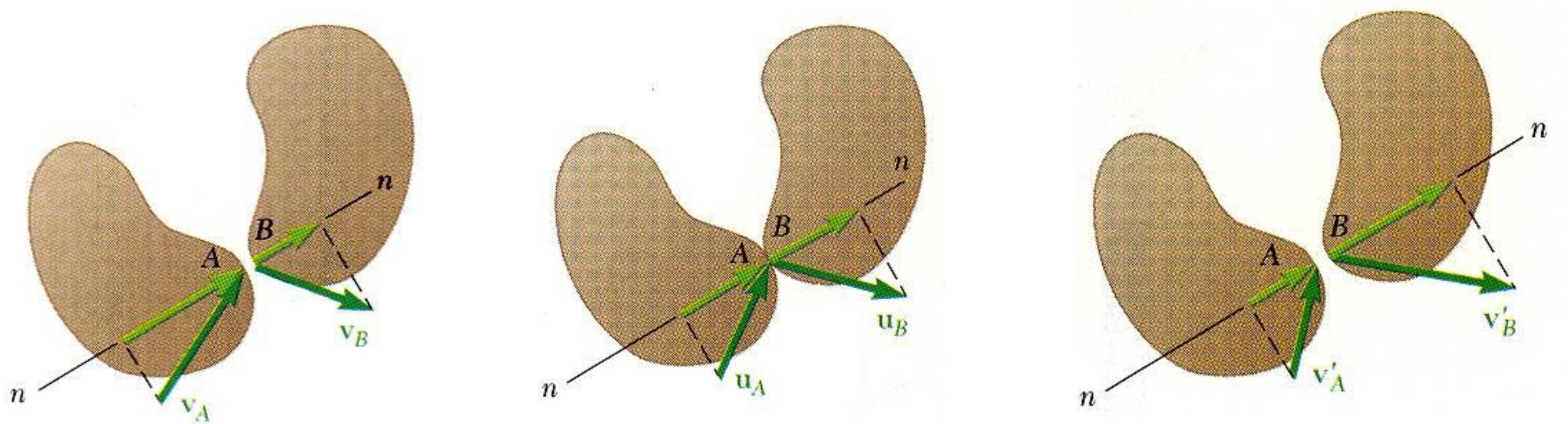
Preview of 17.11

17. 11 Eccentric Impact



$$(\vec{u}_A)_n = (\vec{u}_B)_n$$

Period of deformation

$$Impulse = \boxed{\int \vec{R}dt}$$

Period of restitution

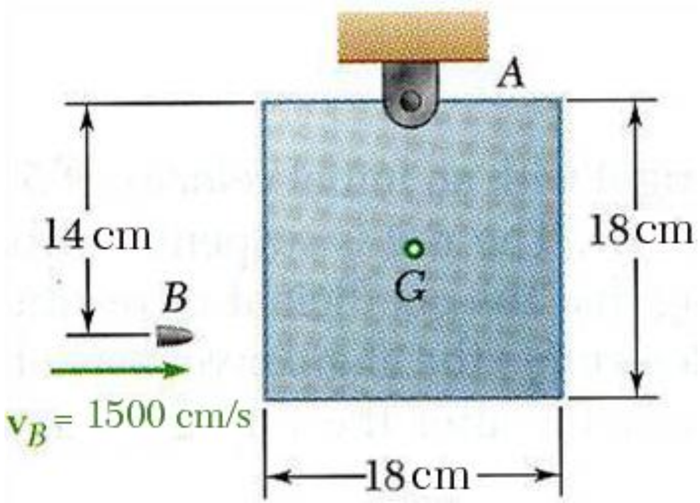
$$Impulse = \boxed{\int \vec{P}dt}$$

- Principle of impulse and momentum is supplemented by

$$e = coefficient\ of\ restitution = \frac{\int \vec{R}dt}{\int \vec{P}dt}$$

$$= \frac{(v'_B)_n - (v'_A)_n}{(v_A)_n - (v_B)_n}$$

Sample Problem 17.9

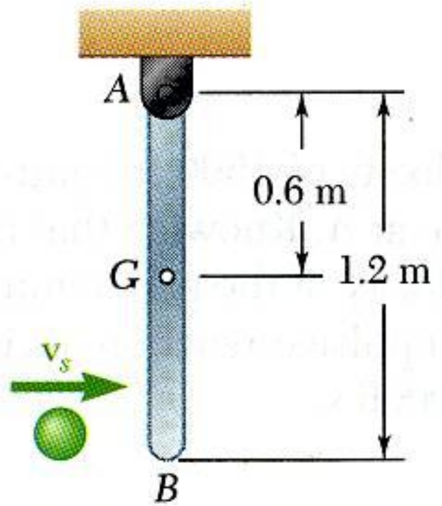


A 0.05-N bullet is fired into the side of a 20-N square panel which is initially at rest.

