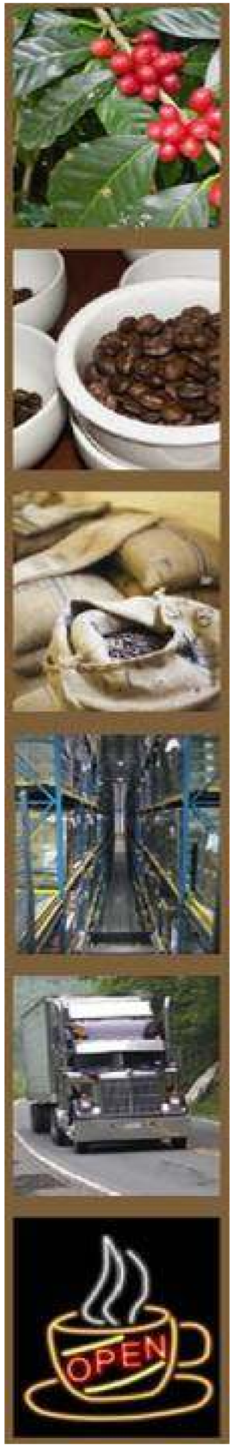
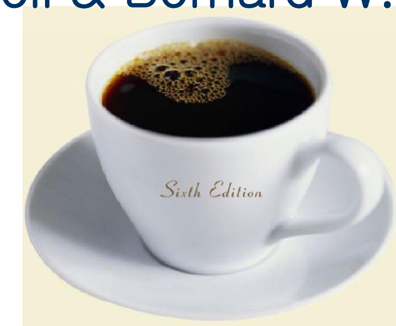


Chapter 3

Statistical Process Control

Operations Management - 6th Edition

Roberta Russell & Bernard W. Taylor, III



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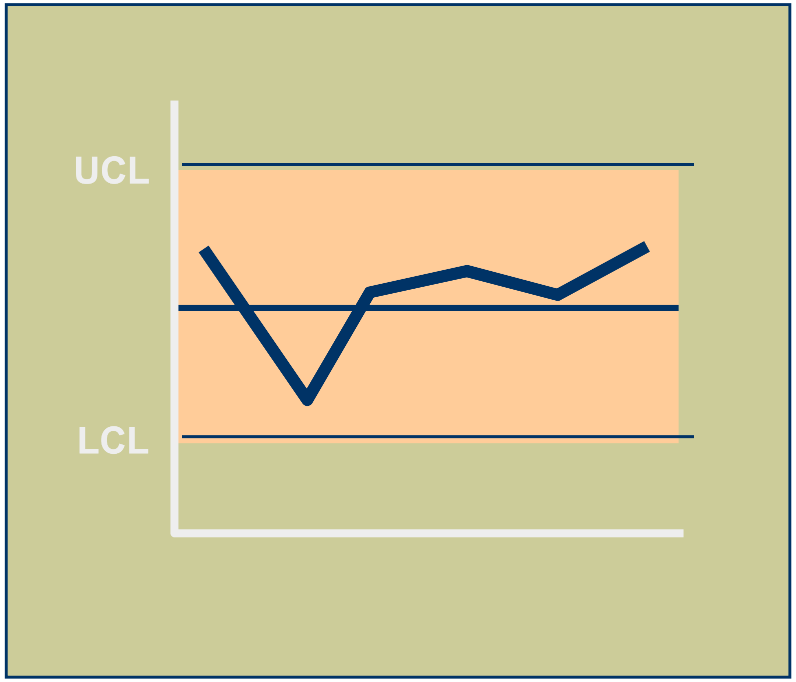
Lecture Outline

- Basics of Statistical Process Control
- Control Charts
- Control Charts for Attributes
- Control Charts for Variables
- Control Chart Patterns
- SPC with Excel and OM Tools
- Process Capability

Basics of Statistical Process Control

- Statistical Process Control (SPC)
  - monitoring production process to detect and prevent poor quality
- Sample
  - subset of items produced to use for inspection
- Control Charts
  - process is within statistical control limits



Basics of Statistical Process Control (cont.)

- Random
  - inherent in a process
  - depends on equipment and machinery, engineering, operator, and system of measurement
  - natural occurrences
- Non-Random
  - special causes
  - identifiable and correctable
  - include equipment out of adjustment, defective materials, changes in parts or materials, broken machinery or equipment, operator fatigue or poor work methods, or errors due to lack of training

SPC in Quality Management

- SPC
  - tool for identifying problems in order to make improvements
  - contributes to the TQM goal of continuous improvements

Quality Measures: Attributes and Variables

- Attribute
  - a product characteristic that can be evaluated with a discrete response
  - good – bad; yes – no
- Variable measure
  - a product characteristic that is continuous and can be measured
  - weight – length

SPC Applied to Services

- Nature of defect is different in services
- Service defect is a failure to meet customer requirements
- Monitor time and customer satisfaction

SPC Applied to Services (cont.)

- Hospitals
  - timeliness and quickness of care, staff responses to requests, accuracy of lab tests, cleanliness, courtesy, accuracy of paperwork, speed of admittance and checkouts
- Grocery stores
  - waiting time to check out, frequency of out-of-stock items, quality of food items, cleanliness, customer complaints, checkout register errors
- Airlines
  - flight delays, lost luggage and luggage handling, waiting time at ticket counters and check-in, agent and flight attendant courtesy, accurate flight information, passenger cabin cleanliness and maintenance

SPC Applied to Services (cont.)

- Fast-food restaurants
  - waiting time for service, customer complaints, cleanliness, food quality, order accuracy, employee courtesy
- Catalogue-order companies
  - order accuracy, operator knowledge and courtesy, packaging, delivery time, phone order waiting time
- Insurance companies
  - billing accuracy, timeliness of claims processing, agent availability and response time

Where to Use Control Charts

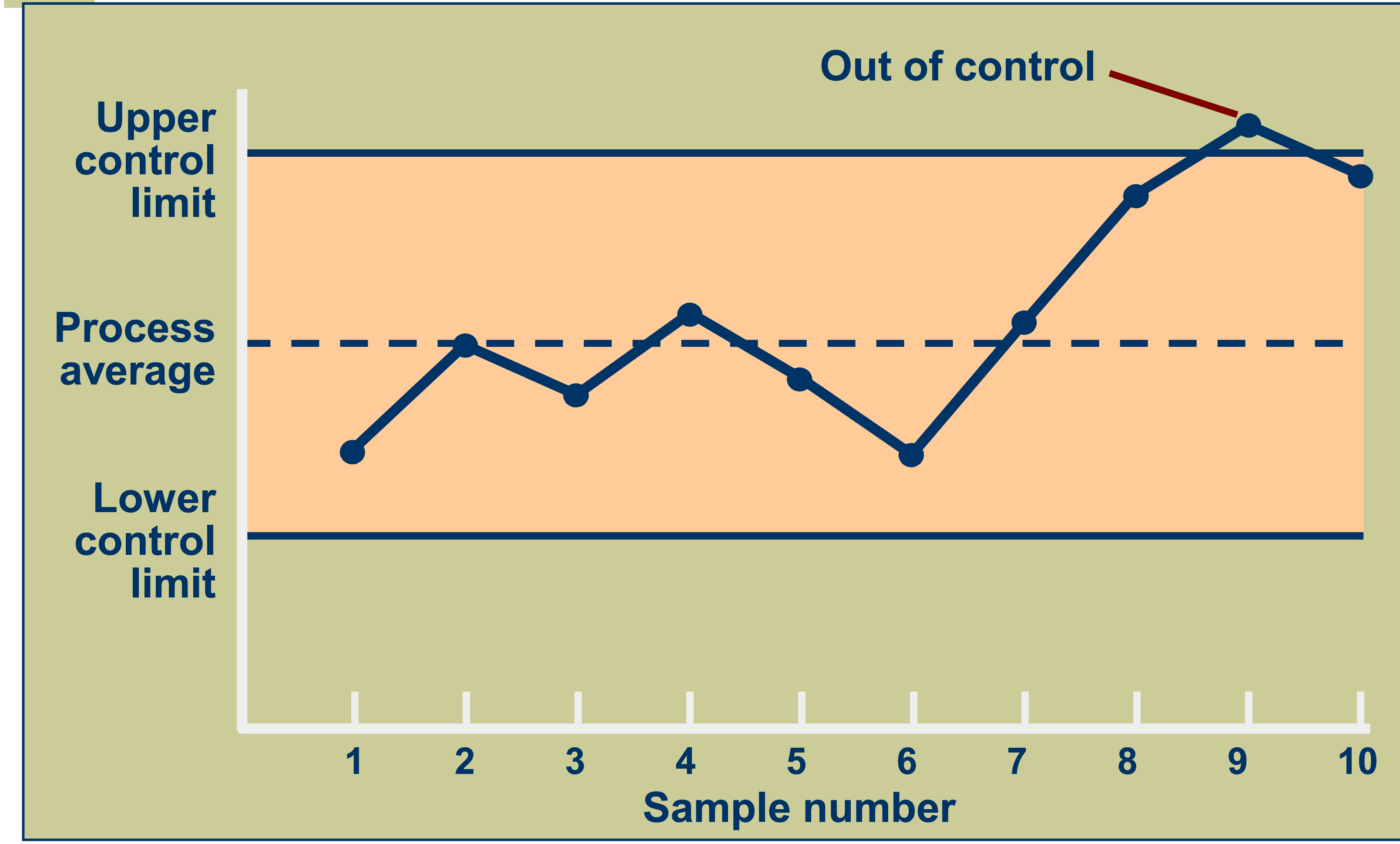
- Process has a tendency to go out of control
- Process is particularly harmful and costly if it goes out of control
- Examples
  - at the beginning of a process because it is a waste of time and money to begin production process with bad supplies
  - before a costly or irreversible point, after which product is difficult to rework or correct
  - before and after assembly or painting operations that might cover defects
  - before the outgoing final product or service is delivered

