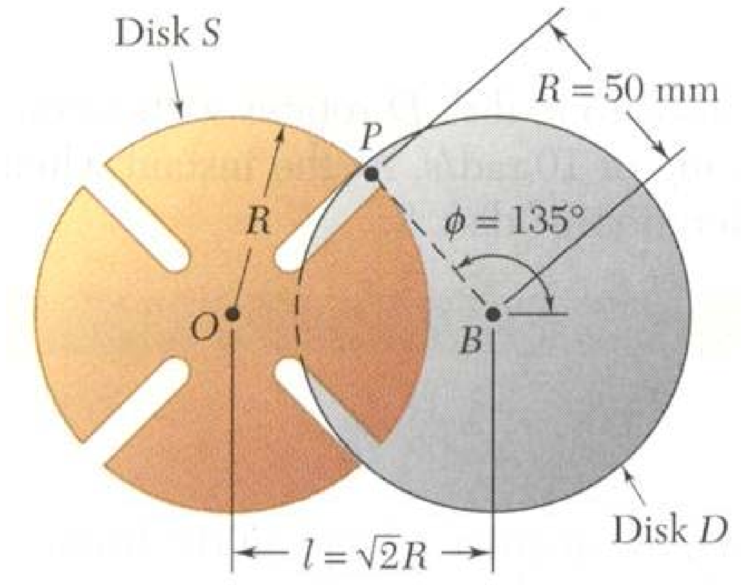
- $\overrightarrow{RR'}$ and $\overrightarrow{T''T'}$ result from combined effects of relative motion of P and rotation of the rod

$$\lim_{\Delta t \to 0}\left(\frac{RR'}{\Delta t} + \frac{T''T'}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(u\frac{\Delta\theta}{\Delta t} + \omega\frac{\Delta r}{\Delta t}\right)$$

$$= u\omega + \omega u = 2\omega u$$

$$\text{recall,} \quad \vec{a}_c = 2\vec{\Omega} \times \vec{v}_{P/\mathfrak{F}} \qquad a_c = 2\omega u$$

Sample Problem 15.9

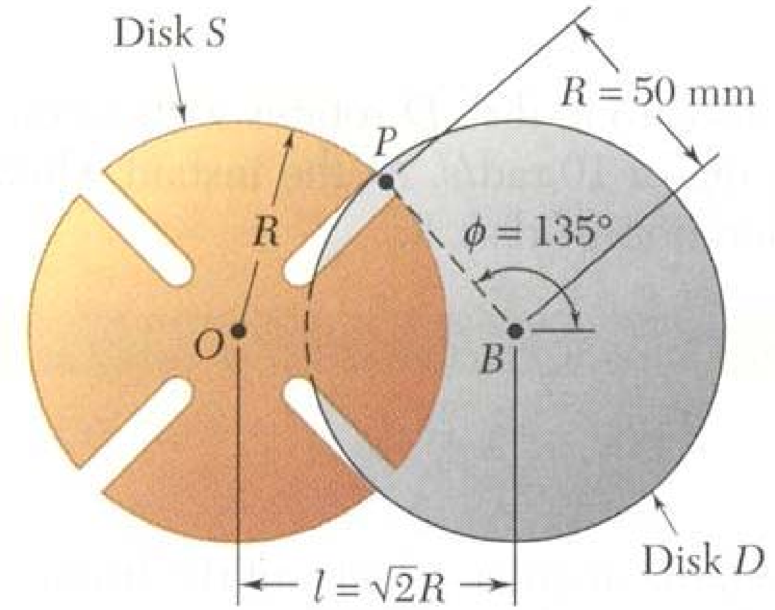


Disk D of the Geneva mechanism rotates with constant counterclockwise angular velocity $\omega_D = 10$ rad/s.

At the instant when $\phi = 150^o$, determine (*a*) the angular velocity of disk *S*, and (*b*) the velocity of pin *P* relative to disk *S*.

Sample Problem 15.9

Sample Problem 15.10



In the Geneva mechanism, disk *D* rotates with a constant counter-clockwise angular velocity of 10 rad/s. At the instant when $\varphi = 150^{o}$, determine angular acceleration of disk *S*.

Sample Problem 15.10