Determine *a*) the angular velocity of the panel immediately after the bullet becomes embedded and *b*) the impulsive reaction at *A*, assuming that the bullet becomes embedded in 0.0006 s.

Sample Problem 17.9

Sample Problem 17.10

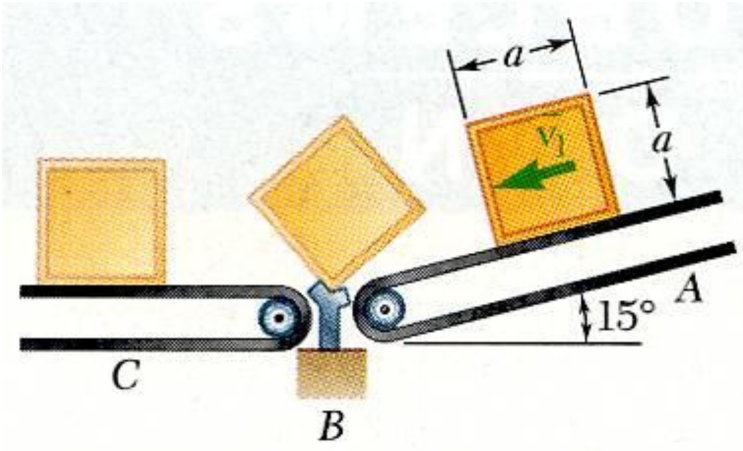


A 2-kg sphere with an initial velocity of 5 m/s strikes the lower end of an 8 kg rod *AB*. The rod is hinged at *A* and initially at rest. The coefficient of restitution between the rod and sphere is 0.8.

Determine the angular velocity of the rod and the velocity of the sphere immediately after impact.

Sample Problem 17.10

Sample Problem 17.11



A square package of mass *m* moves down conveyor belt *A* with constant velocity. At the end of the conveyor, the corner of the package strikes a rigid support at *B*. The impact is perfectly plastic.

Derive an expression for the minimum velocity of conveyor belt *A* for which the package will rotate about *B* and reach conveyor belt *C*.

Sample Problem 17.11

Preview of 15.12- 15.15

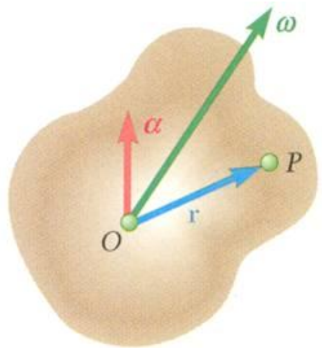
Difference with 2D?

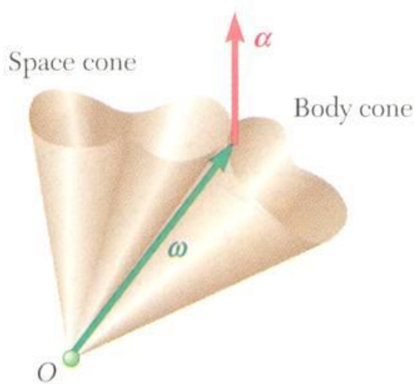
15.12 Motion About a Fixed Point

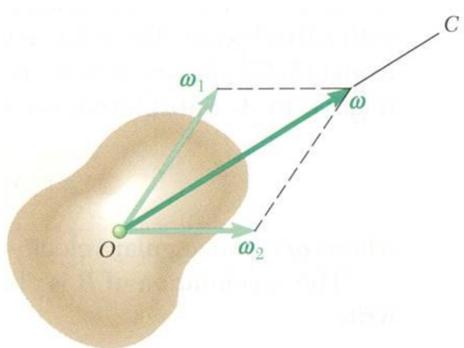
15.13 General Motion

15.14 Coriolis Acceleration.   When does this occur?

15.12 Motion About a Fixed Point







- The most general displacement of a rigid body with a fixed point *O* is equivalent to a rotation of the body about an axis through *O*.
- With the instantaneous axis of rotation and angular velocity $\vec{\omega}$, the velocity of a particle *P* of the body is