Control Charts

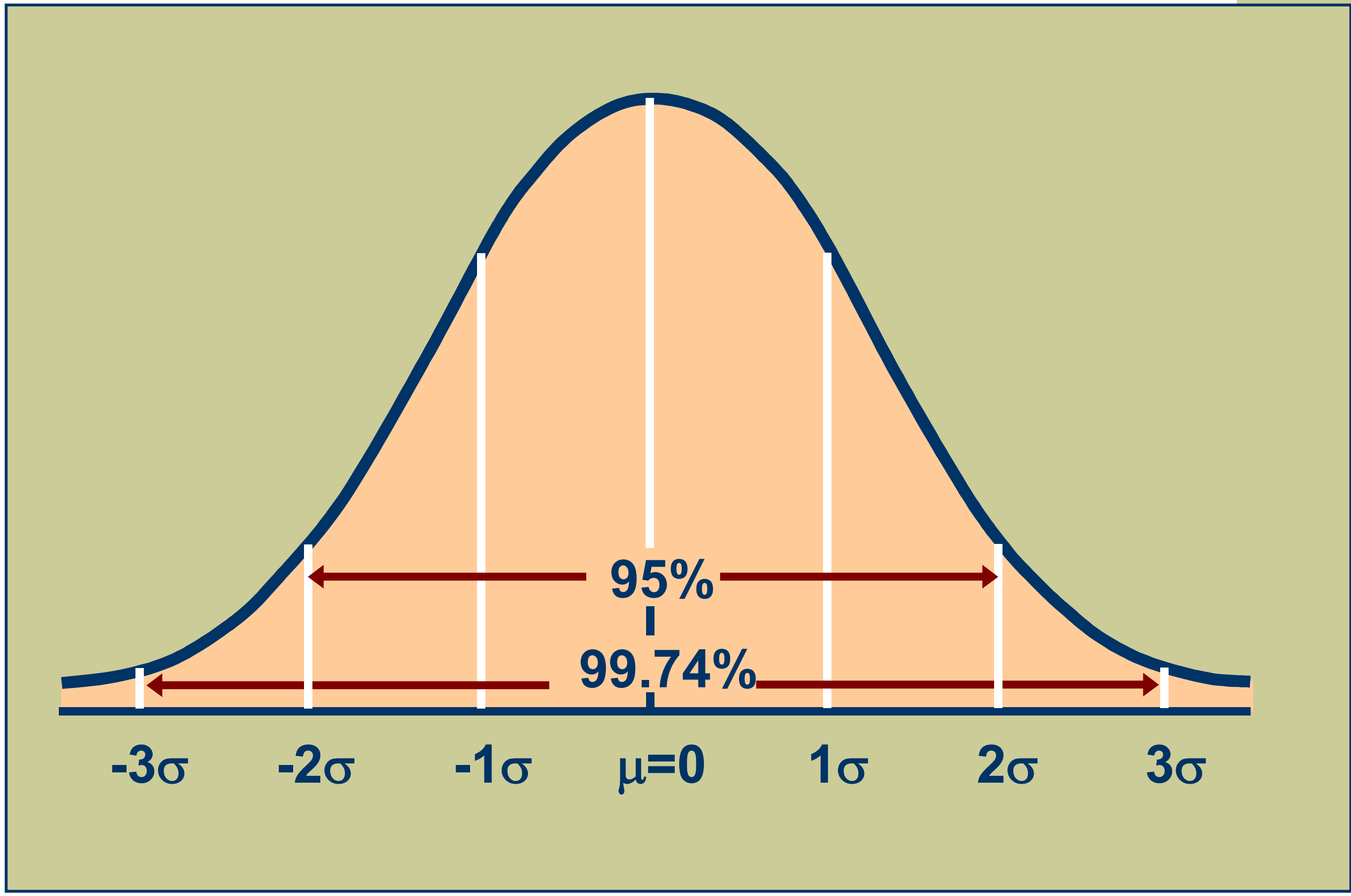
- A graph that establishes control limits of a process
- Control limits
  - upper and lower bands of a control chart

- Types of charts
  - Attributes
    - p-chart
    - c-chart
  - Variables
    - mean (x bar – chart)
    - range (R-chart)

Process Control Chart



Normal Distribution



A Process Is in Control If …

1. … no sample points outside limits
2. … most points near process average
3. … about equal number of points above and below centerline
4. … points appear randomly distributed

Control Charts for Attributes

- p-chart
  - uses portion defective in a sample
- c-chart
  - uses number of defective items in a sample

p-Chart

$$UCL = \bar{p} + z\sigma_p$$

$$LCL = \bar{p} - z\sigma_p$$

z = number of standard deviations from process average

$\bar{p}$ = sample proportion defective; an estimate of process average

σ_p = standard deviation of sample proportion

$$\sigma_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

Construction of p-Chart

SAMPLE	NUMBER OF DEFECTIVES	PROPORTION DEFECTIVE
1	6	.06
2	0	.00
3	4	.04
:	:	:
:	:	:
20	18	.18
	200	

20 samples of 100 pairs of jeans

Construction of p-Chart (cont.)

$$\bar{p} = \frac{\text{total defectives}}{\text{total sample observations}} = 200 / 20(100) = 0.10$$

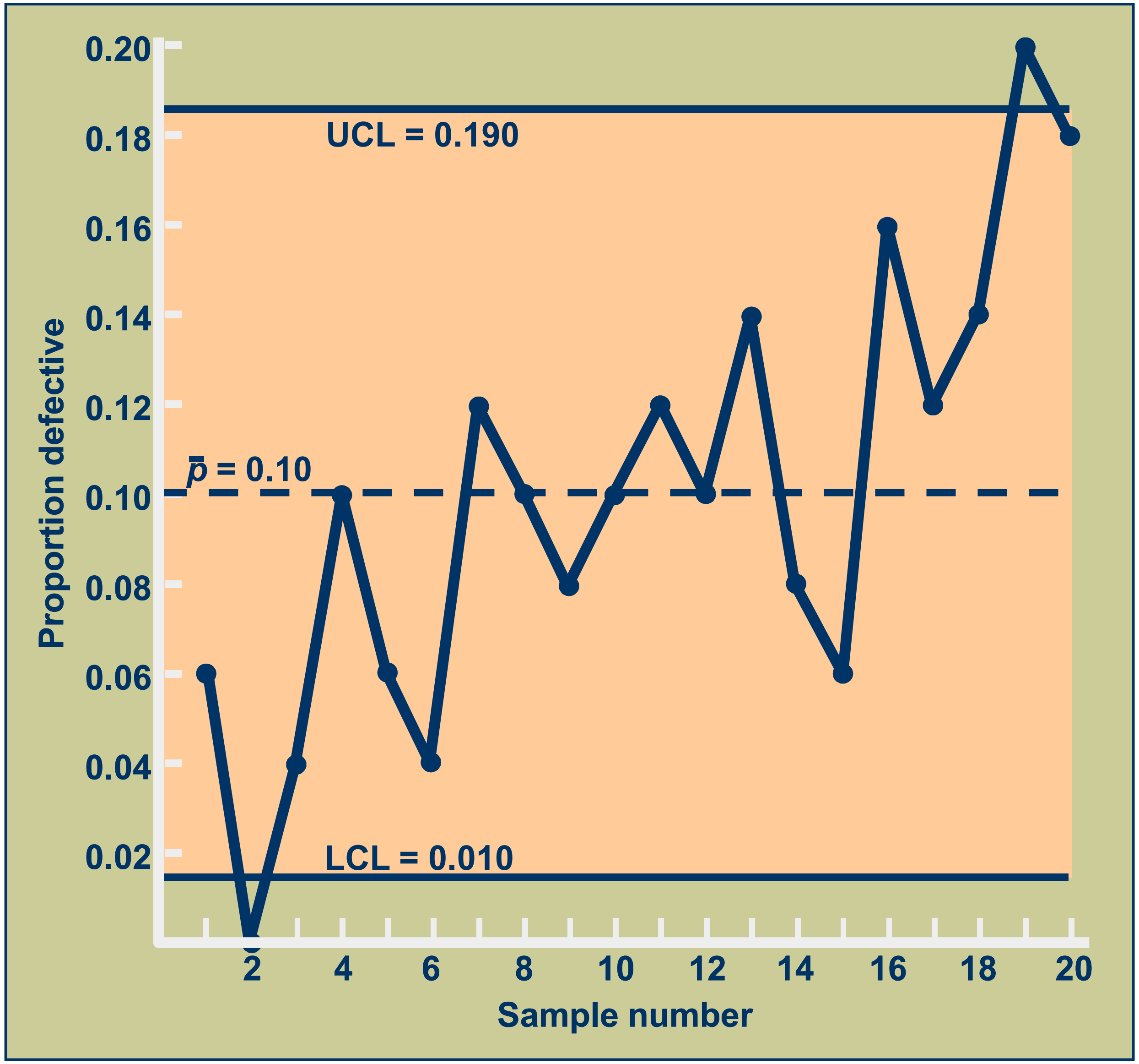
$$UCL = \bar{p} + z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 + 3\sqrt{\frac{0.10(1 - 0.10)}{100}}$$

