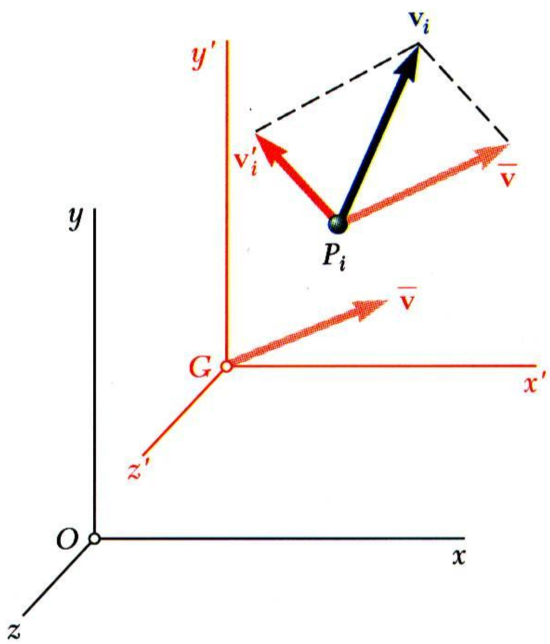
# Preview of 14.7- 14.9

## 14.7 Kinetic Energy




$$\vec{v}_i = \vec{v}_G + \vec{v}'_i$$

- Kinetic energy of a system of particles,

$$T = \frac{1}{2}\sum_{i=1}^{n} m_i \boxed{\phantom{(\vec{v}_i \bullet \vec{v}_i)}} = \frac{1}{2}\sum_{i=1}^{n} m_i v_i^2$$

- Expressing the velocity in terms of the centroidal reference frame,

$$T = \frac{1}{2}\sum_{i=1}^{n}\left[m_i \boxed{\phantom{(\vec{v}_G + \vec{v}'_i)\bullet(\vec{v}_G + \vec{v}'_i)}}\right.$$

$$= \frac{1}{2}\left(\sum_{i=1}^{n} m_i\right) v_G^2 + \vec{v}_G \bullet \sum_{i=1}^{n} m_i \vec{v}'_i + \frac{1}{2}\sum_{i=1}^{n} m_i v_i'^2$$

$$= \frac{1}{2} m \vec{v}_G^2 + \frac{1}{2}\sum_{i=1}^{n} m_i v_i'^2$$

- Kinetic energy is equal to kinetic energy of mass center plus kinetic energy relative to the centroidal frame.

## 14.8 Work-Energy Principle. Conservation of Energy

- Principle of work and energy can be applied to each particle $P_i$ ,

$$T_1 + U_{1\to 2} = T_2$$

where $U_{1\to 2}$ represents the work done by the internal forces $\vec{f}_{ij}$ and the resultant external force $\vec{F}_i$ acting on $P_i$ .

- Principle of work and energy can be applied to the entire system by adding the kinetic energies of all particles and considering the work done by all external and internal forces.

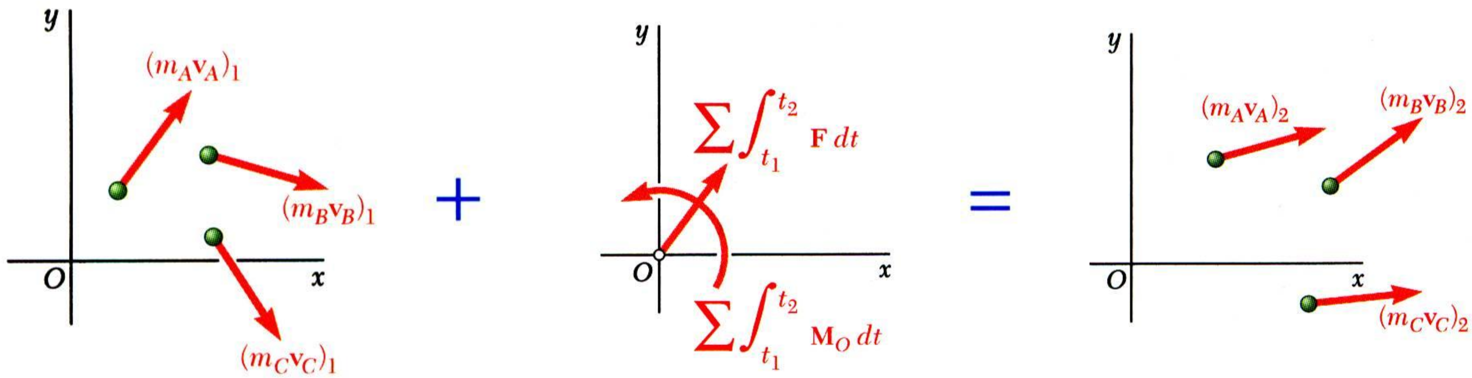
- Although $\vec{f}_{ij}$ and $\vec{f}_{ji}$ are equal and opposite, the work of these forces will not, in general, cancel out.