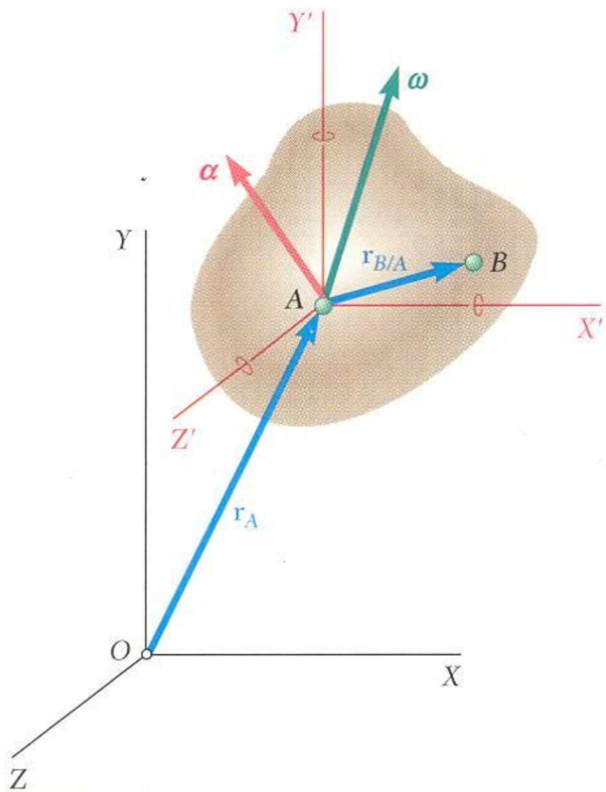
$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

and the acceleration of the particle *P* is

$$\vec{a} = \boxed{\qquad\qquad} \qquad \vec{\alpha} = \frac{d\vec{\omega}}{dt}.$$

- The angular acceleration $\vec{\alpha}$ represents the velocity of the tip of $\vec{\omega}$.
- As the vector $\vec{\omega}$ moves within the body and in space, it generates a body cone and space cone which are tangent along the instantaneous axis of rotation.
- Angular velocities have magnitude and direction and obey parallelogram law of addition. They are vectors.

15.13 General Motion



- For particles A and B of a rigid body,

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

- Particle A is fixed within the body and motion of the body relative to $AX'Y'Z'$ is the motion of a body with a fixed point

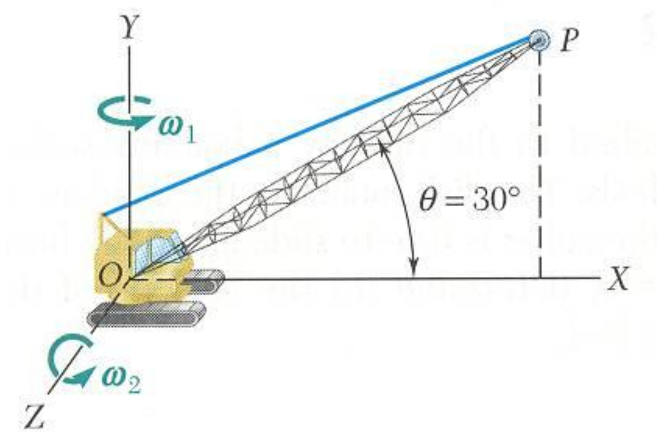
$$\vec{v}_B = \boxed{}$$

- Similarly, the acceleration of the particle P is

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$
$$= \boxed{}$$

- Most general motion of a rigid body is equivalent to:
  - a translation in which all particles have the same velocity and acceleration of a reference particle A, and
  - of a motion in which particle A is assumed fixed.

Sample Problem 15.11



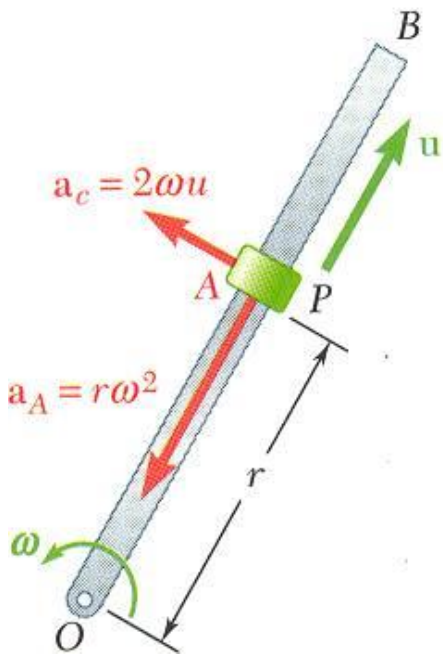
The crane rotates with a constant angular velocity $\omega_1 = 0.30$ rad/s and the boom is being raised with a constant angular velocity $\omega_2 = 0.50$ rad/s. The length of the boom is $l = 12$ m.