UCL = 0.190

$$LCL = \bar{p} - z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 - 3\sqrt{\frac{0.10(1 - 0.10)}{100}}$$

LCL = 0.010

Construction of p-Chart (cont.)



c-Chart

$$UCL = \bar{c} + z\sigma_c$$

$$LCL = \bar{c} - z\sigma_c$$

$$\sigma_c = \sqrt{\bar{c}}$$

where

c = number of defects per sample

c-Chart (cont.)

Number of defects in 15 sample rooms

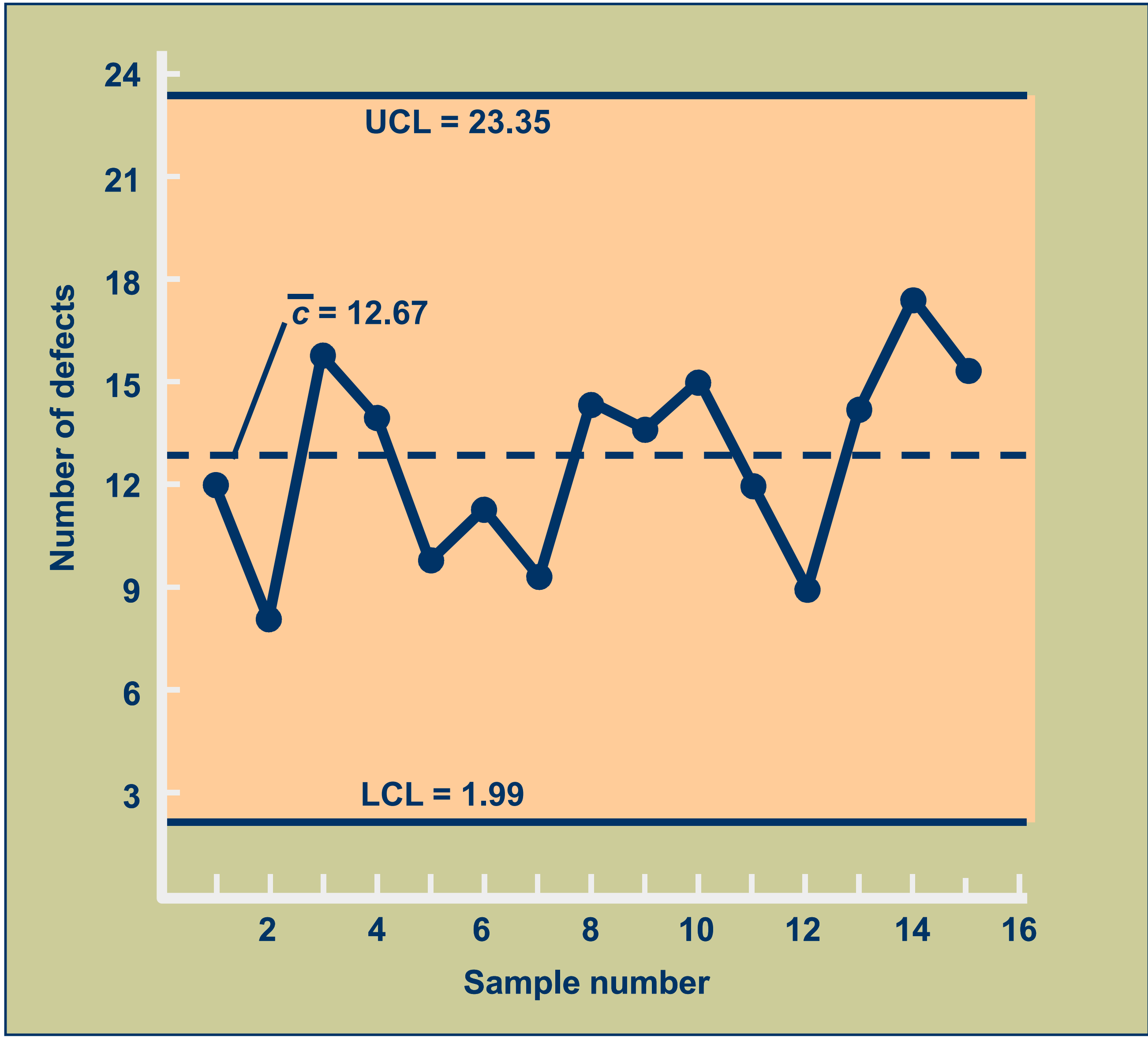
SAMPLE	NUMBER OF DEFECTS
1	12
2	8
3	16
:	:
:	:
15	15
	190

$$\bar{c} = \frac{190}{15} = 12.67$$

$$\begin{aligned} UCL &= \bar{c} + z\sigma_c \\ &= 12.67 + 3\sqrt{12.67} \\ &= 23.35 \end{aligned}$$

$$\begin{aligned} LCL &= \bar{c} - z\sigma_c \\ &= 12.67 - 3\sqrt{12.67} \\ &= 1.99 \end{aligned}$$

c-Chart (cont.)


UCL = 23.35
c̄ = 12.67
LCL = 1.99
Number of defects
24
21
18
15
12
9
6
3
2
4
6
8
10
12
14
16
Sample number

Control Charts for Variables

- Range chart (R-Chart)
  - uses amount of dispersion in a sample
- Mean chart (x -Chart)
  - uses process average of a sample

x-bar Chart: Standard Deviation Known

$$UCL = \bar{\bar{x}} + z\sigma_{\bar{x}} \qquad LCL = \bar{\bar{x}} - z\sigma_{\bar{x}}$$

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \dots \bar{x}_n}{n}$$

where

$\bar{\bar{x}}$ = average of sample means

x-bar Chart Example: Standard Deviation Known (cont.)

	Observations (Slip-Ring Diameter, cm)					
Sample *k*	***1***	***2***	***3***	***4***	***5***	$\bar{x}$
1	5.02	5.01	4.94	4.99	4.96	4.98
2	5.01	5.03	5.07	4.95	4.96	5.00
3	4.99	5.00	4.93	4.92	4.99	4.97
4	5.03	4.91	5.01	4.98	4.89	4.96
5	4.95	4.92	5.03	5.05	5.01	4.99
6	4.97	5.06	5.06	4.96	5.03	5.01
7	5.05	5.01	5.10	4.96	4.99	5.02
8	5.09	5.10	5.00	4.99	5.08	5.05
9	5.14	5.10	4.99	5.08	5.09	5.08
10	5.01	4.98	5.08	5.07	4.99	5.03
						50.09

x-bar Chart Example: Standard Deviation Known (cont.)

$$\bar{\bar{x}} = \frac{50.09}{10} = 5.01$$

$$\begin{aligned}
\text{UCL} &= \bar{\bar{x}} + z\sigma_{\bar{x}} \\
&= 5.01 + 3(.08/\sqrt{10}) \\
&= 5.09 \\
\text{LCL} &= \bar{\bar{x}} - z\sigma_{\bar{x}} \\
&= 5.01 - 3(.08/\sqrt{10}) \\
&= 4.93
\end{aligned}$$

x-bar Chart Example: Standard Deviation Unknown

$UCL = \bar{\bar{x}} + A_2\bar{R}$ $LCL = \bar{\bar{x}} - A_2\bar{R}$

where

$\bar{\bar{x}}$ = average of sample means

Control Limits

Sample Size n	Factor for $\bar{x}$-Chart A_2	Factors for R-Chart D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59
21	0.17	0.43	1.58
22	0.17	0.43	1.57
23	0.16	0.44	1.56
24	0.16	0.45	1.55
25	0.15	0.46	1.54

x-bar Chart Example: Standard Deviation Unknown

	OBSERVATIONS (SLIP- RING DIAMETER, CM)						
SAMPLE *k*	*1*	*2*	*3*	*4*	*5*	$\bar{x}$	*R*
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