- If the forces acting on the particles are conservative, the work is equal to the change in potential energy and

$$T_1 + V_1 = T_2 + V_2$$

which expresses the principle of conservation of energy for the system of particles.

# 14.9 Principle of Impulse and Momentum



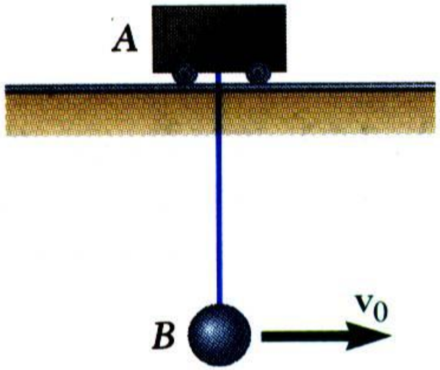

$$\sum \vec{F} = \dot{\vec{L}} \qquad \sum \vec{M}_O = \dot{\vec{H}}_O$$

$$\sum \int_{t_1}^{t_2} \vec{F} dt = \vec{L}_2 - \vec{L}_1 \qquad \sum \int_{t_1}^{t_2} \vec{M}_O dt = \vec{H}_2 - \vec{H}_1$$

$$\vec{L}_1 + \sum \int_{t_1}^{t_2} \vec{F} dt = \vec{L}_2 \qquad \vec{H}_1 + \sum \int_{t_1}^{t_2} \vec{M}_O dt = \vec{H}_2$$

- The momenta of the particles at time $t_1$ and the impulse of the forces from $t_1$ to $t_2$ form a system of vectors *equipollent* to the system of momenta of the particles at time $t_2$ .

## Sample Problem 14.4

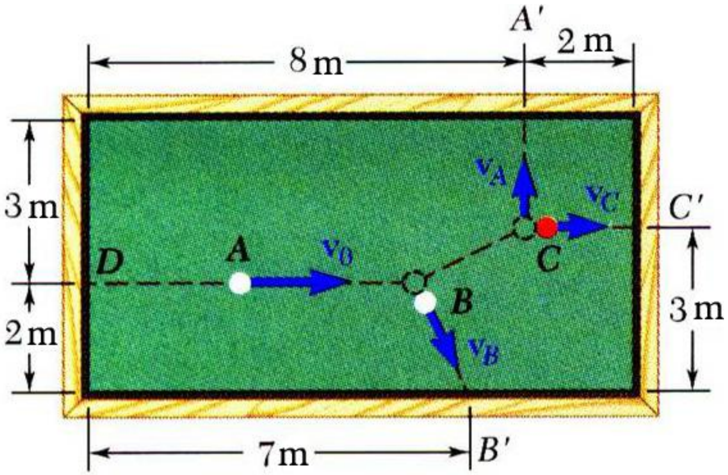



Ball *B*, of mass $m_B$, is suspended from a cord, of length *l*, attached to cart *A*, of mass $m_A$, which can roll freely on a frictionless horizontal tract. While the cart is at rest, the ball is given an initial velocity $v_0 = \sqrt{2gl}$.

Determine *(a)* the velocity of *B* as it reaches it maximum elevation, and *(b)* the maximum vertical distance *h* through which *B* will rise.

## Sample Problem 14.4

## Sample Problem 14.5




Ball *A* has initial velocity $v_0$ = 10 m /s parallel to the axis of the table. It hits ball *B* and then ball *C* which are both at rest. Balls *A* and *C* hit the sides of the table squarely at *A'* and *C'* and ball *B* hits obliquely at *B'*.

Assuming perfectly elastic collisions, determine velocities $v_A$, $v_B$, and $v_C$ with which the balls hit the sides of the table.

## Sample Problem 14.5