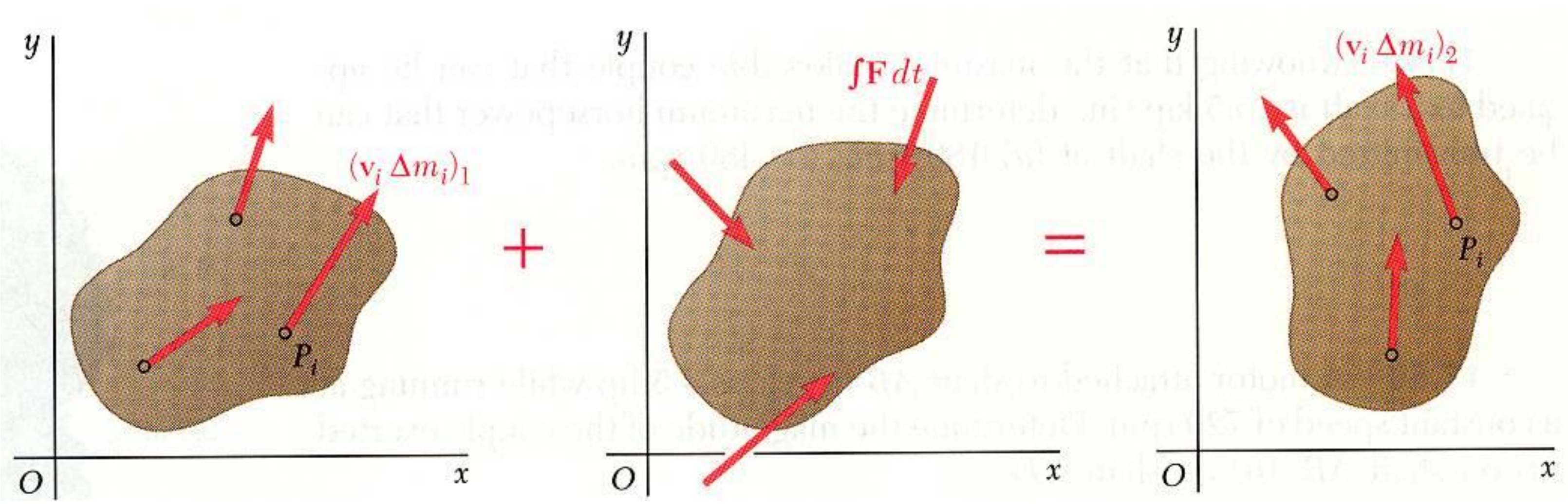
Energy and Momentum Methods for Plane Motion of Rigid Bodies

Preview of 17.8- 17.10

17.8 Principle of Impulse and Momentum

- Method of impulse and momentum:
  - well suited to the solution of problems involving time and velocity
  - the only practicable method for problems involving impulsive motion and impact.



Sys Momenta$_1$ + Sys Ext Imp$_{1\text{-}2}$ = Sys Momenta$_2$

17.8 Principle of Impulse and Momentum

- The momenta of the particles of a system may be reduced to a vector attached to **the mass center** equal to their sum,

$\vec{L} =$ [ ] $=$ [ ]

