Systems of Particles

Preview of 14.1- 14.6

14.1 Introduction

- System of particles: How would you represent system of particles?

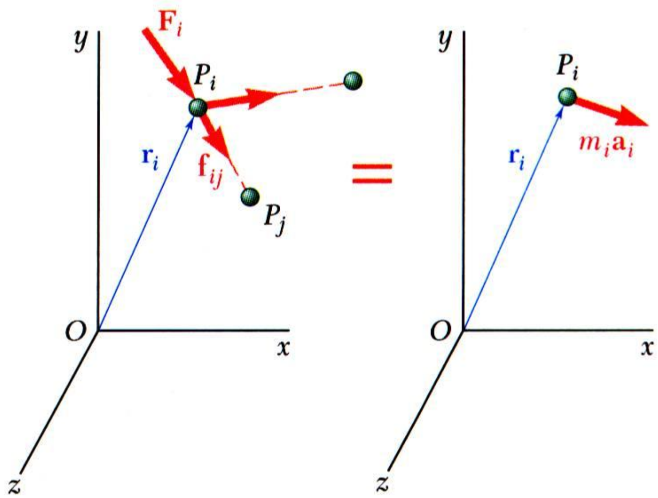
- The ***effective force*** of a particle = defined as the product of it mass and acceleration.

- It will be shown that the *system of external forces* acting on a system of particles is *equipollent* with the *system of effective forces* of the system.

- **The *mass center*** of a system of particles will be defined and its motion described.

- Application of the *work-energy principle* and the *impulse-momentum principle* to a system of particles will be described. Result obtained are also applicable to a system of rigidly connected particles, i.e., a *rigid body*.

14.2 Application of Newton’s Laws. Effective Forces

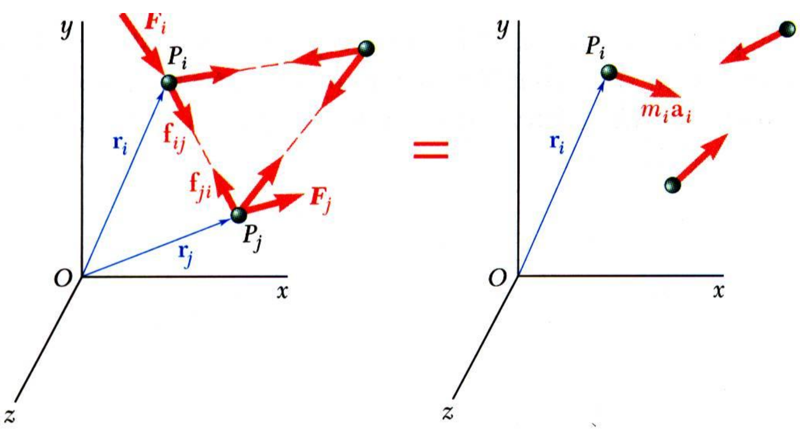


- Newton’s second law for each particle P_i in a system of n particles,

$$\vec{r}_i \times \vec{F}_i + \sum_{j=1}^{n}\left(\vec{r}_i \times \vec{f}_{ij}\right) = \vec{r}_i \times m_i\vec{a}_i$$

$\vec{F}_i$ = external force $\quad\quad$ $\vec{f}_{ij}$ = internal forces

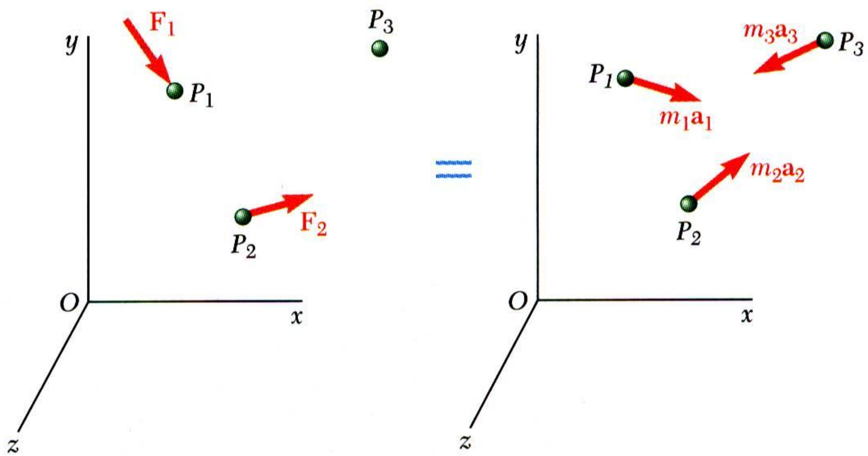
$m_i\vec{a}_i$ = effective force

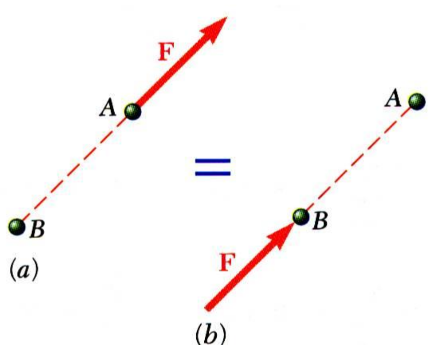


- The system of external and internal forces on a **particle** is *equivalent* to the **effective force** of the particle.