Determine:

- angular velocity of the boom,
- angular acceleration of the boom,
- velocity of the boom tip, and
- acceleration of the boom tip.

Sample Problem 15.11

15.11 Coriolis Acceleration (2D)



- The point A on the rod corresponds to the instantaneous position of P.

$$\vec{v}_P = \vec{\Omega} \times \vec{r} + \left(\dot{\vec{r}}\right)_{Oxyz}$$

$$= \vec{v}_{P'} + \vec{v}_{P/\mathfrak{F}}$$

- Absolute acceleration of the collar is

$$\vec{a}_P = \boxed{\vec{a}_A + \vec{a}_{P/\mathfrak{F}} + \vec{a}_c}$$

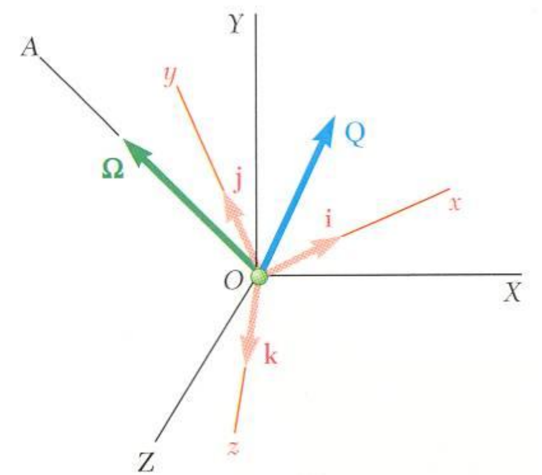
where

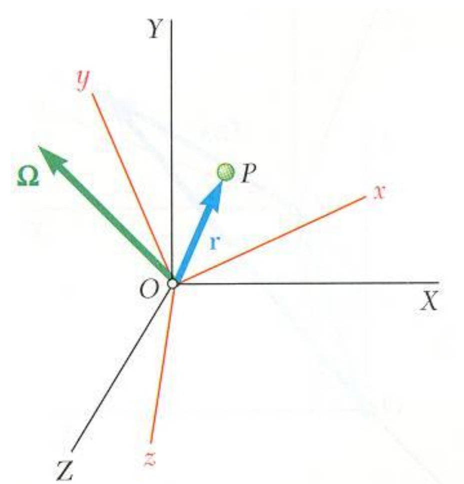
$$\vec{a}_A = \boxed{\dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right)} \qquad a_A = r\omega^2$$

$$\vec{a}_{P/\mathfrak{F}} = \boxed{\left(\ddot{\vec{r}}\right)_{Oxy} = 0}$$

$$\vec{a}_c = \boxed{2\vec{\Omega} \times \vec{v}_{P/\mathfrak{F}}} \qquad a_c = 2\omega u$$

15.14 Three-Dimensional Motion. Coriolis Acceleration





- With respect to the fixed frame *OXYZ* and rotating frame *Oxyz*,

$$\left(\dot{\vec{Q}}\right)_{OXYZ} = \left(\dot{\vec{Q}}\right)_{Oxyz} + \vec{\Omega}\times\vec{Q}$$

- Consider motion of particle *P* relative to a rotating frame *Oxyz* or $\mathfrak{F}$ for short. The absolute velocity can be expressed as