Example 15.4

x-bar Chart Example: Standard Deviation Unknown (cont.)

$$\overline{R} = \frac{\sum R}{k} = \frac{1.15}{10} = 0.115$$

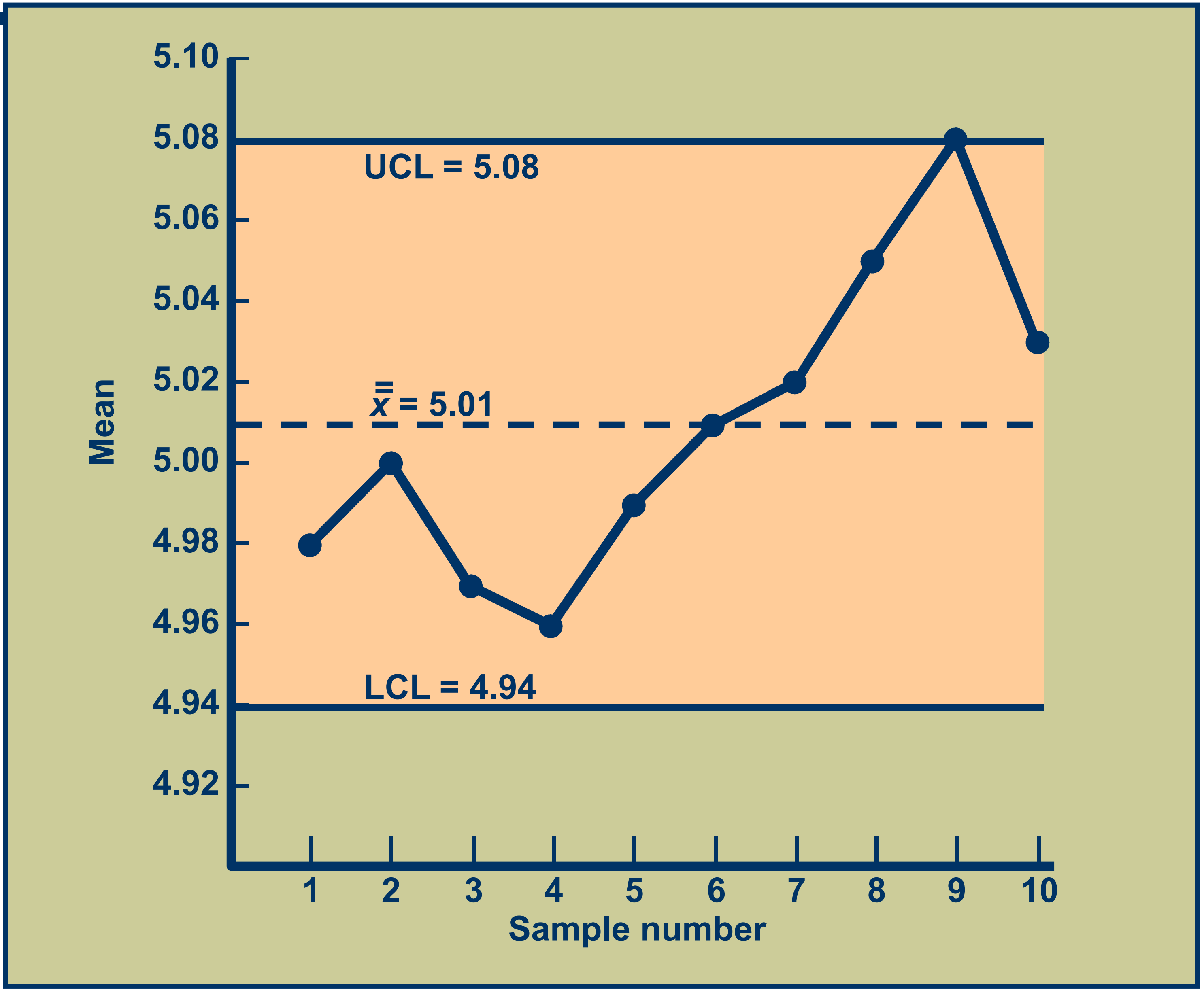
$$\overline{\overline{x}} = \frac{\sum \overline{x}}{k} = \frac{50.09}{10} = 5.01 \text{ cm}$$

$$UCL = \overline{\overline{x}} + A_2\overline{R} = 5.01 + (0.58)(0.115) = 5.08$$

$$LCL = \overline{\overline{x}} - A_2\overline{R} = 5.01 - (0.58)(0.115) = 4.94$$

Retrieve Factor Value A_2

x- bar Chart Example (cont.)



R- Chart

$$UCL = D_4\bar{R} \qquad LCL = D_3\bar{R}$$

$$\bar{R} = \frac{\sum R}{k}$$

where

R = range of each sample
k = number of samples

R-Chart Example

SAMPLE *k*	OBSERVATIONS (SLIP-RING DIAMETER, CM) 1	2	3	4	5	$\bar{x}$	*R*
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

Example 15.3

R-Chart Example (cont.)

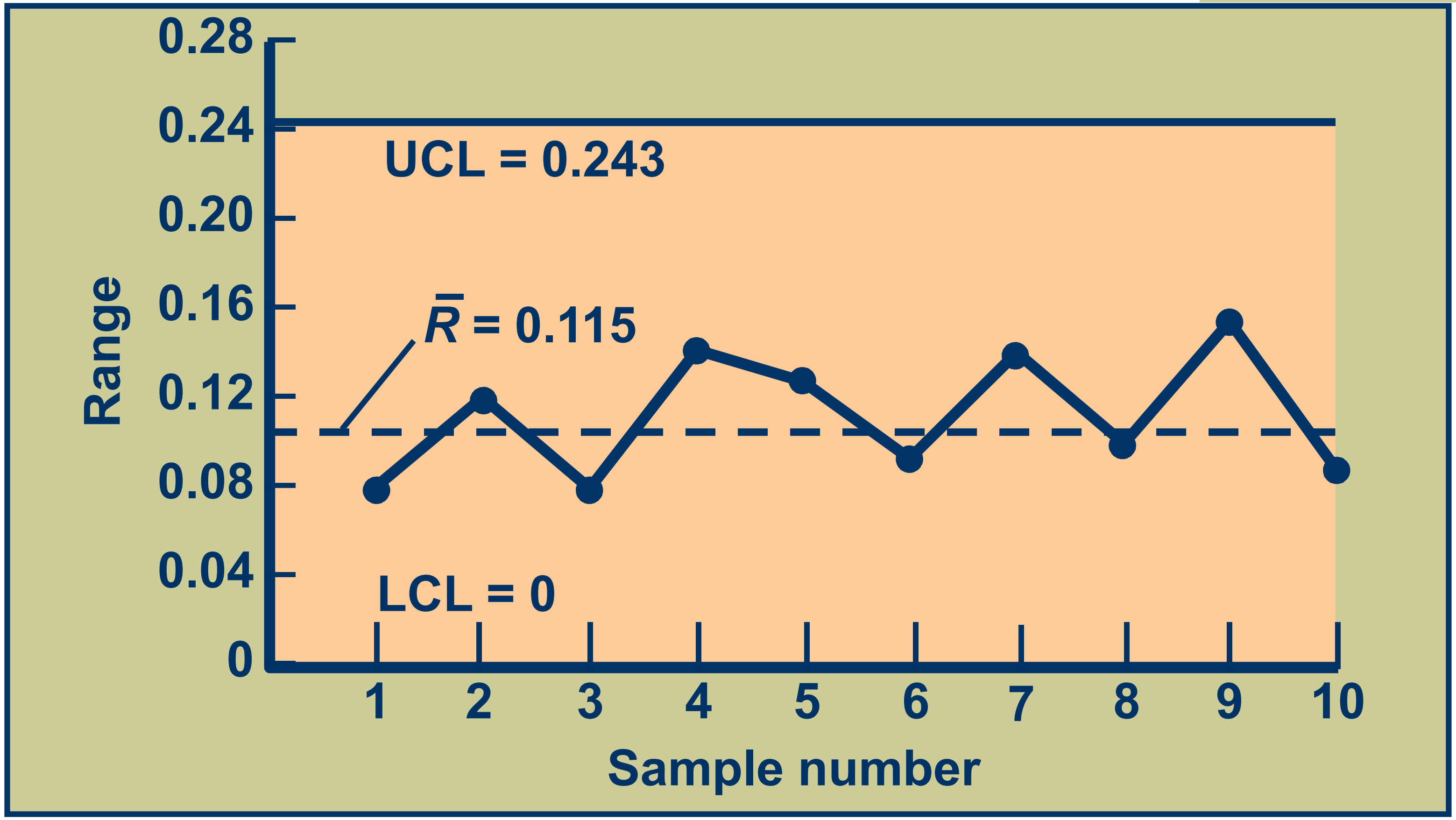
$$UCL = D_4\bar{R} = 2.11(0.115) = 0.243$$

$$LCL = D_3\bar{R} = 0(0.115) = 0$$

Retrieve Factor Values D_3 and D_4

Example 15.3

R-Chart Example (cont.)