and a couple equal to the sum of their **moments about the mass center**,

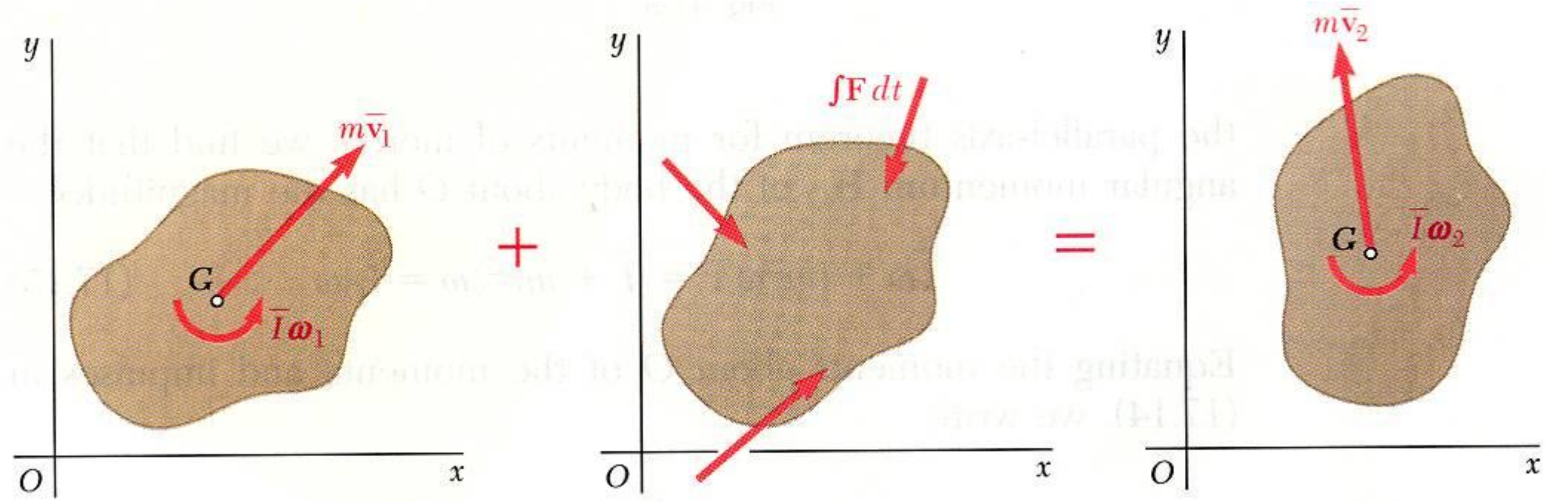
$\vec{H}_G =$ [ ]

- For the plane motion of a rigid slab or of a rigid body symmetrical with respect to the reference plane,

$\vec{H}_G =$ [ ]

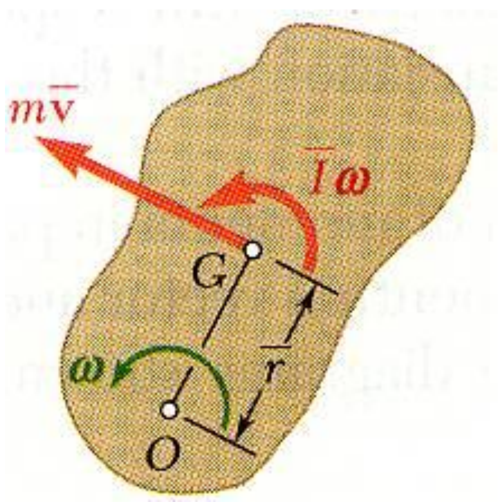
17.8 Principle of Impulse and Momentum

- Principle of impulse and momentum for the plane motion of a rigid slab or of a rigid body symmetrical with respect to the reference plane expressed as a *free-body-diagram equation*,



- Leads to three equations of motion:
  - summing and equating momenta and impulses in the *x* and *y* directions
  - summing and equating the moments of the momenta and impulses with respect to any given point

17.8 Principle of Impulse and Momentum



- Noncentroidal rotation:
  - The angular momentum about O

$$I_O\omega =$$
$$=$$
$$=$$

  - Equating the moments of the momenta and impulses about O,

$$I_O\omega_1 + \sum \int_{t_1}^{t_2} M_O dt = I_O\omega_2$$

17.9 Systems of Rigid Bodies

- Motion of several rigid bodies can be analyzed by applying the principle of impulse and momentum to each body separately.
- For problems involving no more than three unknowns, it may be convenient to apply the principle of impulse and momentum to the system as a whole.
- For each moving part of the system, the diagrams of momenta should include a momentum vector and/or a momentum couple.
- Internal forces occur in equal and opposite pairs of vectors and do not generate nonzero net impulses.