$$\vec{v}_P = \vec{\Omega}\times\vec{r} + \left(\dot{\vec{r}}\right)_{Oxyz}$$

$$= \vec{v}_{P'} + \vec{v}_{P/\mathfrak{F}}$$

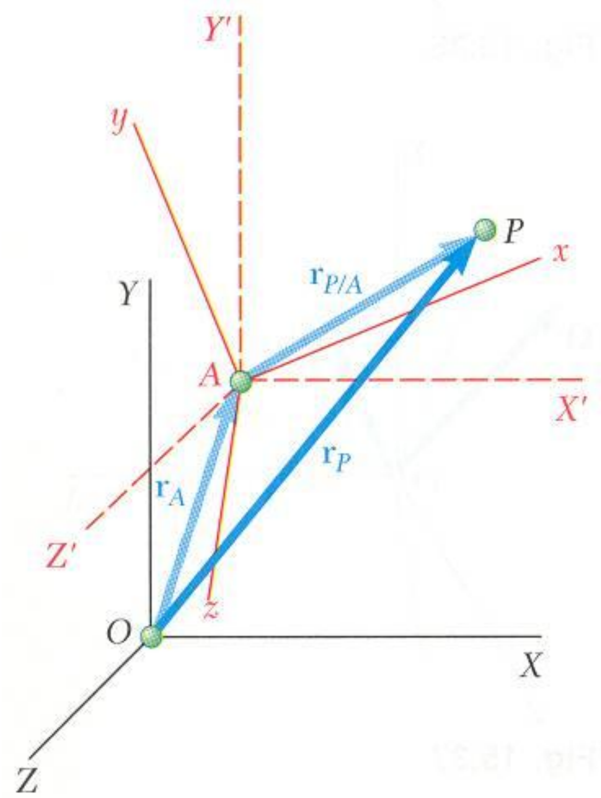
- The absolute acceleration can be expressed as

$$\vec{a}_P = \boxed{}$$

$$= \vec{a}_{p'} + \vec{a}_{P/\mathfrak{F}} + \vec{a}_c$$

$$\vec{a}_c = 2\vec{\Omega}\times\left(\dot{\vec{r}}\right)_{Oxyz} = 2\vec{\Omega}\times\vec{v}_{P/\mathfrak{F}} = \text{Coriolis acceleration}$$

15.15 Frame of Reference in General Motion



Consider:

- fixed frame *OXYZ*,
- translating frame *AX'Y'Z'*, and
- translating and rotating frame *Axyz* or $\mathfrak{F}$.

- With respect to *OXYZ* and *AX'Y'Z'*,

$$\vec{r}_P = \vec{r}_A + \vec{r}_{P/A}$$

$$\vec{v}_P = \vec{v}_A + \vec{v}_{P/A}$$

$$\vec{a}_P = \vec{a}_A + \vec{a}_{P/A}$$

- The velocity and acceleration of *P* relative to *AX'Y'Z'* can be found in terms of the velocity and acceleration of *P* relative to *Axyz*.

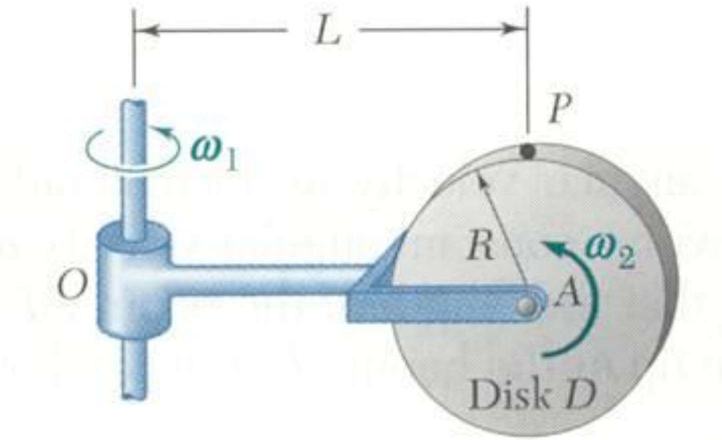
$$\vec{v}_P = \vec{v}_A + \vec{\Omega}\times\vec{r}_{P/A} + \left(\dot{\vec{r}}_{P/A}\right)_{Axyz}$$

$$= \vec{v}_{P'} + \vec{v}_{P/\mathfrak{F}}$$

$$\vec{a}_P = \vec{a}_A + \dot{\vec{\Omega}}\times\vec{r}_{P/A} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{r}_{P/A}\right) + 2\vec{\Omega}\times\left(\dot{\vec{r}}_{P/A}\right)_{Axyz} + \left(\ddot{\vec{r}}_{P/A}\right)_{Axyz}$$

$$= \vec{a}_{P'} + \vec{a}_{P/\mathfrak{F}} + \vec{a}_c$$

Sample Problem 15.15



For the disk mounted on the arm, the indicated angular rotation rates are constant.

Determine:
- the velocity of the point *P*,
- the acceleration of *P*, and
- angular velocity and angular acceleration of the disk.

Sample Problem 15.15