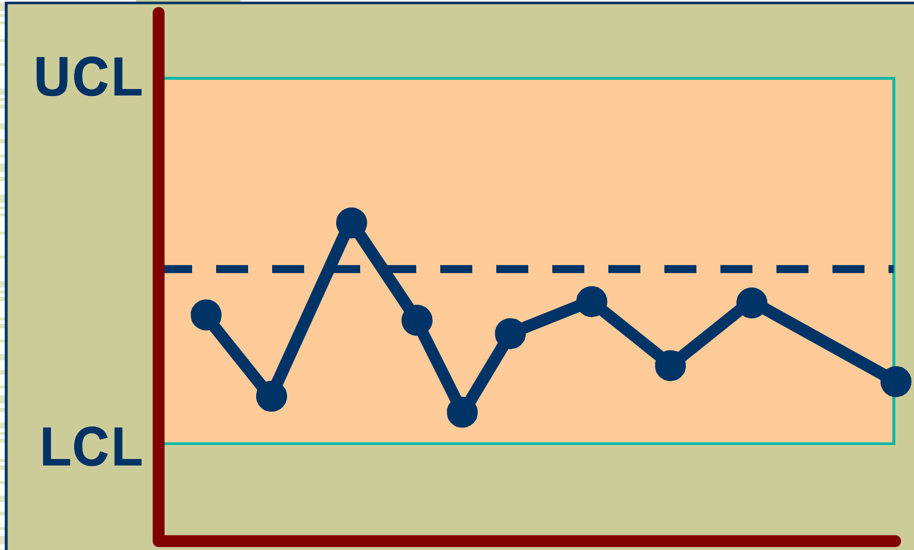
Using x- bar and R-Charts Together

- Process average and process variability must be in control
- It is possible for samples to have very narrow ranges, but their averages might be beyond control limits
- It is possible for sample averages to be in control, but ranges might be very large
- It is possible for an R-chart to exhibit a distinct downward trend, suggesting some nonrandom cause is reducing variation

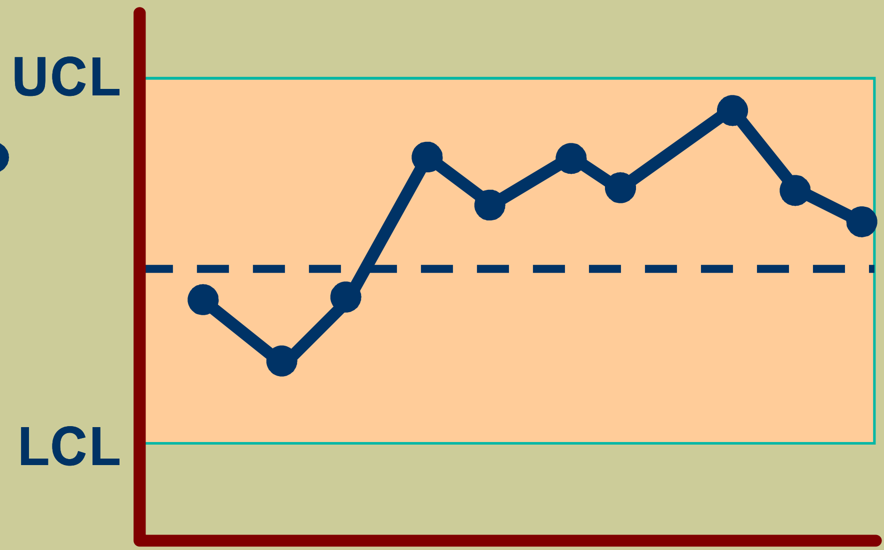
Control Chart Patterns

- Run
  - sequence of sample values that display same characteristic
- Pattern test
  - determines if observations within limits of a control chart display a nonrandom pattern
- To identify a pattern:
  - 8 consecutive points on one side of the center line
  - 8 consecutive points up or down
  - 14 points alternating up or down
  - 2 out of 3 consecutive points in zone A (on one side of center line)
  - 4 out of 5 consecutive points in zone A or B (on one side of center line)

Control Chart Patterns (cont.)