17.10 Conservation of Angular Momentum

- When no external force acts on a rigid body or a system of rigid bodies, the system of momenta at t_1 is equipollent to the system at t_2. The total linear momentum and angular momentum about any point are conserved,

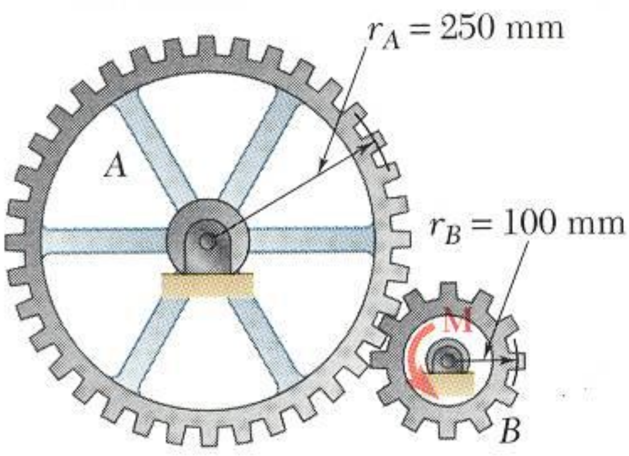
$$\vec{L}_1 = \vec{L}_2 \qquad (H_0)_1 = (H_0)_2$$

- When the sum of the angular impulses pass through *O*, the linear momentum may not be conserved, yet the angular momentum about *O* is conserved,

$$(H_0)_1 = (H_0)_2$$

- Two additional equations may be written by summing *x* and *y* components of momenta and may be used to determine two unknown linear impulses, such as the impulses of the reaction components at a fixed point.

Sample Problem 17.6



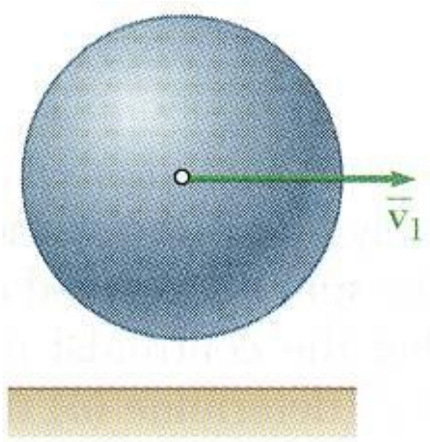
$$m_A = 10\,\text{kg} \quad \bar{k}_A = 200\,\text{mm}$$
$$m_B = 3\,\text{kg} \quad \bar{k}_B = 80\,\text{mm}$$

The system is at rest when a moment of $M = 6\,\text{N}\cdot\text{m}$ is applied to gear *B*.

Neglecting friction, *a*) determine the time required for gear *B* to reach an angular velocity of 600 rpm, and *b*) the tangential force exerted by gear *B* on gear *A*.

Sample Problem 17.6

Sample Problem 17.7

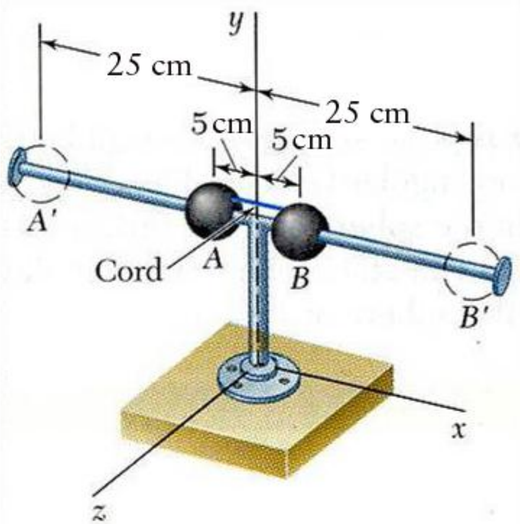


Uniform sphere of mass *m* and radius *r* is projected along a rough horizontal surface with a linear velocity $\bar{v}_1$ and no angular velocity. The coefficient of kinetic friction is μ_k.

Determine *a*) the time t_2 at which the sphere will start rolling without sliding and *b*) the linear and angular velocities of the sphere at time t_2.

Sample Problem 17.7

Sample Problem 17.8



Two solid spheres (radius = 3 cm, $W = 2$ N) are mounted on a spinning horizontal rod ($\bar{I}_R = 0.25\,\text{kg cm}^2$, $\omega = 6$ rad/s) as shown. The balls are held together by a string which is suddenly cut. Determine *a*) angular velocity of the rod after the balls have moved to *A'* and *B'*, and *b*) the energy lost due to the plastic impact of the spheres and stops.

Sample Problem 17.8