- The system of external and internal forces acting on the entire **system of particles** is *equivalent* to **the system of effective forces**.

14.2 Application of Newton's Laws. Effective Forces





- Summing over all the elements,

$$\sum_{i=1}^{n} \vec{F}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \vec{f}_{ij} = \sum_{i=1}^{n} m_i \vec{a}_i$$

$$\sum_{i=1}^{n} \left(\vec{r}_i \times \vec{F}_i\right) + \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\vec{r}_i \times \vec{f}_{ij}\right) = \sum_{i=1}^{n} \left(\vec{r}_i \times m_i \vec{a}_i\right)$$

- Since the internal forces occur in equal and opposite collinear pairs, the resultant force and couple due to the internal forces are zero,

- The system of external forces and the system of effective forces are *equipollent* but not *equivalent*.

14.3 Linear & Angular Momentum

- **Linear momentum** of the system of particles,

$$\vec{L} = \boxed{\quad}$$

$$\dot{\vec{L}} = \sum_{i=1}^{n} m_i \dot{\vec{v}}_i = \sum_{i=1}^{n} m_i \vec{a}_i$$

- **Resultant of the external forces** is equal to rate of change of linear momentum of the system of particles,

$$\sum \vec{F} = \boxed{\quad}$$

- **Angular momentum** about fixed point O of system of particles,

$$\vec{H}_O = \boxed{\quad}$$

$$\dot{\vec{H}}_O = \sum_{i=1}^{n} \left(\dot{\vec{r}}_i \times m_i \vec{v}_i \right) + \sum_{i=1}^{n} \left(\vec{r}_i \times m_i \dot{\vec{v}}_i \right)$$

$$= \sum_{i=1}^{n} \left(\vec{r}_i \times m_i \vec{a}_i \right)$$

- **Moment resultant about fixed point *O*** of the external forces is equal to the rate of change of angular momentum of the system of particles,

$$\sum \vec{M}_O = \boxed{\quad}$$

14.4 Motion of the Mass Center of a System of Particles

- Why Mass Center?

- Mass center *G* of system of particles is defined by position vector $\vec{r}_G$ which satisfies

$$m\vec{r}_G = \boxed{}$$

- Differentiating twice,

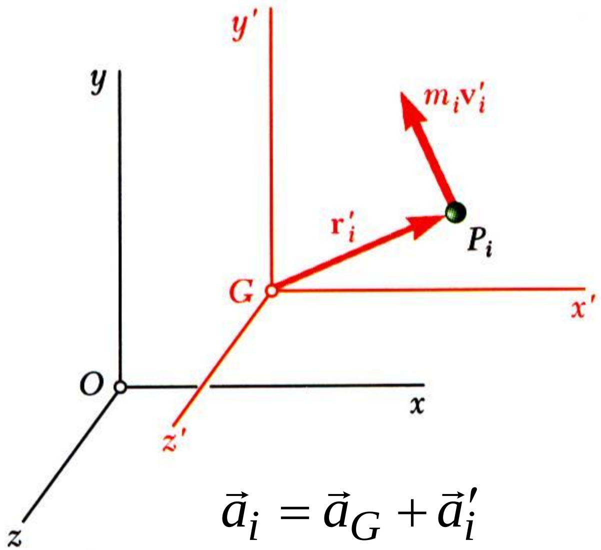
$$m\dot{\vec{r}}_G = \boxed{}$$

$$m\vec{v}_G = \boxed{}$$

$$m\vec{a}_G = \boxed{}$$

- The mass center moves as if the entire mass and all of the external forces were concentrated at that point.

14.5 Angular Momentum About the Mass Center



$$\vec{a}_i = \vec{a}_G + \vec{a}'_i$$

- Consider the centroidal frame of reference *Gx'y'z'*, which translates with respect to the Newtonian frame *Oxyz*.

- The centroidal frame is not, in general, a Newtonian frame.