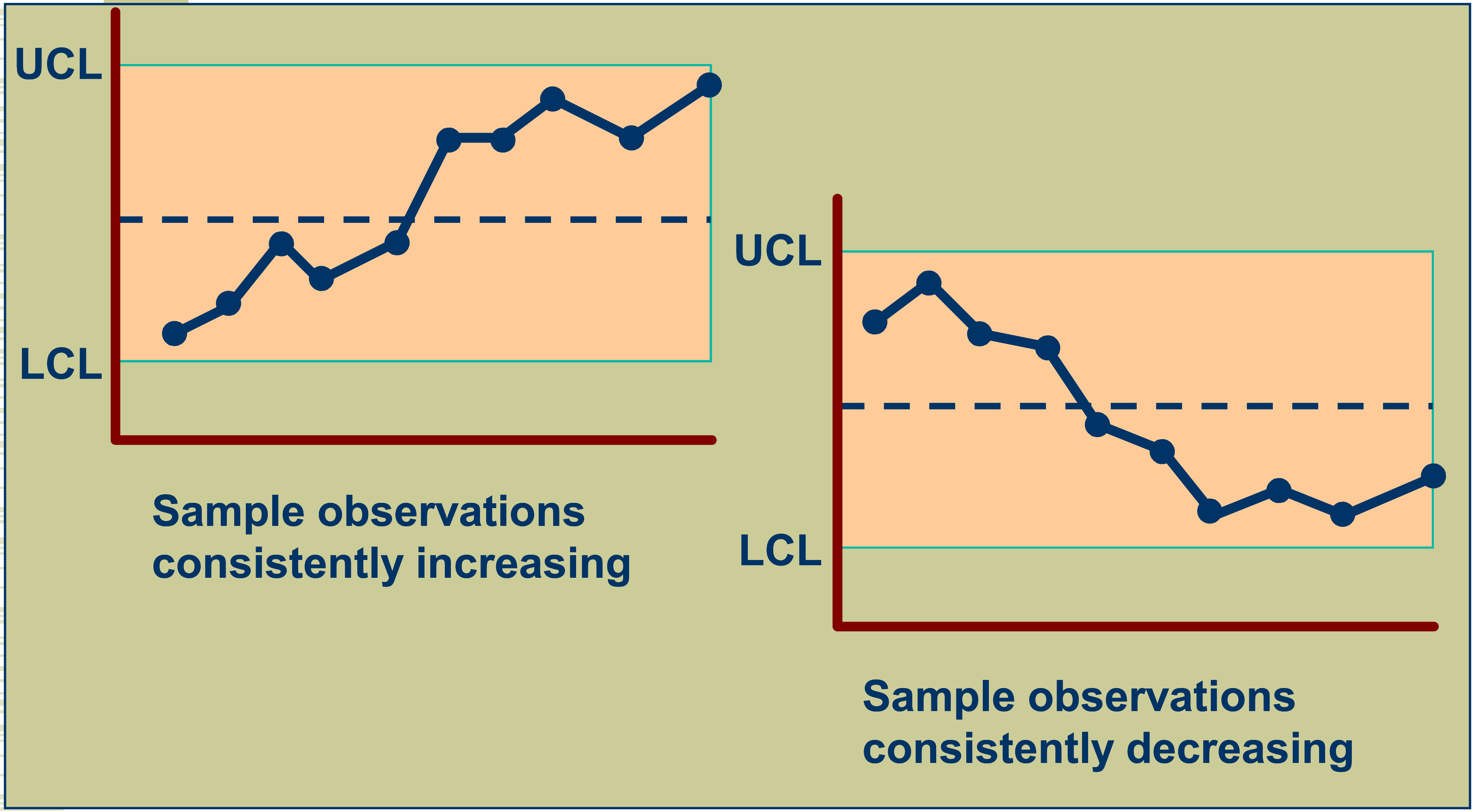


Sample observations consistently below the center line

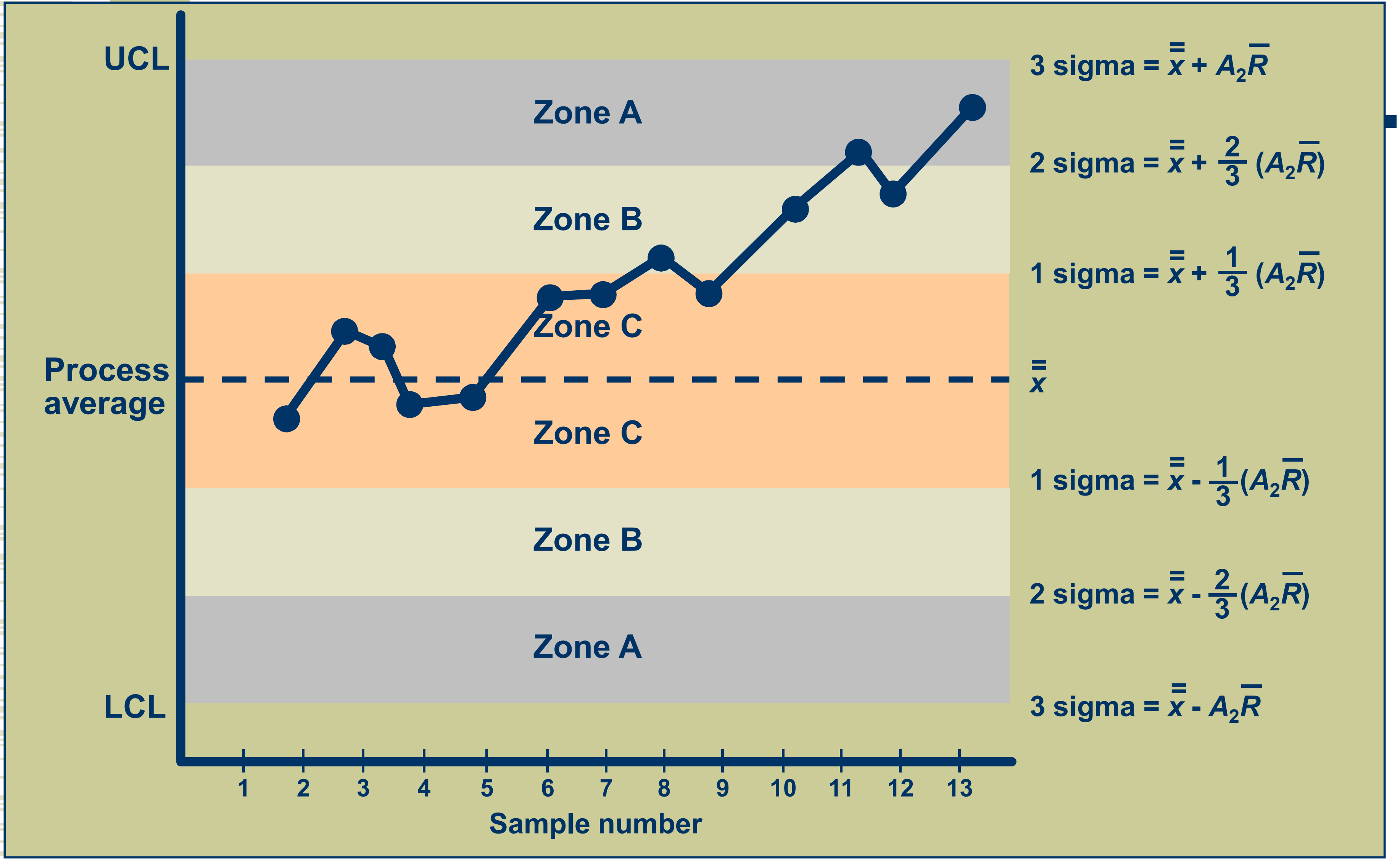


Sample observations consistently above the center line

Control Chart Patterns (cont.)



Zones for Pattern Tests



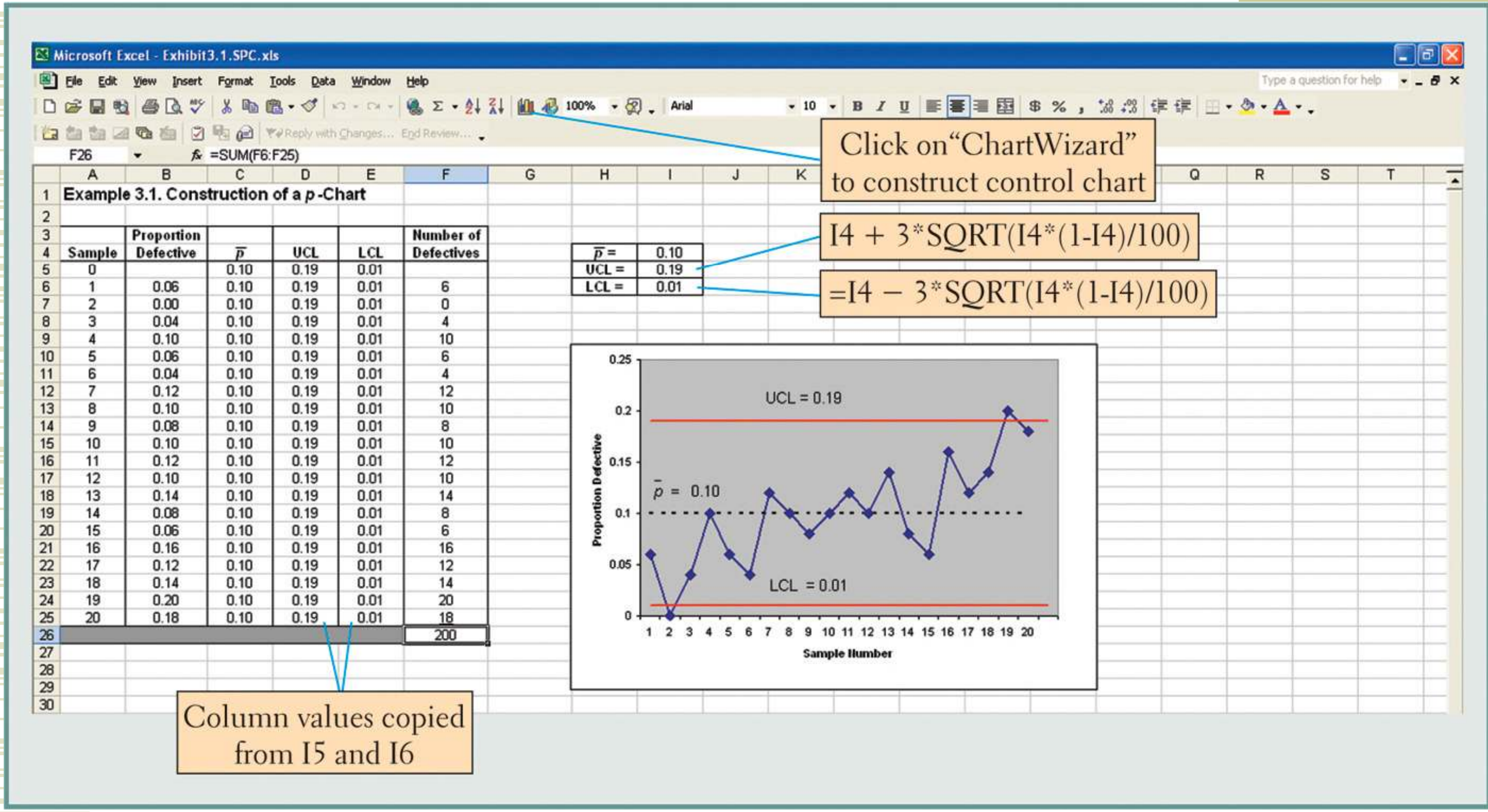
Performing a Pattern Test

SAMPLE	$\bar{x}$	ABOVE/BELOW	UP/DOWN	ZONE
1	4.98	B	—	B
2	5.00	B	U	C
3	4.95	B	D	A
4	4.96	B	D	A
5	4.99	B	U	C
6	5.01	—	U	C
7	5.02	A	U	C
8	5.05	A	U	B
9	5.08	A	U	A
10	5.03	A	D	B

Sample Size Determination

- **Attribute charts require larger sample sizes**
  - **50 to 100 parts in a sample**
- **Variable charts require smaller samples**
  - **2 to 10 parts in a sample**