- The angular momentum of the system of particles about the mass center,

$$\vec{H}'_G = \square$$

$$\dot{\vec{H}}'_G = \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i \vec{a}'_i\right) = \square$$

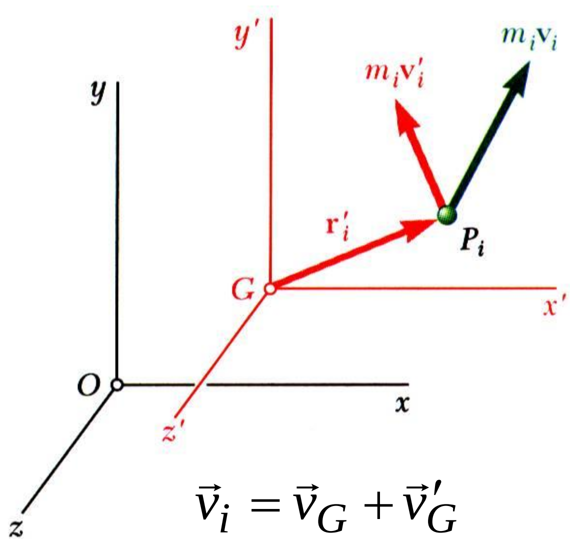
$$= \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i \vec{a}_i\right) - \left(\sum_{i=1}^{n} m_i \vec{r}'\right) \times \vec{a}_G$$

$$= \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i \vec{a}_i\right) = \square$$

$$= \sum \vec{M}_G$$

- The moment resultant about *G* of the external forces is equal to the rate of change of angular momentum about *G* of the system of particles.

14.5 Angular Momentum About the Mass Center



$$\vec{v}_i = \vec{v}_G + \vec{v}'_G$$

- Angular momentum about *G* of the particles in their motion relative to the centroidal *Gx'y'z'* frame of reference,

$$\vec{H}'_G = \boxed{}$$

- Angular momentum about *G* of particles in their absolute motion relative to the Newtonian *Oxyz* frame of reference.

$$\vec{H}_G = \sum_{i=1}^{n}\left(\vec{r}_i{}' \times m_i \vec{v}_i\right)$$

$$= \boxed{}$$

$$= \left(\sum_{i=1}^{n} m_i \vec{r}_i{}'\right) \times \vec{v}_G + \sum_{i=1}^{n}\left(\vec{r}_i{}' \times m_i \vec{v}_i\right)$$

$$\vec{H}_G = \vec{H}'_G = \sum \vec{M}_G$$

- Angular momentum about *G* of the particle momenta can be calculated with respect to either the Newtonian or centroidal frames of reference.

14.6 Conservation of Momentum

- If no external forces act on the particles of a system, then the linear momentum and angular momentum about the fixed point *O* are conserved.

$$\dot{\vec{L}} = \sum \vec{F} = 0 \qquad \dot{\vec{H}}_O = \sum \vec{M}_O = 0$$
$$\vec{L} = \text{constant} \qquad \vec{H}_O = \text{constant}$$

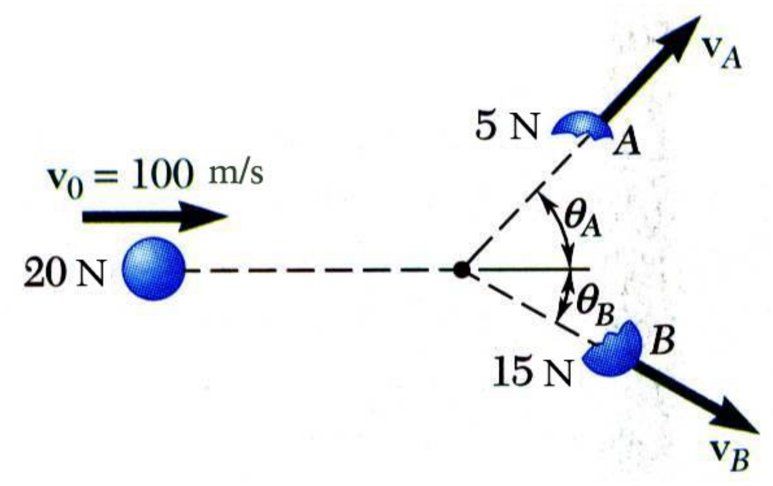
- In some applications, such as problems involving central forces,

$$\dot{\vec{L}} = \sum \vec{F} = 0 \qquad \dot{\vec{H}}_O = \sum \vec{M}_O = 0$$
$$\vec{L} = \text{constant} \qquad \vec{H}_O = \text{constant}$$

- Concept of conservation of momentum also applies to the analysis of the mass center motion,

$$\dot{\vec{L}} = \sum \vec{F} = 0 \qquad \dot{\vec{H}}_G = \sum \vec{M}_G = 0$$
$$\vec{L} = m\vec{v}_G = \text{constant}$$
$$\vec{v}_G = \text{constant} \qquad \vec{H}_G = \text{constant}$$

Sample Problem 14.2



A 20 N projectile is moving with a velocity of 100 m/s when it explodes into 5 and 15 N fragments. Immediately after the explosion, the fragments travel in the directions $\theta_A = 45^o$ and $\theta_B = 30^o$.

Determine the velocity of each fragment.