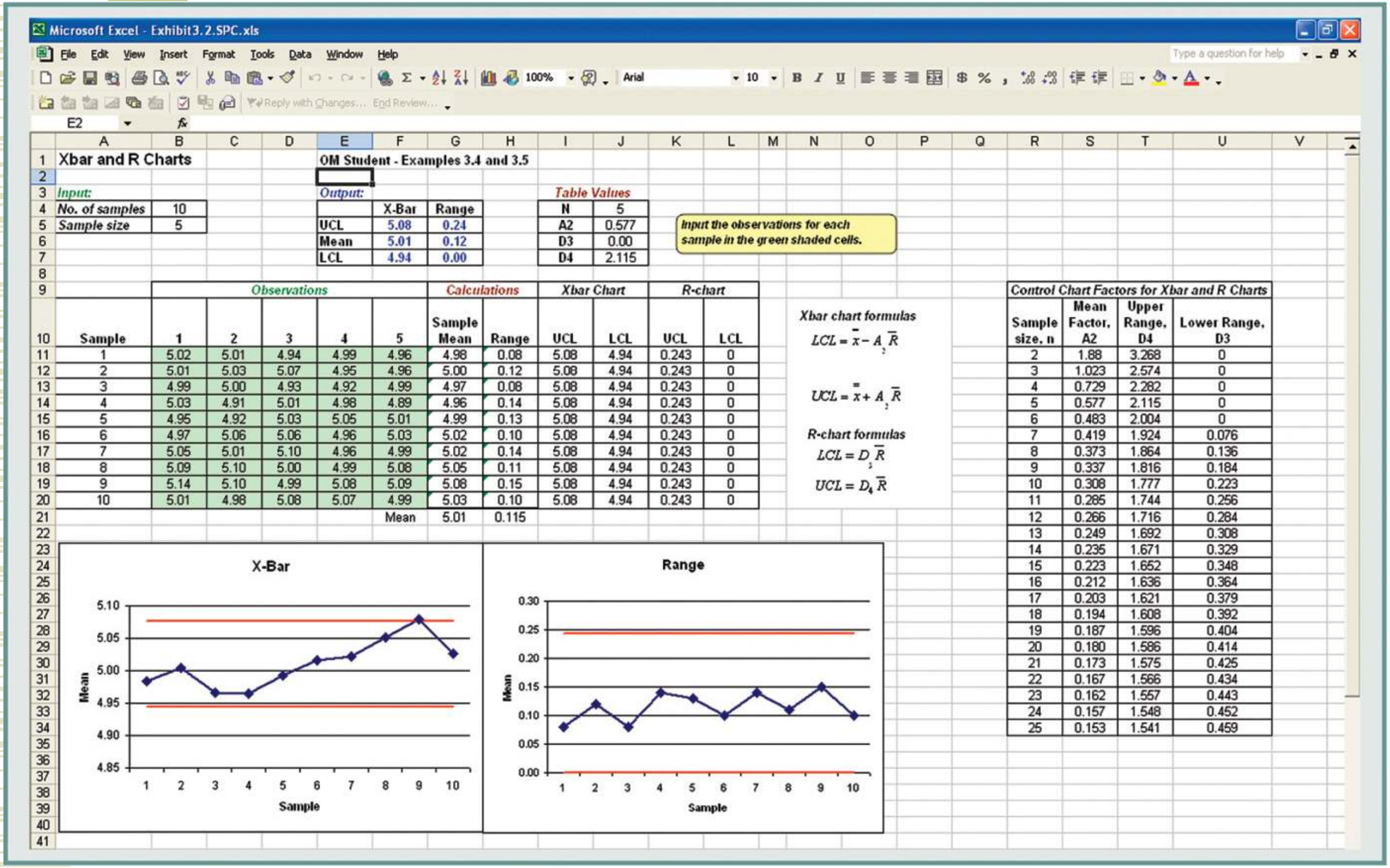
SPC with Excel



Sample	Proportion Defective	$\bar{p}$	UCL	LCL	Number of Defectives
0		0.10	0.19	0.01	
1	0.06	0.10	0.19	0.01	6
2	0.00	0.10	0.19	0.01	0
3	0.04	0.10	0.19	0.01	4
4	0.10	0.10	0.19	0.01	10
5	0.06	0.10	0.19	0.01	6
6	0.04	0.10	0.19	0.01	4
7	0.12	0.10	0.19	0.01	12
8	0.10	0.10	0.19	0.01	10
9	0.08	0.10	0.19	0.01	8
10	0.10	0.10	0.19	0.01	10
11	0.12	0.10	0.19	0.01	12
12	0.10	0.10	0.19	0.01	10
13	0.14	0.10	0.19	0.01	14
14	0.08	0.10	0.19	0.01	8
15	0.06	0.10	0.19	0.01	6
16	0.16	0.10	0.19	0.01	16
17	0.12	0.10	0.19	0.01	12
18	0.14	0.10	0.19	0.01	14
19	0.20	0.10	0.19	0.01	20
20	0.18	0.10	0.19	0.01	18
					200

$\bar{p}$ =	0.10
UCL =	0.19
LCL =	0.01

SPC with Excel and OM Tools



Input:	
No. of samples	10
Sample size	5

Output:	X-Bar	Range
UCL	5.08	0.24
Mean	5.01	0.12
LCL	4.94	0.00

Table Values	
N	5
A2	0.577
D3	0.00
D4	2.115

	Observations					Calculations		Xbar Chart		R-chart	
Sample	1	2	3	4	5	Sample Mean	Range	UCL	LCL	UCL	LCL
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08	5.08	4.94	0.243	0
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12	5.08	4.94	0.243	0
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08	5.08	4.94	0.243	0
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14	5.08	4.94	0.243	0
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13	5.08	4.94	0.243	0
6	4.97	5.06	5.06	4.96	5.03	5.02	0.10	5.08	4.94	0.243	0
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14	5.08	4.94	0.243	0
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11	5.08	4.94	0.243	0
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15	5.08	4.94	0.243	0
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10	5.08	4.94	0.243	0
					Mean	5.01	0.115				

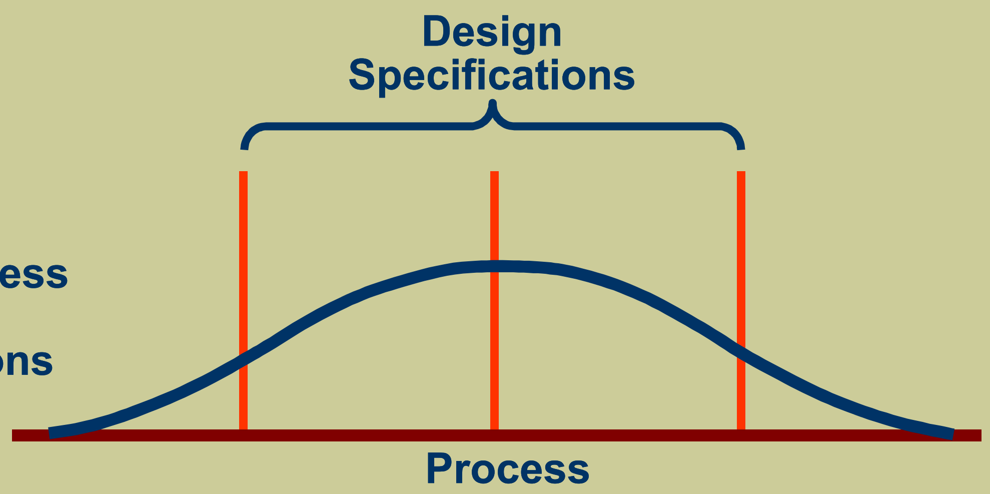
Control Chart Factors for Xbar and R Charts			
Sample size, n	Mean Factor, A2	Upper Range, D4	Lower Range, D3
2	1.88	3.268	0
3	1.023	2.574	0
4	0.729	2.282	0
5	0.577	2.115	0
6	0.483	2.004	0
7	0.419	1.924	0.076
8	0.373	1.864	0.136
9	0.337	1.816	0.184
10	0.308	1.777	0.223
11	0.285	1.744	0.256
12	0.266	1.716	0.284
13	0.249	1.692	0.308
14	0.235	1.671	0.329
15	0.223	1.652	0.348
16	0.212	1.636	0.364
17	0.203	1.621	0.379
18	0.194	1.608	0.392
19	0.187	1.596	0.404
20	0.180	1.586	0.414
21	0.173	1.575	0.425
22	0.167	1.566	0.434
23	0.162	1.557	0.443
24	0.157	1.548	0.452
25	0.153	1.541	0.459

Process Capability

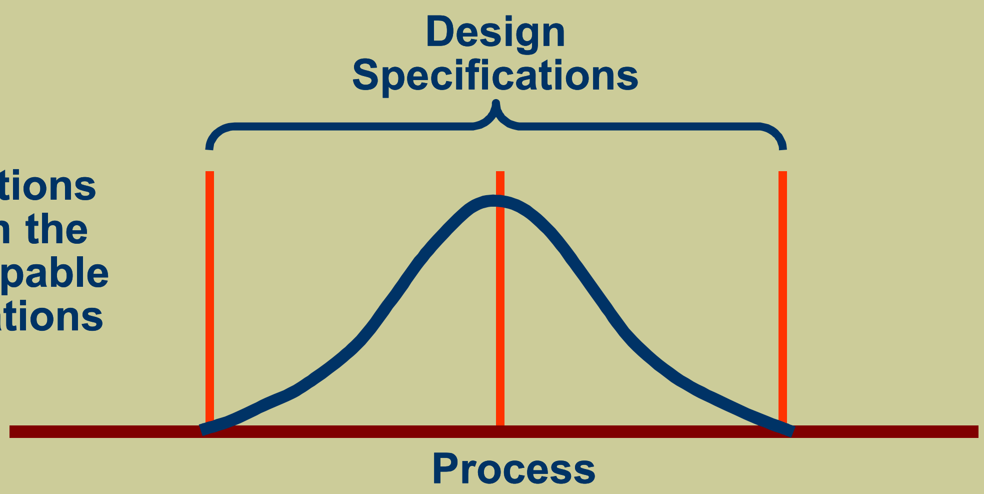
- Tolerances
  - design specifications reflecting product requirements
- Process capability
  - range of natural variability in a process—what we measure with control charts

Process Capability (cont.)

(a) Natural variation exceeds design specifications; process is not capable of meeting specifications all the time.

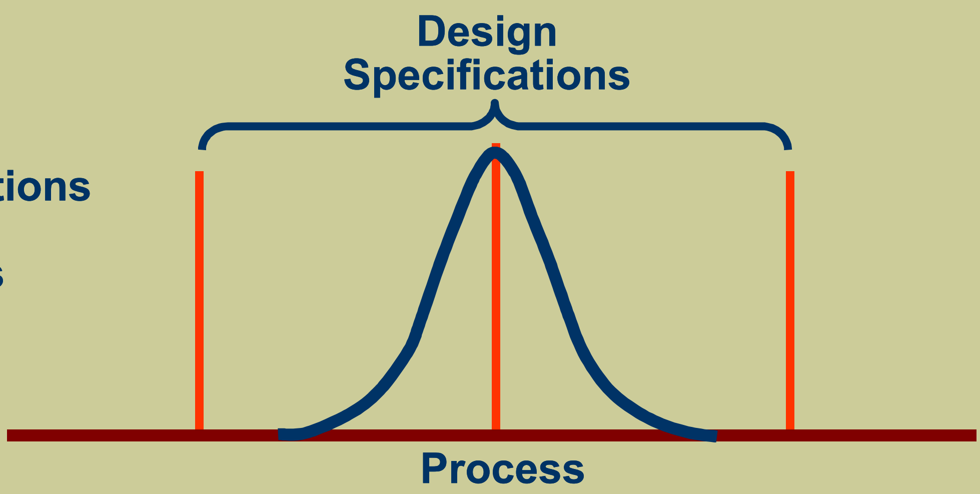


(b) Design specifications and natural variation the same; process is capable of meeting specifications most of the time.

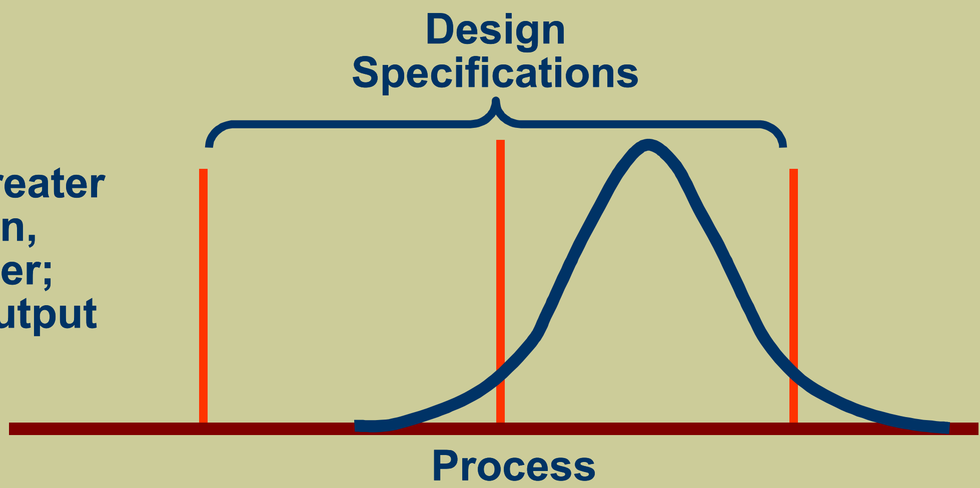


Process Capability (cont.)

(c) Design specifications greater than natural variation; process is capable of always conforming to specifications.



(d) Specifications greater than natural variation, but process off center; capable but some output will not meet upper specification.



Process Capability Measures

Process Capability Ratio

$$C_p = \frac{\text{tolerance range}}{\text{process range}}$$

$$= \frac{\text{upper specification limit - lower specification limit}}{6\sigma}$$

Computing C_p

Net weight specification = 9.0 oz ± 0.5 oz
Process mean = 8.80 oz
Process standard deviation = 0.12 oz

$$C_p = \frac{\text{upper specification limit - lower specification limit}}{6\sigma}$$

$$= \frac{9.5 - 8.5}{6(0.12)} = 1.39$$

Process Capability Measures

Process Capability Index

$$C_{pk} = \text{minimum}\left[\frac{\bar{\bar{x}} - \text{lower specification limit}}{3\sigma}, \frac{\text{upper specification limit} - \bar{\bar{x}}}{3\sigma}\right]$$

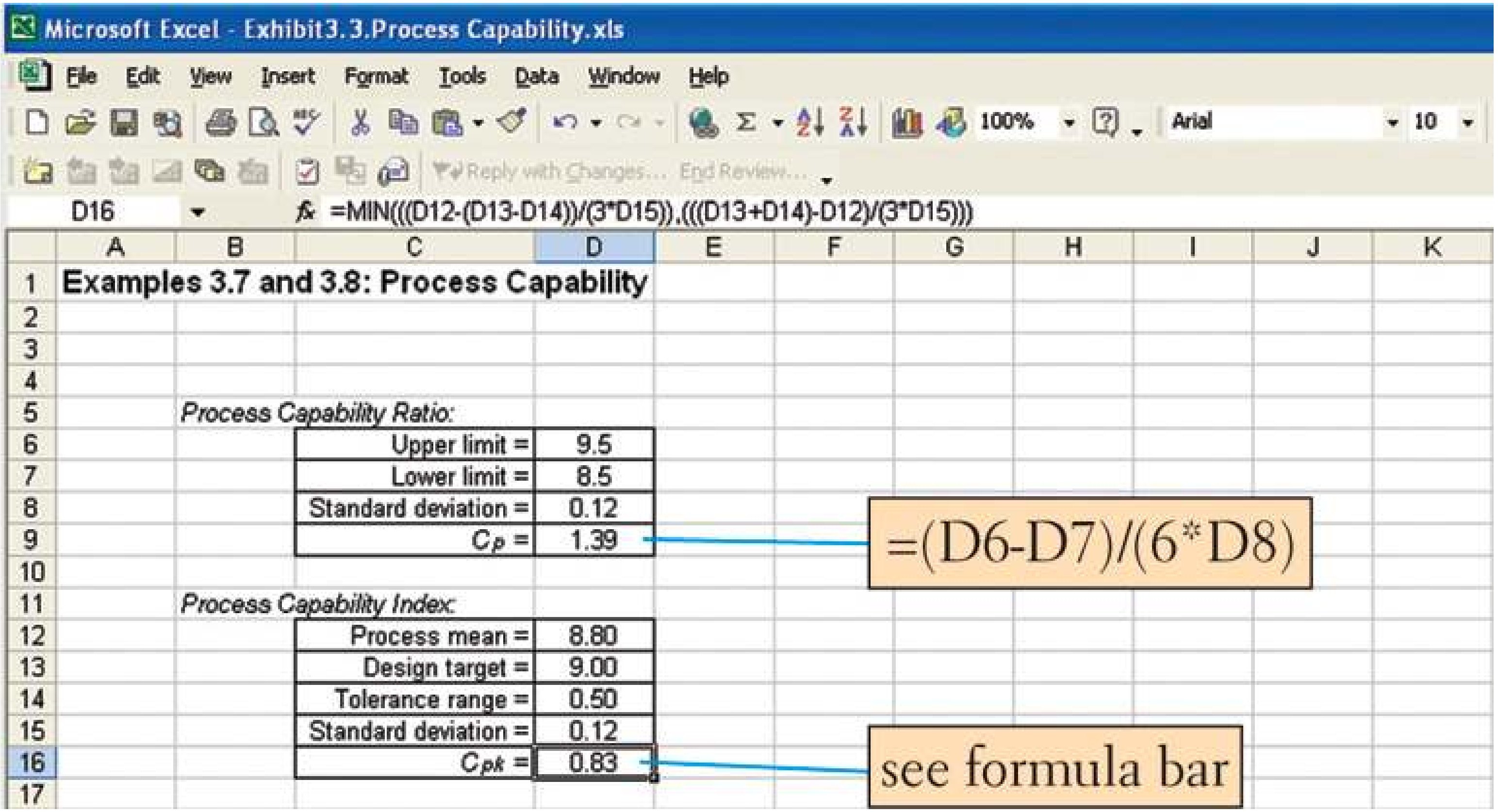
Computing C_{pk}

Net weight specification = 9.0 oz ± 0.5 oz
Process mean = 8.80 oz
Process standard deviation = 0.12 oz

$$C_{pk} = \text{minimum}\left[\frac{\bar{\bar{x}} - \text{lower specification limit}}{3\sigma}, \frac{\text{upper specification limit} - \bar{\bar{x}}}{3\sigma}\right]$$

$$= \text{minimum}\left[\frac{8.80 - 8.50}{3(0.12)}, \frac{9.50 - 8.80}{3(0.12)}\right] = 0.83$$

Process Capability with Excel



Process Capability with Excel and OM Tools